

# 8TH GRADE

## MATH STANDARDS GUIDANCE

*WI Math Standards*

Domain: The Number System (8.NS)		
CLUSTER	STANDARD	EXAMPLES
A. Know that there are numbers that are not rational and approximate them by rational numbers.	M.8.NS.A.1: Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and use patterns to rewrite a decimal expansion that repeats into a rational number.	
	M.8.NS.A.2: Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line, and estimate the value of expressions (e.g., $2$ ).	For example, by truncating the decimal expansion of $\sqrt{2}$ , show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5 and explain how to continue on to get better approximations.
Domain: The Expressions and Equations (8.EE)		
CLUSTER	STANDARD	EXAMPLES
A. Work with radicals and integer exponents.	M.8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions.	For example, $3^2 \times 3^{-5} = 3^{-3} = 1/33 = 1/27$ .
	M.8.EE.A.2: Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$ , where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	
	M.8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other.	For example, estimate the population of the United States as $3 \times 10^8$ and the population of the world as $7 \times 10^9$ and determine that the world population is more than 20 times larger.
	M.8.EE.A.4: Use technology to interpret and perform operations with numbers expressed in scientific notation. Choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading).	
B. Understand the connections between proportional relationships, lines, and equations.	M.8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.	For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
	M.8.EE.B.6: Use similar triangles to explain why the slope $m$ is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $b$ .	
C. Analyze and solve linear equations and pairs of simultaneous equations. (M)	M.8.EE.C.7: Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into equivalent forms. b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	
	M.8.EE.C.8: Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables by graphing and analyzing tables. Solve simple cases represented in algebraic symbols by inspection. c. Solve real-world and mathematical problems leading to two linear equations in two variables.	(b) For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6. (c) For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.
Domain: Functions (8.F)		
CLUSTER	STANDARD	EXAMPLES
A. Define, evaluate and compare functions.	M.8.F.A.1: Understand that a function is a rule that assigns to each input exactly one output. The graph of a numerically valued function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8.	
	M.8.F.A.2: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
	M.8.F.A.3: Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.	For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4), and (3,9), which are not on a straight line.
B. Use functions to model relationships between quantities. (M)	M.8.F.B.4: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two $(x, y)$ values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values.	
	M.8.F.B.5: Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear, continuous or discrete). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	

<b>Domain: Geometry (8.G)</b>		
CLUSTER	STANDARD	EXAMPLES
A. Understand congruence and similarity using physical models, transparencies, or geometry software.	M.8.G.A.1: Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	
	M.8.G.A.2: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	
	M.8.G.A.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	
	M.8.G.A.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	
	M.8.G.A.5: Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line and give an argument in terms of transversals why this is so.
B. Understand and apply the Pythagorean Theorem. (M)	M.8.G.B.6: Justify the relationship between the lengths of the legs and the length of the hypotenuse of a right triangle and the converse of the Pythagorean Theorem.	
	M.8.G.B.7: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	
	M.8.G.B.8: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	
C. Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. (M)	M.8.G.C.9: Know the relationship among the formulas for the volumes of cones, cylinders, and spheres (given the same height and diameter) and use them to solve real-world and mathematical problems.	
<b>Domain: Statistics and Probability (8.SP)</b>		
CLUSTER	STANDARD	EXAMPLES
A. Investigate patterns of association in bivariate data. (M)	M.8.SP.A.1: Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	
	M.8.SP.A.2: Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	
	M.8.SP.A.3: Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.	For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.
	M.8.SP.A.4: Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.	For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?
<b>Math Practice 1: Make sense of problems and persevere in solving them. (M)</b>		
CLUSTER	STANDARD	
	6-8 Mathematically proficient middle school students set out to understand a problem and then look for entry points to its solution. Students identify questions to ask and make observations about the situation by using noticing and attending to aspects of the problem that look familiar. Students make assumptions where needed to make the problem more clearly defined. They analyze problem conditions and goals, translating, for example, verbal descriptions into equations, diagrams, or graphs as part of the process. They consider analogous problems, and try special cases and simpler forms of the original in order to gain insight into its solution. For example, to understand why a 20% discount followed by a 20% markup does not return an item to its original price, they might translate the situation into a tape diagram or a general equation; or they might first consider the situation for an item priced at \$100. Mathematically proficient students can explain how alternate representations of problem conditions relate to each other. For example, they can identify connections between the solution to a word problem that uses only arithmetic and a solution that uses variables and algebra; and they can navigate among verbal descriptions, tables, graphs, and equations representing linear relationships to gain insights into the role played by constant rate of change. Mathematically proficient students check their approach, continually asking themselves “Does this approach make sense?” and “Can I solve the problem in a different way?” Students ask themselves these types of questions as a way to persevere through problem solving. While working on a problem, they monitor and evaluate their progress and change course if necessary. Students will reflect and revise their solution as needed. They can understand the approaches of others to solving complex problems and compare approaches.	

