CALCULUS AB

PRACTICE EXAM
Section I, Part A
Time – 55 minutes
Number of questions – 28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM. (Questions 1-28)

1. $\int \cos(3x)\,dx =$ $u = 3x$ $\frac{1}{3}\int \cos u\,du$

(A) $-3\sin(3x)+C$ (B) $-\frac{1}{3}\sin(3x)+C$ (C) $\boxed{\frac{1}{3}\sin(3x)+C}$

(D) $\sin(3x)+C$ (E) $3\sin(3x)+C$

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2. $\lim_{x\to 0}\frac{2x^6+6x^3}{4x^5+3x^3}$ is $\frac{\cancel{x^3}(2x^3+6)}{\cancel{x^3}(4x^2+3)} = $ Plug 0 in $\frac{0+6}{0+3} = 2$

(A) 0   (B) $\frac{1}{2}$ (C) 1   (D) $\boxed{2}$ (E) nonexistent

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$$f(x)=\begin{cases} x^2-3x+9 & \text{for } x\le 2 \\ kx+1 & \text{for } x>2 \end{cases}$$

$x^2-3x+9 = kx+1$
$4-6+9 = 2k+1$

3. The function f is defined above. For what value of k, if any, is f continuous at $x=2$?

(A) 1   (B) 2   (C) $\boxed{3}$ (D) 7   (E) none $7 = 2k+1$

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4. If $f(x)=\cos^3(4x)$, then $f'(x)=$ $3\cos^2 4x \cdot (-\sin 4x)\cdot 4$

(A) $3\cos^2(4x)$ (B) $\boxed{-12\cos^2(4x)\sin(4x)}$ (C) $-3\cos^2(4x)\sin(4x)$

(D) $12\cos^2(4x)\sin(4x)$ (E) $-4\sin^3(4x)$

5. The function f given by $f(x)=2x^3-3x^2-12x$ has a relative minimum at $x=$

A) -1 (B) 0 $\boxed{\text{(C) 2}}$ (D) $\frac{3-\sqrt{105}}{4}$ (E) $\frac{3+\sqrt{105}}{4}$

$f'(x) = 6x^2-6x-12 = 6(x^2-x-2)$
$(x-2)(x+1)$

$+ \quad - \quad +$; $-1 \quad 2$

---

6. Let f be the function given by $f(x)=(2x-1)^5(x+1)$. Which of the following is an equation for the line tangent to the graph of f at the point where $x=1$?

Product Rule

(A) $y=21x+2$
(B) $y=21x-19$
(C) $y=11x-9$
(D) $y=10x+2$
(E) $y=10x-8$

$f(1)=2$

$f' = 5(2x-1)^4\cdot 2(x+1)$
$+ (2x-1)^5$

$y-2 = 21(x-1)$
$y = 21x-19$

$f'(1) = 5\cdot 4+1$
$=21$

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7. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx=$

$u=\sqrt{x}$, $du=\frac{1}{2}x^{-1/2}dx$, $dx = 2\sqrt{x}\,du$

$\int \frac{e^u}{\cancel{\sqrt{x}}}\cdot 2\sqrt{x}\,du$

$\boxed{\text{(A) } 2e^{\sqrt{x}}+C}$ (B) $\frac{1}{2}e^{\sqrt{x}}+C$ (C) $e^{\sqrt{x}}+C$

(D) $2\sqrt{x}e^{\sqrt{x}}+C$ (E) $\frac{e^{\sqrt{x}}}{2\sqrt{x}}+C$

$2e^{\sqrt{x}}+c$

---

x	0	2	4	6
$f(x)$	4	k	8	12

$\frac{2}{2}\left(4+2k+2\cdot 8+12\right)=$
52

8. The function f is continuous on the closed interval [0, 6] and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)dx$ found with 3 subintervals of equal length is 52. What is the value of k?

$32+2k=52$

(A) 2 (B) 6 (C) 7 (D) 10 (E) 14

9. A particle moves along the x-axis so that at any time $t > 0$, its velocity is given by $v(t) = 4 - 6t^2$. If the particle is at position $x = 7$ at time $t = 1$, what is the position of the particle at time $t = 2$?

