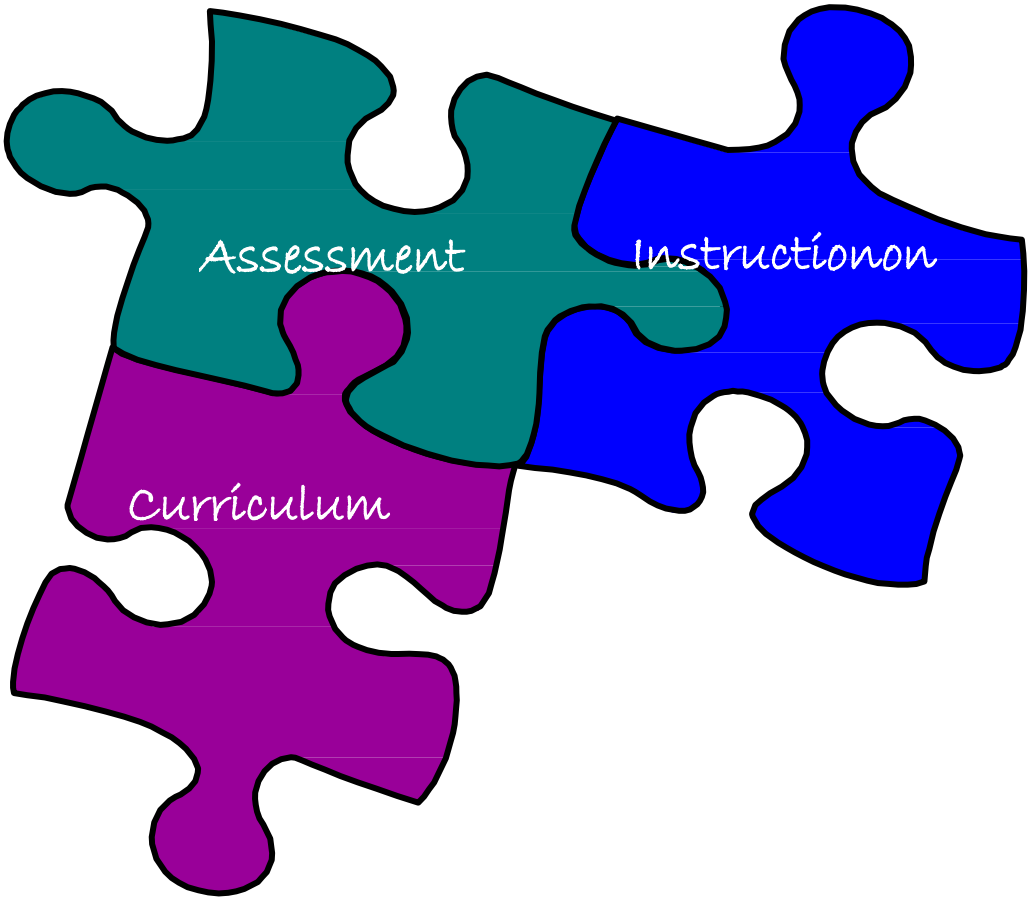
ROCK HILL Schools
YORK COUNTY DISTRICT THREE

Math Expectation Guide



"Putting the pieces together for success in mathematics!"

Math Expectation Guide

Kindergarten through Grade 5

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Written 2008, Updated 2016

Mathematics Instruction in Rock Hill Schools
Kindergarten through Grade 5

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The cover was designed by **Chris Beard**, the principal at Ebenezer Elementary School.

The committee began meeting in the spring of 2008 and continued through the fall of 2008 and early spring of 2009. Once the draft was complete, input was solicited from all committee members and the review team listed above. All comments and recommendations were considered before publishing this draft.

Preface

The Mathematics Expectation Guide is a reference document,that has been prepared to guide educators through the processes that have been deliberately designed to improve student achievement and close achievement gaps. This guide is a resource for teachers not a curriculum. Research shows that teachers directly affect student learning through the design of work that has those qualities that are most engaging to students. Therefore, our instructional goal is to increase, enhance, and meet the academic needs of all students through authentic assessment, standards-based curriculum, and engaging instruction.

This guide revolves around differentiated instruction and learning through an inquiry-based approach. It consists of 3 main sections: assessment, curriculum, and instruction. Each section operates interdependently of the other in order for students to attain their greatest mathematical potential.

To get started, there are a number of strategies and tools that need to be put in place to find out specifics about students. This can be done through the interpretation and evaluation of several different types of assessment and some of these assessment strategies should be on-going throughout the instruction process. In essence, in order to close the achievement gap and increase student achievement, we, as educators, need to understand that assessment assists in determining where to begin instruction. To promote equity, students should have many opportunities to demonstrate THEIR understanding of math concepts and skills and should be given immediate feedback as teachers monitor and reflect on the learning process of students. Assessment is also a valuable tool for teachers to use as they make decisions regarding instruction while continuing to meet the academic needs of all students.

Curriculum is the part of this equation that determines what teachers are to teach as they facilitate students and encourage them to take ownership of their learning. This reference guide provides teachers with a framework of clear and concise explanations of vertically aligned standards as well as sample item representations. The curriculum framework can also be used to differentiate instruction, create flexible groupings, and develop common assessments.

Instruction should be authentic, engaging, and differentiated to meet the academic needs and interests of students through the mathematical processes that develop students' abilities to think critically, problem solve, make connections, communicate their ideas, represent what they have learned in various ways, and make reasonable estimations and provide proof of solutions.

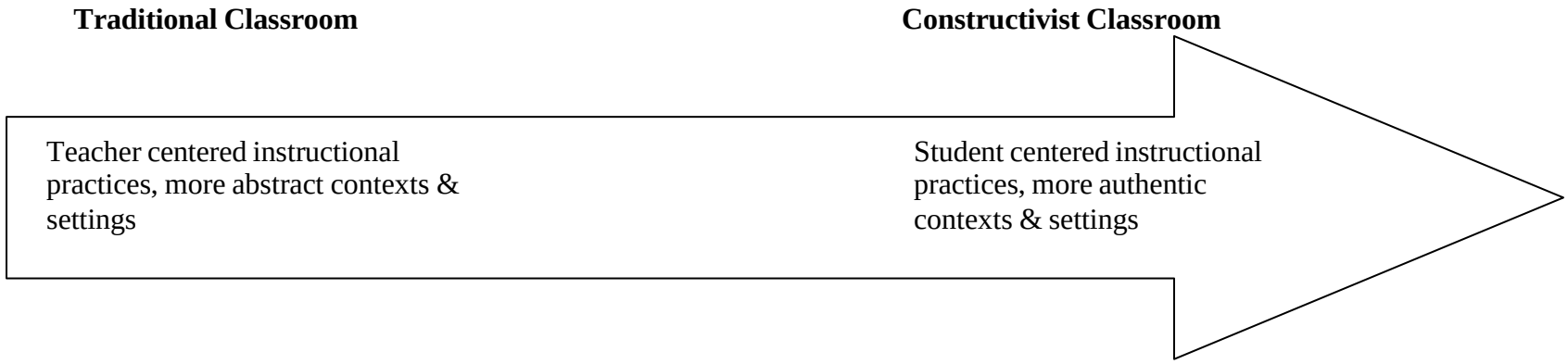
It is our intention that teachers will use this guide to meet the ever demanding and challenging mathematical needs of students as we work towards increasing student achievement and closing the achievement gap.

The Rock Hill School District Professional Code: *Put Students First, Nurture Relationships, Work Together for a Shared Vision, Grow Professionally, Continuously Find Ways to Improve*

Learn+Grow+Connect+Thrive

Traditional Classroom vs. Constructivist Classroom

Through the support of the Rock Hill Schools' Instruction Department and this Math Expectation Guide, mathematics instruction should transition from the traditional classroom approach to learning to the constructivist classroom approach to learning. The teacher's role is not to transmit knowledge to the students, but to create an environment in which students actively explore mathematical ideas through standards-based, engaging investigations. As students investigate and solve problems, important mathematical ideas and concepts will be learned with understanding, not just through memorization. The student's role is to be actively involved in doing mathematics, thinking and reasoning about mathematics, searching for answers and ultimately, taking responsibility for their own learning.



http://www.coe.uga.edu/ctl/theory/external.pdf

The figure on the following page was modified by Shake Seigel from **"Constructivism as a Paradigm for Teaching and Learning"**. http://www.thirteen.org/edonline/concept2class/constructivism/index.html July 2004

Traditional Classroom vs. Constructivist Classroom

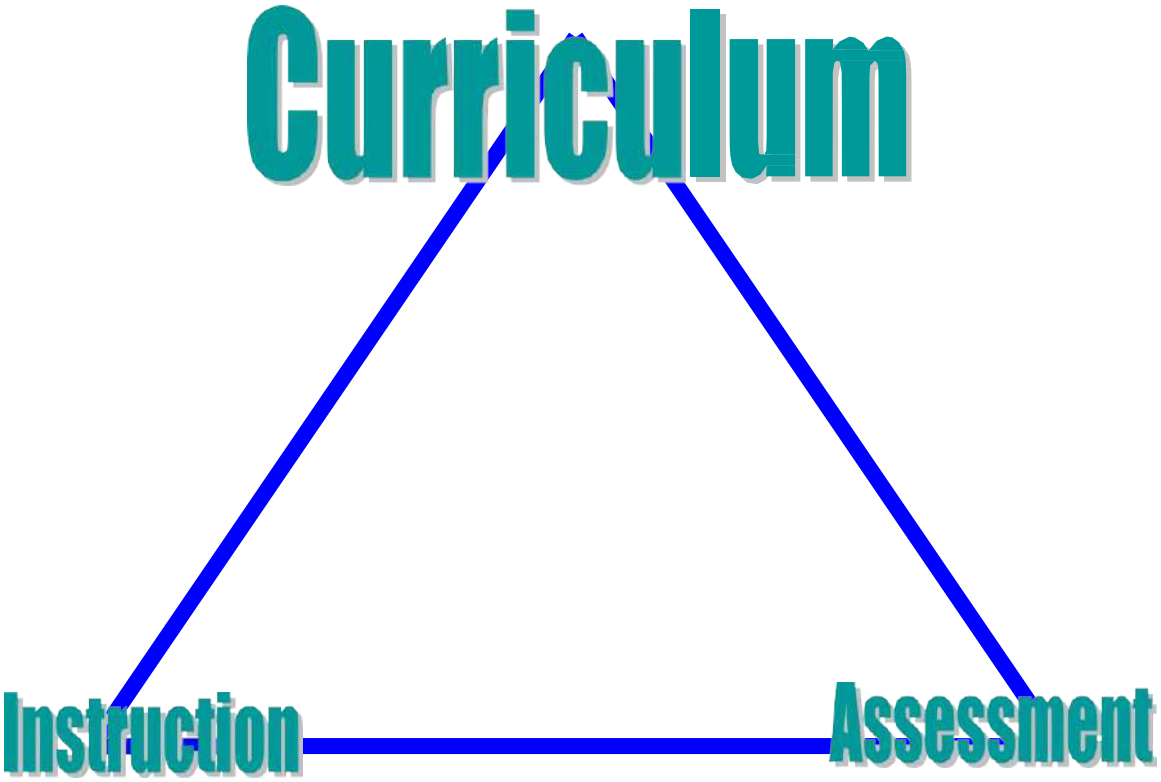
Traditional Classroom	Constructivist Classroom
Curriculum begins with the parts of the whole. Emphasizes basic skills.	Curriculum emphasizes big concepts, beginning with the whole and expanding to include the parts.
Strict adherence to fixed curriculum is highly valued.	Pursuit of student questions and interests is valued.
Materials are primarily textbooks and workbooks.	Materials include primary sources of material and manipulative materials.
Learning is based on repetition.	Learning is interactive, building on what the student already knows.
Teachers disseminate information to students; students are recipients of knowledge.	Teachers have a dialogue with students, helping students construct their own knowledge.
Teacher's role is directive, rooted in authority.	Teacher's role is interactive, rooted in negotiation.
Assessment is through testing, and correct answers are expected.	Assessment includes student works, observations, and points of view, as well as tests. Process is as important as product.
Knowledge is inert.	Knowledge is seen as dynamic, ever changing with our experiences.
Students work primarily alone.	Students work primarily in groups.

Teaching and Learning Mathematics with Understanding

The table outlines the five important dimensions of a classroom that promotes teaching and learning mathematics with understanding and the core features regarding each dimension.
Adapted from: Hiebert et al. *Making Sense: Teaching and Learning Mathematics with Understanding*

DIMENSIONS	CORE FEATURES
Nature of Classroom Tasks	• Make mathematics problematic • Connect with where students are • Should be interesting/engaging • Should encourage communication and reflection • Should allow students to use tools • Leave behind something of mathematical value
Role of the Teacher	• Select tasks with goals in mind • Facilitate, not direct, learning • Encourage a variety of strategies • Encourage mathematical conversations • Differentiate instruction • Establish a mathematical community of learners
Social Culture of the Classroom	• Ideas and methods are valued • Students choose and share their methods • Collaboration and communication are essential • Mistakes are learning sites for everyone • Correctness rests in mathematical argument – not the teacher
Mathematical Tools as Learning Supports	• Meaning for tools must be constructed by each user • Used with purpose – to solve problems • Used for recording, communicating, and thinking
Equity and Accessibility	• Tasks are accessible to all students • Tasks reflect consideration of diverse cultures • Every student contributes and every student is heard

Curriculum



"The curriculum is to be thought of in terms of activity and experience rather than knowledge to be acquired and facts to be stored." Haddow Report UK 1931

The South Carolina College and Career Readiness Standards were implemented in Rock Hill Schools during the 2015-2016 school year. "The content standards in combination with the Mathematical Process standards represent a balance of conceptual and procedural knowledge and specify what mathematics students will master in each grade level. The content standards and process standards work together to enable all students to develop the world class knowledge, skills, and life and career characteristics identified in the *Profile of the South Carolina Graduate*."
To access the complete standards for each grade level and support documents developed by the state, click here:
http://ed.sc.gov/instruction/standards-learning/mathematics/

Rock Hill Schools Support Documents were developed to outline a suggested pacing for each grade level, as well as standards rationales, lesson ideas, tasks, and number sense routines/talks. These documents are aligned to the Rock Hill Schools curriculum maps that tell teachers when specific standards are to be reported on the report card.

Standards to be Reported on Report Card			
Quarter 1	**Quarter 2**	**Quarter 3**	**Quarter 4**
K.NS.1 (by ones to 20) K.ATO.6 K.G.1 K.G.2 K.MDA.1 K.MDA.2	K.NS.1 (by ones to 50) K.NS.4 K.NS.5 K.NS.9 K.G.3 K.G.4 K.MDA.3 K.MDA.4	K.NS.1 (by ones and tens to 100) K.NS.3 K.NS.6 K.NS.7 K.NS.8 K.G.5 K.ATO.1	K.NS.2 K.ATO.1 K.ATO.2 K.ATO.3 K.ATO.4 K.ATO.5

Mathematical Process Standards

Standards Rationale:

Mathematical Processes are the foundation for which students develop a mathematical mindset. These processes are embedded in content, but in kindergarten these will specifically be addressed as classroom procedures, routines, and norms/expectations are being taught at the beginning of the year.

1.Make sense of problems and persevere in solving them.

a. Relate a problem to prior knowledge.

b. Recognize there may be multiple entry points to a problem and more than one path to a solution.

c. Analyze what is given, what is not given, what is being asked, and what strategies are needed, and make an initial attempt to solve a problem.

d. Evaluate the success of an approach to solve a problem and refine it if necessary.

2. Reason both contextually and abstractly.

a. Make sense of quantities and their relationships in mathematical and real-world situations.

b. Describe a given situation using multiple mathematical representations.

c. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation.

3. Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others.

a. Construct and justify a solution to a problem.

b. Compare and discuss the validity of various reasoning strategies.

c. Make conjectures and explore their validity.

4. Connect mathematical ideas and real-world situations through modeling.

a. Identify relevant quantities and develop a model to describe their relationships.
b. Interpret mathematical models in the context of the situation.
c. Make assumptions and estimates to simplify complicated situations.
d. Evaluate the reasonableness of a model and refine if necessary.

5. Use a variety of mathematical tools effectively and strategically.

a. Select and use appropriate tools when solving a mathematical problem.
b. Use technological tools and other external mathematical resources to explore and deepen understanding of concepts.

6. Communicate mathematically and approach mathematical situations with precision.

a. Express numerical answers with the degree of precision appropriate for the context of a situation.
b. Represent numbers in an appropriate form according to the context of the situation.
c. Use appropriate and precise mathematical language.
d. Use appropriate units, scales, and labels.

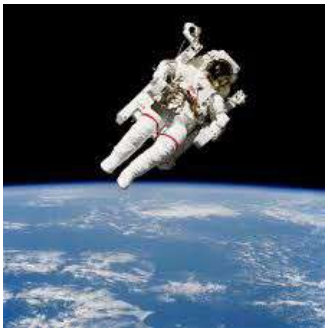
7. Identify and utilize structure and patterns.

a. Recognize complex mathematical objects as being composed of more than one simple object.
b. Recognize mathematical repetition in order to make generalizations.
c. Look for structures to interpret meaning and develop solution strategies.

The development of Mathematician statements will provide students the opportunity to practice these process standards. Students in kindergarten are at various levels when it comes to understanding numbers, quantities, and their uses. Therefore, they will be provided opportunities in the first week or so of school to demonstrate what they already know, and apply it to some new situations.

Sample Task: We Are All Mathematicians!

Students will understand that we are all mathematicians and that there are various characteristics and behaviors of mathematicians. Tell students, “We are all mathematicians! What do you think a mathematician is? What do you think they do? Do you know any mathematicians? Let’s look at some pictures of mathematicians.” Display/pass around various pictures of people who are mathematicians, such as the ones below.



Provide directions for how students will look at the pictures in pairs or small groups. Provide enough time for students to look at the pictures and talk about what they see amongst themselves.

As students look at and talk about pictures, the teacher should observe students, looking to see if students use any mathematical language (informal or formal), looking to see if students

count things in the pictures, looking to see how students interact with one another, observing students and taking notes on how they "mathematize" the pictures.

After students have had some time to look at pictures, gather the whole group and ask, "What did you notice?" It's important at this age for students to develop the understanding that we talk about math. Ask students, "What are you wondering?" You might record students' wonders, as something to refer back to as you continue the first week with Mathematician Statements.

See more detailed Week 1 Lesson Plan here.

Sample Task: Developing Mathematician Statements
Spend the first 3-5 days of school creating a culture of learning and developing mathematical mindsets through mathematician statements with students. Here are some examples of mathematician statements. It is important not to get stuck with these, but to let your students help you develop the statements for your classroom.

We've provided sample activities to introduce each statement. (Introduce 1 statement per day, then allow students to "practice" it as a mathematician.)

- We are all mathematicians! What is a mathematician and what do they do? (Picture Activity)
- Mathematicians are curious! (What are you curious about? Around the room)
- Mathematicians talk about their work and share their thinking! Mathematicians tell stories. (Read Aloud a counting book and/or use a picture book with only pictures (no text) and make up a mathematical story to go with it.)
- Mathematicians use tools! Tool exploration lets students explore and talk about how they might be used (Put math tools in tubs/toolboxes and provide time for students to explore with the tools. Then ask, "How might we use these tools?")
- Mathematicians make mistakes, and learn from them!

More information on developing mathematician statements to build mathematical identities can be found here.

The development of mathematical identities and mindsets is a process and should not stop after these activities. These statements, that you develop with your students, should be reviewed and referred to in lessons daily. Point out when a student demonstrates one of the statements.

Number Sense Standards and Suggested Instructional Strategies

Standards Rationale: The kindergarten day should lend itself to multiple opportunities for students to use math throughout the day. There might be more structured math lessons,

however a majority of Number Sense concepts should be embedded throughout the day with meaningful opportunities for students to count.

Calendar Routines: Calendar routines are a great way to embed many different number sense concepts on a daily basis. The calendar routine does not need to be complicated or take too long, in fact it should only take 15 minutes or less.

Sample Calendar Activities:

A recommendation is to purchase a blank desk calendar from an office supply store, and display up to 3 months at a time, one month before, current month, and next month. This will allow students to see the progression of things over time. To introduce the calendar at the beginning of the year, gather students in the class meeting area and display the calendar so students can easily view it.

Ask students, “What do you notice about the calendar? What do you wonder about the calendar?” Take all suggestions equally, students will probably notice things like the numbers, but they also might notice things such as lines or “boxes”, or colors on the calendar. If students do not comment on the important features then mention them as your notice/wonder.

Explain to students that your class will use the calendar all year to keep track of what day it is and important events happening at school. Help students understand where the month is located, what the letters and numbers on the calendar represent, and ask students to come point/locate the different parts of the calendar.

Show students a “Today” marker (such as a star or other die cut). Tell students that this “marker” will help to keep track of which day it is. Ask students to help you figure out what number today is by looking for the “Today” marker. Finally ask students to count up to that number with you.

As the year progresses, add special dates to the calendar (student birthdays, important events happening at the school, holidays, etc.)

Different Calendar Prompts/Questions

- Read today’s date.
- Count the days in this month.
- What is an important date we should put on our calendar. Have a student locate the date and help write it in the appropriate place on the calendar.
- If __(month)____ ends on a Monday, what day of the week will _(next month)____ begin on ? What about _____?
- How many days until _____?
- How many weeks (or months) until your birthday? (or other exciting days)
- Is it three or four days until your birthday?
- If Monday is _____(insert date, for example March 17th), what day of the week will it be on the _____ (20th)?
- Find the 9.
- Find the number that represents this amount (show a card with 5 dots).
- Find the number that is one less/more than _____.

- Find a number more than 5.
- Find a number less than 10.
- What patterns do you notice? How does that help you read the calendar? Does it make sense?

K.NS.1 Count forward by ones and tens to 100.

Sample Activities:

- Read Aloud Counting Books
- Count Around the Circle (begin with ones, as the year progresses count by tens)
    - While it's not expected in the standard, students should be exposed to counting backwards and this can easily be done through Count Around the Circle. (This could be done later in the year, not at the beginning.)
- Calendar

The Counting Jar
(Investigations Unit 1, Session 2.1)
At the beginning of the year, read a book such as Mouse Count by Ellen Stoll Walsh. Introduce a jar with 7 cubes inside and tell students this is the class Counting Jar. Ask students, "What do you notice?" What do you wonder?" (Possible student response: "I notice there are cubes inside. I wonder how many cubes are inside?")

Ask student volunteers to show how they would count the cubes. Ask the rest of the class to watch closely and tell what they notice about how each person counts.

Tell students that you will put out some other objects (pennies, tiles, buttons, beans, etc.) and that you will count out another set of seven objects/things to match the set in the Counting Jar.

Count out another set of seven objects and ask students to re-count and check your work. Then pass out various objects to students and ask them to count out a pile of seven objects to match yours and the Counting Jar.

Ask some students to share how they counted their objects. "I counted_____."
Close this lesson by telling students that the Counting Jar will be a station for them to work at throughout the year. They will be practicing counting the objects in the jar and then count another set of objects to match how many are in the Counting Jar.

Formative Assessment:
When students visit the Counting Jar station, check to see:

1. Who knows the names of the numbers in order? (count sequence)
2. Who counts objects once and only once? (1:1 correspondence)

3. Who has a system for keeping track? (1:1 correspondence)
4. Who double checks their count?

*To find out more about the complexity of counting see **Investigations, Unit 1, page 127**.

K.NS.2 Count forward by ones beginning from any number less than 100.

- Read Aloud Counting Books
- Count Around the Circle
- Calendar

K.NS.3 Read numbers from 0-20 and represent a number of objects 0-20 with a written numeral.

- Read Aloud Counting Books

It is very important that written numerals are ***never*** separated from quantity. Students should always build an understanding of quantity before they begin to match them with abstract numerals.

Counting Objects:
Materials: Bags of objects for each student, number cards 0-20 differentiated based on students' working number at first, then all, index cards or sticky notes
Directions:

1. Students draw a number card from the stack.
2. Then count out that many objects.
3. Students then write the number to match the set of objects they counted on an index card or sticky note.

K.NS.4 Understand the relationship between number and quantity. Connect counting to cardinality by demonstrating an understanding that:

a. The last number said tells the number of objects in the set (cardinality);
b. The number of objects is the same regardless of their arrangement or the order in which they are counted (conservation of number);
c. Each successive number name refers to a quantity that is one more and each previous number name refers to a quantity that is one less.

There is a large and important difference between rote counting (NS.1) and purposeful counting (NS.4 & NS.5). Purposeful counting happens when students have constructed an understanding of quantity and connected the number name with a perception of the size (or amount) of a specific quantity.

It is very important to ask students after they have counted a set of object, "How many did you count?" We should never assume that they know the amount just because they used one to

one correspondence and counted in the sequence correctly. Unfortunately, some students can mimic the counting sequence and the actions seen in counting activities, but still have no idea what they just did. The question, “How many did you count?” helps us determine if a student has an understanding of quantity.

Students must also recognize that arrangement, size, placement, or position has no impact on quantity. They can rearrange a pile and know that as long as no objects were added or taken away, they quantity of that pile remains the same.

Finally, students begin to think and count operationally as objects are added or taken away, one at a time, resulting in a change in the position within the number sequence. If one more is added, students know that the quantity has now shifted up one on the number sequence.

None of these skills can be directly taught. They can only be assessed by asking strategic questions (e.g. “How many did you count?” or “How many would there be if I rearranged them like this?” or (add one more) “Now how many? How do you know?”

K.NS.5 Count a given number of objects from 1-20 and connect this sequence in a one-to-one manner.

Counting objects is much more difficult than rote counting for a variety of reasons. First, students have to remember the rote sequence as they are counting. Next, they must make sure that they assign one number name to exactly one object (all while building their fine motor skills). Finally, they need to remember which objects they have already “touched” or “counted” to ensure that they are keeping track. This proves to be much more challenging than simply reciting a counting sequence.

It is important to note that students will not learn by watching the teacher. They might begin to mimic certain actions, but true learning happens when they get to physically count objects, estimate, question, and discuss their thinking and strategies. Therefore, students should be given plenty of opportunities to do so.

Sample Activities

- The Counting Jar (see above)
- Counting Collections

K.NS.6 Recognize a quantity of up to ten objects in an organized arrangement (subitizing).

There are two types of subitizing with which kindergarten teachers need to be familiar: perceptual and conceptual subitizing. Perceptual subitizing is the ability to quickly identify up to 5 objects without counting or even adding. Students build their perceptual subitization gradually, from 2 to 5, over the course of time. Students who are able to perceptually subitize will instantly recognize small groups of dots and represent small quantities without counting using fingers, blocks, or other manipulatives.

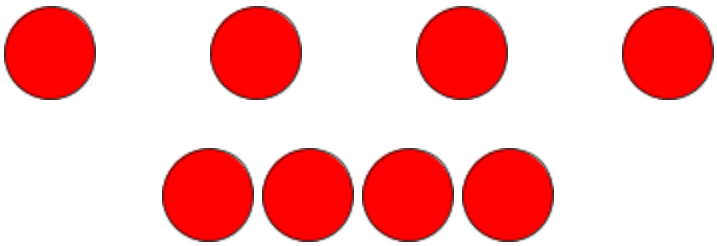
Conceptual subitizing, on the other hand, requires that students perceptually recognize smaller groups and combine those small groups to determine the total amount of a larger group. Students should point out what groups they used to determine the sum and match it with an equation.

Sample Activities:

- Quick Images (up to 3 objects- perceptual subitizing to begin) Build from there as students demonstrate readiness for larger quantities.
- Ten Frame Flash (subitizing and combinations to ten)

K.NS.7 Determine whether the number of up to ten objects in one group is more than, less than, or equal to the number of up to ten objects in another group using matching and counting strategies.

Many kindergarteners might be able to identify “more” or “less” in some cases based on previous experiences. However, we must make sure that their assignment of the adjectives “more” and “less” (or “fewer”) are based on quantity and not physical size. In the classic Piaget Conservation Experiment, students identified a row of four that was spread out had “more” than a row of four that was close together:



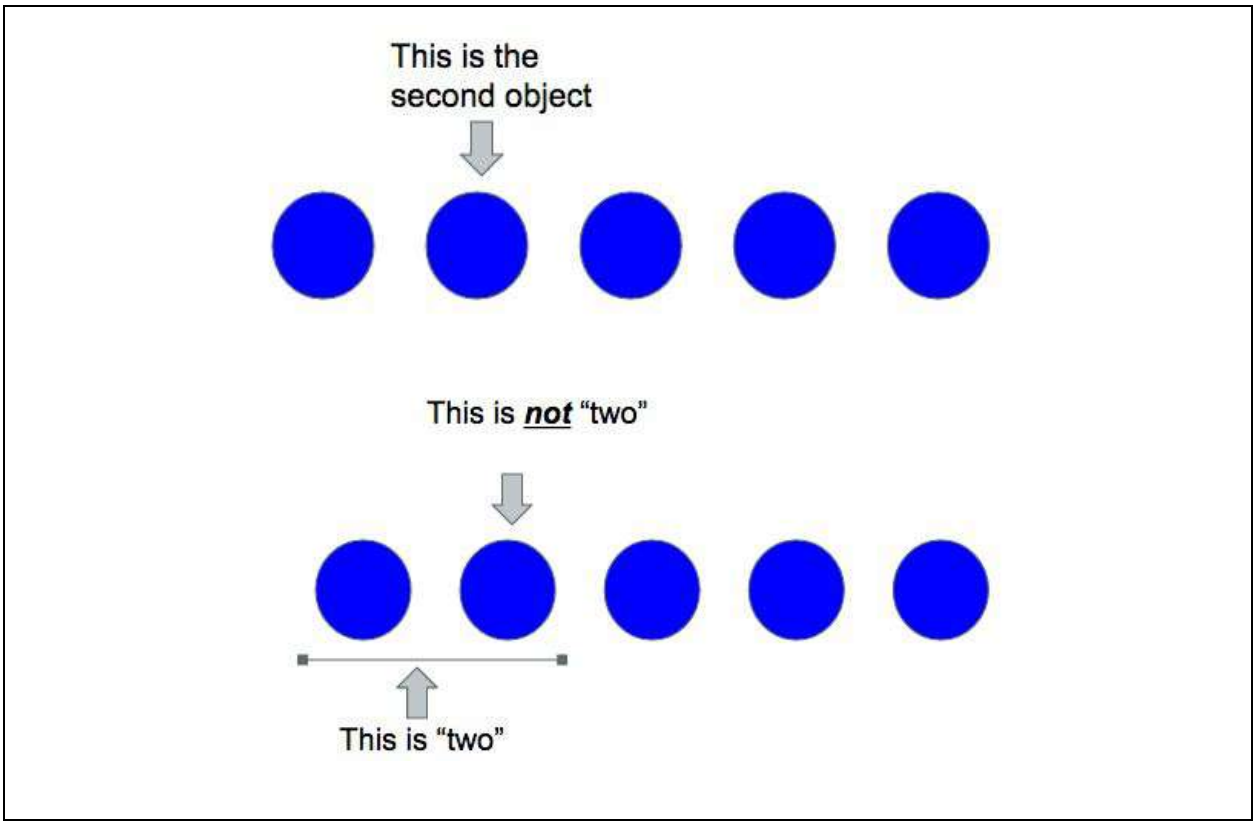
The problem is that students equate “more” with “larger”. Our goal in this standard is to have students understand that “more” and “less” relate to the quantity (NS.4). Once students have an abstract understanding of quantities 1 - 10, comparing will be simple.

K.NS.8 Compare two written numerals up to 10 using *more than, less than, or equal to.*

See standard above.

K.NS.9 Identify first through fifth and last positions in a line of objects.

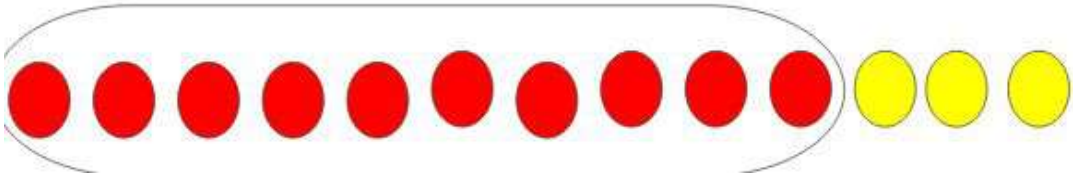
It is very important that students recognize the difference between “first” and “one”, “second” and “two”, etc… First through fifth establish position while one - five establish quantity. When referring to something as the “second” in a certain order, they are only referring to one object. When using the adjective “two”, however, students are referring to a group of objects.



Number Sense in Base Ten Standard and Suggested Instructional Strategies

K.NSBT.1 Compose and decompose numbers from 11-19 separating ten ones from the remaining ones using objects and drawing.

Standard Rationale:
This standard is the beginning foundation for students to begin developing number sense in base ten. However, this standard does not expect students to unitize. Students will be expected to do that in first grade (1.NSBT.2). In kindergarten, students are just learning to count 1:1. Therefore, many will still be developing that skill, and to introduce units of 10 would be confusing. While this standard is not reported on the report card until the 4th quarter, it's highly recommended to introduce this concept as soon as students are confident composing and decomposing ten (K.ATO.3). Students will use pre-place value understanding and apply a "count by ones" approach to quantity. For example 18 means 18 ones to students at this stage. If they count 18 bears, they might think that the 1 in the number is 1 bear, and the 8 is 8 bears. At this level students will not understand place or value of numbers in regards to their place, and that is NOT what this standard expects. For this standards students are expected to count out **a group of 10 ones and extras**, or singles. See example below:



Sample Task: Counting Squares (from Graham Fletcher)
https://gfletchy.com/counting-squares/
Introduce students to the task by showing the picture of squares from the link above. Ask, "What do you notice? What do you wonder?"
After accepting student responses equally, ask students to estimate how many squares are in the pile.
Then ask students, "How many red squares? How many yellow squares?"
Show students the video for Act 2.
Then provide students yellow and red tiles to count (they will need 19).
Allow students to count their tiles. (Watch to see if students separate the colors or if they just count all of the squares/tiles.)
After students have had time to count their squares, Show Act 3.
Have a brief class discussion to ask students to share how they counted. "Who counted all of the squares? Who separated the yellow and red, and then counted?"
"Which counting strategy would be most efficient to find out the number of red and the number of yellow squares? Why?"

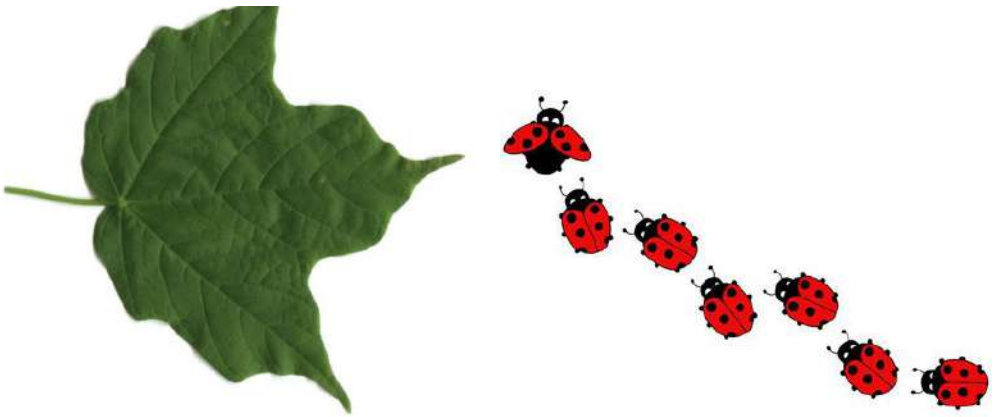
Algebraic Thinking and Operations Standards and Suggested Instructional Strategies

K.ATO.1 Model situations that involve addition and subtraction within 10 using objects, fingers, mental images, drawings, acting out situations, verbal explanations, expressions, and equations.

This standard expects students to demonstrate understanding of situations that involve addition and subtraction in various problem types (part-part-whole, joining, separating, etc.) Notice this standard says "MODEL", while the next standard asks students to solve. That's not to say that students can't do the modeling and solving simultaneously, however initially they may only model. It's the questioning of the students that will push their thinking to the solution.

Sample Activities:

- **Math Story Mats** (Small Group w/ teacher or Center)
    - Provide students with a story mat (great full color real life ones here.)
    - The teacher tells a story, and students model the story with their objects/counters. See example below:



Tell the story orally, “I was cleaning up my yard and found some ladybugs on this leaf.”
Provide students the leaf story mat. (Students can use ladybug counters, printed pictures of ladybugs, or other counters.)
“I counted the ladybugs. How many ladybugs do you think could have been on that leaf?”
Provide time for students to guess and put their guess on the leaf.
“There were 5 ladybugs on the leaf. Show 5 ladybugs on the leaf.”
“2 of the ladybugs flew off the leaf.” Provide time for students to remove 2.
“How many ladybugs are still on the leaf?”
Continue telling stories and have students model the stories on their mat.
Extension: Ask a student to make up a story for others to model. Teacher should also participate in modeling.

- **Daily Math Stories**
    - Ask students to make up a story about a number. The number might be the day on the calendar, the number of students sitting on the carpet, or a “number of the day”.

Example: “Today we have a 6 on the calendar for June 6, 2016. Who would like to tell us a story about 6?” Provide prompts and props (counters) for students to use if needed.

K.ATO.2 Solve real-world/story problems using objects and drawings to find sums up to 10 and differences within 10.

Sample 3-ACT Task: Humpty Dumpty from Graham Fletcher
https://gfletchy.com/humpty-dumpty/
In this engaging task students will view a short video of a girl taking eggs out of the refrigerator for her dad. She drops the eggs. Students will be asked, “What do you wonder?” Then students will inquire about how many eggs did not break. This is a great way to activate students’ thinking and see how they go about solving a real-world problem.

Further story problems to go with this context of eggs:

1. I need 6 eggs to make chocolate chip cookies. My carton of eggs has 10 eggs in it. How many eggs will I have after baking the cookies?
2. On Saturday we used 4 eggs to make breakfast. On Sunday we used 3 eggs to make breakfast. How many eggs did we use over the weekend?

3. Emily has 1 hard boiled egg in her lunch. Sydnee has 2 hard boiled eggs in her lunch. How many hard boiled eggs are in their lunches?

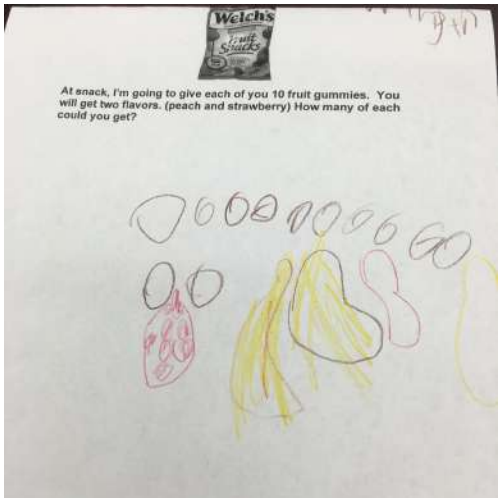
K.ATO.3 Compose and decompose numbers up to 10 using objects, drawings, and equations.

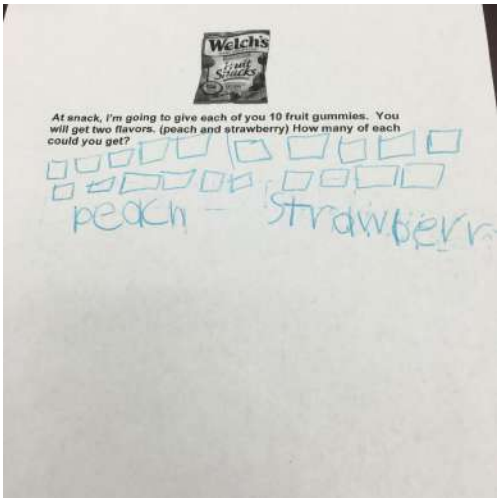
Standard Rationale:
This standard is essential for students to begin looking at how numbers can be composed and decomposed into different parts. As students begin, they will use objects, then progress to drawings, and finally they will represent their compositions/decompositions with equations. In kindergarten, students are only expected to reach the abstract stage with equations within sums of 5, because of the fluency standard (K.ATO.5). However, if students are at a higher level of understanding and ready, they should be encouraged to write equations through whatever number they are ready.

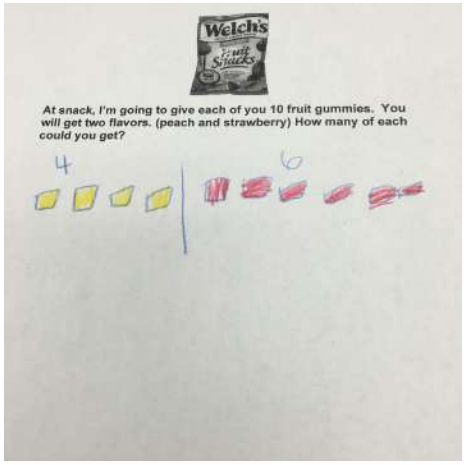
Sample Game:
Mingle, Mingle Game

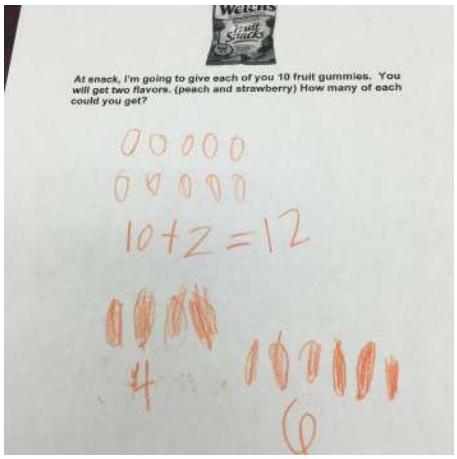
Sample Task: Gummy Snacks
Materials: Objects to count, Fruit Snacks 10 per students, paper & pencil
Directions: Tell students that you are going to pass out gummy fruit snacks today. Everyone will get two different flavors/colors, but everyone will get a total of 10 fruit snacks. How many of each flavor/color could each person get?

Below are student work samples for this task. What information can you gain from each student's response?









K.ATO.4 Create a sum of 10 using objects and drawings when given one of two addends 1-9.

Standard Rationale:
This standard is building on the previous three standards, moving students towards fluency within 5 and composing through 10. Students should continue to use Story Mats and real-world story problems for this standard. The baseline expectation is that students can do this standard with objects and drawings, but if students are ready, they can be encouraged to use equations to represent what they did with objects or drawings. The teacher can model equations that match story problems, but students are not expected to do that abstract level on their own.

Examples:
1. You get 10 gummies for snack. You already have 2. How many more do you get?
2. I need to fill up our Counting Jar with more Hershey Kisses. I already have 5 in the jar, but I need 10 total. How many more Hershey kisses do I need?
3. Mrs. Smith needs 10 student helpers. She wants boys and girls. How many boys will get to help her if she already has 4 girl helpers?

Sample Activity:
Read **Ten Black Dots**
Provide students with a number of black dots less than ten, cut from construction paper.
Ask students, “How many black dots did you get?”
“How many more dots do you need to have Ten Black Dots?”

Sample Ten Frame Activities:
Provide students with a ten frame and some counters. (two color or others)
Tell oral stories similar to those you told earlier using the story mats, but have students model them on the tens frame.

Sample Activity: Make 10 Students have number cards 0 - 10. They lay out 6 cards face up and collect pairs that make 10 (e.g. 6 and 4, 8 and 2, etc…). They then replace those cards

with more from the deck.

K.ATO.5 Add and subtract fluently within 5.

“Fluently and fluency describe a student’s ability to compute with accuracy, flexibility, and efficiency. (Kilpatrick, Swafford, and Findell, 2001) Speed and time are not fluency. This standard is the abstract level of thinking in a concrete-representational-abstract progression, therefore students will develop at different rates. Some might reach fluency earlier than others.” SCCCR Math Support Document.

Fluency is built through an understanding of number positions and number proximity. Students begin by counting on their fingers because it is a convenient and dependable manipulative. They will physically use their fingers to operate within 5 (not fluent) and move to mentally picturing their fingers to operate within 5. For brain research on the use of fingers (physically and mentally), click here.

There are not really any separate tasks for fluency. However, students should be observed and interviewed to identify if they are fluent or not. The games below are ways that students can build on the other ATO standards to practice and develop fluency.

Sample Games:

- Mingle, Mingle Game (see link above)
- Dice Games
    - Partners get 1 die, player 1 rolls a die, Player 2 rolls the die. Students add their two numbers they rolled together to find the sum.
- Card Games
    - Provide digit cards 0-5 to partners, Player 1 draws a card, Player 2 draws a card. Students then add or subtract the digits to get a sum or difference. *Extension:* Students could write the equation to match the cards they drew in their math journals,on paper, or white boards.

K.ATO.6 Describe simple repeating patterns using AB, AAB, ABB, and ABC type patterns.

Students are not expected to name patterns (i.e. “That is an AB pattern.”) but can describe the pattern in the context (i.e. “That is yellow blue, yellow blue…”).

Sample Activity:

Show 2 different types of patterns using concrete objects (such as color tiles) and ask students “What’s the same?” “What’s different?” or “What do you notice?” This helps students make sense of the order of things, which is foundational for the study of mathematics. Patterns are how we organize and give order to situations.

- After students have had multiple experiences with concrete objects, show 2 different pictorial patterns and ask, “What’s the same? What’s different?”

Pattern 1

Pattern 2

Sample Task/Activity: Playdough Kabobs

This task is a way to engage students in the thought of patterns, as well as a way to formatively assess what students already know about patterns. Connect students to the picture activity about mathematicians from earlier. “We learned that chefs use math.” “Today we’re going to pretend that we are chefs!”

Show students the picture and ask:
“What do you notice?” Record student responses.
“What do you wonder?” Record student responses.
Students might recognize there is a pattern, some might say the pattern, “red, green, brown,” or “red pepper, green pepper, steak,” depending on their experiences. Continue with more discussion as necessary based on student notices/wonders.
Tell students that they are going to create their own kabobs using playdough and sticks/skewers.
Provide students with their materials (playdough and skewers) and let them create patterns.
As students work, circulate and ask questions such as, “Can you count your playdough balls on your kebab for me?” “How many playdough balls did you use to create your pattern?” “What pattern did you create?”
Additional questions could be, “What comes first in your pattern?” “What is second? third? fourth? fifth? last?”

Whole Group Discussion: Gather students together to a central location and discuss how they created their kebabs/patterns, what they noticed, and ask individual students to describe or

share their kebab/pattern with the group. If possible, share 1 kabob of each pattern type (AB, ABC, AAB, ABB).
This activity could be continued over multiple days.

Other Sample Activities:

- Sing nursery rhymes with students such as Old McDonald Had a Farm, ask them, what is repeated? (EI-EI-O)
- Demonstrate different patterns using movements such as, Clap, Snap, Clap and ask students to repeat after you and describe what the pattern is.
- Display various pictures of patterns and ask students to describe what they notice/ see. Make note of students who don't see anything that repeats (these students have not yet realized there is structure in the pictures). Make note of students who describe the different pattern types.

Geometry Standards and Suggested Instructional Strategies

K.G.1 Describe positions of objects by appropriately using terms, including *below, above, beside, between, inside, outside, in front of, or behind.*

Standard Rationale:
This standard can be addressed through the Numbers at School activity described above. It can also be addressed as students describe things in the classroom. This standard should be embedded through daily dialogue with students, *not* isolated through direct teaching.

K.G.2 Identify and describe a given shape and shapes of objects in everyday situations to include two-dimensional shapes (i.e. triangle, square, rectangle, hexagon, and circle) and three-dimensional shapes (i.e. cone, cube, cylinder, and sphere).

Standards Rationale:
Students are ***not*** expected to describe the attributes (number of sides) of shapes, but simply identify the shape in real life and make connections with other similar real-world objects that are that shape. Describing attributes is the baseline expectation for 1st grade (1.G.1).

Sample Activities: Shape Hunt
Provide students with a shape (i.e. triangle, square, rectangle, hexagon, circle, cone, cube, cylinder, sphere).
Ask them to find something in the classroom that is the same shape.

Have students share their shape and the classroom item they found that matches it. Have students describe why they are the same and ask them to name the shapes.
This activity can be done multiple times over several days or throughout the year so that students get to hunt for the different shapes, rather than only one shape.

K.G.3 Classify shapes as two-dimensional/flat or three-dimensional/solid and explain the reasoning used.

This standard integrates well with K.MDA.3. Kindergarteners are not expected to use the terms two-dimensional or three-dimensional, they only need to use the terms “solid” or “flat” when classifying the shapes.

Sample Activity: Shape Sort (Small group, Small group w/ teacher, Center/Independent)
Provide students with a bag of different two- and three- dimensional shapes.
Ask students to sort their shapes, and then ask them why they sorted the shapes as they did.

Students can then create object graphs to see how many of each shape they have in their bag.

K.G.4 Analyze and compare two-and three- dimensional shapes of different sizes and orientations using informal language.

Standard Rationale:
This standard is important for students to begin looking at similarities and differences. Often students can describe things that are “the same”, but have more difficulty with “differences”.

Sample Activity:
Provide students with a bag of different shapes. (5-6 shapes in each bag)
Ask students to take out their shapes.
Ask, “What do you notice?”
On a different day you might only put out 2-3 shapes and ask students, “What do you notice?”

K.G.5 Draw two-dimensional shapes (i.e. square, rectangle, triangle, hexagon, and circle) and create models of three-dimensional shapes (i.e. cone, cube, cylinder, and sphere).

Measurement and Data Standards and Suggested Instructional Strategies

K.MDA.1 Identify measurable attributes (length, weight) of an object.

Although the baseline requirement is *only* to identify the attributes, it would be appropriate to allow students to ask and investigate the actual measures using informal and nonstandard measuring tools (paper clips, blocks, tiles, etc…). Students should ***not*** use rulers or other measuring tools.

K.MDA.2 Compare objects using words such as *shorter/longer, shorter/taller,* and *lighter/heavier.*

There should not be a specific unit or lesson on this standard, but it should be embedded in daily vocabulary when investigating shapes, objects, and having conversations driven by student curiosity.

K.MDA.3 Sort and classify data into 2 or 3 categories with data not to exceed 20 items in each category.

At the beginning of the school year teachers are getting to know their students. Because kindergarten students have varied backgrounds and experiences, teachers helping them get to know "school". This standard lends itself to sorting and classifying information personal to students, as well as to school and the classroom.

Sample Task: Math About Me
"What numbers do you use in your life and how are they used?"

Possible student responses: *my age, number of people in my house, number on my house, phone number, size of shoe, clothing size, etc.)* Teachers can record student responses as a Shared Writing lesson.

Sample Task/Activity: Numbers at School
After students have talked about numbers and math in their lives, start talking about numbers at school. Take a walk around the school having students identify numbers that they see, while you/the teacher records them. Once back in classroom discuss which numbers students saw and what they are used for.
Technology integration idea: Provide students iPads to use to take pictures of the numbers around the school.

Other Questions for Sorting & Classifying Data into 2 or 3 categories:
"Data Analysis in kindergarten is not considered a focus that should encompass a lot of time in the classroom, however it is a way to reinforce counting skills and develop students' problem solving skills, reasoning, and representation processes that are a focus throughout the grades. Therefore, data analysis activities in kindergarten should be connected to the content of numbers, operations, geometry, and measurement as well as contexts

kindergarten students are familiar with in their everyday lives; such as the number of students who are wearing a certain color or favorite colors; how many students ride the bus, walk, car; how many students have pets, etc." *SCCCR Support Document*

- Are you wearing pants, shorts, dress? (3 categories)
- Do you like ice cream? (Yes/No)
- Which students ride the bus, walk, car/van rider?
- Did you bring your lunch? (Yes/No)
- Do you have brother/sister? (Yes/No)
- Who is wearing red, blue, or yellow today?
- Do your shoes tie, buckle, or slip on?
- Did you go to preschool or daycare?
- Which is your favorite restaurant? (only provide 3 categories)

***Teacher Note:** Initially you might just ask the questions and have students sort and classify, but if you record the data, you will want to keep it so that you can refer back to it when addressing K.MDA.4.

K.MDA.4 Represent data using object and picture graphs and draw conclusions from the graphs.

Standard Rationale:
Students are not expected to draw any type of graph in kindergarten. They are simply expected to use actual objects and or pictures on a teacher "created graph", then analyze that graph to draw conclusions.

Sample Object Graph: Do you have shoes that tie, buckle, or slip on?
For object graphs (such as this), obtain a clear or white plastic shower curtain, use duct tape to make 3 columns, and a row at the bottom for up to 3 categories. Have students take off one of their shoes and place it on the "shower curtain" object graph. Ask questions such as, "What do you notice?" "What do you wonder?"
Explore student "wonders" (students will probably wonder "How many more slip ons than ties?", etc.).

Snack mix object graph:
For snack, provide students with a mixture (such as trail mix, snack mix with pretzels crackers, tri-colored goldfish, etc.)
Ask students to sort their mix and then represent it on a graph (could provide a laminate graph mat to each student rather than using a shower curtain).
Ask students: "What do you notice?" "What do you wonder?"
When finished let students eat their snack mix.

Sample Picture Graph: How do you go home?
Provide students with a picture of the transportation they use to go home (car, bus, walk). Have students place their picture on the graph "template" (shower curtain could also be used

for picture graphs).
Ask students, “What do you notice?” “What do you wonder?”

How many boys and girls are in our class?
Take students’ actual pictures and print them out. (or could use female/male clipart)
Have students place their picture on the graph (two columns).
Ask: “What do you notice?” “What do you wonder?”

Follow these types of questions and discussion for any of the data collection above from K.MDA.3.

Standards to be Reported on Report Card			
Quarter 1	**Quarter 2**	**Quarter 3**	**Quarter 4**
1.NSBT.1 1.ATO.1 1.ATO.6 1.ATO.9 1.G.1 1.G.4 1.MDA.1 1.MDA.2	1.NSBT.1 1.ATO.1 1.ATO.2 1.ATO.3 1.ATO.4 1.ATO.5 1.ATO.6 1.ATO.7 1.ATO.8 1.MDA.4 1.MDA.5	1.ATO.1 1.ATO.6 1.NSBT.1 1.NSBT.2 1.NSBT.3 1.NSBT.4 1.NSBT.5 1.NSBT.6 1.G.2 1.G.3	1.ATO.1 1.ATO.6 1.NSBT.1 1.NSBT.2 1.NSBT.3 1.NSBT.4 1.NSBT.5 1.NSBT.6 1.MDA.3 1.MDA.6

Number Sense and Base Ten Standards and Suggested Instructional Strategies

1.NSBT.1 Extend the number sequence to:

a. **Count forward by ones to 120 starting at any number;**
b. **Count by fives and tens to 100, starting at any number;**
c. **Read, write, and represent numbers to 100 using concrete models, standard form, and equations in expanded form.**
d. **Read and write in word form numbers zero through nineteen, and multiples of ten through ninety.**

For this standard, parts a and b can be addressed and assessed through number sense routines, especially Count Around the Circle. Part c will require more “kid watching”, which can be done through simple contexts, where students are asked to build a number using concrete models (such as ten frames, base ten blocks), then write the numbers they build in standard form, adding an equation in expanded form. In kindergarten students composed and decomposed ten, as well as composed the teen numbers (11-19), therefore it’s recommended to start with 1-25, and build from there as students are ready. This standard is essential for students in order to move onto all the other NSBT standards. Below we provide some examples of numbers to provide students and watch to see how they model the numbers. While students have not yet been expected to read and write number words, these should be integrated with the other activities as a way to ask students to represent the numbers, rather than taught in isolation or as “spelling” words.

Sample Contexts for Number Representation:

- There are ____ crayons in a box. (Provide number for students or let them fill in the blank) Show how many and explain your thinking.
- There are 19 crayons in the box. Use your ten frames to model this number. Model this number using the base ten blocks and explain your thinking.
- In a larger box, there are 33 crayons. Model this number using the base ten blocks and explain your thinking.
- There are 99 crayons in an extra large box. Model this number using the base ten blocks and explain your thinking.

* Initially you could put out unifix cubes, ten frames, and base ten blocks and see which tool students choose with which to model. The grade level expectation is that they do use groups of tens (using any manipulative) and ones by the end of the year.

Sample Task: Building Numbers (Small group or Whole group)

Similar to the counting contexts above, provide students with some tools (color counters, color tiles, unifix cubes, tens frames, base ten blocks), tell or show them a number on an index card and ask them to "build the number" with a tool. Observe students to see which tool they choose and how they count their tools. For example, if they choose color tiles or unifix cubes, do they count them 1:1 or do they count in groups (such as 2, 5)? Which students use a ten frame to organize/group their counters? Which students use the base ten blocks? How do they use them? Do they use them accurately or do they count rods individually?

- Possible Numbers to Build: 8, 13, 25, 46, 101

Sample Game: I Have, Who Has? (Whole Group)
Provide students with 2 sided index cards. On 1 side is a number (0-19), and on the other side is a number word (zero through nineteen) Play a game where a student says, "I have ___" (holds up card), and the student who has the matching number/word says, "I have ___" and holds up card).

Sample Activity: Matching (Partners)
Provide number cards for the numbers 0-19 and word cards with the number words (zero, one, two, three…. nineteen). Each card is placed face down and students play a matching game by pairing a number with its word form.

Sample Task: Things that Come in Groups
Have students find items in the classroom that are in groups, draw a picture to represent the items, and make a corresponding number label. Students can draw their groups on paper (or in a math journal/notebook) and write the number of items next to each group. Once students have had time to represent their groups of objects, share and discuss their drawings. Look for how students think about "groups", anything more than one could be thought of as a group, so pay attention to see if students "invented" groups such as students, or something that wasn't as obvious. Ask how they thought of this.

Sample Task:
Materials needed: 0-99 chart, book 1,2,3, Sassafras by Stuart Murphy, base ten blocks, number cards (1 set per person), recording sheet
Directions: Read aloud 1,2,3, Sassafras by Stuart Murphy. Discuss all the different ways the family could be arranged in the picture. Ask students, "What are different ways numbers can be arranged?" (For example, 1 and 9 could be 19, or 91.) Ask students to build each number arrangement with their base ten blocks, then ask, and ask "How do your numbers differ when you build them with the blocks?" "What patterns do you notice when you build the numbers with base ten blocks?" Show students a 0-99 chart, ask students to find their number on the chart, then find another number using the same digits, ask students "What do you notice?"

Differentiation: For students who struggle to unitize (count 1 ten as 1) have them put ones cubes on a ten frame, then trade those ones for a ten rod when building their numbers.

Variations:

1. Students can play a game, using the digit cards. Each player has their own stack of cards, each player draws two cards, then decides a number to make with the 2 digits. Players can decide to make the largest/smallest number possible. Each player builds the number with base ten blocks. Players record their numbers on the recording sheet.
2. After students have been able to play with a partner, ask students to identify the largest number they built. Have students use their base ten blocks to build that number (decompose) using as many combinations of base ten blocks they can. (For example, if 91 was the largest number, students would build it with 9 tens and 1 one, 8 tens, and 11 ones, 7 tens and 21 ones, etc.)
3. As students are ready, they can then use the numbers they built and decomposed in a variety of ways to write them as expanded form equations. (91 = 9 tens + 1 one, 91 = 90 + 1, etc) It's important for students to understand how our verbal and written language connects to the mathematical symbols (+, =) therefore it's important in teaching to write it out in word form for students to see, such as 91 is the same as 9 tens and 1 one.) Students don't necessarily need to write it out this way, but as often as possible, the teacher should model the words and move to the mathematical symbols.

1.NSBT.2 Understand place value through 99 by demonstrating that:

a. **Ten ones can be thought of as a bundle (group) called a "ten";**
b. **The tens digit in a two-digit number represents the number of tens and the ones digit represents the number of ones;**

c. Two-digit numbers can be decomposed in a variety of ways (e.g. 52 can be decomposed as 5 tens and 2 ones, or 4 tens and 12 ones, etc.) and record the decomposition as an equation.

"In order for students to develop an understanding of place value concepts they will need ample opportunities to explore tens and ones groupings using concrete models and math drawings. Students need repeated experiences in ***building*** *2-digit numbers with strong visual support before extended place value concepts to add with 1- and 2-digit numbers."* SCCCR Math Support Document.

