Mixed Practice #2 for Quiz 2 Name: Answers

AP Calculus

This 50-ish point quiz will cover: 1) Derivatives using all shortcuts with exponential and trig functions; 2) Proofs/derivations of trig derivatives; 3) Equations of tangent and normal lines;
4) Continuity and differentiability of functions; and 5) Extrema and inflection points of functions

Work on these problems as well as former home works, daily checks, and textbook problems.

1) Is the function $f(x) = 5x^3 - 7x^{-1}$ differentiable on the open interval $(-10,10)$? Explain.

$f(x) = 5x^3 - \frac{7}{x}$ is not continuous at $x=0$ so it's not differentiable

2) Let f be the function defined by: $f(x) = \begin{cases} 2x+1, & \text{for } x \le 2 \\ \frac{1}{2}x^2 + k, & \text{for } x > 2 \end{cases}$

a) For what value of k will f be continuous at x = 2? Justify your answer. $K=3$

b) Using the value of k found in part (z), determine whether f is differentiable at x = 2. Justify your answer. Yes. Slopes (ie derivs) are equal at $x=2$

c) Let k = 4. Determine whether f is differentiable at x = 2. Justify your answer.

a) $2x+1 = \frac{1}{2}x^2 + K$ at $x=2$

$5 = 2 + K$

$3 = K$

b) $f(x) = \begin{cases} 2x+1 \\ \frac{1}{2}x^2+3 \end{cases}$

$f'(x) = \begin{cases} 2 \\ x \end{cases}$ at $x=2$ equal

c) $f(x)$ is NOT diff. if $K=4$ since it would not be continuous

3) Suppose the following piece-wise linear/quadratic/linear function is continuous. Determine the values of A and B

$$f(x) = \begin{cases} Ax + 8, & x < 2 \\ \frac{5}{12}x^2 - 4x + \frac{31}{3} & 2 \le x \le 8 \\ -11 + Bx & x > 8 \end{cases}$$

$-2x+8$

$-11+2B$

at $X=2$:

$2A + 8 = \frac{5}{12}(4) - 4(2) + \frac{31}{3}$

$2A = -4$

$A = -2$

at $x=8$:

$\frac{5}{12}(64) - 4(8) + \frac{31}{3} = -11 + 8B$

$16 = 8B$

$2 = B$

Is the function above differentiable? Show the work that leads to your conclusion

$$f'(x) = \begin{cases} -6 & x<2 \\ \frac{10}{12}x - 4 & 2 \le x \le 8 \\ 2 & x > 8 \end{cases}$$

at $x=8$ $\frac{80}{12} - 4$ not equal

at $x=2$ -6 $\frac{10}{12}(2) - 4 = -2.\overline{3}$

Slopes are NOT the same so $f(x)$ is NOT diff.

$(2x^3-1)^{1/3}$

4) Take the derivative of each function (attempt to clean it up when applicable)

a. $f(x) = 3x(7x^2-2)^4$

$f'(x) = 3(7x^2-2)^4 + 3x \cdot 4(7x^2-2)^3(14x)$

$f'(x) = 3(7x^2-2)^3(7x^2-2+56x^2)$ factor to simplify

$f'(x) = 3(7x^2-2)^3(63x^2-2)$

b. $f(x) = \sqrt[3]{2x^3-1}$

$f'(x) = \frac{1}{3}(2x^3-1)^{-2/3}(6x^2)$

$f'(x) = 2x^2(2x^3-1)^{-2/3}$

c. $y = \dfrac{3x-1}{\sqrt{x}}$

$\dfrac{dy}{dx} = \dfrac{3\sqrt{x} - (3x-1)\cdot\frac{1}{2}x^{-1/2}}{x} \cdot \dfrac{(x^{1/2})}{(x^{1/2})}$ ← trick to get rid of neg. expon

$= \dfrac{3x - \frac{3}{2}x + \frac{1}{2}}{x^{3/2}} \cdot \dfrac{(2)}{(2)} = \boxed{\dfrac{3x+1}{2x^{3/2}}}$

