Measurement and Significant Figures

Accuracy is about whether a measurement agrees with the true value. If a measurement is *accurate* then it is *correct*. This can be difficult to determine unless it is possible to look up a result in a trusted source.

Precision is about whether several measurements of something agree *with each other*. Precision can be measured using the range of values. One half of this range is the amount by which the true value may vary above and below the average value (this is called the plus-or-minus amount—the \pm amount). In lab activities we make several measurements in order to be able to calculate just how precise you are. Chances are good, but not certain, that if your measurements have a small range then your results are accurate, too.

The precision of a result is reflected in the number of digits reported. If you have a bathroom scale and measure the mass of a book with it you might find it weighs 7 pounds. A more precise scale might reveal that the weight of the book is actually 7.15 pounds. This second number is more precise. When a measurement is expressed with more digits it is generally more precise than a measurement of the same thing showing fewer digits. The **significant figures** of a measurement are all the digits known for sure plus one estimated digit.

Errors in the lab come in three types. First, **human error**. Human error is when you make a boneheaded mistake. For example, say you intend to measure out 1.0 g of a chemical but instead measure out 10.0 g. This type of mistake can be avoided and will be noticed while you are working if you are paying attention. Second, there is systematic error. Systematic error leads to measurements being always high or always low. The change is in the same direction for each measurement and is always the same size. This type of error occurs, for example, if you use a ruler to make measurements by pushing the end of the ruler up against something. Many rulers do not have their zero mark right at the end. This results in all your measurements being too long by the extra bit of ruler. Third, there is random error. Random error can be seen in a set of data that represent measurements of the same quantity. The quantity is a constant and has some true value but the measurements of it are usually a bit too high or a bit too low. For each individual measurement the amount that the measurement is too high or low is random. It can be safely assumed that random error will affect every set of measurements that anyone makes. It occurs because we estimate the last digit of a measurement and different people estimate that last digit differently. Random error may also occur because the value displayed by an electronic instrument may fluctuate and it is up to the observer to make a judgment about the value. Random error can be measured by using the range of measurements to calculate a plus-or-minus amount. The plus-or-minus amount is one-half of the range. It can also be measured using the percent random error. This is calculated by dividing the plus-of-minus amount by the average and multiplying by 100%.

Unless otherwise noted the final significant figure (the last digit) in a measurement is assumed to be ± 1 . So 9.45 kg is 9.45 ± 0.01 kg—that is, the true value is assumed to be between 9.44 kg and 9.46 kg. Random error's influence over results can be reduced by taking multiple measurements of the same thing and averaging the reults. This evens out the too-high measurements with the too-low measurements.

Estimating the Last Digit

The **maximum possible precision** is defined as one tenth (1/10 or 0.1 times) the smallest division on the measuring instrument. On the first ruler shown below there are marks only every centimeter. So the best precision comes by estimating the last digit which is 0.1×1 cm = 0.1 cm. The maximum possible precision can be expressed as a plus-or-minus amount and is ± 0.1 cm. For the second ruler the plus-or-minus amount is 0.1×0.1 cm = 0.01 cm. This ruler is ten times more precise than the one above it at ± 0.01 cm.

Look at the rulers below and use each one in turn to measure the gray line. Remember, precision is limited to one tenth of the smallest division. For each ruler estimate the last digit. That is, for the first ruler, estimate to the tenths place. For the second ruler estimate to the hundredths place.



To do the estimating you must imagine that the space between the markings on the ruler can be divided into ten smaller spaces. Use these imaginary spaces to measure the last digits

Here is a key point: When you estimate the last digit you are actually making your measurement *more* precise. First, you have added a significant figure: more significant figures means better precision. Second, you are making the digit before the last digit more secure due to the fact that you are observing the last digit so closely.

Counting Significant Figures

There are three rules for counting the number of significant figures in a measurement.

- 1. Nonzero integers are always significant.
- 2. Zeros are sometimes significant:
 - 1. Leading zeros are never significant figures. The number 0.0042 has only two significant figures.
 - 2. Captive zeros are zeros between nonzero integers. These are always significant figures. The number 1.004 has four significant figures.
 - 3. Trailing zeros are significant only if they are after a decimal point. The number 4,000 has one significant figure. The number 3.00×10^8 has three significant figures.
- 3. Exact numbers have an infinite number of significant figures. Exact numbers include things like numbers that come from counting: 14 beakers is an exact number. Other exact numbers come about because of definitions: one inch is *defined* as exactly 2.54 cm.

It is always easiest to tell the number of significant figures if you use scientific notation. In scientific notation you never have trailing zeros or leading zeros. Those zeros only tell you how far the measurement is from the decimal point. In scientific notation this job is taken care of by the exponent.

Calculations with Significant Figures

When you do a calculation using measurements you must be sure that the result is never expressed with more significant figures than you started with. If you do then you are pretending you know more than you really know. The number of significant figures tells you how *precise* your data are. If you use too many figures you are exaggerating your precision. Here are two simple rules:

1. For *Multiplication and Division* the result of a calculation may have no more digits in it than the least precise measurement in the calculation. For example:

 $2.48 \div 5.4 = 0.459259$ must be rounded to 0.46 because the number 5.4 has just two significant figures.

2. For *Addition and Subtraction* the result has the same number of digits after the decimal as the least precise measurement.

For example: 14.56 + 12.1 + 1.019 = 27.679 must be rounded to 27.7 because the number 12.1 has only one digit past the decimal point.

Important: Calculations often end up producing answers that have a lot more digits (in the calculator) than the measurements you started with. **These digits are not all significant!** It is impossible to increase the precision of measurements by doing calculations with them. Just because the calculator says there are more digits doesn't mean they are all meaningful.