(A) -10 (B) -5 (C) -3 (D) 3 (E) 17

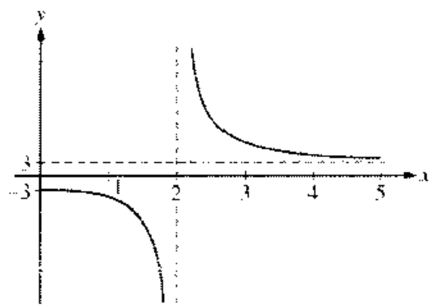
change

$7 - 10$

initial

$\int v(t)$ = change in position, $\int_1^2 4 - 6t^2 = 4t - 2t^3 \Big|_1^2 = (8-16) - (2)$

-10



10. The function f is given by $f(x) = \dfrac{ax^2 + 12}{x^2 + b}$. The figure above shows a portion of the graph of f. Which of the following could be the values of the constants a and b?

~~(A) $a = -3,\ b = 2$~~

(B) $a = 2,\ b = -3$

(C) $a = 2,\ b = -2$

(D) $a = 3,\ b = -4$

(E) $a = 3,\ b = 4$

when $x = 2$, bottom $= 0$, ~~$b = 2$~~ $b = -4$

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11. What is the slope of the line tangent to the graph of $y = \dfrac{e^{-x}}{x+1}$ at $x = 1$?

QUOTIENT

(A) $-\dfrac{1}{e}$ (B) $-\dfrac{3}{4e}$ (C) $-\dfrac{1}{4e}$ (D) $\dfrac{1}{4e}$ (E) $\dfrac{1}{e}$

$$\frac{(x+1)(-e^{-x}) - e^{-x}(1)}{(x+1)^2} = \frac{2(-e^{-1}) - e^{-1}}{4} = \frac{-\frac{2}{e} - \frac{1}{e}}{4} = \frac{\frac{-3}{e}}{4} = \frac{-3}{4e}$$

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12. If $f'(x) = \dfrac{2}{x}$ and $f\left(\sqrt{e}\right) = 5$, then $f(e) =$

$f(e) = 2\ln e + 4$

(A) 2 (B) ln 25 (C) $5 + \dfrac{2}{e} - \dfrac{2}{e^2}$ (D) 6 (E) 25

$\int f'(x) = f(x)$, $\int \frac{2}{x} = 2\ln x + C$, then plug in pt to find C.

$2\ln\sqrt{e} + C = 5$

$2\ln x + 4$

13. $\int (x^3+1)^2\,dx =$ Foil : $\int x^6 + 2x^3 + 1\,dx,$

(A) $\frac{1}{7}x^7 + x + C$ (B) $\frac{1}{7}x^7 + \frac{1}{2}x^4 + x + C$ (C) $6x^2(x^3+1)+C$

(D) $\frac{1}{3}(x^3+1)^3 + C$ (E) $\frac{(x^3+1)^3}{9x^2} + C$

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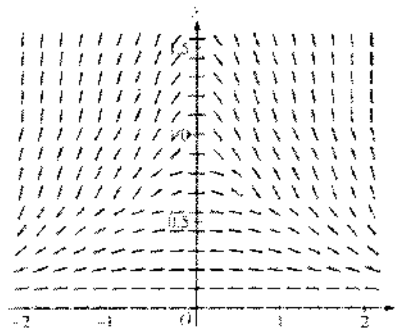
14. $\lim_{h\to 0} \frac{e^{(2+h)} - e^2}{h} =$ $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ is asking for derivative of e^x when $x = 2$

(A) 0 (B) 1 (C) $2e$ (D) e^2 (E) $2e^2$



which ~~graph~~ function below looks like the slope field?

15. The slope field for a certain differential equation is shown above. Which of the following could be a solution to the differential equation with the initial condition $y(0) = 1$?

(A) $y = \cancel{\cos x}$ (B) $y = \cancel{1 - x^2}$ (C) $y = \cancel{e^x}$ (D) $y = \sqrt{1-x^2}$ (E) $y = \frac{1}{1+x^2}$

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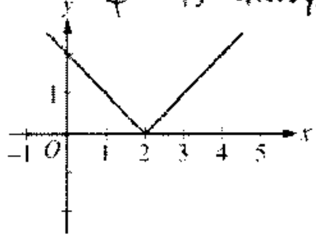
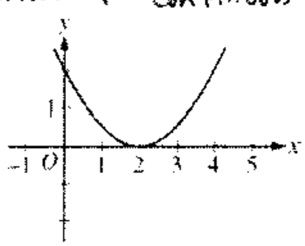
16. If $f'(x) = |x-2|$, which of the following could be the graph of $y = f(x)$.

f' is always positive + continuous

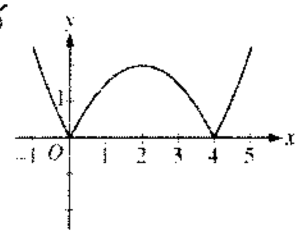
(A)