This standard is the foundation for students to understand the concept of our base ten system. It's important for students to make the connection between our number words, and the digits in the tens and ones place. After students have explored counting by ones with groupings of objects, we can start to push their thinking to the need for groups of ten. Part **a** of this standard is a bridge between what kindergarten students understand, teen numbers are composed of **a group of ten ones and some more,** and the idea that all those ones are now 1 ten, or a ten. The tasks below address this idea of **a group (bundle) of ten**. This is where students are expected to be at the unitizing stage of development, however it's important to note that some students won't be there yet, however, through experiences like this they will get there.

Students should be provided opportunities to physically bundle and trade 10 ones for a ten through a variety of meaningful activities to help them develop the understanding that our numbers are "nested" inside of one another, meaning ten ones are nested inside a ten.

Sample Task: Counting in Groups Part 2 (Whole Group)
Provide a collection of objects that children are interested in counting (the number of shoes in the classroom, a tub of unifix cubes/counters, a chain of paper links, or the number of crayons or markers in the classroom). The quantity should be between 25 and 100. Ask, "How could we count these in some way that makes it easier than counting by ones every time?" Take all suggestions and try them with students. Discuss what works well, what doesn't work well and why students think that way.

Sample Task: Groups of Ten (Individual or Partner) Possibly 2 days
Prepare bags of different objects such as toothpicks, buttons, beans, plastic chips, unifix cubes, popsicle sticks or other countable items. Share with students that the quantity of items in their bag is a mystery to be solved. First, count your objects in a way that makes sense to you. Record it as a number word. Then count the objects, making a bundle/group of ten objects in the cups, and extras that don't make a group of ten out of the cup. Record the number of cups/tens and the number of extras. Is it still the same amount?

After students have time to count and record their groups of ten, have a group discussion. Ask students to share their objects in their bag, the number word, and groups of ten and extras. Record responses on a chart, similar to this one, for all students to see:

Student Name	Object in bag	Number Word	Groups/Bundles of Ten	Extras	Standard form number

What do you notice? What do you wonder?
If no one offers, “I notice the paper clips, there were 13, it has a 1 and a 3 and it had 1 group/bundle of ten and 3 extras.” --pose as a question or share with students as something you notice. Continue to analyze the standard form numbers with the bundles of ten and extras. The goal of this task is for students to notice that the number of groups of ten matches the digit on the left and the number of extras matches the digit on the right. It might be necessary to let students rotate counting the bags over several days and keep recording and noticing/wondering. Once students see this, you can tell them we call the digit that represents the group(s) of ten, “a ten” and the digit that represents the extras is called “ones”.

Once students have experienced groupings of objects and demonstrated they are ready to begin “bundling” and counting a ten (unitizing), they should be encouraged to do so. While not all students will reach this point at the same time, it’s important to expose students who are still grouping objects, to help them move their thinking forward.

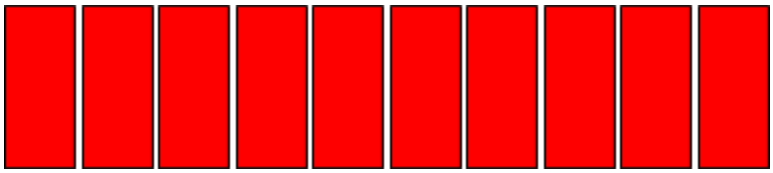
Sample Task: Candy Shop *(Approximately 3 days)*
Materials: large bags of store bought hard candy, wrapped individually, such as Jolly Ranchers, tape, 3x5 index cards, 2 or 3 large bowls that can hold 100 candies in each, containers to hold candies

Introduce this lesson by showing students the large bags of candy or pictures like this:

Ask, “What do you notice?” “What do you wonder?” Record student notices & wonders on a chart. Ask students to make predictions about how many candies they have altogether. Record student predictions. (You might also point out that the bag states that there are “about” a certain number of candies, but that means it’s a rough estimate, they don’t always have an exact count when they make these large bags.) Ask students, “How might you easily count these candies?” (Students might offer, count by 2’s, 5’s or 10’s).

Discuss with students what might be the more efficient way to count the candies (by 10’s). Explain to students that your class will be making packages of candy for other classes in the school (you can decide how you want to distribute the candies, or give more context to this depending on your school). First you need to make packages of 10 candies, and then you will need to assemble bags of candy with enough for each student in those other classes to which you’ll be distributing the candies. For example, you might need 23 candies for a 2nd grade class, but you might need 27 for a fifth grade class, so you need to know how many candies to put in each class’ bag. By first organizing the candies into ten packs, you will more easily and efficiently be able to fill the orders for each class.

Show students how to tape a line of 10 candies together for a ten pack.



(Place a long strip of tape on the top of this row and the back of this row to hold the 10 pack of candies together.)

Split students into small groups or partners and provide them with a large bowl of candies, and tape. (Keep about 30 loose candies separate for following days.) Instruct students to grab a handful of candies, count to make sure they have exactly 10, then tape the ten together as a pack.

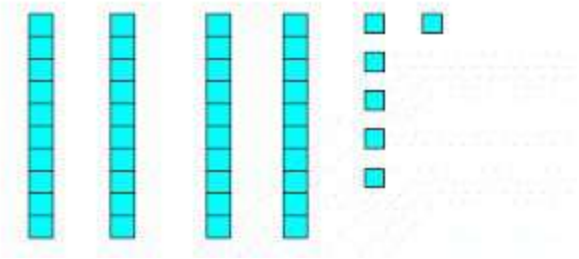
Students will need ample time to create their ten packs, therefore this task should be done over multiple days. After creating their ten packs, have each table figure out how many candies they have at their table. This is a good place to stop the task for Day 1.
Day 2: Gather students together to look at all the ten packs of candies they created the previous day. Ask students, "What do you notice?" "What do you wonder?" (If nobody offers it, offer up "I wonder how we should count these?") Take student suggestions to count and try it out, but stop to comment how long it might take if you're counting by 1's, 2's, or 5's. As a group, count the candies by their groups of ten and record the numbers on the board. When you reach 100 (or 10 tens), put those into a new bowl and call it a "hundred". (Label that bowl 100.) Each time you reach another hundred, put them in a bowl and label it 100. Continue counting until all candies have been counted including the 30 loose ones you withheld on Day 1.
Discuss the "approximate" number of candies on the bags with the number you got as you counted the candies. Are they the same/equal? Do you have more or less than is suggested on the bag?
Day 3: Provide small groups or partners with a bag labeled with a teacher's class, and the number of candies needed so that each student in that class gets 1. Have groups fulfill the "orders" and then deliver to those respective classes.

A note about what to notice through this task:

1. *Did students recognize that they can count a large quantity by tens or by ones and the total will remain the same? (conservation of number)*
2. *Did students know the count sequence when counting by tens?*
3. *When students grabbed a handful of candies and checked to make sure they had ten, did they use number relationships to adjust their counts of ten? (For example, if a student grabbed a handful, counted and found they had 12, do they say, "I need to put 2 back." or if they grabbed only 7, do they say, "I need to get 3 more to make 10."?)*
4. *When counting all the candies, are students confident counting by tens and ones, or are they only trusting when they count the candies by ones?*
5. *Did students make the connection between the written numbers and the number of ten packs and loose ones?*

Sample Activity: Build it a Different Way (Whole or Small Group w/ teacher)
This activity is to help students see that numbers can be represented multiple ways to address 1.NSBT.2 part **c**. Show students a base ten representation on the board, and ask them to build the same number a different way.

Example:



Students then would build 46 differently than this. (3 tens and 16 ones, 2 tens and 26 ones, etc.)

Repeat for several different numbers.

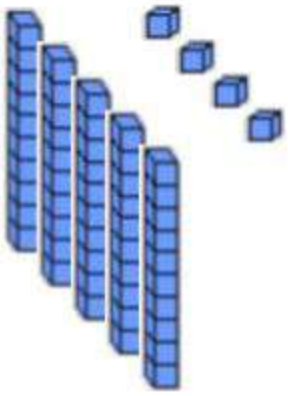
Part **c** of this standard is essential for students to begin regrouping conceptually later in the year (and on into 2nd and 3rd grades). For example, in order to add 67 + 8 later in the year, students will need to recognize that the sum could be 60 + 15 by combining the ones or regrouping to make another ten (70 + 5) and that 75 can be represented by either method.

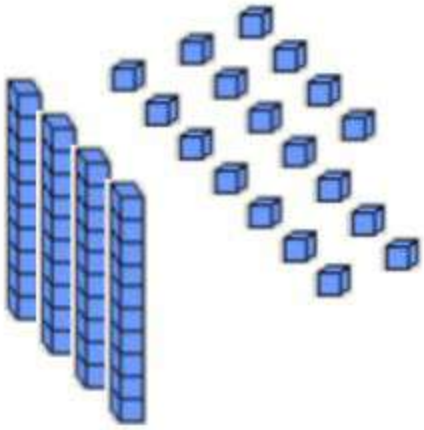
1.NSBT.3 Compare two two-digit numbers based on the meanings of the tens and ones digits, using the words greater than, equal to, or less than.

This can be combined with the above task, as well as used with any task students do for 1.NSBT.1 and 1.NSBT.2. This standard can also be addressed through daily number sense routines and number talks.

Sample Activities: Build It, Compare It! (Whole Group)
Show students a number, have them build their number with base ten blocks. Then show students a different number and have them build that number. Then ask them which is greater? Which is less? Ask, “How do you know?” During student explanations, it is suggested that the teacher records as the student explains. (For example, if numbers were 16 and 36, student might say, “I have 3 tens here in 36 and only 1 ten here in 16, so I know that 36 is greater than 16.”) Make sure you do an example where one number has more tens and the other has more ones and ask, “Why does having more tens make this number greater?”

Greater than, Less than, or Equal? (Whole Group)





Does the picture on the right have more, less, or the same as the picture on the left? How do you know?

Compare and Explain:

1. Compare 34 and 38. Which is greater/less? How do you know?
2. Compare 68 and 86. Which is greater/less? How do you know?

Possible Student Responses:

1. 34 is less than 38. I know because 34 has only 4 ones and 38 has 8 ones. They both have 3 tens. I also know because when I count, 34 comes before 38.
2. 68 is less than 86 because there are 6 tens in 68 and 8 tens in 86. I also know that 68 comes before 86 when I count, and when I count by tens, I count 60 before I count 80.

1.NSBT.4 Add through 99 using concrete models, drawings, and strategies based on place value to:

a. **Add a two-digit number and a one-digit number, understanding that sometimes it is necessary to compose a ten (regroup);**
b. **Add a two-digit number and a multiple of 10.**

Initially students should be provided real-world/story problems to make sense of numbers, and determine when to regroup or compose a ten. This standard will require students to demonstrate fluency within 10, as well as mental/numerical strategies through 20. They will build on this to begin adding through 99.

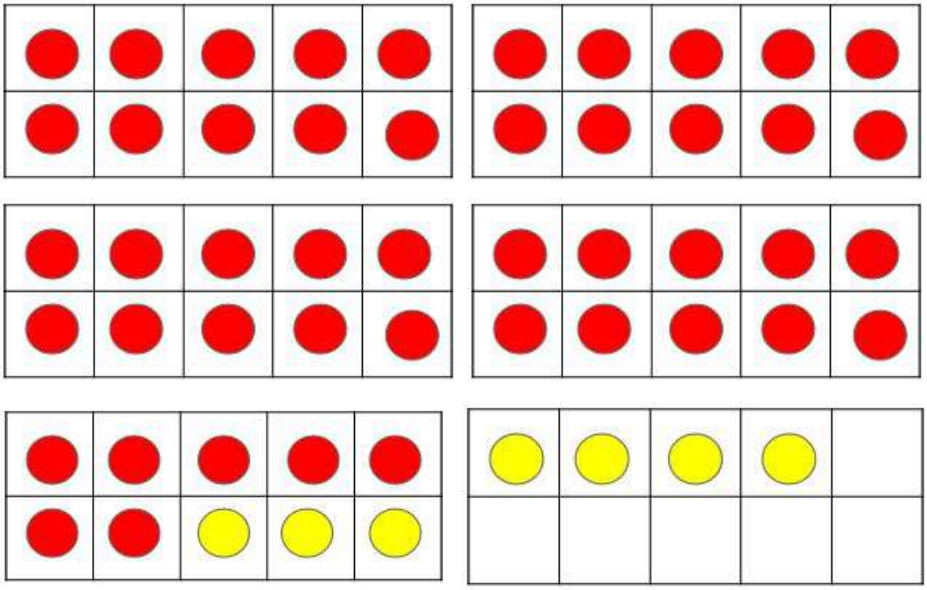
The standard itself is written in a concrete → representational → abstract progression. Therefore, to begin, students should be using concrete models (ten frames, base ten blocks --tens and ones, and rekenreks), then translate those into drawings, and finally by the end of

the year students should have abstract strategies they derive that are based on place value understanding. This is a standard that can be introduced earlier in the year, but by Quarter 3, students should be applying their understanding from previous ATO standards to now operate within 99. Notice this standard *only* requires students to add, NOT SUBTRACT. It's also imperative to let students initially use strategies that make sense to them, and then move them to more efficient strategies. It is NOT necessary, nor is it expected to directly teach students the different strategies.

Sample Task: I have been running out of crayons lately. I needed to come up with a way that I can always know how many crayons I have in each group at all times. So, I decided to put all of my crayons into cups with 10 in each cup. I collected all of my crayons and found that I have 64 crayons. How many cups should I fill with 10 crayons? What should I do with the 4 extras? Mrs. _____ found 9 more crayons outside in the hallway and gave them to me. Do I have enough to make a new cup? How many leftovers do I have? With this information, how many crayons do I have now? How do you know? I bought a pack of 20 crayons to add to my collection. What should I do with those? How many crayons do I have now?

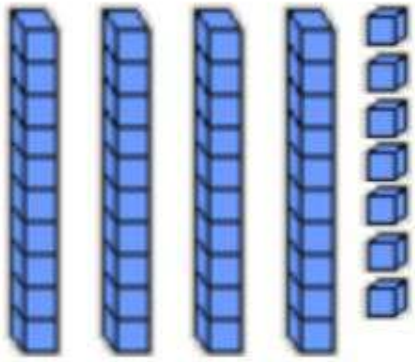
Sample Task: Greg is saving up his money for a video game. He has saved 47 dollars so far. For his 7th birthday, his grandmother gave him 7 dollars. How much money does he have now? How do you know? His parents also gave him 40 dollars for his birthday. How much money does he have now? How do you know?

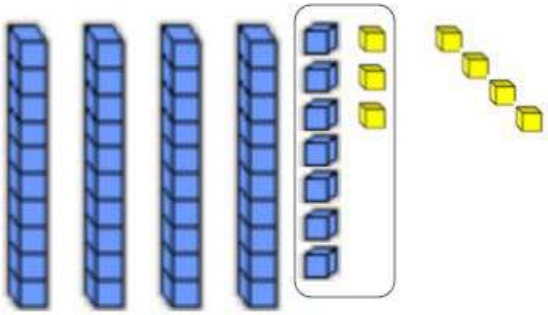
Sample Student Response: I made ten frames to show the amounts:



I have 5 full ten frames filled with 4 left over on another ten frame, so I have 54.

Sample Student Response: I used base-ten blocks to model.





I added 7 more cubes to the 47. I noticed that I needed 3 more to make another rod, so I grouped those together to make 5 tens and 4 ones. 5 tens and 4 ones equals 54.

More Sample Tasks:
See “Cookie Monster 3-Act Task” by Graham Fletcher here.
See “The Juggler 3-Act Task” by Graham Fletcher here.
See “The Whopper Jar 3-Act Task” by Graham Fletcher here.

1.NSBT.5 Determine the number that is 10 more or 10 less than a given number through 99 and explain the reasoning verbally and with multiple representations, including concrete models.

This standard will be reported on the report card in the 3rd and 4th quarters, therefore it makes sense to let students demonstrate understanding through numbers they are proficient at working within, then move onto larger numbers. The place value chart is a great tool to use for this standard.

Sample Activities with Place Value Chart: (Whole Group/Small Group/Partner)
Display a hundreds chart such as the one below (best not to have the label “Hundreds Chart” at top).
Variation: Display a 0-99 chart and do the same.

***Note: A 99 chart is preferable to a 100 chart since it better illustrates the idea of “0 ones” and regrouping.

Ask students, “What do you notice?” “What do you wonder?” “What patterns do you see?”

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

After students have had some time to look at the chart, ask them to locate a number, build that number with their base ten blocks, then find a number that is 10 less than that number, build it, find a number that is 10 more than the original number, and build it.
Example:
Teacher “Locate 23 on the chart. What number is 10 less than 23? Build it!
What number is 10 more than 23? Build it!”

Variation: Guess My Number
Teacher: “I’m thinking of a number that is 10 more than 38. What number am I thinking of?”
“I’m thinking of a number that is 10 less than 70. What’s my number?”

The student “strategies” shouldn’t have anything to do with “addition” or “subtraction” necessarily. The goal is for students to understand our place value system and work strategically with it. For example, instead of adding “10”, students should increase the digit in the tens place by one. (The same applies for the standards NSBT.4 & NSBT.6).

1.NSBT.6 Subtract a multiple of 10 from a larger multiple of 10, both in the range 10 to 90, using concrete models, drawings, and strategies based on place value.

This standard works in tandem with the other NSBTs. As always providing a story or real-life context initially will help students make sense of subtraction by multiples of 10. Students should use strategies that make sense to them. It is not necessary to tell students how to solve these types of problems. Tools such as ten frames, base ten blocks, and hundreds charts should be available for them to model their strategies. Finally, students who *are* ready will model their thinking more abstractly with numbers and equations based on place value.

Sample Problems:

1. Kevin made 20 pizzas for Emma’s birthday party. 10 pizzas were eaten. How many pizzas are left?
2. Pam opened a package of Skittles and counted them. There were 50 Skittles. Over the next two days she ate lots of Skittles, 30 to be exact. How many Skittles did Pam not eat?
3. Mrs. Shaver in the front office ordered 90 pencils. She gave 30 pencils to Mrs. Stokes’ class. How many pencils does Mrs. Shaver have now?

Algebraic Thinking & Operations Standards and Suggested Instructional Strategies

1.ATO.1 Solve real-world/story problems using addition (as a joining action and as a part-part-whole action) and subtraction (as a separation action, finding parts of the whole, and as a comparison) through 20 with unknowns in all positions.

"Research has demonstrated that when kindergarten and first grade children are regularly asked to solve word problems, not only do they develop a collection of number relationships, but they also learn addition and subtraction facts based on these relationships. The key is to allow them to figure out ways to solve the problems." (*Teaching Student Centered Mathematics,* Van de Walle, 2006)

It is important to remember that there is a semantical way to set up the problem and a strategy that can be used to solve the problem. For example, a different equation can be used to solve the problem than the equation that would represent the problem. For this standard, we are focused on how students solve the problem, not solely on the equation matching the story. It is ***never*** acceptable to teach students to solve word problems using key words. As students move on to higher levels and more complex concepts, keywords no longer apply. It is most important to help students model situations, discuss what a story problem is saying (comprehend), and then let them use productive struggle to solve it.

Sample Task: How Many of Each?
For snack, I'm going to give each of you 10 fruit gummies. You will get two flavors (peach and strawberry). How many of each could you get?
On the next day, pose the same problem but ask, "What if I gave you 20 gummies each?"
On another day, pose a similar problem, saying, "I'm going to give you 3 different flavors and a total of 20 gummies." and ask, "How many of each could you get?"

In Kindergarten, students compose and decompose 10 with one addend known.
This task will allow you to formatively assess students knowledge of combinations to 10, as well as fluency within 5.

What are we looking for in student strategies and responses?
We are looking to see which students used tools to represent the problem, which students made a drawing (What does that drawing look like?, How does it connect to the situation/context? *Direct Modeling*), which students wrote numbers, and which students wrote equations to represent the problem. It's not as important that students show every combination of 10 as it is to notice how students model and think about this problem. This will

clue you into their mathematical reasoning. The formative difference among each of these phases in student responses is that some students will still be unsure of a concept and need more concrete or visual clues, while other students who are using abstract numbers or equations have a grasp of the concept.

How to respond to:

Modeling with Concrete Tools:

Prompt the student, “Tell me about your model.”

Ask questions such as: “What tool did you choose? Why?” “What does it represent from the problem?” “Why did you use that many?” “What did you do with your tools?” Think aloud with the student, repeat their reasoning. “Could you make a drawing on paper to represent how you did this problem with your tools?”

Direct Modeling:

Prompt the student: “Tell me about your model.”

Ask questions such as: “Where is that information in the problem?” “What does this __ represent from the problem?” “What are you trying to figure out?” Think aloud with the student, repeat their reasoning. “Is there a way you can use numbers to make your thinking more clear to others?”

Drawing/Representations:

Prompt the student, “Tell me about your model.”

Ask questions such as: “What were you trying to figure out?” “What does __ mean?” (ask about numbers or drawings specifically) (e.g. “What do these circles represent?) “Can you think of a way to represent this with an equation?”

Abstract/Equations:

Prompt the student, “Tell me about your model.”

Ask questions such as: “What were you trying to figure out?” “What does this number represent?” “Could you translate your equation into words?” “How would you model this a different way?” “Can you think of another situation/story that could be modeled this way?”

Sample Problem Types:

Joining Problems with unknowns in all positions:

1. Eva had 5 cookies. Antonio gave her 10 more cookies. How many cookies does Eva have now?
2. Eva has 5 cookies. How many more cookies does she need to have 15 cookies?
3. Eva had some cookies. Antonio gave her 10 more cookies. Now she has 15 cookies. How many cookies did Eva have to start with?

Separation Problems with unknowns in all positions:

1. Eva had 15 cookies. She gave 5 cookies to Antonio. How many cookies does she have now?
2. Eva had 15 cookies. She gave some to Antonio. Now she has 5 cookies left. How many cookies did Eva give to Antonio?
3. Eva had some cookies. She gave 5 to Antonio. Now she has 10 cookies left. How many cookies did Eva have to begin with?

Part-Part Whole Problems with whole unknown, with one part unknown:

1. Michael has 5 lemon cookies and 10 chocolate chip cookies. How many cookies does Michael have?
2. Michael has 15 cookies. Five are lemon and the others are chocolate chip. How many chocolate chip cookies does Michael have?

Compare Problems with unknowns in all positions:

1. Emma has 15 cookies. Ayden has 5 cookies. How many more cookies does Emma have than Ayden?
2. Ayden has 5 cookies. Emma has 10 more cookies than Ayden. How many cookies does Emma have?
3. Emma has 15 cookies. She has 10 more cookies than Ayden. How many cookies does Ayden have? *

*Note: Problem #3 shows the futility in teaching key words to solve word problems. If students always recognized “more” as addition, they will add 15 and 10 and incorrectly solve the problem. If students have an opportunity to model the problem in their own way, they will be more likely to comprehend the entire problem and solve it correctly.

1.ATO.2 Solve real-world/story problems that include three whole number addends whose sum is less than or equal to 20.

This standard is foundational for students to develop fluency within 20 and then to have strategies to add and subtract through 99 (in 2nd grade). It’s through the real-world/story problems that students will initially invent their strategies and then be ready to move on to more efficient strategies.

Sample Problems:

1. Our class is collecting cans for a canned food drive. We collected 3 cans on Monday, 5 cans on Wednesday, and 2 more cans on Friday. How many cans have we collected?
2. I’m trying to eat more vegetables. On Saturday I ate 3 carrots. On Sunday I ate 7 cucumber slices. On Monday I ate 2 tomatoes. How many vegetables have I eaten this week?

3. I picked tulips, daisies, and roses from the garden and want to mix them in different vases. I have 3 flower vases, a small, a medium, and a large vase. I put 8 flowers in the large vase, 6 flowers in the medium vase, and 2 flowers in the small vase. How many flowers did I pick?

The focus should be on encouraging students to make a ten and some more. Some fast counters might solely rely on their fingers and counting on as a strategy and not grow from there. To prove the inefficiency of this strategy, ask "What about 37 + 38? Would it be efficient to count up on our fingers then? So what should we do *(invent more efficient strategies)*?"

1.ATO.3 Apply Commutative and Associative Properties of Addition to find the sum (through 20) of two or three addends.

Students ***do not*** need to know the names of these properties, nor do they need to match them to a problem. We should still use the proper mathematical terminology when referring to it in class (i.e. "commutative property" instead of "turn-around property"). The commutative property is helpful for students who are still working towards automaticity with their facts within 10. For example, a student might still be counting up on his/her fingers from 2 for 2 + 7. We should work to *help them see* (as opposed to *telling them*) that they could simply count up from 7 since 2 + 7 = 7 + 2. This kind of recognition *only* happens through repeated interaction and not direct instruction.

The associative property helps students within 20 when adding 2 or 3 addends. For example, for the addition expression 8 + 7, students could decompose the 7 to be 8 + (2 + 5). By applying the associative property, (8 + 2) + 5, they can then make a ten and some more.

When adding three addends (e.g. 6 + 8 + 4), instead of adding straight from right to left, students need to recognize that they *could* add the 6 and 4 first to make a 10 and then add the 8 leftovers.

1.ATO.4 Understand subtraction as an unknown addend problem.

Subtraction is not only deduction or removal. In fact, subtraction is more accurately defined as finding the distance between two points on a number line (which is why the answer to a subtraction problem is the "difference"). Therefore, if we only treat subtraction as a removal, we are only exposing our students to one type of subtraction. For example, the difference can be found by counting backwards from the minuend by the subtrahend, but it can also be found by counting up from the subtrahend to the minuend.

Sample Task: Michael and Kristen read a total of 16 books over the summer. Kristen read 9 books. How many books did Michael read? How do you know?

Students *could* subtract 16 - 9, but it makes more sense in the context to think of it as

9 + __ = 16. (That is not to say we try to dissuade students from counting back in this context). They need to know that it can be modeled as an unknown addend, but also as a subtraction problem (16 - 9 = ___).

1.ATO.5: Recognize how counting relates to addition and subtraction.

Sample Activity:
https://illuminations.nctm.org/Lesson.aspx?id=310
In this lesson, the teacher reads aloud a counting book, while students use connecting cubes to model the count/one more/one less than actions. Students then write the equation in vertical and horizontal formats.

Task: The Very Hungry Caterpillar (individual whole group or partners) ***Multiple Days***
Provide each student with 30 unifix cubes and 3 tens frames. Ask students to estimate how much food they think the caterpillar will eat in the story. Record student estimates. Explain to students that as you read the story, they will place a cube on their tens frame to get an actual count of how many food items the caterpillar eats. (*Differentiation:* For students who are very literal, provide pictures of the food instead of cubes for them to count.)
As you read the story, stop when students place counters on their tens frame and ask students to help you write numbers to represent the cubes they've placed on their tens frame. (For example, when the caterpillar eats 1 apple, write 1, when he eats 2 pears, write 2.) Ask students "How many in all?" and record that number as well, 3.) Teacher recording should look something like this:
1 apple and 2 pears is the same as 3 fruits.
On a subsequent day revisit the recording and ask students how you could make these into mathematical equations.
Go through each recording and have students help you record using pictures, math words, numbers, and then using symbols. You can use a table similar to the one below so students can see connections.

Pictures	**Words**	**Numbers & Words**	**Equation**
	1 apple and 2 pears is the same as 3 fruits	1 plus 2 equals 3	1 + 2 = 3

1.ATO.6 Demonstrate:

a. **Addition and subtraction through 20;**
b. **Fluency with addition and related subtraction facts through 10.**

This standard is one that can be differentiated for individual students all year. Continue using small group math lessons, partner games, and individual practice tasks for students to develop and demonstrate fluency. Continue to assess students' fluency as described in Quarter 1. Fluency is developmental and students pass through the following phases as they develop fluency. It's important to help students develop their thinking, rather than force them to memorize facts.

Students should use strategies that make sense to them to add and subtract through 20 initially. As they become more fluent within 10, they will use more efficient strategies. Fluency has nothing to do with speed and therefore should ***never*** be assessed using timed tests. Fluency is described as "a student's ability to compute with accuracy, flexibility, and efficiency." (Kilpatrick, Swafford, & Findell, 2001)

Fluency can be assessed in a variety of ways. Analysis of student strategies in journals, number sense routines/number talks, performance tasks, as well as one-on-one student interviews provide much more information about a student's fluency, the same way a running record provides information about a student's reading fluency.

Examples of Ways to Assess Fluency

Sample 1: Assessing fluency, flexibility, and strategy through 1-1 Interview

1. Write 3 + 4 = on a card. Ask student to tell you answer. (assessing fluency)
2. How can you use 4 + 4 to help you solve 8 + 2? (assessing strategy and flexibility)

Sample 2: Assessing fluency and use of an appropriate strategy through 1-1 Interview

1. What is 7 + 3?
2. How did you figure it out?
3. Record student strategy (recall, automatic within 3 seconds, making 10, near doubles, another derived fact strategy, counting on, counting all, modeling and counting all-such as with fingers or writing it down)

Sample 3: Journal/Writing Prompt:

1. If your friend did not know the answer to 4 + 5, how could he figure it out?
2. Explain how to use the "count on" strategy for 3 + 6.
3. Emily solved 6 + 4 by changing it in her mind to 5 + 5. What did she do? Is this a good strategy, why or why not?
4. Which facts do you "just know"? Which facts do you use a strategy to solve?

Sample 4: Story Problems/Performance Tasks:

1. Rachel sold 4 boxes of Girl Scout cookies on Friday and 6 boxes on Saturday. How many boxes did she sell?
2. Michael made 8 drawings in art class this month. He made 2 drawings in art class last week. How many drawings did he make during the rest of the month?
3. Students who are going on the field trip need to return permission slips. We have 10 students in our class. So far 6 have returned their permission slips. How many students have not returned their permission slips?
4. Joey ate 3 more donut holes than his little brother. His little brother ate 2 donut holes. How many donut holes did Joey eat?

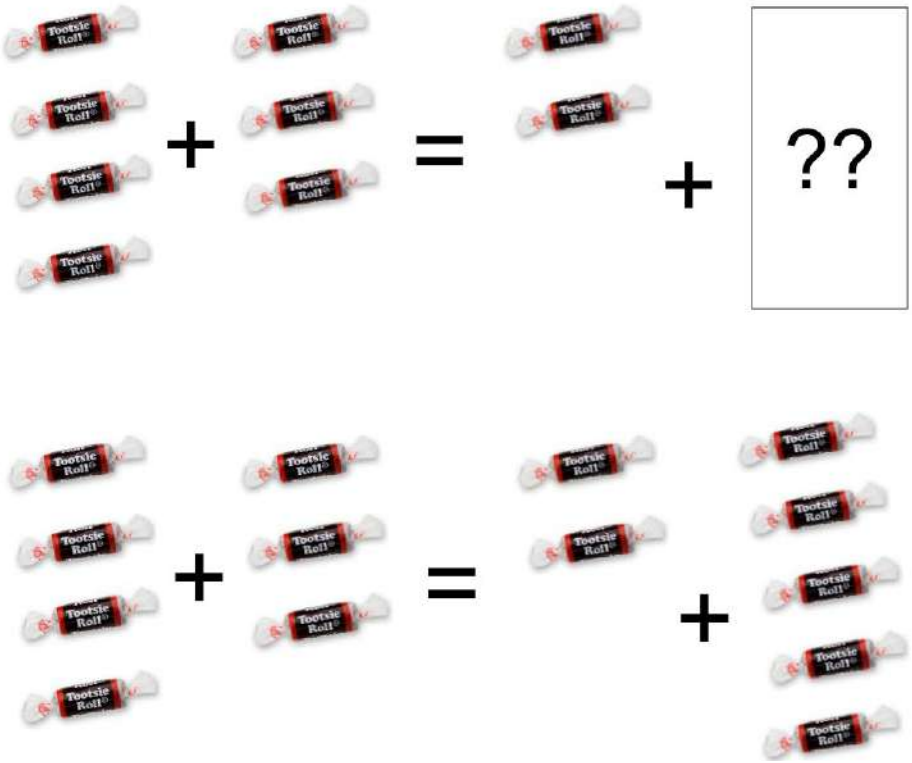
It's through these formative assessments of students basic fact fluency and strategies that teachers can differentiate for small group instruction, as well as provide partner games/activities for students to practice their strategies as they develop fluency.

1.ATO.7 Understand the meaning of the equal sign as a relationship between two quantities (sameness) and determine if equations involving addition and subtraction are true.

There are two parts to this standard when you read it closely. First, students need to understand the meaning of the equal sign. This part should be addressed through word problems in ATO.1. Secondly, students need to determine if equations involving addition and subtraction are true. When posed through equations, the concept becomes more abstract. Therefore, using contexts to scaffold and modeling what the equations would look like in ATO.1 will help students with the second part of this standard.

Misconception: Students (wrongly) believe that the equal sign means "the answer is" rather than showing equality between two quantities or expressions. We must proactively prevent this misconception by presenting equations in both directions (i.e. 5 + 4 = 9 AND 9 = 4 + 5).

Sample Activity: Brendon loves to copy his older brother, Austin. He always wants to make sure that he does the exact same thing his brother does. Austin loves Tootsie Rolls, so Brendon does too. Austin has 4 Tootsie Rolls and he buys 3 more. Brendon only has 2 Tootsie Rolls. How many more does he need to buy to give him the same number of Tootsie Rolls as his brother?



Follow-up: Austin ate 2 of his Tootsie Rolls. How many should Brendon eat to keep them equal? Draw a picture like the one above to match the change and write a new equation. (e.g. 2 + 3 = 0 + 5 or 3 + 2 = 1 + 4)

1.ATO.8 Determine the missing number in addition and subtraction equations within 20.

In order for students to determine the missing number in equations within 20, they need to have experiences with contexts, therefore this standard builds on 1.ATO.1 & 1.ATO.7 with story problems using different problem types, with unknowns in all positions. Students will be more successful with the abstractness of equations if they've had plenty of opportunities to relate equations to different problem contexts. This standard can and should also be addressed through number talks.

Also, this standard is founded in an understanding of ATO.7. Students should be asked, "To make this side equal the other side, what do I need to add (or subtract) to keep them equal?" Students should use manipulatives or draw pictures to represent equations.

Sample Number Talk:
Present students with equations during a number talk and ask, "What's missing? How do you know?"

Example: ____ + 4 = 10, 10 - ___ = 4

Sample Task: Story & Equation Matches (Small Group w/ teacher, or partner)
Provide students with short story problems with unknowns in all positions. Provide students with equations for each story problem. Have students match the context with the equation, and then provide the missing number.

1.ATO.9 Create, extend and explain using pictures and words for:

a. **Repeating patterns (e.g. AB, AAB, ABB, and ABC type patterns);**
b. **Growing patterns (between 2 and 4 terms/figures)**

In kindergarten students describe simple repeating patterns, so they should be able to create, extend, and explain simple repeating patterns. Growing patterns is new material, and it can be related to numbers, shapes, or movements. This standard can be addressed in daily calendar/math meeting time by presenting a pattern for students to explain and extend. Also ask a student each day during calendar/math meeting time to create a pattern, ask another student to extend their pattern, and finally ask another student to describe the pattern.

Sample Task: Eggs on the Farm
I went to the farm last week and visited the chickens. On Monday I found 1 egg. On Tuesday I found 2 eggs. On Wednesday, I found 3 eggs. If I went back on Thursday and Friday, how many eggs do you think I would find? Describe the pattern.
Extension: How many eggs did I find the whole week?

Geometry Standards and Suggested Instructional Strategies

1.G.1 Distinguish between a two-dimensional shape's defining attributes (e.g. number of sides) and non-defining attributes (e.g. color).

Sample Activity: Guess My Shape (Whole Group/Partners)
Place shapes in a brown bag. Choose a shape in the bag, do not show it to students. Provide students with non-defining attributes as clues to your shape. Ask, "Can you guess my shape? Why not?" Then allow students to ask questions with defining attributes.

Sample Task: Shape Sort (Small Group)
Provide each group with about 5 -10 shapes. Ask each child in the group to randomly select a shape and tell one or two things they find interesting (notice) about the shape.
Then ask each group member to select two shapes and describe one thing that is alike about their two shapes, and something that is different. Then the group decides on one shape as

the target shape. Their task is to find other shapes that are like the target shape according to the same rule/attribute.
For example: "This shape is like the target shape because it has 2 short sides and 2 long sides." Then all the other shapes they put in the pile must have these attributes. Repeat with another sort using the same target shape, but a different attribute (e.g. all straight sides).

Sample Question: Hold up a yellow circle and a yellow square and ask, "Are these the same shape? Why or why not?". Hold up a large triangle and a small triangle and ask the same question.

1.G.2 Combine two-dimensional shapes (i.e. square, rectangle, triangle, hexagon, rhombus, and trapezoid) or three-dimensional shapes (i.e.cube, rectangular prism, cone, and cylinder) in more than one way to form a composite shape.

This standard is teaching students the same concept they have been working on with composing numbers. This standard is just an exposure to spatial awareness so that students can think flexibly about shapes.

Sample Activity: Attribute and Pattern Blocks
Provide students with 2-D attribute blocks or pattern blocks and various pattern block puzzles. Ask students to combine their shapes to make something new. Ask students to describe their new shape and count how many of each individual shape they used to make the new shape.

Some sample pattern block puzzles can be found here.

1.G.3 Partition two dimensional shapes (i.e. square, rectangle, circle) into two or four equal parts.

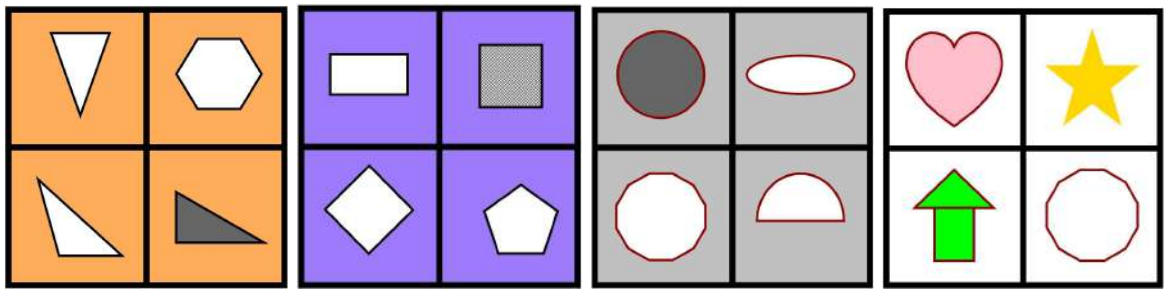
While this standard is a preliminary introduction to fractions, it is really focused on building spatial awareness of equal areas/spaces. Therefore there is NO need to introduce any type of fraction notation or fraction ideas. In 2nd grade, students will build on this understanding to name the equal parts they partition using the terms *half of, halves, fourths,* and *a quarter.* In 1st grade students simply need to partition shapes into 2 or 4 equal parts.

Sample Activity: Provide students with a paper shape (square, rectangle, circle). Each student should have a different shape. Then tell the story, "I'm baking a cake for my sister's birthday. Show me what my cake would look like with 2 equal pieces if I bake it in the same shape as your shape." Gather students together afterwards to discuss the different shapes and how they partitioned it. Repeat the activity telling students you want to have 4 equal pieces. At first, have students use the same shape they did the first time. Over 2-3 days, repeat the activity, but provide students with different shapes than they had the day before.

1.G.4 Identify and name two-dimensional shapes (i.e., square, rectangle, triangle, hexagon, rhombus, trapezoid, and circle).

In kindergarten, students identified and described shapes in everyday situations. Now, students should be able to identify these same shapes in isolation from other objects (that is not to say that we can't use real world pictures to identify shapes). In G.1, the students investigated defining attributes, and that work should come back in this standard as well.

Sample Activity: Which One Doesn't Belong
Students look at four shapes and discuss which shape is out of place based on a certain attribute of the other three shapes. There can be multiple answers for each, so students should have an opportunity to debate amongst one another.



More "Which One Doesn't Belong?" pictures can be found here.

Measurement & Data Standards and Suggested Instructional Strategies

1.MDA.1 Order three objects by length using indirect comparison.

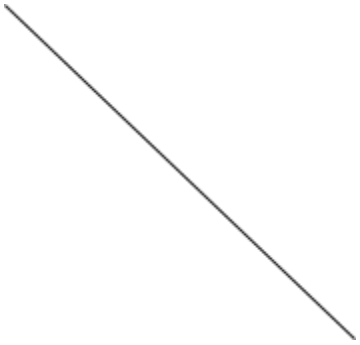
This is students' first experience with the concept that "units are counts that create a number called a measure." In kindergarten, students just used the words "longer/shorter" to describe the relationship between two objects. The only difference here is that students need to consider three objects and sort them from longest to shortest.

Sample Task: I couldn't put my tennis shoes on this morning because I was missing a shoe lace. I found these laces in the junk drawer but I need the shortest one for my shoes. My dad uses the longest one. My sister uses one shorter than my dad's but longer than mine. Show which lace belongs to me, my dad, and my sister?
Provide strings (or actual shoe laces) of varied lengths for students to use and order.

1.MDA.2 Use nonstandard physical models to show the length of an object as the number of same size units of length with no gaps or overlaps.

Sample Task: Nonstandard physical models include anything that is not a standard measurement tool (ruler, yardstick, measuring tape, etc…). Students will not need to measure using those tools until 2nd grade. However, if some of your students are ready to measure using standard units, they should be allowed to do so.

To formatively assess students' understanding of measurement, provide students with a nine inch diagonal line such as the graphic below, and ask them, "What do you notice? What do you wonder?" (Students will probably wonder how long the line is, if not pose it as something you wonder.) Ask students to predict/estimate how long they think the line is before measuring it. Tell students that they may use any of the tools in their toolbox to measure the length of the line. (Provide paper clips, unifix cubes, color tiles, and inchworms if you have them.)



Pay attention to how students line up their tools, do they begin at the end of the line? Do students leave gaps or spaces? Do students line up the tools without gaps or spaces or overlaps? What do they do if the line doesn't end at exactly one unit, how do they count that?

After students have had time to explore measuring the line, ask students to share their strategies for measuring, as well as explain how they actually measured. What tool did they use? How did they get their count/measure?

Sample Task: Read the story Inch by Inch by Leo Lionni (or show the read aloud from Youtube), then discuss with students, "How do you think the worm measured things? Did he have a ruler or tape measure in his pocket?

1.MDA.3 Use analog and digital clocks to tell and record time to the hour and half hour.

This standard will be embedded all year and reported on the report card in the 4th quarter because time is a very abstract concept for students to grasp. Students do not learn to tell

time in a brief unit, but rather over time. It's recommended that each day you ask a student to tell you the time on the clock, starting with time in whole hours, then asking for the time in half hours. By initially introducing the concept of time, and then revisiting it daily, students will become proficient in telling time.

Extension: Provide each student with a recording sheet with clock faces. Secretly set a timer to go off at the hour, and half hour. When the bell rings, tell students to look up and record the time on the clock using numerals on their recording sheet.

Lesson Day 1: To introduce this standard, ask students to draw a picture of 5 activities they do during the day. (Use index cards.) Ask students to put their pictures in order according to the order they complete the activities. (For example, eat breakfast, brush teeth, walk to school, go to recess, go to bed.) Then ask students to write a number on each card according to where it falls in order. 1. Eat breakfast, 2. Brush teeth, 3. Walk to school, 4. Recess, 5. Go to bed.
Ask students if it matters what order they do these activities in a day or if they could do them at any time of the day? Discuss with students that clocks are tools we use to help us know when to do certain activities or at what times we do certain activities. Then show students a real analog clock. Ask, "What do you notice? What do you wonder?" Hopefully someone will notice there are numbers, and someone will wonder "What do the numbers represent/count?" If not, offer it as your own wonder.

Lesson Day 2: Tell students you are going to read a story, The Grouchy Ladybug by Eric Carle and that there is a picture of this tool/a clock throughout the story. Tell students you want to see if they can figure out what the ladybug is using the tool for.
Read the story. Afterwards, ask, "What did the ladybug use the clock for? How could we use a clock for the activities we do each day?"
Ask students about the activities they drew in the previous lesson and have students sort their pictures into Morning/Afternoon/Night piles. Explain to students that while we could just say, "Morning, afternoon, or night," we need to be more precise in telling time, therefore we need to use clocks and other tools to tell time.

Sample Activity: Constructing & Reading a Clock
Explain to students that while it's good to know how to measure time, at this point it's more important for them to learn how to read the tool that measures time, the clock. Explain that there are different types of clocks. (Analog & Digital)

Construct clocks using paper plates and strips of card stock for the hour and minute hands.

Extension Sample Task: Time as a measurement
Using a timer/stopwatch throughout the day (such as building stamina in independent reading) will let students get a feel for how long seconds, minutes, and hours are. Time is the

one measurement that cannot be seen. It has to be experienced. Ask students to compare the different units of time after they've experienced it. Keeping a chart ,such as the one below, in the classroom is a recommendation.

Unit of Time	Activity
Seconds	
Minutes	
Hours	

1.MDA.4 Collect, organize, and represent data with up to 3 categories using object graphs, picture graphs, t-charts, and tallies.

Sample Task: (1.NSBT.1, 1.ATO.1,1.MDA.4, 1.MDA.5) *Spread out over several days.*
"As we begin our year together, I need you to help me take inventory of our pencils, crayons, markers, and paper in the classroom. What do you need to know in order to help me?"

Possible Student Questions/Responses:

- How many students do we have?
- How many pencils do we have? (Ask different students to count the objects.) How many do we need? (Provide this information after students have had the opportunity to count.)
- How many crayons do we have? (Ask different students to count the objects.) How many do we need? (Provide this information after students have had the opportunity to count.)
- How many markers do we have? (Ask different students to count the objects.) How many do we need? (Provide this information after students have had the opportunity to count.)
- How many packs of paper do we have? (Ask different students to count the objects.) How many do we need? (Provide this information after students have had the opportunity to count.)

Provide students just enough information for them to work through the problem.

This task will provide some baseline data for students' counting abilities as well as their problem solving strategies. Pay attention to students who count 1:1, students who keep track of their counts, students who group objects as they count them, and students who constantly have to recount their objects.

Other Activities

Classroom Library Sort: Have students sort and count the books for the classroom library into 3 categories. If you have students help you set up your classroom library, ask them what categories they think you should use, and count to put the books into those categories? If you have books already sorted by categories, ask students to count 3 different categories of books and represent the data using an object and/or picture graph.

Collect, Organize, and Represent Data about Students:

- How do we go home? (Bus, Car Rider/Van Rider, Walker)
- Which is your favorite animal? (Dog, Bird, Fish, or 3 others)
- Which flavor of ice cream is your favorite? (chocolate, vanilla, rainbow)
- How many letters are in your name? (Read story Chrysanthemum, have students write their first name on 1 inch grid paper, then graph the numbers of letters in student names.) (To keep it to 3 categories, you could do less than 5 letters, Equal to 5 letters, and More than 5 letters.)

1.MDA.5 Draw conclusions from given object graphs, picture graphs, t-charts, tallies, and bar graphs.

Use the graphs created for 1.MDA.4 to analyze and draw conclusions.

1. MDA.6 Identify a penny, nickel, dime, and quarter, and write the coin values using a ¢ symbol.

This standard is taught all year, through number sense routines and calendar/math meeting activities. It is not reported on the report card until the 4th quarter. The act of identifying coins and writing their values seems simple, however this is a standard that students will master when provided multiple opportunities throughout the year.

Sample Activity: Coin Grab Bag
Put a collection of coins inside a brown paper bag. Have students sit as a group (in a circle or meeting style together). Pass the bag around and ask each student to grab one coin out of the bag. Student then tell what the name of the coin is, and the value of that coin. As students do this, the teacher can model writing the value with the ¢ symbol.

Other variations

Guess My Coin: As students pass the bag of coins around, ask each student to grab one in their hand, but not to reveal the coin to the other students. The student who grabs the coin

looks at it and uses words to describe it to their classmates. The classmates then try to guess which coin was grabbed.

Small Group Activity: After some whole class discussion of coins, the Coin Grab Bag can be placed in a center/math station for students to play with a partner.

Sample Center/Station: Coin Sort
Place a piggy bank full of coins (real or play) in a learning/math center. Provide students with directions to dump the piggy bank out and sort the coins into marked containers. (Use small plastic containers with lids for different values, 1¢, 5¢, 10¢, and 25¢.)

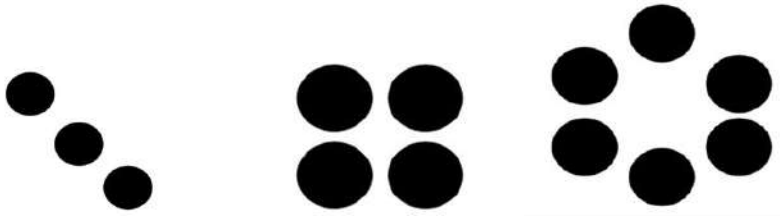
Extension: Students who are ready, can count each collection of similar coins. Count all the pennies, count all the nickels, dimes, etc. and write the value. This is actually not a 1st grade standard, but for students who are ready, pushing them to the next level of thinking is encouraged. Students will solve story problems involving coins or dollars of different denominations in 2nd grade. They will operate with bills and coins and count collections over $1.00 in 4th grade.

Sample Task: Graph the Coins
After students have sorted a collection of coins, have them create an object graph to show the number of each type of coin. Then they can create a picture graph, and/or tally chart to represent and count the amount of each type of coin.

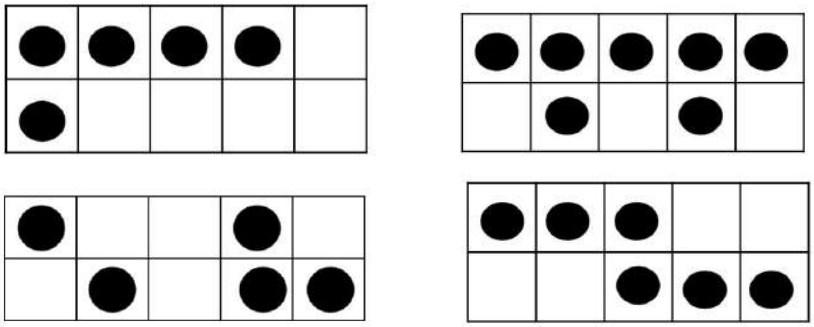
Number Sense Routines/Number Talks:

- Count around the circle (Starting w/any number)
    - Forwards
    - Backwards
    - By 5's
    - By 10's

- Quick Images
    - Dot cards are used to help students subitize, or see quantities quickly without counting 1:1. This is essential for students to have a visual in their mind as they develop number sense. When you flash the dot cards, ask, "How many do you see?" "How do you see them?"

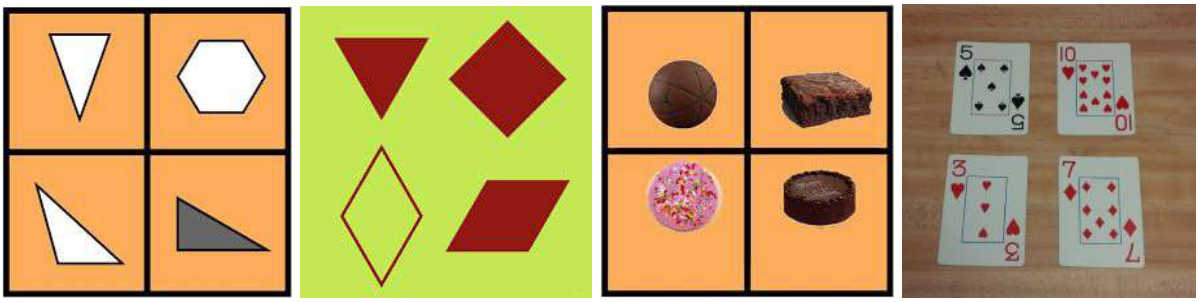


- Ten Frames are used to help students begin to picture quantities of ten. Single ten frames number talks should be used to help students quickly recognize quantities to ten. Here are some examples of single ten frames:



- Which One Doesn't Belong?
  - Students see four objects together and discuss "Which one doesn't belong?". The purpose is to have multiple answers and to practice articulating similarities and differences between shapes, numbers, and other pictures.

More WODBs can be found here.



- Ten Wand
  - The purpose of the Ten Wand is for students to build an understanding of the compositions and decompositions of ten. The teacher holds a "ten wand" (ten unifix cubes connected, 5 of one color and 5 of another) behind his/her back and breaks off a part and shows it to the students. The students must predict how many cubes are still hidden behind his/her back.

- Number of the Day
    - Ask students to write their favorite number on a piece of paper and turn it in. Each day, choose a number at random and ask questions of that number such as:
    - What do you know about this number?
    - When is this number small? When is this number big?
    - What are some ways we can compose/decompose this number into parts & whole?
    - Make up a story that uses this number.

2nd Grade

Unit 1: Data & Shapes (approximately 10 days)

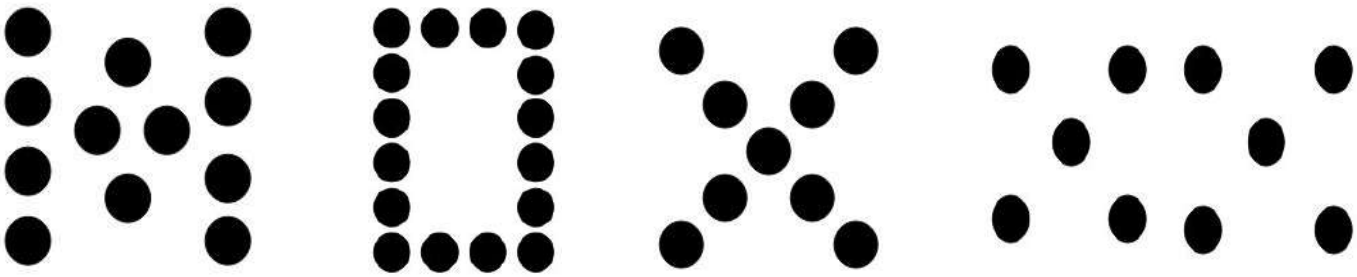
Standards Addressed: 2.MDA.9, 2.MDA.10, 2.G.1, 2.MDA.6

Number Talks/Number Sense Routines:

- Count Around the Circle
    - Start by counting around the circle by 1s, 5s, and 10s. As students become more confident in their abilities, add progressively harder multiples within 10 (e.g. 7s, 8s, etc…).
- Number of the Day
    - Ask students to write their favorite number on a piece of paper and turn it in. Each day, choose a number at random and ask questions of that number such as:
        - How many ones, tens, hundreds, etc… are in this number?
        - Give a context in which this number is small and one where it is large.
        - What are some ways we can decompose this number into two addends? Three addends? A minuend and a subtrahend (subtraction)?

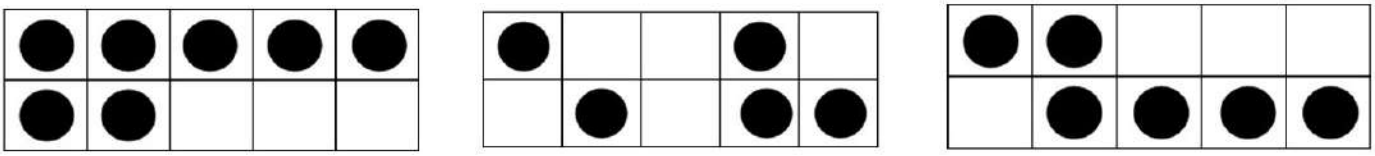
***Note: Number of the Day should be gradually phased out by the end of the 1st or 2nd month.

- Estimation Jar
    - Place a quantity of objects in a jar for students to estimate. You can provide benchmark jars below and above the jar you are asking students to estimate. For example, if the estimation jar is going to have 26 objects, provide a benchmark jar that has 10 objects of the same size, and a benchmark jar has 50 objects of the same size.
    - Give a picture of a jar or another collection of objects and have students estimate how many they see. This builds students' realistic perception of quantity.
- Dot Images
    - This can help students add quantities mentally by looking at those quantities instead of abstract numerals. Here are some examples of some dot cards for 2nd graders.

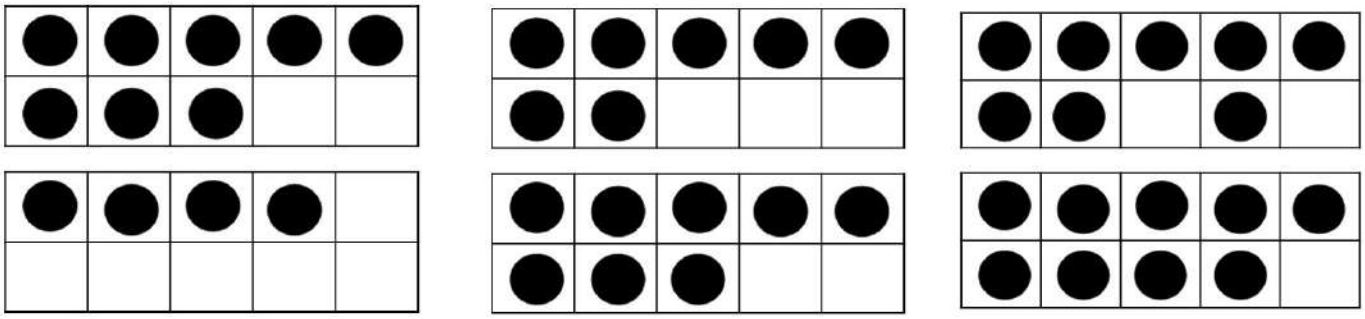


- Ten Frames

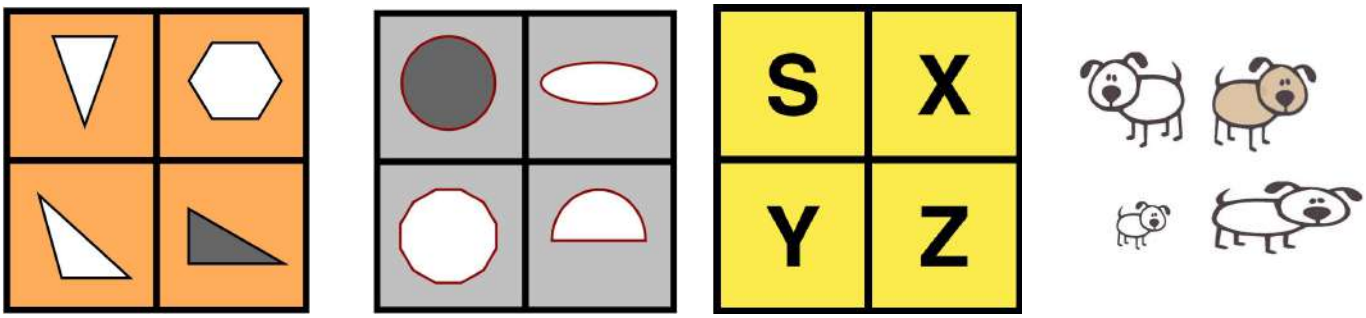
- Ten Frames are used to help students begin to picture quantities of ten. Single ten frame number talks should be used to help students quickly recognize quantities to ten. Double ten frames help students regroup to make a ten and some more.
- Some examples of single ten frames:



- Some examples of double ten frames:



- Rekenreks
  - Rekenreks are also used to help students picture and form quantities to 20 using 5 or 10 as a benchmark.
  - For a video from Graham Fletcher on using rekenreks, click here.
- Which One Doesn't Belong
  - WODB allows students to analyze numbers and use different forms of reasoning to describe why one picture doesn't belong. Each picture within a set can have reasons as to why it doesn't belong. WODBs encourage students to verbalize their thinking and increases the level of discourse in the classroom. More WODBs can be found by clicking here.



Standards Rationale:

These standards were chosen to come first to allow for easier standards to introduce and report on in the 1st quarter, while reviewing 1st grade concepts necessary for future standards during number talks and number sense routines. Since the majority of 2nd grade standards are in place value understanding, this first unit will provide opportunities to review 1st grade number concepts and formatively assess students' level of number sense and fluency while getting to know students through data collection.

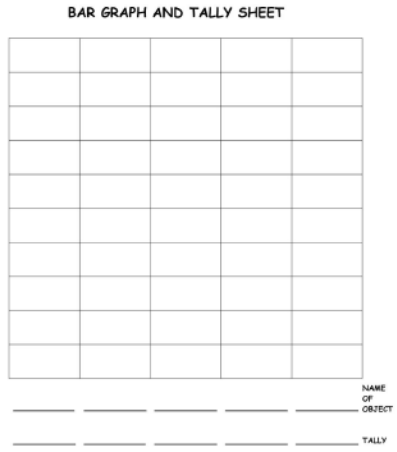
2.MDA.9 Collect, organize, and represent data with up to <u>four categories</u> using picture graphs and bar graphs with a <u>single-unit scale</u>.

