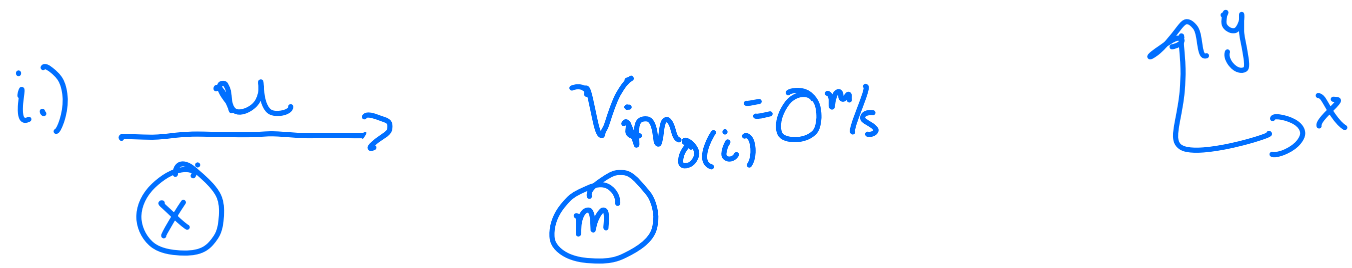
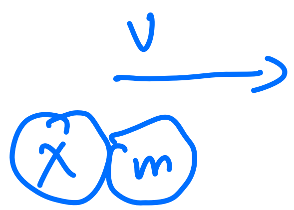
Based on the wording of the problem, the scenario can be modeled by what is depicted in the sketch shown below.

i.)



ii.)



$$P_{x_i} + P_{m_i} + J = P_{x_f} + P_{m_f}$$

For my analysis to work, I'll have to assume that the particle x + test particle system has constant momentum as depicted above. Thus, the initial momentum that object x has becomes split between the two particles after the collision

Modeling the process mathematically, I can write:

$$P_{x_i} = P_{x_f} + P_{m_f}$$

$$Mu = Mv_0 + mv_0$$

As written, the expression above can't be linearized due to the vo term appearing twice.

Legend

M: mass of X

m: mass of known particle

u: velocity of X

V_0: velocity of 2-particle system post-collision

To convert it into a form that works, divide all terms by vo. The equation then becomes:

I'll rearrange this slightly

$$\frac{Mu}{v_0} = M + m \Rightarrow m = \frac{Mu}{v_0} - M$$

From here work off of slope intercept form

$$y = mx + b$$

$$m = \frac{Mu}{v_0} + (-M)$$

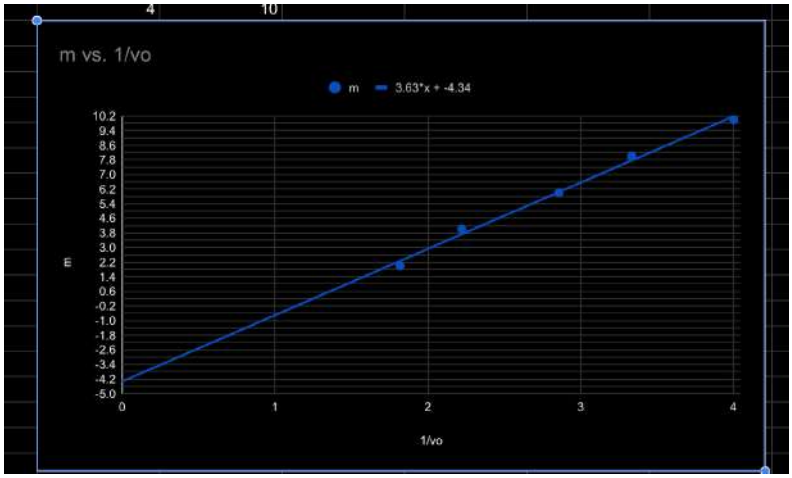
y axis quantity: m

x-axis quantity: $\frac{1}{v_0}$

slope: Mu

y-intercept: $(-M)$

Using google sheets, we can see that the estimated mass is about 4.3 kilograms (I'll note that the problem doesn't actually designate a unit, so I'll just write in standard units for clarity). Since M is proportional to slope, we use it to find u



Solving for u, we find:

from graph

$$\text{Slope} = Mu = 3.63 \, kg \cdot m/s$$

$$4.3 \, kg \cdot u = 3.63 \, kg \cdot m/s$$

$$\boxed{u = \frac{3.63}{4.3} \approx 0.84 \, m/s}$$

Overall, our findings are sensible. The 4.3 kg value and 0.84 m/s value are of the same order as the values in the data table. Likewise, our trend line being fairly linear suggests that my equation successfully

Some general thoughts on the problem and how it can promote learning.

Typically, linearization problems become straightforward if an equation is derived, and the terms in the equation are transcribed successfully onto a graph. This problem is unique from most of the ones I've seen for a couple of reasons.

The most important is that, most of the time (with linearization tasks and problems more generally), there is only one unknown to solve for. Giving students a data table with position vs time data, and plotting x vs t^2 or x vs 1/2 t^2 can be used to find acceleration, if asked.

Here, both the object's mass and it's initial speed were not given. So even knowing how to represent the situation as a process where momentum is constant is not enough to solve it with the usual approach to similar tasks.

So this problem's solution became very involved in a couple of ways.

1. To make the derived expression workable, we had to divide all terms in the equation by one of our givens (vo), which is quite a novel tactic, even for me
2. Once the equation that was derived adopted a form that was "usable", there is still need for extra interpretation. These involved:
   A. Understanding that (at least in the equation I used), that the intercept could help directly solve for mass
   B. Understanding that, once mass was determined, it could be used to find initial speed by relating it to the slope of the line
3. The result of the above is that we end up with a situation where the slope of our line is the object's initial momentum and what's plotted along one axis is the inverse of velocity, neither of which are likely to ever come up in parts of a problem.

So overall, it was pretty demanding. Having students work off of a data table that then becomes linearized adds to the demands associated with derivation. As mentioned earlier, the particulars of this problem required interpretation of the mathematical model at all relevant steps. From a learning standpoint, the mental strain associated with all steps of the problem is what promotes the learning of important practices (mathematical modeling, graphing/linearzing, etc).