

Math: It's All in Your Head

So, how much is 7×28 ? $45 + 28$? 11×54 ? $22 + 63 + 78$? 4200×5 ? Are you picking up your pencil to answer these questions? Are you getting out your calculator? If so, then **wait!** All of these examples and many more like them can be done easily and quickly by simply using mental math.

Cathy Seeley, past president of the *National Teachers of Mathematics* explains.

What does it mean to know mathematics? This is a complex question, but there is strong agreement that facility with numbers and skill in problem solving play important roles. *Principles and Standards for School Mathematics* calls for students to be proficient with tools that include pencil and paper and technology, as well as mental techniques. I would like to make a case for raising the importance of *mental math* as a major component in students' tool kits of mathematical knowledge. Mental math is often associated with the ability to do computations quickly, but in its broadest sense, mental math also involves conceptual understanding and problem solving.

As Ms. Seeley mentions, problem solving plays an important role and the mathematics program at Half Hollow Hills has been increasing its attention to problem solving, kindergarten through twelfth grade. The results of this emphasis can clearly be seen in the increased proficiency of our students in problem solving as well as on test results. So what is involved in "raising the attention to mental math?"

Many of us think of mental math as knowing the basic single-digit addition and multiplication facts. It is important that children have fluency with the basic facts because this knowledge helps students strengthen their number sense and hence problem solving. But facility with mental math goes beyond that. For example, it is useful to know pairs of numbers that add up to 10, 100, or 1000. To compute the sum of $25 + 46 + 75$, students should recognize that the "hundred partner" of 25 is 75. That makes finding this sum a simple mental math computation. Partner 25 and 75 then add 46. The sum is 146. No pencil, no calculator. As is illustrated in this example, mental math problems need not depend so much on rote memorization. To do this problem mentally, the student needs to have a sense of pairs of numbers that add up to 100, and knowledge that addition can be performed in any order (the commutative property of addition).

So...let's learn some more mental math techniques.

Addition of Two Digit Numbers

There are many occasions in life that require us to add. A simple example...John collected 27 seashells and his sister Emily collected 35. How many seashells did they collect altogether?

Most of us learned to do this problem using pencil and paper like so:

$$\begin{array}{r} 27 \\ + 35 \\ \hline 62 \end{array}$$

Relying on the knowledge of place value, we can do the problem in an alternative way that easily lends itself to mental math computation.

27	=	20	+	7	
35	=	<u>30</u>	+	<u>5</u>	
	=	50	+	12	= 62

The basic philosophy....add the tens together, add the ones together, add the two results. A pencil and paper computation turned into a mental math problem.

Here are two more examples.

52 + 23					
	50	+	2		
	<u>20</u>	+	<u>3</u>		
=	70	+	5	=	75

46 + 37					
	40	+	6		
	<u>30</u>	+	<u>7</u>		
=	70	+	13	=	83

Next: Multiplying double digit numbers by a single digit is perfect for mental math.

Suppose you needed to multiply 7×28 . We all know the usual method (called the multiplication algorithm).

$$\begin{array}{r} 28 \\ \times 7 \\ \hline 196 \end{array}$$

Now, consider splitting the two digit number up by its places, that is, $28 = 20 + 8$. We next multiply the 20 by the 7 and the 8 by the 7.

$$20 \times 7 = 140;$$

$$8 \times 7 = 56$$

Now, simply add the results, $140 + 56 = 196$. In mathematics this is an example of using a property of real numbers called the distributive property of multiplication over addition and we usually show the work like this:

$\begin{array}{l} 7 \times 28 = \\ 7 \times (20 + 8) = \\ 7 \times 20 + 7 \times 8 \\ 140 + 56 = \\ 196 \end{array}$
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You're going to have to try some more of these as practice makes you very fast.

Look: 6×42 is simply $6 \times 40 + 6 \times 2$ which is $240 + 12$ or 252

8×35 is $8 \times 30 + 8 \times 5$ which equals $240 + 40$ or 280.