Math Practice 2: Reason abstractly and quantitatively.	<p>Mathematically proficient middle school students make sense of quantities and relationships in problem situations. For example, they can apply ratio reasoning to convert measurement units and proportional relationships to solve percent problems. They represent problem situations using symbols and then manipulate those symbols in search of a solution (decontextualize). They can, for example, solve problems involving unit rates by representing the situations in equation form. Mathematically proficient students also pause as needed during problem solving to validate the meaning of the symbols involved. In the process, they can look back at the applicable units of measure to clarify or inform solution pathways (contextualize). Students can integrate quantitative information and concepts expressed in text and visual formats. Students can examine the constant and coefficient used in a linear function and express the meaning of those numbers related to a contextual situation. They can work with the function in different representations, such as a graph, keeping in mind the slope and vertical. Wisconsin Standards for Mathematics 93 intercept have meaning related to the context. Quantitative reasoning also entails knowing and flexibly using different properties of operations and objects. For example, in middle school, students use properties of operations to generate equivalent expressions and use the number line to understand multiplication and division of rational numbers.</p>
Math Practice 3: Construct viable arguments, and appreciate and critique the reasoning of others.	<p>Mathematically proficient middle school students understand and use assumptions, definitions, and previously established results in constructing verbal and written arguments. They understand the importance of making and exploring the validity of conjectures. They can recognize and appreciate the use of counterexamples. For example, using numerical counterexamples to identify common errors in algebraic manipulation, such as thinking that <math>5 - 2x</math> is equivalent to <math>3x</math>. Conversely, given a pair of equivalent algebraic expressions, they can show that the two expressions name the same number regardless of which value is substituted into them by showing which properties of operations can be applied to transform one expression into the other. They can explain and justify their conclusions to others using numerals, symbols, and visuals. They also reason inductively about data, making plausible arguments that take into account the context from which the data arose. For example, they might argue that the great variability of heights in their class is explained by growth spurts, and that the small variability of ages is explained by school admission policies.</p> <p>While communicating their own mathematical ideas is important, middle school students also learn to be open to others' mathematical ideas. They appreciate a different perspective or approach to a problem and learn how to respond to those ideas, respecting the reasoning of others (Gutiérrez 2017, 17-18). Together, students make sense of the mathematics, asking helpful questions such as "How did you get that?" "Why is that true?" and "Does that always work?" that clarify or deepen everyone's understanding. Mathematically proficient students are able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and if there is a flaw in an argument, explain what it is. Students engage in collaborative discussions, drawing on evidence from problem texts and arguments of others, follow conventions for collegial discussions, and qualify their own views in light of evidence presented. They can present their findings and results to a given audience through a variety of formats such as posters, whiteboards, and interactive materials.</p>
Math Practice 4: Model with mathematics.	<p>"In the course of a student's mathematics education, the word 'model' is used in many ways. Several of these, such as manipulatives, demonstration, role modeling, and conceptual models of mathematics, are valuable tools for teaching and learning. However, they are different from the practice of mathematical modeling. Mathematical modeling, both in the workplace and in school, uses mathematics to answer big, messy, reality-based questions" (Bliss and Libertini 2016, 7).</p> <p>Mathematically proficient middle school students formulate their own problems that emerge from natural circumstances as they mathematize the world around them. They can identify the mathematical elements of a situation and generate questions that can be addressed using mathematics (e.g., noticing and wondering). Middle school students can see a complicated problem and understand how that problem contains smaller problems to be solved. They are comfortable making assumptions as they decide "what matters". Mathematically Wisconsin Standards for Mathematics 94 proficient middle students understand that there are multiple solutions to a modeling problem so they are working to find a solution rather than the solution. Students make judgements about what matters and assess the quality of their solution (Bliss and Libertini 2016, 10). They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving their mathematical modeling approach if it has not served its purpose. As they decontextualize the situation and represent it mathematically, they are also reasoning abstractly (Math Practice 2).</p> <p>In the middle school grades, students encounter mathematical opportunities each and every day at school and at home. Mathematically proficient middle school students might consider how to plan a route to get to school with their friends. Students might then need to make assumptions about travel time and when to leave the house. In the morning they implement their plan and revise it by changing the departure time or including additional friends to the route. As a classroom, students might plan a fundraising event involving selling popcorn after school. In this example, sometimes students will be engaged in only a part of the modeling cycle such as making assumptions about how much to charge or how much popcorn to make (Godbold, Malkevitch, Teague, and van der Kooij 2016, 50). Note: Although physical objects and drawings can be used to model a situation, using these tools absent a contextual situation is not an example of Math Practice 4. For example, drawing an area model to illustrate the distributive property in <math>4(t + s) = 4t + 4s</math> would not be an example of Math Practice 4. Math Practice 4 is about applying math to a problem in context.</p>
Math Practice 5: Use appropriate tools strategically.	<p>Mathematically proficient middle school students strategically consider the available tools when solving a mathematical problem and while exploring a mathematical relationship. These tools might include pencil and paper, concrete models, a ruler, a protractor, a dynamic graphing tool, a spreadsheet, a statistical package, or dynamic geometry software. Proficient students make sound decisions about when each of these tools might be helpful, recognizing both the insights to be gained and their limitations. For example, they use estimation to check reasonableness; graph functions defined by expressions to picture the way one quantity depends on another; use algebra tiles to see how the properties of operations familiar from the elementary grades continue to apply to algebraic expressions; use an area model to visualize multiplication of rational numbers; use dynamic graphing tools to approximate solutions to systems of equations; use spreadsheets to analyze data sets of realistic size; or use dynamic geometry software to discover properties of parallelograms. Students are also strategic about when not to use tools, such as by simplifying an expression before substituting values into it (Math Practice 7), or rounding the inputs to a calculation and calculating on paper when an approximate answer is enough (Math Practice 6). When making mathematical models, students know that technology can enable them to visualize the results of their assumptions, to explore consequences, and to compare predictions with data (Math Practice 4). Mathematically proficient students are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems.</p>

