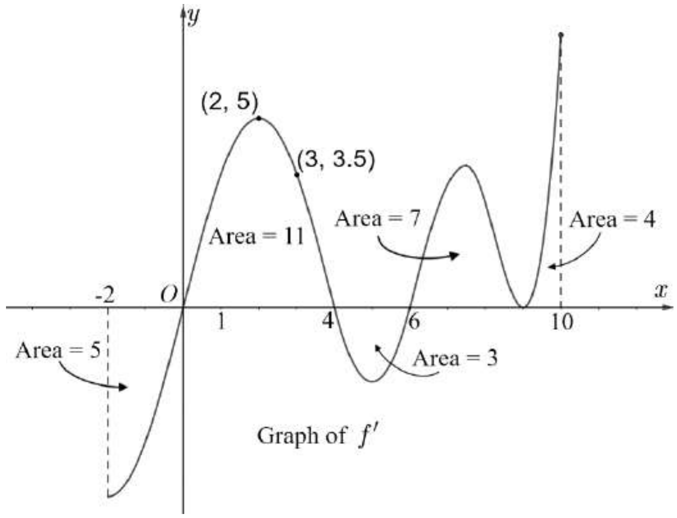


AB1: Let g be the function defined by $g(x) = \begin{cases} f(x^2 - 5), & x \leq 3 \\ 5 - 2x, & x > 3 \end{cases}$

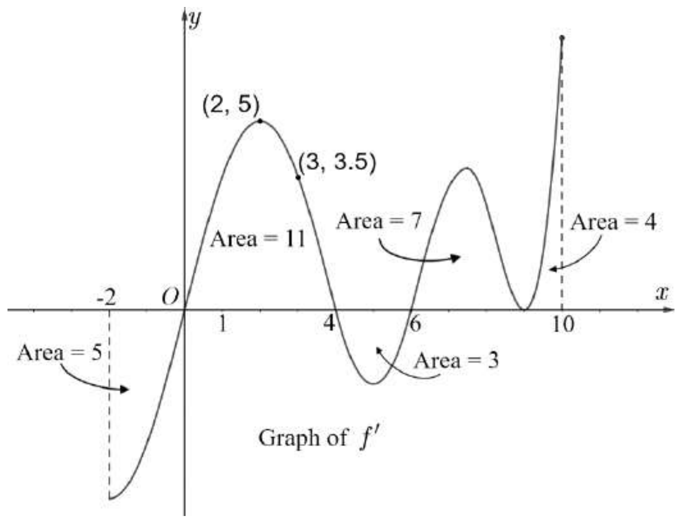
The function f is twice differentiable on the closed interval $[-2, 10]$ and satisfies $f(6) = -4$. The graph of f', the derivative of f, is shown in the figure above. The graph of f' has horizontal tangent lines at $x = 2, x = 5, x = 8,$ and $x = 9$. The areas of the regions between the graph of f' and the x axis are labeled in the figure.

(a) Show that g is continuous at $x = 3$.

(b) Find the absolute maximum value of f on the interval $[-2,10]$. Justify your answer.

(c) For $x \neq 3$, the function k is defined by $k(x) = \dfrac{3\int_4^{3x} f'(t)dt - 4x}{3e^{2f(x)+5} - x}$. It is known that $\lim_{x \to 3} k(x)$ can be evaluated using L'Hospital's Rule. Find $f(3)$ and evaluate $\lim_{x \to 3} k(x)$.

Created by Bryan Passwater bryanpasswater1@gmail.com



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(d) Let $y = h(x)$ be a function such that $\frac{dy}{dx} = f'(2x) \cdot (3y - 1)$ where $y > \frac{1}{3}$. Find $\frac{d^2y}{dx^2}$ and use this expression to determine if the graph of $h(x)$ is concave up or down at $h(1) = 2$.

(e) Consider the function $y = h(x)$ from part (d). For $-1 < x < 5$, on what interval(s) is the function $h(x)$ increasing? Give a reason for your answer.

(f) Consider the differential equation from part (d). Use separation of variables to find a general solution for $y = h(x)$ of the form $ln|3y - 1| = af(bx) + C$.

t (hours)	0	1	4	6	8
$C'(t)$ (100's calls/hour)	5	4.5	2.5	1	0.2

AB2. A large pizza chain delivers pizzas nightly from 6PM to 2AM. On a Friday night, the rate that the pizza chain receives calls for delivery is modeled by the differentiable function $C'(t)$, measured in hundreds of calls per hour, and t is measured in hours since 6PM ($t = 0$). Selected values of $C'(t)$ are given in the table above.