This standard is an extension of kindergarten (K.MDA.3) and 1st grade standards (1.MDA.4). Therefore, it is a good place to begin as students are collecting and organizing data to get to know each other. The only new information in 2nd grade is 4 categories and use of a single unit scale. Collecting, organizing, and representing the data is not new.

The school year naturally begins with rules, procedures and "get to know you" activities. This presents the perfect opportunity to collect, organize, classify, and interpret data with 4 categories. Questions could include:

- "How many pets do you have?"
- "What kind of pet(s) do you have?"
- "In what month does your birthday fall?"
- "What is your favorite flavor of ice cream?"
- "What is your favorite school subject?"
- "What is your favorite place to visit?" (Beach, Mountains, etc.)
- "What is your favorite restaurant?"

Students should be encouraged to collect the data through questionnaires or face-to-face questioning, organize the information using a format that they create, represent the data using picture graphs & bar graphs, and interpret the results for their groups/class. Students could also create their own questions, of interest to them, to ask their classmates. The introduction of a single-unit scale reinforces counting and quantity. It's important to note that students do not have to draw a bar graph from scratch, but they should be given a "template" such as the one below that they then label with the scale and categories.



2.MDA.10 Draw conclusions from t-charts, object graphs, picture graphs, and bar graphs.

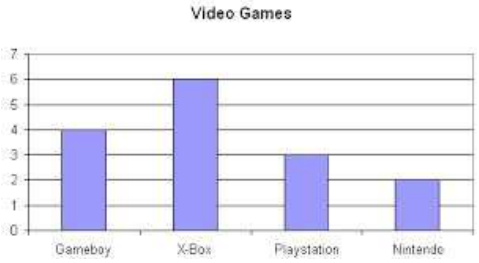
Students have experience with interpreting and drawing conclusions from object, picture, and bar graphs in kindergarten (K.MDA.4) and 1st grade (1.MDA.5). The standard incorporates t-charts, which would only involve 2 categories. When appropriate, represent the same data in different forms (t-chart and bar graph), allowing students to see the similarities and differences to help them draw conclusions. Another opportunity for students to draw conclusions would be to look in nonfiction texts that display data. Science and social studies texts, in and around the room, contain important graphs and charts, so it would be efficient to let students read and interpret those charts and graphs. Additionally, there are multiple websites that have bar graphs and t-charts that need to be interpreted within a relevant and engaging context.

Sample Task: Students in Ms. Tabbie's class were asked about their favorite sports to play. Their answers are given in the following t-chart.

Sport	# of Students
Baseball	5
Soccer	7
Football	3
Volleyball	2
Other	6
No Sports	2

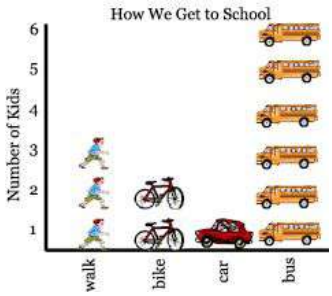
How many students are in Ms. Tabbie's class? How many more students prefer soccer over football? How many students prefer something other than baseball? How many students chose a sport that was listed in the survey?

Sample Task: Mr. Johnson's class was surveyed about the different kinds of video games they play. They put the information in a bar graph as seen below. (*Photo source: kwiznet.com)*



How many students are in Mr. Johnson's class (assuming everyone answered the survey question)? How many more play X-Box over Nintendo?

Sample Task: Mrs. Kingston made a picture graph demonstrating how students in her class get to school. *(Photo credit: beaconlearningcenter.com)*



How many students are in the class? How many students walk, bike, or ride in a car? How many more students ride in a bus than walk?

2.G.1 Identify triangles, quadrilaterals, hexagons, and cubes. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.

After students have collected data about themselves, they can begin to analyze and identify different shapes based on their attributes. In fact, students can begin collecting and organizing data about the attributes of the shapes to present in various graphs and charts.

Sample Task: Take a bucket full of different shapes (e.g. unifix cubes, pattern blocks, tiles, etc…). Close your eyes and grab a handful of shapes and dump it on your desk. Create some sort of display to represent the shapes (triangles, hexagons, cubes, or quadrilaterals). Repeat this same activity again and note the differences in the amount of each shape.

Students have worked with triangles, hexagons, and cubes in kindergarten and 1st grade. The only new shape to students would be the quadrilateral.

Additionally, this is the first time students are explicitly expected to identify angles as an attribute of a shape. They should not have to differentiate between acute, right, or obtuse angles, but identify where the angles are on the shape. They should have opportunities to discuss the attributes of given shapes to form a baseline understanding of those shapes to be able to draw and describe them in a future opportunity.

2.MDA.6 Use analog and digital clocks to tell and record time to the nearest five-minute interval using a.m. and p.m.

This standard is to be reported in the 4th nine weeks, but is relooped continuously throughout the year. At different times in the day, simply ask a student, "What time is it? How do you know?" Teachers are encouraged to use analog and digital clocks to reinforce this concept. It is recommended to cover digital clocks in the room to force students to use analog clocks as students build proficiency with both throughout the year.

Time can also be inserted in as the number sense routine for the day. Students can gather around the carpet and analyze and describe the analog clock as the teacher moves the minute and hour hands.

This standard ***should not*** be isolated into an individual lesson, but should be ongoing.

Unit 2: Place Value (approximately 30 days)
Standards Addressed: 2.ATO.3, 2.NSBT.1, 2.NSBT.2, 2.NSBT.3, 2.NSBT.4, 2.NSBT.8, 2.MDA.6
Number Talks/Number Sense Routines: Continue the number talks from Unit 1. One additional number talk in this unit could be More/Less/Same.

Standards Rationale:
Although these standards address different skills, proficiency in one cannot come without the others. Students must know place value in order to count by tens and hundreds, compare numbers, read numbers, and know ten more/ten less. As a result, these standards should be taught concurrently.

2.NSBT.1 Understand place value through 999 by demonstrating that:

- ***a. 100 can be thought of as a bundle (group) of 10 tens called a "hundred";***
- ***b. the hundreds digit in a three-digit number represents the number of hundreds, the tens digit represents the number of tens, and the ones digit represents the number of ones;***
- ***c. three-digit numbers can be decomposed in multiple ways (e.g., 524 can be decomposed as 5 hundreds, 2 tens, and 4 ones OR 4 hundreds, 12 tens, and 4 ones, etc.).***

NSBT.1 is not reported until the 2nd quarter, but still should be introduced in the 1st quarter. The conceptual understanding of place value takes time and a complete understanding likely will not develop until the 2nd quarter.

These standards are building on students' understanding that a bundle of 10 ones becomes a new unit called a 10 ("ten") (1.NSBT.1). In 2nd grade students should come to see a set/group of 10 tens as a new unit called a 100 ("hundred"). This is an important concept leading up to developing strategies for addition and subtraction based on place value.

Students should be provided opportunities to physically bundle and trade 10 tens for a hundred through a variety of meaningful activities.

Sample Activity: Count around the circle by tens. Students count around the circle by tens and every time they reach 10 tens, they physically bundle themselves together to become a hundred. What do you think happens if we count by hundreds? When should we bundle again? Is it the same as when we counted by tens? By ones? Why?

The importance of conceptual understanding of these standards cannot be understated. In order to be successful with operations with whole numbers, they must understand our place value system. Similarly, they must understand the concept of "nesting" (that ten ones are "nested" inside of a ten and 10 tens are "nested" inside a hundred, etc…). They must have multiple experiences breaking up (decomposing) tens into 10 ones AND decomposing a hundred into 10 tens.

A great manipulative for this work would be the mini connecting cubes since students can physically "break up" a rod instead of "trading" the rod.

One activity frequently used to reinforce part ***b*** of the standard would be to randomly assign numbers to the ones, tens or hundreds places based on drawing cards or rolling dice. These activities imply that the numbers are disconnected from one another rather than working together to form a number. We do *not* recommend using this activity to reinforce part ***b*** of the standard.

Instead, try some of these activities:

Sample Activity: Grab Bag

Fill a bag with base-ten blocks and have students pull a random assortment of blocks from the bag. If they pull more than ten of any block, they must regroup those ten into the next value. This helps students understand the “nesting” concept.

Sample Activity: More/Less

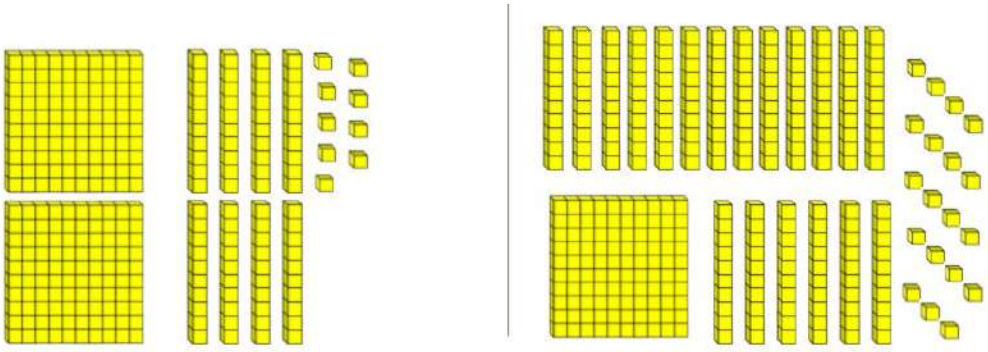
Place a three-digit number on cards in the front of the class. Have students build that number with base ten blocks. Once students have built the number, show a new number with only one place value changed. (For example, go from 364 to 374 to see if students add a rod to their pile or recount.) This helps students relate the place value with the amount representing that digit.

This standard lends itself to being addressed in the number sense routine, Count Around the Circle. The routine should begin by counting by 1’s, then count by tens up to 120, then by tens beyond 120. Then as the year progresses count by 100’s to 1,000.

Decomposition of numbers in multiple ways (part ***c*** of the standard) is absolutely vital in helping students understand the process of regrouping. This reinforces the idea that there are 10 tens inside of a hundred and the hundred can be referred to as a hundred, 10 tens, or 100 ones. Similarly, 5 hundreds could be refer to as 50 tens or 500 ones.

Sample Activity: More/Less/Same?

Show students two side-by-side collections of base ten blocks and ask if the one on the right has more, less, or the same amount as the one on the left. See below for an example:



Sample Response: "I think the one on the right has more because we can bundle ten of the rods together to make a flat (or a hundred), leaving us with 9 tens left over. We can also bundle 10 of the ones to make another ten. We now have another ten rod so we can make another flat. So we have 309 on the right and 289 on the left."

In first grade students are required to compare two two-digit numbers using the words "greater than, equal to, or less than". In 2.NSBT.3, the standard is expanded to include three-digit numbers and the symbols. It is important for students to connect the words to symbols and **NOT** memorize a trick (ex: the alligator eats the bigger number). We want students to understand the language of mathematical symbols and their meanings. Begin instruction using the words and then show the students the symbols for the words.

2.NSBT.2 Count by tens and hundreds to 1,000 starting at any number.

This standard doesn't need a separate lesson, but can be covered through Count Around the Circle and Number of the Day routines throughout the year. Student progress should still be documented throughout the year.

2.NSBT.3 Read, write, and represent numbers through 999 using concrete models, standard form, and equations in expanded form.

Students could represent the numbers in Number of the Day using base ten blocks, standard form, and expanded form. Repeated meaningful experiences within the other standards is the best way to teach this standard. (**Note: Expanded form is any combination of addends that make up a certain number. The traditional place value expansion is the most common, but other ways are possible.)

2.NSBT.4 Compare two numbers with up to three digits using words and symbols (i.e., >, =, or <) Students should be required to compare numbers with words (greater than, less than, equal to) as well as symbols.

When building numbers in standard 2.NSBT.3, students should get a chance to compare two or more three-digit numbers by building, using base ten blocks or their understanding of place values.

Sample Question: Which number is larger 189 or 302? Why is 302 larger since 189 has an 8 and 9, which is higher than the 0 and the 2 in "302"? Explain.

2.NSBT.8 Determine the number that is 10 or 100 more or less than a given number through 1,000 and explain the reasoning verbally and in writing.

This standard is *not* about addition or subtraction, but about a complete understanding of our place value system. For example, if we were to add 100 to 432, there is no need to line the numbers up or use any other algorithm or "strategy". Students must understand that they simply have to add one to the hundreds digit to get 532. They must explain their thinking verbally and in writing.

2.ATO.3 Determine whether a number through 20 is odd or even using pairings of objects, counting by twos, or finding two equal addends to represent the number (e.g., 3 + 3 = 6).

The focus of this standard is based on the conceptual understanding of even and odd numbers. An even number is an amount that can be made of two equal parts with no leftovers. An odd number is one that is not even or cannot be made of two equal parts. The number endings of 0, 2, 4, 6, and 8 are only an interesting and useful pattern or observation and should ***not*** be used as the definition of an even number. (Van de Walle & Lovin, 2006, p. 292)

This standard also marks the beginning of an understanding of equal sharing. If a set of up to 20 items can be divided equally into 2 groups *or* groups of 2, then the number is even. This opens the door to other explorations in division (groups of 3, 3 equal groups, groups of 4, etc…).

Unit 3: Adding and Subtracting within 100 (Approx. 40 days)

Standards Addressed: 2.ATO.1, 2.ATO.2, 2.NSBT.1, 2.NSBT.5, 2.MDA.7, 2.MDA.6

Number Talks/Number Sense Routines: To this point, students have investigated addition using ten frames, dot cards, and some place value concepts. Now, we are moving students to more of an abstract understanding of addition and subtraction within 20 using place value understanding.

If some students still need ten frames or other manipulatives to see and make ten, they should still have them available. We would like to wean students off the use of concrete or representational

manipulatives, but there is no need to rush them through it if they're not ready. In fact, moving them to the abstract too soon could do more damage than letting them continue with fingers or ten frames.

Therefore, during number talks in Unit 3, some students could have ten frames, some could have base ten blocks, and some could only use mental strategies. It is up to teacher discretion and student autonomy to determine if those things are needed.

Some sample number talks could include:

Sample Question: 8 + 7

Sample Student Response: "I know that 8 + 8 is 16, so one less would be 15."
Sample Student Response: "I know that 7 + 7 is 14, so one more would be 15."
Sample Student Response: "I gave 2 to the 8 from the 7, making 10 + 5 = 15."
Sample Student Response: "The 8 is two away from 10 and the 7 is three away from 10. 10 + 10 = 20 and 20 - 5 = 15."

Sample Question: 15 - 9

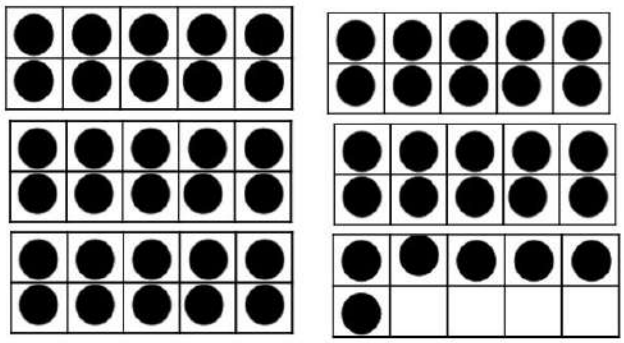
Sample Student Response: "I counted up from 9 to get to 15. I counted 10, 11, 12, 13, 14, 15. So it's 6."
Sample Student Response: "I know that 15 is 5 away from 10 and 9 is 1 more, so it's 6."
Sample Student Response: "I decomposed 9 into 5 and 4 which makes 15 - 5 - 4. 15 - 5 is 10 and 10 - 4 is 6."

Sample Question: 7 + 6 + 5

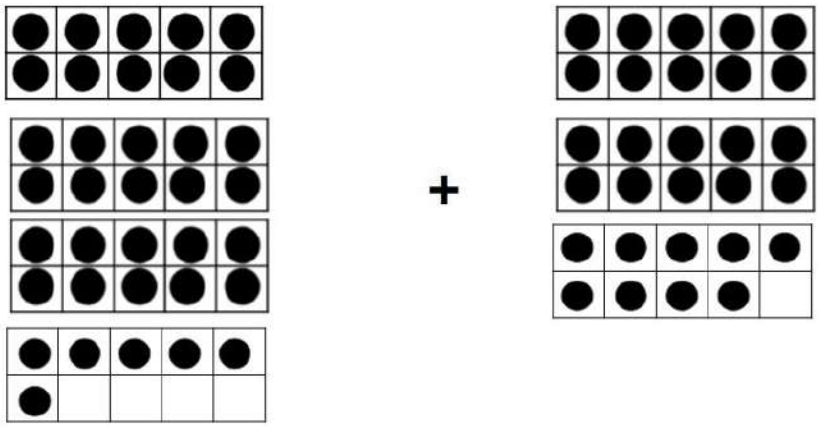
Sample Student Response: "I made ten by taking 3 from 6 and giving it to the 7, so now I have 10 + 3 + 5 or 10 + 8. 10 + 8 = 18."
Sample Student Response: "I made ten by taking 5 from 6 and giving it to the 5 and giving the leftover 1 to the 7, giving me 8 + 10. 8 + 10 = 18."

***Note: For problems like this one, the facilitator should reinforce the associative and commutative properties.

Sample Question: Quick Image: How many dots do you see? How did you know?



Sample Question: How many dots are there? How did you know?



As your students demonstrate proficiency with adding and subtracting double digit numbers using ten frames and other manipulatives, they should begin using abstract strategies. Once again, if some students aren't ready to use mental strategies to solve, they can have manipulatives to guide them, but the ultimate goal (fluency) is to use mental strategies.

Problem sets should include addition and subtraction with and without regrouping.

Mental strategies for addition (e.g. 37 + 38) could include:

- Borrowing to Make a Ten (i.e. "compensation") → (37 + 38 → 40 + 35 = 75)
- Adding by Place Value → (30 + 7 + 30 + 8 → 30 + 30 + 7 + 8 → 60 + 15 → 75)
- Adding in Chunks → (37 + 30 + 8 → 67 + 8 → 67 + 3 + 5 → 70 + 5 = 75)
- Friendly Number → (37 + 38 → 37 + 3 + 38 → 40 + 38 = 78 → 78 – 3 = 75)

Mental strategies for subtraction (e.g. 61 – 28) could include:

- Subtracting in Chunks → (61 – 20 – 8 → (61 – 20) – 8 → 41 – 8 = 33)
- Adding Up → (28 + 2 = 30 + 30 = 60 + 1 = 61 → 2 + 30 + 1 = 33)
- Keeping a Constant Difference → (61 – 2 – 28 – 2 → 59 – 26 = 33 (by place value with no regrouping))

- “Negative” Numbers → (60 – 20 = 40; 1 – 8 = -7 → 40 – 7 = 33)

These strategies shouldn’t be directly “taught” but derived through exploration and student curiosity driven by strategic teacher questioning. Strategic teacher questions can come in the form of purposeful problems or improvisational questioning. Improvisational questions cannot be planned, but can be developed through careful reflection, feedback, and peer collaboration. Purposeful problems are given with the intent of allowing students to recognize more efficient methods of adding and subtracting. For example:

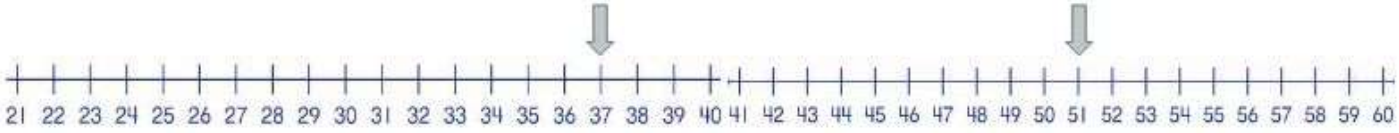
Friendly Numbers:
If 40 + 37 = 77, what is 38 + 37? How do you know?

Compensation:
If 42 + 15 = 57, what is 39 + 18? 35 + 22? 29 + 28? What do you notice about these numbers?

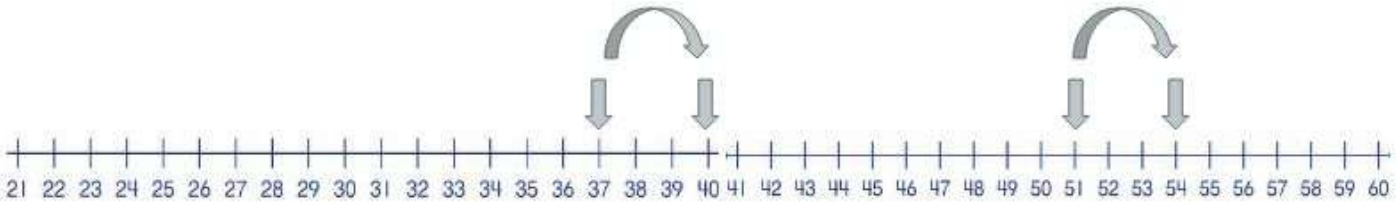
Adding in Chunks & Place Value:
Is 35 + 37 the same as 35 + 30 + 7? What about 30 + 5 + 30 + 7? How do you know? Why would it be convenient for some people to add this way?

Subtracting in Chunks:
81 - 37 = ? (After that one is solved), 81 - 30 - 7 = ? Why are those differences the same? Why would it be more convenient to subtract this way?

Adding Up & Keeping a Constant Difference:
51 - 37 = ? (After it is solved). We know that subtraction is the distance between points. How would you find the distance between the points? (Spend time digging into strategies and asking questions to make their strategies more efficient.)



Is the distance between these two points the same? So would 54 - 40 = 51 - 37? How would you know?



Standards Rationale:

2.ATO.1 Solve one- and two-step real-world/story problems using addition (as a joining action and as a part-part-whole action) and subtraction (as a separation action, finding parts of the whole, and as a comparison) through 99 with unknowns in all position.

2.ATO.1 is nearly similar to the first grade standard 1.ATO.1 with the only difference being the limit of 99 instead of 20. Therefore, students have experience with joining action, part-part-whole, separation action, parts of the whole, and comparison problems from 1st grade.

While addition is a joining action and subtraction a separation action, the operations can be used for a variety of situations. Therefore, students should interact with addition problems that not only model the joining action, but also an analysis of part-part-whole. Similarly, students should have equal access to subtraction problems involving the separation action, finding parts of the whole, and comparison.

Joining Action is exactly like it sounds. Students are taking an existing number and adding a new number to it.

Part-Part-Whole problems involve a total quantity split into two parts. The two parts are given and students must find the sum.

Separation could also be thought of as removal or deduction. It involves the deduction of a quantity from another quantity.

Finding parts of the whole can also be thought of as a missing addend problem. Similar to the part-part-whole relationship, in this type of problem, students are given one of the parts and the total, and they must find the missing part.

Finally, comparison could be thought of as “how many more” or “how much less”. We are given two parts and we are comparing the difference between the two amounts.

The following are sample questions for each type:

Sample Task (Joining Action): The school was having a canned food drive, the goal was for each class to bring over 90 cans in 2 weeks. After the first week of the canned food drive, Mrs. Garrett's class had brought in 46 cans. In the second week, they brought 37 more. Did they reach their goal? If so, how many extras did they have? If not, how many more did they need?

Sample Task (Part-Part-Whole Action): To get to his grandmother's house, Hal must drive 25 miles up Interstate 57 and then over 18 miles on Interstate 68. How many miles does Hal drive to get to his grandmother's house and back?

Sample Task (Separation Action): Matt has saved $87 of his allowance to buy a new video game. The video game costs $49. How much money will Matt have left after he buys the video game?

Sample Task (Finding Parts of the Whole): At Bubba's Burgers, Bubba sold 37 Bubba Burgers on Saturday and some more on Sunday. Over the weekend, they sold a total of 71 burgers. How many burgers did they sell on Sunday?

Sample Task (Comparison): At the Walk-a-Thon, Eva walked 61 miles and William walked 47 miles. How many more miles did Eva walk than William?

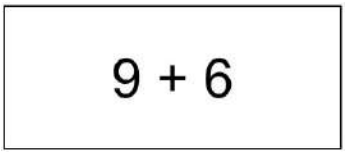
In addition to the 5 types of problems explicitly stated in the standard, students must also solve for unknowns in all positions. The types above illustrate missing sums, missing differences, and missing addends. Students should also work with unknown starts with addition and subtraction (i.e. I saved some money. I spent $37 and had $15 left over. How much money did I start with?)

2.ATO:2 Demonstrate fluency with addition and related subtraction facts through 20.

Fluency is defined as being flexible, efficient, and accurate in computing numbers. Fluency ***is not*** primarily about speed. While speed plays a minor role, fluency is mainly concerned with the *way* that students computed the numbers.

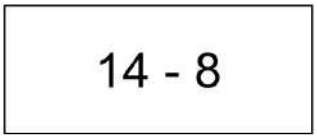
Pencil/paper assessments cannot adequately assess fluency. In order to determine fluency, teachers must make note of student strategies during number talks and tasks. To further supplement the assessments, 1-on-1 interviews could suffice. The following is a vignette demonstrating a possible 1-on-1 interview.

Teacher: I'm going to show you a card and I want you to tell me how you solved it.



Student 1: I made 10 by adding 1 to the 9 from the 6 creating 10 + 5, which is 15.
Student 2: I thought about 6 + 6 which is 12 and since 9 is 3 more than 6, I add 3 to 12 to get 15.
Student 3: I counted up on my fingers. I held up 9 fingers like this and counted up 10, 11, 12, 13, 14, 15.
Student 4: I drew 9 tallies and 6 tallies and then counted all of the tallies and got 15.
Student 5: I like to think about 20. 9 is 1 away from 10 and 6 is 4 away from 10, which is a total of 5. 15 is 5 away from 20, so it's 15.

Teacher: I'm going to show you a new card and I want you to tell me how you solved it.



Student 1: I broke down 8 into 4 and 4 and subtracted 14 - 4 - 4. 14 - 4 = 10 and 10 - 4 = 6.
Student 2: I added up from 8 to get to 14. I said 2 more to get to 10 and 4 more to get 14 so 2 + 4 = 6.
Student 3: I drew 14 circles and marked out 8 of them, leaving me with 6 circles.
Student 4: I counted back from 14 on my fingers. I said 13, 12, 11, 10, 9, 8, 7, 6 and kept track on my fingers.
Student 5: I know that 8 is 2 away from 10. I took 10 from 14 to get 4. But I actually needed to take away 2 less, so my answer is 6.

Let's evaluate each student's answer to determine if he/she is fluent within 20.

Student 1: This student used his/her understanding of 10 to add and subtract efficiently. This student ***is*** fluent.
Student 2: This student used his/her knowledge of doubles to add, and understanding of the relationship between addition and subtraction to solve subtraction problems. This student ***is*** fluent.
Student 3: This student counted on his/her fingers and used pictures to solve the subtraction problem. While he/she understands the concepts of addition and subtraction, he/she is *not* yet fluent.
Student 4: This student also used visual methods to solve the addition and subtraction problems (tallies and fingers). This student is *not* yet fluent.
Student 5: This student used a unique strategy involving his/her knowledge of tens to make the computations convenient. Although this isn't a typical strategy, the student is using mental

strategies to solve problems within 20. He/she could use the same strategies to solve within 100. This student ***is*** fluent.

For students 3 & 4 who are *not* fluent, they should continue to work with ten frames, rekenreks, and other manipulatives to sure up their foundation. They could be given other problems that encourage the making of tens to solve. For example:

Sample Task: Mrs. Miller wants to make cups of crayons for each table. To keep up with the crayons, she decides to put 10 crayons in each cup. She finds 8 crayons on the floor and then Ms. Levine gives her 6 more that she found. How should she organize the crayons?

Sample Question: Mrs. Miller (from the problem above) found 8 crayons. How many more does she need to make a cup of 10? If Ms. Levine found 6 crayons and gave them to Mrs. Miller, does she have enough to make a cup of 10? If so, how many leftover crayons does she have? How many more before she can make another cup?

When writing equations in this and other standards, students should see the equations with the sums and differences at the beginning and end. For example 9 + 8 = 17 should also be represented as 17 = 8 + 9.

2.NSBT.5 Add and subtract fluently though 99 using knowledge of place value and properties of operations.

Although both standards use the word “fluently”, the application of the two words have slightly different meanings in standards ATO.2 and NSBT.5. In ATO.2, fluency means that students perform the operation mentally. In NSBT.5, students do not necessarily have to compute mentally, but can record their thinking to keep track of the numbers. Some of the strategies can be found in the Number Talks section above.

Teachers should not directly teach these strategies but give students the opportunity to derive and construct them through engaging and problematic contexts. Here are some examples:

Sample Task (Additive Strategies): Mifflin Elementary School is collecting cans for a canned food drive. To keep track of how many cans have been collected, Mr. Beasley puts every ten cans in a paper bag. Mr. Halpert’s class collected 4 bags and 7 extra cans. Mrs. Scott’s class collected 3 bags and 4 extra cans. How many cans have they collected all together?

Sample Task (Adding Up to Subtract): Bayview Elementary School is raising money for the American Heart Association. If a student raises $90, they will get a $20 Target gift card. Melody has raised $65. How much more money does she need to raise to get the $20 gift card?

***Note: Students could still deduct to find the difference, but the context calls for adding up.

2.MDA.7 Solve real-world/story problems involving dollar bills using the $ symbol or involving quarters, dimes, nickels, and pennies using the ¢ symbol.

A close read of the standard reveals that students should solve problems involving EITHER dollar bills OR quarters, dimes, nickels, and pennies. Since students should add one or the other, there is no need to involve decimals. See the following examples for the baseline expectations for 2nd graders.

Sample Task: On a recent shopping trip, Jan spent $26 on a new skirt and $35 on a new sweater. How much did she spend in all? If she paid with 4 $20 bills, how much change did she get back?

Sample Task: A candy bar at the store costs 84¢. Dwight found 69¢ in his car. How much more change does he need to find to buy the candy bar? What coins could he use?

At no point did we need to use decimals nor did we need to use dollars and cents in the same problem.

Unit 4: Expanding Addition & Subtraction (Approx. 20 days)
Standards Addressed: 2.ATO.1, 2.ATO.2, 2.NSBT.5, 2.NSBT.6, 2.NSBT.7, 2.MDA.6
Number Talks/Number Sense Routines: Continue number talks from Unit 3.
Standards Rationale: Teachers should continue working on mental fluency within 20 and computational fluency within 99 using problematic and engaging contexts.

2.NSBT.6 Add up to four two-digit numbers using strategies based on knowledge of place value and properties of operations.

Students should apply their understanding of adding two-digit numbers within 99 to add up to four two-digit numbers. Students are to use knowledge of place value and properties of operations. Students can employ the same strategies from previous standards by making tens or adding the total tens and ones.

Sample Task: The following table shows how many students the four 2nd grade teachers have at Sims Elementary School.

Teacher	# of Students
Lappin	19
Nunez	22
O'Neil	27
Demsick	17

How many students are in the 2nd grade?

Sample Student Response: "I wanted to make even tens where I could. I gave one to the 19 from the 22 to make 20 and 21. I gave 3 from the 17 to the 27 making 30 and 14. So, 20 + 30 = 50, 50 + 21 = 71 and 71 + 14 = 85."

Sample Student Response: "I thought of it as 20 + 20 + 20 + 20 = 80. 19 is one less than 20, 22 is 2 more than 20, 27 is 7 more than 20 and 17 is 3 less than 20. (-1, +2, +7, -3) The answer will be 5 more than 80 which is 85."

Sample Student Response: "I added by place value. I added 10 + 20 + 20 + 10 to get 60. Then I added 9 + 2 to get 11, 7 more to get 18 and then 7 more to get 25. 60 + 25 = 85."

Sample Task: Graham Fletcher has a Three Act Task involving adding 5 two-digit numbers. The task can be found by clicking here.

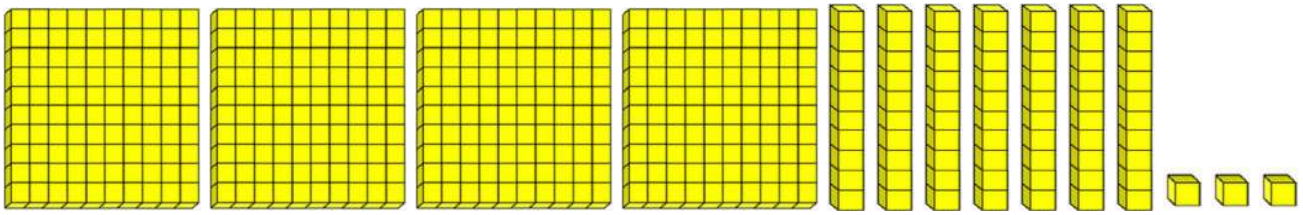
2.NSBT.7 Add and subtract through 999 using concrete models, drawings, and symbols which convey strategies connected to place value understanding.

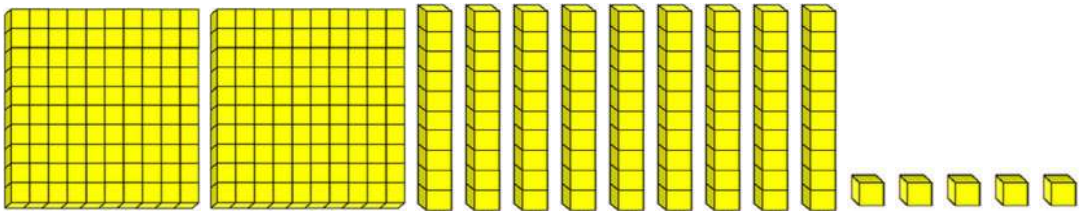
Students should progress from concrete to representational to an abstract understanding of addition and subtraction within 999. Students should be given problematic and engaging contexts that require students to operate with three-digit numbers. Students are *not* required to be "fluent" so being able to add and subtract using ***any*** method is sufficient.

Sample Task: Liz and her family are driving from Charlotte to Chicago to visit her aunt. They plan to drive from Charlotte to Louisville on Friday and then Louisville to Chicago on Saturday. According to Google Maps, it is 473 miles from Charlotte to Louisville and it's 295 miles from Louisville to Chicago. What is the total distance from Charlotte to Chicago?

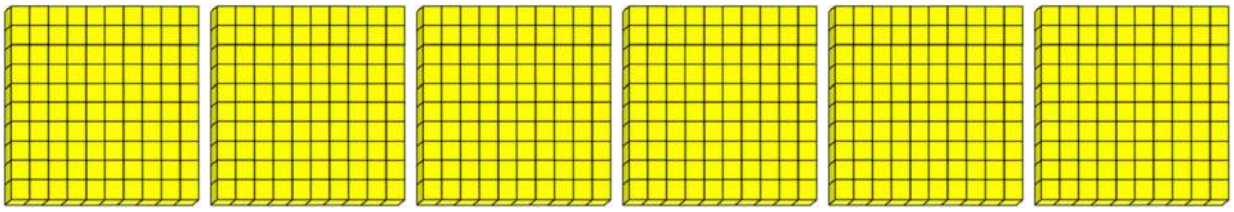
Concrete

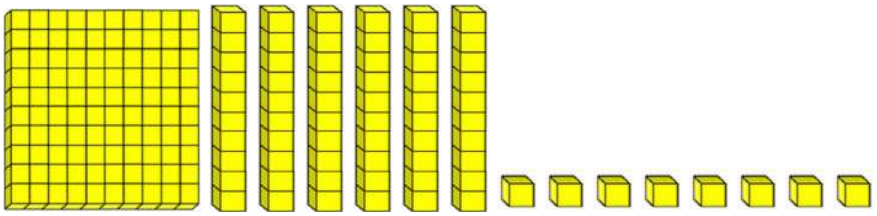
Sample Student Response: "I made 473 and 295 with base ten blocks."





"Next, I looked to see if I could regroup anything to a larger unit. I know that 10 tens make a hundred, so I regrouped those into another flat."

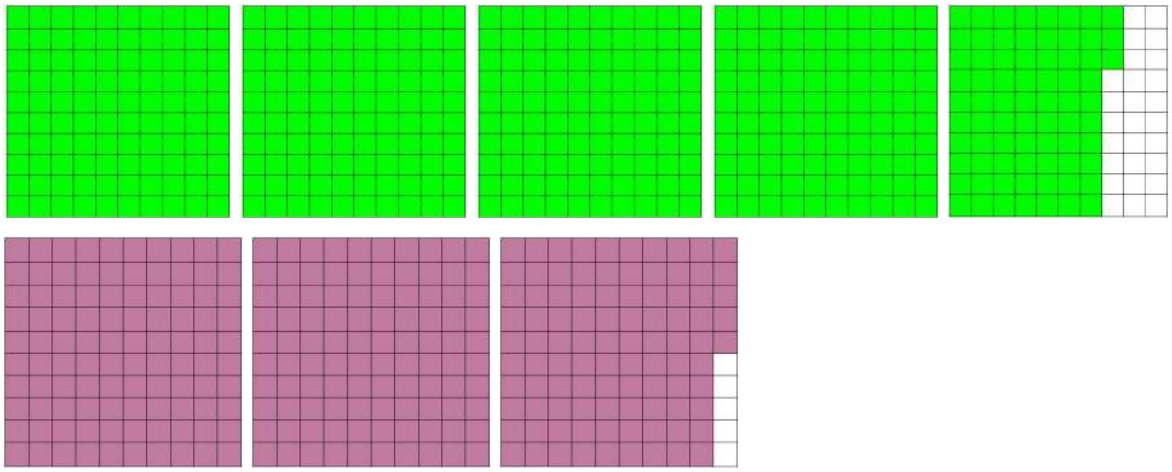




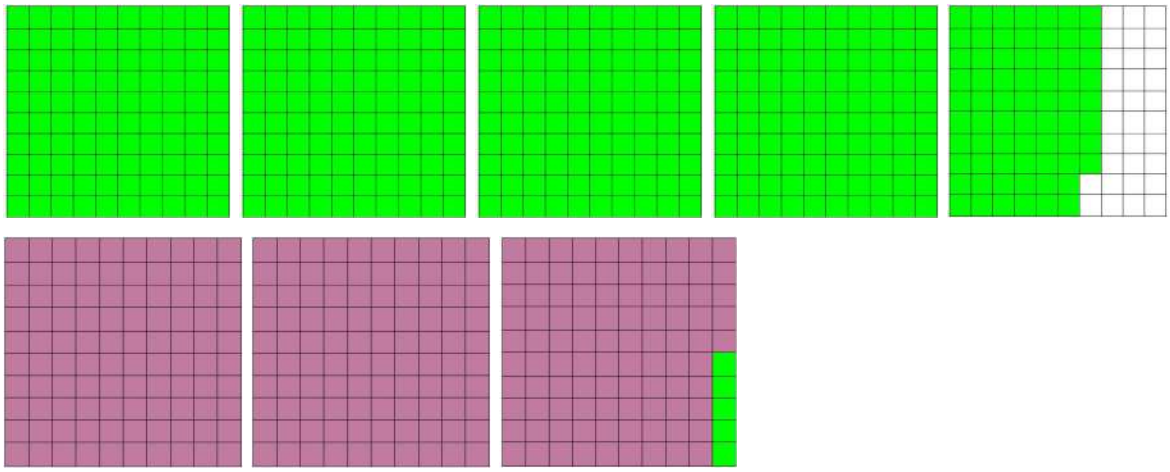
"After regrouping, I have 768 miles."

Representational

Sample Student Response: "I took some hundred grids and colored in the 473 (green) and 295 (purple)."



"I noticed that I only needed 5 more purples to make a full hundred, so I took them from the other grids."



"Now, I have a total of 7 full grids, 6 full columns, and 8 small squares, making 768."

Abstract

Sample Student Response: "I added by place value."

400 + 200 = 600
70 + 90 = 160
3 + 5 = 8
600 + 160 + 8 = 768

Sample Student Response: "I compensated by giving 5 to 295 from 473, making 300 + 468 = 768."

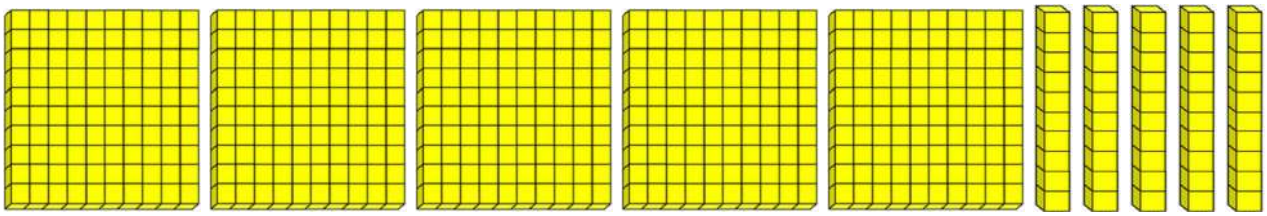
Sample Student Response: "I added up by place value. I added 473 + 200 to get 673. I added 673 + 90 = 763 and 763 + 5 to get 768."

Sample Student Response: "I noticed that 295 was close to 300 so I just rounded it up to 300 and added it to 473 to get 773. Since I added 5 to get to 300 when I rounded, I took 5 away from 773 to get 768."

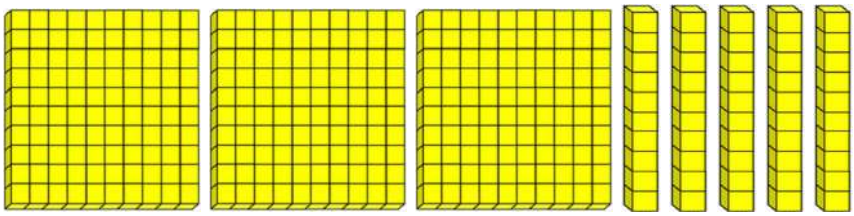
Sample Task: The city community center donated 550 bookbags to local elementary schools. Rockefeller Elementary School received 287 bookbags. How many were left for the other schools?

<u>Concrete</u>

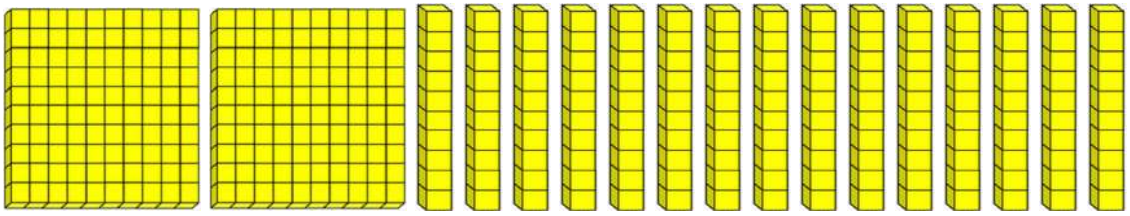
Sample Student Response: "I made 550 with base ten blocks. I used 5 flats and 5 rods."

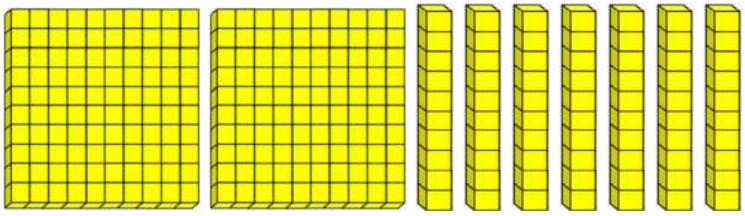


"Next, I took away 2 flats."

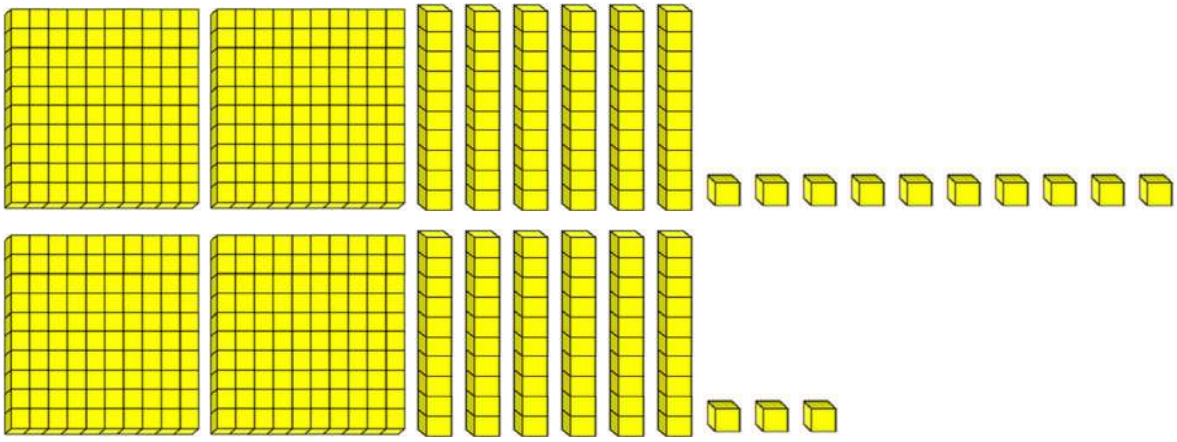


"Next, I wanted to take away 8 rods, but I didn't have enough so I decomposed a flat to make 10 rods and then took 8 of those away."





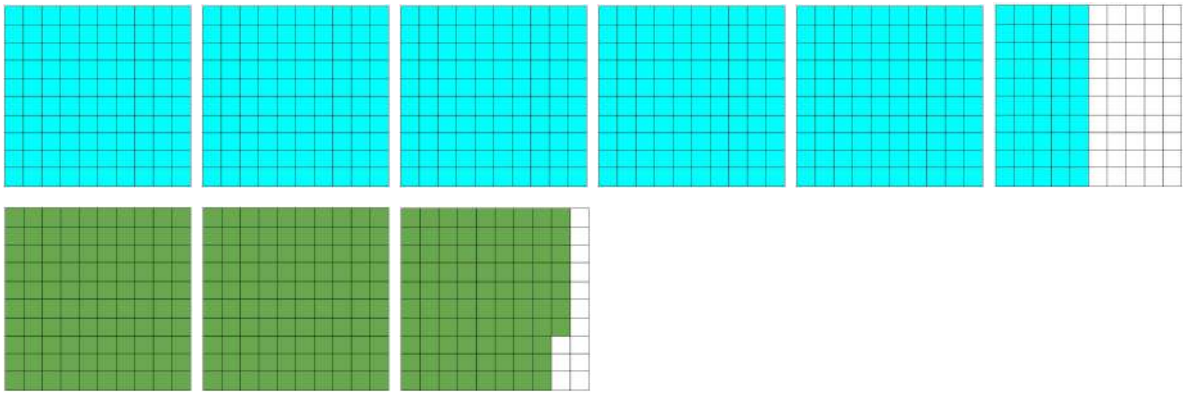
“Finally, I needed to take 7 individual cubes away, but I needed to decompose a rod to take those away.”



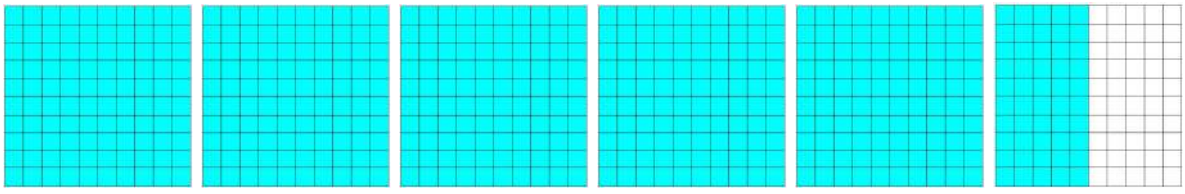
“Then looking at the blocks I had left, I saw the answer was 263.”

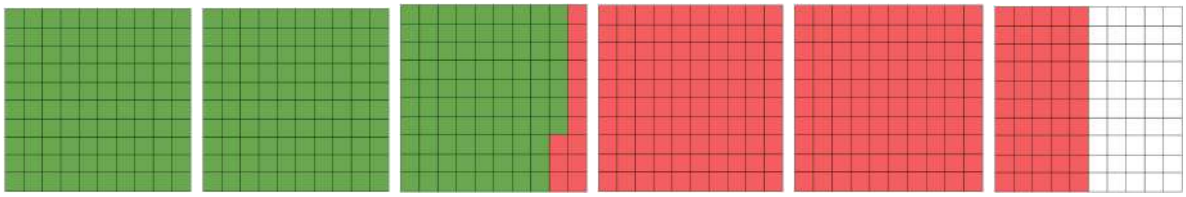
<u>Representational</u>

Sample Student Response: “I filled out hundreds grids for 550 (blue) and 287 (green).”



“To find the difference, I tried to figure out how much I needed to add to make 550.”





“I counted up the red blocks which represented the difference between 550 and 287. I counted the 2 full grids (200) and each column (60) and then the leftover 3 to make 263.”

Abstract

Sample Student Response: “I added up from 287 to get to 550. I added 13 to get to 300, 200 to get to 500 and then 50 more to get to 550. Then I added up 13, 200, and 50 to get to 263.”

Sample Student Response: “I subtracted in chunks. I subtracted 200 from 550 to get to 350. Then I subtracted 80 from 350 to get to 270. Finally, I subtracted 7 more to get to 263.”

Sample Student Response: “I added 13 to each number to make 563 - 300, which equals 263.”

Sample Student Response: “I wanted to subtract by place value, but I was unable to take away.

5 hundreds - 2 hundreds

5 tens - 8 tens

0 ones - 7 ones

So, I regrouped one of my hundreds into 10 tens and one of my tens to make 10 ones.”

4 hundreds - 2 hundreds = 2 hundreds

14 tens - 8 tens = 6 tens

10 ones - 7 ones = 3 ones

263

Unit 5: Measurement (Approx. 25 days)
Standards Addressed: 2.MDA.1, 2.MDA.2, 2.MDA.3, 2.MDA.4, 2.MDA.5, 2.MDA.8, 2.MDA.6
Number Talks/Number Sense Routines: Continue number talks from Units 3 & 4.
Standards Rationale: ***2.MDA.1 Select and use appropriate tools (e.g. rulers, yardsticks, meter sticks, measuring tapes) to measure the length of an object.***

In order to be able to select an appropriate tool, students must have a general idea of the length of different measuring tools. Students should have opportunities to measure different objects inside and outside of the classroom with a variety of tools at their disposal.

Sample Question: What tool would you use to most accurately measure the length of your driveway? The length of your arm? The width of your desk? Explain in writing why you chose your tool.

The difficult part of assessing this standard is the subjectivity of the possible answers. Students, not the teachers, are the ones that determine which tool is most appropriate for them. To be clear, there are situations that are *obviously* inappropriate (e.g. measuring a football field with a ruler), but many measureable objects could be measured with a variety of tools. Teachers should accept a variety of answers as long as there are detailed explanations of their choices.

2.MDA.2 Measure the same object or distance using a standard unit of one length and then a standard unit of a different length and explain verbally and in writing how and why the measurements differ.

Students have difficulty understanding how an object could, hypothetically, be 3 feet long *and* 36 inches long. “36 is larger than 3”, they might say, “How is that the same length?”. One way to help students understand is highlighting the difference between hundreds, tens, and ones.

Sample Discussion Questions: Which is larger? Inches, feet, or yards? How do you know? Show with your hands an estimate of each length. Which is larger: 4 tens or 8 ones? Why? Which is larger 5 inches or 3 feet? How do you know?

Sample Student Response: “I think that 3 feet is larger even though 5 is a larger number. Feet are much longer than inches and so 3 of these lengths (shows feet with hands) is longer than 5 of these lengths (shows inches with fingers).”

Students should continue to measure different objects inside and outside the classroom, but this time in three different units. They must take note and explain why the numbers get larger as the units get smaller, and why the numbers get smaller as the units get larger.

It is important to note that the baseline expectation for 2nd grade is *standard units (or customary units)*. Therefore, there is no need to compare measurements in metric units. However, it would not be developmentally inappropriate to inquire about those measurements in the discussions or tasks.

2.MDA.3 Estimate and measure length/distance in customary units (i.e., inch, foot, yard) and metric units (i.e., centimeter, meter).

The clear output of the standard is for the students to estimate and measure. When interacting with a length to measure, students should estimate first and then measure to check their estimates. The more opportunities they have to estimate and check, the more realistic and accurate their estimates will become.

In order to be able to estimate units, students must have some idea of what each length looks like. Through repeated interaction and engaging activities, students will begin to build their conceptual understanding of what inches, feet, and yards look like, as well as centimeters and meters.

Sample Task: Look at the following pictures: How long do you think each item is? What unit did you choose and why did you choose that unit? Is it possible for multiple people to have a different answer, but still all be right? Explain.

2.MDA.4 Measure to determine how much longer one object is than another, using standard length units.

In order to determine “how much longer” students must have experience with comparison problems. Students do not necessarily have to subtract, but could add on to the shorter length until they get to the greater length. Once again, the units are standard units instead of metric units.

Additionally, students should only compare using the same units (e.g. 4 inches vs. 7 inches or 5 feet vs. 11 feet, but NOT 5 feet vs. 11 inches).

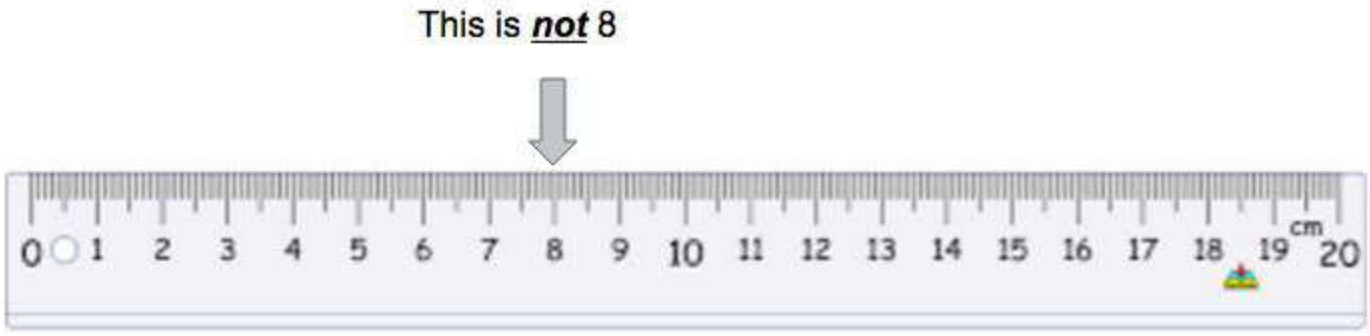
2.MDA.5 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences through 99 on a number line diagram.

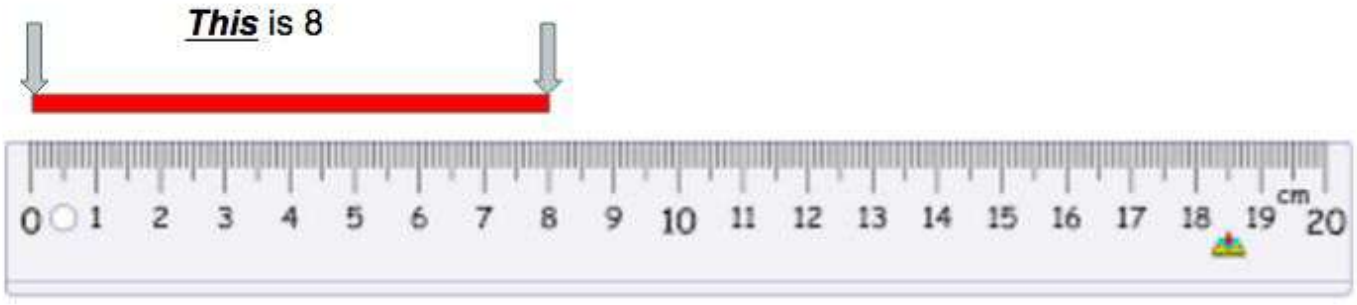
Traditionally, the number line is directly taught as a "strategy" for addition and subtraction. It is recommend that students understand the numbers on a number line represent lengths *first*. From there, if students recognize that the number line can help them solve addition and subtraction problems through 99, they can use it. It is not recommended to "directly teach" the number line method for adding and subtracting. Instead, show the number line and ask "How can this tool help us add and subtract more efficiently?"

This is the first time students have explicitly interacted with a number line diagram. In kindergarten and 1st grade, students operated with open and organic number lines, but likely did not work with equally spaced number lines that resembled measurement tools.

This standard also addresses other functions of numbers. Up until this point, students have interacted with numbers as quantities. Now, in 2nd grade, students are seeing that numbers can be used to represent measurements (lengths) in addition to quantities. These numbers can be manipulated in the same way that quantities can be manipulated, so long as they have the same units.

The most important component of this standard is the understanding that the numbers on the measurement tool represent the distance from 0 to that point. That point itself is *not* the number. For example:

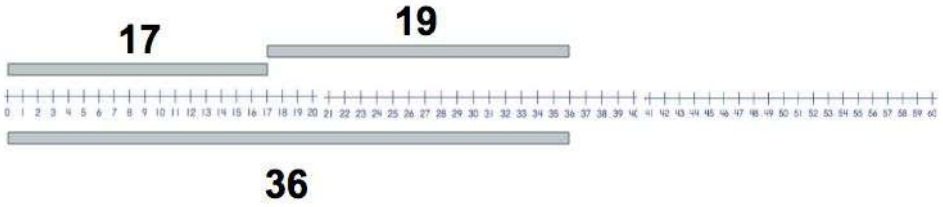




To represent sums and differences on a number line diagram, students must consider the different types of addition and subtraction contexts from ATO.1. They can be represented in the photos below:

Joining Action or Part-Part-Whole

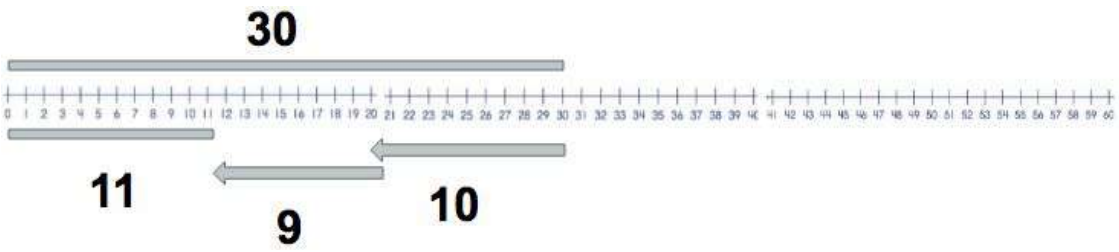
Margie read 17 books in the month of September. Dawn read 19 books in September. How many books did they read altogether?



Separation Action

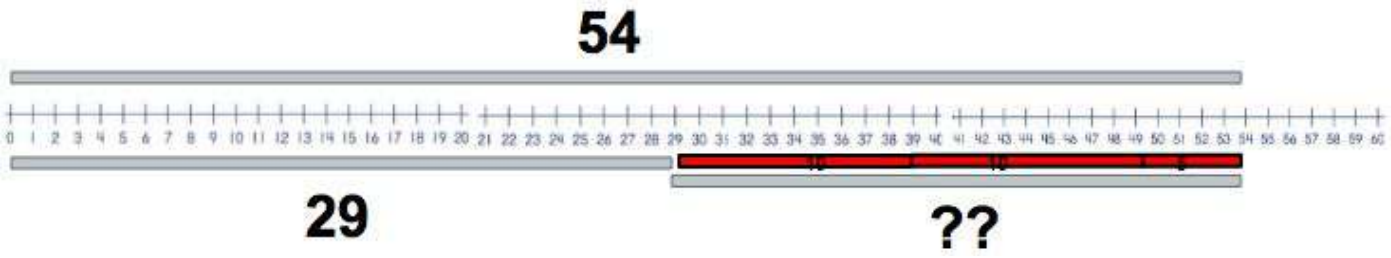
Bruno was given $30 for mowing the lawn on Saturday. He went to a movie with his friend and spent $19 on a movie ticket and popcorn. How much money does he have left?

***Teacher note, students might break down subtrahends and addends into 10s and some more ones to break up the distance needed for “jumps”.



Finding Parts of the Whole or Comparison

Justin is checking his cell phone bill. His two children sent a total of 54 text messages last week. His daughter sent 29 texts. How many did his son send? How do you know?