↑ get rid of fraction

d. $g = \dfrac{e^x}{7} - \dfrac{7}{e^x} = \frac{1}{7}e^x - 7e^{-x}$

$g' = \frac{1}{7}e^x - 7e^{-x} \cdot -1$

$g' = \frac{1}{7}e^x + 7e^{-x}$

e. $f(x) = \ln(x^2+1)$

$f'(x) = \dfrac{1}{x^2+1} \cdot (2x)$

$f'(x) = \dfrac{2x}{x^2+1}$

f. $y = 2^{\ln x}$

$\dfrac{dy}{dx} = (\ln 2)(2^{\ln x}) \cdot \left(\dfrac{1}{x}\right)$

$\dfrac{dy}{dx} = \dfrac{(\ln 2)(2^{\ln x})}{x}$

g) $f(x) = \sqrt{\ln(\sin x) + \ln(\cos x)}$

$f'(x) = \frac{1}{2}\left(\ln(\sin x) + \ln(\cos x)\right)^{-1/2} \cdot \left(\dfrac{1}{\sin x} \cdot \cos x + \dfrac{1}{\cos x} \cdot -\sin x\right)$

$f'(x) = \frac{1}{2}\left(\ln(\sin x) + \ln(\cos x)\right)^{-1/2}(\cot x - \tan x)$

h $f(x) = \tan(\sin^3 x)$

$f'(x) = \sec^2(\sin^3 x) \cdot 3\sin^2 x \cdot \cos x$

$f'(x) = 3\sec^2(\sin^3 x)\sin^2 x \cdot \cos x$

5) Which expression below is the derivative of $f(x) = 2\csc^3(4-x)$?

a) $-6\csc^2(4-x)\cot(4-x)$
b) $-3\csc(4-x)\cot^2(4-x)$
c) $6\csc^3(4-x)\cot(4-x)$
d) $-6\csc(4-x)\cot(4-x)$
e) $24\csc^2(4-x)\cot^2(4-x)$

$f'(x) = 2 \cdot 3(\csc(4-x))^2 \cdot -\csc(4-x) \cdot \cot(4-x)(-1)$

$f'(x) = 6\csc^2(4-x)\csc(4-x)\cot(4-x)$

$= 6\csc^3(4-x)\cot(4-x)$

6) Given $f(x)=\sin(2x)\cos(x)$, then $f'(\frac{\pi}{6})=\ldots$ $\boxed{\frac{\sqrt{3}}{4}}$

a) $\frac{\sqrt{3}-3}{4}$ $\quad$ $\boxed{\text{b)} \ \frac{\sqrt{3}}{4}}$ $\quad$ c) $\frac{\sqrt{3}}{2}$ $\quad$ d) $\frac{2\sqrt{3}+3}{4}$ $\quad$ e) 0

$$f'(x)=\cos(2x)\cdot 2\cdot\cos(x)+\sin(2x)\cdot(-\sin(x))$$

$$f'(\tfrac{\pi}{6}) = 2\cos(\tfrac{\pi}{3})\cos(\tfrac{\pi}{6}) + -\sin(\tfrac{\pi}{3})(\sin(\tfrac{\pi}{6})$$

$$= 2\left(\tfrac{1}{2}\right)\left(\tfrac{\sqrt{3}}{2}\right) - \left(\tfrac{\sqrt{3}}{2}\right)\left(\tfrac{1}{2}\right) = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{4} = \frac{2\sqrt{3}-\sqrt{3}}{4}$$

7) Supply the "proof" that shows how the derivative of f(x) = sec(x) is f'(x) = sec(x)tan(x).

$$\frac{d}{dx}\sec = \sec(x)\cdot\tan(x)$$

$$\frac{d}{dx}\left(\frac{1}{\cos x}\right)$$

$$\frac{0-1(-\sin x)}{(\cos x)^2}$$

$$\frac{\sin x}{(\cos x)^2}$$

$$\frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}$$

$$\tan x\cdot\sec x = \tan x\cdot\sec x$$

QED

8) Be sure to know how to derive the derivatives of tangent and cosecant as well!

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\frac{\sin x}{\cos x}$$

$$\frac{\cos x\cdot\cos x - \sin x\cdot(-\sin x)}{(\cos x)^2}$$

$$\frac{\cos^2 x+\sin^2 x}{\cos^2 x}$$

$$\frac{1}{\cos^2 x}$$

$$\sec^2 x = \sec^2 x$$

9) Determine the equation of the line that would be normal (perpendicular) to the function

y = sec(x) at $x=\frac{\pi}{4}$.

point: $y=\sec\left(\frac{\pi}{4}\right)=\frac{1}{\cos(\frac{\pi}{4})}=\frac{2}{\sqrt{2}}$