(a) Use the data in the table to approximate $C''(7)$. Using correct units, interpret the meaning of $C''(7)$ in context of the problem.

(b) Do the data in the table support the conclusion that there is a time t, $0 \le t \le 8$, at which the pizza chain receives 300 delivery calls per hour? Give a reason for your answer.

(c) Using correct units, explain the meaning of the definite integral $\int_0^8 C'(t)dt$ in context of the problem. Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate the value of $\int_0^8 C'(t)dt$.

(d) The total number of calls that the pizza chain receives for delivery on Friday night, in $100's$ of calls, can be modeled by the function F, given by $F(x) = \dfrac{x^2}{200 - 2x}$ where x is the number of delivery drivers. When there are 60 drivers, the number of drivers is decreasing at a rate of $\frac{7}{24}$ drivers per hour. According to this model, what is the rate of change of total delivery calls with respect to time, in $100's$ pizzas per hour, at the time when there are 60 drivers delivering pizzas.

t	0	25	30	45	65	80
$W(t)$	2	10	13	30	60	81

BC1: Workers at a local office building are expected to be at work by 9AM ($t = 60$) each day. Let the twice differentiable function $y = W(t)$ be the amount of workers in the building at time t, in minutes, satisfying the logistic differential equation $\frac{dy}{dt} = \frac{1}{15}y\left(1 - \frac{y}{100}\right)$ where $W(0) = 2$. Selected values of $W(t)$ are given in the table where $\mathrm{t} = 0$ is 8AM.

(a) Find $W'(65)$ and $W''(65)$. Using correct units, interpret the meaning of $W''(65)$ in context of this problem.

(b) Evaluate $\int_0^\infty W'(3t + 25)dt$. Show the work that leads to your answer.

(c) Use a left Riemann sum with three subintervals indicated by the data in the table to approximate the value of $\frac{1}{45}\int_0^{45} W(t)dt$. Using correct units, explain the meaning of $\frac{1}{45}\int_0^{45} W(t)dt$ in the context of the problem.

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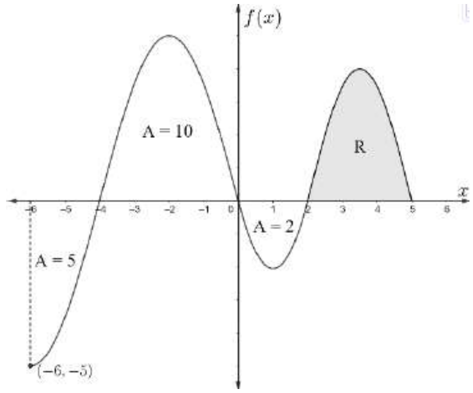
(d) Use Euler's method with two steps of equal size starting at $t = 45$ minutes to approximate the number of workers inside the office building by 9: 25AM ($t = 85$ minutes).

(e) Is there a time c, such that $25 < c < 45$, when $W'(c) = 1$. Justify your answer.

(f) Let $P_2(t)$ be the second degree Taylor polynomial for $W(t)$ centered at $t = 65$. Find $P_2(t)$ and use it to approximate the number of workers inside the office building at 9: 25AM ($t = 85$).



BC2: The function f is twice differentiable for $x \geq -6$. A portion of the graph of f is given in the figure above. The areas formed by the bounded regions between the graph of f and the x axis are labeled in the figure. Let the twice differentiable function g be defined by $g(x) = \int_2^x f(t)dt$.

(a) Find $\lim_{x \to -2} \dfrac{g(2x) - 4x}{e^{x^2-4} + x + 1}$.

(b) Find the absolute maximum of g on the interval $[-6,2]$. Justify your answer.

(c) Evaluate $\int_{-3}^{1} xf'(2x)dx$.

(d) The region R is the base of a solid whose cross sections perpedicular to the x axis are squares. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(e) For $x \geq 5$, the function f is not explicitly given, but $f'(x) = -4\left(\frac{1}{2}\right)^{x-5}$. If $a_n = f'(n)$, find $\sum_{n=5}^{\infty} a_n$ or show that the series diverges.