Mixed Practice #1 for Quiz 2 **Name:** Answers

AP Calculus

1)

If $f(x)=(x-1)(x^2+2)^3$, then $f'(x)=$

(A) $6x(x^2+2)^2$

(B) $6x(x-1)(x^2+2)^2$

(C) $(x^2+2)^2(x^2+3x-1)$

(D) $(x^2+2)^2(7x^2-6x+2)$

(E) $-3(x-1)(x^2+2)^2$

$$f'(x)=1(x^2+2)^3+(x-1)(3)(x^2+2)^2(2x)$$
$$=(x^2+2)^2(x^2+2+6x^2-6x)$$
$$=(x^2+2)^2(7x^2-6x+2)$$

2)

$$f(x)=\begin{cases}\dfrac{x^2-4}{x-2} & \text{if } x\neq 2\\ 1 & \text{if } x=2\end{cases}$$

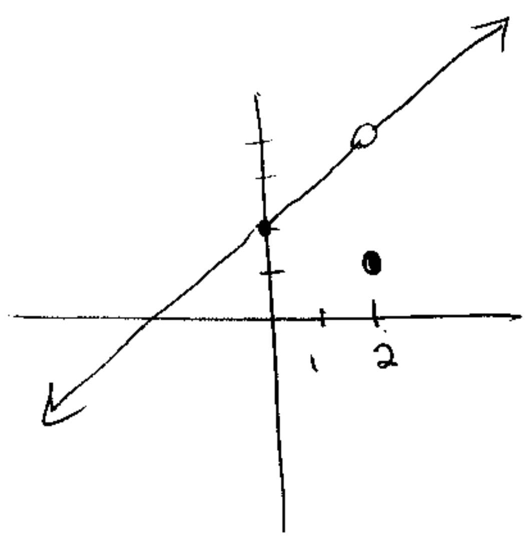
$$\frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}=x+2$$

Let f be the function defined above. Which of the following statements about f are true?

I. f has a limit at $x=2$. ✓

II. f is continuous at $x=2$. X

III. f is differentiable at $x=2$. X



(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

3)

If $f(x)=\cos(3x)$, then $f'\left(\frac{\pi}{9}\right)=$

$$f'(x) = -\sin(3x)\cdot 3$$

$$f'\left(\tfrac{\pi}{9}\right) = -3\sin\left(3\cdot\tfrac{\pi}{9}\right)$$

(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$

4)

$$f(x) = e^{2x^{-1}}$$

If $f(x)=e^{(2/x)}$, then $f'(x)= (\ln e)\left(e^{\frac{2}{x}}\right)\left(-2x^{-2}\right) = \frac{-2e^{\frac{2}{x}}}{x^2}$

(A) $2e^{(2/x)}\ln x$ (B) $e^{(2/x)}$ (C) $e^{(-2/x^2)}$ (D) $-\frac{2}{x^2}e^{(2/x)}$ (E) $-2x^2e^{(2/x)}$

5)

If $f(x)=x^2+2x$, then $\frac{d}{dx}(f(\ln x))=$

(A) $\frac{2\ln x+2}{x}$ (B) $2x\ln x+2$ (C) $2\ln x+2$ (D) $2\ln x+\frac{2}{x}$ (E) $\frac{2x+2}{x}$

$$f(\ln x) = (\ln x)^2 + 2(\ln x)$$

$$f' = 2(\ln x)\left(\tfrac{1}{x}\right) + 2\cdot\frac{1}{x}$$

$$= \frac{2\ln x}{x} + \frac{2}{x}$$

6)

In the xy-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k?

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

$y' = 2x + 3 = \text{slope}$

The slope of the line $x + y = K$ is -1

since $y = -x + K$

So $2x + 3 = -1$

$2x = -4$

$x = -2$

$x = -2$, find y

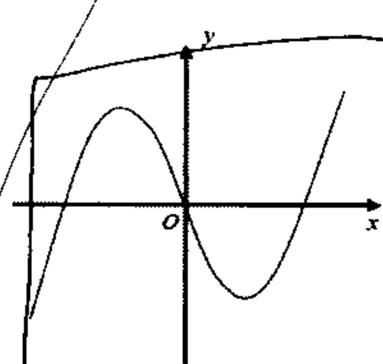
$y = (-2)^2 + 3(-2) + 1$

$y = -1$

$x + y = K$

$-2 + -1 = K$

$K = -3$

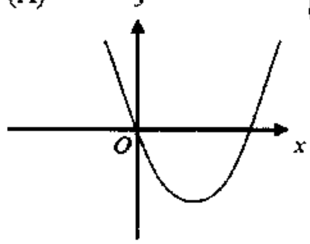


Graph of f

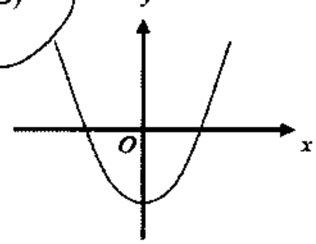
7)

The graph of a function f is shown above. Which of the following could be the graph of f', the derivative of f?

