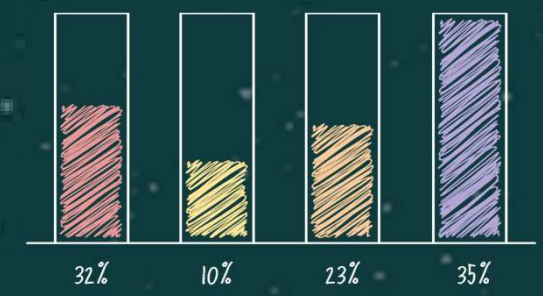
Math Strategies



Use the Key Words to unlock the Operation

- total
- together
- sum
- altogether
- plus
- increase
- join
- add
- more
- how many in all?
- combined
- and
- both
- perimeter

- remains
- left
- minus
- subtract
- take away
- "er" words-fewer, shorter, etc.
- how many more?
- less than, more than
- difference

- quotient
- divided by
- how many per
- each group
- shared
- divide into parts
- how many groups?

- product
- each
- multiply
- times
- twice as many
- three times, four times, etc.
- multiple
- find the area
- per

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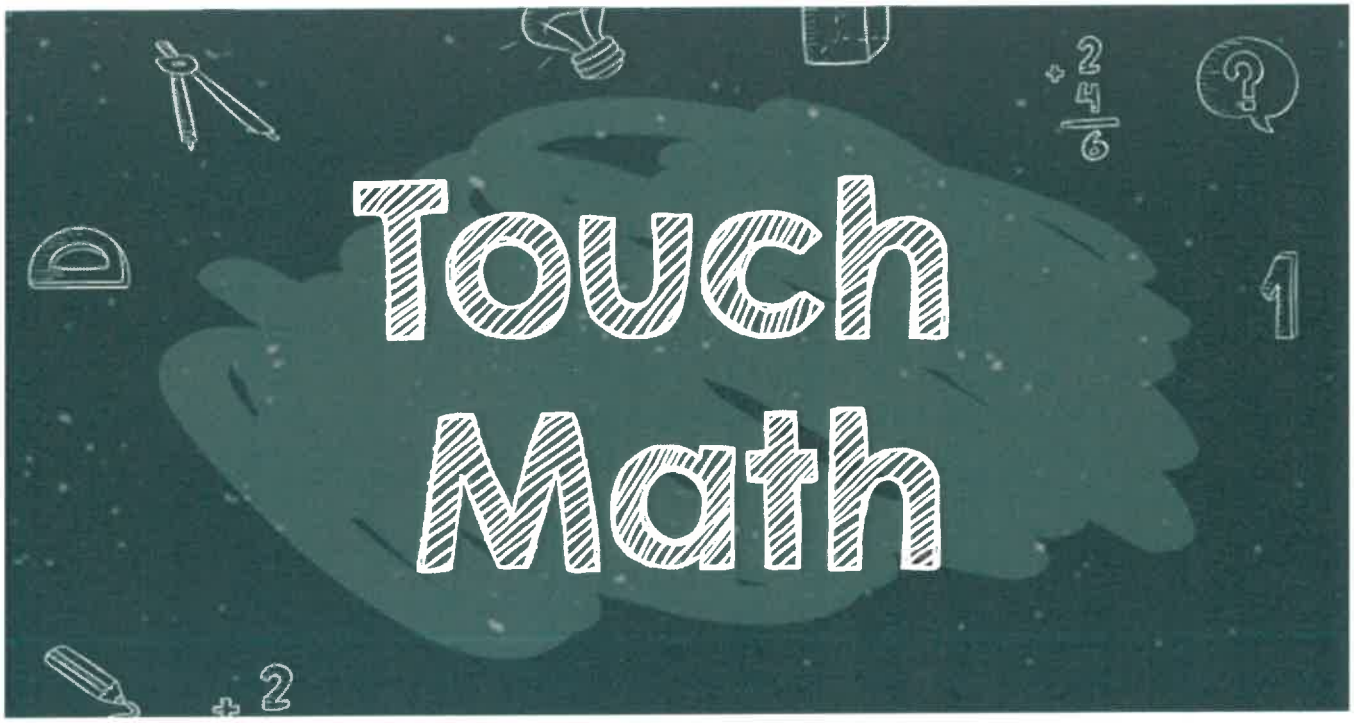
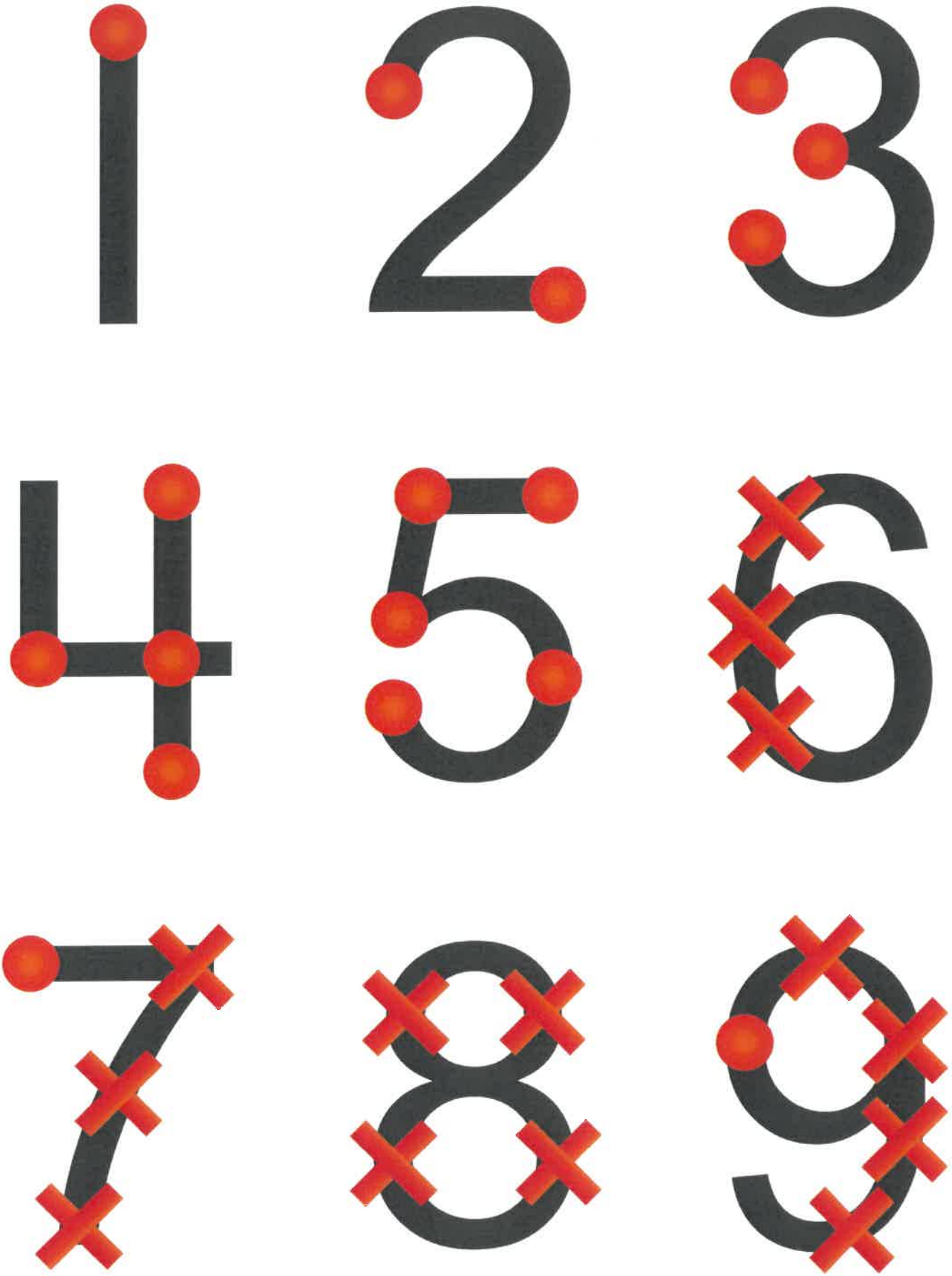
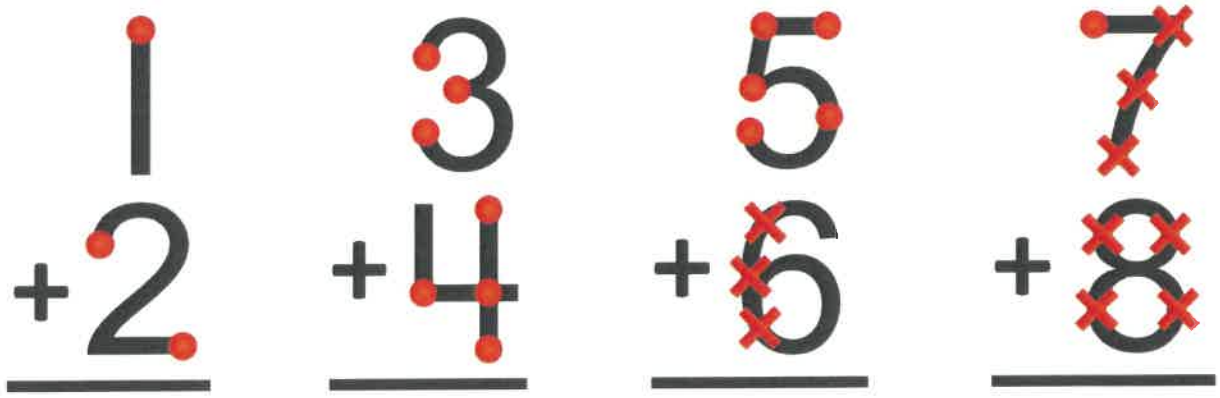

Touch
Math

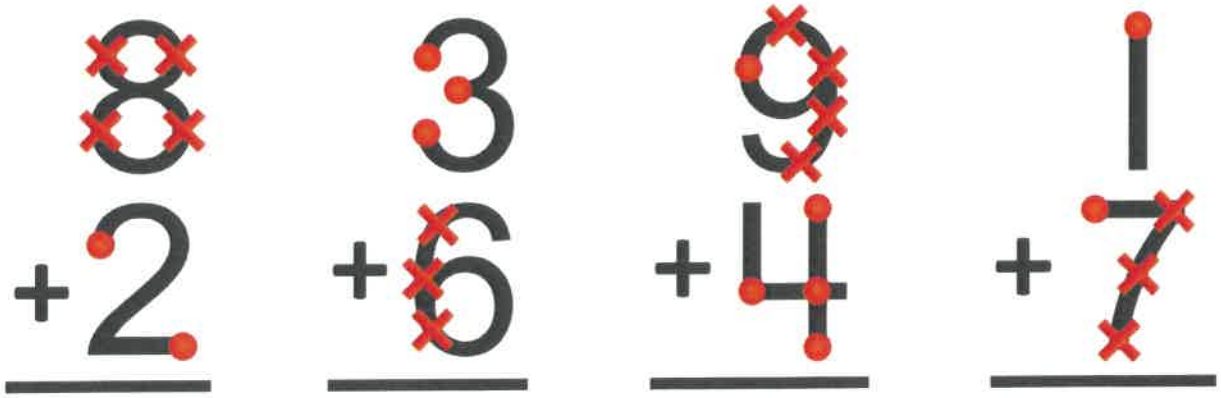

1
2
3
4
5
6
7
8
9

Name_____ Date_____

DIGITDOTS

Addition 1



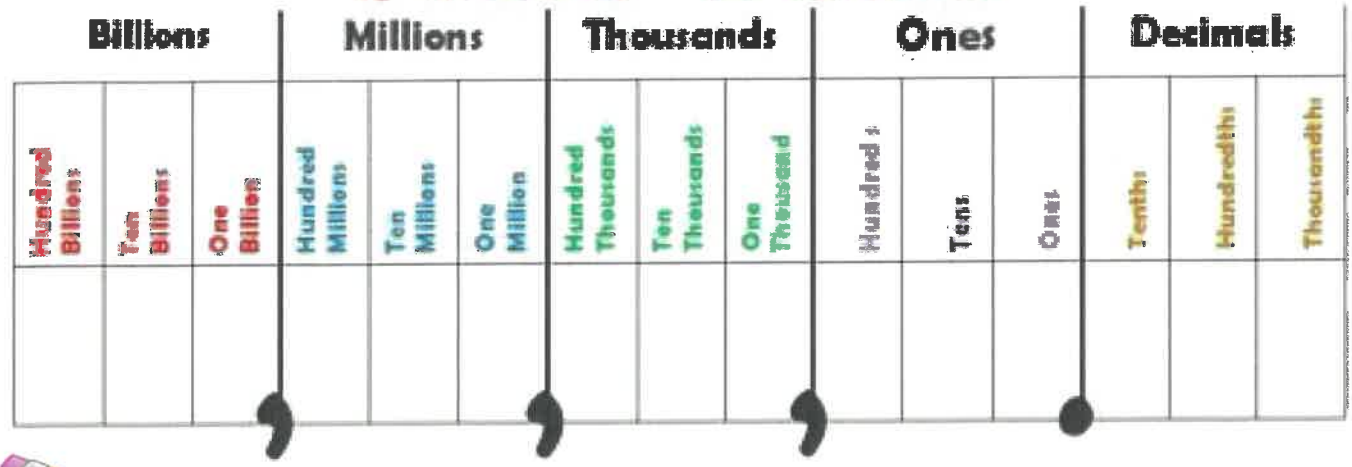


Touch Point Money: Each ● = count by 5s

To count money using touch points, touch each coin at the red dot and count by 5s. When you get to a penny, drag your finger across the penny and count on by 1s.


Place
Value

Place Value



Numbers are grouped in periods or families.

The number families are ones, thousands, millions, billions (and decimals.)
There are three places in each number family: ones, tens, and hundreds.

Numbers can be expressed in 3 different forms:

Standard Form

Numbers written as you would normally see them

7,986,345.62

Word Form

Numbers written in words (as they would be read)

seven million, nine hundred eighty-six thousand, three hundred forty-five, and sixty two hundredths

Expanded Form

Numbers written by adding the value of each number separately.

