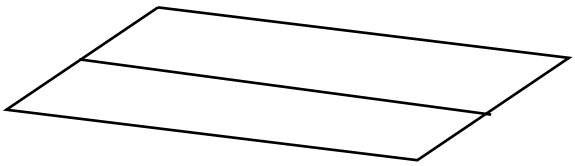
Plane Geometry

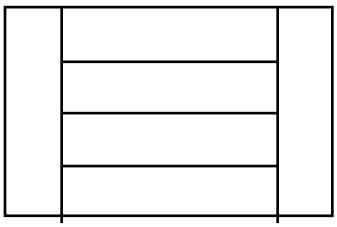
1) On a number line, the coordinates of P and Q are 8 and 48 respectively. The midpoint of $\overline{PQ}$ is B, the midpoint of $\overline{BQ}$ is C and the midpoint of $\overline{PC}$ is D. What is the coordinate of D? (2) *2004-WU3-4*

2) Ken and Maria are planning to plant rectangular gardens of the same length, side by side with fencing all around and dividing the two plots. The total amount of fencing is 100 feet. If the total area of the two plots is 336 square feet and the dimensions are integers, what is the length of the fence that divides the two plots? (2) *2004-WO3-8*



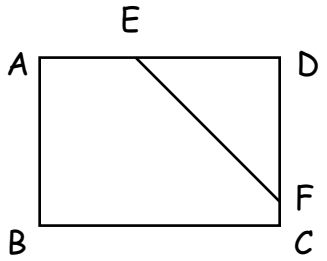
3) Danny has five sticks measuring 5 cm, 5 cm, 8cm, 14 cm, and 14 cm. Using exactly three sticks as the sides of a triangle, how many non-congruent triangles are possible if the sticks are joined only at their endpoints? (2) *2004-WU10-8*

4) Jack is tiling his floor with concrete pavers that have a pattern in which six congruent rectangles are arranged. One such paver is shown here. If the area of the paver is 600 square inches, what is its perimeter? (2) *2004-WU11-9*

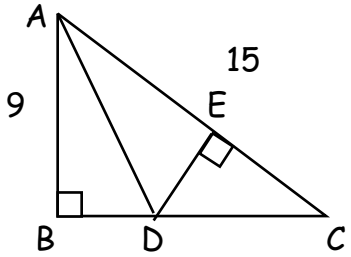


5) A regular hexagon has a side length of 6 units. What is its area? Express your answer in simplest radical form. (3) *2004-WU13-6*

6) In rectangle ABCD, AB = 6 cm, BC = 8 cm, and DE = DF. The area of triangle DEF is one-fourth the area of rectangle ABCD. What is the length of segment EF? Express your answer in simplest radical form. (3) *2004-WU16-8*



7) Right triangle ABC is divided as shown, such that angle BAC is bisected by $\overline{AD}$. $\overline{DE} \perp \overline{AC}$. What is the length of $\overline{AD}$? Round to nearest hundredth. (3) *2004-WO8-8*



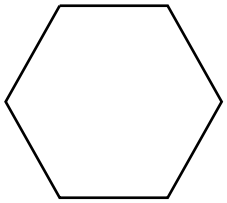
8) If the measure of the interior angle of a regular polygon is 162°, and this polygon is the base of a prism, how many edges does the prism have? (3) *2004-WU17- 6*

9) What is the ratio of the number of degrees in the complement of a 60° angle to the number of degrees in the supplement of a 60° angle? (1) *1999-WU1-9*

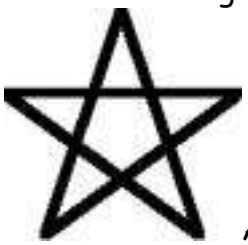
10) What is the side length of the largest square that can be inscribed in a circle with a radius of 8 inches? Express in simplest radical form. (3) *2003-WU12-5*

11) The perimeter of a square lot is lined with trees, and there are three yards between the centers of adjacent trees. There are eight trees on a side, and a tree is at each corner. What is the perimeter of the lot? (1) *1999-WU4-5*

12) From a regular hexagon, three vertices are selected at random. What is the probability that these vertices form an equilateral triangle or an isosceles triangle? (3) *2003-WO7-5*

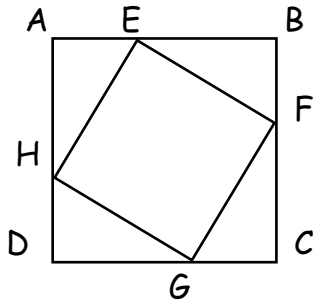


13) The sides of a regular pentagon are extended to form congruent isosceles triangles as shown. What is m<A? (3) *1999-WU5-9*