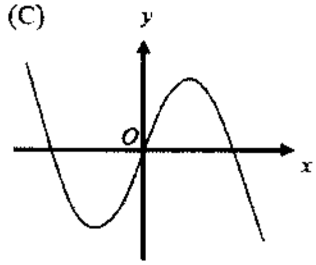
(A)



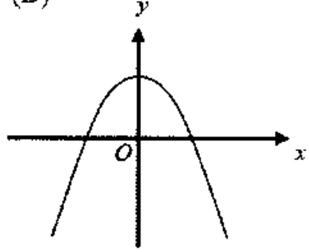
(B)



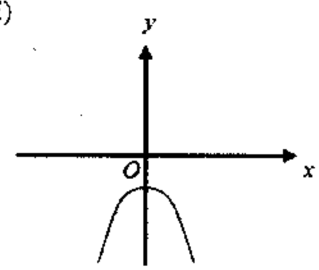
(C)



(D)



(E)



8)

$$f(x)=\begin{cases} cx+d & \text{for } x\le 2 \\ x^2-cx & \text{for } x>2 \end{cases}$$

$$f'(x)=\begin{cases} c \\ 2x-c \end{cases}$$

Let f be the function defined above, where c and d are constants. If f is differentiable at $x=2$, what is the value of $c+d$?

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

continuous at $x=2$

$cx+d=x^2-cx$

$2c+d=4-2c$

$2(2)+d=4-2(2)$

$d=-4$

diff at $x=2$

$c=2x-c$

$2c=2x$

$c=x$

since $x=2$, $c=2$

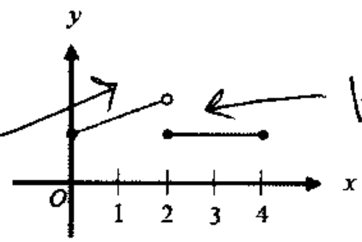
$c+d$

$2+-4$

-2

9)

limit exists from left



limit exists from right

Graph of f

left $\neq$ right

so $\lim_{x\to 2}$ DNE

The figure above shows the graph of a function f with domain $0\le x\le 4$. Which of the following statements are true?

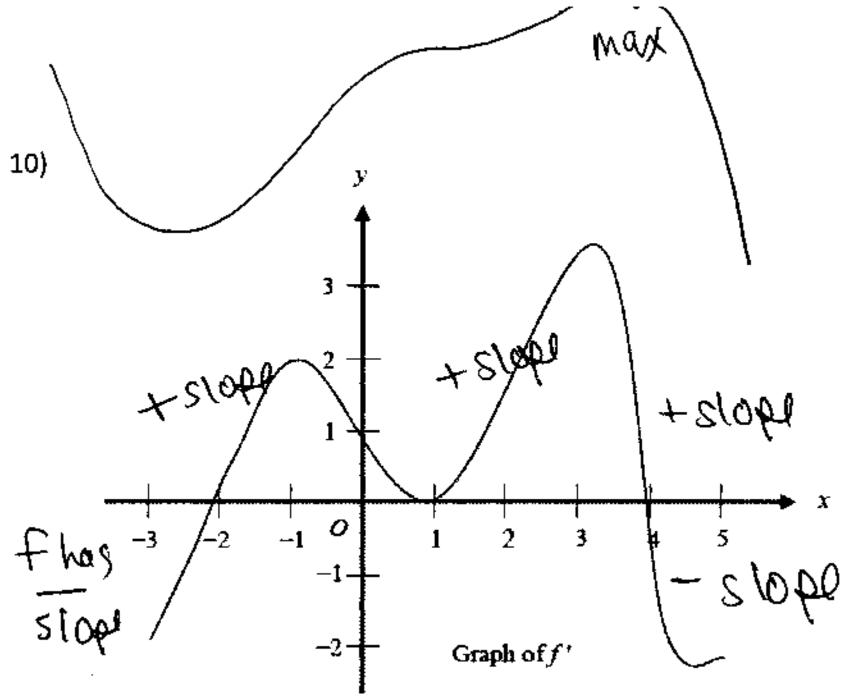
I. $\lim_{x\to 2^-} f(x)$ exists. ✓

II. $\lim_{x\to 2^+} f(x)$ exists. ✓

III. $\lim_{x\to 2} f(x)$ exists. ✗

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

10)



The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x=-1$, $x=1$, and $x=3$. At which of the following values of x does f have a relative maximum?