7,000,000 + 900,000 + 80,000 + 6,000 + 300 + 40 + 5 + 0.6 + 0.02

Standard Form

182

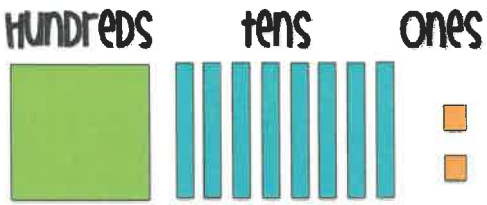
hundreds tens ones

Expanded Form

100+80+2=182

hundreds + tens + ones = number

Base Ten Blocks



Word Form

182

one hundred eighty two

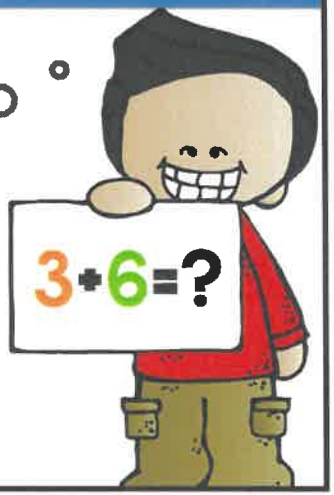

Addition

Count on

Start with the BIGGER number and count on

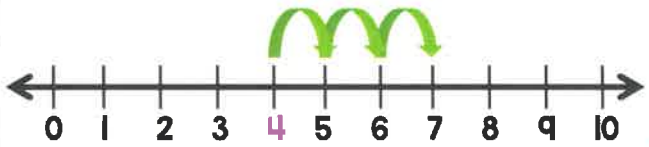
6, 7, 8, 9

3 + 6 = 9

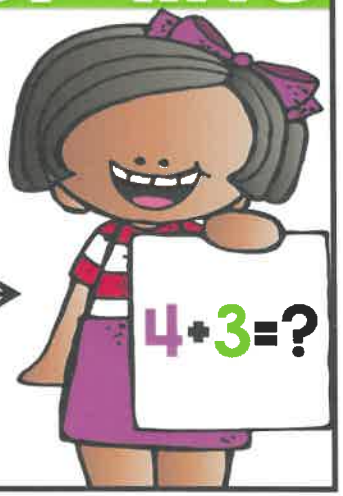


Use a number line

Start with the greater number and count on by jumping to the right

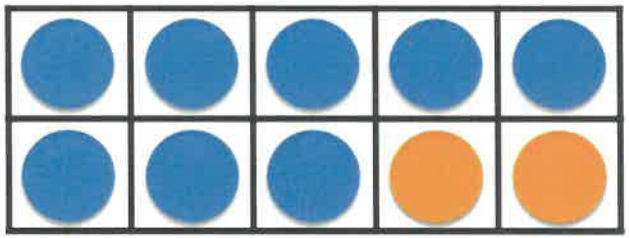


4 + 3 = 7

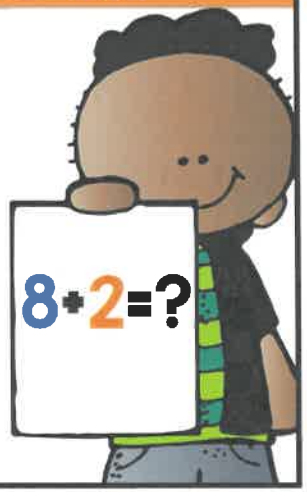


Make a ten

Use numbers that make a group of ten to solve



8 + 2 = 10



Use Tally Marks To Add

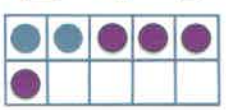
4 + 2 = 6

|||| + || = |||| |

Make tally marks for each number.
How many altogether?

Use a Ten Frame To Add

2 + 4 = 6



Fill the ten frame with dots
for both numbers.
How many altogether?

Add with Friends of Ten

10 + 0 = 10	6 + 4 = 10	2 + 8 = 10
9 + 1 = 10	5 + 5 = 10	1 + 9 = 10
8 + 2 = 10	4 + 6 = 10	0 + 10 = 10
7 + 3 = 10	3 + 7 = 10	

Think about which two
numbers equal ten.

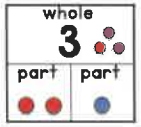
Add : Commutative Rule

If 2 + 1 = 3 Then 1 + 2 = 3

You can add numbers
in any order.

Add with Part Part Whole

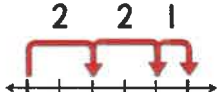
2 + 1 = 3



Add two parts together.
The answer equals the whole.

Add Doubles Plus 1

2 + 2 + 1 = 5



Start with a double. (2 + 2 = 4)
Then add one more. (4 + 1)
How many altogether? (5)

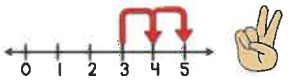
Adding With Doubles

2 + 2 = 4

Add the same number
two times.

Count On To Add

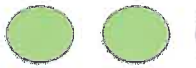
3 + 2 = 5



Start on the first number. (3)
Count forward as many as the
second number. (2)
What number are you on? (5)

Draw A Picture To Add

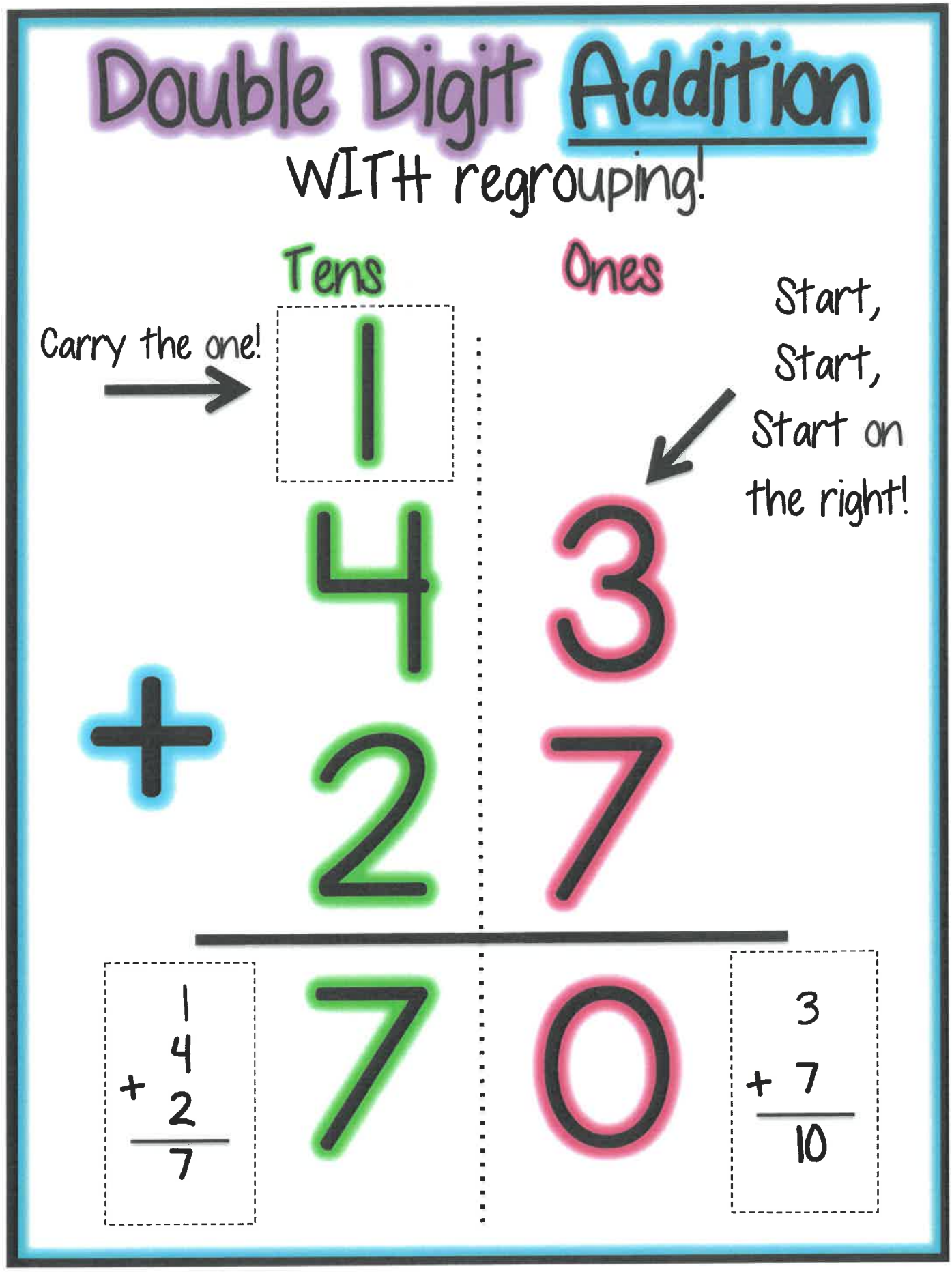
2 + 1 = 3



Draw how many you start with. (2)
Draw how many more are added. (1)
How many altogether? (3)

Double Digit Addition

WITH regrouping!



Addition Strategies

Expanded Form:

116 + 118

(100 + 10 + 6) + (100 + 10 + 8)

100 + 100 = 200

10 + 10 = 20

6 + 8 = 14

200 + 20 + 14 = **234**

Making Friendly Numbers:

116 + 118

+2

116 + 120 = 236

236 − 2 = **234**

Addition Strategies

Compensation:

116 + 118

a. 116 + 2 = 114; 118 − 2 = 120; 114 + 120 = **234**

b. 116 + 4 = 120; 118 − 4 = 114; 120 + 114 = **234**

Making Tens:

116 + 118

a.

(110 + 4 + 2) + (110 + 8)

110 + 110 + (2 + 8) + 4

230 + 4 = **234**

b.

(110 + 6) + (110 + 4 + 4)

110 + 110 + (6 + 4) + 4

110 + 110 + 10 + 4 = **234**

Standard Algorithm:

116 + 118

Step 1: Add the ones
6 + 8 = 14 (Regroup? Yes!)

Step 3: Add the hundreds
100 + 100 = 200

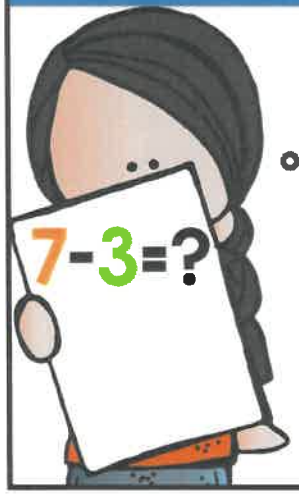
Step 2: Add the tens
10 + 10 + 10 = 30

```
  1
 116
+118
----
```

Step 4: 200 + 30 + 4 = **234**


Subtraction

Count back



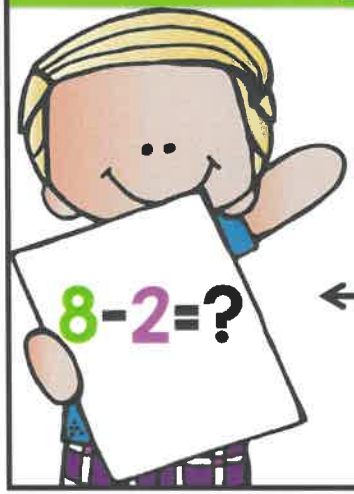
Start with the BIGGER number and count backwards

4, 5, 6, 7

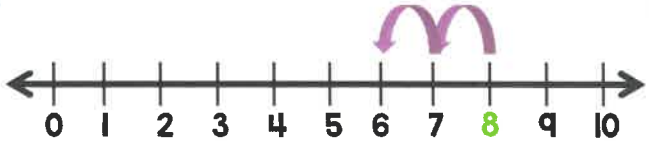
7 - 3 = 4

Ashley Dodson 2016

Use a number line



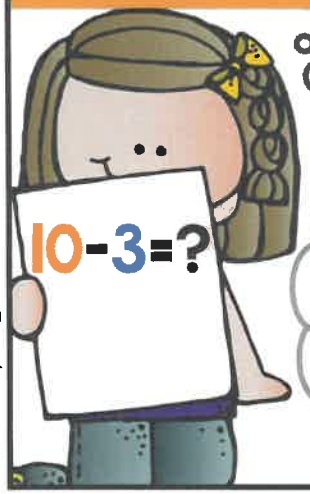
Start with the greater number and count back by jumping to the left



8 - 2 = 6

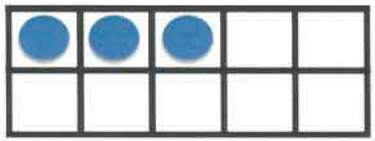
Ashley Dodson 2016

Make a ten



Use groups of ten to solve.
Think: 3 plus what is 10?

I know 3 plus 7 is 10, so 10 minus 3 is 7.



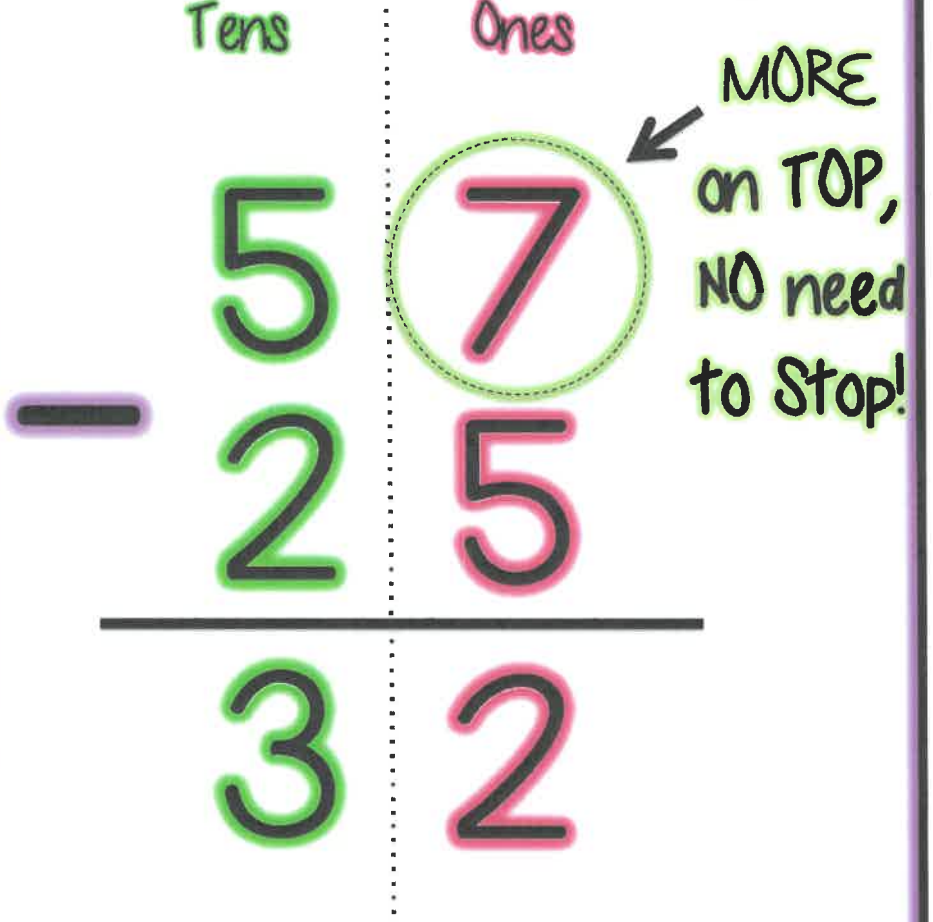
10 - 3 = 7

Ashley Dodson 2016

Double Digit Subtraction

Line up your tens and ones!

Start, Start, Start on the right!