2.MDA.8 Generate data by measuring objects in whole unit lengths and organize the data in a line plot using a horizontal scale marked in whole number units.

This is the first time students have experienced line plots. As students measure different objects around the room, they can plot those points on a line plot. The purpose of a line plot is to show data frequency and multiple instances of the same data point (e.g. measurements).

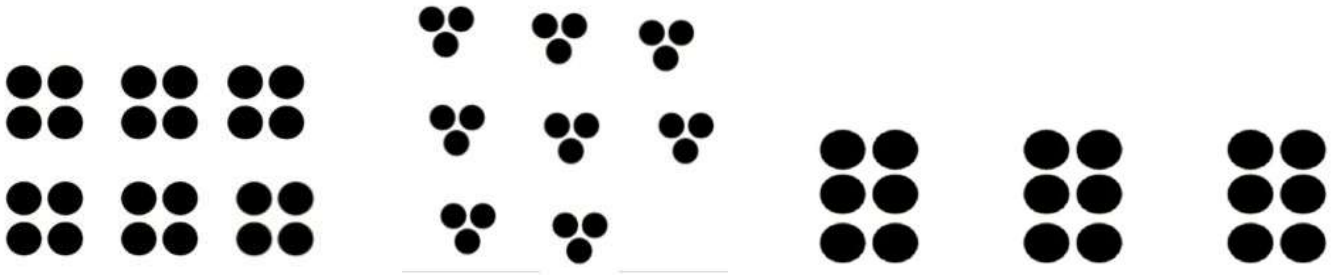
Students should interact with activities what will likely produce not only a variety of measurements, but also some of the same measurements so that they can see duplicated results on the line plot.

Sample Task: Measure the lengths of the pencils of everyone in your group and a nearby group and mark those lengths on a line plot.

Unit 6: Spatial Reasoning (Approx 20 days)

Standards Addressed: 2.G.2, 2.G.3, 2.ATO.4, 2.MDA.6

Number Talks/Number Sense Routines: Students should continue building on their abstract understanding of addition and subtraction within 100. If they are ready to abstractly consider operations within 1,000, the teacher can use those number talks also. In addition, if you would like students to think multiplicatively, you could start doing multiplication dot cards. 2nd grade is not a time to teach multiplication notation, but students can begin to think multiplicatively if they are ready. Here are some examples of multiplication dot cards:



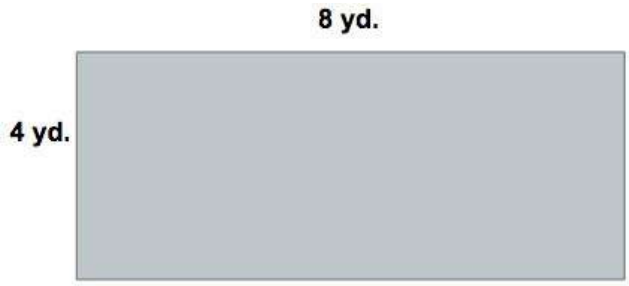
Students could skip count and combine groups (i.e. instead of 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3, think of it as 9 + 9 + 6).

Standards Rationale:

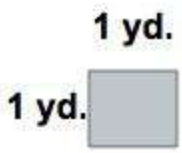
2.G.2 Partition a rectangle into rows and columns of same size squares to form an array and count to find the total number of parts.

This standard is the prerequisite to area in 3rd grade. Area is expressed as the number of unit squares that cover a specific shape, in this case a rectangle. In 2nd grade, students practice "tiling" the rectangles and counting the squares within the shape. Students are permitted to skip count, multiply, or use other, more efficient strategies to determine the total. They are not limited to *just* counting.

Sample Task (Opening/Intro.): I want to make a concrete patio in the shape of a rectangle in my backyard.

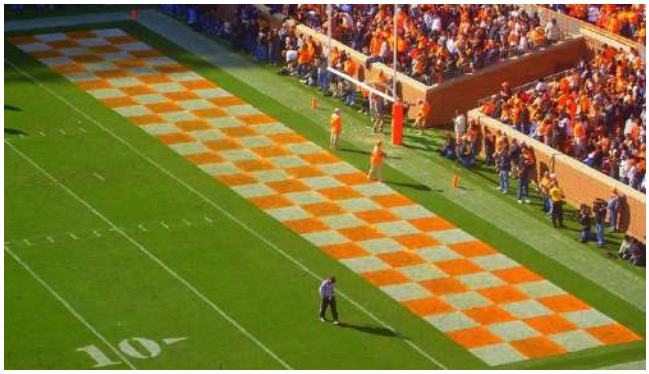


I asked a concrete service how much they charge, but they only told me how much they charge for squares that are 1 yard on each side. How many of those 1 yd. squares will it take to fill my patio space?



Do not tell your students *how* to partition the rectangles. Instead, give them a context and let them figure out how to do it. For differentiation, students could be given the price of each square yard and have to skip count to determine the price.

Sample Task: The Tennessee Volunteers football team has a checkerboard end zone as seen in the picture below.



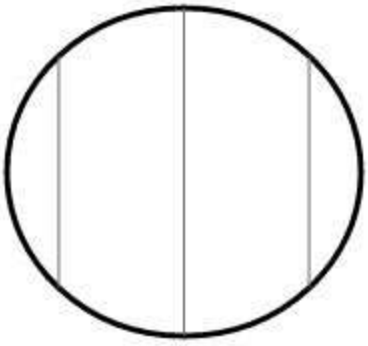
Each square is about 1 yard on each side. If the length of the space for the checkerboard stretches 30 yards and the width is 4 yards, how many squares are painted in the space? How many are orange? How many are white? Recreate the picture in the space below.

2.G.3 Partition squares, rectangles, and circles into two or four equal parts, and describe the parts using the words halves, fourths, a half of, and a fourth of. Understand that when partitioning a square, rectangle, or circle into two or four equal parts, the parts become smaller as the number of parts increases.

While on the surface, this standard seems to set students up for the fraction work in upcoming grades, this standard actually is intended to build students' spatial reasoning. The major focus is recognizing what lines would create equal parts in squares, rectangles, and circles. The other major focus is understanding the seemingly contradictory idea that the parts become smaller as the number of parts increases. The only way that students will understand this is by investigating it on their own accord. The teacher can ask questions such as, "What do you notice about the pieces when we went from halves to fourths? Why do you think that happens?" and "If you had a brownie, would you get more if you split it with 1 other person or with 3 other people? How can 2 pieces be larger than 4 pieces?"

Secondarily, the vocabulary of *halves*, *fourths*, *a half of*, and *a fourth of* helps students begin to think about fractional reasoning. ***However***, students are not expected to use fraction notation to express the sizes of the parts. There is no need to introduce students to ½ or ¼. Additionally, "thirds" are no longer in the standards. The baseline expectation of the standard is for students to split squares, rectangles, and circles, into *equal* parts and describe using fractional *unit words*.

Sample Question: Is the following circle partitioned into four equal parts? Explain. If not, what can I do to make it equal parts?



Sample Task: I made a small rectangular cake for a party. There are 4 people who want to eat equal parts of the cake. How many different ways can I cut this cake into 4 equal pieces?

Sample Student Responses:

***Note: The fifth picture above created two triangles that were half of the rectangles, then the two triangles were cut in half from different vertices. Even though the shapes are different, the sizes are the same.

2.ATO.4 Use repeated addition to find the total number of objects arranged in a rectangular array with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

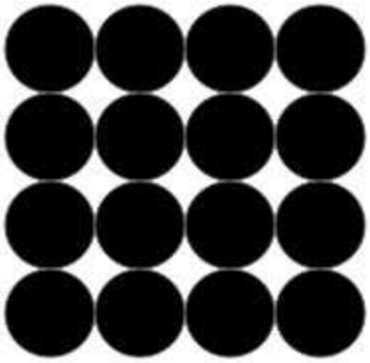
This is the first time that students have been introduced to the word “array”, specifically a rectangular array. An array is defined as a set of items arranged in a particular way. In order for it to be a rectangular array, there must be an equal number of rows and columns.

The way that the standard is worded emphasizes that the objects are *already arranged* in a rectangular array and the requirement of the student is to use repeated addition to determine the total number of objects. It is *not* required that students physically arrange a group of objects into an array. However, it could be developmentally appropriate to allow students to arrange a prepared number of counters or cubes into different arrays (e.g. 12 counters).

Similarly, the standard restricts the number of rows and columns to 5 at the max. While this represents the baseline expectation, it wouldn’t be developmentally inappropriate to let students investigate more rows and columns.

The standard refers to rows and columns, but it is not a requirement that students recall the difference between rows and columns. The teacher should always use correct mathematical vocabulary in all student interactions, but a student’s inability to differentiate between rows and columns should not prevent him/her from demonstrating proficiency in the standard. The bottom line proficiency is being able to recognize an array and use a strategy other than one to one counting to determine the total, namely repeated addition.

Sample Task: Quick dot arrays can encourage students to add instead of count. Some dot arrays can be seen below:



Students should express these sums in an equation (e.g., 4 + 4 + 4 + 4 =16).

Sample Task: When I was in Boston, I looked out my hotel window and saw this building.

I determined the total number of windows without counting them. How did I do it? How would you do it?

Sample Task: Where do you see arrays? Take pictures with a cell phone or your iPad as you see arrays around the school or in life and write a repeated addition equation for each picture.

Note: While this standard is building students up to operate with groups of objects instead of individual objects, this standard ***is not about multiplication. Repeated addition is *not* multiplication. They are two completely different operations. This is *not* a time where teachers should teach multiplication or multiplication notation. The repeated addition and skip counting in this standard prime the ground to begin to fluently multiply, but repeated addition is a completely different operation from multiplication.

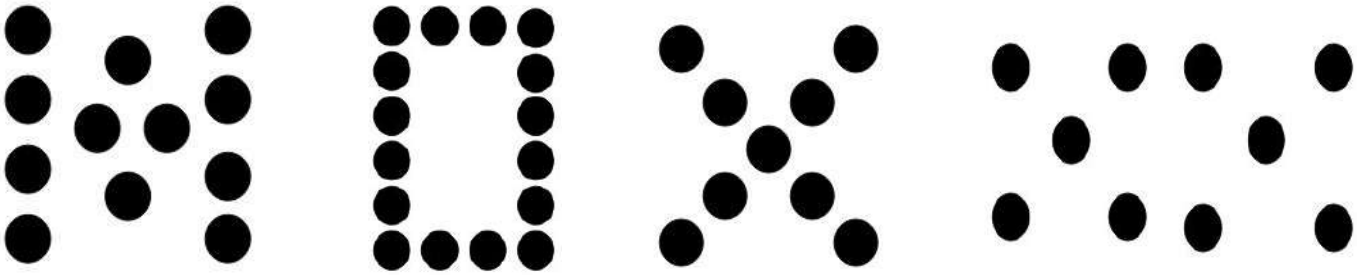
3rd Grade

Unit 1: Data, Patterns, and Introductions (Approx. 10 days)

Standards Addressed: 3.MDA.3, 3.ATO.9, 3.MDA.1

Number Talks/Number Sense Routines:

- Making Tens on two ten frames (e.g. 8 + 5, 4 + 9, 6 + 7, 8 + 9). Students should derive their own strategies to include doubles/near doubles, adding by 5s, moving dots to make a ten, and other efficient strategies.
- Dot Images/Arrangements within 20



- Number of the Day
- Count Around the Circle (by numbers 1 - 9)

Standards Rationale:

These standards were chosen to come first to allow for some easier standards to introduce while reviewing 2nd grade concepts necessary for future standards during number talks and number sense routines.

3.MDA.3 Collect, organize, classify, and interpret data with multiple categories and draw a scaled picture graph and a scaled bar graph to represent the data.

The school year naturally begins with rules, procedures and "get to know you" activities. This presents the perfect opportunity to collect, organize, classify, and interpret data with multiple categories. Questions could include:

- "How many pets do you have?"
- "In what month does your birthday fall?"
- "What is your favorite flavor of ice cream?"
- "What is your favorite school subject?"
- "Who was your teacher last year?"
- "How do you get to/from school?"

Questions should be formed to allow for multiple categories. Students should be encouraged to collect the data through questionnaires or face-to-face questioning, organize the information using a format that they create, classify the data by desired category, and interpret the results for their groups/class. Students could also create their own questions, of interest to them, to ask their classmates.

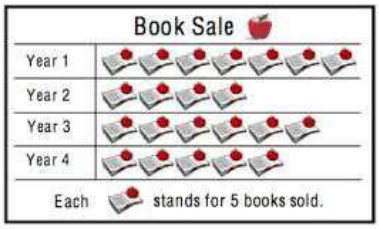
Students were required to do the same activities in 2nd grade (2.MDA.9). The difference in 3rd grade is the addition of scaled graphs. The numbers used shouldn't be so large that they are difficult for the students to reason mathematically.

In addition to creating tables and graphs, students should also be able to analyze given graphs and tables to draw conclusions and perform simple operations with the data. Examples of scaled picture graphs and scaled bar graphs with sample questions are shown below.

***Source: Excelmath.com

Sample questions for the graph above:

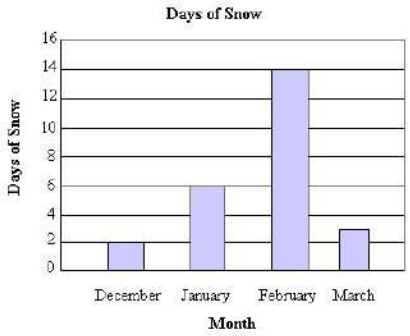
- How many more newspapers did Maxine sell than Lillian?
- How many newspapers did Nick sell?
- What was the total number of newspapers sold by all four people?



***Source: Jamesville Dewitt School District

Sample questions for the graph above:

- How many more books did they sell in year 1 than in year 2?
- How many did they sell in years 3 and 4 combined?



***Source: Basic-mathematics.com

Sample questions for the graph above:

- How many more days did it snow in February than in March?
- Did it snow more in February than in all the other months combined? Explain.

Scaled picture graphs require multiplicative reasoning. However, some students may not have developed multiplicative thinking by this point. Students can still use repeated addition to determine totals and begin to think of single objects representing multiple units (unitizing).

3.ATO.9 Identify a rule for an arithmetic pattern (e.g. patterns in the addition table or multiplication table).

An arithmetic pattern is a pattern that increases or decreases by a fixed amount each time.

Examples: 4, 9, 14, 19… ; 453, 461, 469, 477, 485… ; 78, 69, 60, 51…
Non-Example: 4, 8, 16, 32, 64… (This is a geometric pattern. It is scaled by a *multiple* each time.)
Non-Example: 8, 11, 15, 20, 26, 33… (The rule for this pattern is add one to each difference. However, the difference does not remain the same for each number and therefore is NOT an arithmetic pattern.)

In this standard, students only have to identify the rule for the pattern (e.g. “add 5” or “subtract 9”). Students are *not* required to continue the pattern or even create their own pattern. Those skills are added in 4th grade (4.ATO.5). However, it isn’t *wrong* to encourage students to explore and predict numbers that may come next.

The abbreviation “e.g.” literally means “for example”. “E.G.” provides examples of what it *could* look like, but is not an exhaustive list of what it can only look like. When the standard says to use the multiplication table, it is not referring to multiplicative patterns (e.g. Rule: Multiply by 3). Students should look at a multiplication table (see the picture below) and see arithmetic patterns within the multiplication table.

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

With the multiplication table, broad questions such as, “What patterns do you see?” or specific questions such as, “What do you notice about the numbers in the “8” column? What about the “8” row? What does that tell you?”, could be asked.

3.MDA.1 Use analog and digital clocks to (a) determine and record time to the nearest minute, (b) using a.m. and p.m, (c) measure time intervals in minutes and solve problems involving addition and subtraction of time intervals within 60 minutes.

3.MDA.1 should be an ongoing standard all year. Students do not learn to tell time through direct instruction, but through multiple experiences with the analog clock. Throughout the day, ask a student to read the clock, tell the time, and make predictions. Instead of setting timers on phones, etc., have students remind the class/teacher about stop times. The ***(a)***, ***(b)*** and ***(c)*** components of this standard should *not* be taught in isolation.

Sample Discussions:
Record what time it is when morning announcements begin.*
Provide a reasonable time that you might eat dinner?* Include a.m. or p.m. in your response.
Look at the clock. We go to lunch in 30 minutes.* What time will that be?
*Offer a variety of scenarios for practice.

This standard should be addressed in each unit, but not reported until the 4th quarter.

Unit 2: Numbers and Place Value (Approx. 15 days)
Standards Addressed: 3.NSBT.1, 3.NSBT.4, 3.NSBT.5, (3.MDA.1 continued…)
Number Talks/Number Sense Routines: If students have demonstrated proficiency with double ten frames, they can begin to move to discussing how to make 10 abstractly through operating mentally with numeral expressions. (e.g. 18 + 7: “I took 2 from the 7 and gave it to the 18 to make it

20. Then I added the 5 left over."). Students should be weaned off the use of fingers to count on and encouraged to break apart numbers efficiently. Students that still need concrete objects or representations (ten frames, base-10 blocks, etc…) should be allowed to use them to support their thinking. Problem sets should include addition and subtraction with and without regrouping.

Mental strategies for addition (e.g. 37 + 38) could include:

- Borrowing to Make a Ten (i.e. "compensation") → (37 + 38 → 40 + 35 = 75)
- Adding by Place Value → (30 + 7 + 30 + 8 → 30 + 30 + 7 + 8 → 60 + 15 = 75)
- Adding in Chunks → (37 + 30 + 8 → 67 + 8 → 67 + 3 + 5 → 70 + 5 = 75)
- Friendly Number → (37 + 38 → 37 + 3 + 38 → 40 + 38 = 78 → 78 – 3 = 75)

Mental strategies for subtraction (e.g. 61 – 28) could include:

- Subtracting in Chunks → (61 – 20 – 8 → (61 – 20) – 8 → 41 – 8 = 33)
- Adding Up → (28 + 2 = 30 + 30 = 60 + 1 = 61 → 2 + 30 + 1 = 33)
- Keeping a Constant Difference → (61 – 2 – 28 – 2 → 59 – 26 = 33 (by place value with no regrouping))
- "Negative" Numbers → (60 – 20 = 40; 1 – 8 = -7 → 40 – 7 = 33)

***Note: These strategies should not be directly "taught" but derived through exploration and student curiosity driven by strategic teacher questioning.

Standards Rationale:

3.NSBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

Students begin their investigation into place value by rounding whole numbers to the nearest 10 or 100. The goal of rounding is to encourage students to investigate number proximity, (i.e. how close one number is to another). When rounding to the nearest ten, students should identify the distance from the two closest decade numbers. For example, if students were rounding 67 to the nearest ten, they would need to consider 67s proximity to 70 and 60. It's 3 away from 70 and 7 away from 60 and is therefore rounded to 70. The same thought process should be applied to numbers rounded to the nearest hundred.

Tricks, rhymes, and mnemonic devices ***should be avoided*** because they only lead students to the right answer rather than conceptual understanding. Students will eventually use this understanding of number proximity to think efficiently with operations such as addition and multiplication.

Sample Question: Name two numbers that round to 80. (Ask students to give numbers greater than and less than 80).

Sample Question: Olivia says that her number 58 can round to 60 and 100. Is she correct? Use what you know about rounding to explain her reasoning.

3.NSBT.4 Read and write numbers through 999,999 in standard form and equations in expanded form.

Students will read and write numbers to 100 while working on the rounding unit in addition to already having some exposure in 2nd grade. The expansion in 3rd grade is to the next period of place values (the thousands period; "one" thousand, "ten" thousands, "hundred" thousands). The connection should be made between the pattern in the thousands period and the ones period.

One of the investigations of this standard could be to notice what happens when the digit in the place value to the right reaches and surpasses "9". For example, one could say, "When we count and record numbers, what happens when we reach 9? 1, 2, 3, 4, 5, 6, 7, 8, 9, ?. We reach 10, we put a one in the tens place, which tells us that we have one ten and no leftover ones. When we continue counting 11, 12, 13, 14, 15, 16, 17, 18, 19, ?, we make another ten with our ones, making us place a 2 in the tens place since we have two tens and no leftover ones. With that in mind, what happens when we get to the 90s? 95, 96, 97, 98, 99,? Not only can we make another ten, we can bundle ten of those tens to make a hundred. We have one hundred all together and no leftover tens or ones."

All of this should be a review of the 2nd grade place value standard. However, this understanding of "regrouping" when we reach the next decade or century will help when teaching elapsed time, fractions, and addition/subtraction. Students should develop the understanding that similar to a bundle of ten tens is a "hundred", ten hundreds make a "thousand", ten thousands make a "ten-thousand", and ten ten-thousands make one "hundred thousand". This will help students understand each place value digit and develop a conceptual understanding of more abstract numbers.

It should be noted that this standard only requires students to read the numbers and write them in standard form (e.g. 55,432) and in expanded form (50,000 + 5,000 + 400 + 30 + 2 = 55,432). (**Note: Expanded form could be considered any combination of addends that makes 55,432. Breaking it down by place value is the most common way to write a number in expanded form). Students are *not* required to write the numbers in word form (e.g. Fifty-five thousand four hundred thirty-two).

3.NSBT.5 Compare and order numbers through 999,999 and represent the comparison using the symbols >, =, or <.

Students should have been exposed to each of the comparison symbols in 2nd grade. They should draw on their knowledge of place value as bundles or packages of smaller units. If students are familiar with money, they could be asked "Would you rather have 9 pennies or 9 dimes? We have the same amount of both, why would you rather have 9 dimes? How many pennies would we need in order to have the same value as 9 dimes?" If students are unfamiliar with coin values, M&Ms could be substituted. "I love peanut butter M&Ms. If someone said 'I could give you 3 peanut butter M&Ms or one full package of peanut butter M&Ms', which one should I choose? Why would I choose one of something instead of choosing three of something?" This connection should be made to place value and how each digit contains ten of the value to its right, and therefore is always more valuable than the digit to its right.

After making some real-world comparisons involving higher order units, students should begin their investigation of numeral values by comparing collections of different units and describing which one is bigger. For example, one could ask, "If I have one flat and you have 8 rods, which one of us has more?" Students who struggle unitizing (i.e. knowing 1 of a higher order unit is worth more (or is different) than 1 of a lower order unit) will struggle with this comparison. These students should spend more time building and regrouping across decade and century numbers and trading groups of units for higher order single units. They will not understand through direct instruction, but through their own constructed understanding.

3.MDA.1 Use analog and digital clocks to (a) determine and record time to the nearest minute, (b) using a.m. and p.m, (c) measure time intervals in minutes and solve problems involving addition and subtraction of time intervals within 60 minutes.

3.MDA.1 should be an ongoing standard all year. Students do not learn to tell time through direct instruction, but through multiple experiences with the analog clock. Throughout the day, ask a student to read the clock, tell the time, and make predictions. Instead of setting timers on phones, etc., have students remind the class/teacher about stop times. The ***(a)***, ***(b)***, and ***(c)*** components of this standard should not be taught in isolation.

Sample Questions provided above in Unit 1.

This standard should be addressed in each unit, but not reported until the 4th quarter.

Unit 3: Addition and Subtraction

Standards Addressed: 3.NSBT.2, 3.ATO.8 (Addition & Subtraction ONLY), (3.MDA.1 continued…)

Number Talks/Number Sense Routines: Students should continue adding and subtracting two-digit numbers with and without regrouping. Students that still need concrete objects or representations (ten frames, base-10 blocks, etc…) should be allowed to use them to support their thinking. For possible strategies, see the Number Talks section from Unit 2.

Standards Rationale:

3.NSBT.2 Add and subtract whole numbers fluently to 1,000 using knowledge of place value and properties of operations.

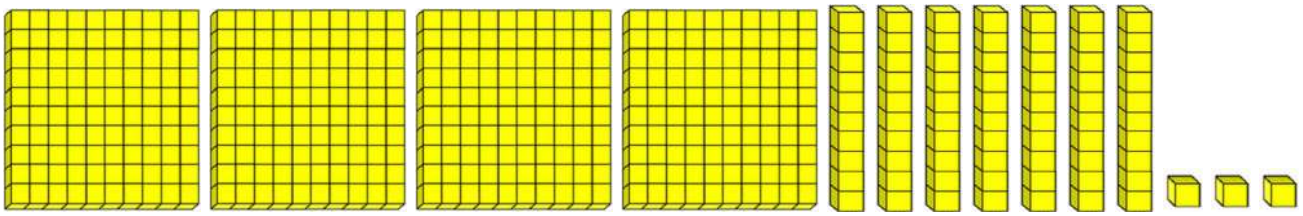
This standard refers to fluency. Fluency means accuracy, efficiency (using a reasonable amount of steps and time) and flexibility (using derived strategies). Third grade students are *not* expected to use the standard algorithm, which is a 4th grade expectation. Instead, there should be an emphasis on place value strategies. In the problem 63 - 21, teachers should emphasize correct mathematical terminology such as “60 minus 20” or “6 tens minus 2 tens”, not “6 minus 2.” Problems should be presented to students in both vertical and horizontal forms, and opportunities for students to apply the commutative and associative properties should also be presented. The emphasis is on using the properties as a number sense strategy rather than learning the formal names of the properties.

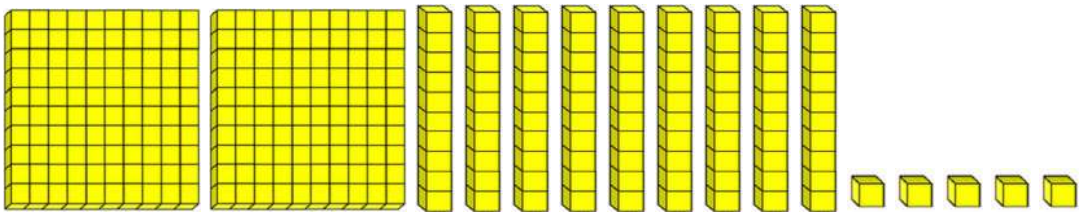
In 2nd grade, students used objects, drawings, and strategies based on place value and properties of operations. It made no difference whether they were at the concrete, representational, or abstract stage. In 3rd grade, the baseline expectation is that students are at an abstract understanding of addition and subtraction within 1,000 using derived strategies. However, we should still allow students to model concretely and representationally with addition and subtraction to eventually build abstract fluency.

Sample Task: Liz and her family are driving from Charlotte to Chicago to visit her aunt. They plan to drive from Charlotte to Louisville on Friday and then Louisville to Chicago on Saturday. According to Google Maps, it is 473 miles from Charlotte to Louisville and it’s 295 miles from Louisville to Chicago. What is the total distance from Charlotte to Chicago?

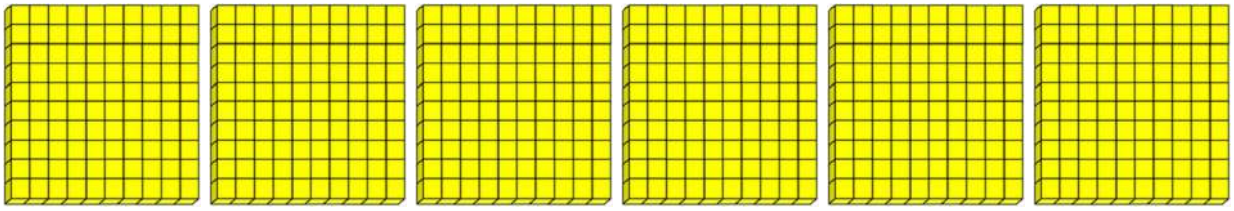
Concrete

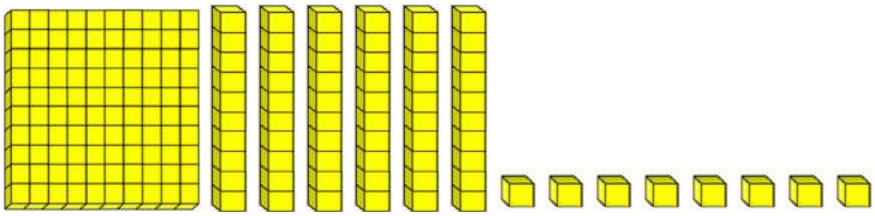
Sample Student Response: "I made 473 and 295 with base ten blocks."





"Next, I looked to see if I could regroup anything to a larger unit. I know that 10 tens make a hundred so I regrouped those into another flat."

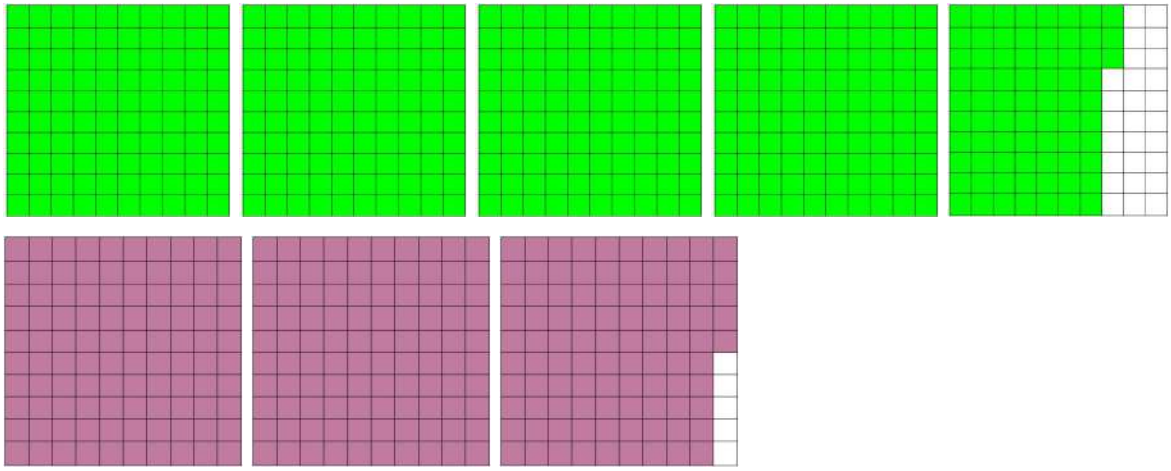




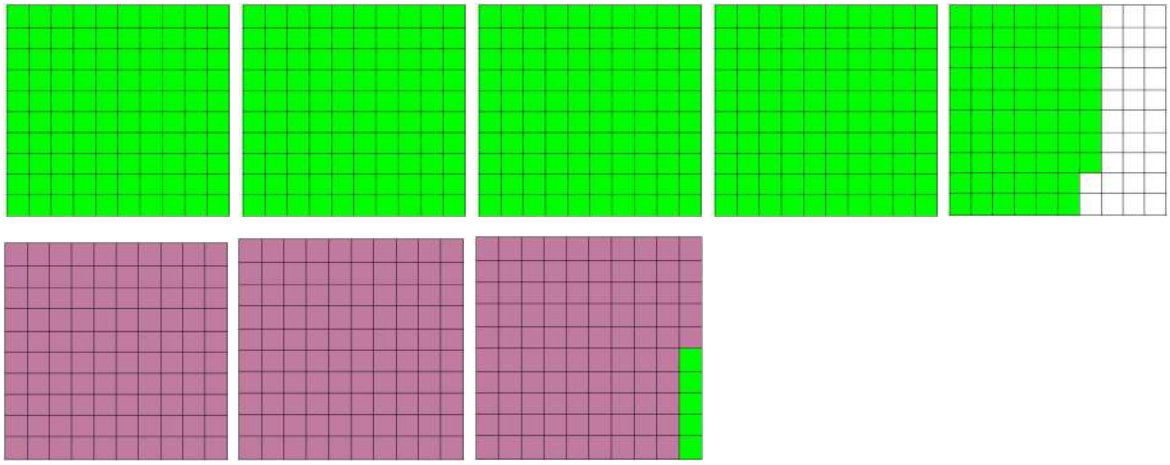
"After regrouping, I have 768 miles."

Representational

Sample Student Response: "I used hundred grids and colored in the 473 (green) and 295 (purple)."



"I noticed that I only needed 5 more purples to make a full hundred, so I took them from the other grids."



"I have a total of 7 full grids, 6 full columns, and 8 small squares, making 768."

Abstract

Sample Student Response: "I added by place value."

400 + 200 = 600
70 + 90 = 160
3 + 5 = 8
600 + 160 + 8 = 768

Sample Student Response: "I compensated by giving 5 to 295 from 473, making 300 + 468 = 768."

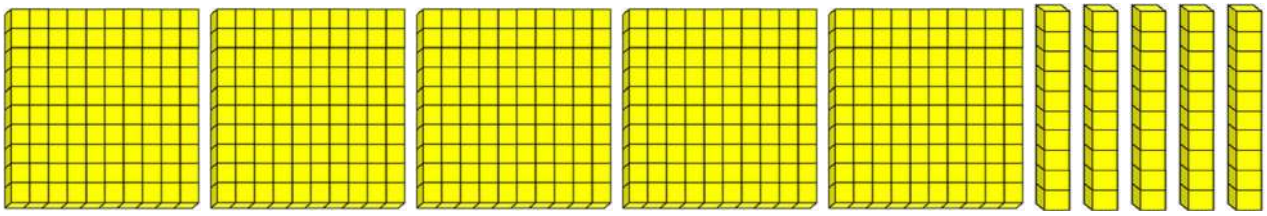
Sample Student Response: "I added up by place value. I added 473 + 200 to get 673. I added 673 + 90 = 763 and 763 + 5 to get 768."

Sample Student Response: "I noticed that 295 was close to 300 so I just rounded it up to 300 and added it to 473 to get 773. Since I added 5 to get to 300, I took 5 away from 773 to get 768."

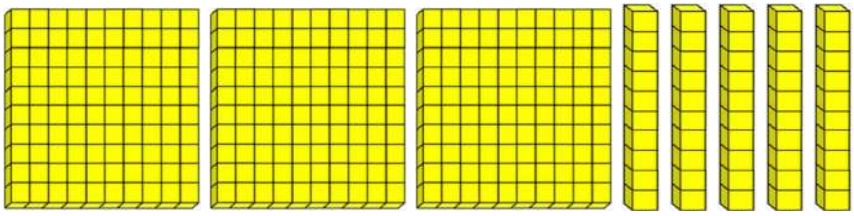
Sample Task: The city community center donated 550 bookbags to local elementary schools. Rockefeller Elementary School received 287 bookbags. How many were left for the other schools?

<u>Concrete</u>

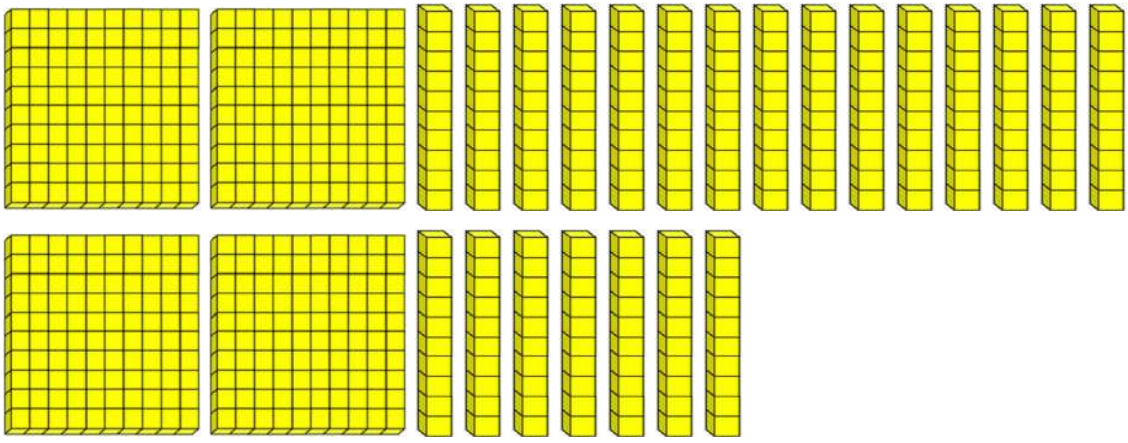
Sample Student Response: "I made 550 with base ten blocks. I used 5 flats and 5 rods."



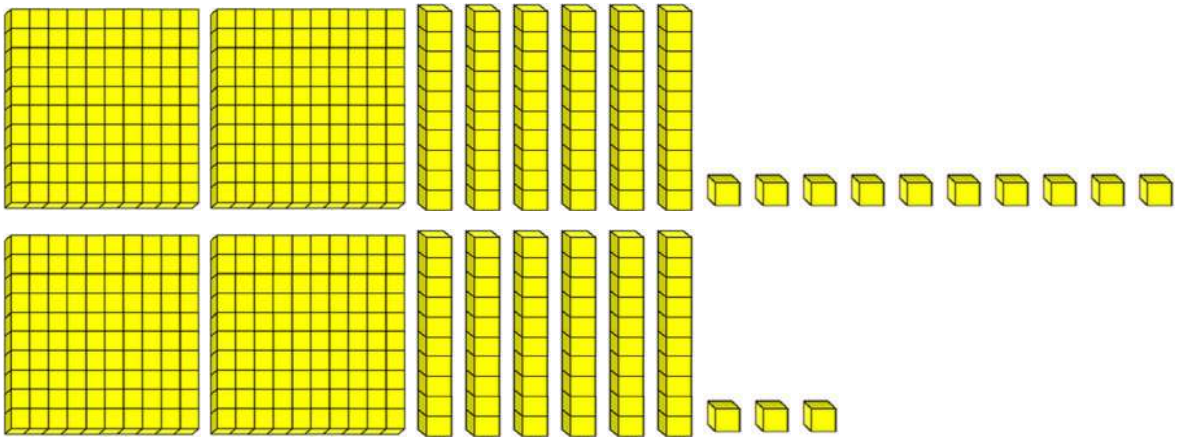
"Next, I took away 2 flats."



"Next, I wanted to take away 8 rods, but I didn't have enough so I decomposed a flat to make 10 rods and then took 8 of those away."



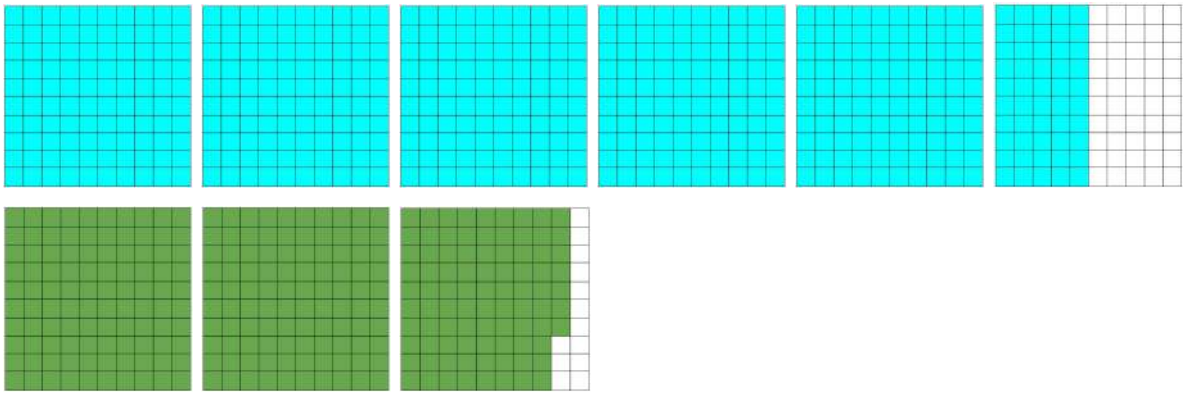
"Finally, I needed to take 7 individual cubes away, but I needed to decompose a rod to take those away."



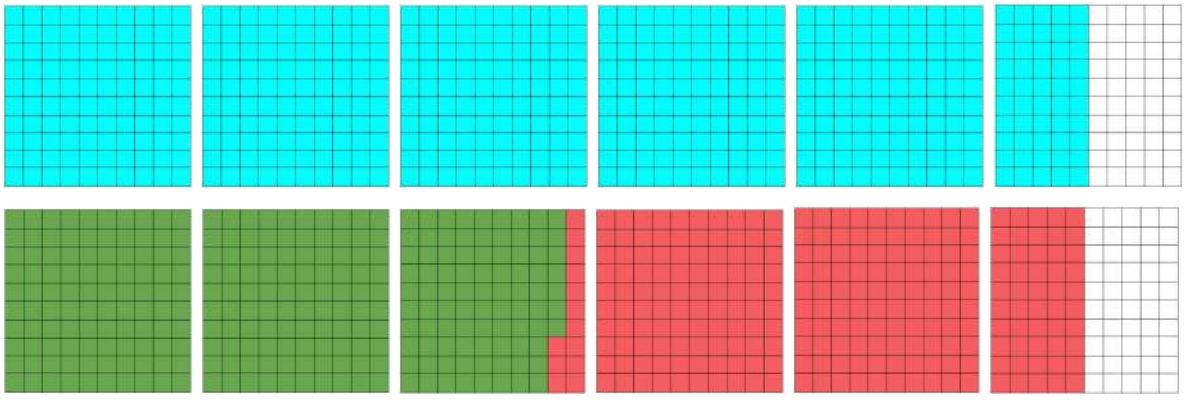
"Then looking at the blocks I had left, I saw the answer was 263."

Representational

Sample Student Response: "I filled out hundreds grids for 550 (blue) and 287 (green)."



"To find the difference, I tried to figure out how much I needed to add to make 550."



"I counted up the red blocks which represented the difference between 550 and 287. I counted the 2 full grids (200) and each column (60) and then the leftover 3 to make 263."

Abstract

Sample Student Response: "I added up from 287 to get to 550. I added 13 to get to 300, 200 to get to 500 and then 50 more to get to 550. Then I added up 13, 200, and 50 to get to 263."

Sample Student Response: "I subtracted in chunks. I subtracted 200 from 550 to get to 350. Then I subtracted 80 from 350 to get to 270. Finally, I subtracted 7 more to get to 263."

Sample Student Response: "I added 13 to each number to make 563 - 300 which equals 263."

Sample Student Response: "I wanted to subtract by place value, but I was unable to take away.

5 hundreds - 2 hundreds

5 tens - 8 tens

0 ones - 7 ones

So, I regrouped one of my hundreds into 10 tens and one of my tens to make 10 ones."

4 hundreds - 2 hundreds = 2 hundreds

14 tens - 8 tens = 6 tens

10 ones - 7 ones = 3 ones

263

Sample Task: Mr. Jackson is organizing the cans his school collected in a recent canned food drive. Since his school collected so many cans, he decided to develop a strategy to organize all of the cans to keep track of the number of cans at all times. He decided that they should put groups of 10 cans in a paper bag. Once they have collected 10 bags full of 10 cans each, they would put those 10 bags in a box. After collecting from each classroom in October, his school had 5 boxes, 4 bags, and 8 cans left over. How many cans did they collect in October? In November, his school collected 3 boxes, 8 bags, and had 4 cans left over. How should he organize the cans from October and November when sticking with his strategy? How many cans did they collect in all in October and November?

In this task, students should model the act of regrouping into boxes (hundreds) and bags (tens) for questions involving both addition and subtraction. Once they concretely (or representationally) model the actions happening in the problem, they should try to connect their models to numerical abstract representations. Teachers should create multiple tasks for students to be able to see and model the regrouping process and represent it abstractly using strategies based on place value and properties and operations, each derived from their experiences.

Sample Assessment Question: Marcus is saving up to buy a new mountain bike. The bike he wants is $906. He has saved $619 so far. How much more money does he need to save in order to have enough to buy the bike? Show your work and describe in words your process.

3.ATO.8 Solve two-step real-world problems using addition, subtraction, ~~multiplication~~ and ~~division~~ of whole numbers and having whole number answers. Represent these problems using equations with a letter for an unknown quantity.

Since multiplication and division have not yet been introduced, the two step problems here should only include addition and subtraction. However, these questions should be embedded within the NSBT.2 lessons and should not be "introduced" separately.

Sample Task: There are 126 students in 3rd grade at Rock Hill Elementary School. The following table shows the attendance on the last three days of school:

Day	Attendance
Wednesday	109
Thursday	87
Friday	48

How many student absences did they have over the last 3 days?

Sample Task: Chris has a jar in his kitchen that holds dog treats. He emptied a large box of 300 treats into the jar. In January, he gave his dog 89 treats, and in February he gave him 65 treats. How many treats are left in the jar at the end of February?

***Note: For both problems, there are two possible strategies *(that students should derive!)*. One of the strategies would be to subtract multiple times from the total. The other strategy would be to add up all of the numbers being subtracted and then subtract that sum. There could be other strategies derived by students and each one should be explored and expanded in the whole group discussion after the task.

3.MDA.1 Use analog and digital clocks to (a) determine and record time to the nearest minute, (b) using a.m. and p.m, (c) measure time intervals in minutes and solve problems involving addition and subtraction of time intervals within 60 minutes.

3.MDA.1 should be an ongoing standard all year. Students do not learn to tell time through direct instruction, but through multiple experiences with the analog clock. Throughout the day, ask a student to read the clock, tell the time, and make predictions. Instead of setting timers on phones, etc., have students remind the class/teacher about stop times. The ***(a)***, ***(b)***, and ***(c)*** components of this standard should not be taught in isolation.

Sample Questions provided above in Unit 1.

This standard should be addressed in each unit, but not reported until the 4th quarter.

Unit 4: Multiplication and Division (Approx 40 days)

Standards Addressed: 3.ATO.1 -- 3.ATO.8, 3.NSBT.3, 3.MDA.1

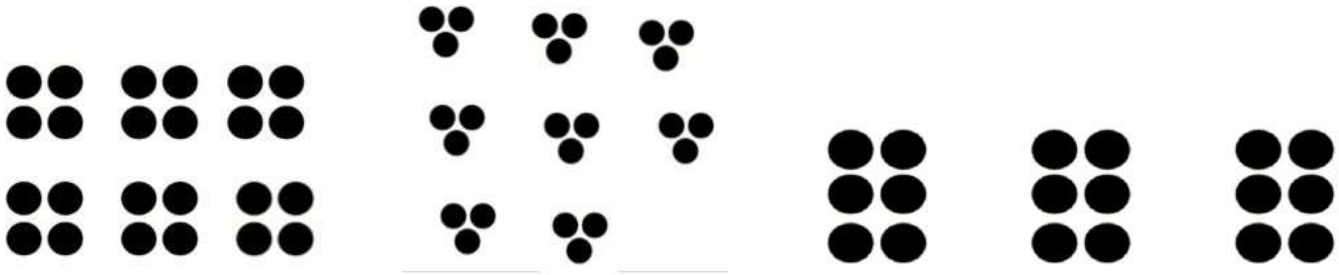
Number Talks/Number Sense Routines: If students are showing consistent proficiency with addition and subtraction of two-digit numbers, move to abstract reasoning with three-digit numbers, using the same strategies from previous units. Feel free to use creative number talks as well. For example:

If 356 + 575 = 931, then 356 + 576 = ???. How do you know?
If 173 + 478 = 651, then 173 + 578 = ???. How do you know?
If 671 - 174 = 497, then 671 - 164 = ???. 661 - 174 = ??? How did you answer change? Why?
If 555 + 289 = 844, then 455 + 389 = ???. How do you know?
If 813 - 645 = 168, then 913 - 545 = ???. How do you know?
If 813 - 645 = 168, then 814 - 167 = ???. How do you know?

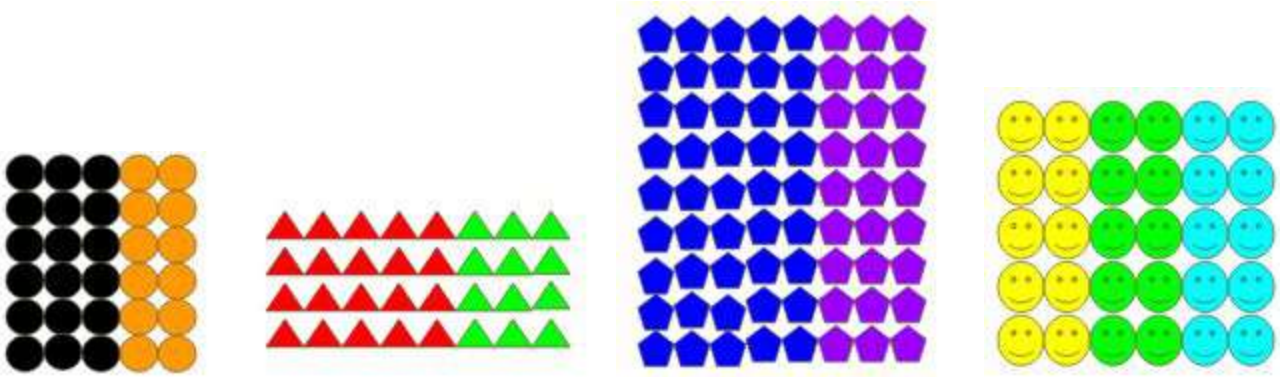
In these number talks, students should solve without calculating or “using strategies”. Instead, they should practice using what they know to solve problems about what they don’t. These problems will be useful in reinforcing conceptual understanding of the properties of addition and subtraction and will ultimately improve students’ mental computation abilities.

If students begin to get bored with addition and subtraction number talks, feel free to move to some early multiplication number talks. Some of those number talks could include:

- Multiplication Dot Cards:



- Quick Arrays (to reinforce distributive property or partial products):



Standards Rationale:

Standards ATO.1 through ATO.8 should not be taught in isolation from one another, but within one another. Students should be given tasks that require them to consider all relationships involving multiplication and division and how those relationships can be used to solve problems.

3.ATO.1 Use concrete objects, drawing and symbols to represent multiplication facts of two single-digit whole numbers and explain the relationship between the factors (0-10) and the product.

3.ATO.2 Use concrete objects, drawings and symbols to represent division without remainders and explain the relationship among the whole number quotient (0-10), divisor (0-10), and dividend.

3.ATO.3 Solve real-world problems involving equal groups, array/area, and number line models using basic multiplication and related division facts. Represent the problem situation using an equation with a symbol for the unknown.

Multiplicative thinking (which includes division) begins when students begin to recognize groups of objects instead of individual objects. As a result, multiplicative thinking begins in first grade when students begin to work with place value concepts and groups of ten. In 3rd grade, they move beyond groups of ten to groups of any number 0 - 10.

Although multiplication can be modeled by repeated addition, the operation of multiplication is different from the operation of addition. Addition is the combination of two or more like units (ex: 6 crayons + 7 crayons = 13 crayons). In order to add, the units must be the same.

In contrast, multiplication is the scaling of a group of units (or part of a unit) by a scale factor (ex: 6 boxes of 8 crayons = 8 crayons x 6 boxes/groups = 48 crayons). The units are not the same.

As a result, the focus should be on the development of multiplicative thinking rather than teaching "strategies" to solve a problem. The only way to develop multiplicative thinking is to pose a problematic context and ask questions of students based on their derived strategies.

In 2nd grade, students were provided arrays (up to 5 x 5) and were encouraged to use repeated addition to determine the total number of objects in an array. Now, they should work with up to 10 x 10 arrays. They *could* use repeated addition to determine the number of objects in the array, but the goal should be to move them to more efficient strategies. Students could identify smaller groups and add them if they are unable to determine the product from memory.

Some quick arrays can be found above in the Number Talks section. Some real-world pictures can be used to let students investigate multiplicative relationships. For example: "How many eggs are in this carton? Can you figure it out without counting or adding?"

Other real-life arrays could include:

Sample Tasks and/or Sample Assessment Questions:

- Katie and Brian are planning for their wedding. They have decided to rent tables that can fit 8 people each. If they have 72 people coming to the wedding reception, how many tables do they need to rent? Show your thinking using pictures or numbers and explain in writing.

- Mrs. Thompson wants to do a project for the 3rd grade classes. She needs at least one marker for each student. Her assistant got her 6 boxes of markers with 8 markers in each box. How many markers does she have? If there are 57 students in the 3rd grade, does she have enough markers? If not, how many more markers does she need? Show your thinking using pictures or numbers and explain in writing.

- Mr. Sandburg loves Oreo cookies, but he is trying to watch his weight by cutting out his sugar intake. Each Oreo cookie has 6g of sugar. If he eats 9 Oreos, how many grams of sugar will he have eaten? Show your thinking using pictures or numbers and explain in writing.

With each of these tasks, students should model using concrete objects (unifix cubes, blocks, counters, etc…) and/or draw out their thinking. Teachers should always be looking at strategies and asking students how to make their thinking more efficient. For example, if a student is drawing out circles, putting Xs in each circle, and counting all of the Xs, the teacher could ask, “Why did you recount? You already counted out ___ (number) of Xs in these circles. How could you use that information to be more efficient?”

Teachers are *not* encouraged to teach “strategies” but ask questions geared toward letting students construct their own understanding of multiplicative relationships.

3.ATO.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is a missing factor, product, dividend, divisor, or quotient.

3.ATO.6 Understand division as a missing factor problem.

These standards should be taught in conjunction with ATOs 1, 2, & 3 when making abstract connections with the mathematics. For example, for the sample wedding task above, students should know that they can represent the problem as 72 ÷ 8 = _?_ ***or*** 8 x _?_ = 72. This kind of conversation can come during the task debriefs where the teacher can ask, "All of our groups represented this problem as division, but could we represent it as multiplication? How could we do that?" OR "Some groups used division to model this problem and some used multiplication. Who is right? How can they both be right?" Students can make connections to the inverse relationship between addition and subtraction to explain the similar inverse relationship between multiplication and division. Explanations to these questions should satisfy the requirements within 3.ATO.6. As a result, formal assessments on ATO.6 should include written explanations of the flexibility in modeling multiplication/division problems as the inverse of the selected operation.

3.ATO.4 calls for an abstract demonstration of understanding the relationship between multiplication and division.

Sample Assessment Questions:

- 6 x __ = 42
- 27 = 9 x __
- 8 = __ ÷ 4
- 36 ÷ __ = 9

3.ATO.6 calls for students to "understand". Therefore, the only way to formally assess ATO.6 would be to ask students to explain and justify the inverse relationship of multiplication and division.

Sample Assessment Question: For the previous context, Sam represented the problem using the equation 5 x __ = 30 and used his knowledge of multiplication facts to know the answer was 6. Beatrice, on the other hand, represented the problem using the equation 30 ÷ 5 = ___ and counted up by 5s 6 times to get to 30. Whose strategy was correct? Were they both correct? Explain.

3.ATO.5 Apply properties of operations (Commutative Property of Multiplication, Associative Property of Multiplication, Distributive Property).

Students are *not* required to know the names of the three different properties. They need to internalize the properties and be able to apply them when necessary.

Commutative Property: A x B = B x A.
Associative Property: (A x B) x C = A x (B x C)
Distributive Property: A x (B + C) = (A x B) + (A x C)

Students have used the Commutative and Associative Properties to help them with fluency of addition and subtraction facts in earlier grades. Teachers should take advantage of that existing familiarity with these properties to reinforce their effect on multiplication. These are not facts that should be copied into the students' notebooks, but should be investigated organically within problematic tasks and strategic teacher questioning.

The *Commutative Property* helps students generalize about multiplication facts, with which they are familiar, to quickly know and understand unfamiliar facts. For example, if a student knows "8 groups of 5", they can use that understanding to know "5 groups of 8" would be the same. Students should explain in words and with drawings (number lines, etc…) why this property works.

The *Associative Property* is seldom used to solve one-digit by one-digit multiplication facts, but is important when larger place values are introduced (see 3.NSBT.3). Students *could* think of an expression like 9 x 8 as 9 x 4 x 2, but this strategy wouldn't necessarily be more efficient than others.

The *Distributive Property* is an extremely important property for helping students conceptually understand and fluently solve one-digit multiplication facts. However, the Distributive Property should not be directly taught, but informally derived through challenging tasks and student discovery. For example, for the student that sees 5 x 7 as 5 + 5 + 5 + 5 + 5 + 5 + 5, they can reason that the expression contains two groups of 5 and five groups of 5.

Sample Tasks:

- The grocery store has two different sized bags of apples for sale. The smaller bag holds 4 apples and the larger bag holds 6 apples. If Greg needs 24 apples for a carnival game, how many small bags should he buy? How many large bags should he buy? What do you notice? (Once students notice 4 x 6 = 6 x 4) Does that work with all multiplication facts? Why?
- Greg and Abby are both trying to earn money to buy tickets to a football game. The tickets are $35. Greg's parents agree to give him an allowance of $5 per week for 7 weeks. Abby's parents agree to give her an allowance of $7 per week for 5 weeks. Who got the better deal? Why? (Once they realize they got the same amount of money ask the following.) How did this work? Abby was getting more money each week...how did they get the same? (Model thinking on a number line.)

- Mr. Abbott has 3 children. Last month, he bought each of them ice cream cones from the ice cream truck 3 different days. This month, he bought ice cream cones for them 4 different days. How many ice cream cones did he buy over the last two months?
    - *Teacher note: This problem can be solved a variety of ways, one of which is using knowledge and understanding of the Distributive Property. It could be reasoned that he bought a total of 9 cones last month and 12 cones this month and add them together (3 x 3) + (3 x 4). It can also be reasoned that he bought each of them ice cream over a total of 7 days: 3 x (3 + 4). Illustrate with students that both methods are sufficient, but the latter is the application of the Distributive Property.*

Sample Assessment Questions: If you know 24 x 31 = 744, is it possible to know 31 x 24? How do you know? Explain.

****Teacher Note: The standard is best assessed through frequent interaction with students and NOT on a paper/pencil assessment. Take note as to how they use strategies to solve 1 digit multiplication problems and if they are able to quickly apply the properties when appropriate.*

Which of the following statements is TRUE? (Circle all that apply)

- 5 x (3 + 4) = 7 x 5
- 2 x 7 x 4 = 8 x 7
- 8 x (2 x 3) = 8 x 5
- 6 x (5 - 2) = (4 + 2) x 3
- (7 - 1) x 4 = (7 x 4) - (7 x 1)

3.ATO.7 Demonstrate fluency with basic multiplication and related division facts of products and dividends through 100.

Fluency can be defined as being flexible, efficient, and accurate in computation. Fluency should ***NOT*** be assessed through timed tests/assessments, as this can create anxiety in students and push insufficient and incomplete strategies in solving problems. In her book, About Teaching Mathematics (2007), Marilyn Burns said,

"Teachers who use timed tests believe that the tests help children learn the basic facts. This makes no instructional sense. Children who perform well under time pressure display their skills. Children who have difficulty with skills, or who work more slowly, run the risk of reinforcing wrong learning under pressure. In addition, children can become fearful and negative toward their math learning."

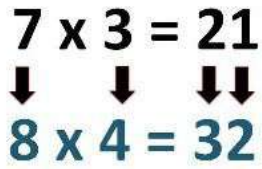
Students develop fluency through meaningful interactions within problematic contexts. They look for patterns and use structure to become more efficient in their multiplication calculations. As a result, students should not be *taught* strategies for multiplication, but the strategies should be derived.

Fluency for multiplication is relatively quick recall using memory from repeated meaningful experiences (not flashcards) or the Distributive Property. Fluency for multiplication *is not* drawing pictures, moving blocks, fingers, or other manipulatives, or using repeated addition. Students who are in this stage of thinking should be pushed beyond these methods of calculation when they demonstrate a need for more efficient strategies. Again, these strategies should never be directly taught, but derived when students are interacting within a context in which the strategy is necessary.

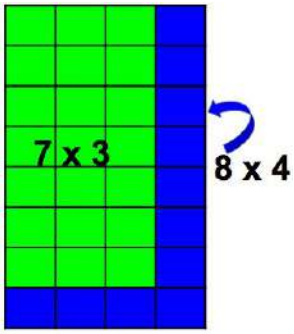
Sample Tasks: Sample tasks for this unit can be found in ATOs 1, 2, & 3. The way ATO.7 is reinforced is through strategy recording and the search for efficiency.

Sample Assessments: Fluency is not easily assessed using a paper/pencil assessment. Teachers could however pose multiplication problems and create a space for students to show their strategies or "prove" their products.

Sample Task: Look at the following picture. What do you notice? Does it always work? When does it work? Why does it work in those cases?



Sample Student Response: "I notice that you added one to each digit. I tried it with other numbers, but it didn't work. For example 8 x 7 = 56 but 9 x 8 = 72...the product's digits didn't increase by one. I thought about it in an array:



In order for the tens and ones digit in the product to increase by 1, I have to add 11 and only 11. So I had to think about what other arrays would require me to add 11 more squares. In the above array, I added 7 more by adding a new column and 3 more by adding a new row. The new row and column also added another block in the bottom right corner. Therefore, it will only work if the two factors in the initial problem add to be 10 (e.g. 4 x 6 = 24 → 5 x 7 = 35)."