Procedure:Part I

For the following measurements and calculations pay close attention to the number of significant figures and use the rules for doing calculations with significant figures.

- 1. As an individual please use a ruler or meter stick to measure the following items: the length of a paperclip
- the length, width, and height of a book (everyone in class will use the same book)
- 2. Compute the area of the top of the lab bench
- 3. Compute the volume of the book
- 4. With at least four other students put all of your results into one neat table; make sure everyone makes his/her own table
- 5. For your group's data compute an Average for each of the things you had to compute above
- 6. For each measurement compute the Range: take the largest value and subtract the smallest value from it
- 7. For each measurement express the range as an amount above or below the average: divide the range by twothis is your ± Amount (Plus-or-minus Amount)
- 8. Report **Results** in this way: 4.2 ± 0.2 cm. Round the plus-or-minus amount to one digit. Round the last digit of the average to match the place value of the remaining digit in the plus-or-minus amount. This is how you use your data to decide the appropriate number of significant figures.
- 9. Compute the **Percent Error** for each result (divide the plus-or-minus amount by the average and multiply by 100%). This value tells how big the random error is relative to the size of the measurement.

Use the following data table to collect your data. If you find that the boxes are too small then make the data table in your lab notebook. Remember, every individual student **must** have their own copy of these data so everyone must write it down. The numbers 1 - 4 in this data table represent different individuals making measurements of exactly the same object.

Papercli Lab Bench Book р Widt Area Sink Area Useabl Widt | Sink

Here are some sample data I collected for the paperclip in cm: 3.30, 3.28, 3.29, 3.30. Average: 3.29 cm, Range: 0.02 cm, \pm Amount: 0.01 cm, Result: 3.29 \pm 0.01 cm, Percent Error: 0.30%.

	(cm)	h (cm)	h (cm)	t (cm)	e (cm ³)	h (cm)	h (cm)	(A_1) (cm^2)	Lengt h (cm)	h (cm)	(A_2) (cm^2)	$\begin{array}{c} e \ Area \\ A_1 - A_2 \\ (cm^2) \end{array}$
1												
2												
3												
4												
Averag e												
Range												
± Amount												
Result												
Percent Error												

Check-in

When you have completed your work in this section check in with your teacher. Initials will be given for a complete set of data and calculations.

Part II

Your teacher has provided several objects for you to weigh.

- 1. Have each member of the group measure the mass of every item at least once. Each and every person must make their own measurements!
- 2. Construct a table of the group's data and find the average, range, plus-or-minus amount, result and percent error for each item; every member of the group must do this

Check-in _____

Part III

There are several graduated cylinders at the back of the room. These instruments are used to measure the volume of liquids. Each one has a limit to its precision. See if you can find out whether there is a relationship between the size of a graduated cylinder and its precision.

- 1. Determine the maximum precision of each graduated cylinder, expressed in mL. Do this by writing down the digit or decimal point that is estimated for each one.
- 2. Have each member of the group carefully read the volume in each of the graduated cylinders
- 3. Construct a table of the group's data and find the average, range, plus-or-minus amount, result and percent error for each cylinder; every member of the group must do this

Check-in _____

For this lab you must turn in your **lab handout with all check-ins**, the **completed data tables** and **answer the lab questions** at the end of this handout. Answer the questions on a separate piece of paper. All numbers must be expressed with the correct number of significant figures. If you must type your tables to make them legible then please use Excel or another spreadsheet program. Be sure to use borders on your tables.

You will be graded on the quality of your writing, the professionalism of your work's appearance, and the quality of the answers to the following questions.

Questions

- 1. The precision of an instrument is limited by how close together the lines are on that instrument. The maximum precision is the decimal digit that is estimated when you use the instrument. What is the precision of the metersticks you used in class? In other words, what is the last decimal digit that can be estimated using the lines marked on it? Report your answer in centimeters and explain why it is what it is.
- 2. What is the plus-or-minus amount for the meterstick based on the data collected by the members of your group? Is this better or worse than the theoretical plus-or-minus amount you found in the previous question?
- 3. Compare the precision of your area and volume calculations with the precision of your length measurements. Which is more precise? Why?
- 4. Put the graduated cylinders in order by increasing theoretical precision based on the spacing between lines. Remember that you estimate to about 1/10 of the size of the smallest division. Explain your reasoning.
- 5. What is the plus-or-minus amount for each graduated cylinder based on the data collected by the members of your group? Is this better or worse than the theoretical plus-or-minus amount you found in the previous question?
- 6. What is the theoretical precision of the three-beam balance based on the spacing between lines? Explain your reasoning.
- 7. What is the plus-or-minus amount for the lab balance based on the data collected by the members of your group? Is this better or worse than the theoretical plus-or-minus amount you found in the previous question?
- 8. Explain why the members of your group were seldom (if ever) able to get exactly the same measurement even when you were all carefully measuring the same thing the same way.
- 9. What are significant figures and what do they tell you about a measurement?
- 10. Consider the following set of data: 456.324 cm², 455.995 cm², 456.198 cm², 455.884 cm² Calculate the average, range, plus-or-minus amount, and percent error. Report the result showing the correct number of significant figures with the ± amount.
- 11. How can you use a set of data you collected to figure out the proper number of significant figures? Consider the calculations you just completed in the previous question.
- 12. What is human error? Give three examples.
- 13. What is systematic error? Give two examples.
- 14. What is random error? Describe a method you can use to measure the amount of random error in your data collection.
- 15. The **percent error** in a measurement is calculated by dividing the plus-or-minus amount by the average measurement and multiplying by 100%. You have already calculated the percent error for all of the measurements in this lab. For each one evaluate whether you had excellent, good or poor precision. Excellent precision is 1% or less. Good precision is between 1% and 3%. Poor precision is when the result is greater than 3%.