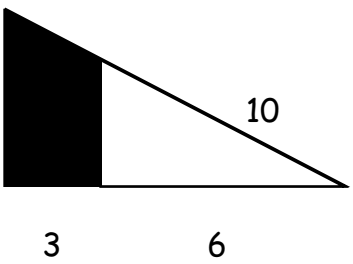


14) Given five line segments of length 2, 3, 5, 8 and 13, what is the number of distinct triangles that can be formed using any three of the segments? (2) *1999-WU6-6*

15) The vertices of square EFGH lie on the edge of square ABCD. $\frac{AE}{EB} = \frac{1}{2}$. What is the ratio of the area of square EFGH to the area of square ABCD? (3) *1999-WU9-8*



16) Find the area of the shaded region: (2) *1999-WU10-8*



18) A billiard ball is hit at a 45° angle from a corner of a 4-foot by 7-foot billiard table. How many times will the ball rebound off an edge of the table before landing in a corner? (3) *1999-WU13-1*

19) 28 circular pepperoni slices, each 1" in diameter, are placed on a circular pizza. The slices neither hang off the edge nor overlap. What is the area of the pizza not covered by the pepperoni slices? Express your answer in terms of π. (2) *1999-WU15-3*

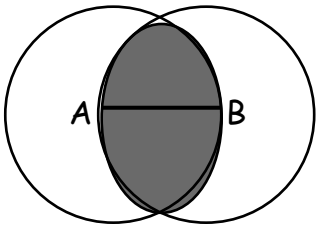
20) The perimeter of a regular hexagon is 48 inches. What is the positive difference between the areas of the circumscribed and the inscribed circles of the hexagon? Express your answer in terms of π. (3) *1999-WU16-6*

21) In a right triangle, the longer leg measures three times the shorter leg. What percent of the hypotenuse is the longer leg? Round to nearest whole percent. (3) *1999-WO1-8*

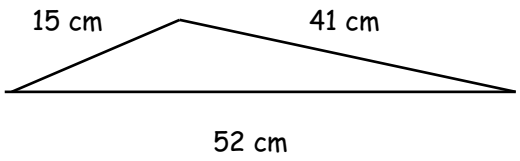
22) A pizza parlor offers a circular 14-inch diameter pizza for $7.99. The pizza is cut into 17 slices. What is the mean cost per slice? (2) *1999-WO2-5*

23) In the diagram, the curve is an arc of a circle having a center at a vertex of the square with edge length 4. What percent of the square is shaded? (3) *1999-WO3-5*

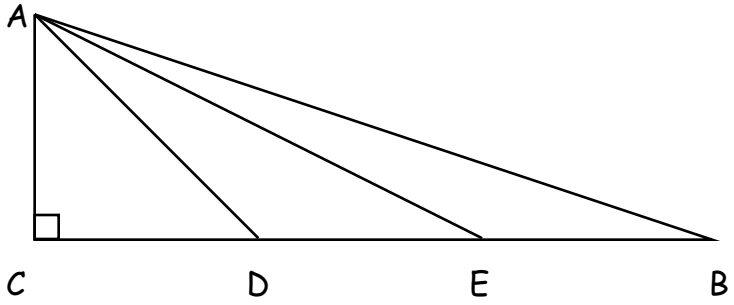
24) Congruent circles A and B overlap such that the line segment that connects their centers ($\overline{AB}$) measures 6 cm. What is the area of their overlapping region? Use π = 3.14 and round to the nearest tenth. (3) *1999-WO5-3*



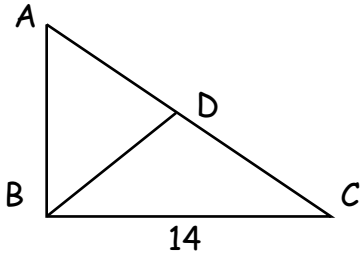
25) What is the area of the triangle shown? (3) *1999-WO6-3*



26) In the figure shown, AC = CD = DE = EB, and AE = $4\sqrt{5}$. What is the area of Δ ADB? (3) *1999-WO7-8*

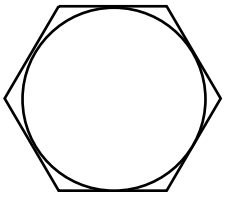


27) In the diagram, AB = AD = CD = BD. Find the area of ΔABD. Round to the nearest whole number. (3) *1999-WO8-4*



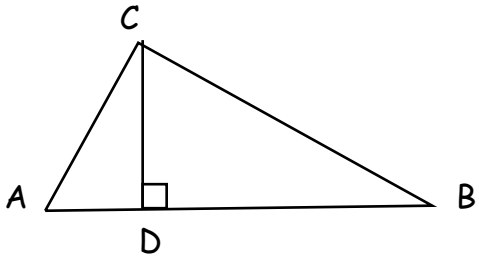
28) The exterior wall of a building forms a right angle with the ground at its base. A 25-foot ladder is placed against the wall so that the foot of the ladder is 7 feet from the base of the wall. The ladder slips and its upper end slides 4-feet down the wall. How many feet did the foot of the ladder slide along the ground? (3) *1999-WO8-6*