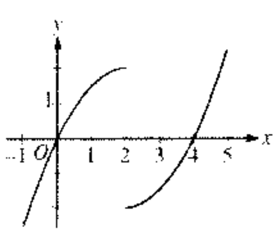
(B)



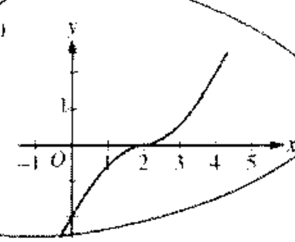
(C)



(D)



(E)



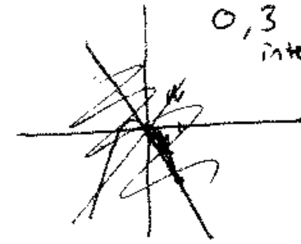
$\frac{9}{2} - 18 + \frac{45}{2} = 27 - 18$ $x - 2x^2 = -5x, \; 6x - 2x^2 = 0$

17. What is the area of the region enclosed by the graphs of $f(x) = x - 2x^2$ and $g(x) = -5x$? $2x(3-x)$

$0, 3$ intersecti

(A) $\frac{7}{3}$ (B) $\frac{16}{3}$ (C) $\frac{20}{3}$ (D) 9 (E) 36

$\int_0^3 x - 2x^2 - (-5x)\,dx, \;\; \frac{1}{2}x^2 - \frac{2}{3}x^3 + \frac{5}{2}x^2 \Big|_0^3$



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18. For the function f, $f'(x) = 2x + 1$ and $f(1) = 4$. What is the approximation for $f(1.2)$ found by using the line tangent to the graph of f at $x = 1$? $f'(1) = 3$

(A) 0.6 (B) 3.4 (C) 4.2 (D) 4.6 (E) 4.64

$y - 4 = 3\,(\underset{1.2}{x} - 1) \;\; = \;\; y - 4 = .6$

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19. Let f be the function given by $f(x) = x^3 - 6x^2$. The graph of f is concave up when

(A) $x > 2$ (B) $x < 2$ (C) $0 < x < 4$ $3x^2 - 12x$

(D) $x < 0$ or $x > 4$ only (E) $x > 6$ only $6x - 12$

$x = 2$ $- \;|\; +$ at 2

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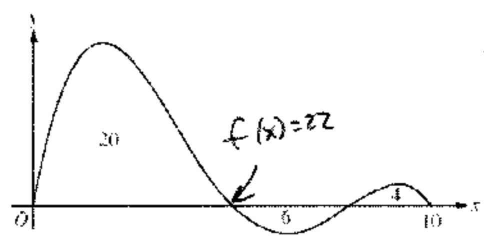
20. If $g(x) = x^2 - 3x + 4$ and $f(x) = g'(x)$, then $\int_1^3 f(x)\,dx =$ $g(3) - g(1) = 9 - 9 + 4 - (1 - 3 + 4)$

$4 - (2)$

(A) $-\frac{14}{3}$ (B) -2 (C) 2 (D) 4 (E) $\frac{14}{3}$



Graph of f'

$f(0) = 2$
$+ 20$
22

21. The graph of f', the derivative of the function f, is shown above for $0 \le x \le 10$. The areas of the regions between the graph of f' and the x-axis are 20, 6, and 4, respectively. If $f(0) = 2$, what is the maximum value of f on the closed interval $0 \le x \le 10$?

(A) 16 (B) 20 (C) 22 (D) 30 (E) 32

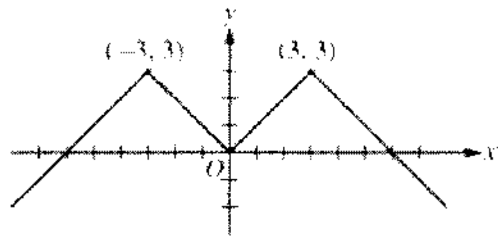
22. If $f'(x) = (x-2)(x-3)^2(x-4)^3$, then f has which of the following relative extrema?

I. A relative maximum at $x = 2$ ✓
II. A relative minimum at $x = 3$
III. A relative maximum at $x = 4$

$+ \quad - \quad - \quad +$
$2 \quad 3 \quad 4$

(A) I only (B) II only (C) I and III only

(D) II and III only (E) I, II, and III



23. The graph of the even function $y = f(x)$ consists of 4 lines segments, as shown above. Which of the following statements about f is false?