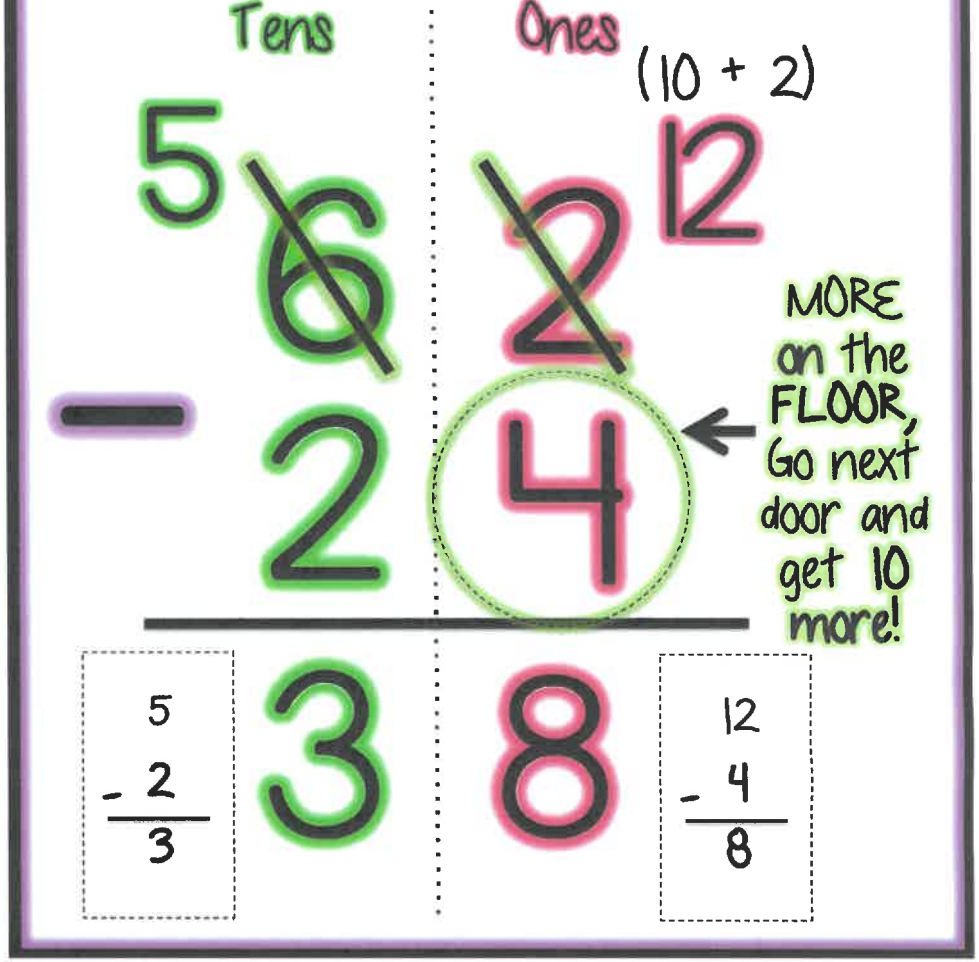


Double Digit Subtraction

WITH regrouping!

Line up your tens and ones!

Start, Start, Start on the right!



Subtraction Strategies

Place Value:

223 - 159

200	20	3
100	50	9

13 - 9 = 4 — Step 1: Subtract the ones (3 - 9) Can you do it? If not, take 10 from the tens in the same number

110

~~20~~ - 50 = 60 — Step 2: Subtract the tens (10 - 50) Can you do it? If not, take 100 from the hundreds in the same number

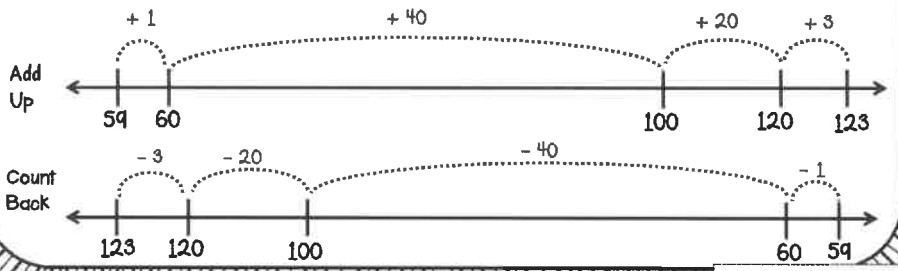
100

~~200~~ - 100 = 0 — Step 3: Subtract the hundreds (100 - 100)

Step 4: Add the ones, tens, and hundreds — 60 + 4 = **64**

Add Up or Count Back:

123 - 59



Subtraction Strategies

Adjusting One Number:

123 - 59

a. 123 - 59 +1 = 123 - 60

63 +1 = **64**

b. 123 +6 = 129 - 59

70 -6 = **64**

Keeping a Constant Difference:

123 +1 = 124

- 59 +1 = - 60

64

Standard Algorithm:

223 - 159

Step 1: Subtract the ones
13 - 9 = 4 (Regroup? Yes!)

1 11 13
~~223~~
- 159

Step 3: Subtract the hundreds
100 - 100 = 0

Step 2: Subtract the tens
110 - 50 = 60 (Regroup? Yes!)

Step 4: 60 + 4 = **64**

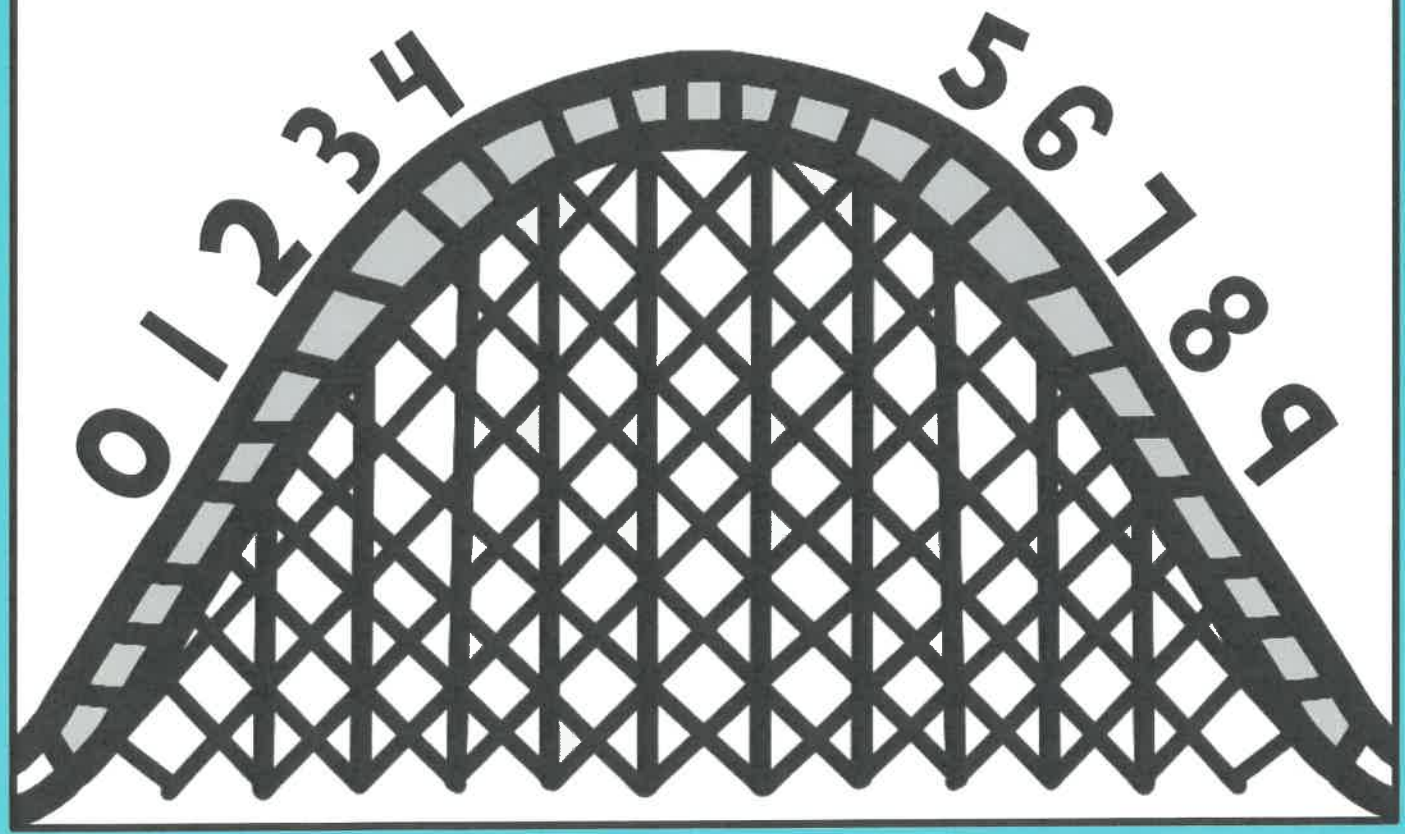

Rounding

ROUNDING

Picture the rounding rollercoaster to help you determine how to round a number. Numbers 0, 1, 2, 3, and 4 don't make it up the hill so the number stays the same. Numbers 5, 6, 7, 8, and 9 make it up and over the hill so the number rounds up one digit.

WEAK
NUMBER STAYS

STRONG
NUMBER ROUNDS UP



ROUNDING

- ❑ Rounding is used when the exact number isn't needed.
- ❑ Rounding makes the number easier to understand and work with.

FOR EXAMPLE:

Round the cost of each item below to quickly and easily add up the numbers. The answer will give a rough idea what the total bill will be at the checkout.

Round the prices to the nearest ten

Game $28 Candy $11 Hat $17

It will approximately cost = $60

The result is less accurate, but still gives a good idea how much money is needed to pay for all of the items.

ROUNDING

Step 1. Underline the place being rounded.

Step 2. Look to the number next door.

Step 3.

If the number is **4** or **lower**, keep the underlined number the **SAME**.

If the number is **5** or **higher**, **RAISE** the underlined number.

Step 4. Change all of the numbers in the place value positions after the underlined digit to zero.

EXAMPLE

*Round **3,432** to the nearest ten.*

Step 1. 3,43̲2

Step 2. Look next door - 3,43̲2

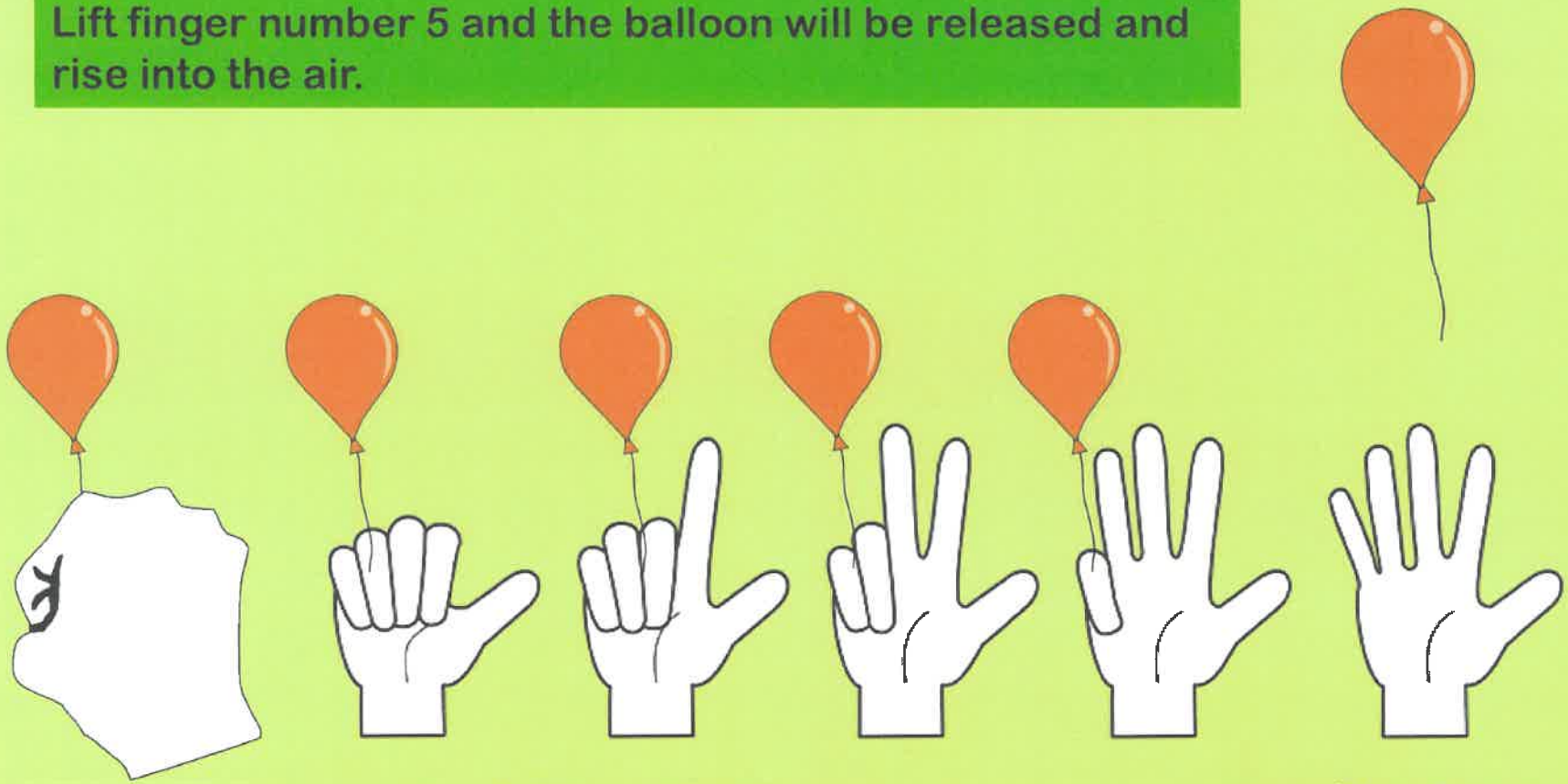
*Step 3. 2 is lower than 4, so the underlined number will stay the **same**.*

Step 4. Change all the numbers after the underlined digit with a zero 3,430.

ROUNDING

0, 1, 2, 3, 4 fingers lifted are able to hold on to the balloon and keep it at the same level.

Lift finger number 5 and the balloon will be released and rise into the air.



0 – 4 Stays the Same

5 or More Goes up

Rounding Numbers

- Rounding to the nearest tens place -

1. **Underline the place value being rounded**

Round to the nearest ten place:

244

2. **Look at the number to the RIGHT next door**

Round to the nearest ten place:

244

3. **If the number to the RIGHT is 4 or less, just ignore**

Round to the nearest ten place:

244 → 240

Rounding Numbers

- Rounding to the nearest tens place -

4. **If the number to the RIGHT is 5 or more, add 1 more to the underlined number**

Round to the nearest ten place:

247 → 250

5. **Change all the numbers to the right of the underlined number to zeros**

Round to the nearest ten place:

244 → 240

Round to the nearest ten place:

247 → 250

Rounding Numbers

- Rounding to the nearest hundreds place -

1. **Underline the place value being rounded**

Round to the nearest hundred place:

244

2. **Look at the number to the RIGHT next door**

Round to the nearest hundred place:

244

3. **If the number to the RIGHT is 4 or less, just ignore**

Round to the nearest hundred place:

244 → 200

Rounding Numbers

- Rounding to the nearest hundreds place -

4. **If the number to the RIGHT is 5 or more, add 1 more to the underlined number**

Round to the nearest hundred place:

267 → 300

5. **Change all the numbers to the right of the underlined number to zeros**

Round to the nearest hundred place:

244 → 200

Round to the nearest hundred place:

267 → 300

Rounding Rules

0-4

Do no more!

Ex: 743 = 700

2,527 = 2,500

5-9

Climb the vine!