29) How many square inches are in the area of a circle inscribed in a regular hexagon with side length 12 inches? Express your answer in terms of π. (3) 2000-WU7-2 (3)

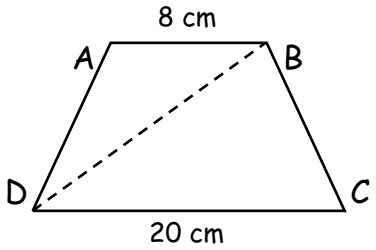


30) The measures of the three angles of a triangle form an arithmetic sequence. If the smallest angle measures 45°, what is the measure of the largest angle? 2000-WU8-5 (1)

31) In $\triangle ABC$, D is a point on $\overline{AB}$ such that CD = 6 and DB = 8. If $\angle CAD = \angle BCD$, what is the perimeter of $\triangle ACD$? (3) *2000-WU8-4*



32) For trapezoid ABCD, $\overline{AB} \parallel \overline{CD}$, AB = 8 cm and CD = 20 cm. The area of triangle BCD is 100 cm^2. What is the area of trapezoid ABCD? (2) *2003-WO2-7*

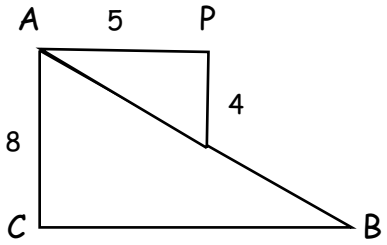


33) What is the length of the longest side of a rectangle whose area is 108 cm^2 and whose perimeter is 42 cm? (1) *2000-WU2-9*

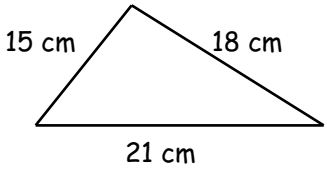
34) The length of one side of a square is increased by 2 and its other side is decreased by 2. What is the difference in areas of the original rectangle and the new rectangle? 2000-WU6-2 (3)

35) Chris wants to tile a rectangular floor with congruent square tiles. Blue tiles will form the border, and white tiles will cover the interior. The number of blue tiles will equal the number of white tiles. What is the maximum area that can be tiled? 2000-WU11-1 (3)

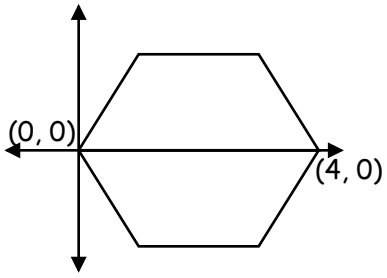
36) In the figure, AP || BC. What is the perimeter of $\triangle ABC$? Round to the nearest whole number. 2000-WU11-5 (2)



37) Find the area of the triangle shown. Express in simplest radical form (3) *2000-WU12-2*



38) What is the number of units in the perimeter of the regular hexagon shown here? (3) *2003-WU5-1*



39) The shortest side of a scalene triangle measures 6 m and the longest side of the triangle measures 14 m. The third side is an integral numbers of meters in its length. What is the difference between the maximum and minimum possible perimeters of the triangle? *2000-WU13-8* (2)

40) What is the number of degrees in the acute angle measure between the hands of a clock at 4:30? *2000-WU18-7* (2)

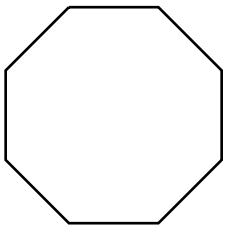
41) What is the number of degrees in the interior angle measure of a regular decagon? *2000-WU18-8* (2)

42) When spelled with capital letters, each letter in HAWAII has vertical symmetry. Find the name of the state whose letters all have vertical and horizontal symmetry. 2000-WO1-1 (1)

43) A rectangular room has a perimeter of 42 feet and an area of 108 square feet. What is the length of the shorter side? (2) *2003-WU7-10*

44) 36 fence posts were used to fence a rectangular plot. One post was placed at each corner, and the distance between adjacent posts was 5 meters. What is the greatest possible area of the plot? 2000-WO4-10 (3)

45) The largest square possible is inscribed in a regular octagon. The longest diagonal of the octagon is 20 cm. What is the area of the square? 2000-WO8-8 (2)