$x \to 0,\ y \to 0$, true

slope at $0 = 0$, no

it's $\emptyset$

slope between a pt. on the right + the left is 0, true b/c graph is symmetrical

(A) $\lim_{x\to 0}\left(f(x) - f(0)\right) = 0$ $\boxed{\text{(B)}}$ $\lim_{x\to 0}\frac{f(x)-f(0)}{x} = 0$ (C) $\lim_{x\to 0}\frac{f(x)-f(-x)}{2x} = 0$

(D) $\lim_{x\to 2}\frac{f(x)-f(2)}{x-2} = 1$ (E) $\lim_{x\to 3}\frac{f(x)-f(3)}{x-3}$ slope does not exist.

Slope at $2 = 1$, true

True, sharp turn

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24. The radius of a circle is increasing. At a certain instant, the rate of increase in the area of the circle is numerically equal to twice the rate of increase in its circumference. What is the radius of the circle at that instant?

(A) $\frac{1}{2}$ (B) 1 (C) $\sqrt{2}$ (D) 2 (E) 4

$C = 2\pi r$ $\frac{dC}{dt} = 2\pi\, dr/dt$

$A = \pi r^2$ $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$, $\frac{dA}{dt} = 2\frac{dC}{dt}$, $2\pi r \frac{dr}{dt} = 2 \cdot 2\pi \frac{dr}{dt}$

$r = 2$

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25. If $x^2y - 3x = y^3 - 3$, then at the point (-1, 2), $\frac{dy}{dx} =$

$2xy + x^2\, dy/dx - 3 = 3y^2\, dy/dx$

(A) $-\frac{7}{11}$ (B) $-\frac{7}{13}$ (C) $-\frac{1}{2}$ (D) $-\frac{3}{14}$ (E) 7

$\frac{3 - 2(-1)(2)}{1 - 12}$

$dy/dx = \frac{3 - 2xy}{x^2 - 3y^2}$

26. For $x > 0$, f is a function such that $f'(x) = \frac{\ln x}{x}$ and $f''(x) = \frac{1 - \ln x}{x^2}$. Which of the following is true? $\ln x$ is neg. when $x < 1$, so $f' < 0$ if $x < 1$ or $f' > 0$ when $x > 1$

(A) f is decreasing for $x > 1$, and the graph of f is concave down for $x > e$. ← False

(B) f is decreasing for $x > 1$, and the graph of f is concave up for $x > e$. ← False

(C) f is increasing for $x > 1$, and the graph of f is concave down for $x > e$.

(D) f is increasing for $x > 1$, and the graph of f is concave up for $x > e$. False

(E) f is increasing for $0 < x < e$, and the graph of f is concave down for $0 < x < e^{3/2}$.

when $x > e$, $1 - \ln e$ will be negative

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27. If f is the function given by $f(x) = \int_4^{2x} \sqrt{t^2 - t}\, dt$, then $f'(2) =$ Area Acc.

(A) 0 (B) $\frac{7}{2\sqrt{12}}$ (C) $\sqrt{2}$ (D) $\sqrt{12}$ (E) $2\sqrt{12}$

$\left(\sqrt{(2x)^2 - 2x}\right) \cdot 2$

$\left(\sqrt{16 - 4}\right) \cdot 2$

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28. If $y = \sin^{-1}(5x)$, then $\frac{dy}{dx} =$

(A) $\cancel{\frac{1}{1+25x^2}}$

(B) $\frac{5}{1+25x^2}$

(C) $\cancel{\frac{-5}{\sqrt{1-25x^2}}}$

(D) $\cancel{\frac{1}{\sqrt{1-25x^2}}}$

(E) $\frac{5}{\sqrt{1-25x^2}}$

My bad, we didn't go over this... But,

$\sin^{-1}$ graph is

slope > 0. Also, since there is a $(5x)$, 5 must be in the derivative

Section I, Part B	*Time – 50 minutes*	*Number of questions – 17*

A CALCULATOR IS REQUIRED ON THIS PART OF THE EXAM. (Questions 29-45)

29. A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = t^2 \ln(t+2)$. What is the acceleration of the particle at time $t = 6$?