Ex: 763 = 800

2,597 = 2,600


Multiplication

Multiplication Facts

1

1 x 0 = 0
1 x 1 = 1
1 x 2 = 2
1 x 3 = 3
1 x 4 = 4
1 x 5 = 5
1 x 6 = 6
1 x 7 = 7
1 x 8 = 8
1 x 9 = 9
1 x 10 = 10
1 x 11 = 11
1 x 12 = 12

2

2 x 0 = 0
2 x 1 = 2
2 x 2 = 4
2 x 3 = 6
2 x 4 = 8
2 x 5 = 10
2 x 6 = 12
2 x 7 = 14
2 x 8 = 16
2 x 9 = 18
2 x 10 = 20
2 x 11 = 22
2 x 12 = 24

3

3 x 0 = 0
3 x 1 = 3
3 x 2 = 6
3 x 3 = 9
3 x 4 = 12
3 x 5 = 15
3 x 6 = 18
3 x 7 = 21
3 x 8 = 24
3 x 9 = 27
3 x 10 = 30
3 x 11 = 33
3 x 12 = 36

4

4 x 0 = 0
4 x 1 = 4
4 x 2 = 8
4 x 3 = 12
4 x 4 = 16
4 x 5 = 20
4 x 6 = 24
4 x 7 = 28
4 x 8 = 32
4 x 9 = 36
4 x 10 = 40
4 x 11 = 44
4 x 12 = 48

5

5 x 0 = 0
5 x 1 = 5
5 x 2 = 10
5 x 3 = 15
5 x 4 = 20
5 x 5 = 25
5 x 6 = 30
5 x 7 = 35
5 x 8 = 40
5 x 9 = 45
5 x 10 = 50
5 x 11 = 55
5 x 12 = 60

6

6 x 0 = 0
6 x 1 = 6
6 x 2 = 12
6 x 3 = 18
6 x 4 = 24
6 x 5 = 30
6 x 6 = 36
6 x 7 = 42
6 x 8 = 48
6 x 9 = 54
6 x 10 = 60
6 x 11 = 66
6 x 12 = 72

7

7 x 0 = 0
7 x 1 = 7
7 x 2 = 14
7 x 3 = 21
7 x 4 = 28
7 x 5 = 35
7 x 6 = 42
7 x 7 = 49
7 x 8 = 56
7 x 9 = 63
7 x 10 = 70
7 x 11 = 77
7 x 12 = 84

8

8 x 0 = 0
8 x 1 = 8
8 x 2 = 16
8 x 3 = 24
8 x 4 = 32
8 x 5 = 40
8 x 6 = 48
8 x 7 = 56
8 x 8 = 64
8 x 9 = 72
8 x 10 = 80
8 x 11 = 88
8 x 12 = 96

9

9 x 0 = 0
9 x 1 = 9
9 x 2 = 18
9 x 3 = 27
9 x 4 = 36
9 x 5 = 45
9 x 6 = 54
9 x 7 = 63
9 x 8 = 72
9 x 9 = 81
9 x 10 = 90
9 x 11 = 99
9 x 12 = 108

10

10 x 0 = 0
10 x 1 = 10
10 x 2 = 20
10 x 3 = 30
10 x 4 = 40
10 x 5 = 50
10 x 6 = 60
10 x 7 = 70
10 x 8 = 80
10 x 9 = 90
10 x 10 = 100
10 x 11 = 110
10 x 12 = 120

11

11 x 0 = 0
11 x 1 = 11
11 x 2 = 22
11 x 3 = 33
11 x 4 = 44
11 x 5 = 55
11 x 6 = 66
11 x 7 = 77
11 x 8 = 88
11 x 9 = 99
11 x 10 = 110
11 x 11 = 121
11 x 12 = 132

12

12 x 0 = 0
12 x 1 = 12
12 x 2 = 24
12 x 3 = 36
12 x 4 = 48
12 x 5 = 60
12 x 6 = 72
12 x 7 = 84
12 x 8 = 96
12 x 9 = 108
12 x 10 = 120
12 x 11 = 132
12 x 12 = 144

Associative Property

20 x 30 =

(20 x 3) x 10 =

60 x 10 =

600

Place Value

20 x 30 =

20 x 3 tens =

60 tens =

600

Halve then Double

20 x 30 =

10 X 30 = 300

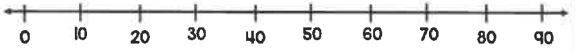
10 x 30 = 300

300 + 300 = 600

Number Line

20 x 9 =

2 x 9 = 18



2 x 90 = 180

Estimate Products-Round

32 → 30

x 31 → x 30

900

Estimate Products-Compatible Numbers

26 → 25

x 48 → x 50

1,250

Area Model

43

x 25

	40	3
20	40 x 20 = 800	3 x 20 = 60
5	40 x 5 = 200	3 x 5 = 15

800

200

60

+ 15

1,075

Partial Product

43

x 25

800 40 x 20 = 800

60 20 x 3 = 60

200 5 x 40 = 200

+ 15 5 x 3 = 15

1,075

Regrouping Method

43

x 25

215 5 x 3, then 5 x 40

+ 860 20 x 3, then 20 x 40

1,075

Area Model for Multiplication

1. Expand each number

$$36 \rightarrow 30 + 6$$
$$\times 25 \rightarrow 20 + 5$$

2. Set up numbers and multiply

×	30	+	6
20	20x30= 600		20x6= 120
+			
5	5x30= 150		5x6= 30

2. Add the products for the answer

$$\begin{array}{r} {\scriptstyle 1}\\ 600\\ 120\\ 150\\ +\ 30\\ \hline 900 \end{array}$$

Partial products

$182 \rightarrow 100 + 80 + 2$
(100: 1 hundred; 80: 8 tens; 2: 2 ones)

$\times 26$

6 x 2 →	12
6 x 80 →	480
6 x 100 →	600
20 x 2 →	40
20 x 80 →	1600
20 x 100 → +	2000
	4732

1. Multiply 6 by each digit in 182
2. Multiply 20 by each digit in 182
3. Find the partial products
4. Add to find the product

THE AREA MODEL

This is a box model for a two-digit number times a two-digit number.

Step 1 Write your multiplication problem.

$$\begin{array}{r} 58 \\ \times\ 45 \\ \hline \end{array}$$

Step 2 Write an addition problem using the tens and the ones value of the top number.

$$\begin{array}{r} 58 \\ \times\ 45 \\ \hline \end{array} \qquad 50 + 8$$

Step 3 Draw a box model starting with a vertical line under the addition sign and write the 2nd addition problem using the tens and the ones value of the bottom number.

$$\begin{array}{r} 58 \\ \times\ 45 \\ \hline \end{array}$$

	50	8
40		
5		

Step 4 Multiply: 40 x 50 and write the answer inside the 1st box; 40 x 8 and write the answer in the 2nd box; 5 x 50 and write the answer in the 3rd box; 5 x 8 and write the answer in the 4th box.

$$\begin{array}{r} 58 \\ \times\ 45 \\ \hline \end{array}$$

	50	8
40	2000	320
5	250	40

Step 5 Add the four boxes together.

$$\begin{array}{r} 2000 \\ 320 \\ 250 \\ +\ \ 40 \\ \hline 2610 \end{array}$$

Therefore, the answer is: **2610**

Multiplication Strategies
Traditional Algorithm

```
  2 1
  4 2
  453
x  48
 3624
+18120
 21,744
```

Multiplication Strategies
Partial Products

453

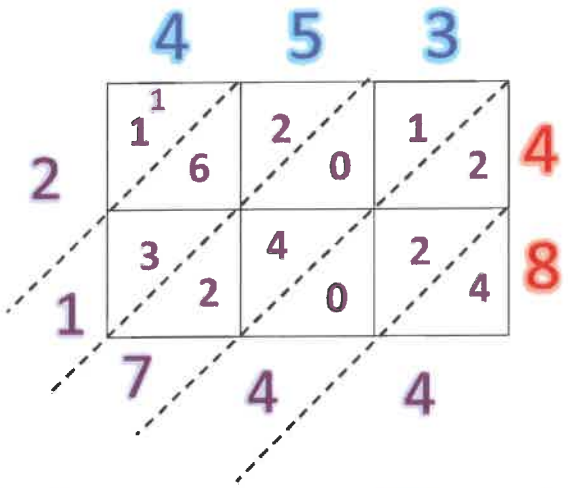
x 48

24	8 x 3
400	8 x 50
3,200	8 x 400
120	40 x 3
2,000	40 x 50
16,000	40 x 400
21,744	

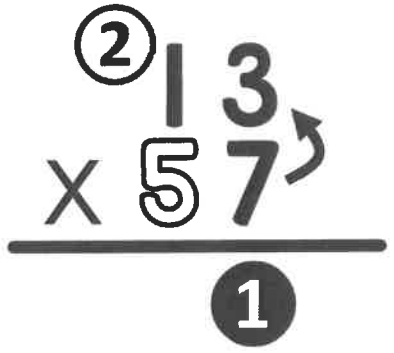
Multiplication Strategies
Lattice

453

x 48

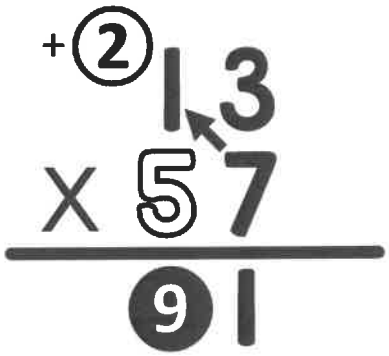


Multiplication Algorithm



What's 7 x 3? Yes! 21!

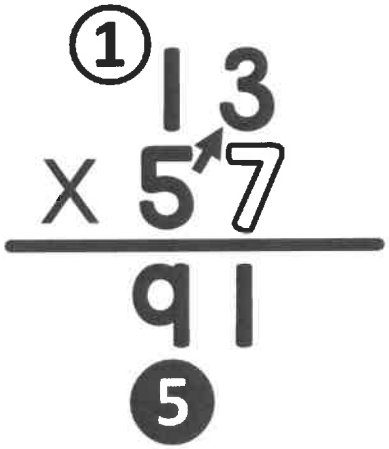
2 1



1: What's 7 x 1? Yes! 7!

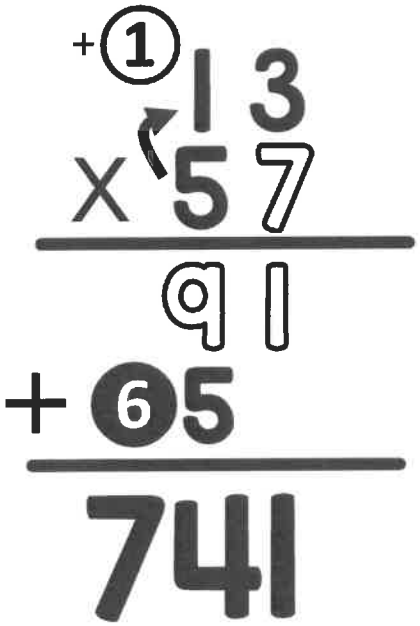
2: Then add that 2

3: 7 x 1 + 2 = 9



What's 5 x 3? Yes! 15!

1 5



1: What's 5 x 1? Yes! 5!

2: Then add that 1

3: 5 x 1 + 1 = 6

Now add em up!

YOU DID IT! ☺


Division

Divisibility Rules

Divisible by	Rule	Example	Non - Example
2	If the last digit is even, (2, 4, 6, 8, or 0)	724	423
3	If the sum of the digits is divisible by 3	345	317
4	If the last 2 digits are divisible by 4	712	710
5	If the number ends with 5 or 0	45	92
6	If the number is divisible by BOTH 2 and 3	144	517
7	No Rule	n/a	n/a
8	If the last 3 digits are divisible by 8	3,840	6,428
9	If the sum of the digits is divisible by 9	6,939	6,923
10	If the number ends in 0	7,200	4,258

Division Words

Division Symbol

Equal sign

Remainder
The number that is left over and does not fit into an equal group.

37 ÷ 6 = 6 R1

Dividend
The large number that is being separated into smaller groups.

Divisor
The number of groups that the dividend is being separated into.

Quotient
The answer. The number of items in each group.

Division Words

Division Symbol

7
6 | 43
−42
1R

Quotient
The answer. The number of items in each group.

Dividend
The large number that is being separated into smaller groups.

Divisor
The number of groups that the dividend is being separated into.

Remainder
The number that is left over and does not fit into an equal group.

Use Multiplication Facts

When you read a division question - ask yourself a multiplication question.

21 ÷ 7 = ?

Ask yourself:

7 x ? = 21

Skip Pattern Count!

This is a great strategy to use when dividing by 2, 3, 5 or 10.

Skip pattern count by the small number stop when you reach the large number.

21 ÷ 3 =

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

Use Related Facts It's A Fact Family

Remember there are 4 members in each family.

When multiplying the greatest number is always the answer.

When dividing the greatest number is always the number you are dividing into smaller groups.

Example:

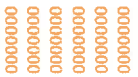
7 x 4 = 28 4 x 7 = 28

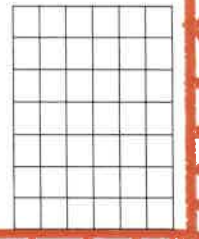
28 ÷ 7 = 4 28 ÷ 4 = 7

Make An Array

When dividing make an array to find the answer. The big number tells you the total number in the array or when to stop. The smaller number tells you how many in one row. The answer is how many in one column.

Example: 42 ÷ 6 =





Nifty Nines

When dividing by 9 remember your 9 addition facts.

Ask yourself "What is the biggest number?" – add one to the tens digit in the biggest number and that is your answer.

27 ÷ 9 =
2 + 1 =3

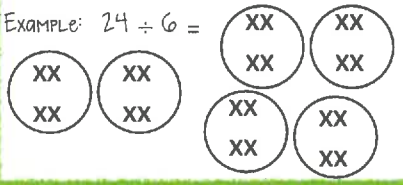
45 ÷ 9=
4 + 1 = 5

Draw A Picture

When dividing make a picture of equal groups to find the answer. The big number tells you the total number to put into groups or when to stop. The small number tells you how groups to make. The answer is how many in one group.

Remember to put one in a group at a time.

Example: 24 ÷ 6 =



Use Base Ten Blocks

Use base ten blocks to model the question. Model the big number. Then separate the blocks into equal groups – the smaller number tells you how many equal groups. The answer is the number represented in one group. You may have to exchange a ten for ones to make the groups equal and you may have some leftover.

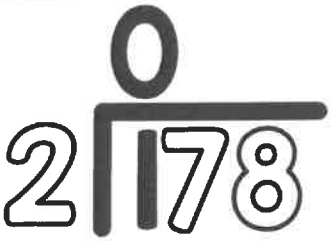
Example 46 ÷ 4 =

Make a Base 10 Model

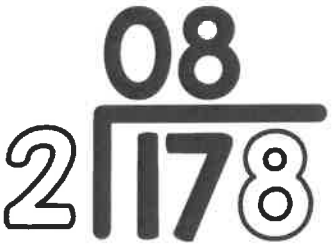
Make a Base 10 model of the question. Model the big number. Then circle the blocks using the small number. The answer is the number of circles. You may have to exchange a ten for ones to make the groups equal and you may have some leftover.

Example 46 ÷ 4 =

Long Division Algorithm

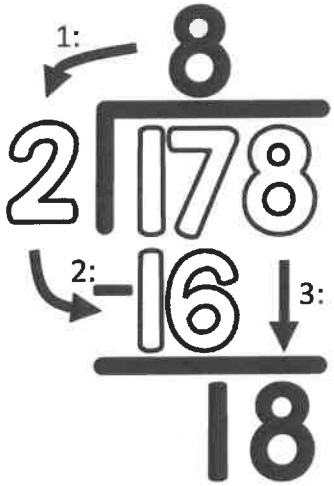


Does 2 fit into 1? No way, it's too big!
(so we put a "0" above 1 as a placeholder)



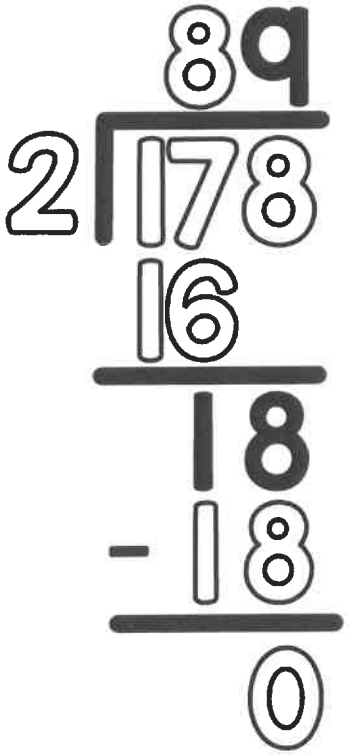
Does 2 fit into 17? YES! **8** times. 2 x **8** = 16
(one more 2 would be too big)

The **8** goes above the last digit in 17.