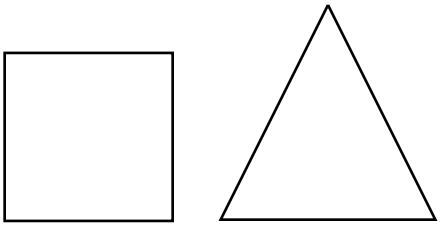


46) A rectangular tabletop is tiled with a 3 by 5 array of small tiles. The area of each tile is 36 square inches. What is the perimeter of the tabletop? (1) *2003-WU9-5*

47) The ratio of the length of the base to the length of the height of a triangle is 2:3. The area of the triangle is 81 cm^2. What is the height of the triangle? Express in simplest radical form. (3) *2003-WU1O-6*

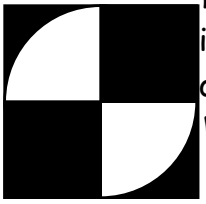
48) The total area of four squares, each with whole-number side measurements, is 23 square inches. What is the positive difference between the perimeter of the largest square and the perimeter of the smallest square? (2) *2003-WU10-9*

49) This square and equilateral triangle each have a perimeter of 36. What is the positive difference in their areas? Round to the nearest whole number. (3) *2003-WO5-8*



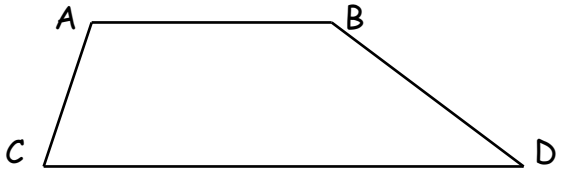
50) A weather balloon rises vertically at a rate of six feet per second. The wind is blowing east at a rate of 8.56 miles per hour. There are 5280 feet in one mile. After twenty minutes, how many miles from the launch point is the balloon? Round to the nearest hundredth. (3) *2003-WO7-6*

51) A dart is randomly thrown and lands within the boundary of a 6-foot by 6-foot square. The unshaded regions are each a quarter of an inscribed circle. What is the probability that the dart lands in one of the shaded regions? Express in terms of π. (3) *2003-WU15-3*

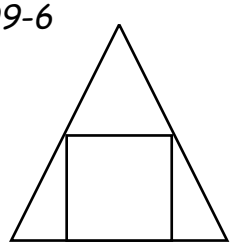


52) A square blanket measuring x by x feet was folded in half, folded in half again and finally folded in half one last time. After these three successive folds, without ever unfolding, the blanket covers an area of 8 square feet. What is the value of x? (2) *2003-WU17-2*

53) In the trapezoid ABCD, $\overline{AB}$ || $\overline{CD}$, AB = ½CD, and point P lies on $\overline{AB}$. What is the ratio of the area of triangle DPC to the area of trapezoid ABCD? (3) *2003-WU18-2*



54) A square with side length s is inscribed as shown in an equilateral triangle with side length t. What is the ratio t:s? Express as a decimal to the nearest thousandth. (3) *2003-WO9-6*



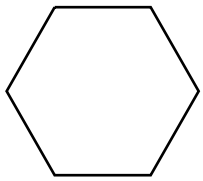
55) How many different isosceles triangles have integer side lengths and perimeter of 81 units? (2) *2002-WU4-3*

56) The measures of the three angles of a triangle are in a ratio of 4:5:6. What is the measure of the greatest supplement of these three angles? (2) *2002-WU8-9*

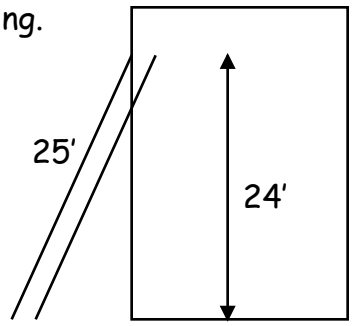
57) The perimeter of a rectangle is 112 and its area is 640. What is the product of its diagonals? (3) *2002-WO4-6*

58) The ratio of the length to the width of a rectangle is 12 to 5 and the area of the rectangle is 540 square units. What is the length of the rectangle? (3) *2002-WU9-1*

59) How many lines of symmetry does a regular hexagon have? (2) *2002-WU10-6*



60) A 25-foot ladder reaches 24 feet up the side of a building. Then the top of the ladder slides down 4 feet. How many additional feet does the bottom of the ladder slide out from the base of the building? (3) *2002-WU12-6*

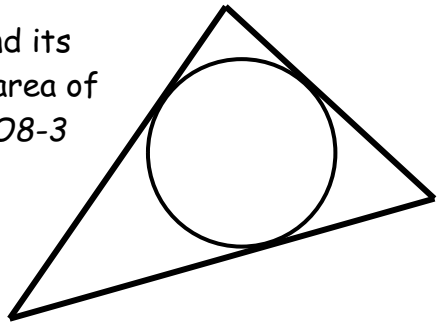


61) One angle of a triangle has a measure of 70 degrees. The other two angles have degree measures in a ratio of 5 to 6. What is the sum of the measures of the two larger angles? (2) *2002-WU5-7*