Multiplying two digit numbers times 11.

OK, try to figure out why this next mental math shortcut works...

To multiply any two digit number by 11 using mental math: (for this example I will use 35×11)

- Separate the tens and ones digit creating a space between them as such (3__5)
- Add the 3 and the 5 together ($3+5 = 8$)
- Put the result in the space between the 3 and the 5. 385.
- That is the answer to 11×35 ; $11 \times 35 = 385$

Let's try another.

$45 \times 11 = 495$. Add the 4 and the 5. Put the sum between the two original digits.

One thing to beware of... if the sum of the two digits is greater than 9, you must add the extra 1 to the first digit.

Here is an example.

57×11

- Separate the tens and ones digit creating a space between them as such (5__7)
- Add the 5 and the 7 together ($5+7 = 12$)
- Put the 2 (from the 12) in the space between the 5 and the 7, and add the one (from the 12) to the five giving you 627.
- So $11 \times 57 = 627$.

Moving on...

One last thing...have you ever seen this easy way to multiply by 5?

There is a great shortcut to multiply by 5

$460 \times 5 = 2300$. I did not use a calculator; I did not use pencil and paper. I did simple mental math by following this rule.

To multiply by 5, divide by two, and move the decimal point one place to the right.

So 460 divided by 2 is 230. Move the decimal point one place to the right and you get 2300.

Try 686×5 .

Yes the answer is 3430. That only took about 3 seconds to do. Just divide by 686 by 2 which is 343 and move the decimal point one place to the right, 3430.

If the number you start with is an odd number you will get .5 at the end when you divide by 2. So, 125×5 would be done like this.

125 divided by 2 is 62.5. Move the decimal point and you get 625 for the answer.

When you have some time, see if you can figure out the mathematics behind why this shortcut works. Also, can you figure out a similar shortcut to **divide** a number by 5???

Summary and Samples to Try.

The *National Council of Teachers of Mathematics* states,

Students should become proficient at using mental math shortcuts, performing basic computations mentally, and generating reasonable estimates involving size, distance, and magnitude.

In this article we took a look at a few mental math shortcuts that can enhance children's ability to compute as well as improve their number sense and understanding of math concepts such as the commutative and distributive properties. Each of the mental math shortcuts presented has validation and proof in math concepts that children can try to figure out, i.e., why does this shortcut work? Therefore, not only are the techniques good for computation, but also critical thinking and mathematical discourse.

Well, it's time to become a mental math whiz. Try to do the following computations as fast as you can. **All of them are based on the ideas presented in this article.** Have fun!

Compute using mental math.

1. $35 + 78 + 65$
2. 424×5
3. 25×11
4. $26 + 47$
5. 63×11
6. $45 + 67$
7. 8×34
8. $18 + 89 + 82$
9. $4 + 97 + 6$
10. 7×36
11. 8400×5
12. 3×86

Solutions on next page.

Solutions (answers in bold)

1. $35 + 65 + 78 = \mathbf{178}$ (use the hundred partner)
2. 424 divided by 2 is 212. Move the decimal point. Answer = **2120**
3. 2__5. Add the 2 and 5, place in the blank. Answer **257**.
4. $20 + 40 = 60$; $6 + 7 = 13$; Answer $60 + 13 = \mathbf{73}$.
5. **693** (use the x 11 shortcut)
6. $40 + 60 = 100$; $5 + 7 = 12$; $100 + 12 = \mathbf{112}$
7. $8 \times 30 + 8 \times 4 = 240 + 32 = \mathbf{272}$
8. $18 + 82 + 89 = \mathbf{189}$ (use the hundred partner)
9. $4 + 6 + 97 = \mathbf{107}$ (use the ten partner)
10. $7 \times 30 + 7 \times 6 = 210 + 42 = \mathbf{252}$
11. 8400 divided by 2 = 4200; move the decimal point = **42000**
12. $3 \times 80 + 3 \times 6 = 240 + 18 = \mathbf{258}$

Congratulations on your success with this mental math quiz!