(To simplify the look of things, let's drop that 0 placeholder since **08 = 8** anyway.)

1: Now we multiply **8** x 2.
2: Subtract (we get 1 here when we subtract).
3: And bring down the **8**.



Does 2 fit into 18? YES! **9** times. 2 x **9** = 18
The **9** goes up top next to **8**.

Now repeat the three steps above.
1: Multiply **9** x 2.
2: Subtract.
3: There's nothing to bring down! We're done!

178 ÷ 2 = 89

YOU DID IT! ☺

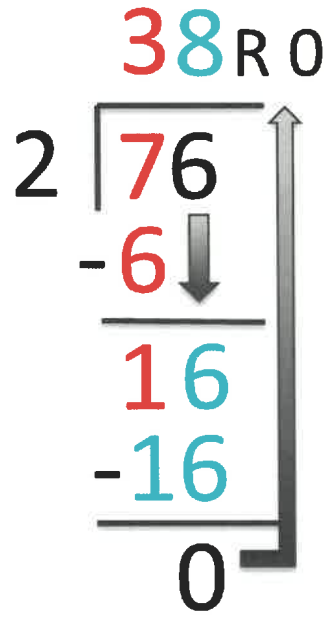
Does
Divide

Mc**D**onald's
Multiply

Serve
Subtract

Cheese
Check

Burgers
Bring down (or up!)

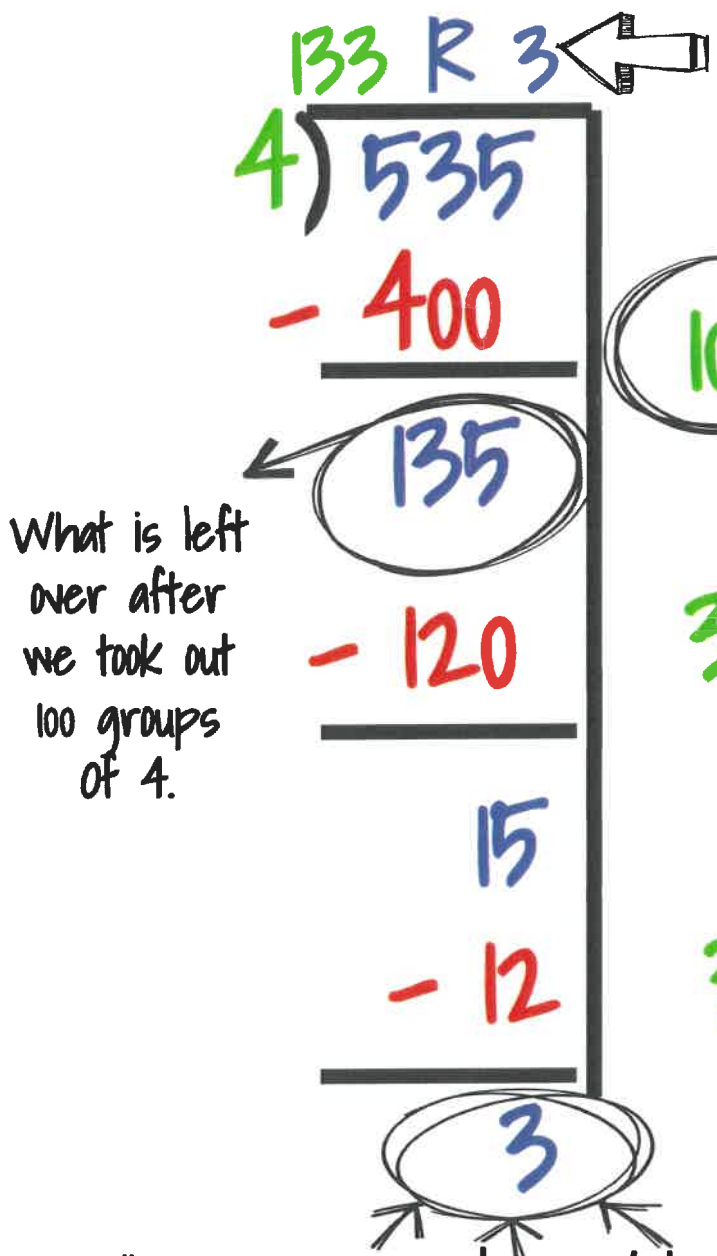


Suggested language:

- What "2" fact gets you close to "7" without going over? (2x3=6)
- What "2" fact gets you close to "16" without going over? (2x8=16)

Teaching Paws-itively

Partial Quotients Division



How many groups of 4 we took out total, and the remainder (or leftovers).

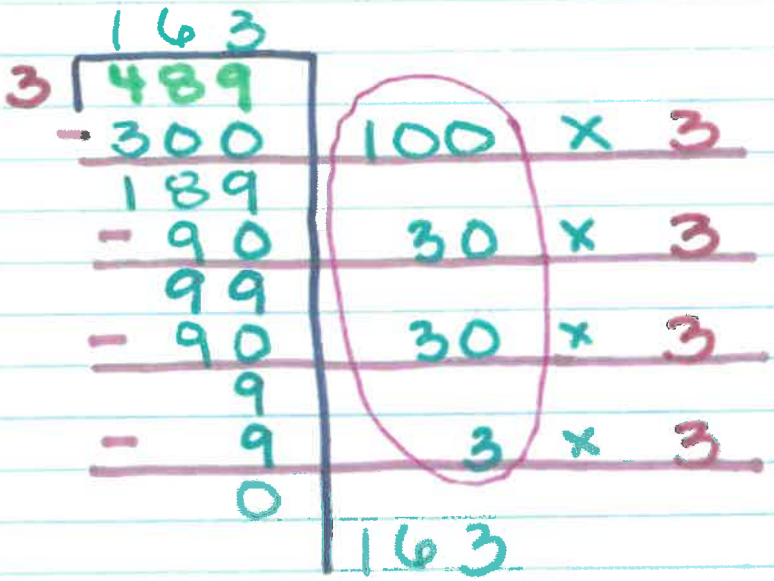
This side shows how many groups of 4 we have taken out each time.

What is left over after we took out 100 groups of 4.

When we can no longer take out any more groups of 4, we are done subtracting. The number that remains, or is leftover, becomes our remainder.

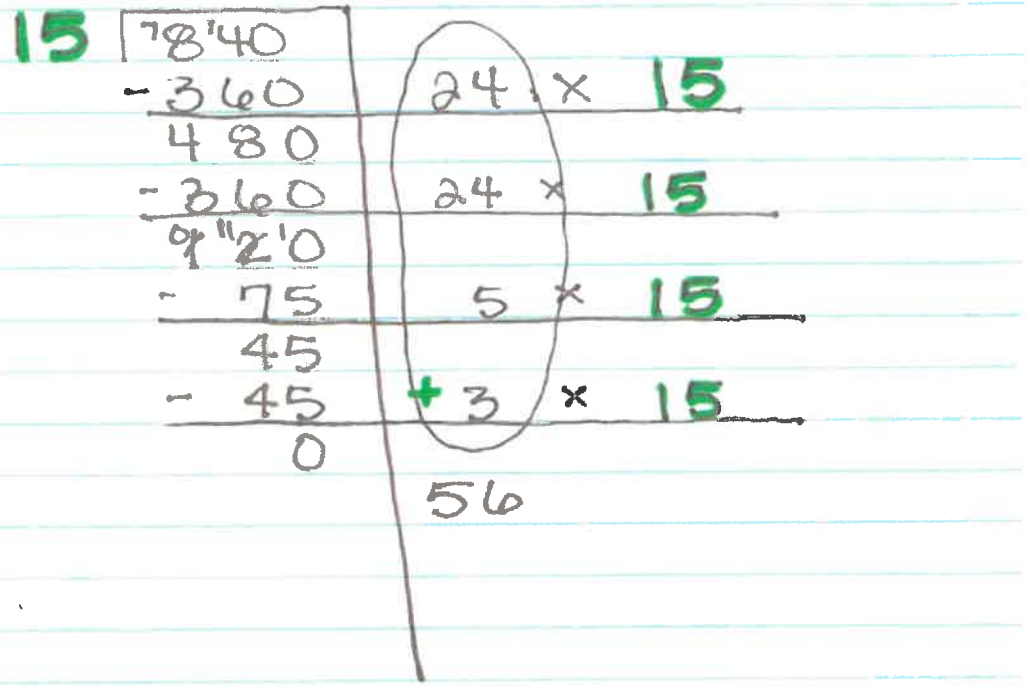
Big "7" Division

489 ÷ 3



Big "I" Division

(Double Digit)



66000
- 46000
20000

AREA MODEL FOR DIVISON

This is an area model dividing with a two-digit divisor.

Step 1 Write your division problem.

$12\overline{)463}$ OR $463 \div 12 =$ (quotient)

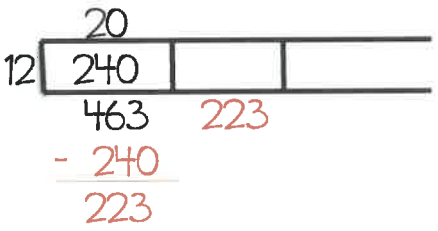
(dividend) (divisor)

Step 2 Draw an open-ended rectangle- leave it open to expand. Write the divisor number on the left side of the rectangle. Write the dividend number underneath.

Step 3 Think "What is the highest number I can easily multiply by the divisor with a value less than the dividend?" Write that number on the top of the rectangle. (Hint: Try 100, 50, 20, 10, 5 or 1)

Thinking box
12 x 20 = 240

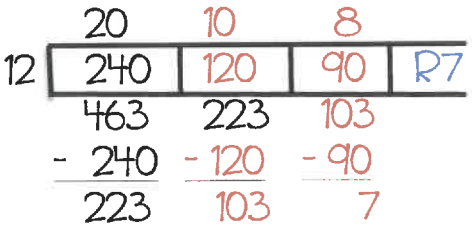
Step 4 Then subtract the value of your multiplication problem (20 x 12 = 240) from your dividend. Write the value of the subtraction problem under the next box.



Thinking box
20 x 12 = 240

Step 5 Think "What is the highest number I can easily multiply by the divisor with a value less than the value of my subtraction problem?" Write that number on top of the next box. Then subtract that answer from 223. Continue.

Repeat this step until you can't multiply any number by the divisor and it's value is less than the value of your **subtraction problem**



Thinking box
12 x 20 = 240
12 x 10 = 120
12 x 8 = 90

Step 6 Add the numbers on top of the rectangle. (These are the numbers you multiplied by the divisor.) If there is a remainder, write it next to the value for your answer: 38 R7

20 + 10 + 8 = 38 R7


Least Common Multiple (LMC) and Greatest Common Factor (GCF)

Divisibility Rules

A number is divisible by....	When....	Examples
2	even: last digit is 0, 2, 4, 6, 8	60 98 52
3	sum of digits is divisible by 3	60 75 207
4	can be divided by 2 twice	60 28 108
5	ends in a 0 or 5	60 75 325
6	divisible by both 2 and 3	60 48 162
9	sum of digits is divisible by 9	63 27 711
10	ends in 0	60 90 170
100	ends in two zeros	600 300 2700

Prime Numbers

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

A prime number is a whole number with only two factors: 1 and itself.

least common multiple

used when comparing 2 numbers

start listing the multiples for both until you find the smallest number they both have in common

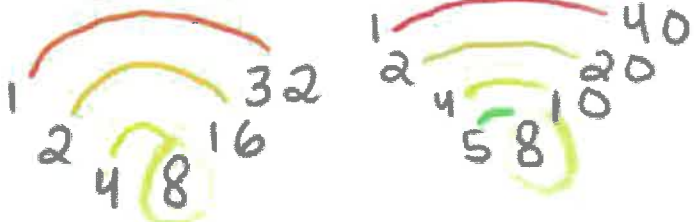
Ex: 3 - 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33..
10 - 10, 20, 30, 40...

greatest common factor (gcf)

used when comparing 2 numbers, or reducing a fraction

Find GCF of . . .

32 & 40



list all the factors and find the biggest number that both have in common.

1, 2, and 4 are common factors, but 8 is the largest.

9/30 find the factors for both numbers

Once you've found the GCF divide both numbers by it to reduce your fraction

9 ÷ 3 = 3 30 ÷ 3 = 10

3/10

7.2 PRIME FACTORIZATION

FACTOR TREES

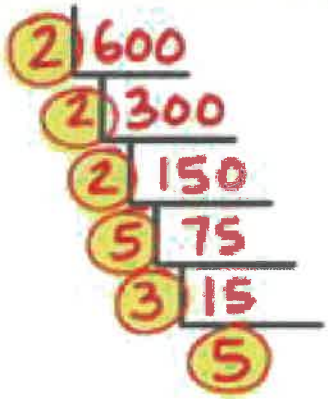
Prime number - a number greater than one and has exactly two factors 1 and itself

Prime numbers are the building blocks of all numbers.

FIND the PRIME FACTORIZATION of 600

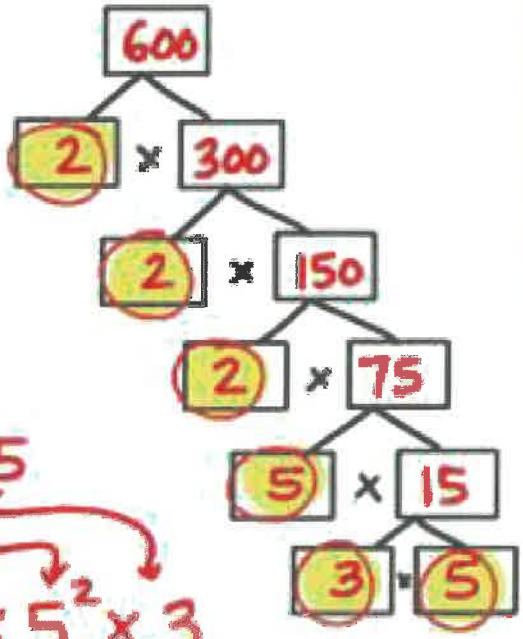
Two WAYS to SHOW prime factorization

1. Ladder Diagram



means divide

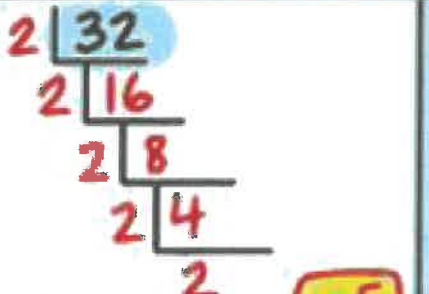
2. Tree Diagram



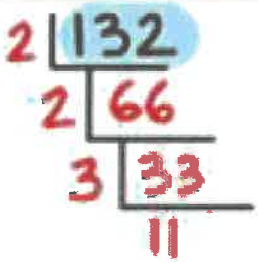
$2 \times 2 \times 2 \times 5 \times 3 \times 5$

Prime Factorization of 600 → $2^3 \times 5^2 \times 3$

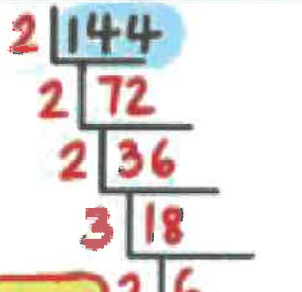
Find the Prime Factorization of these numbers.