62) A triangle has sides of integer lengths 3, 6 and x. For how many values of x will the triangle by acute? (3) *2002-WO6-7*

63) One leg of a right triangle is two meters longer than twice the length of the other leg. The hypotenuse is eight meters longer than the shorter of the two legs. What is the perimeter of the triangle? (3) *2002-WU13-6*

64) The area of the triangle shown is 336 square units and its perimeter is 84 units. How many square units are in the area of the inscribed circle? Express in terms of π. (3) *2002-WO8-3*



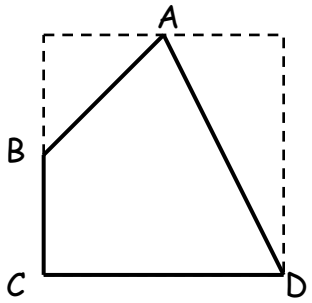
65) A formula for the area of a triangle is:

$$\sqrt{s(s-a)(s-b)(s-c)}$$

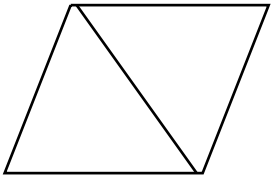
Where s is the semi-perimeter (half perimeter) and a, b, c are side lengths. What integer is closest to the area of a triangle whose sides are 5, 6 and 7? (2) *2002-WU17-10*

66) Rebecca celebrated her graduation by going on a hot air balloon ride. The wind first blew the balloon $\frac{1}{2}$ mile due east. Then the balloon was blown $\frac{3}{4}$ mile southwest. Finally, it was blown $\frac{1}{2}$ mile north and then landed. How many miles did they land from the point where the balloon was launched? Express the answer as a decimal to the nearest hundredth. (3) *2002-WO9-8*

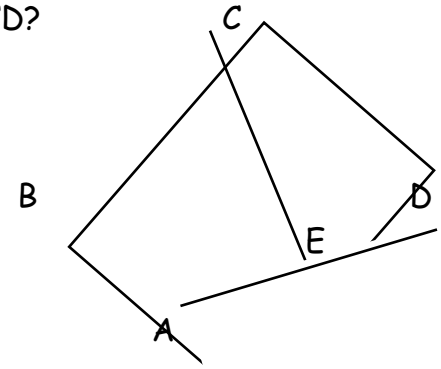
67) Quadrilateral ABCD has been formed from a square. Points A and B are midpoints of the sides of the square. What is the ratio of the area of ABCD to the area of the original square? (3) *1997-WU3-7*



68) When the short diagonal of an equilateral triangle is drawn, two equilateral triangles are formed. If each side of the rhombus is 6 inches long, what is the length of the other diagonal? Express in simplest radical form. (3) *1997-WU3-10*



69) In the diagram, $\overline{AB} \perp \overline{BC}$ and $\overline{BC} \perp \overline{CD}$. AB = 8", BC = 12" and CD = 16". E is the midpoint of $\overline{AD}$. What is the area of ABCD? (3) *1997-WU8-2*

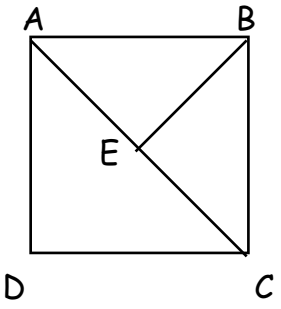


70) A rectangle is 10 inches long and 6 inches wide. P, Q, R and S are midpoints of the sides of the rectangle. Determine the area of quadrilateral PQRS. (1) *1997-WU9-8*

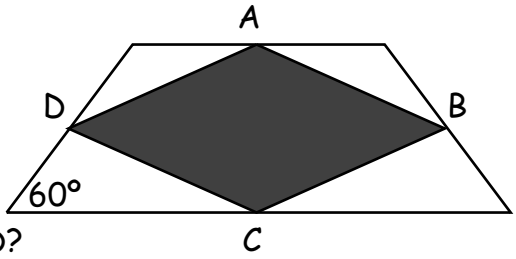
71) The area of a rectangle is 240 ft^2. The ratio of the length to the width of the rectangle is 12:5. What is the length of the diagonal of the rectangle? (3) *1997-WU13-2*

72) One leg of a right triangle is 6 inches long. The hypotenuse is 12 inches long. Find the area of the triangle. Express the answer in simplest radical form. (3) *1997-WU13-8*

73) Square ABCD has sides of length 6 in. $\overline{BE}$ bisects $\overline{AC}$. Find the length of BE. Express the answer in simplest radical form. (3) *1997-WU14-2*