(A) -2 only

(B) 1 only

(C) 4 only

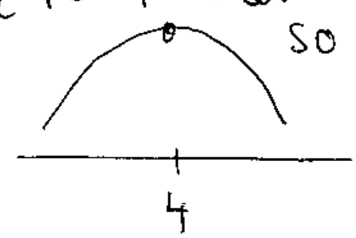
(D) -1 and 3 only

(E) -2, 1, and 4

C.P are at $x = -2, 1, 4$

slope is + then −

so max at $x=4$



11)

If $f(x) = \ln\left(x + 4 + e^{-3x}\right)$, then $f'(0)$ is

(A) $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) nonexistent

Can do in calculator! or by hand

$$f'(x) = \frac{1}{x+4+e^{-3x}} \cdot \left(1 - 3e^{-3x}\right)$$

$$f'(0) = \frac{1}{0+4+e^{0}} \cdot \left(1-3e^{0}\right) = \frac{1}{5} \cdot -2 = \frac{-2}{5}$$

12) Determine the slope of the line that would be normal (perpendicular) to the function

$y = e^{3\sin(2x)}$ at $x = \pi$.

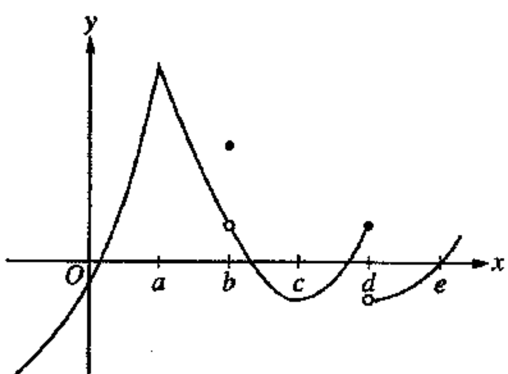
$$\frac{dy}{dx} = e^{3\sin(2x)} \cdot 3\cos(2x) \cdot 2$$

at $x = \pi$, $\frac{dy}{dx} = 6e^{3\sin(2\pi)} \cdot \cos(2\pi) = 6e^0 \cdot (1) = 6$

so slope $= 6$ at $x = \pi$

point: $y = e^{3\sin 2\pi} = e^0 = 1$ $(\pi, 1)$

$$\boxed{y - 1 = 6(x - \pi)}$$

13) 

Graph of f

The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) a (B) b (C) c (D) d (E) e

14)

If $y = x^2 \sin 2x$, then $\frac{dy}{dx} =$

$2x \cdot \sin(2x) + x^2 \cdot \cos 2x \cdot 2$

$2x\left(\sin(2x) + x\cos(2x)\right)$

(A) $2x \cos 2x$

(B) $4x \cos 2x$

(C) $2x(\sin 2x + \cos 2x)$ ← Whoops Look carefully!

(D) $2x(\sin 2x - x \cos 2x)$

(E) $2x(\sin 2x + x \cos 2x)$

15)

$$f(x) = \begin{cases} x + 2 & \text{if } x \le 3 \\ 4x - 7 & \text{if } x > 3 \end{cases} \quad \begin{matrix} (3,5) \\ (3,5) \end{matrix}$$

Let f be the function given above. Which of the following statements are true about f ?

I. $\lim_{x \to 3} f(x)$ exists. ✓

II. f is continuous at $x = 3$. ✓

III. f is differentiable at $x = 3$. X

$f'(x) = \begin{cases} 1 \\ 4 \end{cases}$ slopes are not equal

(A) None

(B) I only

(C) II only

(D) I and II only

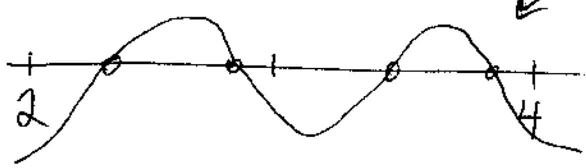
(E) I, II, and III

16)

Let f be the function with derivative given by $f'(x) = \sin(x^2 + 1)$. How many relative extrema does f have on the interval $2 < x < 4$?

(A) One (B) Two (C) Three (D) Four (E) Five

$\sin(x^2+1) = 0$



4 places where $f'(x) = 0$ + slope

$\therefore$ 4 c.p.

+ since slopes change (from − to + or + to −) they're all rel. extrema

17)

Let f be a differentiable function with $f(2) = 3$ and $f'(2) = -5$, and let g be the function defined by $g(x) = xf(x)$. Which of the following is an equation of the line tangent to the graph of g at the point where $x = 2$?

(A) $y = 3x$

(B) $y - 3 = -5(x - 2)$

(C) $y - 6 = -5(x - 2)$

(D) $y - 6 = -7(x - 2)$

(E) $y - 6 = -10(x - 2)$

point: $(2, 3)$

slope at $x = 2$ is -5

$g(x) = x\,f(x)$

$g'(x) = 1\,f(x) + x\,f'(x)$

$g'(2) = 1\,f(2) + 2\,f'(2)$

$= 1 \cdot 3 + 2 \cdot -5$

$= 3 + -10$

$= -7$