Math Practice 6: Attend to precision.	Mathematically proficient middle school students communicate precisely to others both verbally and in writing. They present claims and findings, emphasizing salient points in a focused, coherent manner with relevant evidence, sound and valid reasoning, well-chosen details, and precise language. They use clear definitions in discussion with others and in their own reasoning and determine the meaning of symbols, terms, and phrases as used in specific mathematical contexts. For example, they can use the definition of a rational number to explain why $\sqrt{2}$ is irrational. Middle school students and describe congruence and similarity in terms of transformations in the plane. They decide which parts of a problem need to be defined by a variable, state the meaning of the symbols, consistently and appropriately, such as independent and dependent variables of a linear equation. They are careful about specifying units of measure, and label axes to display the correct correspondence between quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate to the context. For example, they accurately apply scientific notation to large numbers and use measures of center to describe data sets. In statistics and probability, students must attend to precision in the manner they write their statistical questions, the manner in which they collect their data, and the process they use to develop a simulation. Mathematically proficient middle school students care that an answer is right or reasonable; they attend to precision when they check their work; they solve the problem another way; they make revisions where appropriate.
Math Practice 7: Look for and make use of structure.	Mathematically proficient middle school students look closely to discern a pattern or structure. They might use the structure of the number line to demonstrate that the distance between two rational numbers is the absolute value of their difference, ascertain the relationship between slopes and solution sets of systems of linear equations, and see that the equation $3x = 2y$ represents a proportional relationship with a unit rate of $3/2 = 1.5$ . They might recognize how the Pythagorean Theorem is used to find distances between points in the coordinate plane and identify right triangles that can be used to find the length of a diagonal in a rectangular prism. They also can step back for an overview and shift perspective, as in finding a representation of consecutive numbers that shows all sums of three consecutive whole numbers are divisible by six. They can see complicated things as single objects, such as seeing two successive reflections across parallel lines as a translation along a line perpendicular to the parallel lines or understanding $1.05a$ as an original value, $a$ , plus 5% of that value, $0.05a$ . They can evaluate numeric expressions without combining each term in the order they are given.
Math Practice 8: Look for and express regularity in repeated reasoning.	Mathematically proficient middle school students notice if calculations are repeated, and look for both general methods and general and efficient methods. Working with tables of equivalent ratios, they might deduce the corresponding multiplicative relationships and make generalizations about the relationship to rates. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, students might abstract the equation $(y - 2)/(x - 1) = 3$ . Noticing the regularity with which interior angle sums increase with the number of sides in a polygon might lead them to the general formula for the interior angle sum of an $n$ -gon. As they work to Wisconsin Standards for Mathematics 96 solve a problem, mathematically proficient students maintain oversight of the process while attending to the details. They continually evaluate the reasonableness of their results throughout all stages of the process.