74) The lengths of the bases of the isosceles trapezoid shown are 6 inches and 10 inches, and one base angle measures 60°. If A, B, C and D are the midpoints of the sides of the given trapezoid, what is the area of quadrilateral ABCD? Express the answer in simplest radical form. (3) *1997-WU15-3*



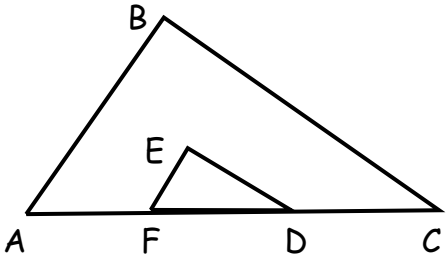
75) An isosceles triangle with an area of 192 in^2 has a height equal to 2/3 of the length of its base. What is the perimeter of the isosceles triangle? (2) *1997-WU16-4*

76) A rectangular garden is separated into four smaller rectangles. The area of rectangle B is 60 m^2, the area of square C is 16 m^2 and the area of rectangle D is 48 m^2. What is the perimeter of the garden? (2) *1997-WU17-3*

A	B
C	D

77) What is the number of degrees in the acute angle made by the hands of a clock at 7:24? (2) *1997-WU18-3*

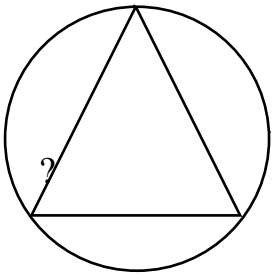
78) AF = FD = DC = 5 inches. A, F, D and C are collinear. <B = <E = 90°. EF = 3 inches, and ED = 4 inches. AB || EF and BC || ED. What is the area of Polygon ABCDEF? (2) *1997-WU18-7*



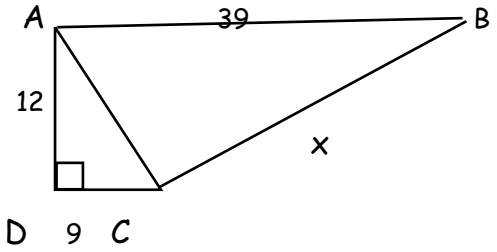
79) A triangle has integral length sides of 8 feet, 13 feet and n feet. What is the median of all possible values of n? (3) *1997-WO4-7*

80) A square and an equilateral triangle have equal perimeters. The area of the triangle is $\sqrt{3}$ square inches. What is the length of the diagonal of the square? Express as a decimal to the nearest tenth. (3) *1997-WO9-1*

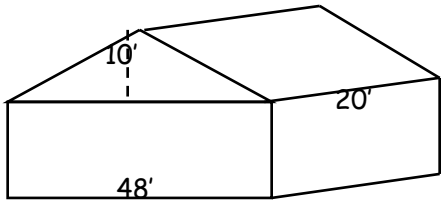
81) An equilateral triangle and a square are inscribed in a circle as shown. $\triangle ABC$ is isosceles. The triangle and the square share a common vertex. What is the measure of the angle indicated by the question mark? (3) *1997-WO9-6*



82) What is the perimeter of quadrilateral ABCD? <D = < ACB = 90° (2) *2001-WU5-2*



83) A rancher is planning to build a new barn for their horses and want to cover the roof with tile. The roof will have a slope of 5/12 known as a 5-12 pitch. If each tile covers an 8 x 12 inch region, what is the number of tiles needed to cover the portion of the roof shown? (3) *1997-WU9-4*



84) A rectangle has integer side lengths and its area is equal to 24 square units. The length of each side of the rectangle is increased by one unit. What is the largest possible number of square units in the area of the new rectangle? (1) *1997-WU9-5*

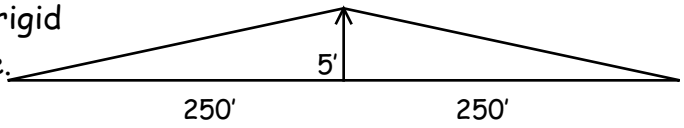
85) A regular hexagon ABCDEF has each side equal to 1 unit. What is the area of quadrilateral ABCE? Express the answer in simplest radical form. (3) *1997-WU10-7*

86) A square has area of 144 cm^2. What is the length of each diagonal? Express the answer in simplest radical form. (3) *1997-WU11-1*

87) Given six distinct points on a line, how many distinct segments can be named using the six points? (2) *1997-WU14-5*

88) The hypotenuse of a right triangle is twice the length of one leg of the triangle. The length of the other leg is 12 cm. What is the area of the right triangle? Express your answer in simplest radical form. (3) *1997-WO5-3*

89) A flat steel bridge is built from two rigid 250-foot long beams joined at the middle. On a hot day, the beams expand equally causing the joint to rise 5 feet. By how many inches did one of the beams expand? Round to the nearest tenth. (3) *1997-WO8-2*