$2 \times 2 \times 2 \times 2 \times 2 = 2^5$



$2 \times 2 \times 3 \times 11 = 2^2 \times 3 \times 11$



$2^4 \times 3^2$


Decimals

Handout/Printable or file to use on projector/overhead to teach/review decimal operations

Rules of Decimals

Addition

> Find the decimal
> Line up the decimals
> Fill in empty spots with zero
> Add
> Bring down the decimal in your answer

EXAMPLE

10.5 + 11.74

Rewritten with decimals lined up...

$$\begin{array}{r} 10.50 \\ +\ 11.74 \\ \hline 22.24 \end{array}$$

Subtraction

> Find the decimal
> Line up the decimals
> Fill in empty spots with zero
> Subtract
> Bring down the decimal in your answer

EXAMPLE

12.7 - 9.23

Rewritten with decimals lined up...

$$\begin{array}{r} {\scriptstyle 6\ 10} \\ 12.\not{7}\not{0} \\ -\ 9.23 \\ \hline 3.47 \end{array}$$

Multiplication

> The number with most digits goes on top
> Decimals do not have to line up
> Multiply like normal
> Count how many places in first number the decimal is moved over
> Count how many places in 2nd number the decimal is moved over
> This is how many places you move the decimal in your answer

EXAMPLE

$$\begin{array}{rl} 1.201 & \text{< 3 DECIMAL PLACES} \\ \times\ .25 & \text{< 2 DECIMAL PLACES} \\ \hline 6005 & \\ 24020 & \\ \hline .30025 & \text{< 5 DECIMAL PLACES} \end{array}$$

Division

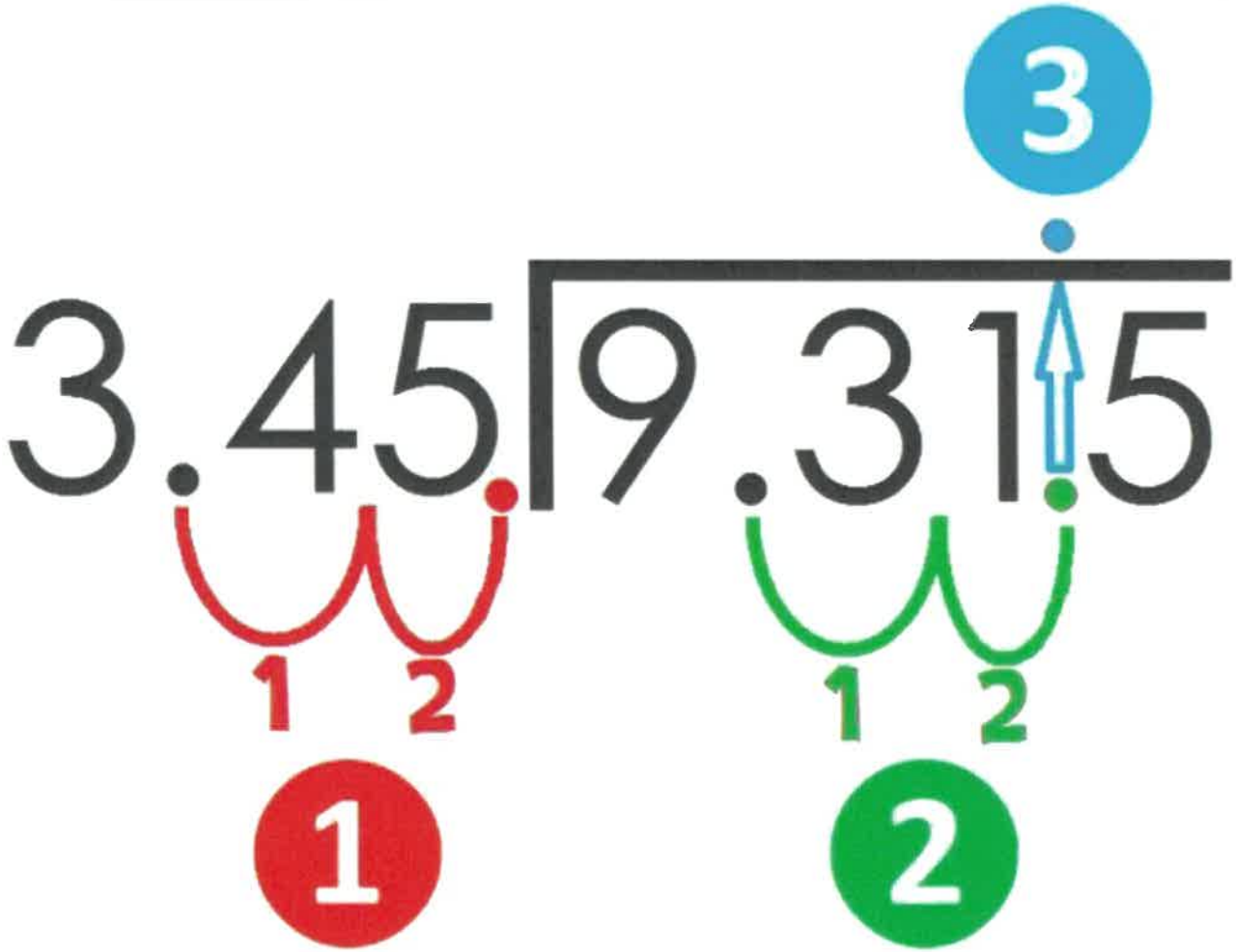
> Divisor can not have a decimal
> Move the divisor decimal so it is a whole number
> Move the same amount of places in dividend
> Place a decimal straight up where you write your answer, rewrite problem
> Divide like normal

EXAMPLE

DIVISOR > $0.3\overline{)1.41}$

$$\begin{array}{r} 4.7 \\ 3\overline{)14.1} \\ -12 \\ \hline 21 \\ -21 \\ \hline 0 \end{array}$$

Dividing with Decimals




Fractions

Adding Fractions

1. **Find the LEAST COMMON DENOMINATOR of the two fractions (LCD).**
2. **Make equivalent fractions.**
3. **Add the numerators.**
4. **Keep the denominators the same.**
5. **Simplify (Reduce).**

$$\frac{2}{3} + \frac{2}{5} =$$

Common denominators

$$\frac{10}{15} + \frac{6}{15} = \frac{16}{15}$$

SIMPLIFY!

$$1\frac{1}{15}$$

Subtracting Fractions

1. **Find the LEAST COMMON DENOMINATOR of the two fractions (LCD).**
2. **Make equivalent fractions.**
3. **Subtract the numerators.**
4. **Keep the denominators the same.**
5. **Simplify (Reduce).**

$$\frac{5}{8} - \frac{1}{4} =$$

Common denominators

$$\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$$

Multiplying & Dividing Fractions

Multiplying

Go straight across!

$$\frac{2}{9} \rightarrow \times \rightarrow \frac{1}{5}$$

$$= \frac{2}{45}$$

*Reduce if possible!

Dividing

Keep Change Flip!
after you change all numbers to fraction form

$$\frac{2}{9} \div 1\frac{1}{3}$$

*Around the world!

$$\frac{2}{9} \div \frac{4}{3}$$

KCF

$$\frac{2}{9} \times \frac{3}{4}$$

$$\frac{6}{36} = \frac{1}{6}$$

*Reduce if possible!

Fractions

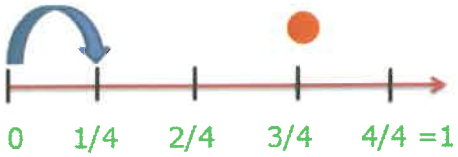
A fraction is a part of a whole number.

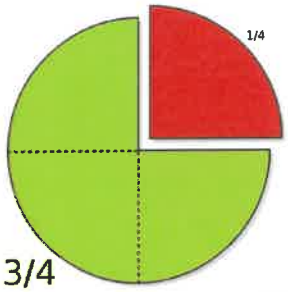
$$\frac{\text{PART}}{\text{WHOLE}} \qquad \frac{\text{NUMERATOR}}{\text{DENOMINATOR}}$$

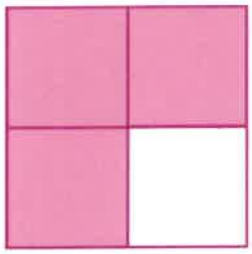
$$\frac{1}{4} \quad 4\overline{)1}$$

To convert a fraction to a decimal use long division.

$$\frac{3}{4} =$$

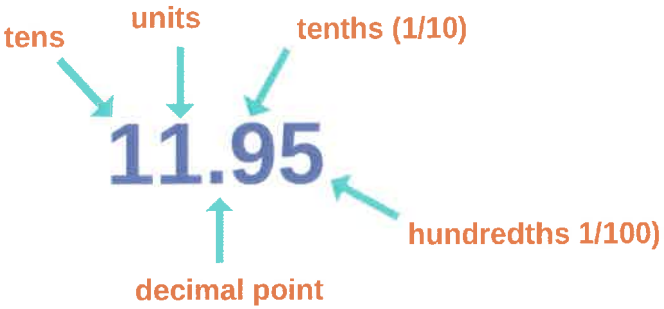






Decimals

A decimal is part of a whole.
A decimal point is used to separate whole numbers from values less than one.



$0.46 = 46\%$

To convert a decimal to a percent, move the decimal point two place to the right.

$$\frac{53}{100} = 0.53$$

To convert a decimal to a fraction remove the decimal and write as a fraction (over 100).

Percent

Percents are a ratio of a number to 100.
They can be written as a decimal or a fraction.

('Per'= for every. 'Cent' = 100.)

$12\% = 0.12$

To convert a percent to a decimal divide by 100 (move the decimal 2 spaces to the left).

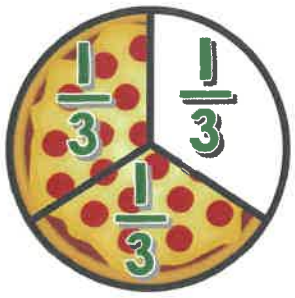
To convert a percent into a fraction write the percent divided by 100 and simplify.

$$75\% = \frac{75}{100} \div \frac{25}{25} = \frac{3}{4}$$

$$\frac{\%}{100} \times \frac{\text{PART}}{\text{WHOLE}}$$

To find the percent of a number multiply the percent with the amount then divide by 100.

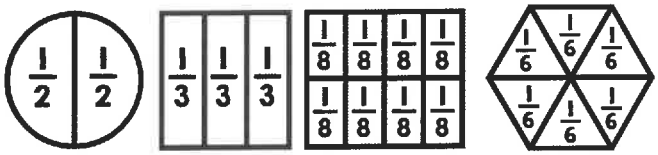
What is a **fraction**?



one third $\frac{1}{3}$ two thirds $\frac{2}{3}$

One third of the pizza has been eaten.
Two thirds of the pizza remain.

A fraction is a numerical quantity that is not a whole number.

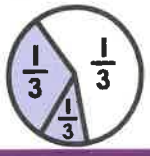


What is the **numerator**?

$\frac{2}{3}$ ← The **numerator** is the number above the line in a fraction.

The numerator tells us how many equal parts of the whole are being considered.

Correct!



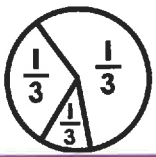
Incorrect!

What is the **denominator**?

$\frac{2}{3}$ ← The **denominator** is the number below the line in a fraction.

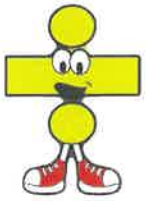
The denominator tells us how many equal parts make up the whole.

Correct!



Incorrect!

How do we **divide fractions**?



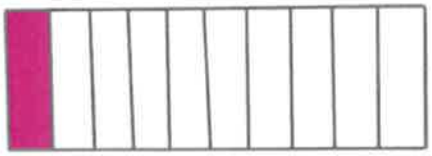
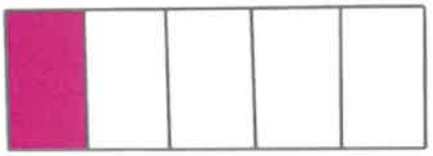
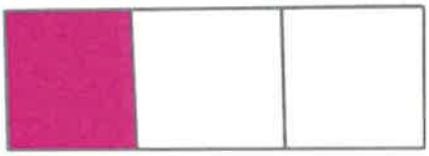
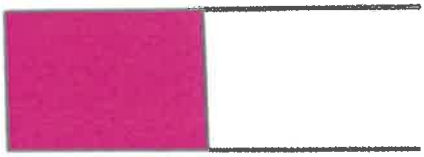
$\frac{1}{2} \div \frac{2}{3} = ?$

RECIPROCAL

$\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$

To divide fractions, multiply the first fraction by the reciprocal of the second fraction.

fractions, decimals and percents

FRACTION	DECIMAL	PERCENT	PICTURE
$\frac{1}{10}$	0.1	10%	
$\frac{1}{5}$	0.2	20%	
$\frac{1}{4}$	0.25	25%	
$\frac{1}{3}$	$0.\overline{33}$	$33.\overline{3}\%$	
$\frac{1}{2}$	0.5	50%	
$\frac{2}{3}$	$0.\overline{66}$	$66.\overline{6}\%$	
$\frac{3}{4}$	0.75	75%	
1	1.00	100%	

CONVERT DECIMALS, FRACTIONS, & PERCENT

DECIMAL	PERCENT	• Multiply by 100. • Write a percent sign.	$0.3 \cdot 100 = 30$ 30%
	FRACTION	• Write it as a fraction, then multiply by the smallest place value.	$0.3 = \frac{0.3}{1} \cdot \frac{10}{10} = \frac{3}{10}$ ↑ TENTHS
FRACTION	DECIMAL	• Divide the numerator by the denominator.	$\frac{3}{10} = 3 \div 10 = 0.3$ 0.3 10) 3.0 3 0 0
	PERCENT	• Divide the numerator by the denominator. • Multiply by 100. • Write a percent sign.	$\frac{3}{10} = 3 \div 10 = 0.3$ $0.3 \cdot 100 = 30$ 30%
PERCENT	FRACTION	• Write it as a fraction with a denominator of 100. • If the numerator has a decimal, multiply by the smallest place value.	$30\% = \frac{30}{100} = \frac{3}{10}$ $3.5\% = \frac{3.5}{100} \cdot \frac{10}{10} = \frac{35}{1000} = \frac{7}{200}$
	DECIMAL	• Divide by 100.	$30\% = 30 \div 100 = 0.3$
Use GCFs to express fractions in their simplest form.			$\frac{30}{100} \div \frac{10}{10} = \frac{3}{10}$ $\quad \frac{35}{1000} \div \frac{5}{5} = \frac{7}{200}$

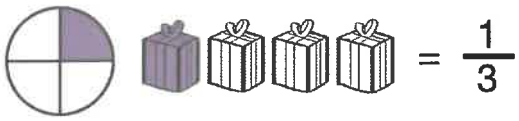
Misconceptions: Fractions, Decimals & Percentages

Sometimes, it pays to be aware of some common mistakes that students may encounter when learning how about Fractions, Decimals & Percentages.

$\frac{3}{12}$ is simplified

$\frac{9}{12} \overset{\div 2}{\underset{\div 2}{}} = \frac{4.5}{6}$

Student assumes you half in order to simplify. Does not divide by 3 or other possible numbers.



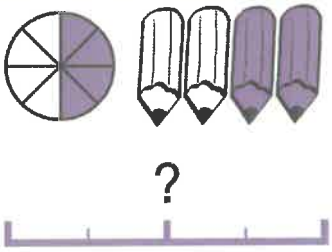
Student thinks that Numerator is total shaded, and Denominator is total unshaded.