$\left(\frac{\pi}{4}, \frac{2}{\sqrt{2}}\right)$

Slope:

$$\frac{dy}{dx}=\tan(x)\sec(x)$$

at $x=\frac{\pi}{4}$

$$\frac{dy}{dx}=\tan\left(\frac{\pi}{4}\right)\cdot\frac{1}{\cos(\frac{\pi}{4})}$$

$$=1\cdot\frac{2}{\sqrt{2}}=\frac{2}{\sqrt{2}}$$

so slope of $\perp$ line is $-\frac{\sqrt{2}}{2}$

$$\boxed{y-\frac{2}{\sqrt{2}}=-\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)}$$

10) Find the absolute extrema of the function f(x) on the closed interval $[-2, 3]$

$$f(x) = -\frac{1}{4}x^4 + x^3 - 6$$

$f'(x) = -x^3 + 3x^2$

$0 = x^2(-x+3)$

$x = 0,\ x = +3$

x	$f(x)$
-2	$-\frac{1}{4}(-2)^4 + (-2)^3 - 6 = -18$
0	$-\frac{1}{4}(0)^4 + (0)^3 - 6 = -6$
3	$-\frac{1}{4}(3)^4 + (3)^3 + 6 = .75$

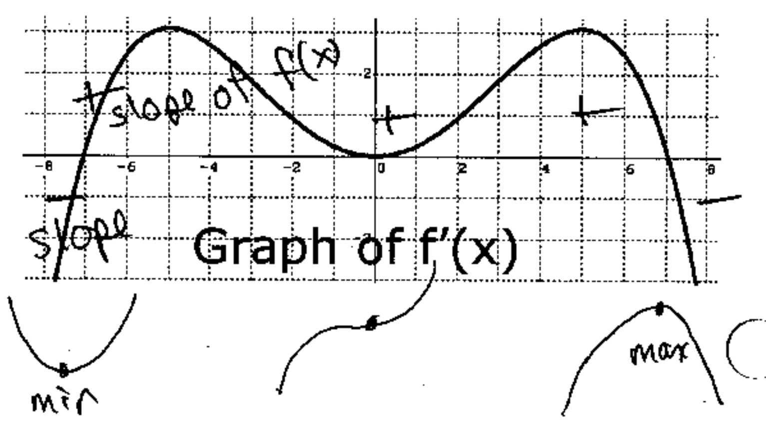
$(3, .75)$ is absolute max and $(-2, -18)$ is absolute min

11) Use the graph of **f'(x)** shown to determine **at which x-values** the original function f(x) has...

Critical point(s) at x = $-7, 0, 7$

Relative maximum(s) at x = 7

Relative minimum(s) at x = -7



12) Given the graph of **f''(x) below**, how many inflection points does the original have?

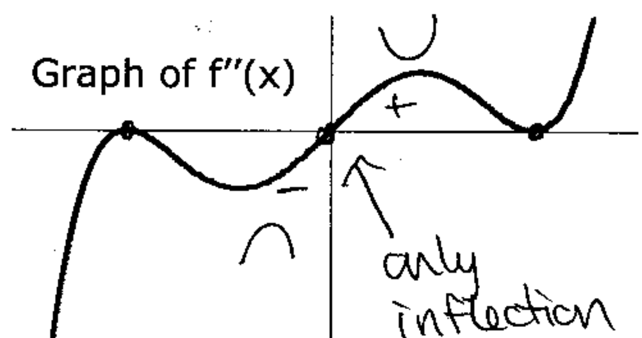
(a) f(x) has no inflection points

(b) f(x) has 1 inflection point

(c) f(x) has 2 inflection points

(d) f(x) has 3 inflection points

(e) f(x) has 4 inflection points

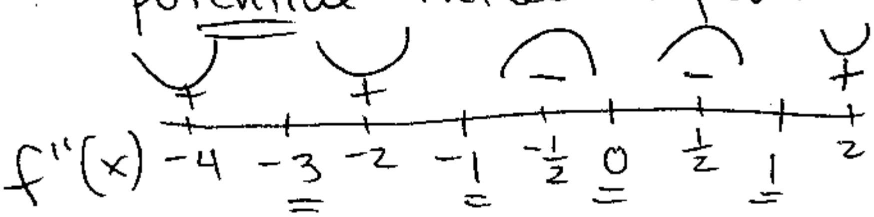


13) Given the **second** derivative of a function, $f''(x) = x^2(x^2-1)(x+3)^2$

$0 = x^2(x-1)(x+1)(x+3)(x+3)$

determine the x-values of inflection points for the **original** function.

potential inflection points at $x = 0, \pm 1, -3$



$\boxed{\text{Inflection points at } x=-1,\ x=1}$