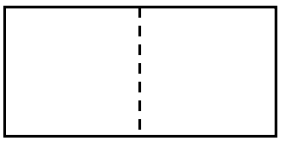


90) For how many positive integers p does there exist a triangle with sides of length $3p - 1$, $3p$ and $p^2 + 1$? (2) *1997-WO9-6*

91) The lengths of two sides of a triangle are five inches and seven inches. What is the positive difference between the greatest possible whole number length and the smallest whole number length of the third side? (1) *1996-WU3-3*

92) A regular pentagon is rotated n° clockwise around its center until it coincides with its original image. What is the smallest positive value of n? (1) *1996-WU3-6*

93) The rectangle pictured consists of two squares placed side by side. The perimeter of the rectangle is 60 cm. What is the area of the rectangle? (1) *1996-WU5-7*

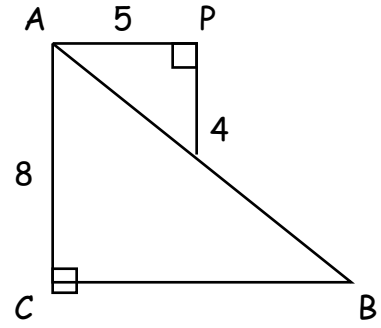


94) Point B is the midpoint of $\overline{PQ}$. $\overline{PQ}$ is 8 cm longer than $\overline{PB}$. What is the length of $\overline{QB}$? (1) *1996-WU5-9*

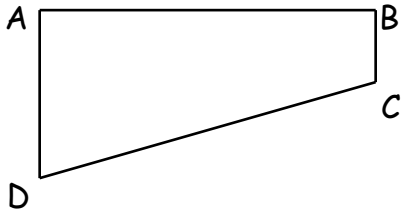
95) The measure of one angle of an isosceles triangle is twice another. What is the positive difference between the greatest possible degree measure and the least possible degree measure of the remaining angle? (2) *1996-WU9-8*

96) The lengths of the legs of a right triangle are in the ratio of $4 : 2\sqrt{5}$. What is the ratio of the length of the shorter leg of the triangle to the length of the hypotenuse? (3) *1996-WU10-2*

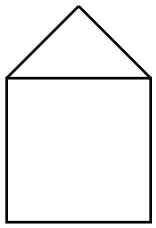
97) In the figure, what is the perimeter of $\triangle ABC$ if $\overline{AP} \parallel \overline{BC}$? Express the answer in simplest radical form. (3) *1996-WU16-5*



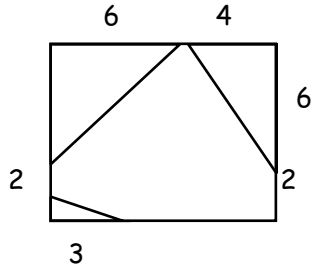
98) $\overline{AD} \perp \overline{AB}$, $\overline{BC} \perp \overline{AB}$, AD = 8, BC = 3 and AB = 12. What is the perimeter of ABCD? (2) *1996-WU17-3*



99) A pentagon is drawn by placing an isosceles triangle on top of a square as pictured. What percent of the area of the pentagon is the area of the right triangle? (3) *1996-WU16-5*

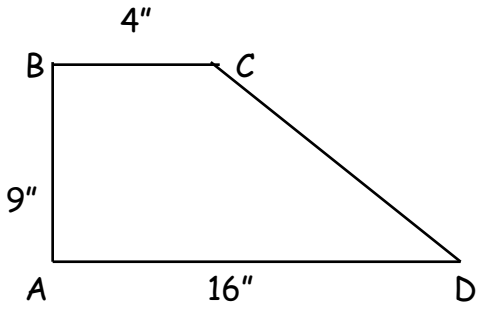


100) The outer figure in the diagram is a rectangle. What is the area of the shaded polygon? Round to the nearest tenth. (3) *1996-WO8-2*



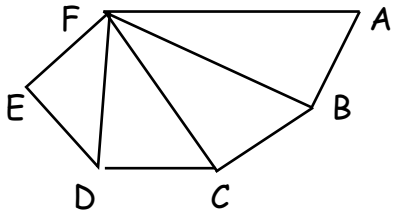
101) Find the length of segment CD in trapezoid ABCD shown.