3.ATO.8 Solve two-step real-world problems using addition, subtraction, multiplication and division of whole numbers and having whole number answers. Represent these problems using equations with a letter for an unknown quantity.

This is the continuation of the ATO.8 standard that began in the previous unit with addition and subtraction. Now, students should apply their understanding of all 4 operations to operate within multi-step problems. This standard should *not* be taught in isolation from the other standards, but should be embedded within them. Students should be given problems in ATOs 1, 2, & 3 that require multiple steps.

The standard also requires students to represent the multi-step problems using equations with a letter for the unknown quantity. This *does not* necessarily mean that students use variables like "*x*" and "*y*" for all unknowns, but letters that make sense in the context of the problem (e.g. the first letter of the unknown value).

Sample Task: Greg and Thomas are selling doughnuts to raise money for a class trip. Each box sold raises $6 for their class trip. If Greg sells 8 boxes and Thomas sells 6 boxes, how much money will they raise for their class trip?

Sample Response*:* 6 x 8 + 6 x 6 = *M* (where *M* represents "money")

Even though it is not explicitly stated in the standard, students should understand that in expressions without grouping symbols, multiplication and division are calculated first and then addition and subtraction. This ***does not*** mean that we introduce "PEMDAS" or any other mnemonic device. Instead, we discuss *why* we multiply and divide first.

Take the expression 4 + 5 x 8. If they were to add first, they would change the meaning of the multiplication part from "5 groups of 8" to "9 groups of 8". If the context calls for adding first, it must be indicated with grouping symbols (i.e. parentheses) to indicate the total number of groups changed. Students do not *have* to use grouping symbols in 3rd grade, but it would be appropriate to introduce the idea of them to be accurate with our expressions.

3.NSBT.3 Multiply one-digit whole numbers by multiples of 10 in the range of 10 - 90, using knowledge of place values and properties of operations.

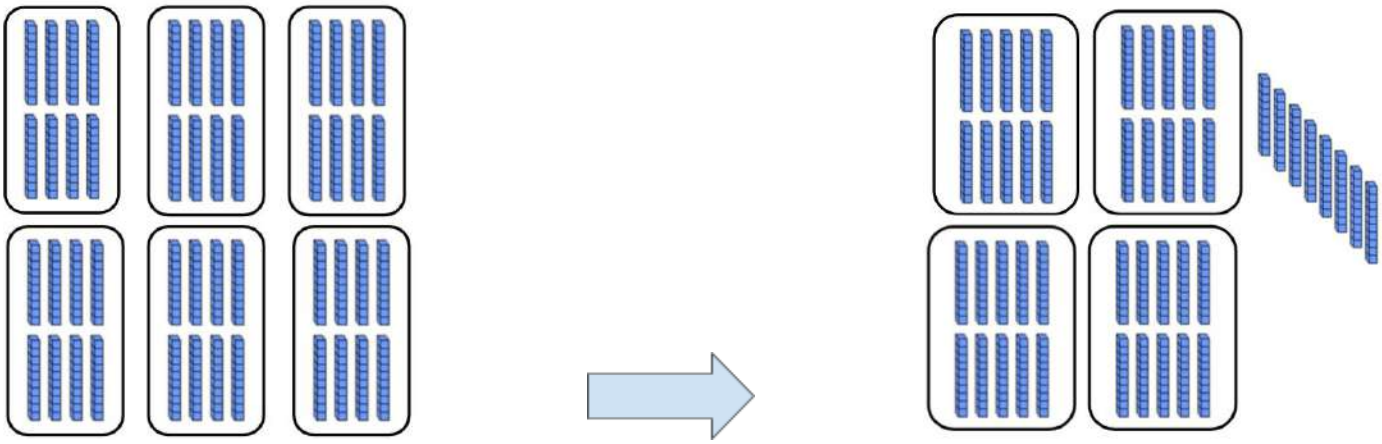
It is important that students don't simply "multiply by the front digit and add a zero" but truly understand what they are doing multiplicatively. In many ways, students are applying the Associative Property and their understanding of place value (see student responses below for examples).

Sample Assessment Question: Southern Charm Nursery is running a special for Mother's Day. They are selling vases of 6 roses for $12.99. On the Saturday before Mother's Day, they sold 80 vases. How many flowers did they sell?

Sample Student Response*:* "I know that 80 is the same as 8 tens. 6 x 8 tens would be 48 tens. 48 tens = 480 ones, so the answer is 480."

Sample Student Response: "I can rewrite 80 as 8 x 10, which means I can write the expression as 6 x 8 x 10. If I multiplied the 6 x 8 first, I get 48. Then, I can multiply 48 x 10 and get 480."

Sample Student Response*:* "I took out 8 rods to represent the 80 vases. Then I made 6 groups of those 8 rods. I regrouped every ten rods into hundreds. I ended up with 4 flats and 8 rods which is 480."



Students should have opportunities to explore each of the responses above. They should use base ten blocks, drawings, and other representations to reason about their answers and multiplying using place value concepts. They should also explain their thinking verbally and/or in writing.

3.MDA.1 Use analog and digital clocks to (a) determine and record time to the nearest minute, (b) using a.m. and p.m, (c) measure time intervals in minutes and solve problems involving addition and subtraction of time intervals within 60 minutes.

3.MDA.1 should be an ongoing standard all year. Students do not learn to tell time through direct instruction, but through multiple experiences with the analog clock. Throughout the day, ask a student to read the clock, tell the time, and make predictions. Instead of setting timers on phones, etc., have students remind the class/teacher about stop times. The ***(a)***, ***(b)***, and ***(c)*** components of this standard should not be taught in isolation.

Sample Questions provided above in Unit 1.

This standard should be addressed in each unit, but not reported until the 4th quarter.

Unit 5: Fractions & Partitioning (Approx. 35 days)

Standards Addressed: 3.G.2**,** 3.NSF.1, 3.NSF.2, 3.NSF.3 , 3.MDA.1*, 3.ATO.7*
(*Ongoing Standards)

Number Talks/Number Sense Routines:
Number talks should now begin to reloop to cover addition, subtraction, multiplication and division. They should change frequently to prevent students from getting bored or regurgitating strategies without truly understanding. Use any of the number talks in the previous units in addition to some new number talks:

- How Do You See It?
  - Example: 8 x 7 *"I used 7 groups of 10 which is 70 and took away two groups to make 56."* or *"I did 5 groups of 8 which is 40. 2 more groups of 8 is 16 and so we get 56."*
  - (7 x 10) - (2 x 7) = 70 - 14 = 56
  - (5 x 8) + (2 x 8) = 40 + 16 = 56
- If, Then...
  - If 16 x 5 = 80, then 17 x 5 = ???
  - If 16 x 5 = 80, then 26 x 5 = ???
  - If 16 x 5 = 80, then 16 x 15 = ???
  - If 20 x 20 = 400, then 20 x 21 = ???
  - If 14 x 18 = 252, then 14 x 19 = ???

- If 14 x 18 = 252, then 15 x 18 = ???
- If 60 ÷ 5 = 12, then 120 ÷ 5 = ???
- If 60 ÷ 5 = 12, then 60 ÷ 10 = ???
- If 112 ÷ 16 = 7, then 112 ÷ 8 = ???
- If 112 ÷ 16 = 7, then 128 ÷ 16 = ???
- If 112 ÷ 16 = 7, then 96 ÷ 16 = ???

Standards Rationale:

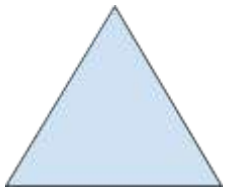
In 1st and 2nd grades, students partitioned shapes into two and four equal parts and used the words “halves” and “fourths” (in 2nd grade) to describe the partitioned areas. Continue with this same understanding of partitioning and its relationship to fractional reasoning.

3.G.2 Partition two-dimensional shapes into 2, 3, 4, 6, or 8 parts with equal areas and express the area of each part using the same unit fraction. Recognize that equal parts of identical wholes need not have the same shape.

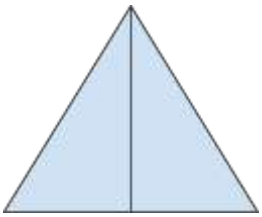
It is important that students are not “told” how to partition these shapes, but are given opportunities to investigate and test ideas. Students should begin with squares and rectangles and should progress to more challenging shapes, such as triangles, circles, trapezoids, and irregular shapes. As they partition these shapes, avoid discussing fraction notation. Keep discussions focused on the number of equal parts and referring to those as “halves, thirds, fourths, sixths, and eighths”. Teachers could even highlight multiple partitioned regions and have students refer to those parts as “three sixths” or “3 sixths” but *NOT* “3/6”. This keeps a focus on the parts as units rather than abstract numbers. At the end of the unit, circle back to the partitioning of shapes and the labeling of each section with its respective unit fraction.

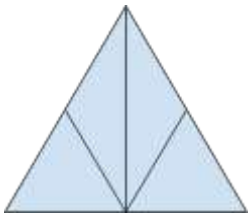
Sample Task: How many different ways can you partition this rectangle into four equal parts? Six equal parts? Eight equal parts?

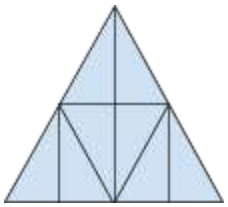
Sample Task: How many different ways can you partition this triangle into 2 equal parts? 4 equal parts? 8 equal parts? Explain.



(Possible Solutions)







3.NSF.1 Develop an understanding of fractions (i.e. denominators 2, 3, 4, 6, 8, 10) as numbers.

- ***a. A fraction 1/b (called a unit fraction) is the quantity formed by one part when a whole is partitioned into b equal parts.***
- ***b. A fraction a/b is the quantity formed by a parts of size 1/b.***
- ***c. A fraction is a number that can be represented on a number line based on counts of a unit fraction.***
- ***d. A fraction can be represented using set, area, and linear models.***

After focusing on partitioning of shapes and referring to the parts as "halves, fourths, etc…", students should continue thinking about counts of units, but now in the context of groups of objects. For example, "If I had 18 cookies to distribute among 6 people...", 3rd grade students can (at least by this point in the year) quickly identify that each would receive 3 cookies. This would make 6 groups (or units) of 3. It is ***very*** important to underscore this idea of units.

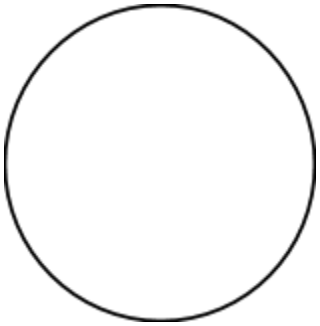
Sample Task: (continued) But what do I do if I have 19 cookies to distribute to 6 people? I can give each person 3 cookies, but I still have that one extra cookie to split. How do I split it between three people? What do I call each piece of cookie?

In this problem students should think of the pieces of the cookie as "thirds" instead of ⅓. So, if a student was looking at two pieces of the cookie, they would look at it as 2 "thirds" rather than ⅔.

Focusing on units will help students make connections to previous knowledge (units within place value & measurement) and reduce some of the anxiety with numbers that seem different.

Before even moving to fraction notation, students should practice splitting area, linear, and set models into “halves”, “thirds”, “fourths”, “sixths”, “eighths”, and “tenths”. Here are examples of each:

Sample Area Task: I baked a frozen pizza for my children. My son ate five eighths of the pizza, my daughter ate two eighths of the pizza, and I ate the rest. In the following picture, show how much each person ate.



***Note: The circle wasn’t partitioned for a reason. Students need to practice partitioning shapes into equal parts (division) and labeling the parts (G.2).

Sample Linear Task: For an activity, Mrs. Harper needs five sixths of an inch of string. 1 inch of string is shown below. Indicate where she should cut to create a length of string that is five sixths of an inch long.

Sample Set Task: Mrs. Bridges provided cookies on a platter for her teachers during lunch. The picture is shown below. The teachers ate two thirds of the cookies. How many cookies are left over?

Sample Student Response: “I know that “thirds” means that there are three equal groups. I noticed that I could split them into three equal groups like this:

If I took two thirds away, that means I took two groups away, leaving me with one group of 6, or 6 cookies."

There are three main functions of fractions in 3rd grade: as measurement, unfinished division, or parts of a whole. Traditionally, 3rd grade fractions have primarily been taught and assessed as parts of a whole. However, we must give students different contexts that incorporate each of the 3 functions.

Consider how many different ways the number ⅝ *could* be represented:

1.) "We ran ⅝ of a mile today."
2.) "We ran ⅝ of the 3 mile race without walking."
3.) "We ran a 5 mile relay race with 6 people on our team. Each person ran ⅝ of a mile."

In the first context, which is a measurement context, ⅝ is referenced as a measurement, or a length. ⅝ is the distance from 0 miles to ⅝ of a mile. In the second context, which is a part of a whole context, the whole is 3 miles and ⅝ of the whole is referenced. Finally, in the third context, an unfinished division problem, a quantity or a total of 5 miles is being divided between 6 people. The meaning of the 5 in the unfinished division is different than the meanings of the 5 in the measurement and part/whole contexts. In that case, the 5 represents the numerator being divided, while the "5" in the other two contexts represents the counts of ⅛ units.

As a result of these differences, teachers must intentionally give all different types and not focus on just one.

Before we continue, let's provide clarity into the ***a-d*** parts of the standard:

a. ***A fraction 1/b (called a unit fraction) is the quantity formed by one part when a whole is partitioned into b equal parts.***
b. ***A fraction a/b is the quantity formed by a parts of size 1/b.***
c. ***A fraction is a number that can be represented on a number line based on counts of a unit fraction.***
d. ***A fraction can be represented using set, area, and linear models.***

To clarify some of the algebraic terminology in this standard, assign values to *a* and *b*. Let *a* = 3 and *b* = 8. Then rewrite parts ***a*** and ***b*** to reflect these substitutions.

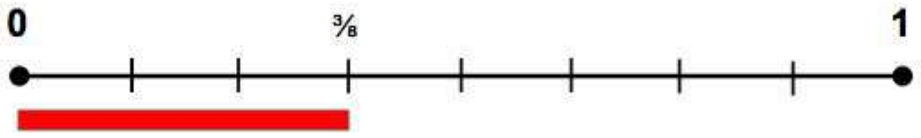
a. ***A fraction 1/8 (called a unit fraction) is the quantity formed by one part when a whole is partitioned into 8 equal parts.***
b. ***A fraction 3/8 is the quantity formed by 3 parts of size ⅛.***

So, ⅛ references 1 part of a whole split into 8 pieces. ⅜, therefore, would represent 3 parts of ⅛. However, ⅜ *could also* represent 3 wholes split into 8 equal parts. While the solution to that unfinished division *is* ⅜, the meanings of the numerator and denominator can only be defined by a context.

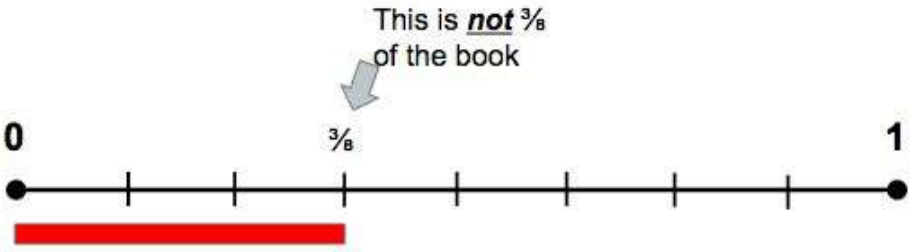
Part ***d*** reinforces all of these types of fractions with denominators of (only) 2, 3, 4, 6, 8, and 10 by modeling them within set, area, and linear models. A sample context and the representations of each context are below.

Sample Context: Elizabeth is reading a new book. In one sitting, she read ⅜ of the book. Create a model representing how much of the book she has read.

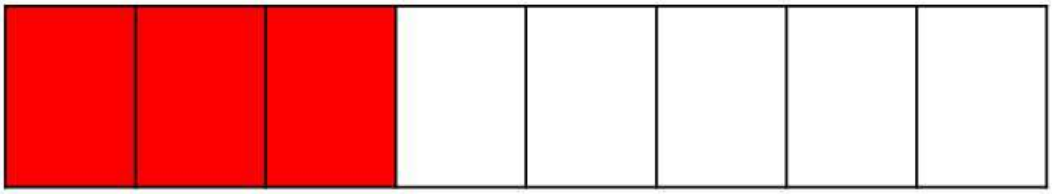
Linear:



Note: The ***red bar is the ⅜ in the context. ⅜ is a distance when modeled on a number line.



Area Model

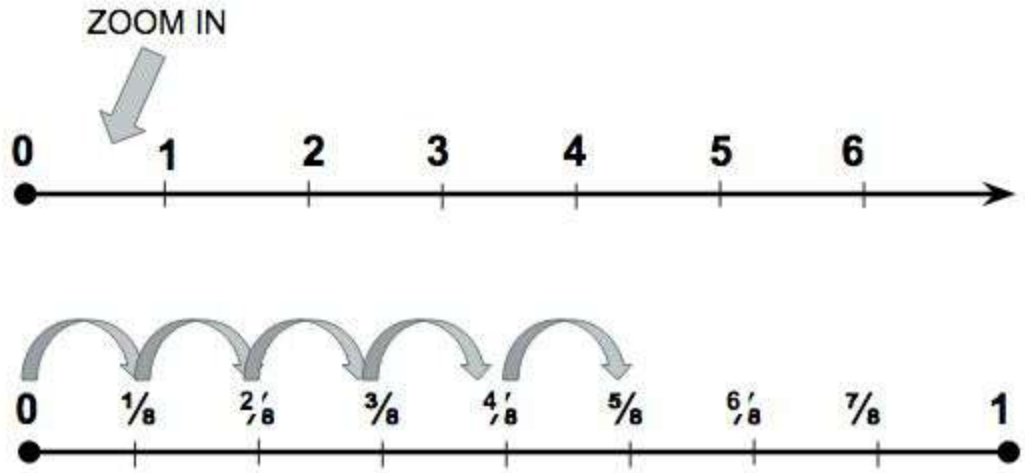


Set Model

A set model for this context is a little tricky. Pretend that there are 800 pages in the book. For the set model, split the 800 pages into 8 equal groups. Each of the following pictures have 100 pages:

Finally, part ***c*** requires that students understand and represent fractions on a number line as counts of a unit fraction. As discussed before, *a/b* isn't *always* counts of a unit fraction, but in some contexts, it is. The phrase "counts of a unit fraction" means that students should recognize ⅕, ⅖, ⅗, ⅘… as counting the number of "fifths" (i.e. "one, two, three, four...four fifths").

A number line is usually partitioned into whole number lengths. When investigating fractions, the space *between* integers (whole numbers) is what is being investigated. Between those integers, partitioning is happening again, depending on the unit(s) that are being represented. For example, ⅝ could be represented this way:



The end goal of this standard is for students to have a holistic understanding of fractions as a number. However, that doesn't mean that instruction should begin with fractions as a number. Instead, as stated before, investigations of fractions should start as *units* and once students have a grasp of units written in word form, they can transition to this understanding of fractions as a number that can be represented on a number line.

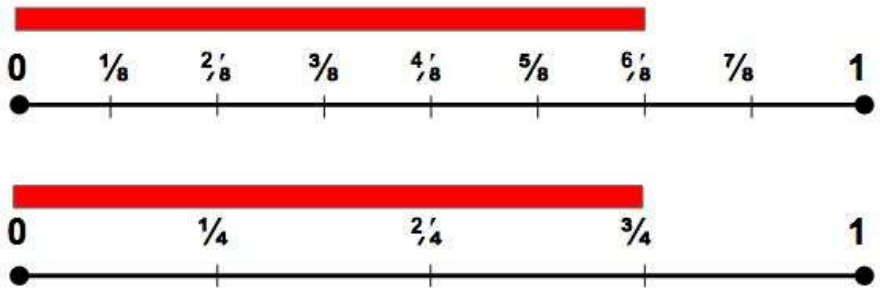
3.NSF.2 Explain fraction equivalence (i.e. denominators 2, 3, 4, 6, 8, 10) by demonstrating an understanding that:

- ***a. Two fractions are equal if the are the same size, based on the same whole, or at the same point on a number line.***
- ***b. Fraction equivalence can be represented using set, area, and linear models.***
- ***c. Whole numbers can be written as fractions (e.g., 4 = 4/1 and 1 = 4/4).***
- ***d. Fractions with the same numerator or same denominator can be compared by reasoning about their size based on the same whole.***

Fraction equivalence is not about "reducing" fractions. Rather, it is about investigating the difficult idea that numerals consisting of different numerals and units can be equal. How is a number made of 2 and 3 the same as a number made of 4 and 6? How is 4/6 equivalent to ⅔ if 4 and 6 are larger than 2 and 3? It is the teacher's job to facilitate a discussion to help students understand how it is possible to have different numbers work together to make the same number.

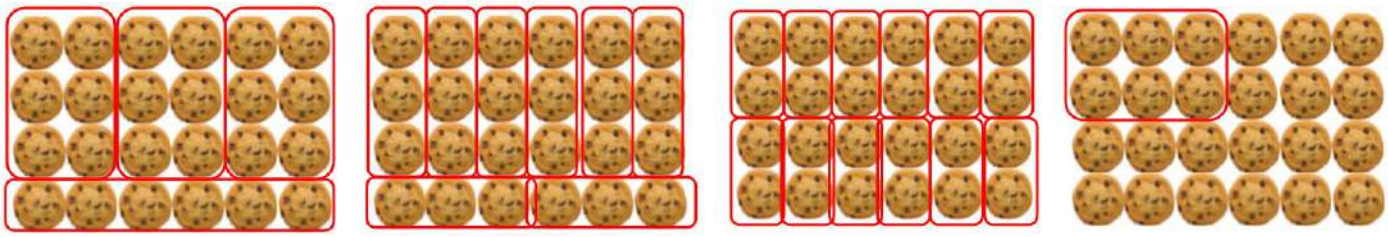
Sample Activity: Equivalent Fraction Race - Students are in two or three teams and lined up in a straight line. The first student in each line draws a unit fraction out of a hat (i.e. ½, ⅓, ¼, ⅙, ⅛, 1/10). When the teacher says "go" the students go down the line of teammates naming equivalent fractions for the fraction that was drawn. No fractions can be repeated. The first line to finish wins. For example, if the first person in line draws "¼", the sequence *could* look something like this: "4/16, 3/12, 5/20, 8/40, 2/8…". The teacher serves as judge to ensure that students do not repeat equivalent fractions and give correct equivalent fractions. Students from other teams also assist in accountability.

Just like NSF.1, students should represent equivalent fractions using a set, area, and linear models. Linear models that represents equivalent fractions represent the same distance between points on a number line, usually from 0 to that number. For example,



Students' spatial reasoning has not been fully developed yet, so it is not uncommon for students to make 6/8 and ¾ at two different lengths. When beginning to investigate equivalent fractions on a number line, provide students with grid paper or other ways to ensure that their spacing is equal. When making eighths and fourths on grid paper, however, a common mistake would be to make two different sized number lines. Ensure students know that equivalent fractions (as stated in the standard) must be at the same point on the same number line or on a number line of equivalent length.

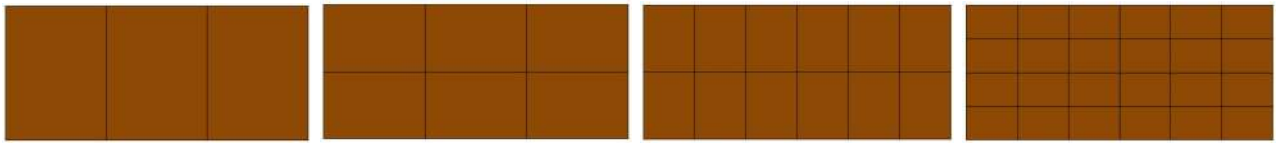
A set model has been modeled a few times in the earlier pages. It is essential that students understand fractions as division in order to use set models to demonstrate fraction equivalency. For example, "How could this set be used to model that ¼ = 2/8 = 3/12 = 6/24?"



¼ 2/8 3/12 6/24

In the set models above, ¼, 2/8, 3/12, and 6/24, all included 6 cookies.

Finally, modeling equivalent fractions using area models are intended to demonstrate that a certain section of a two dimensional figure is the same no matter how it is cut. For example, ⅓ of a pan of brownies would be the same amount of brownies if it was cut into 3, 6, 12, or 24 pieces.



Part ***c*** of the standard involves relating the understanding of fractions as division and an understanding of whole numbers. If students understand that $4 \div 1 = 4$ and $4 \div 4 = 1$, then they will have no problems understanding that 4/1 = 4 and 4/4 = 1. This ***should not*** be an isolated lesson, but should be integrated within equivalent fractions activities.

Part ***d*** begins students' understanding of the magnitude of fractions. Fortunately, in 3rd grade, students only have to compare fractions with common numerators or common denominators. Students must be able to express WHY larger denominators have a smaller magnitude (if the numerators are the same) and WHY larger numerators have a larger magnitude (if the denominators are the same). Additionally, if students understand fractions as units, they will understand that ⅜ < ⅝ since 3 units of ⅛ would be less than 5 units of ⅛.

3.NSF.3 Develop an understanding of mixed numbers (i.e., denominators 2, 3, 4, 6, 8, 10) as iterations of unit fractions on a number line.

This standard is new to the South Carolina State Standards. The goal is for students to understand mixed numbers as a combination of a whole number and a fractional distance to the next whole number in order to operate with mixed numbers in 4th and 5th grades. This understanding can be built through an understanding of regrouping into whole numbers. Just as 10 ones make a "ten" and 10 tens make a "hundred", 6 sixths make a "one" or a "whole". One way to highlight this concept is to "Count Around the Circle" by certain unit fractions. As students get to the whole numbers, they highlight the fact that they regroup the collection (or iteration) of unit fractions into the wholes.

Since this standard stresses student understanding, it is not easily assessed using pencil and paper. Students can explain (verbally or in writing) how a mixed number like 2 ⅙ can be composed and decomposed. The composition (or decomposition) should simply be a collection of the unit fraction ⅙ and not combinations of other fractions or mixed numbers like 1 ⅚ and 2/6. It would be appropriate to ask students how many ⅙s make up 2 ⅙ by having students count up by ⅙ to get to 2 ⅙.

Although it is *not* the baseline requirement that students convert back and forth between mixed numbers and improper fractions, it would be developmentally appropriate to let students investigate their relationships. Students can explain why 33/6 is equal to 5 3/6 since they can count up by 6s to

30 and explain that it would make 5 wholes with 3 sixths left over. Again, students *should not* be assessed and reported on their ability to convert, but it would make a nice extension for students.

3.ATO.7 Demonstrate fluency with basic multiplication and related division facts of products and dividends through 100.

Even though the multiplication unit is "finished", students are still developing and improving their fluency with multiplication and related division facts through 100. Fluency can be developed through number talks and other activities meant to strengthen strategic student thinking.

3.MDA.1 Use analog and digital clocks to (a) determine and record time to the nearest minute, (b) using a.m. and p.m, (c) measure time intervals in minutes and solve problems involving addition and subtraction of time intervals within 60 minutes.

3.MDA.1 should be an ongoing standard all year. Students do not learn to tell time through direct instruction, but through multiple experiences with the analog clock. Throughout the day, ask a student to read the clock, tell the time, and make predictions. Instead of setting timers on phones, etc., have students remind the class/teacher about stop times. The ***(a)***, ***(b)***, and ***(c)*** components of this standard should not be taught in isolation.

Sample Questions provided above in Unit 1.

This standard should be addressed in each unit, but not reported until the 4th quarter.

Unit 6: Capacity & Measurement (Approx 15 days)
Standards Addressed: 3.MDA.2, 3.MDA.4, 3.MDA.1
Number Talks/Number Sense Routines: Continue activities such as "Count Around the Circle" (by unit fractions or equivalent fractions), multiplication fluency problems, and three-digit addition and subtraction problems.
Standards Rationale: ***3.MDA.2 Estimate and measure liquid volumes (capacity) in customary units (i.e. c., pt., qt,. gal.) and metric units (mL, L) to the nearest whole unit.***

In 2nd grade, students began measuring lengths using rulers, yard sticks, meter sticks, and measuring tapes in whole number units. In 3rd grade, they begin to measure using fractional lengths and expand to liquid volumes by expanding their understanding of measurement to three-dimensional capacities as measured by measuring cups or other capacity measuring tools.

In order for students to gain an understanding of cups, pints, quarts, and gallons as measurements, they must practice pouring water (or sand & rice) into measurement containers. It is *not* necessary for students to convert between customary units, but it would be appropriate for students to investigate how many cups can fit inside of a pint and how many pints fit inside of a quart, etc…

In the Common Core Standards, students had to perform operations with the measurements. While not explicitly stated in the South Carolina standards, students should be extended by comparing measurements to determine "how much more?" or "how much less?" or "how much more do we need for ___?" Students should *not* be assessed on this skill, but only on their estimation and ability to measure. Anything else would be supplemental.

3.MDA.4 Generate data by measuring length to the nearest inch, half-inch, and quarter-inch and organize the data in a line plot using a horizontal scale marked off in appropriate units.

Students began line plots in 2nd grade by measuring to the nearest whole unit and plotting them on a line plot. The only difference in 3rd grade is that they measure to the nearest half-inch and quarter-inch.

Line plots should not be approached as "something we have to do" but as something we have/use to make statistical analysis easier. As Dan Meyer has famously asked, "If ____ is the Aspirin, what is the headache?" So, if line plots are the Aspirin, what is the headache? The following opening task, is used to investigate how teachers can get students to realize the usefulness of line plots.

Sample Task: Mrs. Rupert needed to collect the heights (in feet) of all of her students. She measured each student to the nearest full, ½, or ¼ foot. Here are all of the measurements (in feet): 4 ¾, 5, 4 ¾, 5 ½, 5, 5 ¼, 5, 4 ½, 5, 5 ½, 5, 5 ¼, 5 ¾, 4 ¾, 4 ½, 5, 5 ½, 5 ¼, 5 ¾, 4 ¾, 4 ½, 5, 5, 5 ¾. What do you notice? What do you wonder? Can you help her organize this data? How would you organize this data? Why would we need to organize this data?

Sample Student Response: "I notice that many of the measurements repeat, but I don't know how much. We could put a number next to each measurement to show how many times it occurred (frequency table) (i.e. 4 ½ - 3, 4 ¾ - 4, etc…)."

By creating the *need* for line plots, students are allowed to form their own understanding of the use of line plots. Now, line plots are no longer something that is being done *to* them, but something that gets them to create models that display data efficiently.

Sample Activities: Put strips of paper or ribbon with predetermined lengths in a baggie and have students measure each and plot the results on a line plot.

3.MDA.1 Use analog and digital clocks to (a) determine and record time to the nearest minute, (b) using a.m. and p.m, (c) measure time intervals in minutes and solve problems involving addition and subtraction of time intervals within 60 minutes.

3.MDA.1 should be an ongoing standard all year. Students do not learn to tell time through direct instruction, but through multiple experiences with the analog clock. Throughout the day, ask a student to read the clock, tell the time, and make predictions. Instead of setting timers on phones, etc., have students remind the class/teacher about stop times. The ***(a)***, ***(b)***, and ***(c)*** components of this standard should not be taught in isolation.

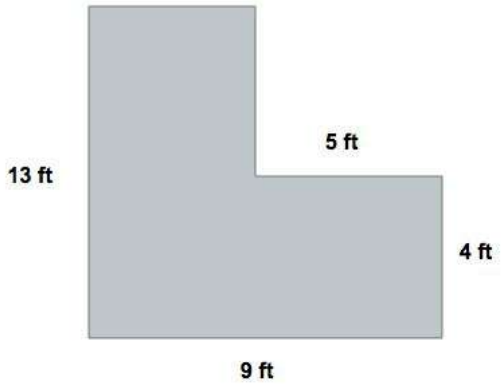
Sample Questions provided above in Unit 1.

This standard should be addressed in each unit, but not reported until the 4th quarter.

Unit 7: Area & Perimeter (Approx. 15 days)
Standards Addressed: 3.MDA.5, 3.MDA.6, 3.MDA.1, 3.ATO.7
Number Talks/Number Sense Routines: Continue the same number talks from previous units.
Standards Rationale: ***3.MDA.5 Understand the concept of area measurement.*** ***a. Recognize area as an attribute of plane figures.*** ***b. Measure area by building arrays and counting standard unit squares.*** ***c. Determine the area of a rectilinear polygon and relate to multiplication and addition.***

In 2nd grade, students partitioned rectangles into equal sized squares and counted to find the total number of squares. In 3rd grade, students understand this sum as the total amount of space inside the rectangle, also known as the “area”. It is very important that students are *not* taught the formula for area, but are given the opportunity to derive it. It is recommended to start with a task that allows students to recognize the need to find the space inside of a figure.

Sample Task: I am putting a new patio in my backyard by pouring cement in some space in my yard. The cement company charges by how many 1 ft. squares of concrete I need. What do we need to know? (After students have an opportunity to think about the needed information), Here is a picture of the space I need to fill:



Do I have enough information? How can I figure out how many 1 ft. squares can fit in this space? If they charged $10 per square, how much will it cost? How do you know?

Some students will recognize an array and others will simply count all of the squares. Either strategy is sufficient for 3rd graders. Ask the counters if there are more efficient ways to determine the number of squares. Some might use partial products and others might multiply the entire thing. Some might need to physically put squares on their paper and count. They should be allowed to do that too.

The shape in the above task is what is considered a rectilinear polygon. A rectilinear polygon is defined as any shape whose edges ***all*** create 90 degree angles. It could also be considered a shape that is a composite of multiple rectangles. Students are not explicitly required by the standard to find unknown side lengths using their knowledge of other side lengths. However, it would be appropriate to extend student thinking and problem solving.

3.MDA.6 Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and

exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Once again, this standard is more about the conceptual understanding of perimeter and the need for perimeter as a measurement than simply computing the perimeter. There are three parts of the standard that should be analyzed.

"...involving perimeters of polygons, including finding the perimeter given the side lengths"

This standard doesn't limit perimeter to rectangles, but to all polygons, including rectilinear shapes. Instead of telling students to add all of the side lengths, give students contexts that require them to use the perimeter.

Sample Task: (From the task above) Once I put the concrete in my patio area, I needed to put some rope around the outside of the space to keep people out. How much rope did I need? How do you know?

"...finding an unknown side length"

This part of the standard *is not* requiring students to use deductive reasoning to determine missing side lengths of rectilinear polygons (though appropriate), but using the perimeter to find unknown lengths. Once again, it would be most meaningful if students are given problematic contexts that make them think critically to determine the length.

Sample Task: Give students the cutout of a rectangle. Ask: "If I told you that the perimeter of this rectangle was 36 inches, could you figure out the lengths of all of the other sides by measuring only one side?"

Sample Question: Do we know the dimensions of a square with a perimeter of 28 feet?

"...exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters."

Sample Task: Give students 24 inches of string and have them make a rectangle with whole number side lengths with the largest area. Why did you choose those dimensions? What do you notice as the dimensions get closer together? What if I gave you 37 inches of string? What would be the whole number dimensions of that rectangle?
*Note: Students could also use geo-boards for this task.

Sample Task: Give students 16 colored tiles. Make a rectangle that would have the largest perimeter. What do you notice? What do you wonder? What connections can you make to the previous task?

3.MDA.1 Use analog and digital clocks to (a) determine and record time to the nearest minute, (b) using a.m. and p.m, (c) measure time intervals in minutes and solve problems involving addition and subtraction of time intervals within 60 minutes.

3.MDA.1 should be an ongoing standard all year. Students do not learn to tell time through direct instruction, but through multiple experiences with the analog clock. Throughout the day, ask a student to read the clock,tell the time, and make predictions. Instead of setting timers on phones, etc., have students remind the class/teacher about stop times. The ***(a)***, ***(b)***, and ***(c)*** components of this standard should not be taught in isolation.

Sample Questions provided above in Unit 1.

This standard should be addressed in each unit, but not reported until the 4th quarter.

Unit 8: Shapes (Approx 20 Days)
Standards Addressed: 3.G.1, 3.G.3, 3.G.4, 3.MDA.1, 3.ATO.7
Number Talks/Number Sense Routines: Continue number talks from previous units.
Standards Rationale: ***3.G.1 Understand that shapes in different categories (e.g., rhombus, rectangle, square, and other 4-sided shapes) may share attributes (e.g., 4-sided figure), and the shared attributes can define a larger category (e.g., quadrilateral). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.*** In 2nd grade, student identified triangles, quadrilaterals, hexagons, and cubes and drew shapes based on specified attributes (2.G.1). Now, in 3rd grade, students explore the possibility that shapes

in different categories could share the same attributes. For example, squares and trapezoids are completely different shapes, although they both have four sides (i.e. are quadrilaterals).

Although students do *not* have to recognize the hierarchies within quadrilaterals, it is important to be accurate in the way shapes are spoken about. For example, squares ***<u>are</u>*** rectangles since they have four sides and four right angles. Similarly, a square ***<u>is</u>*** a rhombus since they have four equal sides with two pairs of parallel sides (students do *not* need to recognize and use the words parallel or perpendicular).

The students outputs of this standard should consist of:

- recognizing any quadrilateral based on the presence of 4 sides,
- drawing irregular quadrilaterals,
- identifying a shape as having different classifications (e.g. a square is also a rectangle, rhombus, quadrilateral, and polygon).

3.G.3 Use a right angle as a benchmark to identify and sketch acute and obtuse angles.

Students had informal interactions with angles in 2nd grade by drawing shapes based on a given number of angles. Therefore, they should have a basic understanding of an angle, but not the measures or classificatory qualities of angles. In 3rd grade, while not explicitly finding the measures of the angles, they will use their informal understanding of angles to determine whether or not an angle is larger or smaller than a "right angle". A right angle is easy for a student to identify because of the shape that it makes. Therefore, students should only use this informal understanding to determine if an angle is "obtuse" (larger than a right angle) or "acute" (smaller than a right angle).

Students should *not* interact with exact degree measurements such as 90 or 180 degrees. Measurements are not expected until 4th grade.

3.G.4 Identify a three-dimensional shape (i.e. right rectangular prism, right triangular prism, pyramid) based on a given two-dimensional net and explain the relationship between the shape and the net.

Students have worked with three-dimensional shapes since kindergarten. However, they were not required by the standards to interact with three-dimensional shapes in 2nd grade, so students may need a refresher as to what qualifies a shape as three-dimensional.

The "i.e." in the standard indicates an exhaustive list of three-dimensional shapes and nets with which students are expected to interact. To begin, students should interact with actual right

rectangular and triangular prisms and pyramids, investigating the shapes formed by the base and the sides. Students should work with cereal boxes, Toblerone candy boxes, individual pizza slice boxes (from Harris Teeter), and pyramid shaped gift boxes, by tearing them apart at the seams and investigating the shapes formed when completely flattened.

Furthermore, they should be given two-dimensional nets and be allowed to build three-dimensional shapes given the nets. Students should recognize that prisms can be long and narrow or short and wide and should note the changes that result in the sides of the shape.

3.MDA.1 Use analog and digital clocks to (a) determine and record time to the nearest minute, (b) using a.m. and p.m, (c) measure time intervals in minutes and solve problems involving addition and subtraction of time intervals within 60 minutes.

3.MDA.1 should be an ongoing standard all year. Students do not learn to tell time through direct instruction, but through multiple experiences with the analog clock. Throughout the day, ask a student to read the clock, tell the time, and make predictions. Instead of setting timers on phones, etc., have students remind the class/teacher about stop times. The ***(a)***, ***(b)***, and ***(c)*** components of this standard should not be taught in isolation.

Sample Questions provided above in Unit 1.

4th Grade

Unit 1: Reading, Writing, and Rounding Numbers (Approx. 15 days)

Standards Addressed: 4.NSBT.1, 4.NSBT.2, 4.NSBT.3

Number Talks/Number Sense Routines: Begin the year by adding and subtracting two-digit numbers with and without regrouping. Students should not be encouraged to do the algorithm during this time. Instead, for addition, they should use strategies based on place value, compensation, friendly numbers, or adding in chunks.

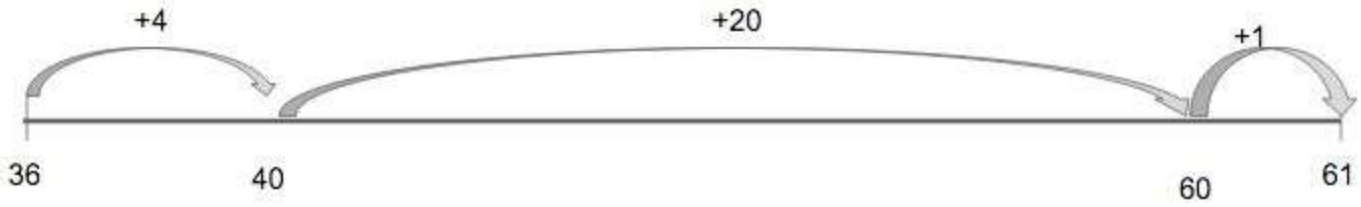
Example: 38 + 37

Place Value: 30 + 30 = 60; 8 + 7 = 15; 60 + 15 = 75
Compensation: 38 + (2 + 35) → (38 + 2) + 35 → 40 + 35 = 75
Friendly Numbers: 38 + 40 = 78 → 78 - 3 = 75 OR 40 + 37 = 77 → 77 - 2 = 75
Adding in Chunks: 38 + 30 = 68; 68 + 7 = 75

For subtraction, students should use strategies to include adding up, subtracting in chunks, changing a number, or keeping a constant difference.

Example: 61 - 36

Adding up: 36 **+ 4** = 40 **+ 20** = 60 **+ 1** = 61 → **4 + 20 + 1 = 25**



Subtracting in Chunks: 61 - 30 = 31 → 31 - 6 = 25
Changing a Number: 61 - (36 **+ 4)** = 21 → 21 **+ 4** (because we closed the difference by 4) = 25
Keeping a Constant Difference: (61 **- 2**) - (36 **- 2**) → 59 - 34 = 25
(61 **+ 4**) - (36 **+ 4**) → 65 - 40 = 25

Students should not be directly taught these strategies. They should bring their own strategies to the table and describe their thinking and rationales.

Standards Rationale:

The following standards make up Unit 1. They should not be taught in isolation, but in cohesion with one another, using understanding of one to understand the others.

4.NSBT.1 Understand that, in a multi-digit whole number, a digit represents ten times what the same digit represents in the place to its right.

This standard is not easily assessed on a paper/pencil assessment. In fact, the only way to know if a student truly understands the way our place value system works is if they can explain it using a number provided or a number they create.

For example, the statement "81 is greater than 18" is true only because the values in their respective digits. Both numbers are made up of the same individual numerals, but since the 8 in 81 is in the tens place and therefore represents 8 tens, and the 8 in 18 represents eight ones, 81 is greater than 18. Students should use this understanding to generalize about larger, abstract numbers.

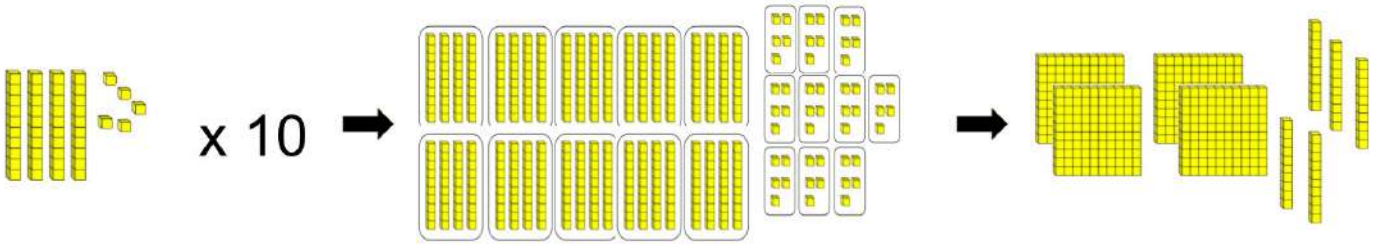
Sample Question: Explain why 72,571 is greater than 27,571 using place value terminology.

Sample Task: Explain the following statement using words and numbers: In a multi-digit number, a digit represents ten times what the same digit represents in the place to its right.

Sample Task: How many hundreds are in a thousand? How many hundred-thousands are in a million? How many tens are in a ten-thousand? Explain.

In addition to simply knowing that the digits to the left are worth more than the digits to the right, students must also know that they are worth ten times the digit to the right. Students should understand our place value system consists of values containing lesser values. In other words, that there are 10 ones in a ten, 10 tens in a hundred, 10 hundreds in a thousand, etc…

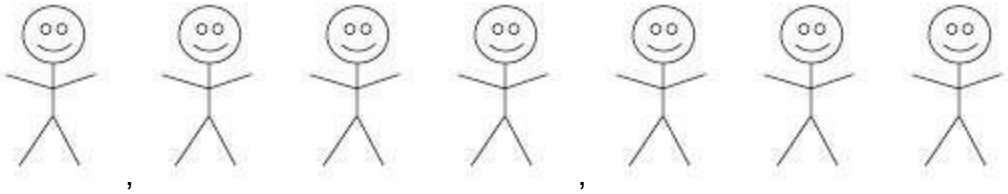
One way to illustrate this point would be to break down a multiplication expression to see *exactly* what is happening to the numbers. Take 45 x 10 for example:



When we multiply by ten, it literally means multiplying both sets of base-ten blocks (the 4 rods and 5 cubes) by 10. In the 2nd picture, there are 10 groups of 4 rods (or 40 rods) and 10 groups of 5 cubes (or 50 cubes). In the 3rd picture, those rods and cubes are getting regrouped into the next place value to make 4 flats and 5 rods (or 450).

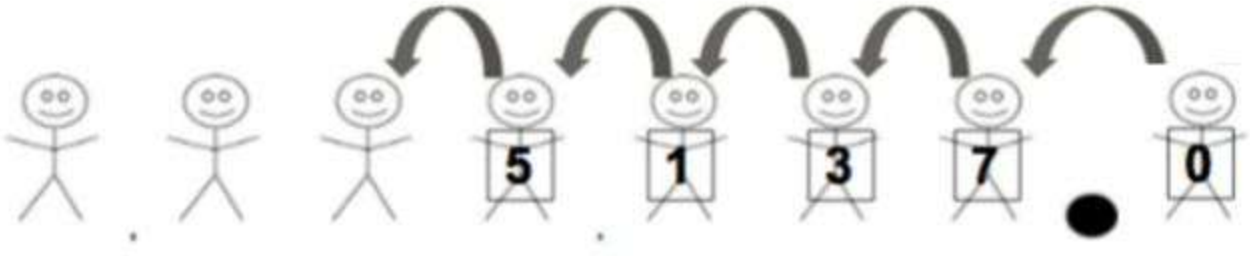
If 45 x 10 = 450, does that mean that when we shift the digits one place to the left every time we multiply by 10? Students should take a day to investigate this question with different examples and a variety of manipulatives. What do you think happens when we multiply by 100? 1,000?

Sample Activity: Line up some students at the front of the room. Each student represents a place value based on the placement of the decimal point.

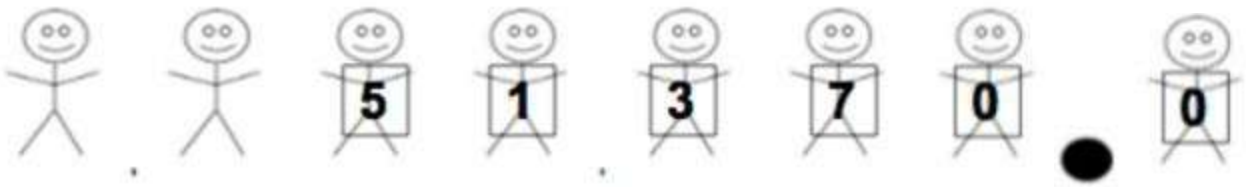


Give some students a large card with a number on it. Discuss what each place value means.

After discussing the place values, ask what happens if that number is multiplied by 10.



Each student can pass their number one space to the left, creating a new number. The student in the ones place doesn't have a digit, so they would get their digit from the tenths place, so feel free to add another student and a decimal to illustrate this movement. It is important to note that the decimal does not move, just the numbers in each digit.



4.NSBT.2 Recognize math periods and number patterns within each period to read and write in standard form large numbers through 999,999,999.

In order to be able to read and write large numbers in standard form, students must understand and recognize math periods and the patterns within those periods. Within each period, the pattern remains the same (hundreds, tens, & ones of each period).

Students should have familiarity with numbers to one million from 3rd grade.

Students are only expected to read and write these numbers. There is no standard expectation to write the numbers in expanded or word form.

4.NSBT.3 Use rounding as one form of estimation and round whole numbers to any given place value.

Students should ***not*** be taught rhymes and raps about the rules for rounding. Instead, they should work with and review number proximity within 1,000 to draw generalizations about more abstract and larger numbers.

Example: Round 352 to the nearest hundred.

Sample Student Response*:* "I thought about the nearest two hundreds to 352, which would be 300 and 400. 352 is 52 spaces away from 300 and 48 spaces away from 400, so it must be closer to 400."

Example: Round 671,963 to the nearest ten-thousand.

Sample Student Response: "670,000 and 680,000 are the two nearest ten-thousands. Anything between 670,000 and 674,999 would be closer to 670,000. Anything higher would be closer to 680,000. 671,963 is less than 674,999 so 671,963 rounded to the nearest ten-thousand would be 670,000."

Example: Give four numbers that would round to 47,000. Choose two numbers less than 47,000 and two numbers greater than 47,000. Explain why you chose those numbers.

Sample Student Response: "I chose 46,677 and 46,988 since they are greater than 46,500 and will round up to 47,000. I also chose 47,143 and 47,344 since they are both less than 47,500 which would round up to 48,000, so they round down to 47,000."

Unit 2: Adding and Subtracting Multi-Digit Numbers (15 Days)
Standards Addressed: 4.NSBT.4, 4.ATO.3, 4.MDA.2
Number Talks/Number Sense Routines: If students have demonstrated proficiency with mental operations with two-digit numbers, they can begin to move to three-digit numbers. They should still employ mental strategies to solve. The challenge is holding larger numbers in their heads while operating. Problem sets should include addition and subtraction with and without regrouping. Strategies from Unit 1 should still be used.
Standards Rationale: The following standards make up Unit 2. They should be taught simultaneously using a variety of strategies and algorithms, using students' understandings of place value. Being able to break numbers apart aids in computation strategies. (For example: 234 is 200, 30, and 4; *not* 2, 3, and 4.) ***4.NSBT.4 Fluently add and subtract multi-digit whole numbers using strategies to include a standard algorithm.***

Fluently does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Students should be exposed to a variety of strategies and algorithms to solve addition and subtraction problems, not only the traditional algorithms. Students should be able to explain why the algorithm works and justify their steps.

Sample Question: The local college football stadium holds 86,000 fans. For the 1st game of the season, they have sold exactly 63,561 tickets. How many more tickets can they sell before they sell out?

Sample Student Response: "I added up from 63,561. I added 9 to get to 63,570, 30 to get to 63,600, 400 to get to 64,000 and 22,000 to get to 86,000. I added all of my previous addends (9 + 30 + 400 + 22,000) to get 22,439."

Sample Student Response: "To prevent regrouping, I changed both numbers by subtracting one. My new problem became 85,999 - 63,560, and I just subtracted by place value to get 22,439."

$$\begin{array}{rcl} 86{,}000 & -1 \rightarrow & 85{,}999 \\ \underline{-\ 63{,}561} & -1 \rightarrow & \underline{63{,}560} \\ 22{,}439 & \leftarrow & 22{,}439 \end{array}$$

Sample Student Response: "I lined them up vertically and subtracted by place value. Instead of regrouping, I used negative numbers."

$$\begin{array}{r} 86{,}000 \\ -\ \underline{63{,}561} \\ -1 \\ -60 \\ -\ 500 \\ 3000 \\ 20{,}000 \end{array}$$

"Finally, I combined my results (20,000 + 3000 - 500 - 60 - 1). 23,000 - 500 = 22,500. 22,500 - 60 = 22,440. 22,440 - 1 = 22,439."

Sample Student Response: "I lined them up vertically and regrouped my place values to be able to subtract. I took one of my thousands and decomposed it to 10 hundreds. I took one of those

hundreds and decomposed it to make 10 tens. Finally, I took one of those tens and decomposed it into 10 ones. 85 thousands, 9 hundreds, 9 tens, and 10 ones is equivalent to 86,000."

$$\begin{array}{r} {}^{5}\,{}^{9}\,{}^{9}\,{}^{10} \\ 86{,}000 \\ -\ 63{,}561 \\ \hline 22439 \end{array}$$

***Note: Students *must* be able to explain these strategies before and after using them to be deemed as "fluent". A student who blindly follows a series of steps is not fluent.

4.ATO.3 Solve multi-step real-world problems using the four operations. Represent the problem using an equation with a variable as the unknown quantity. *(Addition and subtraction only at this time.)*

This standard should not be taught in isolation, but should be used with the addition and subtraction standard (NSBT.4). To help students better understand the problem solving process and internalize the problems, it is recommended to begin with numberless word problems.

Sample Task: Mike works as a concert manager for Time Warner Cable Arena in Charlotte. They had three concerts over a three day weekend. He noticed a lot of empty seats and needed to provide a report to his bosses about the empty seats. How can he determine how many empty seats there were at the concerts? (Once students inquire about the number of seats the arena holds and how many attended the concert each day, give them the information to operate.)

- Arena holds 17,000
- Friday had 14,345 people attend
- Saturday had 15,897 people attend
- Sunday had 13,956 people attend

The standard also calls for students to "represent the problem using an equation with a variable as the unknown quantity." This part of the standard will look different for every problem depending on the problem. The goal of this part of the standard is to get students to begin to think in terms of unknown information. The goal is *not* for the students to write elaborate equations with multiple variables. For example, in the sample task above, if the entire problem was represented with an equation with variable(s) representing the unknown quantity, the equation could look like this (depending on how the student decided to solve the problem):

$$(14{,}345 + x) + (15{,}897 + y) + (13{,}956 + z) = 51{,}000$$
$$x + y + z = t$$

This is above student requirements in 4th grade. Instead, students could model parts of the problem with variables, but not necessarily the entire problem in one big equation. For example:

	Attending	Missing	Total
Friday	14,345	*x*	17,000
Saturday	15,897	*y*	17,000
Sunday	13,956	*z*	17,000

They could also write three separate equations to represent the information:
14,345 + *x* = 17,000
15,897 + *y* = 17,000
13,956 + *z* = 17,000

The purpose of this part of the standard is to encourage students to begin to model their thinking and the way they see the problem algebraically. It is the first time they have been introduced to the concept of a "variable". In 3rd grade, students had to use an equation with a letter representing the missing information. The difference implied is the concrete idea of a letter in 3rd grade compared to the more abstract idea of a variable in 4th grade. 3rd graders usually used the first letter in the word to represent the missing information, while 4th graders are encouraged to use more abstract letters and traditional algebraic notation, such as "*x*" and "*y*".

4.MDA.2 Solve real-world problems involving distance/length, intervals of time within 12 hours, liquid volume, mass/weight, and money using the four operations.

Since this standard is in the addition and subtraction unit, the real-world problems should be limited to addition and subtraction problems. However, students are familiar with some simple multiplicative concepts from 3rd grade, so integration of simple multiplication and division is encouraged.

This standard should be on-going throughout the year as students begin to think about and investigate different operations such as multi-digit multiplication and division, measurement conversions, and fractions.

When students solve problems involving lengths of time within 12 hours, the focus should be on connecting the regrouping in our system of time with regrouping within base ten (i.e. 60 seconds can be regrouped into a minute, 60 minutes can be regrouped into an hour, etc…).

Sample Task: Tina wakes up for work at 6:35 each morning. She takes 50 minutes to get ready, 25 minutes for breakfast, and drives for 35 minutes to work. What time does she arrive at work each morning?

Sample Student Response: "I added all of the minutes together to get a total of 110 minutes. I know 60 minutes is an hour, so it is the same thing as an hour and 50 minutes after when she wakes up. I added an hour to get to 7:35. I need another 25 minutes to get to 8:00, so I take that from the 50 minutes and have 25 minutes left over. Therefore, she arrives to work at 8:25."

Sample Student Response: "I solved it in the order in which it came. She wakes up at 6:35 and gets ready for 50 minutes. She has 25 more minutes in that hour so I take 25 from 50 to get to 7:00 and have 25 minutes leftover. So at 7:25, she eats breakfast for 25 minutes which takes us to 7:50. She needs 10 minutes until 8:00, so I take that away from her 35 minute drive to get to 8:00 with 25 minutes leftover. Therefore, she arrives to work at 8:25."

Sample Student Response: "I noticed that 110 minutes is 10 minutes away from 120 minutes, which is 2 hours. 2 hours from 6:35 is 8:35. If I take the 10 minutes away, I get 8:25."

While students should interact in this unit with addition and subtraction problems (with some simple multiplication), the bulk of this standard will come in Unit 4 when students investigate conversions and collections of coins. See Unit 4 for more problems for this standard.

Unit 3: Factors, Multiples, and Patterns (Approx. 15 Days)

Standards Addressed: 4.ATO.4 & 4.ATO.5

Number Talks/Number Sense Routines: In this unit, students should review their multiplication facts within 100. HOWEVER, students should ***not*** review using timed tests, flashcards, or other strategies that emphasize memorization and speed over conceptual understanding and flexible, strategic thinking.

Example: 8 x 7

Sample Student Response: “I know that 7 x 10 = 70. So 70 is 10 groups of 7 and I need 8 groups of 7 so I just took away two more groups of 7. That’s 14. So 70 - 14 is 56.”

Sample Student Response: “I know that 5 groups of 8 is 40. 2 more groups of 8 would be 16 and 40 + 16 is 56.”

Sample Student Response: “Four groups of 7 is 28 and eight groups of 7 would just be two of those four groups of 7. So 28 + 28 = 56.”

If students are demonstrating consistent proficiency, you can move to more challenging number talks:

Example: 14 x 6

Sample Student Response: “I know 10 x 6 is 60 and 4 x 6 is 24 and 60 + 24 is 84.”

Sample Student Response: “I know 6 x 7 = 42 and I need two of those, so 42 + 42 = 84.”

Standards Rationale:

4.ATO.4 Recognize that a whole number is a multiple of each of its factors. Find all factors for a whole number in the range 1 – 100 and determine whether the whole number is prime or composite.

This is the first time that students are explicitly determining multiples and factors. Students should have experience with multiples from skip counting. This unit should begin skip counting by a variety of numbers.

Sample Activity: Count around the room by 6s, 9s, 12s, & 18s. Teacher writes down the multiples on the board as the students count. Give students ample time (or paper) to work out the next number in the sequence. Once all of the numbers are written on the board, define multiples and outline patterns seen in the 4 lists of multiples.

Students should also look for and analyze patterns in the multiplication chart, which was a standard in 3rd grade, but applicable in this standard as well.

Sample Activity:
Take the following 99 chart and place a:

- red unifix cube on all the multiples of 2.
- green cube on all the multiples of 3.
- blue cube on all the multiples of 4.
- yellow cube on all the multiples of 6

What do you notice? What do you wonder?

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Repeat this activity with different multiples and analyze other patterns within the 99 chart.

Sample Activity: NCTM has a wonderful strategy game for factors and multiples. Plus, there's opportunity for differentiation: https://illuminations.nctm.org/Activity.aspx?id=4134

4.ATO.5 Generate number or shape pattern that follows a given rule and determine a term that appears later in the sequence.

The two action verbs in this standard are "generate" and "determine". Students must create a pattern from a *given rule* (meaning a rule they don't have to generate) and determine a later term in the pattern. This is the baseline expectation for 4th grade.

Even though the standard can seem rote and operationally based, there are plenty of contexts that could go with this standard:

Sample Task: Walter makes $9 per hour in his job at the grocery store. Create a pattern representing how much he makes when he works 1, 2, 3, 4, 5, 6, and 7 hours. How much will he make in a week where he worked 38 hours?

Sample Task: For a movie rental subscription, Winnie pays a $49 shipping fee each year and then $12 per month to rent an unlimited amount of DVDs. Write a pattern to show how much Winnie has paid in the first 24 months.