(A) 1.500 (B) 20.453 (C) 29.453 (D) 74.680 (E) 133.417

$y_1 = v(t)$

2^{nd}, trace, dy/dx

$x = 6$

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30. If $\int_0^3 f(x)dx = 6$ and $\int_3^5 f(x)dx = 4$, then $\int_0^5 (3 + 2f(x))\,dx =$

$\int_0^5 f(x) = 10$

(A) 10 (B) 20 (C) 25 (D) 35 (E) 50

$\int_0^5 3dx = 15$

$15 + 2 \cdot 10$

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31. For $t \geq 0$ hours, H is a differentiable function of t that gives the temperature, in degrees Celsius, at an Arctic weather station. Which of the following is the best interpretation of $H'(24)$?

$H = {}^\circ$, $H' = {}^\circ/\text{hour}$

(A) The change in temperature during the first day

(B) The change in temperature during the 24th hour

(C) The average rate at which the temperature changed during the 24th hour

(D) The rate at which the temperature is changing during the first day

(E) The rate at which the temperature is changing at the end of the 24th hour

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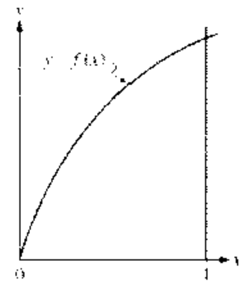
32. A spherical tank contains 81.637 gallons of water at time t = 0 minutes. For the next 6 minutes, water flows out of the tank at the rate of $9\sin\left(\sqrt{t+1}\right)$ gallons per minute. How many gallons of water are in the tank at the end of the 6 minutes? Subtract

(A) 36.606 (B) 45.031 (C) 68.858 (D) 77.355 (E) 126.668

$\int_0^6 9 \sin \sqrt{t+1} =$ total change



33. A left Riemann sum, a right Riemann sum, and a trapezoidal sum are used to approximate the value of $\int_0^1 f(x)dx$, each using the same number of subintervals. The graph of the function f is shown in the figure above. Which of the sums give an underestimate of the value of $\int_0^1 f(x)dx$?

a. Left sum yes
b. Right sum no
c. Trapezoidal sum yes

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

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34. The first derivative of the function f is given by $f'(x) = x - 4e^{-\sin(2x)}$. How many points of inflection does the graph of f have on the interval $0 < x < 2\pi$?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

$Y_1 = f'$
look for max/mins

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35. If f is a continuous function on the closed interval $[a, b]$, which of the following must be true?

(A) There is a number c in the open interval (a, b) such that $f(c) = 0$. no
(B) There is a number c in the open interval (a, b) such that $f(a) < f(c) < f(b)$. no
(C) There is a number c in the open interval [a, b] such that $f(c) \geq f(x)$ for all x in $[a, b]$. yes, EVT
(D) There is a number c in the open interval (a, b) such that $f'(c) = 0$. NO
(E) There is a number c in the open interval (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$. close, but must be [a,b]

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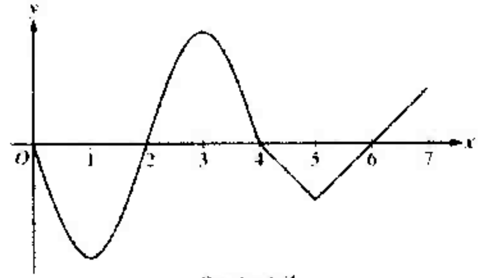
x	2.5	2.8	3.0	3.1
$f(x)$	31.25	39.20	45	48.05

slope: 26.5 29 30.5

36. The function f if differentiable and has values as shown in the table above. Both f and f' are strictly increasing on the interval $0 \leq x \leq 5$. Which of the following could be the value of $f'(3)$?

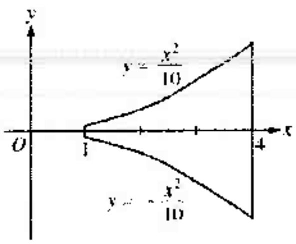
(A) 20 (B) 27.5 (C) 29 (D) 30 (E) 30.5

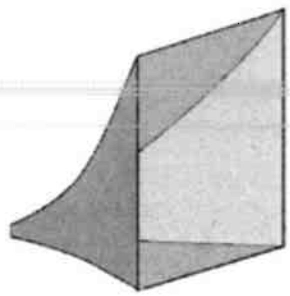


Graph of f'

37. The graph of f', the derivative of the function f, is shown above. On which of the following intervals is f decreasing?