(2) *1993-WU3-7*



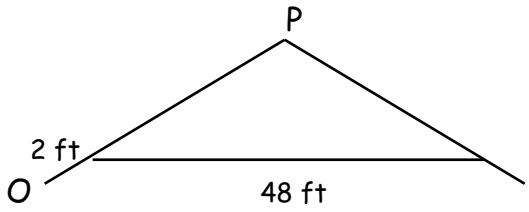
102) A square table can seat four people. If two tables are joined together 6 people can be seated. How many people can be seated if twenty such tables are joined? (1) *1993-WU11-1*

103) In the diagram all of the triangles are right triangles and all the segments AB, BC, CD, DE and EF have length 3. What is the length of segment AF? (3) *1993-WO3-9*



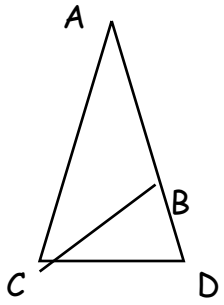
104) A traveler went 10 miles south, 7 miles east, 2 miles south and 2 miles west. How many miles is the traveler from the starting point? (2) *1993-WO6-4*

105) A 48-foot wide building has a roof that rises 4 feet for each 12-foot horizontal change. If the roof has a 2-foot overhang, what is the distance from the peak, P, to the edge of the overhang, O? Express in simplest radical form. (3) *1993-WO7-5*



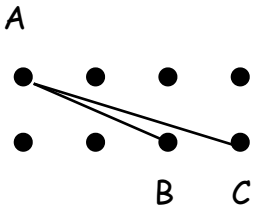
106) The length of the diagonals of a square is 10 inches. Find the perimeter of the square. Express in simplest radical form. (3) *1993-WU7-5*

107) $\overline{CB}$ bisects <ACD. $\overline{AB} \cong \overline{BC} \cong \overline{CD}$. Find m<A (2) *1995-WU15-9*

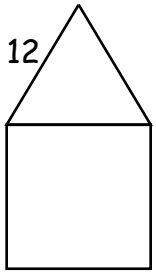


108) Mr. Macgregor plants cabbages in a square grid so that the number of rows and columns are equal. He increases the size of his cabbage patch equally in rows and columns to create a new patch, which could grow 211 additional cabbages at the same spacing. How many cabbages did he plant along one row in the old garden? (2) *1995-WU19-6*

109) The dots in the grid are equally spaced horizontally and vertically, and $\overline{AB} = 3\sqrt{5}$. Find AC. Express the answer in simplest Radical form. (3) *1995-WU19-7*



110) An equilateral triangle with side 12 is placed on top of a square so they share an edge to form a pentagon. What is the length of the longest diagonal of the pentagon. Round to the nearest hundredth. (3) *1995-WO9-8*



111) A square piece of paper is folded along a diagonal to create a triangle with an area of 144.5 cm^2. What is the perimeter of the piece of paper? (3) *1995-WO10-4*

Answers

1) 23

2) 24 ft

3) 3

4) 100 in

5) $54\sqrt{3}$ u^2

6) $4\sqrt{3}$ cm

7) 10.06

8) 60

9) 1:4

10) $8\sqrt{2}$ in

11) 84

12) 2/5

13) 36º

14) 0

15) 5:9

16) 30

17) 72π

18) 9

19) 42π

20) 16π

21) ≈95%

22) 47 cents

23) 21.5%

24) 44.2 cm^2

25) 234 cm^2

26) 16

27) 28

28) 8 ft

29) 108π

30) 75º

31) 18

32) 140 cm^2

33) 12 cm

34) 4

35) 60

36) 31

37) $54\sqrt{6}$ cm^2

38) 12

39) 4 meters

40) 45º

41) 144º

42) OHIO

43) 9 ft

44) 2,025 m^2

45) 200 cm^2

46) 96 in

47) $9\sqrt{3}$

48) 8 in

49) 19

50) 3.16 mi

51) $\frac{8-\pi}{8}$

52) 8

53) 2/3

54) 2.155

55) 20

56) 132º

57) 1,856

58) 36

59) 6

60) 8 ft

61) 130

62) 1

63) 30

64) 64π

65) 15

66) .04 mi

67) 5:8

68) $6\sqrt{3}$ in

69) 144 in^2

70) 30 in^2

71) 26 ft

72) $18\sqrt{3}$ in^2

73) $3\sqrt{2}$ in

74) $8\sqrt{3}$

75) 64 in

76) 50 m

77) 78º

78) 48 in^2

79) 13

80) 2.1 in

81) 75º

82) 96

83) 780

84) 50 u^2

85) $\sqrt{3}$ u^2

86) $12\sqrt{2}$ cm

87) 16

88) $24\sqrt{3}$ cm^2

89) 0.6”

90) 5

91) 8”

92) 72º

93) 200 cm^2

94) 8 cm

95) 27º

96) 2:3

97) $2\sqrt{41} + 18$

98) 36

99) 20%

100) 51.1

101) 15”

102) 42

103) $3\sqrt{5}$

104) 13 miles

105) $2 + 8\sqrt{10}$ ft

106) $20\sqrt{2}$ in

107) 36º

108) 105

109) $3\sqrt{10}$

110) 23.18

111) 68 cm