In the Common Core Standards, students were expected to "identify patterns that were not evident in the rule itself". While that is not in the SC standard and therefore not part of the baseline expectation for SC students, it is an excellent discussion to have and is not developmentally inappropriate. For example, for a pattern that starts at 0 and increases by five, students can notice the number in the ones place in each number alternates between 0 and 5. If we started that same pattern at 2, the ones place would alternate between 2 and 7.

The baseline expectation also has no mention of determining the rule of a pattern. In 3rd grade, students determined the rule for an arithmetic pattern (pattern with a constant difference). In 4th grade, it would be a great extension for students to determine unique rules (i.e. multiply by 3, add 2 and then multiply by 2, etc…), but *it is not a baseline expectation* and therefore a student's grade should not be determined by this skill.

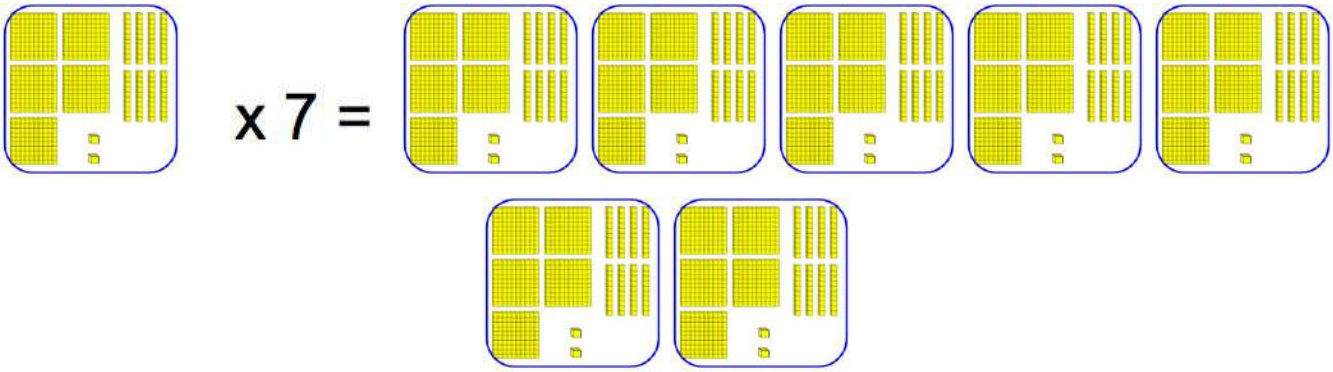
Unit 4: Multiplying Across Contexts (Approx. 30 days)
Standards Addressed: 4.NSBT.5, 4.ATO.1, 4.ATO.2, 4.MDA.8, 4.MDA.3, 4.MDA.1, 4.MDA.2
Number Talks/Number Sense Routines: Continue the same number talks from Units 2 & 3. As students become more comfortable with easier multiplication problems, gradually increase the difficulty and the size of the numbers.
Standards Rationale: While there are 7 different standards in this unit, all of the standards should be taught together. NSBT.5 & ATO.1 directly address the skill of multiplication, MDA.1 requires conversions that ONLY require multiplication, and ATO.2, MDA.8, MDA.3, & MDA.2 are standards that emphasize real world problems where multiplication is applied. These types of problems should be used throughout the unit and not isolated in a certain part of the unit. ***4.NSBT.5 Multiply up to a four-digit number by a one-digit number and multiply a two-digit number by a two-digit number using strategies based on place value and the properties of***

operations. Illustrate and explain the calculation by using rectangular arrays, area models and/or equations.

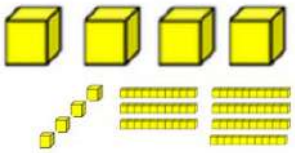
In NSBT.5, students are not required to apply the traditional US algorithm to multiply large numbers. Instead, they are required to use strategies based on place value and the properties of operations. Strategies based on place value and the properties of operations are not two separate strategies, but work together to form strategies. These strategies ***<u>should not be directly taught</u>*** but should be derived through engaging and problematic contexts. To illustrate both of these methods, we will use the following two contexts:

Sample Context: At the county carnival, the price of admission comes with 7 free tickets. If 582 people came to the carnival on Saturday, how many tickets did they give away with the admission price?

Sample Student Response for 582 x 7: "I made 582 using place value blocks to represent the people and pretended that represented 1 ticket. To represent 7 tickets, I made 6 more of those same groups.



With these 7 new groups, I regrouped as much as I could since 10 hundreds make 1 thousand, 10 tens make a hundred and 10 ones make a ten. This was my new picture after regrouping:



Next, I modeled what I did with numbers.

500 x 7 = 3500

80 x 7 = 560

2 x 7 = 14

6 of my groups of 500 regrouped to be 3 thousands with 5 hundreds leftover. However, my 7 groups of 8 tens regrouped to make 5 more hundreds which combined with the other 5 hundreds to make another thousand. I had 6 leftover tens, but my 7 groups of 2 made another ten with 4 leftovers. So I had 4 thousands, 0 hundreds, 7 tens, and 4 ones, or 4,074."

This same strategy can be modeled abstractly using the distributive property.

Sample Student Response for 582 x 7: "I thought about the total number of people in expanded form: 500 people, 80 people, and 2 people. It is easier to distribute 7 tickets to multiples of 10 and 100. I gave 7 tickets to 500 people, which would have been 3500 tickets. 7 tickets to 80 people would have been 560 tickets, and 7 tickets to 2 people would be 14 tickets. I added up all of the tickets to get 4,074 tickets."

582 = (500 + 80 + 2)
(500 + 80 + 2) x 7
(500 x 7) + (80 x 7) + (2 x 7)
3500 + 560 + 14 = 4074

***Note: It is important that students explain their numbers within the context to ensure that they are not mindlessly repeating steps, but truly understand what they are doing.

Sample Student Response for 582 x 7: "I wondered how much it would be if 600 people came since 582 is close to 600. If 600 people came and each got 7 tickets, we would have given away 4200 tickets. But 600 people didn't come...582 did, which is 18 fewer. Therefore, I would need to subtract 18 x 7 from 4,200. 18 x 7 is the same as 10 x 7 and 8 x 7, which is 126. If I take 100 from 4,200, I get 4,100. If I take 20 away, I get 4,080 and then 6 more away, I get 4,074."

582 = (600 - 18)
(600 - 18) x 7
(600 x 7) - (18 x 7)
(600 x 7) - (10 x 7) - (8 x 7)
4,200 - 70 - 56
4,200 - 126 = 4,074

***Note: Just as students are supposed to match their calculations with an explanation, they should also be able to match their explanations with calculations.

Sample Context: Mrs. Hurley's class is going to the museum and a restaurant for lunch. The total cost for each student is $16. If there are 35 people going on the field trip, what is the total cost of the trip?

Sample Student Response for 35 x 16: "I thought about $16 as $10 and $6 together. If the trip was only $10 per person, the total cost would be $350. If it was $6 per person, it would be (35, 70, 105, 140, 175, 210) $210, which make sense since it is less than $350. So, $16 per person would be $350 + $210 = $560."

35 x 16
35 x (10 x 6)
(35 x 10) + (35 x 6)
350 + 210 = 560

Sample Student Response for 35 x 16: "I thought about it as $20 per person. If it was $20 per person, it would cost $700. But the actual cost was $4 less per person, so I would need to take 4 per person (35) away. I know 35 x 4 = 140, since 4 = 2 x 2 and I can rewrite that expression as 35 x 2 x 2. 35 x 2 = 70 and 70 x 2 = 140. 700 - 140 is 560 since 700 - 100 is 600 and 600 - 40 = 560."

35 x 16
35 x (20 - 4)
(35 x 20) - (35 x 4)
700 - 140 = 560

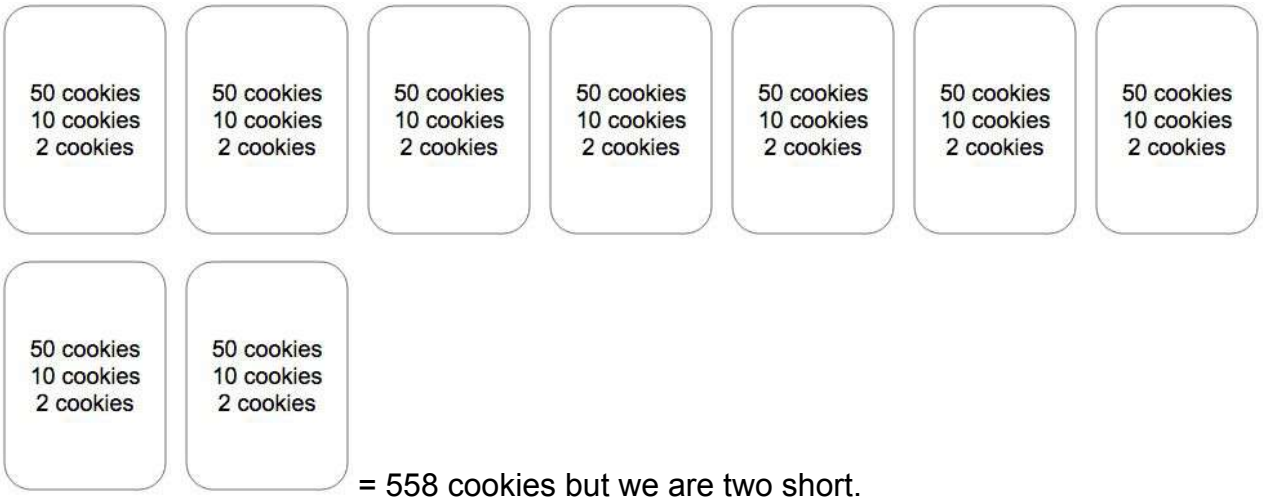
This is not an exhaustive list of how students could find the product of 582 x 7 or 35 x 16. Students could use an area model for both, or even the "doubling/halving" strategy for 35 x 16. However, the context doesn't lend itself to modeling this way. For an example of the area model, see the description of MDA.3 below. A different context would be better to model the idea of doubling/halving.

Sample Student Response for 35 x 16: "I know that 16 is the same as 4 x 4, so I rewrote the expression to be 35 x 4 x 4. I know that 4 can be rewritten as 2 x 2, so I rewrite the expression again to be 35 x 2 x 2 x 2 x 2. So, if I double 35 four times, I will get my answer. 35 x 2 = 70 x 2 = 140 x 2 = 280 x 2 = 560. I am using the associative property of multiplication since I am showing I can multiply in any order."

***Note: The following context involves a lot of division, but also illustrates the opportunity to use "doubling and halving" as a strategy. Even though we haven't "gotten to division" yet, give students the opportunity to model and prep the ground for division investigation.

Sample Context: Lisa is baking cookies for a bake-off. She has to bake exactly 560 cookies. She plans to put 35 cookies on each tray. About how many trays would she need? What number is too high? What number is too low? She realizes she needs 16 trays to put 35 on each, but only has 9 trays. If she wants to put the same number of cookies on each tray, how many should she plan to put on each tray? Can she use exactly 9 trays? Could she use 8? 7? 6? 5? What if she could only fit a maximum of 75 cookies on a tray? Could she fit the same amount on each tray if she couldn't put more than 75 cookies on a tray and only has 9 trays? Explain:

Sample Student Response: "We realized she couldn't fit the same amount on 9 trays since she could she could fit 62 on each but would have 2 cookies left over.



= 558 cookies but we are two short.

Next, we tried 8 by taking away the 9th tray and distributing those and the two left over (64) among 8. We realized we could give each 8 cookies for a total of 70 on each tray. 70 x 8 = 560."

Teacher Follow Up: "So, 35 x 16 = 560 cookies *and* 70 x 8 = 560? What do you notice about these two equations? What do you wonder? Does it work every time? Could we always multiply one factor by 2 and one by half and still keep my product the same? Why does that work? When is it efficient? When is it inefficient? Why?"

Sample Student Response: "I notice that we doubled the first factor and halved the second factor, but kept the same product. It works every time because we are just rewriting the equation to be:

2 x 35 x 16 **x** ½

Because of the associative property, we know we can multiply in any order."

2 x ½ **x** 35 x 16
1 x 35 x 16
35 x 16

It isn't always efficient to double and half, because not all numbers can be easily halved, nor will doubling a number create an easier factor. This only is efficient in some contexts.

***Teacher Note: This is the same philosophy as equivalent fractions since we can also say that we are multiplying by 2/2. Also, this works for any combination whose product makes one (i.e. 3 and ⅓, 4 and ¼, etc…).

Sample Game: Bullseye - Students are in pairs with a stack of number cards 1 - 9 and a bowl full of pre-determined 3, 4, or 5 digit numbers (e.g. 3,426, 846, 5,189, 9,943, 12,458, etc…). Student 1 draws a card from the bowl and student 2 draws 3 or 4 cards from the stack of number cards. Both students try to create multiplication expressions that are closest to the drawn number. For example, if student 1 draws 5,782 and student 2 draws 9, 7, 2, and 1. The students could come up with expressions such as 9 x 721 or 92 x 71. Assume student 1 comes up with 9 x 721 = 6489 and student 2 comes up with 92 x 71 = 6538. Student 1 was 707 away and student 2 was 756 away. The student with the fewest amount of points at the end of 5 rounds wins.

***Note: There are multiple answers in this game. Students are practicing estimation, division, addition, subtraction, and multiplication. There must be a debrief at the end of each game to draw out strategic estimation and game play.

4.ATO.1 Interpret a multiplication equation as a comparison (e.g. interpret 35 = 5x7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.) Represent verbal statements of multiplicative comparisons as multiplication equations.

Multiplicative comparisons are very similar to additive comparisons students learned in 1st and 2nd grades. This time, instead of asking "how many more" we are asking "how many times more" or "how much of".

Take, for example, the following context:

Billy has $32 and Martin has $8,

Two comparison questions could be asked from here. An additive comparison problem could be asked:

"How much more money does Billy have than Martin?"

OR a multiplicative comparisons question could be asked:

"How many ***times*** more money does Billy have than Martin?"

Students struggle most when they have to determine whether to multiply or divide by the given scalar (the scale by which we are multiplying).

Sample Task: Elisa opens her pack of skittles and dumps them out on her desk. She notices that she has 3 times as many greens as red. She also notices that she has twice as many red as yellow. If she has 6 red skittles, how many yellows and greens does she have?

The standard also requires students to represent their thinking in equations. For the sample task above, students could write the following equations:

$G = 6 \times 3$
$Y = 6 \div 2$
$Y = 6 \times \frac{1}{2}$

4.ATO.2 Solve real-world problems using multiplication (product unknown) and division (group size unknown, number of groups unknown).

This standard should be embedded in standards NSBT.5, NSBT.6, and ATO.1. Students should be able to solve problems with unknowns in all positions. Even though a "group size unknown" or "number of groups unknown" problem can be modeled with division, if the problem is posed as a missing factor problem, (e.g. "Brad bought 7 boxes of crayons and dumped them on his table. He counted a total of 56 crayons. How many crayons were in each box?") the students should use an equation that represents the problem (e.g. 7 x ___ = 56).

4.MDA.8 Determine the value of a collection of coins and bills greater than $1.00.

When operating with money in 4th grade (see also: MDA.2), students should ***not*** work with decimal concepts. This standard falls within the multiplication unit because in order to determine the value of the collection, they will need to multiply the number of coins by their respective values.

Sample Task: Jared wanted to buy a meal at Chick-fil-a for lunch. He opens up his piggy bank and dumps out the contents. He counts 13 quarters, 7 dimes, 4 nickels, and 16 pennies. Does he have enough money to buy a Chick-fil-a meal that costs $4.19? Why or why not? What kinds of coin collections could he have that would let him pay for the meal?

Sample Student Response: "I know that 4 quarters is a dollar and there are three groups of 4 inside of the 13 quarters. So that would be $3.25. 7 dimes is another 70 cents and 4 nickels is a total of 20 cents. Ten more pennies would create another dollar, which makes $4.25, so he has enough with 15 cents left over."

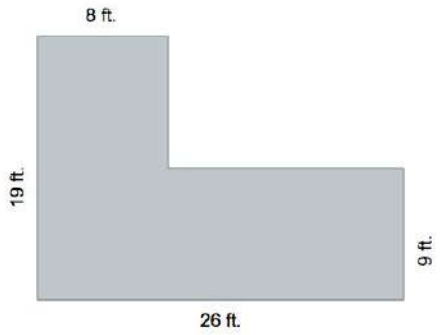
Sample Student Response: "I multiplied 13 x 25 to get 325, 7 x 10 to get 70, 4 x 5 to get 20. I added 325 + 70 + 20 + 16 (the pennies) to get 431. 431 cents is the same as 4 dollars and 31 cents."

Sample Activity: Students have a container filled with fake (or real) coins. They reach in without looking and pull out a handful of coins and the group counts the amount. The next person in the group repeats and compares his/her results with the 1st student.

4.MDA.3 Apply the area and perimeter formulas for rectangles.

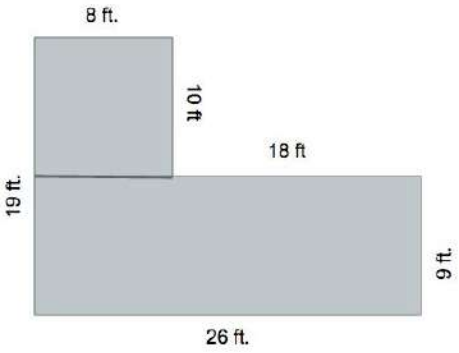
In 3rd grade, students are introduced to the concepts of area and perimeter as attributes of two dimensional shapes. They partition rectangles into unit squares based on the given dimensions and count the total number of squares to come up with a square unit area (units^2). In 4th grade, review should include that area is *not* just a number, but a value that gives the total number of unit squares that can fit into the rectangle. From here, students should recognize that these squares within the rectangle form an array and can be summed using multiplicative reasoning.

Sample Task: I am putting tiles down in my kitchen. How many tiles should I buy at home depot? What do I need to know? (After gathering student "wonders" and "notices", reveal the information.) Well, I found the blueprints to my house buried in the back of a closet. Here is the layout of my kitchen.



The tiles I want to buy are squares that are 1 ft. on each side (NOTE: For a challenge, the squares could be 4 inches or 6 inches on each side). Each tile has been priced at $6 a piece. Do I have all the information I need? Do I need to be given the lengths of all of the sides of my kitchen? Why or why not?

Sample Student Response: “We don’t need to know all of the sides because if we know the left side is 19 ft and that part of the right side is 9 ft, then the rest must be ten feet. Same with the missing part in the middle, if the whole horizontal length is 26 feet and we already know one part is 8 ft, the other part must be 18 feet. So I partitioned the picture into a square and a rectangle.



If I was to put 1 ft squares across the top, I would have 8 squares. If I was to keep making 10 of those rows, I would have 80 tiles in the square. I would do the same thing across the bottom with tiles, making a row of 26 squares. I would then have 9 rows of 26, which is 234 squares. 234 + 80 = 314 squares. If they were $6 a piece, I would need to multiply 314 x 6. I would multiply 300 x 6, 10 x 6, and 4 x 6 to get 1800 + 60 + 24 = $1,884.”

If students need to physically tile with blocks or colored tiles, they should be allowed to do that until they generalize about the arrays. They ***should not*** simply be “told” the formulas.

Follow up activity: My builder told me that I need to put a border around the outside of my kitchen on the floor to cover up the edges of the tiles and cabinets/walls. I went to Home Depot and they

charge $3 per foot of border. How much would the border cost? What do you need to know? (NOTE: For a challenge, the price could be $0.50 per inch or $2 per 6 inches).

The most important part of this standard is allowing students to reason about area and use strategies to *derive* the algorithm instead of being directly taught an algorithm.

4.MDA.1 Convert measurements within a single system of measurement, customary (i.e., in., ft., yd., oz., lb., sec., min., hr.) or metric (i.e., cm, m, km, g, kg, mL, L) from a larger to a smaller unit.

4.MDA.2 Solve real-world problems involving distance/length, intervals of time within 12 hours, liquid volume, mass, and money using the four operations.

Now that students have been introduced to multiplication, they can begin to convert measurements ***from a larger to a smaller unit***. Students are not required to convert from a smaller to a larger unit in 4th grade. Students are not expected to have learned to divide numbers that require a remainder be interpreted. In order to convert from a smaller to a larger unit, in most cases, requires interpreting and converting the remainder into a fraction. However, it is not necessarily developmentally inappropriate to allow students to investigate both directions of conversions. The baseline expectation (what should be reflected on the report card and on formative/summative assessments) should be converting from a larger to a smaller unit.

Additionally, the standard uses the abbreviation “i.e.” to give examples of which measurements to use. “I.E.” comes from a latin phrase meaning “that is”. It is used to indicate a complete and exhaustive list. In this case, the measurements listed are ***all*** of the measurements required. In the Common Core Standards and in previous state standards, 4th graders have been required to convert with cups, gallons, pints, quarts, miles, etc… That is ***no longer required*** in 4th grade. However, once again, it is not developmentally inappropriate to discuss and investigate those measurements. It is just no longer the baseline requirement.

Sample Activity: Take a meter stick and a yardstick and measure different items around the room in all of the required standard and metric units. Have students record the results in a table and draw conclusions about their conversions.

*Note: The following task will require division, which is in Unit 6. However, it would be reasonable for students to think of it as an unknown factor problem and use multiplicative reasoning to solve.

Sample Task: To decorate the homecoming banner, the planning team bought 18 yards of ribbon. The design team decided to put 8 inch long ribbons scattered along the banner. How many 8 inch long ribbons can they make from the purchased ribbon?

Sample Student Response: "I know there are three feet in a yard and twelve inches in a foot. In other words, every yard of ribbon is also 3 feet of ribbon. That would mean that there are 54 feet of ribbon (18 x 3). Every foot of ribbon contains 12 inches of ribbon. That would mean that there's also 648 inches of ribbon. I needed to know how many 8 inch lengths I can cut out of the 648 inches. I thought of it as 8 x ___ = 648. 100 is too high since we would need 800 inches. We could make 50 pieces which would take up 400 inches of ribbon. 20 pieces would be 160 inches for a total of 560 inches of ribbon. 10 more pieces would be another 80 inches, giving us a total of 640 used so far and 8 inches left, or one more piece. So, we made 81 pieces of ribbon."

**There can be variations of this task to help students on their own personal learning trajectories. They may just want to find out how many inches of ribbon they have in all. The task could also be used in or after the division unit. We should not withhold opportunities to investigate operations because we haven't "covered" it yet.

Sample Task: Sam is driving his car 45 kilometers per hour. How many meters does he drive in 36 seconds? How do you know?

Sample Student Response: "There are 60 minutes in one hour and 60 seconds in one minute, so 60 x 60 = 3,600 seconds in 1 hour. Also, I know that there are 1,000 meters in each kilometer, so that would mean that he drives 45,000 meters in one hour, or 3,600 seconds. If I divide 3,600 by 36, I would get 100. Therefore, I would need to divide 45,000 by 100 to get 450. He drove 450 meters in 36 seconds."

***Note: The numbers were strategic to create easier division for students.

Sample Task: Walter bought 2 pounds of ground beef for a BBQ. He wants to split it evenly between his 7 guests and himself to make hamburgers. How many ounces should be in each burger?

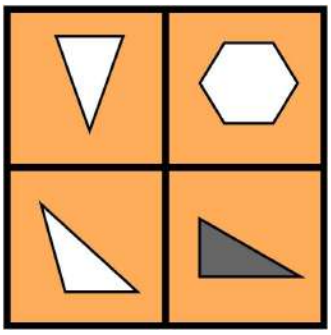
Sample Student Response: "There are 16 ounces in a pound so there would be 32 ounces in 2 pounds. If I split 32 ounces of beef between a total of 8 people, there would be 4 oz for each person."

Unit 5: Geometry (Approx. 10 Days)

Standards Addressed: 4.G.1, 4.G.2, 4.G.3, 4.G.4

Number Talks/Number Sense Routines:
Students should begin to compute mentally with two digit by two digit multiplication and even three digit by one digit. Students should construct their own strategies and not be directly taught strategies. Instead, give students progressions (or "strings") of problems where they "identify and utilize structure and patterns" (Mathematical Practice 7). Sample strings can be found in Sherry Parrish's book "Number Talks" or Cathy Humphries and Ruth Parker's Book "Making Number Talks Matter".

This unit is also a perfect time to do an activity called "Which One Doesn't Belong?". A sample WODB problem could be:



Sample Student Responses:

- "The bottom right doesn't belong because it is the only one shaded."
- "The top right doesn't belong because it is the only one that is not a triangle."
- "The top left doesn't belong because it is the only one with all acute angles."
- "The top right doesn't belong because it is the only one with more than one obtuse angle."

More WODB problems and explanations can be found here.

Standards Rationale:

4.G.1 Draw points, lines, line segments, rays, angles (i.e., right, acute, obtuse), and parallel and perpendicular lines. Identify these in two-dimensional figures.

4.G.2 Classify quadrilaterals based on the presence or absence of parallel or perpendicular lines.

4.G.3 Recognize right triangles as a category, and identify right triangles.

4.G.4 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

The vocabulary introduced in standard 4.G.1 should be new for most 4th graders with the exception of angles. Students were required to identify angles first in 2nd grade and then identify and sketch acute, right, and obtuse angles in 3rd grade. Now, they are explicitly required to identify these (and the other attributes) in two-dimensional figures.

More specifically, 4.G.2 requires students to use their understanding of parallel and perpendicular lines to classify quadrilaterals. For example, they should recognize all of the shapes below as a parallelograms due to their pairs of parallel lines.

***Note: Rectangles, squares, and rhombuses are each parallelograms due to having two pairs of parallel lines.

4.G.3 requires students to recognize right triangles as specific types of triangles. There is no requirement to identify triangles as obtuse, acute, scalene, isosceles, or equilateral, but those classifications could be introduced.

Although 4.G.4 explicitly requires students to identify lines of symmetry, the real goal of the standard is to expand students' spatial reasoning within shapes. Students should be able to explain that even though a line can split a shape into equal parts, it doesn't necessarily mean it is a line of symmetry (e.g. diagonals of rectangles). Students should be given a variety of regular and irregular shapes and be required to identify all of the lines of symmetry by folding.

While the Rock Hill Schools' Math Expectation Guide is clear about constructivist pedagogical being preferred, there are some standards and units that simply need to be taught using direct instruction.

However, this doesn't mean that teachers cannot use engaging and problematic situations to help reinforce student understanding. Some engaging and problematic tasks can be found on the Georgia Standards website here.

Unit 6: Division (Approx. 20 Days)

Standards Addressed: 4.NSBT.6

Number Talks/Number Sense Routines: Continue the number talks and number sense routines from Units 4 & 5.

Standards Rationale:

4.NSBT.6 Divide up to a four-digit dividend by a one-digit divisor using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.

Students should ***not*** be taught an algorithm in this unit. An algorithm includes any "strategy" such as the traditional algorithm, "magic 7" (partial quotients) or other tricks that lead to a step-by-step process. Instead, students should construct their understanding of the division of a 3 or 4 digit number by a 1 digit number within problematic tasks. It is recommended to start with a problematic context like the one below and expand on student strategies as they become more efficient.

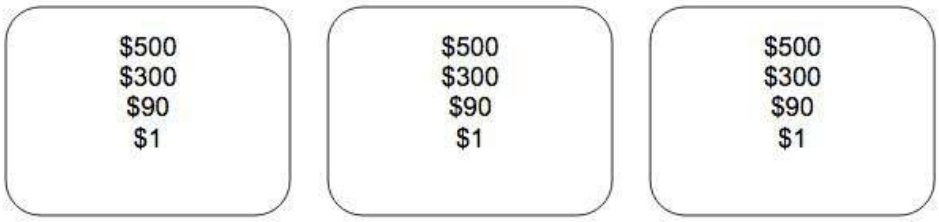
Sample Task: On the game show "Chain Reaction", teams of three solve word puzzles. On one episode, a group of 3 friends won $2,675. How much do you think each person on the team received? What number is too high? What number is too low? Why did you choose these numbers?

Sample Student Response: "I think that each person would get less than $1,000 a piece because they won less than $3000. They would get more than $500 since $500 a piece would be $1,500."

Sample Task Part 2: How much would each person get? How do you know?

Sample Student Response: "First, I drew a picture with three circles which represented the three contestants. I distributed money to each person equally until I was out of money. I gave each person $500 which means I gave away a total of $1,500. I would have $1,175 left over to give out. Each person then got $300 which would be a total of $900 with $275 left over. I couldn't give each $100,

but something close to that, so I gave each $90 for a total of $270. I had $5 left over, so I gave each $1 more and we had $2 left over. After adding it all up, I gave each person $891 with $2 left over."



Sample Task: Marley and her five friends are competing in a 435 yard relay race at school. They want to split it up evenly, but realize they can't divide the yards evenly between 6 people. What do you suggest they do?

Sample Student Response: "I split the yards up between 6 people and then I converted the remaining yards into feet and split those as well. To split up the yards, I realized that each person was running less than 100 yards, so I gave each person 50 yards. If each person ran 50 yards, they would run a total of 300 yards (because 50 x 6 = 300) with 135 yards left over. If I gave each person 20 yards, that would be a total of 120 yards (because 20 x 6 = 120) with 15 yards left over. I gave each person another two yards for a total of 12 yards with 3 yards left over. I couldn't split 3 yards between 6 people, so I converted the yards to feet. 3 yards is the same as 9 feet, so I gave each person a foot, leaving 3 feet. I then converted those feet to inches, giving me 36 inches, that I could split into 6 inches a piece. So each person would need to run a total of 72 yards, 1 foot, and 6 inches."

Sample Student Response: "I converted the yards to feet first. There are 3 feet in a yard so 435 x 3 is 1,305 feet. I gave out 200 feet to each person for a total of 1,200 feet with 105 left over. Then I gave each person 10 feet which would be 60 feet with 45 left over. From there, I gave each person 6 feet for a total of 42 feet with 3 feet left over. I couldn't split the 3 feet between six people, so I converted it into 36 inches. Each person then could get 6 inches for a total of 216 feet and 6 inches."

Task Follow-up: "Which strategy was more efficient? Did both students get the same answer?"

Sample Task: Rock Hill Elementary School is taking a school-wide field trip to the Columbia Zoo. They want to put 8 students into each group with one chaperone. How many chaperones will they need? What information do we need to know? (Students should identify the total # of students as the information needed.) There are 758 students going. How many chaperones do they need?

Sample Student Response: “I know that I need less than 100 chaperones since 100 chaperones could serve 800 students. I know that 50 chaperones would serve 400 students, but we have 358 more students that need chaperones. We could do another 30 chaperones that could serve 240 students, so now we have 118 students that still need a chaperone. If we did another 10 chaperones for 80 students, we will still have 38 students without a chaperone. 4 chaperones would be able to take 32 students, but we still have 6 students that don’t have a chaperone. Even though those 6 students do not make a full group, we still need one more chaperone. So, we need a total of 95 chaperones.”

Sample Student Response: “Let’s pretend that I invited a total of 100 chaperones. They could take 800 students, but I don’t need that many. I have chaperones for 42 extra students, so I could tell 5 of them that I don’t need them, leaving me with 95 chaperones. Even though the 95th chaperone would only have 6 students, I would still need him or her.”

***Note: The following task requires dividing by a double digit divisor, which is a 5th grade standard. This is not the baseline expectation for 4th grade students, but can be accessible to students who have used strategies to this point. This should *not* be used to determine a student’s grade, but should be used to extend student thinking and strategies.

Sample Task: Mrs. Rodriguez wants to meet with each of her students during her math block. She has blocked off 1 hour and 15 minutes to meet with all of her students. How much time should she dedicate to each student? What information do we need to know? If she has 23 students to meet with, how much time should she dedicate to each student?

Sample Student Response: “1 hour and 15 minutes is the same as 75 minutes since there are 60 minutes in an hour. She can give each student 3 minutes since 23 x 3 = 69 minutes. She has 6 leftover minutes that she could split into 360 seconds since there are 60 seconds in a minute (and 60 x 6 = 360). She could give each person an additional 10 seconds for a total of 230 seconds**.** She has 130 seconds left to split up, so she could give each 5 seconds, which would be 115 seconds (since 23 x 5 = 115). She has an extra 15 seconds that she can’t distribute evenly, so she should give each student 3 minutes and 15 seconds. She will have an extra 15 seconds to do other things.”

Sample Student Response: “Since we can’t split 75 minutes evenly, I turned that into seconds. Since 75 x 60 = 4,500 seconds, I split 4,500 seconds between 23 people. Each person could get 100 seconds which would be 2,300 seconds, with 2,200 leftover. That’s really close to 2,300, so I can estimate that each student could get 90 more seconds. 90 x 23 = 2,070 seconds which gives a total of 4,370 seconds. We have 130 seconds left to distribute. I estimated we could give each

student 5 more seconds. 23 x 5 = 115 seconds. We had 15 seconds leftover that couldn't be distributed. So we spent a total of 195 seconds."

Teacher Follow-up: "Did both groups get the same answer? How do you know? Which strategy was more efficient? How do you know?"

Unit 7: Fractions (Approx 30 Days)

Standards Addressed: 4.NSF.1, 4.NSF.2, 4.NSF.3, 4.NSF.4

Number Talks/Number Sense Routines: You can start by continuing the number talks from previous units and progressing to newer, fractional number talks. The number talks in this unit should range from comparing fractions to adding and subtracting fractions and mixed numbers with common denominators.

Comparing Fractions:

Sample Problem: Compare: ⅜ and ⅝

Sample Student Response: "Since ⅜ is literally 3 units of ⅛ and ⅝ is literally 5 units of ⅛, 5 of the same unit would be larger than 3 of the same unit."

Sample Problem: Compare ⅝ and ⅚

Sample Student Response: "⅝ is 5 units of ⅛ and ⅚ is 5 units of ⅙. I know that ⅙ is bigger than ⅛ and so 5 of ⅙ would be larger than 5 of ⅛."

Sample Problem: Compare ⅜ and 7/12

Sample Student Response: "I know that ⅜ is less than ½ since ½ is equivalent to 4/8. Similarly, 7/12 is greater than ½ since ½ is equivalent to 6/12. Therefore, 7/12 > ⅜."

Sample Problem: Compare ⅝ and 7/12

Sample Student Response: "Since 4/8 = ½ and 6/12 = ½, both numbers are one unit above ½ (4/8 + ⅛ = ⅝ AND 6/12 + 1/12 = 7/12). Since ⅛ > 1/12, ⅝ > 7/12."

Sample Problem: Compare 13/14 and 39/40.

Sample Student Response: “They are both one unit away from 1. 1/14 is a larger distance from 1 than 1/40, so 39/40 is closer to one and is therefore greater.”

Sample Problem: Compare ⅔ and ⅝

Sample Student Response: “⅝ is ⅛ larger than ½. I don’t know ⅔’s relationship to ½ without making an equivalent fraction. 4/6 is equivalent to ⅔ and is ⅙ larger than ½. Therefore, ⅔ is ⅙ larger than ½ and ⅝ is ⅛ larger than ½. ⅙ > ⅛ so ⅔ > ⅝.”

<u>Adding and Subtracting Fractions:</u>

Sample Problem: ⅝ + ⅞

Sample Student Response: “⅞ is really close to 1, so I borrowed ⅛ from ⅝ to make it 1. So I now have 1 + 4/8 which is 1 and 4/8 or 1 ½.”

Sample Problem: 4 5/12 + 7 11/12

Sample Student Response: “I need 7 more twelfths to make 4 5/12 a whole number so I took them from the other addend, leaving me with 5 + 7 4/12. My answer is 12 4/12 or 12 ⅓.”

Sample Problem: 7 ¼ - 5 ¾

Sample Student Response: “I subtracted 5 from 7 ¼ to get 2 ¼. Then I subtracted ¾ by decomposing it into ¼ and 2/4. 2 ¼ - ¼ = 2 and 2 - 2/4 = 1 2/4 or 1 ½.”

Sample Student Response: “I added up from 5 ¾. I added ¼ to make it 6, added 1 to get to 7, and then I added another ¼ to get to 7 ¼. If I added all of those addends together, I would get 1 2/4 or 1 ½.”

Sample Student Response: “To keep from regrouping, I added ¼ to both numbers to get 7 2/4 - 6, giving me 1 2/4 or 1 ½.”

***Note: For the previous 4 sample problems, connections should be made to these same strategies when adding and subtracting whole numbers. It is recommended to start with a whole number regrouping problem (e.g. 46 + 38 or 81 - 45) and then connect it to the same ideas for fractions.

Multiplying Fractions by Whole Numbers:

Sample Problem: ¾ x 48.

Sample Student Response: "I split 48 into 4 groups which would put 12 in each group. The three in the denominator tells me that I need 3 of those groups, which would be 36."

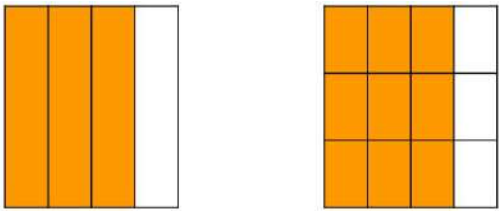
Standards Rationale:

4.NSF.1 Explain why a fraction (i.e., denominators 2, 3, 4, 5, 6, 8, 10, 12, 25, 100), a/b is equivalent to a fraction, (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

One thing to note at the beginning of this unit would be the exhaustive list of denominators in the standard. The abbreviation "i.e." means "for example" which implies a complete list of denominators. While this outlines the baseline expectation for 4th grade and what should be reported on report cards and other official documents, it could still be appropriate to investigate beyond those denominators in class, as long as those are not on a common assessment or factor into the students' grades.

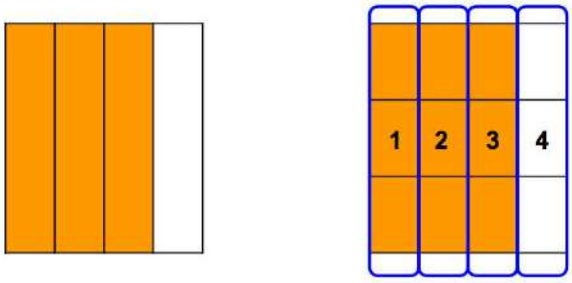
In this standard, students must *explain* fraction equivalency. This includes verbally and in writing. In 3rd grade, students used visual fraction models to investigate fraction equivalency and now 4th graders must explain the relationship between the two fractions using visual fraction models. Take the following task, for example:

Sample Task:

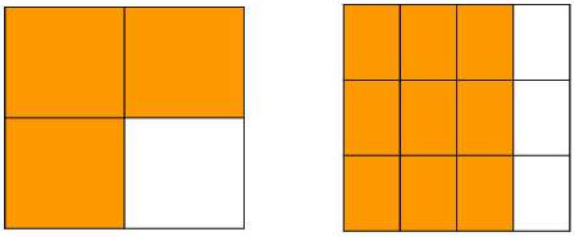


Are the two wholes equally shaded? Do they represent the same fraction? What fraction of the first visual model is shaded? The second model? If the second model (9/12) is equivalent to ¾, where's the "3" and where's the "4"?

Sample Student Response: "The two wholes are equally shaded and so they represent the same fraction. The first model is ¾ and the second model is 9/12, but both fractions are equal since they cover the same amount of the whole. In the second model, you can still split it into 4 columns. Three of the four columns are shaded so we can say 9 out of the 12 boxes are shaded OR 3 of the 4 columns are shaded. Since we can also split the second model into 4 equal groups and 3 of those groups will be completely shaded, ¾ can also be used to represent the second model."



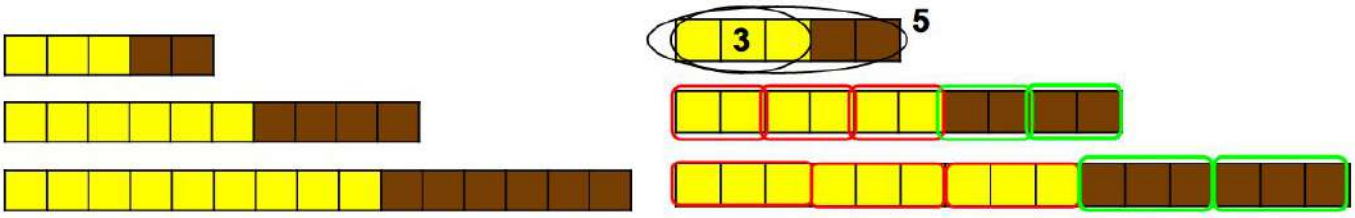
Teacher Follow-up: "Do the following models represent the same fraction even though they do not cover the exact same parts of the model? Explain."



Sample Student Response: "Yes, they are still the same fraction since we can still split the second model into 4 equal groups by columns and 3 of those 4 groups will be shaded."

Sample Task: Take your unifix cubes and represent ⅗ in 3 different ways ***and*** using 3 different amounts of unifix cubes. Identify the "three" and the "five" in each.

Sample Student Response:



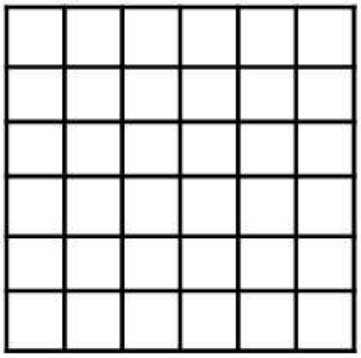
“I split each of my models into 5 equal groups. The first model, each group was one unifix cube. I had 3 yellows out of a total of 5, which is ⅗. In the second one, I split the 10 cubes into 5 groups of two. Three of the groups of two were yellow out of a total of 5 groups which is ⅗. In the last group, I had 5 groups of three cubes, three of which were completely yellow, so it is also ⅗ AND 9/15.”

The standard also points to the abstract a/b = (n x a) / (n x b). Students can use visual fraction models to explain why it is that when multiplying the numerator and denominator by the same number it results in keeping the same fraction.

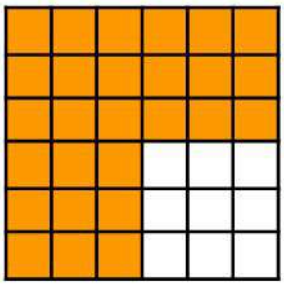
Teacher Follow-up: What do you notice about the fractions ⅗, 6/10, and 9/15?

Sample Student Response: “3 x 2 = 6 and 5 x 2 = 10 and 3 x 3 = 9 and 5 x 3 = 15. We multiplied the numerator and the denominator by the same number. (*T: Why does that create the same fraction instead of a fraction two or three times larger?)*. While I doubled the size of my yellow region in the second model, I also doubled the total number of cubes. I still have three equal parts yellow and two equal parts not yellow. I duplicated the number of yellow regions, but I also duplicated the number of non-yellow regions. You could also say that while my numerator got bigger, the denominator (the number I’m dividing by) also got bigger. For example, I took 6 divided by 2 and 12 divided by 4. They are both equal to 3 even though I made both numbers bigger. The numerator got bigger but the denominator got bigger too, which gave me the same answer.”

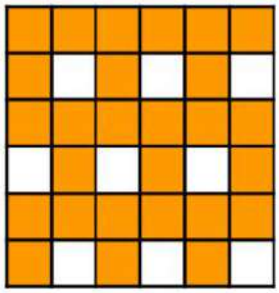
Sample Task: If this grid is the whole, shade in ¾:



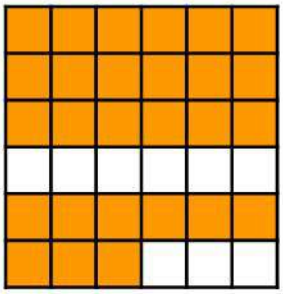
Sample Student Responses:



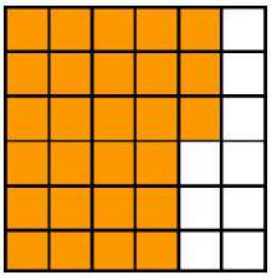
I split the square into four equal groups in the corners. I shaded three of the four squares.



Going up and down, I shaded 3 and then left one unshaded. I noticed a pattern after 4 columns, so I continued that pattern



I shaded in 3 of the first 4 rows. I couldn't shade in 3 out of 4 more rows, so I split the last two rows into 4 groups of three and shadeed 3 of those 4 groups.



Since it's arranged in a 6 x 6 array, I know there are 36 squares. ¾ is just 3 groups of ¼ and ¼ of 36 is 9. 3 groups of 9 is 27, so I just shaded 27 of the squares.

***Note: *a*/*b* = (*n* x *a*)/(*n* x *b*) could be confusing. If numbers are substituted for the letters: 2 for *a* and 3 for *b*, the fraction would be *a*/*b* = ⅔. If *n* = 4, it would be $\frac{2 \times 4}{3 \times 4}$. Then separate both parts to say $\frac{2}{3} \times \frac{4}{4}$. Since 4/4 = 1, the equation is essentially multiplying ⅔ by 1. Anything times 1 is itself and therefore ⅔ = 8/12.

4.NSF.2 Compare two given fractions (i.e., denominators 2, 3, 4, 5, 6, 8, 10, 12, 25, 100), by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½ and represent the comparison using the symbols >, =, or <.

This standard could be completely addressed inside of number talks with strategic debriefs. Students should *not* be taught the "butterfly method" or any other kind of algorithm for comparing fractions. If students derive the algorithm through investigations or an understanding of equivalent fractions or are simply taught at home, they must explain why the algorithm works.

For example, if students were comparing $\frac{4}{5}$ and $\frac{7}{8}$, and they multiplied 5 and 7 and 8 and 4 and concluded that since 35 was greater than 32, ⅞ was larger than ⅘, they must explain why that works mathematically. It works because they are creating common denominators. If they multiplied the numerator and denominator of the first fraction by 8 (32/40) and the numerator and denominator of the second fraction by 5 (35/40), they would have two fractions with common denominators. 32 units of 1/40 would be less than 35 units of 1/40, therefore ⅞ > ⅘. If students cannot explain using common denominator terminology, they should not use that "strategy".

The standard says that students should use common denominators or numerators or benchmark fractions. See the number talk examples above for more examples of students using those strategies.

Sample Question: Compare: ⅞ and 21/25

Sample Student Response: "I created common numerators by multiplying both the numerator and denominator of ⅞ by 3, creating the equivalent fraction 21/24. I have the same amount of units in both 21/24 and 21/25, but the unit 1/24 is larger than 1/25, therefore 21/24 > 21/25 and ⅞ > 21/25."

The strategic planning of these problems is essential. Students must have the opportunity to reason logically about the fractions, which can only happen through strategic problems. When crafting strategic problems, teachers should think about what strategy they would like the students to consider. If wanting them to consider benchmarks or units, they should think about what benchmarks they would like students to consider (½ or 1). If wanting them to consider common numerators or denominators, teachers should create problems where the numerator or denominator is a multiple of the numerator or denominator in the fraction being compared.

4.NSF.3 Develop an understanding of addition and subtraction of fractions (i.e., denominators 2, 3, 4, 5, 6, 8, 10, 12, 25, 100) based on unit fractions.

- ***a. Compose and decompose a fraction in more than one way, recording each composition and decomposition as an addition or subtraction equation.***
- ***b. Add and subtract mixed numbers with like denominators.***
- ***c. Solve real world problems involving addition and subtraction of fractions referring to the same whole and having like denominators.***

When students began investigating whole numbers in kindergarten and first grade, they decomposed whole numbers to understand that numbers are nested within other numbers. For example, 5 contains the numbers 4, 3, 2, & 1. In other words, you can't make a pile of 5 objects without 4, 3, 2, & 1 objects respectively.

If students truly understand fractions as an expression of units, this standard will be simple. If students haven't come to 4th grade understanding that ⅝ is literally 5 units of ⅛, this must be readdressed before decomposing. If ⅝ is 5 units of ⅛, then it could be looked at as 2 units of ⅛ + 3 units of ⅛. As long as the units are the same, the number of units can be directly added together.

Sample Task: On Friday night, Sam ordered a large pizza that he planned to eat for dinner Friday night, lunch on Saturday, and breakfast on Sunday morning. By Sunday night, he had eaten ⅞ of the pizza. What amounts of pizza *could* he have eaten for each meal?

When adding and subtracting mixed numbers (within real world problems!), students should still consider units.

Sample Task: The Martin family is trying to cut down on their water usage to lower their bills and be more environmentally friendly. They have decided to track their water usage over the three day weekend (Friday, Saturday, & Sunday). Their goal is to only use 1,000 gallons of water over the weekend. On Friday, they used 345 ⅞ gallons of water. On Saturday they used 416 ⅝ gallons of water. How much can they use on Sunday to make their goal?

Sample Student Response: "I added 345 and 416 to find the total number of whole gallons they used and got 761 gallons. They also used 7 eighths and 5 eighths of a gallon on Friday and Saturday. That would give me 12 eighths. Since there are 8 eighths in one whole, I can convert that to another whole gallon with 4 eighths left over. So on Friday and Saturday, they used a total of 762 and ½ gallons (since 4/8 = ½). To find out how much they need on Sunday, I added up from 762 ½. They could use ½ gallon to get to 763, 37 to get to 800 and 200 to get to 1,000. Therefore, they could use 237 ½ gallons on Sunday to get to 1,000 gallons for the weekend."

Sample Student Response: "I wrote 345 ⅞ + 416 ⅝. I gave ⅛ to 345 ⅞ from the ⅝ and rewrote my problem to be 346 + 416 ½ (since 4/8 = ½). I added the wholes from here and got 762 ½. I need to know how much more they can use so I subtracted 1,000 - 762 ½. I know that I can subtract the wholes and then just deduct another ½. Instead of borrowing across zeroes, I keep a constant difference by subtracting one from both numbers to get 999 - 761 ½. By subtracting, I get 238. I must subtract the last ½ to get 237 ½."

For more strategies, see the number talks above.

Sample Task: In a relay race, each member of the team runs 1 ¾ laps. If there are 4 members on each team, how many laps did they run?

Sample Task: For a party, Kari bought ¾ lbs of turkey, ¾ lbs of ham, ¼ lbs of fish, and 1 ¼ lbs of roast beef. What was the total weight of the meat that she bought?

Sample Task: To qualify for the state track meet in the long jump, Cameron must jump 23 ⅝ ft. So far, Cameron's best jump was 19 ⅞ ft. How much longer must he jump to qualify?

4.NSF.4 Apply and extend an understanding of multiplication by multiplying a whole number and a fraction (i.e., denominators 2, 3, 4, 5, 6, 8, 10, 12, 25, 100).

- ***a. Understand a fraction a/b as a multiple of 1/b.***
- ***b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.***
- ***c. Solve real world problems involving multiplication of a fraction by a whole number (i.e. use visual fraction models and equations to represent the problem).***

In this standard, not only do students need to multiply a fraction by a whole number, but also understand (and, by implication, explain) a fraction *a*/*b* as a multiple of 1/*b* and understand a multiple of *a*/*b* as a multiple of 1/*b* and ***use this understanding to multiply a fraction by a whole number.***

By stressing the understanding of multiples, the standard is rejecting the traditional method of "putting a one under the whole number and multiplying across." Instead, part ***a.*** and ***b.*** of the standard should be looked at more deeply.

To understand (part ***a.***) a fraction *a*/*b* as a multiple of 1/*b*, students must understand fractions as units. If numbers were to substituted for *a* and *b*, it could make the understanding the standard a little easier. Substitute 3 for *a* and 4 for *b*. Is ¾ a multiple of ¼? Would 3 units of ¼ be a multiple of 1 unit of ¼? Is 3 a multiple of 1? Of course. In fact, any positive whole number is a multiple of 1 as long as the units are the same. ⅝ is a multiple of ⅛, 11/12 is a multiple of 1/12, and ⅘ is a multiple of ⅕.

Part ***b.*** is a little more complicated. Once again, substitute 3 and 4 for *a* and *b*, respectively. "Understand a multiple of ¾ as a multiple of ¼". What are multiples of ¾ (or 3 units of ¼)? 6/4, 9/4, 12/4, 15/4, etc… would be multiples. The denominators (units) stay the same and the numerators (or number of units of ¼) increase by multiples of 3. Are all of those fractions multiples of ¼? Are all of the numerators multiples of 1? Of course they are.

How does this help when multiplying a fraction by a whole number, as the standard states? Here is an example to help illustrate using an understanding of multiples to multiply:

Sample Question: Francis opened a large bag of 80 Starbursts. ⅖ of the Starbursts were pink. How many Starbursts were pink? How many were not pink?

Sample Student Response: "If I think of ⅖ as 2 units of ⅕, I could find out what ⅕ of 80 would be, and multiply that by 2. ⅕ of 80 is the same as 80÷5, which is 16. 2 groups of 16 would be 32."

In this response, instead of multiplying 80 by 2 and then dividing by 5, the student divided first and then multiplied. The difference between the two strategies is that the student's strategy modeled the situation exactly. When we say ⅖ of 80, we are literally saying 2 groups of the ⅕ of 80. At no point did we have 160 starbursts as would've been the case if the ⅖ x 80/1 strategy was used.

Therefore, an understanding of the decomposition of non-unit fractions and the application of such is essential. For example, students must understand that 7/12 is 7 groups of 1/12.

Sample Task: For spirit week, all 96 of the 4th graders were encouraged to wear blue. If more than ⅝ of the 4th graders wore blue, they would get an extra 15 minutes of recess that day. If 82 students wore blue that day, would they get extra recess? How do you know?

Sample Student Response: "Since ⅝ is 5 groups of ⅛, I needed to find ⅛ of 96 or 96÷6, which is 16. 5 groups of 16 or 5 x 16 = 80. So, ⅝ of 96 is 80. Yes, they would get extra recess."

Unit 8: Decimals (Approx 15 days)
Standards Addressed: 4.NSF.5, 4.NSF.6, 4.NSF.7
Number Talks/Number Sense Routines: Continue number talks from Unit 7.
Standards Rationale: ***4.NSF.6 Write a fraction with a denominator of 10 or 100 using decimal notation, and read and write a decimal number as a fraction.*** ***4.NSF.5 Express a fraction with a denominator of 10 as an equivalent fraction with a denominator of 100 and use this technique to add two fractions with respective denominators of 10 and 100.*** ***4.NSF.7 Compare and order decimal numbers to hundredths, and justify using concrete and visual models.*** Conventional wisdom would suggest that teachers follow the order in which the standards are given. However, beginning with a focus on NSF.6 first will allow students to construct their own understanding of NSF.5.

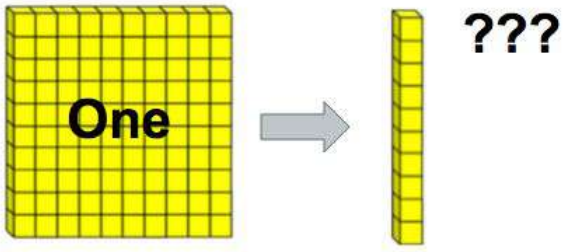
NSF.6 would likely be students' first interaction with any place value to the right of the ones place. However, if students are able to make the connection between integer (whole number) place values and decimal place values, conceptual understanding will come quickly. It is recommend to begin with an activity similar to this one:

Sample Activity: Take your base-ten blocks and build 400 (write 400 on the board). How would you split 400 into ten equal pieces? Represent 400 broken into ten equal pieces with your base-ten blocks. How many do you have? (Write 40 on the board next to 400). Now split that 40 into 10 equal pieces. How would you build that? How many do you have? (Write 4 on the board next to the other two). What do you notice? What do you wonder?

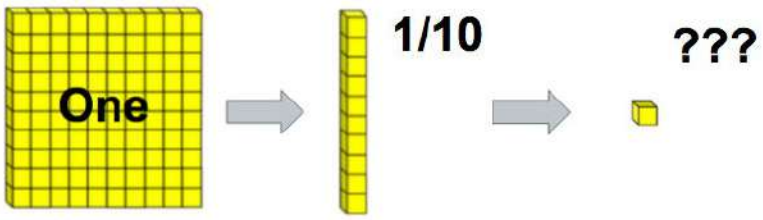
Sample Student Response: "They all still have 4s."

Teacher-follow up: "If they all still have 4s, how has their value changed? If they all have 4s and 0s, are they worth the same? Of course not. We split each of our flats into 10 pieces and took 1 of them to get 4 rods. Then we split our 4 rods into 10 pieces and took one of those pieces from each to get 4 small cubes. Now, how could I split these small cubes into 10 pieces? How much would I have?"

Students should not consider tenths and hundredths as separate from the integer place values, but as portions of them. In order to "zoom in" on the small cubes that represent ones, teachers can replace the small cubes with flats so they can illustrate breaking that apart as well.



What word describes the rod in the above picture? Well, the rod is now one of the pieces if the flat were broken up into ten, or as in the previous unit, 1/10. Similarly, if the rod were to be broken apart into ten equal pieces, that would result in a small cube. How could 1/10 be broken apart into ten pieces?



At this point, it would be important to let students form hypotheses as to what this small cube could be called. Students might reason that it is 1/10 of 1/10 or 1/10÷10, but others might recognize that one hundred of the small cubes make up a flat. Therefore, if a flat represents “one”, then a small cube represents “one hundredth” of the flat.

Students must have an active role in constructing their own understanding instead of copying notes from a flipchart or from a teacher lecture. Also, it is important that they make connections to prior learning in this unit and Unit 1 (place value).

From here, students could begin to interact with multiple tenths or hundredths and investigate the different ways they can be represented numerically. If they recall standard NSF.4, they can recognize that “34 hundredths” is the same as 34 x 1/00 or 34/100. The only new learning here is the way it is read and written in decimal form.

Reading a decimal can be tricky for students if they haven’t had deep and rich experiences with whole numbers. For example, the number 430 could be read as “four hundred thirty” OR “forty-three tens”. Similarly, “6,200” could be read as “six thousand, two hundred” or “sixty-two hundred(s)”. They only name the place values up to the zeros as long as there are no other non-zero digits following that last number.

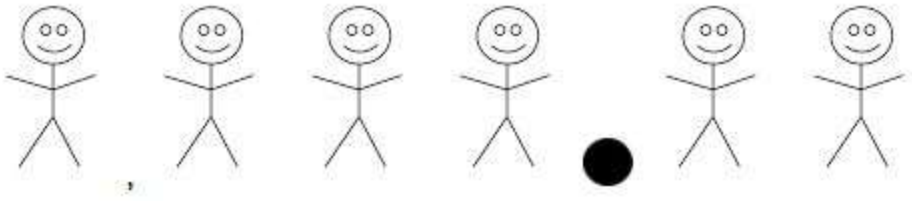
Decimals are read the same way. For example, 0.34 can be read as “34 hundredths” since the hundredths place contains the last non-zero digit in the numeral. Similarly, 0.70 could be read as “seven tenths” but it could also be read as “seventy hundredths”.

This idea leads nicely into the concept of equivalent decimals (through an understanding of equivalent fractions). Once students are proficiently reading and writing decimals within hundredths, try this activity:

Sample Activity: Have students take base ten blocks and make 60. Say: “Did you use rods? Why didn’t you use small cubes? (*Sample Student Response: “Because ten small cubes make a rod and 60 small cubes would make 6 rods.”)* Ok, with that in mind, make 60 hundredths (write 0.60 on the board).”

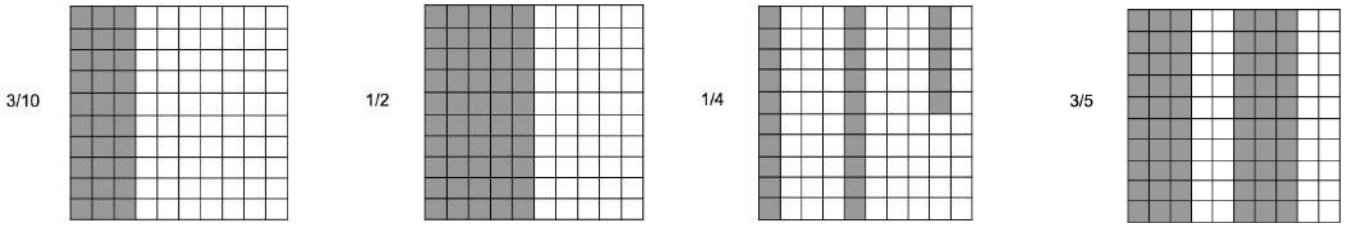
Some students may only rely on the small cube representation of hundredths and make a pile of 60 small cubes, but others will recognize that the relationship between hundredths and tenths is similar to the relationship between ones and tens (and any two adjacent digits).