(A) [2, 4] only
(B) [3, 5] only
(C) [0, 1] and [3, 5]
(D) [2, 4] and [6, 7]
(E) [0, 2] and [4, 6]





s^2

$$\int_1^4 \left(\frac{x^2}{10} - \left(\frac{-x^2}{10}\right)\right)^2 dx$$

38. The base of a loudspeaker is determined by the two curves $y = \frac{x^2}{10}$ and $y = -\frac{x^2}{10}$ for $1 \le x \le 4$, as shown above. For this loudspeaker, the cross sections perpendicular to the x-axis are squares. What is the volume of the loudspeaker, in cubic units?

(A) 2.046 (B) 4.092 (C) 4.200 (D) 8.184 (E) 25.711

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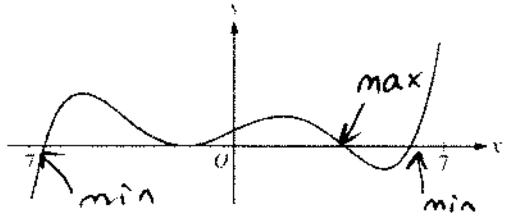
x	3	4	5	6	7
$f(x)$	20	17	12	16	20

39. The function f is continuous and differentiable on the closed interval [3, 7]. The table gives selected values of f on this interval. Which of the following statements must be true?

a. The minimum value of f on [3, 7] is 12. maybe
b. There exists c, for $3 < c < 7$, such that $f'(c) = 0$. yes, MVT, b/c $f(3) = f(7)$ so slope $= 0$
c. $f'(x) > 0$ for $5 < x < 7$. maybe

(A) I only (B) II only (C) III only
(D) I and III only (E) I, II, and III



Graph of f'

40. The figure above shows the graph of f', the derivative of the function f, on the open interval $-7 < x < 7$. If f' has four zeros on $-7 < x < 7$, how many relative maxima does f have on $-7 < x < 7$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

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41. The rate at which water is sprayed on a field of vegetables is given by $R(t) = 2\sqrt{1+5t^3}$, where t is in minutes and $R(t)$ is in gallons per minute. During the time interval $0 \le t \le 4$, what is the average rate of water flow, in gallons per minute?

(A) 8.458 (B) 13.395 (C) 14.691 (D) 18.916 (E) 35.833 $\frac{1}{4}\int_0^4 R(t)\,dt$

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x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	-2	-3	4

$(2(3)+3)(4) + 2(-2)(1+-3)$

42. The table above gives values of the differentiable functions f and g and their derivatives at $x = 1$. If $h(x) = (2f(x)+3)(1+g(x))$, then $h'(1) =$ $h' = (2f(x)+3)(g'(x)) + (2f'(x))(1+g(x))$

(A) -28 (B) -16 (C) 40 $\boxed{\text{(D) 44}}$ (E) 47 $(2f(1)+3)(g'(1)) + 2f'(1)(1+g(1))$

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43. The functions f and g are differentiable, and $f(g(x)) = x$ for all x. If $f(3) = 8$ and $f'(3) = 9$, what are the values of $g(8)$ and $g'(8)$?

They are inverses

$g(8) = 3$

$g'(8) = 1/9$

(A) $g(8) = \frac{1}{3}$ and $g'(8) = -\frac{1}{9}$

(B) $g(8) = \frac{1}{3}$ and $g'(8) = \frac{1}{9}$

(C) $g(8) = 3$ and $g'(8) = -9$

(D) $g(8) = 3$ and $g'(8) = -\frac{1}{9}$

(E) $g(8) = 3$ and $g'(8) = \frac{1}{9}$

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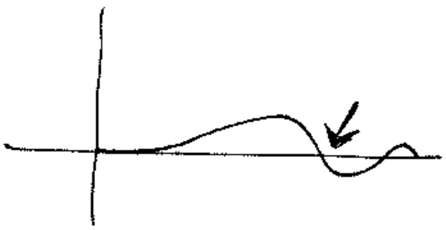
44. A particle moves along the x-axis so that its velocity at any time $t \ge 0$ is given by $v(t) = 5te^{-t} - 1$. At $t = 0$, the particle is at position $x = 1$. What is the total distance traveled by the particle from $t = 0$ to $t = 4$?

(A) 0.366 (B) 0.542 (C) 1.542 (D) 1.821 (E) 2.821 $\int_0^4 |v(t)|$

45. Let f be the function with first derivative defined by $f'(x) = \sin\left(x^3\right)$ for $0 \le x \le 2$. At what value of x does f attain its maximum value on the closed interval $0 \le x \le 2$.

(A) 0 (B) 1.162 $\boxed{\text{(C) 1.465}}$ (D) 1.845 (E) 2



$y_1 = \sin(x^3)$

max is when area is greatest