$\frac{1}{5} > \frac{1}{2}$

Student thinks one fifth is larger than a half. Misunderstands function of Denominator.

$\frac{3}{4}$ Student has shaded the Numerator.

$\frac{3}{4}$ Student has shaded sum of Numerator & Denominator.



Student can find fraction of amount, but does not understand fraction as a number.

$\frac{1}{5} + \frac{2}{5} = \frac{3}{10}$

Student has added Numerator & Denominator. Misunderstands function of Denominator.

$\frac{1}{2} \overset{+1}{\underset{+1}{}} = \frac{2}{3}$

Student adds 1 to Numerator & Denominator instead of multiplying to find quivalents.

$0.2 < 0.19$ Student sees "2" as smaller than "19". Encourage use of extra 0s in spaces.

Student did not multiply by 100. Assumes numbers after decimal point are percentage.

$0.125 = 125\%$ $0.5 = 5\%$

$\frac{1}{4} = 0.4$ or 1.4

$\frac{2}{6} = 0.26$ or 2.6

$\frac{1}{4} = 0.5$ or 2.6

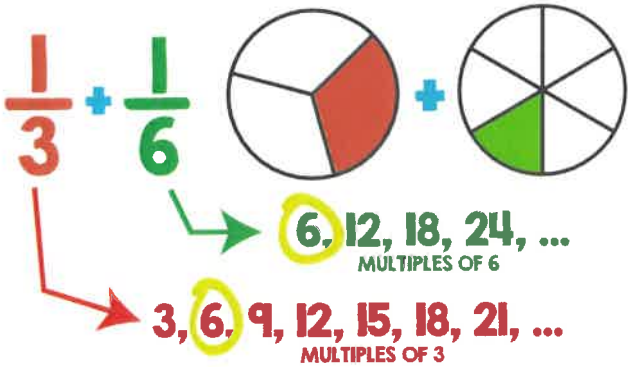
Student may misunderstand the function of the dividing line. Sometimes adds, combines and confuses position of Decimal Point.

Note

There are many more misconceptions, most derive from basic misunderstanding of what a fraction actually is.

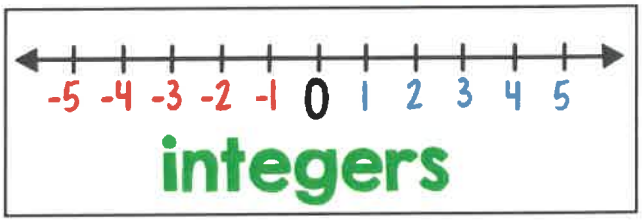
What is the least common denominator?

The **LCD** is the smallest number that is divisible by each of the denominators.



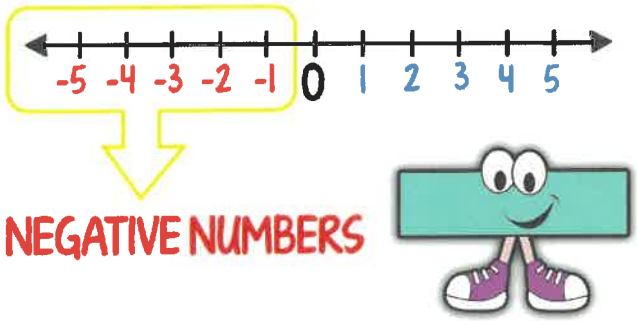
The **LCD** of **3** and **6** is **6**

What are integers?



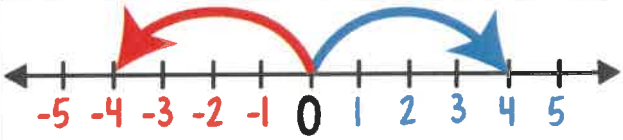
All the whole numbers and all the negative numbers are integers. Integers can be positive (1, 2, 3, 4, 5, ...), negative (-1, -2,-3, -4, -5, ...), or zero (0). Fractions are not integers.

What are negative numbers?



A negative number is any number with a value less than zero.

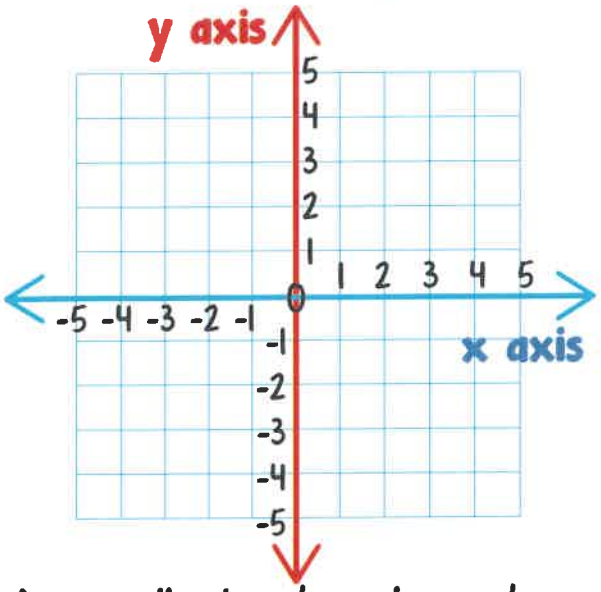
How can I use a number line to determine a number's opposite?



The opposite of 4 is -4.
The opposite of -4 is 4.

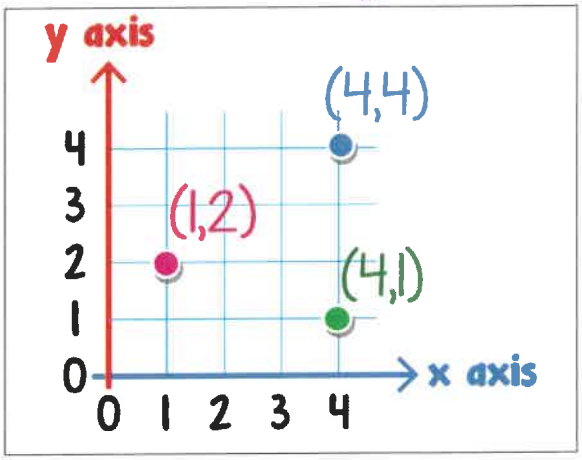
A number's opposite is the number that is an equal distance from zero on the opposite side of the number line.

What is a coordinate plane?



A coordinate plane is a plane divided by the x axis and y axis.

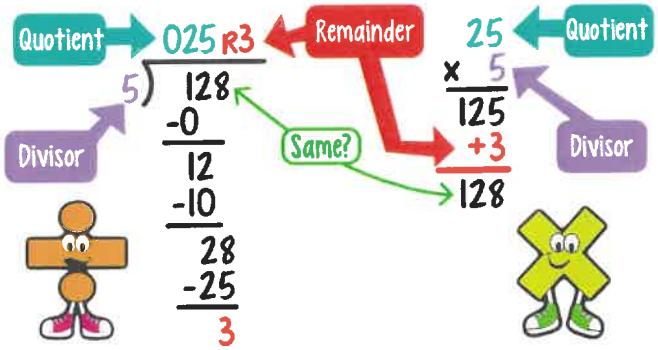
What is an ordered pair?



An ordered pair is a set of two numbers. The first number represents a position on the x axis. The second number represents a position on the y axis. Together, they

How can I check a division problem?

Multiply the quotient by the divisor, then add the remainder.



If the result is the same as the dividend, your answer is correct!

What is a factor?

1 x 12 = 12
FACTOR FACTOR PRODUCT

2 x 6 = 12
FACTOR FACTOR PRODUCT

3 x 4 = 12
FACTOR FACTOR PRODUCT

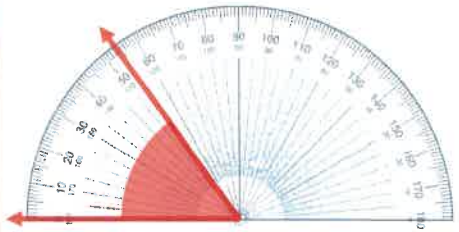
Factors are numbers multiplied together.

1, 2, 3, 4, 6, and 12 are factors of 12

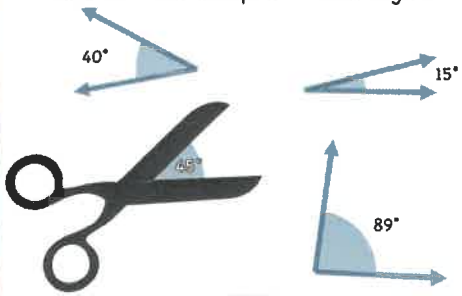

Geometry

Acute Angle

An acute angle is less than 90°

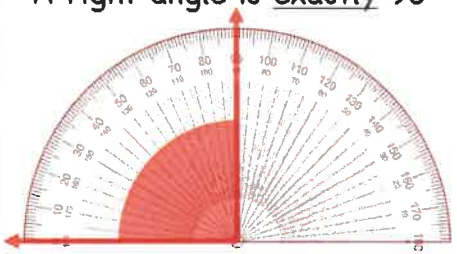


Here are some examples of acute angles:

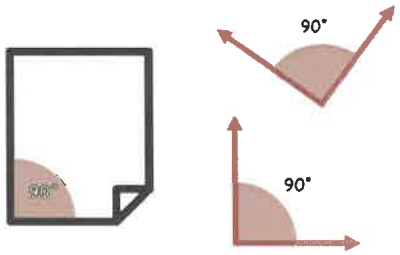


Right Angle

A right angle is exactly 90°

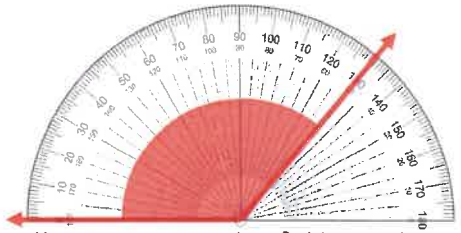


Here are some examples of right angles:

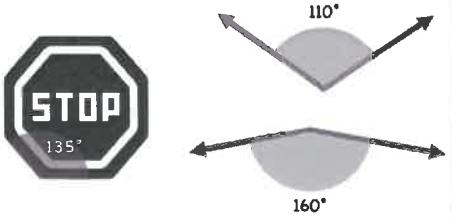


Obtuse Angle

An obtuse angle is greater than 90°, but less than 180°

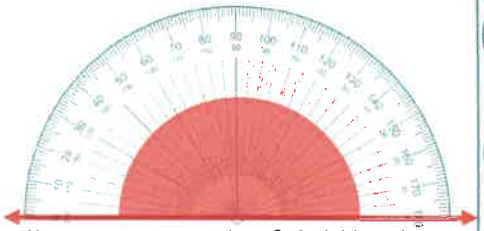


Here are some examples of obtuse angles:

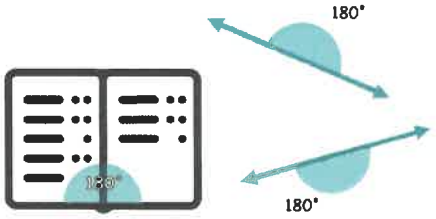


Straight Angle

A straight angle is exactly 180°

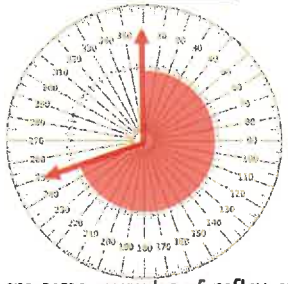


Here are some examples of straight angles:

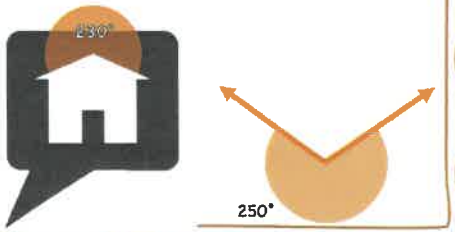


Reflex Angle

A reflex angle is greater than 180°, but less than 360°

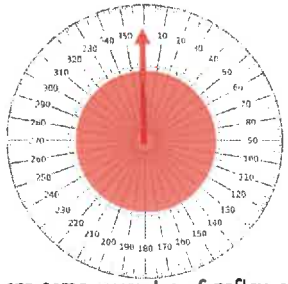


Here are some examples of reflex angles:

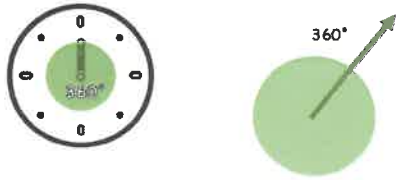


Revolution

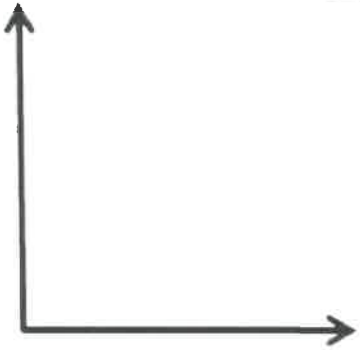
A revolution is exactly 360°



Here are some examples of reflex angles:

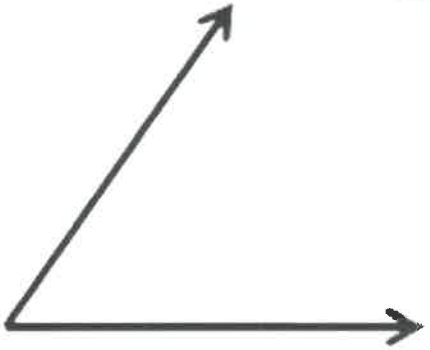


Right Angle



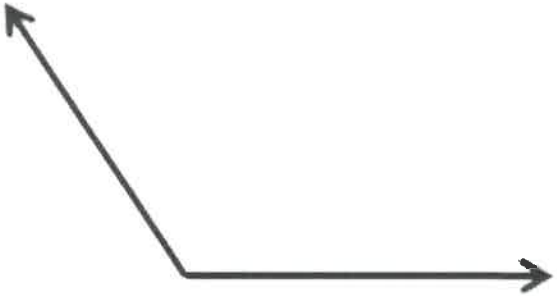
Measures exactly 90 degrees.

Acute Angle



Measures less than 90 degrees.

Obtuse Angle

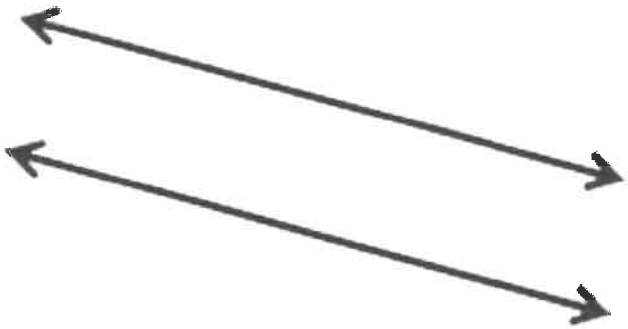


Measures more than 90 degrees but less than 180 degrees.

Straight Angle

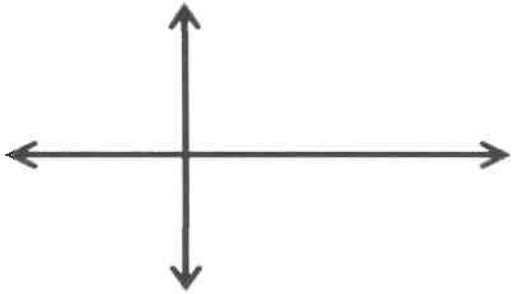
Measures exactly 180 degrees.

Parallel Lines



Lines that will never intersect.