Sample Activity: This activity is similar to the activity from Unit 1. Bring 6 students to the front and give each a basket or equivalent container (to hold their blocks). Line the students up this way:



Have each student identify their respective place values. Say: “What if I was to give my hundredths place 4 of these small cubes. What would our number be? (0.04). What if I gave our hundredths 7 more? *(Sample Student Response: “You can’t have 11 in one place value, so that student will have to give some to the tenths place.”)* How many would he/she have to give to the tenths place? *(“He/she would have to give ten of the cubes, but they would form to make a rod.”)* So now, what would my number be? I gave the hundreds place 11 hundredths...is that the same as one tenth and one hundredth? What if I gave the hundredths place 9 more? What would happen? I’ve given a total of 20 hundredths, but they formed two tenths. Is 2 tenths the same as 20 hundredths?” Continue this activity with however many numbers make sense for the flow of the lesson.

Sample Activity: Give students fractions and have them shade hundreds grids to represent those fractions. They may have trouble shading in fractions that are not counts of tenths, so begin with easier fractions and build up. For example:



Ask: “How many squares are in your grid? So each square represents how much? For each fraction, how many hundredths are shaded? What do you notice? What do you wonder? Which ones had full columns shaded (or could have full columns shaded)? What fraction would a full column represent? So how many tenths are in each fraction? What do you notice about the fractions that didn’t have full tenths shaded?” (We can’t make an equivalent fraction with a denominator of ten.)

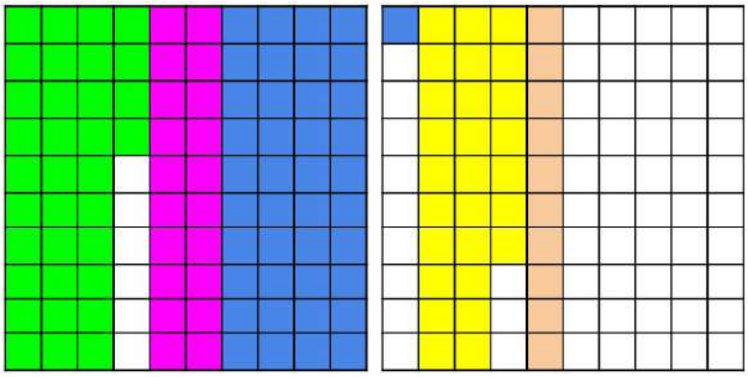
Sample Activity: Once students have had experience shading fractions on the hundredths grid, they can repeat the same activity with given decimal numbers. An extension of this activity would be comparing decimals, especially comparing tenths and hundredths (e.g. 7 tenths and 54 hundredths).

Sample Activity: Fill Two (from *Investigations*): Create decimal cards for students to draw from a deck. Example cards are shown below:

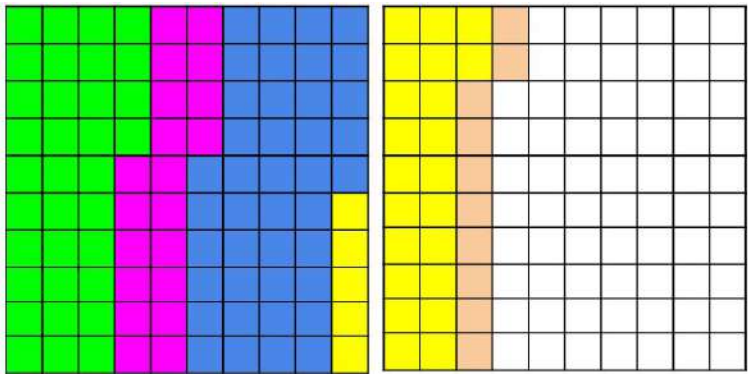
0.34	0.2	0.41	0.27	0.1

As a student draws a card, he/she colors in his/her hundredths grid. The goal is to fill up every square in the grid. Two examples are shown below:

Game Board 1



Game Board 2



Ask: “What is the difference between the two game boards? How is the pink one in the second grid two tenths? It looks like 6 hundredths + 1 tenth + 4 hundredths...how is that 2 tenths? Is it correct?”

Sample Activity: Students compare two fractions by converting them to tenths or hundredths. For example: Compare 4/10 and 37/100 and compare 4/10 and 6/25 by making them both decimals.

Sample Student Response for 4/10 and 6/25: “I made 4/10 equal to 0.4. I couldn’t make 6/25 into a decimal until I gave it a denominator of 10 or 100 (or 1). So I multiplied the numerator and denominator by 4 to get 24/100 or 0.24. 0.4 > 0.24.”

Once students have a solid foundation in decimal to fraction conversions (and vice-versa), then investigations with addition and subtraction of fractions with denominators of 10 and 100 (NSF.5) can be done. The connection between decimal and fraction representations should not be separated, but embraced within this process. Additionally, teachers can capitalize on students’ understanding of integer place value units.

Sample Task: “Add 3 tens and 45 ones. What would be the sum? Why isn’t it 48? 3 + 45 = 48 right? *(Sample Student Response: “No! 3 tens is the same as 30 ones and so it is 45 + 30 which is 75. You could also think of it as increasing the digit in the tens place by 3 to also get 75.”)*. If that’s the case, what do you think 3 tenths plus 45 hundredths would be? 48 hundredths? Why or why not?” Continue to represent these addition problems in decimal notation*, fraction notation, and in word form so that students can practice converting between units.

*Note: Even though students are not required to operate with decimals in 4th grade, they can use their understanding of place value concepts and properties of operations to add with decimals.

4.NSF.7 should be embedded within every activity in this unit. Any opportunity to ask “which one is larger and which is smaller, and why?” should be taken when appropriate.

Unit 9: Angles & Line Plots (Approx. 15 Days)
Standards Addressed: 4.MDA.4, 4.MDA.5, 4.MDA.6, 4.MDA.7
Number Talks/Number Sense Routines: Continue number talks from previous units.

Standards Rationale:

4.MDA.4 Create a line plot to display a data set (i.e., generated by measuring length to the nearest quarter-inch and eighth-inch) and interpret the line plot.

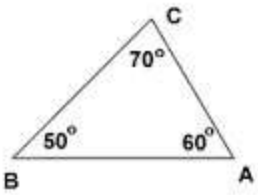
Students began operating with line plots in 2nd grade by measuring to the nearest whole number unit. In 3rd grade, they measured to the nearest whole, half, and quarter inch. Now in 4th grade, they add in eighth inch measurements and the interpretation of the line plot. Interpretation of the line plot does not necessarily mean that students must operate with the data (although certain questions could be appropriate, such as adding/subtracting with like denominators). Students must answer questions such as "which one is longer?" or "Which measure describes the length of the *most* pencils?"

4.MDA.5 Understand the relationship of an angle measurement to a circle.

4.MDA.6 Measure and draw angles in whole number degrees using a protractor.

4.MDA.7 Solve addition and subtraction problems to find unknown angles in real-world and mathematical problems.

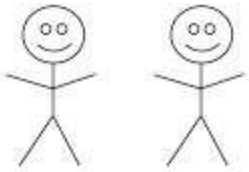
It is very important that students have a solid, conceptual understanding of angles. Some resources define an angle as "the union of two rays, *a* and *b*, with the same initial point P". *(Source: North Carolina 4th Grade Common Core Unpacking Document)* There are two problems with this definition: An angle is not necessarily the union of two *rays*. A ray is defined as a line with one distinct beginning point that continues to infinity. However, angles could be formed by line segments. For example:



All three angles are formed by line segments and not rays. In the picture below, the ground, ladder and wall each have finite endings and are not rays, yet they still form an angle.

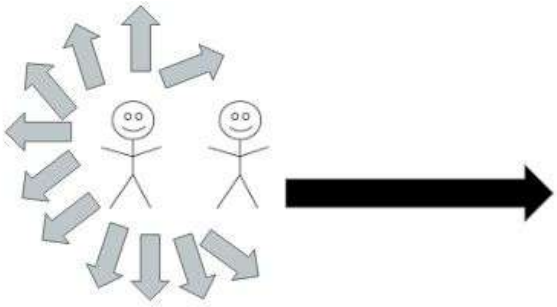
Secondly, the *union* between the two is <u>not</u> an angle, it is a point of intersection. An angle measure is determined by the relationship of the ***<u>directions</u>*** of the two rays or line segments. Take the following picture, for example:

The angle between the ladder and the ground is determined by the direction of the ground and the direction of the ladder. In the picture, the ladder is leaning against the wall in a direction that is different from the ground. The *<u>difference in their directions</u>* is the angle. If the ladder was standing straight up, the difference between the directions would increase. Take another context to help illustrate this point: Lily and Marshall had lunch in the park.



When they were finished with their lunch, Marshall needed to get back to work, so he walked in this direction:

Lily went in a different direction. Lily could literally go in any direction from Marshall:



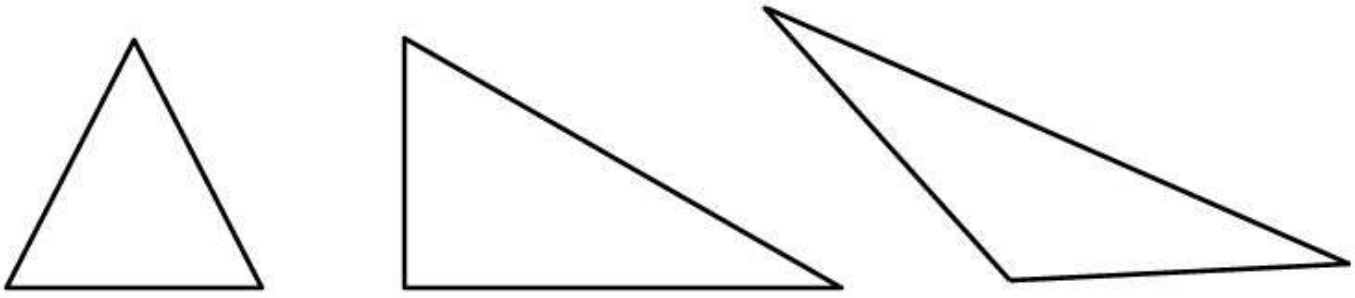
The difference in their directions is known as the angle. Notice that Lily could walk in *any* direction and all of the directions form a circle. That is what is meant when the standard says that angle measurements should be related to circles.

In future mathematics, students will use angle measures to determine lengths, but that isn't required until high school.

In 4th grade, students must have an accurate definition of an angle, be able to measure and draw it with a protractor, and solve addition and subtraction problems involving angle measurements. In order to measure with a protractor, students need multiple opportunities to use one. There is no magic bullet.

To add and subtract with angle measurements, students must understand angle measurements as additive. If angles are the union of two rays, as defined in the North Carolina document, unions are not additive. In fact, they aren't even measurable. If angles are known as the difference between the measurements two directions (as determined by that protractor they just used), then students can recognize the measurements as additive.

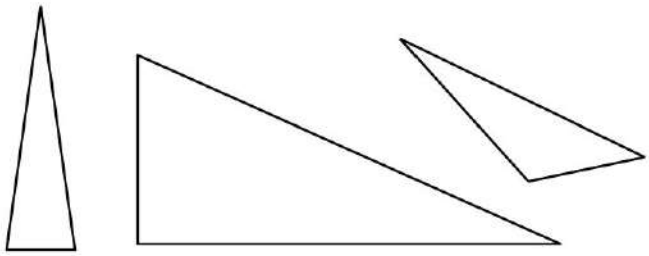
Sample Task: Use your protractor to determine all of the angles in the following triangles:



What do you notice about all of the angles in each of the triangles? (They add up to 180^0.)

**Note: It is not essential that students know that the angles in a triangle add up to 180. Similarly, they don't *have* to know vocabulary like complementary or supplementary. It wouldn't be wrong to teach those words, but students should not be assessed on their ability to apply those words and concepts.

Do you think that's true about all triangles? Without a protractor, estimate the angles in the following three triangles. Why did you choose those numbers? Do they add to be 180?



Sample Task: Watch the first archer in this clip from the 2012 Summer Olympics: https://www.youtube.com/watch?v=g8hMeN7Z_yo

Why do you think it takes them so long to determine their shot? What are some things that affect their shot? Could you do this well? Why is it an olympic sport if everyone could do it? What do you think would happen if his aim was just a little too high? By one degree?

Sample Student Response: "They have to get the perfect angle on their shot. If their angle is a off by one degree in any direction, the arrow might miss the target altogether. Other things that might change the shot are wind, the speed of the arrow, and gravity."

Sample Task: Harry is building a triangular frame. He measures one angle to be 64^0 and the other to be 72^0. Does he need to measure the third angle? What could he do instead?

5th Grade

Unit 1: Patterns, Tables, & Graphing (Approx. 15 days)

Standards Addressed: 5.ATO.3, 5.G.1, 5.G.2

Number Talks/Number Sense Routines: Start the year off by facilitating addition and subtraction number talks with sums and differences within 100. The focus here should be on understanding and operating within our base-ten place value system. Students should use mental strategies such as making ten, place value addition, or compensation.

Mental strategies for addition (e.g. 37 + 38) could include:

- Borrowing to make a ten (i.e. "compensation") → (37 + 38 → 40 + 35 = 75)
- Adding by place value → (30 + 7 + 30 + 8 → 30 + 30 + 7 + 8 → 60 + 15 = 75)
- Adding in chunks → (37 + 30 + 8 → 67 + 8 → 67 + 3 + 5 → 70 + 5 = 75)
- Friendly number → (37 + 38 → 37 + 3 + 38 → 40 + 38 = 78 → 78 – 3 = 75)

Mental strategies for subtraction (e.g. 61 – 28) could include:

- Subtracting in chunks → (61 – 20 – 8 → (61 – 20) – 8 → 41 – 8 = 33)
- Adding up → (28 + 2 = 30 + 30 = 60 + 1 = 61→ 2 + 30 + 1 = 33)
- Keeping a constant difference → (61 – 2 – 28 – 2 → 59 – 26 = 33 (by place value with no regrouping))
- "Negative" numbers → (60 – 20 = 40; 1 – 8 = -7 → 40 – 7 = 33)

***Note: These strategies shouldn't be directly "taught" but derived through exploration and student curiosity driven by strategic teacher questioning.

If this is too easy for students, feel free to expand to sums and differences within 1,000.

Standards Rationale:

For these standards, there is no prior knowledge or skills necessary other than patterns with basic addition and subtraction. These standards come first to allow time for review and remediation of the four operations within number talks and small groups.

5.G.1 Define a coordinate system.
a.The x and y axes are perpendicular number lines that intersect at 0 (the origin);
b. Any point on the coordinate plane can be represented by its coordinates;
c. The first number in an ordered pair is the x - coordinate and represents the horizontal distance from the origin;

d. The second number in an ordered pair is the y - coordinate and represents the vertical distance from the origin.

5.ATO.3 Investigate the relationship between two numerical patterns.
a. Generate two numerical patterns given two rules and organize in tables;
b. Translate the two numerical patterns into two sets of ordered pairs;
c. Graph the two sets of ordered pairs on the same coordinate plane;
d. Identify the relationship between the two numerical patterns.

5.G.2 Plot and interpret points in the first quadrant of the coordinate plane to represent real-world and mathematical situations.

Before diving into the rules and procedures of the coordinate system, it is important to illustrate the purpose of the Cartesian System (or coordinate plane). Dan Meyer famously says, "If _____ is the Aspirin, what is the headache?" In other words, if the Cartesian Plane is indeed a solution to a problem, what is the problem? Why do we need a Cartesian System in the first place? This can be answered through a complex task.

Sample Task: Netflix costs $8 per month. Create a picture that shows the total cost for 1, 2, 3, … 12 months of Netflix service. Plot that information on a number line.

Teacher Note: Plotting on a number line is insufficient for representing the information in the problem. Students must transfer the information to the Cartesian Plane to represent both variables (i.e. the number of months (x-axis) and the total cost (y-axis). Also, note that the task didn't say "table", but said create a picture. This creates an opportunity to discuss the best way to model and organize the information in the problem.

These standards are far more than "walking 4 steps east and 2 steps north". We use the Cartesian Plane to predict and model values in problems with two variables (or unknowns). Students should be given multiple problems in which there are two variables (informally) and a reason to model the information. Although making predictions is not required in the standard, students should be given the opportunity to reason about the graph and use it to make predictions.

Once students have modeled on the Cartesian Plane, we can start to teach some of the conventions of graphing and Cartesian Planes (e.g. the definition of the axes, format of coordinates). However, it is important that we allow students to construct as much as possible through strategic questions and discussions.

In standard 5.ATO.3, part ***d***, it says for students to "identify the relationship between the two numerical patterns". This means that students should recognize the effect that one pattern has on the other. Student responses should be "As this pattern increases by 1, the other increases by 3." or its equivalent. Students should *NOT* have to recognize the "rule" for the patterns (i.e. "We

multiply the x-coordinate by 3 and add 1."). This is algebraic reasoning that is reserved for finding the slopes and writing the equations in slope-intercept form 8th and 9th grades.

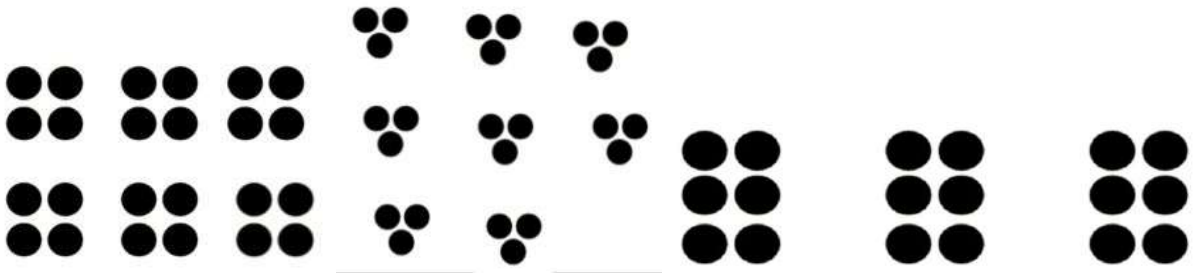
A video was made to provide more clarification on the standard. To access that video, click here.

Unit 2: Place Value (Approx. 10 days)

Standards Addressed: 5.NSBT.1, 5.NSBT.2 (a)

Number Talks/Number Sense Routines: Once students have begun demonstrating proficiency with addition and subtraction strategies, including mentally regrouping, they can begin multiplication and some division number talks:

- How do you see it?
  - Example: 8 x 7 *"I used 7 groups of 10 which is 70 and took away two groups to make 56."* or *"I did 5 groups of 8 which is 40. 2 more groups of 8 is 16 and so we get 56."*
  - (7 x 10) - (2 x 7) = 70 - 14 = 56
  - (5 x 8) + (2 x 8) = 40 + 16 = 56
- If, then
  - If 16 x 5 = 80, then 17 x 5 = ???
  - If 16 x 5 = 80, then 26 x 5 = ???
  - If 16 x 5 = 80, then 16 x 15 = ???
  - If 20 x 20 = 400, then 20 x 21 = ???
  - If 14 x 18 = 252, then 14 x 19 = ???
  - If 14 x 18 = 252, then 15 x 18 = ???
  - If 60 ÷ 5 = 12, then 120 ÷ 5 = ???
  - If 60 ÷ 5 = 12, then 60 ÷ 10 = ???
  - If 112 ÷ 16 = 7, then 112 ÷ 8 = ???
  - If 112 ÷ 16 = 7, then 128 ÷ 16 = ???
  - If 112 ÷ 16 = 7, then 96 ÷ 16 = ???
- Multiplication Dot Cards:



Standards Rationale:

5.NSBT.1 Understand that, in a multi-digit whole number, a digit in one place represents 10 times what the same digit represents in the place to its right, and represents 1/10 times what the same digit represents in the place to its left.

Students have experience with a similar standard in 4th grade. The only difference is that in 5th grade, students are expected to understand the relationships when moving to the right. The key to understanding this standard, and our number system for that matter, is understanding the concept of nesting. “Nesting” is the idea that 10 counts of one unit is nested inside another (i.e. ten ones are nested inside of a ten, 10 tens are nested inside of a hundred, 10 hundreds are nested inside of a thousand…).

Similarly, if we were to move the other direction down the number, if we were to break apart one place value into ten equal parts, one of those parts would be equal to the digit to the right. For example, if 6 hundreds were broken into 10 equal pieces, each piece would be made up of 6 tens. (600 x 1/10 = 60).

Sample Task: 5,624 x 10 = (5 thousands x 10) + (6 hundreds x 10) + (2 tens x 10) + (4 ones x 10). True or false? Explain. What do you notice about the product of both?

Sample Student Response: True, because the expression on the right represents the distributive property when multiplying by 5,624 written in expanded form. The product is 56,240, which is each digit shifted to the place to its immediate left. 5 thousands x 10 = 50 thousands. 6 hundreds x 10 = 6 thousands, etc…

Notice that the same result happens when you multiply by 1/10.

5.NSBT.2 Use whole number exponents to explain:
a. patterns in the number of zeroes of the product when multiplying a number by powers of 10;
b. patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.

This standard should be an expansion of what was discussed in standard 5.NSBT.1. Students should begin to understand that powers of 10 (100, 1,000, 10,000…) have the same effect, just more movement depending on how many “10” factors we have. For example 100 = 10 x 10 so each digit will now move two spaces instead of one. The same works for all powers of 10.

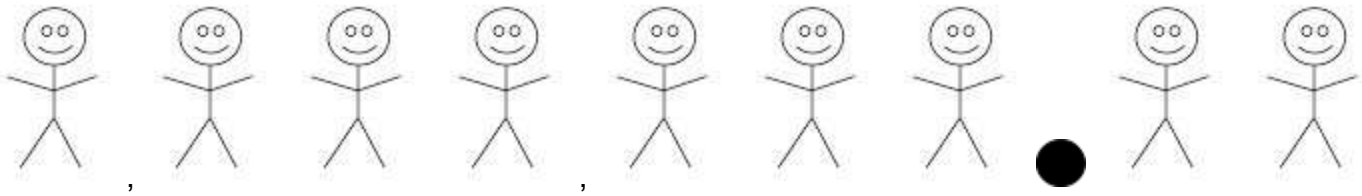
Students should investigate movement to the right and to the left by multiplying or dividing by powers of ten.

It is important to note that the decimal does not move, the digits move based on the power of ten being multiplied or divided.

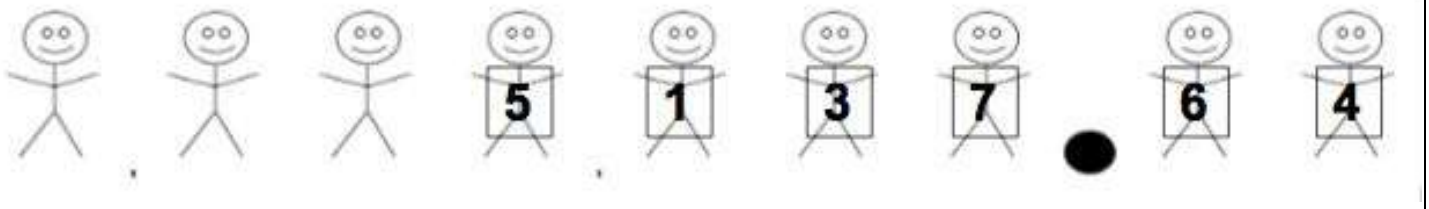
Sample Question: If we know 5,243 x 10 = 52,430 because we moved each digit one place to the left, how does that help us solve 5,243 x 100? 5,243 x 1,000?

Exponential notation should be introduced once students realize that writing 10 x 10 x 10 x 10 x 10 x 10 as supremely inefficient. We create the need to shorten the notation by using the exponent. It is important *not* to tell students that exponentiation is repeated multiplication. While whole number exponents can be represented by repeated multiplication, it is an incorrect definition for when we have non-whole number exponents.

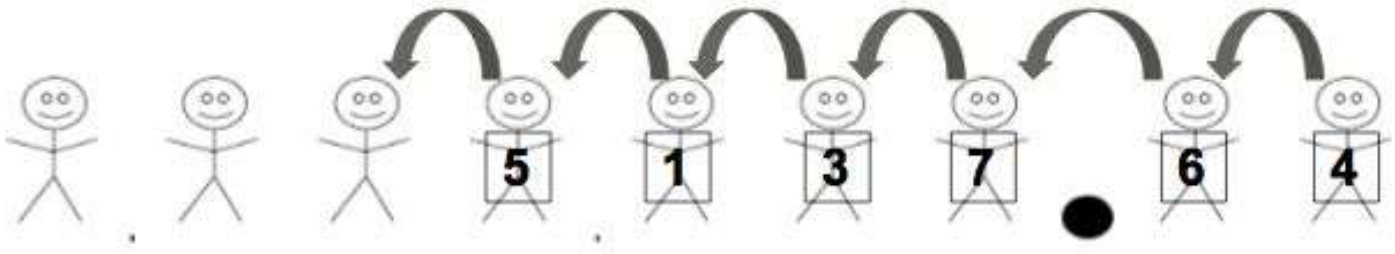
Sample Activity: Line 10 students at the front of the room. Each student represents a place value based on the placement of the decimal point.



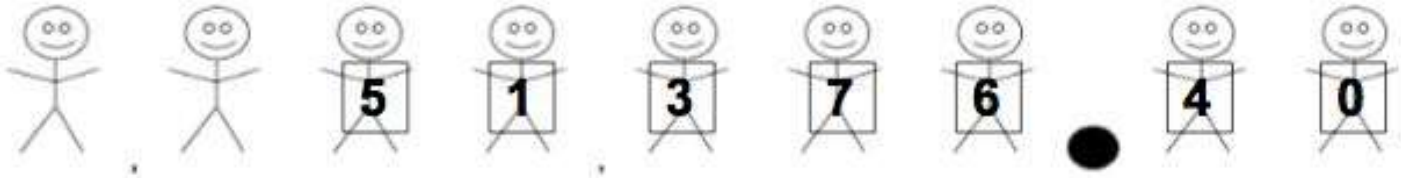
Give some students a large card with a number on it. Discuss what each place value means.



After discussing the place values, ask what happens if we multiplied that number by 10.



Each student can pass their number one space to the left, creating a new number. It is important to note that the decimal does not move, just the numbers in each place.



You can repeat this activity in both directions by multiplying and dividing by powers of 10.

Unit 3: Decimals (Approx. 20 days)

Standards Addressed: 5.NSBT.3, 5.NSBT.4, 5.NSBT.7

Number Talks/Number Sense Routines: Continue the multiplication and division number sense routines from Unit 2. If students have demonstrated proficiency in those number talks, move up to larger multiplication and division number talks:

Mental Strategies for Multiplication: Example: 35 x 12

Distributive Property: 35 x (10 + 2) → (35 x 10) + (35 x 2) → 350 + 70 = 420
(30 + 5) x 12 → (30 x 12) + (5 x 12) → 360 + 60 = 420
(30 + 5) x (10 + 2) → (30 x 10) + (30 x 2) + (5 x 10) + (5 x 2) →
300 + 60 + 50 + 10 = 420
Doubling & Halving: (35 x 2) + (12 x ½) → 70 x 6 = 420

Friendly Division:
3,600 ÷ 9 = ?; 3,600 ÷ 90 = ?; 3,600 ÷ 900 = ???; 36,000 ÷ 90 = ??; 36,000,000 ÷ 9,000 = ?

Division Estimation: Example: 4,678 ÷ 8; What number is too high? What is too low? Why?

***Answers will not be given in these number talks...only estimates.

Sample Response: “I know that it has to be less than 600 since 4,800 ÷ 8 = 600. It has to be more than 500 since 4,000 ÷ 8 = 500. Our dividend is closer to 4,800, so it’s probably somewhere around 575.

Standards Rationale:

5.NSBT.3 Read and write decimals in standard and expanded form. Compare two decimal numbers to the thousandths using the symbols >, =, or <.

In 4th grade, students were expected to read, write, and compare decimals to hundredths. In 5th grade, the expectations expand to the thousandths. The challenge here is the idea that “thousandths” is a smaller unit than “hundredths”, which is lunacy to an 11 year old that just spent 2 years solidifying their understanding of whole number place value. If they understand, however, that the word “thousandth” means that one is broken up into a thousand pieces, it would be easier to understand.

Sample Question: Which is larger: 0.65 or 0.065? Both have 65? Why is one larger than the other?

Sample Student Response: “Because the “6” in 0.65 is worth 10 times the value of the “6” in 0.065. There are 10 “6 hundredths” nested inside of “6 tenths”.

Sample Student Response: “0.65 is 65 hundredths, which is 1 whole broken into 100 pieces. 0.65 would be 65 of those pieces. 0.065 is 65 thousandths, which is 1 whole broken into 1000 pieces. 65 thousandths would be 65 of those pieces. Since the hundredths pieces are larger, 65 hundredths is larger than 65 thousandths.”

5.NSBT.4 Round decimals to any given place value within thousandths.

Rounding began in 3rd grade when students were expected to round whole numbers to the nearest ten or hundred. In 4th grade, students were expected to use place value understanding to round any whole number to any place value. In 5th grade, we add in the decimal component.

It is important that we do *not* use rhymes and raps to reinforce this skill, but conceptual understanding of number proximity. Students should reason about how close one number is from another, possibly calculate that difference, and use those differences to round.

Sample Question: Round 546.835 to the nearest tenth.

Sample Student Response: "The two tenth benchmarks that are closest to 546.835 would be 546.800 and 546.900. The number is .035 from 546.800 and 0.65 from 546.900. It is closer to 546.800 or 546.8.

Sample Question: Give two numbers that could round to 45.6. Why did you choose those two numbers?

5.NSBT.7 Add, subtract, multiply, and divide decimal numbers to hundredths using concrete area models and drawings.

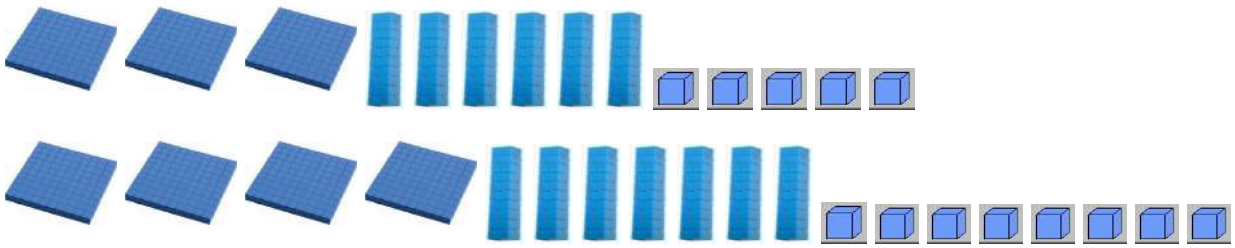
This is the first time students are expected to operate with decimals. It is important to note that students are ***not*** expected to apply a standard algorithm (that is a 6th grade standard). Instead, we are building conceptual understanding and relating operational strategies from whole numbers to decimal operations. We should use base-ten blocks, pictures and drawings, and strategies based on properties of operations and place value understanding. See below for examples of modeling with each operation.

Addition & Subtraction:

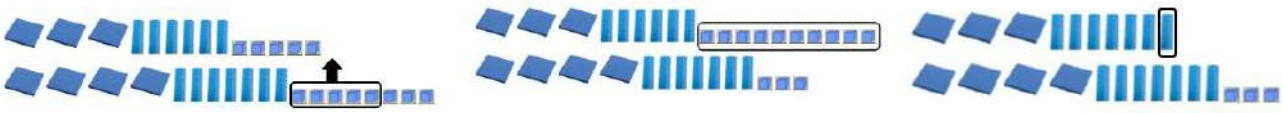
Sample Task: At the store, I bought a box of cereal for $3.65 and a gallon of milk for $4.78 (both tax inclusive). If I planned to pay with a $10 bill, do I have enough? If so, how much change will I get back?

Concrete models

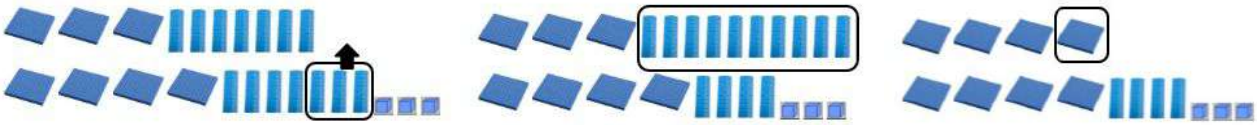
If you have money manipulatives, it is recommended that you use that first on problems like this. If you do not have money manipulatives, you can use base ten blocks.



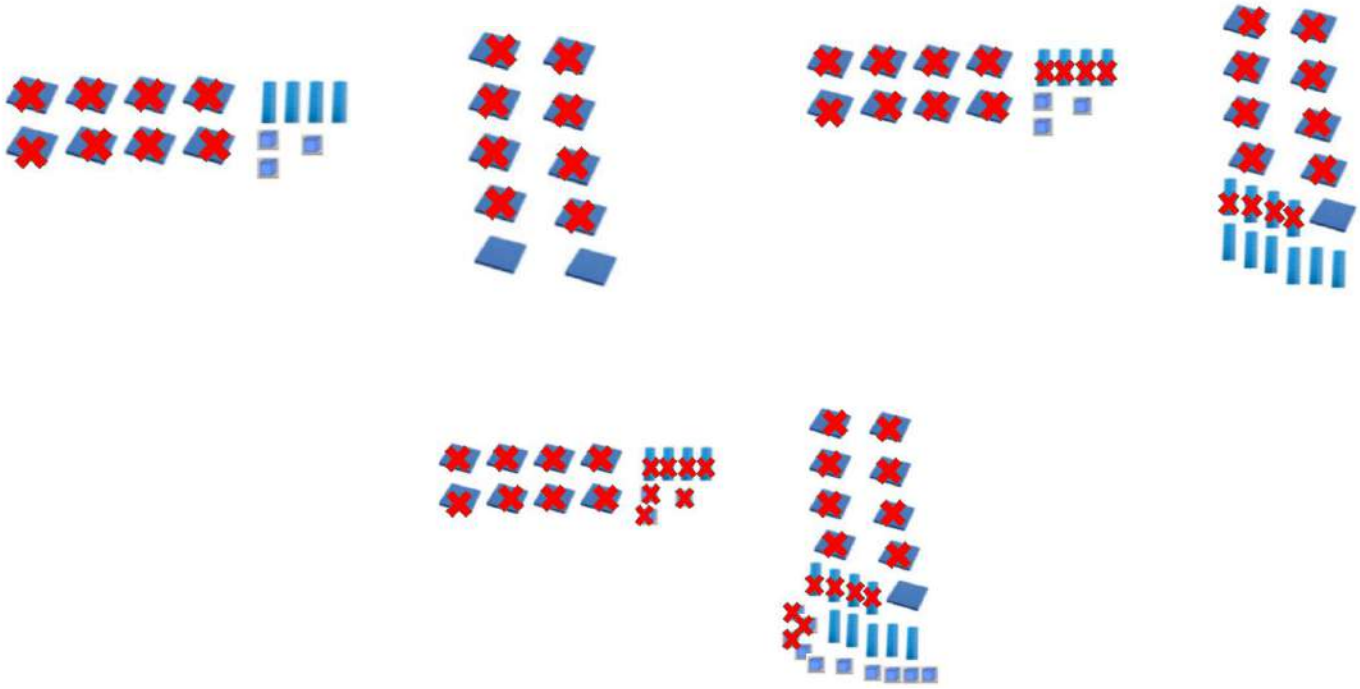
Students can model both amounts using base ten blocks with a flexible understanding of the assigned values of each (flats = 1, rods = 0.1, small cubes = 0.01). They can regroup ten of the hundredths (small cubes) into a tenth (rod).



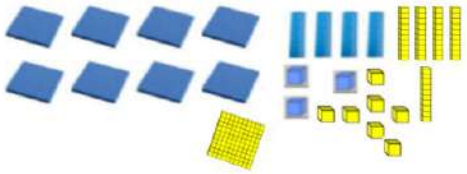
Next, they regroup ten of the rods into another flat.



Leaving us with 8 ones, 4 tenths, and 3 hundredths or 8.43. In order to figure out how much change will be made, students will need to know the difference between 8.43 and 10.00. This could be done using deduction or adding up to find the difference. Students could remove 8.43 from 10 flats by regrouping one flat into ten rods and one of the rods into 10 smaller cubes, then removing the necessary pieces.



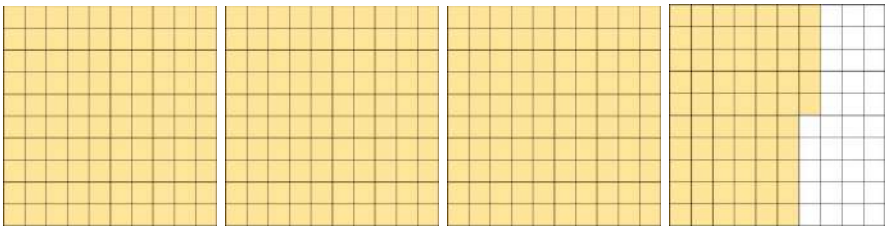
Another strategy would be to add the necessary cubes and rods to make 10 flats. For example, we need 7 more cubes to make another rod, 5 more rods to make another flat, and then another flat to have ten total flats. It is important to connect this concrete strategy to the abstract operations (see below).



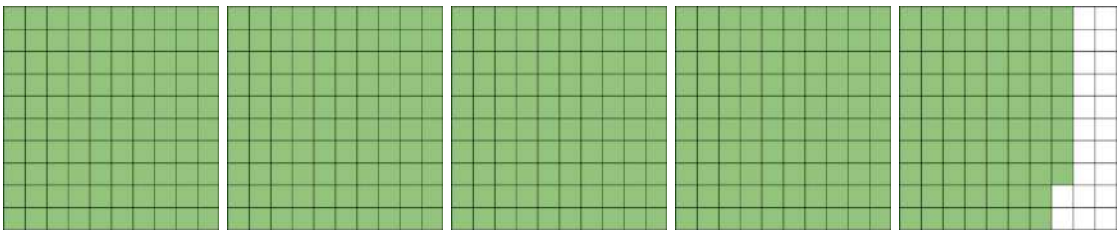
Representational

Students can use the same regrouping strategies as before, but this time using pictures or hundreds grids.

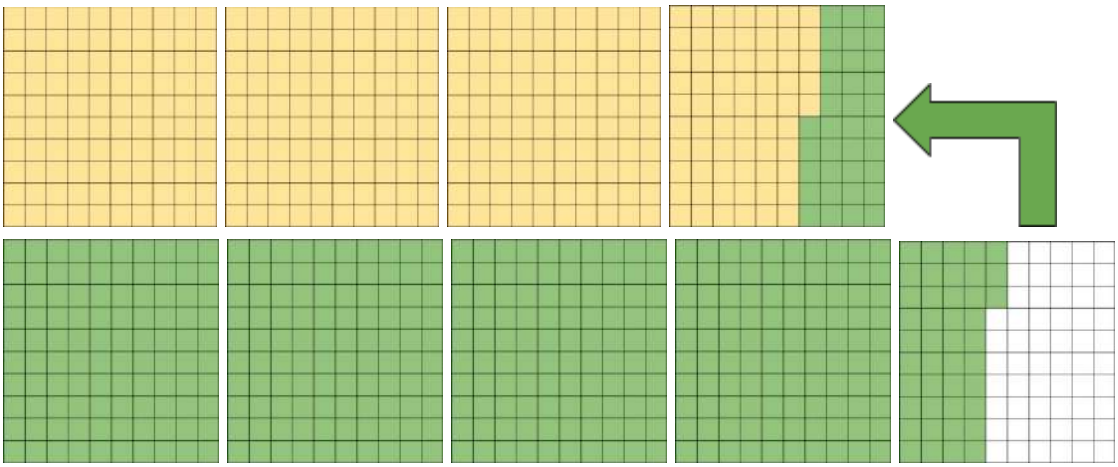
$3.65



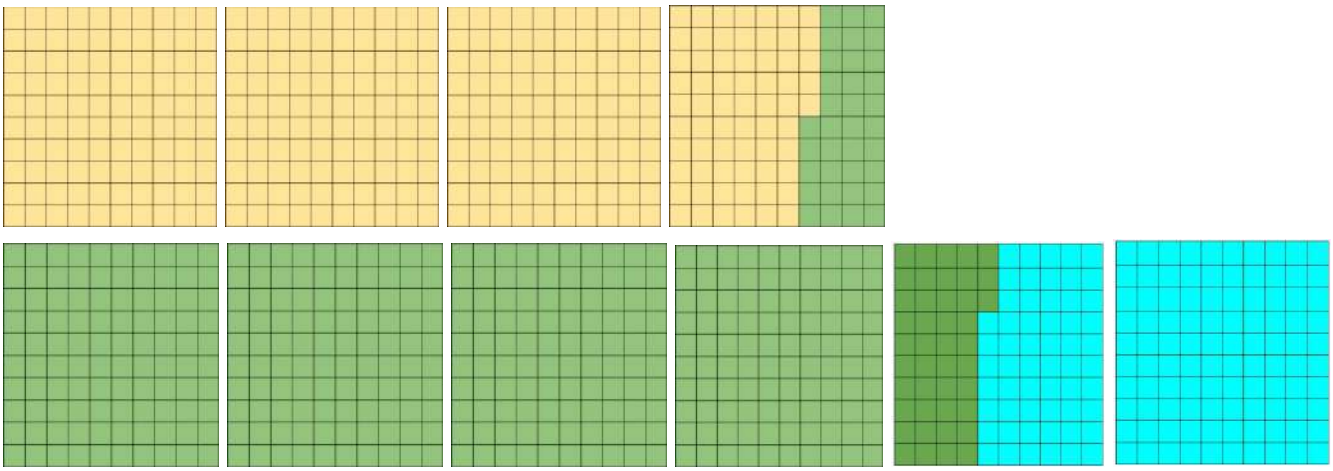
$4.78



Knowing that in order to make a full yellow flat, I need to take 35 green (hundredths) and give it to the $3.65 to make an even $4.



This compensation leaves us with 8 full wholes and 43 hundredths remaining, or a sum of $8.43. If we wanted to determine the difference between $8.43 and $10.00, we can add up to see how much more to get to 9 and another whole, helping us find our difference.



Abstract

Although not explicitly stated in these standards, students should use the properties of operations and understanding of place value to add and subtract decimals within hundredths. These strategies should resemble the same strategies that students applied in number talks with whole numbers.

Adding:
Compensation: 3.65 + 4.78 → 3.65 - 0.22 + 4.78 + 0.22 → 3.43 + 5 = 8.43
3.65 + 4.78 → 3.65 + 0.35 + 4.78 - 0.35 → 4 + 4.43 = 8.43

Adding up in chunks: 3.65 + 4 = 7.65 → 7.65 + 0.7 = 8.35 → 8.35 + 0.08 = 8.43
Adding by place value: 3 + 4 = 7
0.6 + 0.7 = 1.3
0.05 + 0.08 = .13
7 + 1.3 + .13 = 8.43
Friendly number: 3.65 + 4.78 → 4 + 4.78 = 8.78 → 8.78 - 0.35 = 8.43
3.65 + 4.78 → 3.65 + 5 = 8.65 → 8.65 - 0.22 = 8.43

Subtracting:
Adding up: 10.00 - 8.43 → 8.43 + 0.07 = 8.50 → 8.50 + 0.50 = 9.00 → 9.00 + 1 = 10. 1 + 0.5 + 0.07 = 1.57
Subtracting in chunks: 10.00 - 8.00 = 2.00 → 2.00 - 0.40 = 1.60 → 1.60 - 0.03 = 1.57
Keeping a constant difference: 10.00 (-0.01) - 8.43 (-0.01) = 9.99 - 8.42 = 1.57

***Note that the algorithm was not discussed as a strategy. Students will learn the algorithm in 6th grade.

Multiplication:

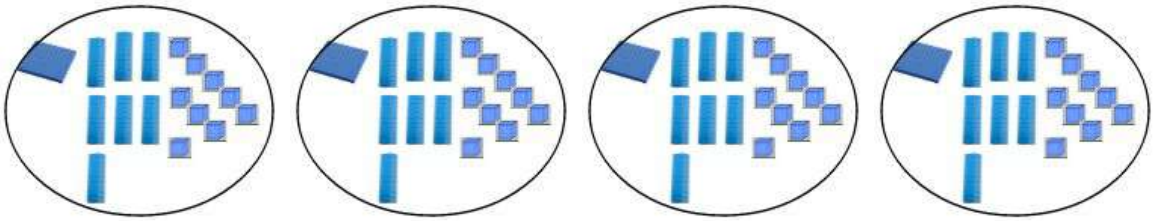
Since the standard specifies that students should work within hundredths, the types of multiplication expressions that can be used are limited. For example, to keep the product within hundredths, we should only multiply a whole number by a tenth or a hundredth (and vice-versa) or a tenth by a tenth. Again, students should model using concrete, representational, and abstract methods.

Sample Question: Ralph is buying hamburgers for each of his 4 children at a local fast-food place. Each burger is $1.79. How much will the 4 burgers cost (excluding tax)?

Concrete

In order to model concretely, students should use base ten blocks or equivalent base ten manipulatives. Students could use an area model or simply duplicate a representation of 1.79.

Duplicating:

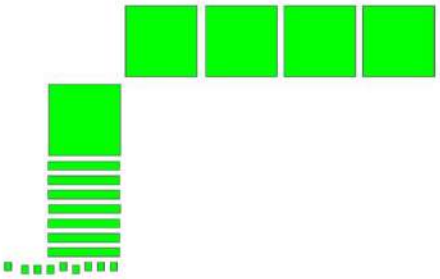


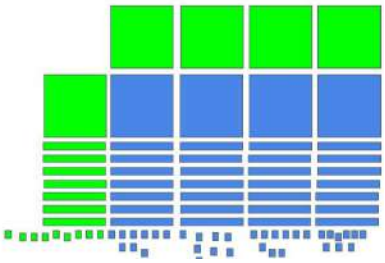
The duplication results in 4 flats, 28 rods, and 36 small cubes. Students can use their knowledge of the place value system (NSBT.1) to regroup ten rods into a flat and ten small cubes into a rod. 20 rods regroups to 2 flats with 8 rods left over, and 30 small cubes regroup to 3 rods with 6 small cubes left over. The 8 rods and 3 rods can regroup for another flat with one rod leftover.

Since, in this case, the flats represent my ones, the rods represent tenths, and the small cubes represent hundredths, I have 7.16.

Area Model

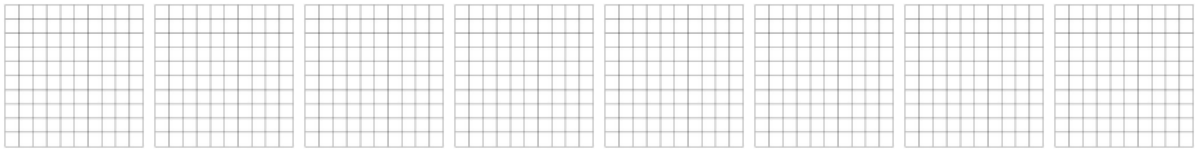
Students can also use an area model broken down by place value to solve. This is very similar to the duplicating strategy. The difference is that it is organized in a rectangle.



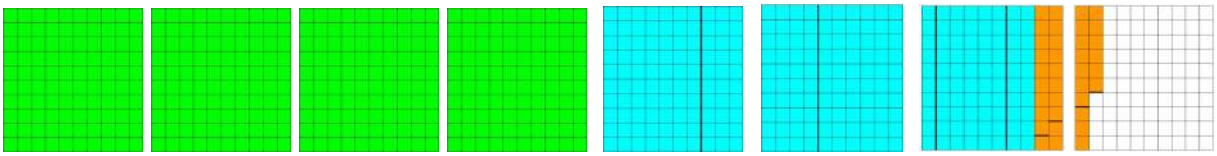


Representational

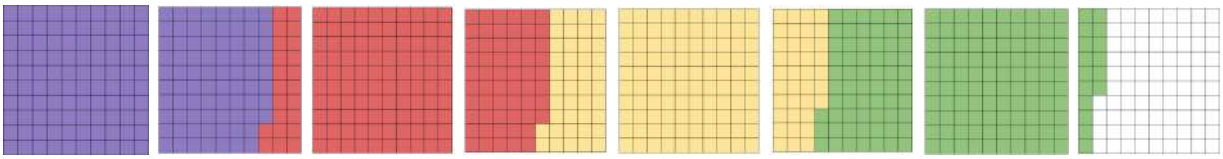
Students can use a blank 100 chart to color in 4 groups of 1.79. Students can reason that the answer must be less than 8.00, so they will need at most 8 blank hundreds charts.



Students should have the opportunity to use strategic planning and thinking to efficiently shade their representations. Some might group all of their ones, tenths, and hundredths and shade them separately. Others might skip count (shading in 1.79 at a time) to determine the product (or sum).



**This student colored the ones first (green), then each tenth (blue), then each hundredth (orange).*

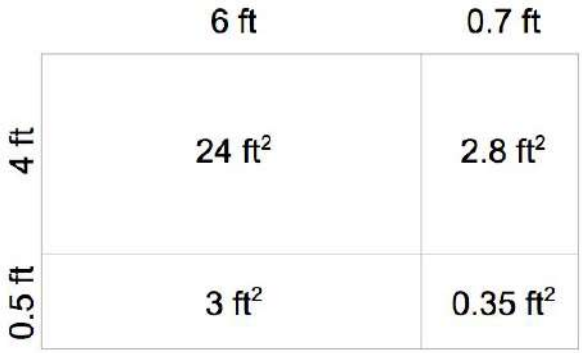


*This student skip counted up by 1.79 four times.

Students could use other representational methods to evaluate and they should be encouraged to investigate in ways that make sense to them.

Sample Task: Gretchen is planting flowers in a rectangular flowerbed outside of her house. She needs to buy some soil to spread over her entire flowerbed. The width of her flowerbed is 6.7 ft. and the length is 4.5 ft. Each bag of soil covers an area of 3 square feet. How many bags should she buy? (***NOTE: This example might be better as a 3-act task with information withheld so that students estimate and inquire about the missing information).

Students might draw an area model to represent this problem.



By adding 24 + 3 + 2.8 + 0.35, students will find the final area of 30.15 ft^2 which would lead students to reason that she would need 10 bags of soil (if she could make due with the extra 0.15 sq ft. left uncovered)

Abstract

Student should apply place value and operational strategies from number talks to help them multiply decimals. The examples we will use are from the previous garden soil problem. It's important ***NOT*** to directly teach students any of these strategies, but to ask questions that delve deeply into their thinking and let students use their own strategies and reason about those strategies. If students provide a strategy, it's ok to give a name to their strategy, but we do *not* want to directly teach students the strategies. Direct instruction does not provide them the opportunity to reason mathematically.

Multiplying by Place Value: 6 x 4 = 24
0.7 x 4 = 2.8
6 x 0.5 = 3
0.5 x 0.7 = 0.35
24 + 2.8 + 3 + 0.35 = 30.15
***Note: This strategy can be modeled concretely using an area model and base ten blocks and representationally by drawing the area model shown above.

Distributive Property: 6.7 x 4.5 = (7 - 0.3) x 4.5 → 7 x 4.5 - (0.3 x 4.5) = 31.5 - 1.35 = 30.15
***Note: Students could reason using a variety of strategies throughout this problem. It is important to draw out students' thinking about the multiple products in this strategy.

Doubling & Halving: (6.7 x ½) x (4.5 x 2) → 3.35 x 9 → 9 x 3 + 9 x 0.3 + 9 x 0.05 →
27 + 2.7 + 0.45 = 30.15

Division:

There are few contexts that require that we divide by a decimal. Most problems that require dividing by a number that isn't a whole number can be more realistically represented using fractions. One type of problem that would require students to divide by decimals would be if the students are given the area and the width and are required to find the length (or the perimeter!). We will not investigate those relationships here, but it is possible.

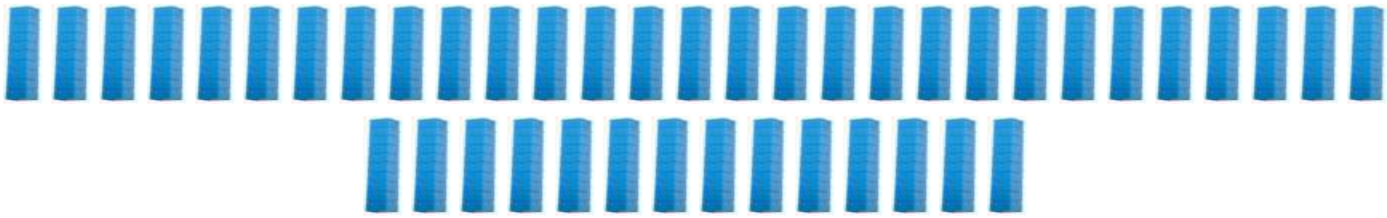
Sample Question: For a party, Selena bought 4.3 lbs of candy that she wants to split evenly between 5 bowls. How much should she put in each bowl? Would it be less than or more than 1 lb? How do you know? Find the exact weight that should be in each bowl to be an even split.

***Note: The numbers in the problem should be strategically crafted so that the quotients remain within hundredths.

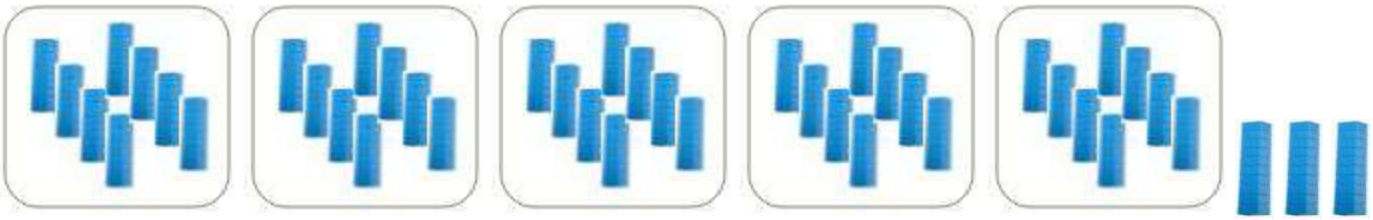
<u>Concrete</u>

Students could model again using base ten blocks and splitting them into 5 equal groups.

In order to split them into 5 equal groups, each flat would need to be decomposed into ten tenths, leaving us with 43 tenths.



From here, students can split those 43 tenths into 5 equal groups.

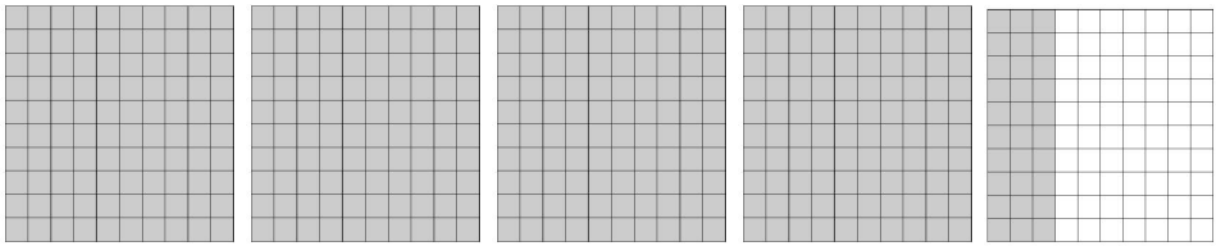


The remaining 3 tenths should be split into 30 hundredths (10 hundredths each) and divided into those 5 groups.

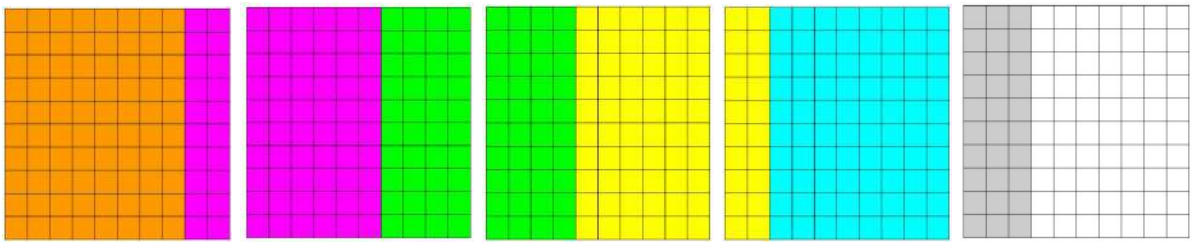
Students can use this concrete modeling to determine that there are 0.86 lbs in each bowl. Students could model concretely in other ways and should be encouraged to do so freely.

Representational

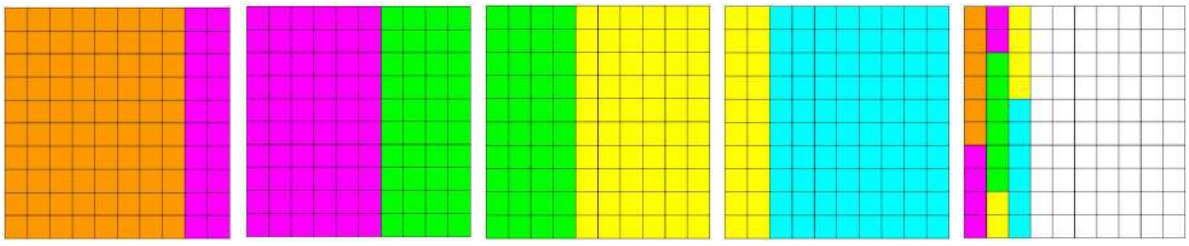
Students who want to model this problem representationally will likely model 4.3 and figure out how to split it into 5 equal groups.



Sample Student Response: "I can't split this up into 5 wholes, but I could probably split it into 5 groups of 0.8."



"Next, I need to split those 3 tenths or 30 hundredths into 5 equal groups. 5 equal groups of 30 hundredths would be 6 hundredths each."



"There are a total of 0.86 of each color, so the quotient would be 0.86 in each bowl."

Abstract

The abstract reasoning would be similar to the concrete and representational reasoning in the above examples. The only difference is that students rely on written numerals and manipulating those numerals instead of needing pictures or objects.

Sample Student Response: "I know that if I put 0.6 of a lb in each bowl, I would have distributed a total of 3 lbs. If I distributed another 0.2 lbs to each, that would be another pound, which would give me a total of 4 lbs and I know I have 0.3 lbs remaining. I know that 3 tenths is equivalent to 30 hundredths and 30 can be split into 5 equal groups of 6 hundredths each, giving me a total of 0.86 lbs distributed."

***Note: Students can reason abstractly using many different strategies. Although it is important that students *not* be taught the algorithmic approach to dividing decimals, if students can accurately relate an algorithmic approach with which they are familiar from their work with whole numbers, that would be appropriate. The key is student derivation and not teacher delivery.

Unit 4: Multiplication and Division (Approx. 30 days)

Standards Addressed: 5.NSBT.5, 5.NSBT.6

Number Talks/Number Sense Routines: Continue the same number talks from Unit 3.

You can sprinkle in some decimal estimation and logic into these number talks as well. Some of those number talks could include:

Count around the circle by certain tenths, hundredths, and thousandths and investigating the regrouping taking place as we count.

Example: (Counting by 0.4) 0.4, 0.8, 1.2, 1.6, 2.0, 2.4, 2.8 (What do you notice?)
Example: (Counting by 0.03) 0.03, 0.06, 0.09, 0.12, 0.15, 0.18, 0.21… (What do you notice?)
Example: (Counting by 0.007) 0.007, 0.014, 0.021, 0.028…(What do you notice?)

Estimation: 4.5 x 7.2 will likely land between which two whole numbers? Why?

Sample Student Response: 4 x 7 = 28 and 5 x 7 is 35. Halfway in between 28 and 35 is about 31.5 so 4.5 x 7 would be about 31.5. 4.5 x 7.2 would be a little more than that, so the product would fall somewhere between 31.5 and 32. (The answer is 31.9)

Estimation: 5.82 + 3.95 = ? Is the sum less than 10 or greater than 10? Why?

Sample Student Response: 5.82 is about 6 and 3.95 is about 4 and 6 + 4 = 10. Both numbers were less than that so our sum would be less than 10.

Estimation: 9.5 - 5.8 = ? Is the difference greater than 4 or less than 4? Why?

Sample Student Response: The difference between 5.5 and 9.5 is 4, but the distance between the two is a little smaller since our subtrahend is actually 5.8 so it will be less than 4.

Sample Student Response: I took away 6 from 9.5 to get 3.5. But I took away a little too much so I add back 0.2 more so it's about 3.7 which is less than 4.