Perpendicular Lines



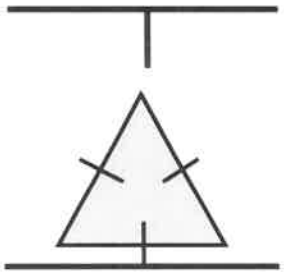
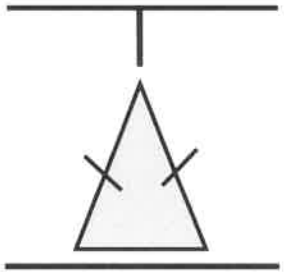
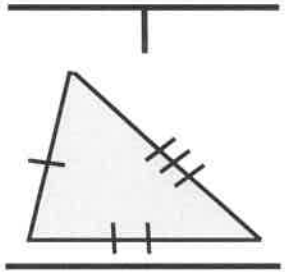
Lines that form square corners.

Intersecting Lines

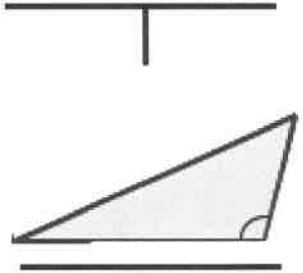
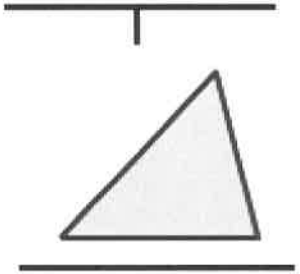
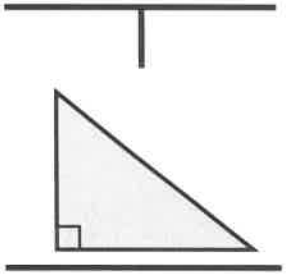
Lines that pass through the same point.

Types of Triangles

Classified by Side Lengths

- Equilateral Triangle
  - Three equal sides
  - 
- Isosceles Triangle
  - Two equal sides
  - 
- Scalene Triangle
  - No equal sides
  - 

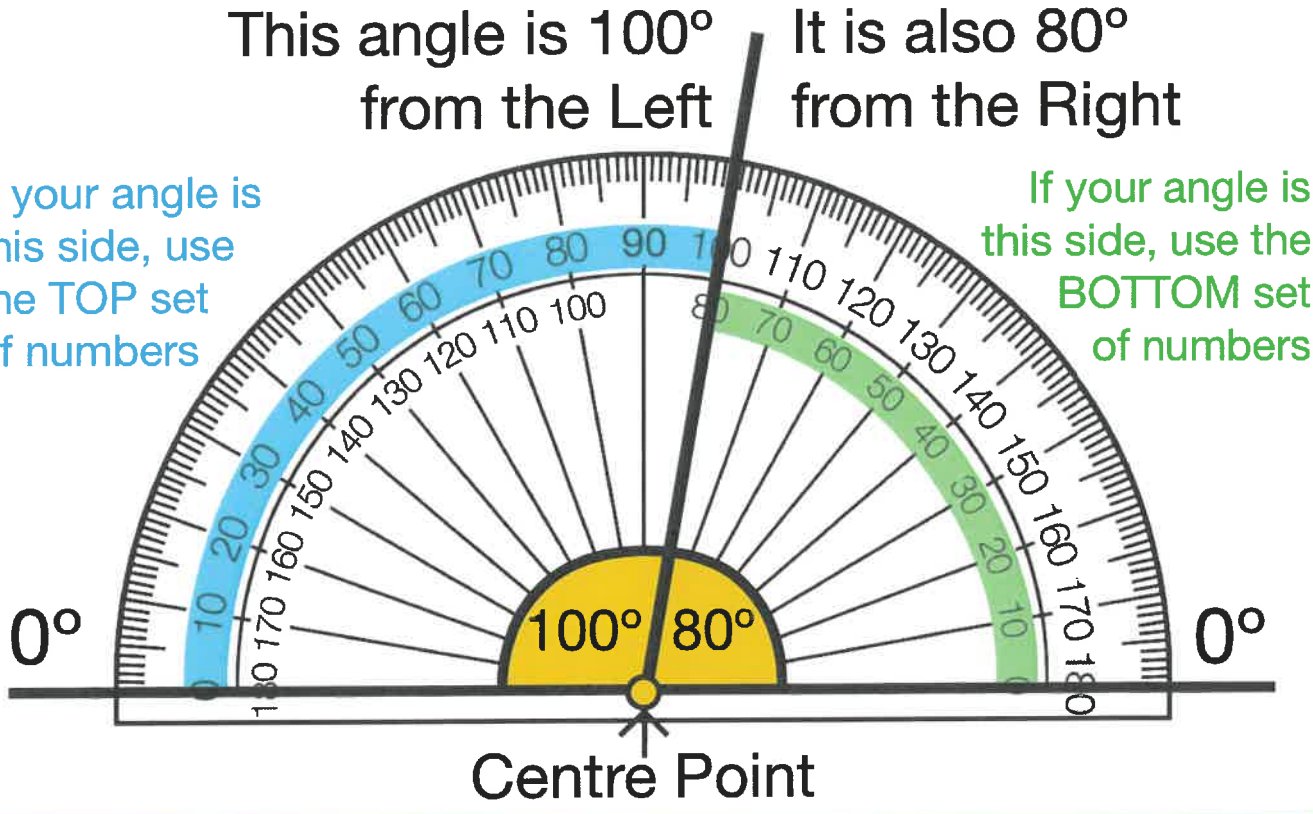
Classified by Angles

- Obtuse Triangle
  - Has one angle greater than 90°
  - 
- Acute Triangle
  - All angles less than 90°
  - 
- Right Triangle
  - Has one 90° angle
  - 

How to Use a Protractor

A protractor is used to measure the number of degrees in an angle

The symbol for degree is °
Example:-
45 degrees is 45°



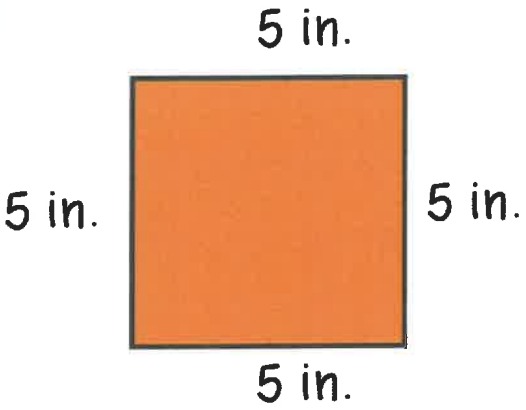
1. Place the centre point of the Protractor on the corner of the angle.
2. Line up the zero line of the Protractor with the bottom line of the angle.
3. Measure the angle that the top line goes through on the Protractor.
4. Well done! You should have your angle measurement!

Review

Well done! You should now know how to measure a basic angle using a protractor!

Fantastic Formulas!

Perimeter: measurement of the distance around an object

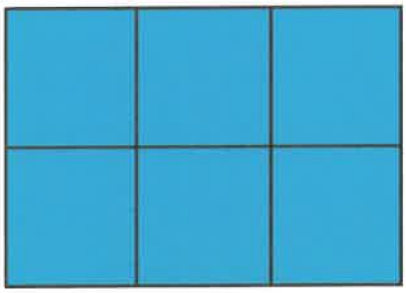


$p = s + s + s + s$

$p = 5 + 5 + 5 + 5$

$p = 20$ in.

Area: measurement of 2D space inside an object

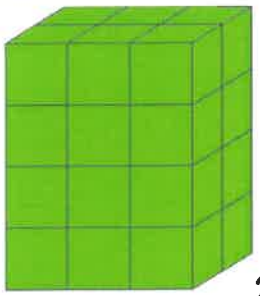


$a = l \times w$

$a = 3 \times 2$

$a = 6$ units^2

Volume: measurement of 3D space inside an object

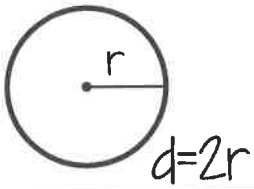
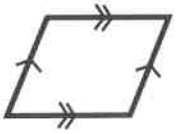
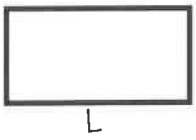
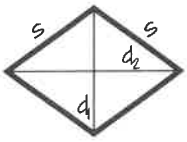
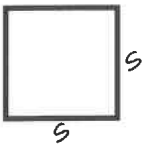
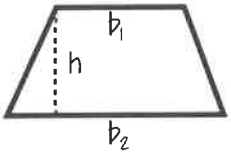
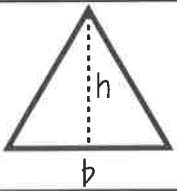


$v = l \times w \times h$

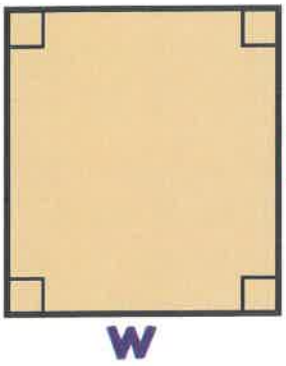
$v = 3 \times 2 \times 4$

$v = 24$ meters^3

Area & Perimeter Formulas

Shape	Area	Perimeter
Circle 	$A = \pi r^2$	Circumference $C = \pi d = 2\pi r$
Parallelogram 	$A = b \times h$	$P = 2b + 2s$
Rectangle 	$A = L \times W$	$P = 2L + 2W$
Rhombus 	$A = \frac{(d_1)(d_2)}{2}$	$P = 4s$
Square 	$A = s^2$	$P = 4s$
Trapezoid 	$A = \frac{(b_1+b_2)h}{2}$	$P = b_1 + b_2 + s_1 + s_2$
Triangle 	$A = \frac{bh}{2}$	$P = s_1 + s_2 + s_3$

How can I find the area of a rectangle?

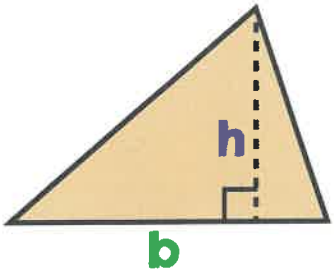


w = width
h = height

$$\text{area} = w \times h$$

Geometry

How can I find the area of a triangle?



b = base
h = height

$$\text{area} = \frac{1}{2} \times b \times h$$

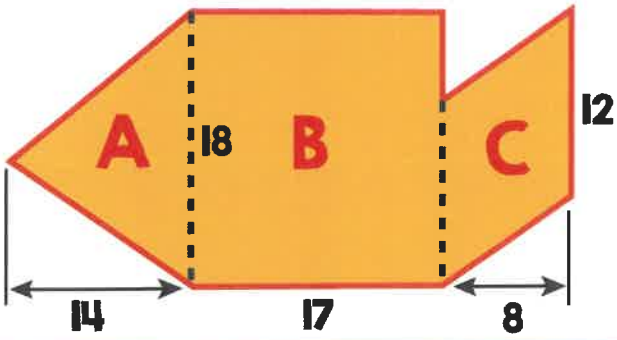
$$\text{area} = \frac{b \times h}{2}$$

Note The base and height must be perpendicular.

Geometry

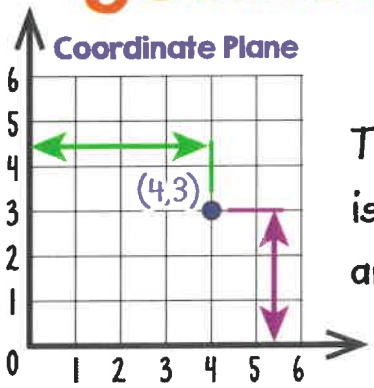
How can I find the area of an irregular polygon?

Divide the polygon into triangles, rectangles, and parallelograms (a, b, c, ...). Calculate the area of each and add them together.



Geometry

What is coordinate geometry?



The point (4,3) is 4 units right and 3 units up.

Coordinate geometry is a system of geometry where the position of points on a plane is described using an ordered pair of numbers.

Geometry

8

Area= L x W

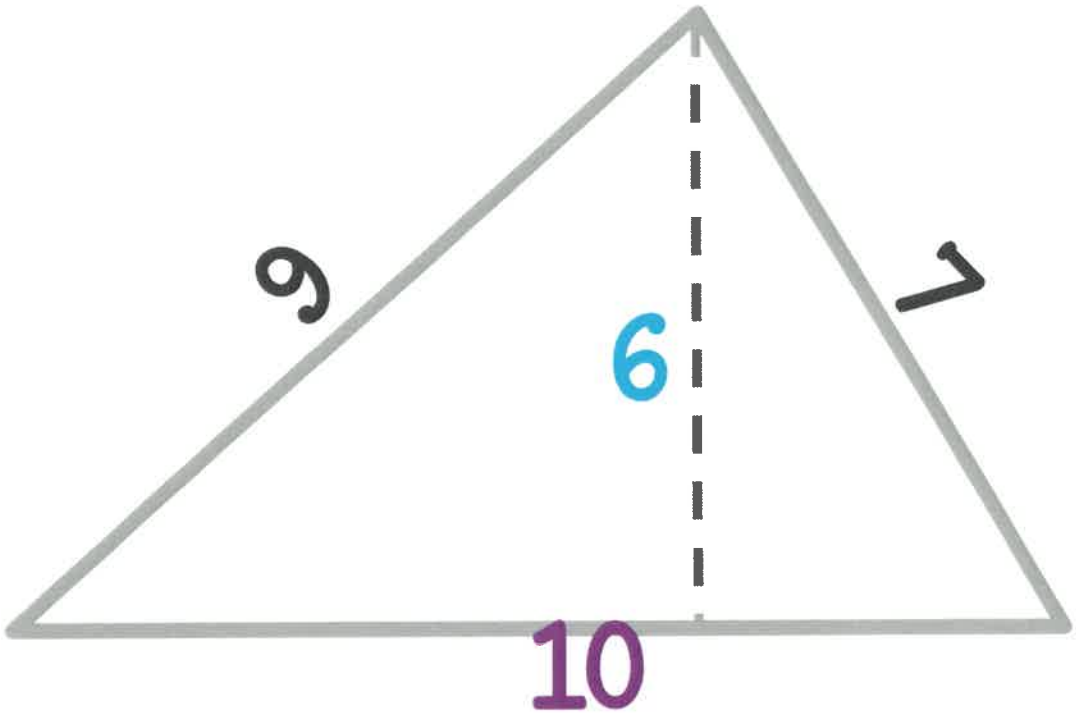
Area = 8x7=56

7 Perimeter= total 7

length around (+)

Perimeter = 8+7+8+7=30

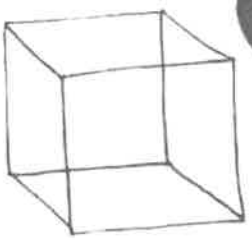
8



Area=(B x H)÷2 (10x6)÷2=30

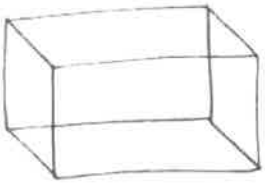
Perimeter= 9+7+10=26

CUBE



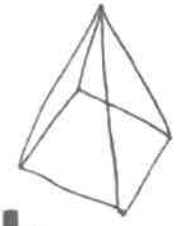
$V = Bh$

Rectangular Prism



$V=Bh$

Rectangular Pyramid

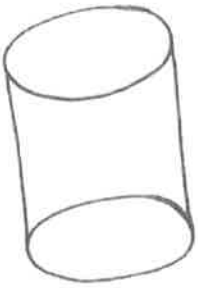


$V=\frac{1}{3}Bh$

Cone

$V=\frac{1}{3}Bh$

Cylinder



$V=Bh$

Sphere

$V=\frac{4}{3}\pi r^3$