Estimation: 6.9 x 5.83 = ? Is the product greater than or less than 42?

Sample Student Response: 6.9 is a little less than 7 and 5.83 is a little less than 6 and 7 x 6 = 42 so our product must be less than 42.

Estimation: 18 x 1.9 = ? Is the product greater than or less than 36?

Sample Student Response: 18 x 2 is 36 but 1.9 is less than 2 so the product must be less than 36.

Estimation: 18 ÷ 2.9 = ? Is the quotient greater than or less than 6?

Sample Student Response: 18 ÷ 3 is 6 but we are splitting 18 into groups smaller than 2.9 so we will have more groups than 6.

Sample Student Response: If I counted up by 3s (3, 6, 9, 12, 15, 18) and then counted up by 2.9 (2.9, 5.8, 8.7…). I realize that it will take more counts to get to 18. So the quotient must be more than 6.

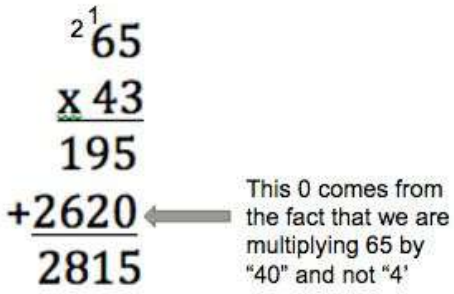
If/Then: If 54 x 21 = 1,134, what is 5.4 x 2.1? Explain.
If/Then: If 4,602 ÷ 78 = 59, what is 4.602 ÷ 0.78 = ? Explain.

Standards Rationale:

5.NSBT.5 Fluently multiply multi-digit whole numbers using strategies to include a standard algorithm.

An algorithm is defined as a series of steps used to carry out a computation that would work for all types of problems. Students have been working to derive algorithms in 3rd and 4th grade. Students should continue that work in 5th grade where they have opportunities to derive their own algorithms to apply to all numbers.

If students use the traditional US algorithm, they should be able to explain the meaning behind the zeros that are added after each place value.



Students should be provided engaging tasks that require them to multiply multi-digit numbers and *not* worksheets with decontextualized numbers.

Sample Task: The art teacher at Rock Hill Elementary School wants to do an activity that requires each student in his 5th grade class to make a design with colored beads. He wants to make sure that he has enough for all of his students. What information does he need to know? (After students figure out that they need to know how many students and how many per student) There are 24 students in his 5th grade class and each student will need 165 beads.

An algorithm could look like the one in the photo above, but other algorithms could include multiplying by place value, lattice, or expanded notation.

The lattice method is an algorithm that removes place value considerations from the distributive property (or the area model). Lattice is an appropriate algorithm for a 5th grader to use, but should only be used by students who can explain it in detail.

For a detailed look at the Lattice method, please watch the video below.
https://vimeo.com/165343472

Sample Game: Bullseye - Students are in pairs with a stack of number cards 1 - 9 and a bowl full of pre-determined 3, 4, or 5 digit numbers (e.g. 3,426, 846, 5,189, 9,943, 12,458, etc…). Student 1 draws a card from the bowl and student 2 draws 3 or 4 cards from the stack of number cards. Both students try to create multiplication expressions that are closest to the number drawn by student 1. For example, if student 1 draws 5,782 and student 2 draws 9, 7, 2, and 1. The students could come up with expressions such as 9 x 721 or 92 x 71. Assume student 1 comes up with 9 x 721 = 6,489 and student 2 comes up with 92 x 71 = 6,538. Student 1 was 707 away and student 2 was 756 away. The student with the fewest amount of points at the end of 5 rounds wins.

***Note: There are multiple answers in this game. Students are practicing estimation, division, addition, subtraction, and multiplication. There must be a debrief at the end of each game to draw out strategic estimation and game play.

5.NSBT.6 Divide up to a four-digit dividend by a two-digit divisor, using strategies based on place value, the properties of operations, and the relationship between multiplication and division.

In this standard, students are supposed to use "strategies based on place value, properties of operations, and the relationship between multiplication and division." The word "algorithm" is nowhere to be found. In fact, the expectation that students divide using a standard algorithm is in 6th grade. Therefore, we should avoid teaching students a step-by-step process using the traditional division symbol (⟌) and traditional mnemonic devices ("Does McDonalds Sell Cheeseburgers" or "Dad, Mom, Sister, Brother").

Instead, students should use estimation strategies and other strategies investigated in 4th grade to expand to double digit divisors.

Sample Task: Mr. Kingston wants to do an activity in his class where they use beads to create geometric designs. He bought a large bag of different colored beads and he wants to make sure that each student in his class gets the same amount of beads. What information does he need in order to figure out the number of beads each student will get? (After students seek out information) He has 24 students in his class and the bag he bought holds 5,623 beads. How many should each student get?

Sample Student Response: "First, I gave each "student" 100 beads, which means I gave out 2,400 beads and leaves me with 3,223 beads. I gave each student 100 more beads which leaves me with 823 beads left. Next, I gave each student 30 beads each which means I gave out 720 (30 x 24) leaving me with 103 beads. I can only give each student 4 more beads each (a total of 96 beads). I have 7 beads left over and I gave each student a total of 234 beads (100 + 100 + 30 + 4)."

Sample Student Response: I know that 24 x 200 is 4,800 and 24 x 30 is 480. 4,800 + 720 = 5,520. 24 x 4 = 96 and 5,520 + 96 = 5,616 which means we have 7 beads left over and our quotient is 234.

Sample Task: Mr. Garcia is excited about his upcoming science experiment. He filled 64 20-ounce bottles of water for the experiment. After re-reading the instructions, he realizes that he needed to use 16-ounce bottles for the experiment to work. He needs to take all the water that is in the 20-ounce bottles and put them into the 16 ounce bottles. How many 16-ounce bottles does he need? Will every bottle be filled to the top?

Sample Task: When I am driving on the highway with my cruise control set, I use one gallon every 33 miles I drive. I have a cross-country trip planned where I need to drive 3,795 miles. How many gallons of gas will I use? Gas is currently $2.29 per gallon. How much will I need to budget for gas?

Students may derive abstract strategies that closely resemble or may even be the standard US algorithm. The important thing to remember is that the students are owning the strategy, not the teacher.

Unit 5: Measurement Conversion (Approx. 10 days)

Standards Addressed: 5.NSF.3, 5.MDA.1

Number Talks/Number Sense Routines: Continue the same number talks from Unit 3, gradually increasing in difficulty.

If students are demonstrating proficiency with these number talks and are ready to move on, you could start with some fraction number talks to get students prepared for Unit 6. These number talks could consist of fraction comparisons or adding/subtracting with like denominators, requiring students to regroup.

Comparison with the same denominator: Compare ⅝ and ⅜

Sample Student Response: I know I can think of ⅝ as 5 units of ⅛ and ⅜ as 3 units of ⅛. 5 units of anything is always larger than 3 units of the same unit (note: this is true if we are dealing with positive numbers).

Comparison with the same numerator: Compare ⅜ and 3/12

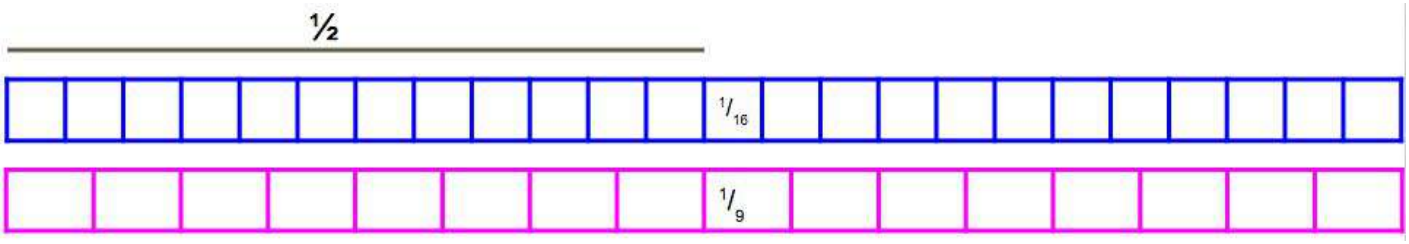
Sample Student Response: We have the same amounts of the units ⅛ and 1/12. ⅛ are larger than 1/12, so 3 units of ⅛ would be larger than 3 units of 1/12.

Comparison using a benchmark: Compare ⅜ and 7/12

Sample Student Response: I know that ⅜ is a little less than ½ since 4/8 is equivalent to ½. 7/12 is a little larger than ½ since ½ is equivalent to 6/12. So 7/12 is larger than ⅜.

Other comparison problems: Compare 9/16 and 13/24

Sample Student Response: I know that both numbers are one unit higher than ½ since ½ = 8/16 and ½ = 12/24. 9/16 is 1/16 more than ½ and 13/24 is 1/24 more than ½. Since 1/16 is more than 1/24, 9/16 is more than 13/24.



Compare: 18/19 and 24/25

Sample Student Response: Both numbers are one of their respective units away from 1. 18/19 is 1/19 away from 1 and 24/25 is 1/25 away from 1. Since 1/25 is a smaller distance away from 1, 24/25 is closer to 1 and is greater than 18/19.

Compare: ⅓ and $^{5}/_{16}$

Sample Student Response: "⅓ is equivalent to 5/15. 5/15 > 5/16 so ⅓ > 5/16"

Adding Fractions with Regrouping: Add: 5 ¾ + 7 ¾

Sample Student Response (1): I added 5 and 7 to get 12. Then I added ¾ and ¾ to get 1 ½. 12 + 1 ½ = 13 ½

Sample Student Response (2): I recognized that 5 ¾ is ¼ away from 6 so I took ¼ from 7 and ¾ to rename the expression as 6 + 7 ½ = 13 ½

Sample Student Response (3): I knew that both numbers were ¼ away from the next whole number or a total of 2/4 (or ½) so I added 6 + 8 = 14 and then subtracted ½ to get 13 ½.

Subtracting Fractions with Regrouping: 7 ⅛ - 3 ⅝

Sample Student Response (1): I added up from 3 ⅝ until I got to 7 ⅛. First I added ⅜ to get to 4, then I added 3 to get to 7 and then ⅛ more to get to 7 ⅛. ⅜ + 3 + ⅛ = 3 ½

Sample Student Response (2): I subtracted 3 from 7 ⅛ first to get 4 ⅛. Then I took away ⅝ by decomposing it into ⅛ and 4/8. I subtracted 4 ⅛ - ⅛ = 4. Then I subtracted 4 - 4/8 to get 3 4/8 or 3 ½. (7 ⅛ - 3 - ⅛ - 4/8 = 3 ½)

Sample Student Response (3): I changed the problem by making it 7 4/8 - 4 by adding ⅜ to both numbers. 7 4/8 - 4 = 3 4/8. *(Alternate)* I changed the problem by making it 6 ⅞ - 3 ⅜ by taking 2/8 away from both to make it so I don't have to regroup. 6 ⅞ - 3 ⅜ = 3 4/8.

Standards Rationale:

We begin our investigations of fractions with NSF.3 since it is a little easier than NSF 1 & 2 which will give us a chance to gradually ease into fractions. Students can apply their understanding of fractions as division to convert from a smaller to a larger unit.

5.NSF.3 Understand the relationship between fractions and division of whole numbers by interpreting a fraction as the numerator divided by the denominator (i.e., $\frac{a}{b}$ = ÷).

Students should operate within contexts that make use of remainders with division and some that do not.

Sample Task: I want to share 7 donuts among my two friends and me. Draw a picture showing how I would split the 7 donuts fairly. Write an equation to match the picture. I also have 7 coupons to the Ice Cream Shack to split between us also. Draw a picture showing how I would share the 7 coupons equally. How is your answer the same and how is it different for the two situations? Explain.

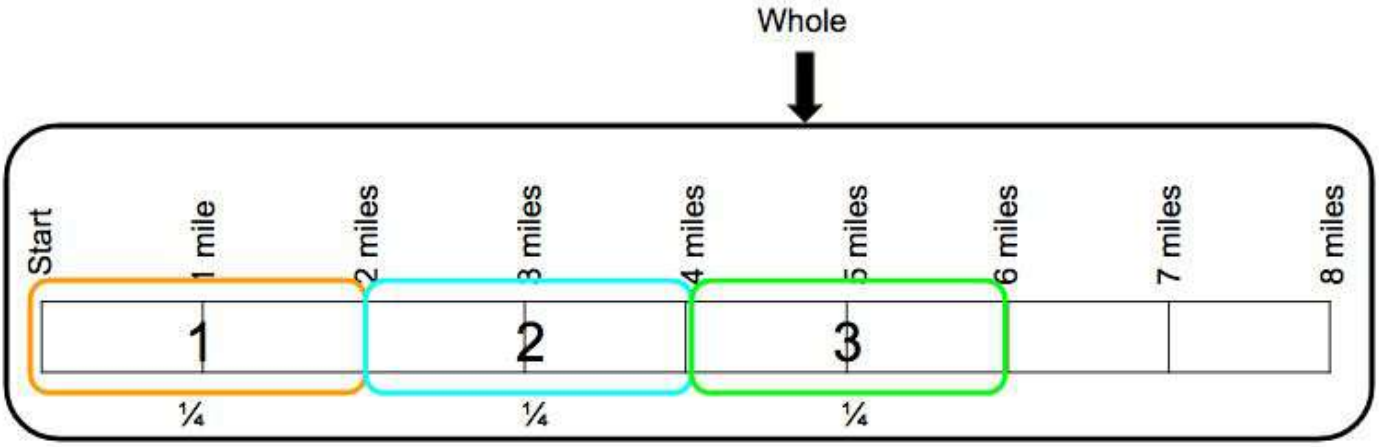
In one of the situations above, it makes sense to split the remainder among the three people. In the other situation (coupons), it would not be worth it to split up the remainder since splitting the

coupon would make the coupon worthless. Students should have experience considering and interpreting both situations.

A deeper look at the standard also brings us to a greater understanding of the multiple functions of fractions if separated from a context. Therefore, in order to accurately model a fraction, it must be accompanied by a context. For example, "¾" could be a measurement, a part of a whole, or a function of division. It could mean 3 wholes split into 4 pieces (division), 3 iterations of 1 unit split into 4 pieces (measurement), or 3 parts of a specific distance or amount split into 4 pieces (part/whole). **Note: ¾ could also represent a ratio, but that isn't expected until 6th grade.

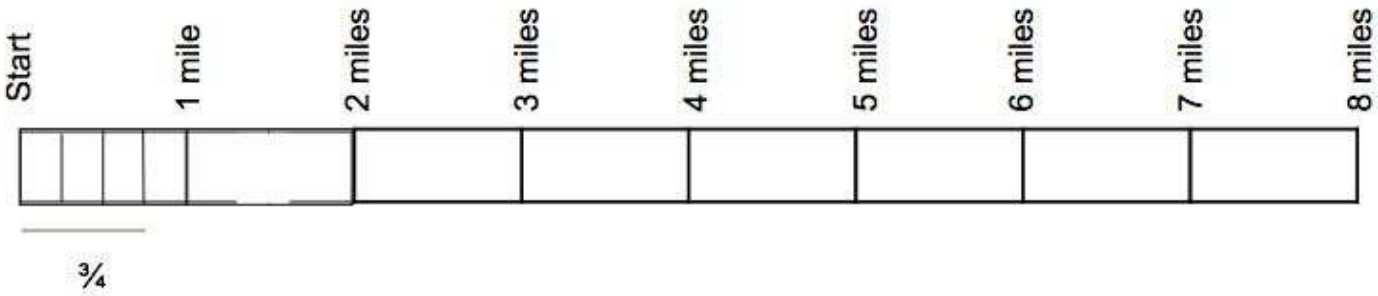
Fractions that are <u>part of a whole</u> represent a portion of an entire amount. The whole is defined as the entire collection of objects or total length, and a part/whole fraction represents part of its entirety. In the context below, the "whole" is the entire 8 mile race.

<u>Part of a whole context</u>: In a local 8 mile race, ¾ of the race takes place on Mission St.



Fractions that are based on <u>measurement</u> represent a length on a number line. ⅘ in a measurement context would be the distance from 0 to ⅘. 2 ⅞ on a number line would be the distance from 0 to 2 ⅞ on a number line. In the context below, the "whole" is still 8 miles, but we are concerned with the distance, or measurement.

<u>Measurement context</u>: In the same race, ¾ of a mile takes place on the Greenway.



Finally, fractions represented as <u>division</u> would be a quantity, represented by the numerator, divided into a certain number of parts, represented by the denominator. While there isn't a clear connection with the race context, we can still take an amount (the numerator, usually greater than 1) and split it into a specific number of parts (denominator).

<u>Division context</u>: At the end of the race, a family of 4 who ran the race got 3 free sandwiches for completing the race. How much of each sandwich does each family member receive?

In third and fourth grade, students primarily work with measurement and part/whole contexts. In 5th grade, they are introduced to division contexts with fractions, where the divisor is not a factor of the dividend, resulting in a remainder or a mixed number.

Sample Question: How many 6 gallon jugs can be filled from a tank holding 38 gallons of water? How much is in the jug that is not filled? What fraction of the jug is filled?

Sample Task: On the reality show Survivor, each "tribe" gets a 50 lb. sack of rice to share evenly. How much should each person get? (After students realize they need the total number of people) There are 8 on each tribe.

Sample Task: "Last year, I took my students on a field trips related to the projects we were working on. We scheduled four field trips in one day. Four students went to the Museum of Natural History, five went to the Museum of Modern Art, eight went with me to Ellis Island and the Statue of Liberty, and the five remaining students went to the Planetarium. The problem is that the school cafeteria staff made 17 submarine sandwiches for the kids for lunch. They gave three sandwiches to the four kids going to the Museum of Natural History, four subs to the five students going to Museum of Modern Art, 7 subs to the students going to Ellis Island, and 3 subs to the students going to the planetarium. Some students complained that they didn't get the same amount of sandwich as other students. Which group got more? How much of a sandwich did students from

each trip get? If we wanted to split them up before the trip, how much should each student get to make sure everyone got the same amount of sandwich?
(From Young Mathematicians at Work by Fosnot & Dolk)

5.MDA.1 Convert measurements within a single system of measurement: customary (i.e., in., ft., yd., oz., lb., sec., min., hr.) or metric (i.e., mm, cm, m, km, g, kg, mL, L) from a larger to a smaller unit and a smaller to a larger unit.

Students had a similar standard in 4th grade. One of the differences is that they were only required to convert from a larger to a smaller unit. Now, in 5th grade, they must do both: convert from a smaller to a larger unit AND a larger to a smaller unit. The other (small) difference is addition of millimeters to the metric requirements.

The abbreviation "i.e." is from a Latin phrase meaning "for example". When the standards use "i.e.", they use it to convey a complete list of required measurement units. In other words, the only measurement units to which they must convert are listed in the standard. That means that students ***do not*** have to convert between cups, (fluid) ounces, pints, quarts, or miles (with length). They can investigate those relationships, but they are *not* the baseline requirement for 5th grade.

Instead of listing and memorizing measurement conversions within a "conversion chart", students should investigate the "nesting" nature of measurement conversions. Just as 10 tenths are "nested" in a one, and 10 hundreds are "nested" in a thousand, metric measurement lengths are also "nested" inside of each other within a "base 10" system. Just as there are 1,000 ones in a thousand, there are 1,000 millimeters inside 1 meter. There are 100 cm nested inside of 1 meter. If we broke a meter into 100 equal pieces, each piece would be exactly 1 cm long. If we broke one of the centimeters into 10 pieces, each piece would be 1 mm long.

Similarly, customary measurement lengths are "nested" inside one another. The difference is that it isn't within a base-10 system. The systems vary depending on the standard measurement. For example, there are 60 seconds nested inside 1 minute. Similarly, there are 60 minutes nested inside of 1 hour. There are 12 inches "nested" inside of 1 ft and there are 3 ft nested inside of 1 yard.

Students should use their understanding of regrouping and nesting to solve problems involving conversions.

Sample Task: The marching band at Columbia High School wants to do a design in their halftime routine between the 30 yard lines on the football field (a total of 40 yards). The plan is to line up members of the band side by side from 30 yard line to 30 yard line. How many band members can fit in that space? What information do you need to know? Draw a picture to help. (Once students determine they need to know how much space each band member takes up, give the information). Each member takes up 5 feet.

Sample Response: I need to know how many feet are in 40 yards. If there are 3 feet in one yard, then there are 120 feet in 40 yards. 120 divided by 5 is 24, so 24 members can fit.

Sample Task: Ms. Dara is planning out her ELA block for Friday. She has 2 hours and 45 minutes set aside for ELA. She wants to interview every student to see what they are reading and how they are enjoying their books. If there are 26 students in her class, about how many *seconds* should she spend with each student?

Sample Task: Mr. Jordan is decorating his classroom. He wants to put some posters side by side to cover his entire wall. Each poster is 34 centimeters wide. If the length of the wall is exactly 9 meters wide, how many posters can he fit on his wall?

Unit 6: Adding and Subtracting Fractions (Approx. 20 days)

Standards Addressed: 5.NSF.1, 5.NSF.2

Number Talks/Number Sense Routines: Continue the number sense routines from Units 3 & 5.

Standards Rationale:

5.NSF.1 Add and subtract fractions with unlike denominators (including mixed numbers) using a variety of models, including an area model and number line.

5.NSF.2 Solve real-world problems involving addition and subtraction of fractions with unlike denominators.

The focus of adding and subtracting fractions should *not* be on unlike vs. common denominators, but instead should focus on adding and subtracting common "units". Similarly, the focus on creating common units should focus *less* on the least common multiple (LCM) and should focus more on equivalent fractions. The primary reason for this distinction is that students are familiar with common units and the futility of adding and subtracting unlike units. Likewise, they are familiar with equivalent fractions. Students have no experience, however, with least common multiples and limited experience with the idea of "common denominators".

For example, if you asked your students to add 5 tens and 6 ones, they would say very quickly that it is 56. Part of the reason is that they have learned that our place value system has digits that show the amount of tens and ones respectively. The other part of the reason they add unlike units so quickly is the implicit conversion of the unlike units into common units. Subconsciously (or

perhaps consciously) they converted 5 tens to 50 ones and added 50 ones and 6 ones for a total of 56 ones.

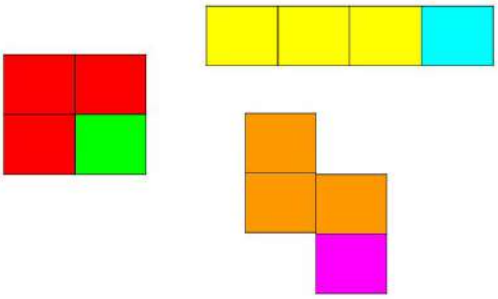
Another familiar example of common units would be operations within measurement. For example, Mr. Thomas' class is comparing heights. Anna and Brian are comparing their heights. Anna measured her height to be 46 inches and Brian measured his height to be 5 ft. 7 in. tall. How much taller is Brian than Anna? Students will recognize that they cannot simply operate with the provided numbers, but instead have to convert first. They may convert Anna's height to feet (and remaining inches) or they may convert Brian's height to inches, but the purpose is to create a common unit to subtract (or add in other contexts).

Before beginning adding and subtracting unlike fractional units, students should have some time to review and rethink their understanding of equivalent fractions. One way to facilitate a review of equivalent fractions would be to do the following activity:

Activity: (Materials needed: Multi-colored unifix cubes; enough for each student to have about 20)

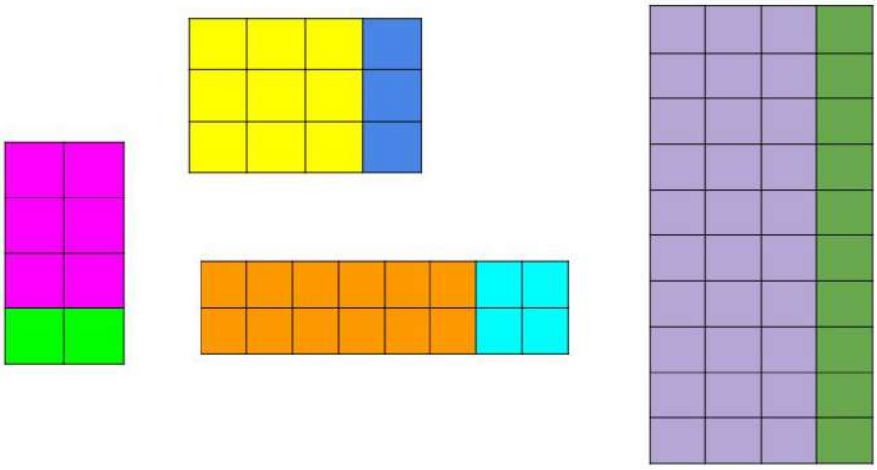
Teacher: "Use the unifix cubes to model the fraction ¾."

Possible answers:

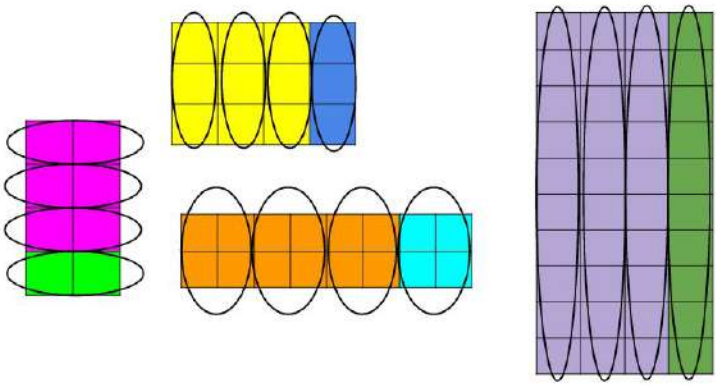


When (or if) students give you one of the above answers, ask them to extend their thinking using more cubes. Is it possible?

Possible answers:



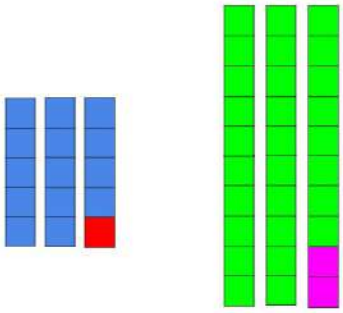
With the answers above, write out the fractions represented (6/8, 9/12, 12/16, 30/40) and ask how are these ¾? ***Where's the 3 and where's the 4?*** Draw out and record student thinking.



Any representation of ¾ can be split into four equal parts, with three of those parts highlighted. Work like this is extremely important for students to see that all fractions can be rewritten and represented in multiple ways. In other words, as we've seen in the picture above, 3 units of ¼ is equivalent to 6 units of ⅛, 9 units of 1/12, 12 units of 1/16, and 30 units of 1/40 (OR 3 fourths = 6 eighths = 9 twelfths = 12 sixteenths = 30 fortieths).

Now make 2 ⅘ with your unifix cubes. You can make it any way you want. (As students make one way, encourage them to make another way.)

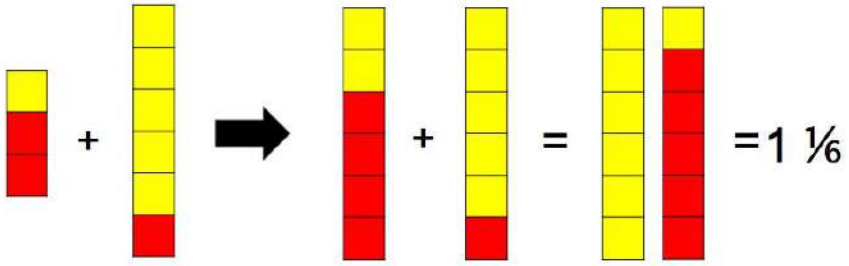
Possible answers:



Students *must* understand that the "full" wholes are the same length or size of the fractional pieces. Students are tempted to always make a "ten" a whole and need some redirection to think differently.

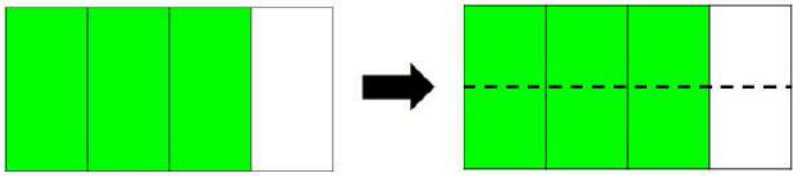
Part 2: Make ⅓ using any combination of cubes you want. (After students make ⅓). Now I want you to make ⅝ using any combination you want. (After students make ⅝). Now, use these models to tell me what ⅓ + ⅝ could be.

Possible Answer:



Sample Task: What do you think 3 eighths + 7 eighths would be? (After students say "10 eighths") What do you think 3 fourths + 7 eighths would be? How does your answer change? What do you need to do?

By this point in 5th grade, students *should* be familiar and at the abstract stage with equivalent fractions. However, we know that some students are not yet at the point where they can reason abstractly with equivalent fractions. Therefore, we should let students who are ready to reason abstractly do it and students who need representations or concrete models to see equivalency do that as well.



Since 3 fourths is equivalent to 6 eighths, we can now say 6 eighths + 7 eighths = 13 eighths.

Click here for more conceptual and contextual tasks as well as more fraction number talks.

Unit 7: Multiplying Fractions (Approx. 25 days)
Standards Addressed: 5.NSF.4, 5.NSF.5, 5.NSF.6
Number Talks/Number Sense Routines: Number talks from all previous units should be relooped within this unit. We can also let students begin to reason mentally with adding and subtracting fractions that are strategically crafted by the teacher. For example: Adding: 2 ½ + ⅞ **Sample Student Response:** I know that I only need ½ more to get to 3 and I know that 4/8 is equivalent to ½. Therefore, I can take 4 of the eighths and give it to 2 ½ to rewrite it as 3 + ⅜ or 3 ⅜. Subtraction: 6 ⅛ - 4 ¾ **Sample Student Response:** I added up from the subtrahend by adding ¼ to get to 5, adding 1 to get to 6 and then ⅛ to get to 6 ⅛. ¼ is equivalent to 2/8 so 2/8 + ⅛ + 1 = 1 ⅜. **Sample Student Response:** To avoid needing to regroup, I added ¼ to both the minuend and the subtrahend to rewrite the problem as 6 ⅜ - 5 (since ¼ = 2/8). 6 ⅜ - 5 = 1 ⅜. **Sample Student Response:** I went ahead and mentally regrouped 6 ⅛ to be 5 9/8 since 1 = 8/8. I also thought of 4 ¾ as its equivalent fraction 4 6/8. 5 9/8 - 4 6/8 = 1 ⅜.
Standards Rationale:

5.NSF.4 Extend the concept of multiplication to multiply a fraction or whole number by a fraction.
a. Recognize the relationship between multiplying fractions and finding the areas of rectangles with fractional side lengths;
b. Interpret multiplication of a fraction by a whole number and a whole number by a fraction and compute the product;
c. Interpret multiplication in which both factors are fractions less than one and compute the product.

To appropriately "extend the concept of multiplication to multiply fractions and whole numbers", one must have an accurate definition of multiplication. Though addition helps us transition to multiplication nicely, it is important to note that multiplication ***cannot*** be accurately defined as "repeated addition".

Multiplication is defined as a multiplicand being scaled by a multiplier. Take the following context: Mrs. Yang sells vases that hold 8 tulips. If she sold 6 vases, how many tulips did she sell? In this context, the 8 is the multiplicand (the number being replicated) and the 6 is the "multiplier". *This problem* can be modeled using repeated addition (e.g. 8 + 8 + 8 + 8 + 8 + 8 = 48).

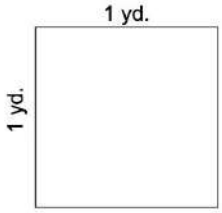
However, what happens when the multiplier is not a whole number? How can we model 8 x 5 ½ using repeated addition? It is true that we *could* model that expression as (8 + 8 + 8 + 8 + 8 + 4 = 44), but it's not *repeated addition* since the entire expression is not repeated. Therefore, multiplication is not repeated addition and should *not* be defined that way in an academic context.

The first part of the standard requires that students construct their understanding of fraction multiplication using an area model. Contexts can prove to be especially helpful to students trying form their understanding.

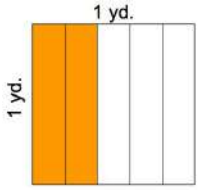
Sample Task: George and his family have built a new house in the country. To start his yard, he has decided to plant rectangular pieces of sod in his front yard. He knows someone that will sell him some sod and he needs to know exactly how many pieces of sod he will need. What information does George need to have before he can buy the sod. (After students inquire…) The sod is ⅝ yd. wide and ¾ yd. long. His yard is 42 square yards. How many pieces of sod should he buy?

**Note: This task would be a multi-day task and require extra time for discussion and collaboration.

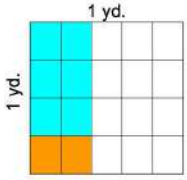
In the task above, students have to reason about a rectangle with ¾ yd. and ⅖ yd. length. When working with these fractions (or any fractions), it is important to ask ¾ and ⅖ of *what*? (1 yd.) Therefore, student should model the dimensions *inside* of a 1 yd. x 1 yd. square.



Within that square, the students should be able to partition the width into two fifths.



Next, they should partition <u>*that shaded section*</u> into three fourths.



The result is 6/20 of the entire square yard (or 3/10). Students should make the connection between the created array and multiplication. Also, students should be able to explain through the process of taking ⅖ of 1 first, and then taking ¾ <u>*of that*</u> . Splitting the ⅖ into 3 fourths created 3/10 of a yard. If fractions within an area model are seen as “part of a whole”, then multiplying fractions within an area model should be seen as “part of a part of a whole”.

Part ***b*** & ***c*** of the standard calls for students to interpret multiplication and compute the product of a whole number and a fraction, and a fraction by a fraction. Most students can calculate the product, but few can interpret the multiplicative action taking place.

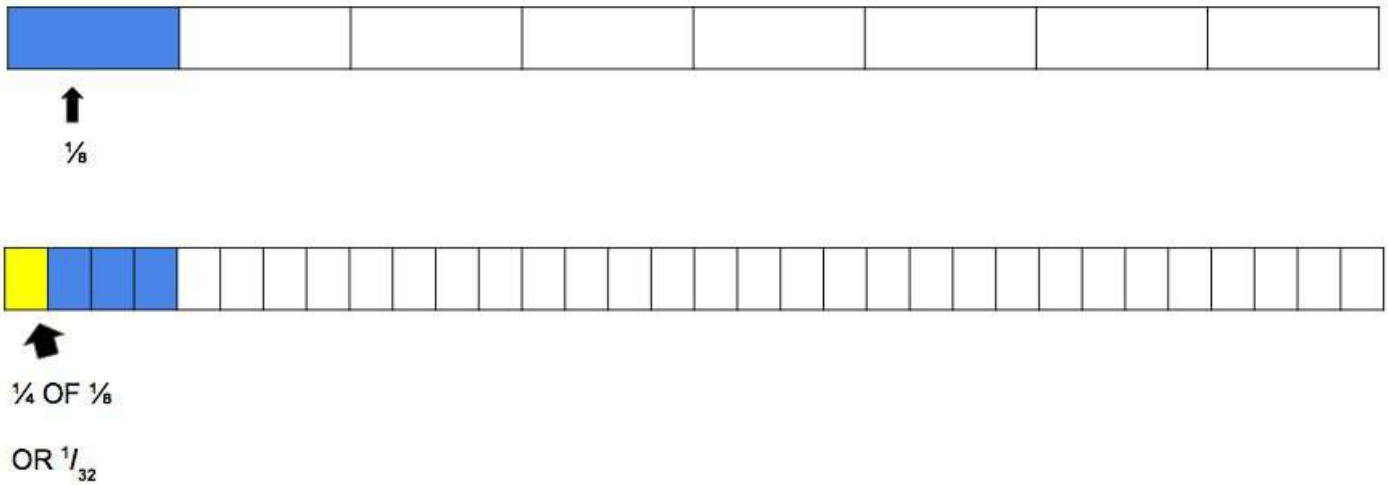
Sample Task: In Ms. Morris' class, ⅗ of the class are boys. Half of those boys have at least one sister. How many students *could* be in the class? How do you know? What fraction of the entire class are boys with sisters?

Sample Response: If ⅗ of the class are boys, then the class has to be a multiple of 5. Since you have to take ½ of the result, the class must be divisible by 5 and then divisible by 2. Therefore, the class could have 10, 20, 30, or 40 students. If there were 20 students, ⅗ would be three of five equal groups. Equal groups would have 4 in each and three of those groups would be 12 students (so ⅗ x 20 = 12). Half of those boys have a sister, so 6 are boys with a sister. 6 out of 20 are boys with sisters or the fraction 6/20 (or 3/10).

Follow-up: Is it $\frac{3}{10}$ no matter how many students (out of the possible totals) are in the class? Why?

Sample Response: It will be 3/10 no matter what since all of the results will be equivalent to 3/10. If there were 10, ½ of ⅗ of 10 is 3, which means 3/10 would be boys with sisters. It would be 9/30 for 30 students.

Before getting into the algorithm for multiplying fractions, students should primarily work with the multiplication of two unit fractions. This will make multiplying by non-unit fractions much more simple. For example, students should consider ¼ x ⅛ ("one fourth *of* one-eighth") and what that might look like with concrete objects (think: cutting paper) or drawings (area models).

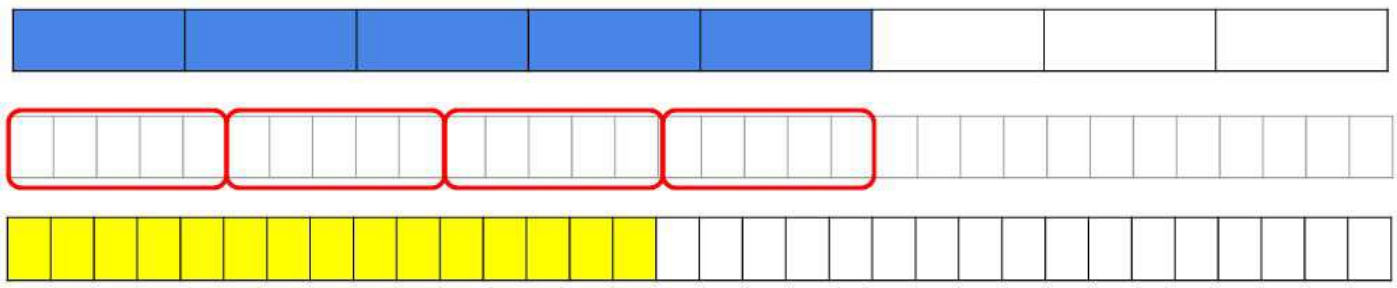


After multiple opportunities to model and think about multiplication of unit fractions, students will begin to generalize (at their own pace) about multiplication of unit fractions and will recognize that the product always ends up being the product of the two denominators.

The standard algorithm for multiplying fractions consists of multiplying the numerators and multiplying the denominators. The explanation, though, lies in an investigation of the units. If we were to use the example from above and write them in units:

3 fourths x 5 eighths

We would multiply 3 x 5 = 15 and find what a fourth of an eighth would be, which would be thirty-seconds (see picture above). Therefore, we have 15/32.This can also be modeled using a picture:



In the blue, we have a whole that is split into eight equal pieces, 5 of which are highlighted in blue. To take 3 fourths of that, we must split that group of 5 eighths into four equal pieces. The easiest way to do that would be to split each eighth into fourths, creating 32 equal pieces in the whole and 20 equal pieces in the blue highlighted region. Finally, we split those 20 equal pieces into fourths and count 3 of those groups, creating 15/32.

In the previous unit (of study), students learned that they cannot add nor subtact unlike units. Is that true for multiplication? Actually, you can *only* multiply unlike units. It would make no sense to multiply 5 oranges and 6 oranges, for example. (*Note: Even though it seems as if we multiply like units when finding the area, we are actually multiplying one of dimensions by a scalar determined by the other dimension.)

It is easy to see that we will have 3 units of *something*.

3 units of _____.

From here, students will be able to remember from the strategic investigations from earlier in the unit when we focused exclusively on the multiplication of unit fractions. If it has solidified to an abstract understanding, they will quickly recognize that one fifth of one half (or one half of one fifth) would be one tenth. The product is:

3 units of 1/10
Or
3/10

The focus of units can also be seen as an application of the associative property of multiplication (e.g. (a x b) x c = a x (b x c)):

(3 x ⅕) x (1 x ½)
(3 x 1) x (⅕ x ½)
3 x 1/10 = 3/10

Students *might* derive the algorithm through repeated interaction and a desire to compute more efficiently, but the *understanding* is essential to interpret the multiplication.

Another important check for understanding would be the ability for students to apply an expression to a context:

Sample Task: Create a context that would require you to multiply ⅜ x ⅝.

Sample Response: At a local restaurant, ⅝ of the items on the menu are less than $20. Of those items, ⅜ of them are less than $15. What fraction of the items on the menu are less than $15? Is it ⅜? Why or why not?

Sample Task: Matthews Mattress Store is running a special closeout sale. They advertise that you can take ¾ off the original price of a mattress on clearance. On some of the mattresses on clearance, there is a sticker that says take an additional ½ off the already reduced price! What fraction of the original price would those mattresses be? Would they be free? Would they pay you to take them? Why or why not?

5.NSF.5 Justify the reasonableness of a product when multiplying with fractions.
a. Estimate the size of the product based on the size of the two factors.
b. Explain why multiplying a given number by a number greater than 1 (e.g., improper fractions, mixed numbers, whole numbers) results in a product larger than the given number.
c. Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number.
d. Explain why multiplying the numerator and denominator by the same number has the same effect as multiplying the fraction by 1.

In previous number talks, students estimated and assessed the reasonableness of operations with decimals. Here, they should do the same with fractions. This could take place within the number talks context or in normal lessons where students have to justify their estimation using drawings or models.

In part ***a*** of the standard, students must estimate the product based on the size of the two factors. For example, they should reason that the product of 5 ⅛ and 7 ⅞ should be between 35 and 48 since 5 x 7 = 35 and 6 x 8 = 48.

This standard is a perfect opportunity to ask, "What number do you know is too high? What number is too low? Why?" It is extremely important to accompany your question with the additional question "Why?". If you don't dig into the justification of their estimations, they will be tempted to suggest extremely high or low numbers without any numerical rationality. Dan Meyer has asked, "Whats a really brave "too high" or "too low"?". In other words, what's a risky answer that is in the ballpark of the right answer?

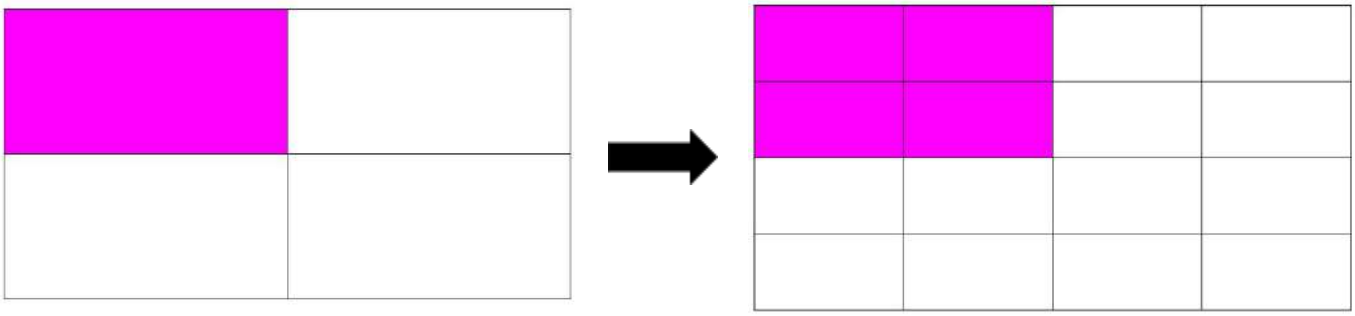
In part ***b*** of the standard, students must ***explain*** why multiplying by a number greater than one results in a product larger than the multiplicand. In part ***c***, students must ***explain*** why multiplying by a number less than one results in a product smaller than the multiplicand.

If students truly understand the operation of multiplication, explaining this concept will be simple. A multiplication expression contains a multiplicand (the number being scaled, or multiplied) and the multiplier (the degree to which the multiplicand is being multiplied). Multiplying by one results in one group of the multiplicand. Multiplying by anything greater than one results in that one multiplicand and more copies depending on the size of the multiplier. Multiplying by anything less than one doesn't quite reach the full product produced by multiplying by one.

Sample Task: Lacy's car holds 16 gallons of gas. Last week, she used ¾ of her tank. This week, after a long road trip, she calculated that she had used 1 ½ tanks of gas. How many gallons of gas did she use last week? This week? What do you notice about the results? Explain why she used less than 16 gallons last week and why she used more than 16 gallons this week.

Finally, part ***d*** of the standard calls for students to ***explain*** why multiplying the numerator and the denominator by the same number has the same effect as multiplying by one. In other words, why does multiplying the numerator and the denominator by the same number result in an equivalent fraction? This identity can be explained with pictures and with abstract numerical reasoning.

<u>Pictures</u>



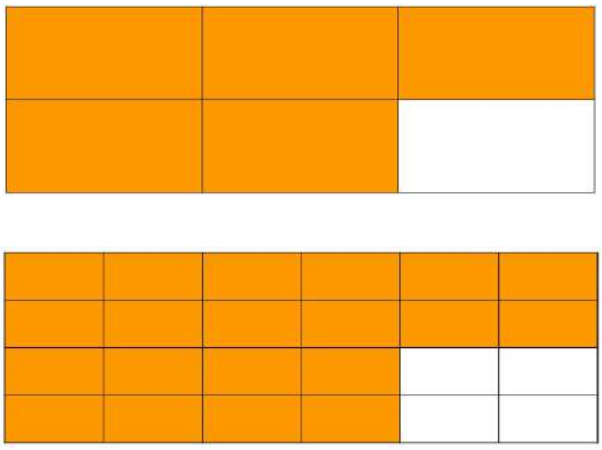
In the picture on the left, ¼ of the rectangle is shaded and in the picture on the right 4/16 is shaded. The same portion of the whole in both rectangles are shaded. Since one of the four pieces (in the picture on the right) were split into 4 equal pieces, in order to maintain equality, all of the four pieces must be split in the same way.

Abstract

$$\frac{1\times4}{4\times4}=\frac{1}{4}\times\frac{4}{4}=\frac{1}{4}\times1=\frac{1}{4}$$

This same identity can be proven abstractly in the above equation. Multiplying the numerator and the denominator by 4 creates a fraction of 4/4. In 3rd grade, students learned that 4/4 is equal to 1. Finally, multiplying any number by 1 results in that same number. Students should explain this identity using any kind of explanation that they derive through multiple opportunities to interact with similar problems and discuss these problems with their classmates.

If we were to look at the picture and equation together, we will notice that each fourth was split into fourths again (the 4 x 4 in the denominator) and we now have 4 times more shaded in the second picture (the 1 x 4). Let's look at this again using another picture and equation:



A whole partitioned into 6 equal pieces with 5 pieces shaded (⅚) has had each piece split into 4 equal pieces (multiplying by ¼ or just a 4 in the denominator). The result is now 20 of those 24 pieces shaded. Since 5 x 4 = 20, we can conclude that the number of shaded regions has now increased by a factor of 4 also:

$$\frac{20}{24}=\frac{5\times4}{6\times4}=\frac{5}{6}\times\frac{4}{4}=\frac{5}{6}\times1=\frac{5}{6}$$

5.NSF.6 Solve real-world problems involving multiplication of a fraction by a fraction, improper fraction and a mixed number.

This standard should be embedded in the previous two standards.

Unit 8: Dividing with Unit Fractions (Approx. 15 days)
Standards Addressed: 5.NSF.7, 5.NSF.8
Number Talks/Number Sense Routines: Continue the same number talks from all of the previous units (including addition, subtraction, multiplication, and division of whole numbers).
Standards Rationale: ***5.NSF.7 Extend the concept of division to divide unit fractions and whole numbers by using visual fraction models and equations.*** ***a.Interpret division of a unit fraction by a non-zero whole number and compute the quotient.*** ***b. Interpret division of a whole number by a unit fraction and compute the quotient.*** ***5.NSF.8 Solve real-world problems involving division of unit fractions and whole numbers, using visual fraction models and equations.*** Once again, students are required to not only compute the final answer (quotient) but interpret the division of a unit fraction by a whole number and a whole number by a unit fraction. It is important

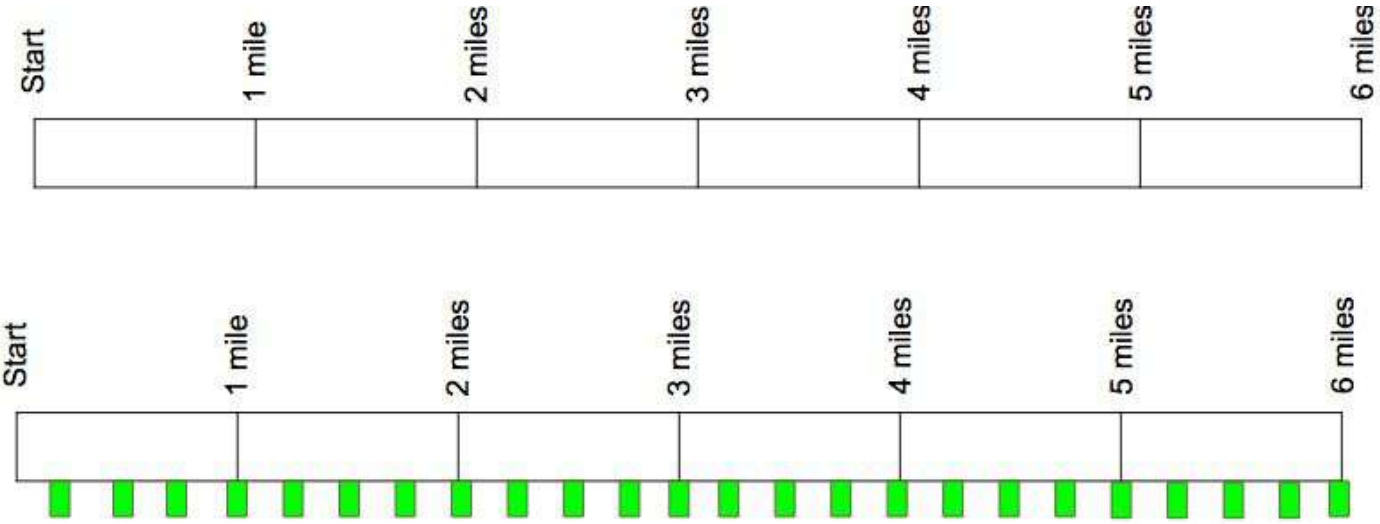
to note that the baseline expectation of the standard limits the operations to whole numbers and unit fractions.

It would be helpful to define a unit fraction before continuing. ⅜, for example, is defined as 3 units of ⅛, where the *unit* is ⅛. Therefore, unit fractions are defined as a fraction whose numerator is 1.

It is important ***not*** to teach the traditional algorithm for dividing fractions (i.e. “invert and multiply”, “keep, change, flip”, etc…). Instead, we should give students contexts in which they can make sense of division.

Sample Task: To better help stranded motorists, the department of transportation has decided to put mile marker signs every ¼ of a mile. How many mile marker signs will they need in a span of 6 miles?

Sample Response:



“By drawing the picture, I noticed that there were four markers in each mile, which makes sense since 4/4 = 1. Since there are 6 miles, we just need to multiply 6 x 4 to get 24.”

Sample Task: A recipe for homemade salad dressing calls for 2 cups of distilled vinegar. Unfortunately, I can only find my ¼ cup measuring cup. How can I ensure that I get exactly 2 cups of vinegar in my mixing bowl? (After students work). I found my ⅓ cup in the dishwasher. Would it be more efficient to use my ⅓ cup or ¼ cup? Explain your reasoning.

Sample Task: At the candy store at the mall, I bought ⅓ lb of jelly beans to share with my friends. How would I split ⅓ lb of jelly beans between 4 people (my friends and I)? Explain using pictures or words.

Sample Task: I'm planning my grocery list for the upcoming week. My dog eats ⅕ cup of dog food each day. Last time at the store, I bought a bag of dog food that holds 12 cups. How many days can I go before I have to buy more dog food? Explain your reasoning using pictures or words.

Sample Task: My 4 kids love chocolate milk. This morning, each of them asked for some. I have ½ of a gallon of milk left. If I give one child more milk than others, the other children will be upset. How can I ensure that each child gets the exact same amount of milk? How much will each child end up getting?

Sample Task: After Jacoby's birthday party, there was ¼ of the original cake left. He and 5 other friends wanted to split the rest of the cake the next day. How much of the leftover cake will each person get? How much of the original cake will each person get? Is there a difference between the two answers? Explain.

Sample Task: My cat eats ⅓ of a can of tuna each day. I have 4 full cans of tuna left. How many days can I feed my cat with the tuna I have left?

With multiple opportunities to interact within meaningful contexts, students might derive the algorithm or a simple "trick" to get them to the answer. The role of the teacher is to challenge these tricks to determine if they will always work or only work in certain situations.

For an explanation of how the algorithm works, click here.

For additional videos and explanations of this standard, click here.

Unit 9: Numerical Expressions (Approx. 15 days)
Standards Addressed: 5.ATO.1, 5.ATO.2
Number Talks/Number Sense Routines: Continue previous number talks.

Standards Rationale:

5.ATO.1 Evaluate numerical expressions involving grouping symbols (i.e., parentheses, brackets, braces).

5.ATO.2 Translate verbal phrases into numerical expressions and interpret numerical expressions as verbal phrases.

This standard is ***not*** intended to teach the students "order of operations". There is no need to even mention "Please Excuse My Dear Aunt Sally", "PEMDAS", or any other mnemonic device used to help students remember the order. The "order of operations" is explicitly left to 6th grade. Additionally, PEMDAS is inaccurate since it implies that multiplication/division and addition/subtraction is in a set order. Since they are inverses of the other, the order doesn't matter. Using PEMDAS confuses students since it implies there is a set order.

The order of operations have been informally taught in 3rd & 4th grades with regards to the four operations. Now, in 5th grade, we are introducing the idea of parentheses, brackets, and braces. The important output of this standard isn't the evaluation of expressions with the grouping symbols, but the understanding of the need of the grouping symbols.

Sample Task: When shopping at the grocery store for her family, Janice buys 5 apples and 7 oranges each week. How many pieces of fruit will she buy over the span of 4 weeks?

For the above task, students will likely add 5 and 7 and multiply by 4 to get 48 pieces of fruit. While the operation required isn't any more advanced than 3rd grade, the expression to represent the situation is more difficult. I know that if I wrote 5 + 7 x 4 on the board, I would multiply 7 and 4 first and then add 5, but that would be 33, which we know is not correct. We have to have some sort of notation that indicates that we need to go out of the traditional order because the context demands it. The solution is to put parentheses around the operation that we need to do first.

Therefore, students should not only be able to evaluate numerical expressions involving grouping symbols, but should also be able to create a context to match a given expression and write an expression from a context that requires grouping symbols.

Once students have understood the *purpose* of grouping symbols, they begin to translate from verbal phrases to expressions.

Sample Task: What is the difference between the following two phrases? How do the differences change the way you operate? What are the expressions for the two phrases?

Phrase 1: Add 5 and 6 and then multiply by 14.
Phrase 2: Multiply 14 and 6 and then add 5.

In addition to translating verbal phrases, they must also translate an expression into a verbal phrase. In other words, they need to accurately read an expression, emphasizing the order in the description. For example, for the expression 64 ÷ (5 + 3), students should make sure they acknowledge that you must add 5 and 3 before you divide. Some sample responses could include:

- "64 divided by the sum of 5 and 3."
- "Add 5 and 3 and divide that into 64."
- "Add 5 and 3 and divide 64 by that sum."

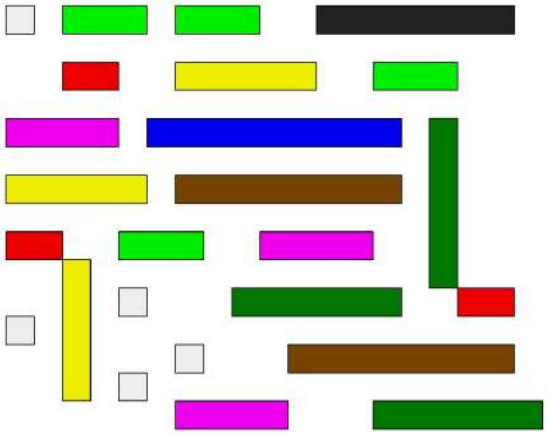
It is important to note that the standard requires students to translate numerical expressions and *not* expressions with variables. There is no need for student to operate with variables yet, as that is a 6th grade expectation. However, it wouldn't be developmentally inappropriate to see what students say in response to variables.

For more information on these standards, click here.

Unit 10: 3D Shapes & Line Plots (Approx. 15 days)
Standards Addressed: 5.MDA.2, 5.MDA.3, 5.MDA.4, 5.G.3, 5.G.4
Number Talks/Number Sense Routines: Continue number talks from previous units.
Standards Rationale: ***5.MDA.2 Create a line plot consisting of unit fractions and use operations on fractions to solve problems related to the line plot.*** Students have interacted with line plots since 3rd grade. However, this is the first time they are expected to operate with the results and the data. There are two expectations in this standard: create and solve problems. Students should be given data to plot on a line plot and then answer questions such as:

- What is the total of all the measurements?
- What is the difference between the totals of the longest and shortest measurements?
- This study was replicated 5 more times. What would be the total lengths after the new studies?
- If we were to take all of the liquid measurements in the line plot and spread them among 8 cups, how much would be in each cup?

Sample Activity: Take cuisenaire rods and put them in a plastic bag and ask students to make 5 rows of equal length using the rods. Have students plot their "lengths" on a line plot and make connections between their answers. See the picture below of a sample collection.



Students will realize that each row will need a combination of different lengths. A helpful strategy would be to collect an inventory of which cuisenaire lengths we have by using a line plot.

For more information and examples for this standard, click here.

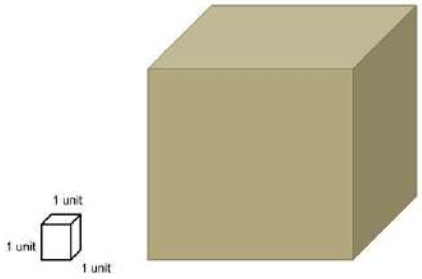
5.MDA.3 Understand the concept of volume measurement.
a. Recognize volume as an attribute of right rectangular prisms;
b. Relate volume measurement to the operations of multiplication and addition by packing right rectangular prisms and then counting the layers of standard unit cubes;
c. Determine the volume of right rectangular prisms using the formula derived from packing right rectangular prisms and counting the layers of standard unit cubes.

5.MDA.4 Differentiate among perimeter, area and volume and identify which application is appropriate for a given situation.

In 3rd grade, students learned about the area of rectangles by packing them with unit squares. Since rectangles are 2-dimensional, to find the total space inside the shape, we must pack them with 2-dimensional unit squares. In the same way, since a rectangular prism is a 3-dimensional shape, the volume, or the space inside the shape, can be described by the number of unit cubes (i.e. cubes with dimensions of 1 unit x 1 unit x 1 unit) that can fit inside.

Students should physically pack different rectangular prisms (boxes) with unit cubes (unifix cubes, snap cubes, etc…) and note the volume by determining the total number of cubes inside the box.

Sample Task: Look at the picture below: How many of those cubes do you think will fit in the brown box? What do you need to know? Make a prediction before I give you the units.



If the dimensions of the box are 5 x 6 x 7, how many will fit?
Extension: How will the volume change if the box were 4 x 6 x 8? 3 x 7 x 9? Why do you think the volume changed in this way?

Sample Task: Graham Fletcher created a fantastic 3-Act Task for this standard. You can find it by clicking here. To find more of Graham Fletcher's Three-Act Tasks, click here.

It should be noted that part ***c*** of the standard says that students should derive the formula by packing the prisms with cubes. Students should *not* be given the formula for volume, but should come to that realization through meaningful opportunities to think about and interact with packing prisms with cubes.

They should investigate the Associative Property's impact on the order in which you can multiply. For example, if the dimensions of the prism were 5 x 6 x 7, you could find the number of cubes on the 5 x 6 layer and multiply it up 7 times OR you could find the number of cubes that fit on the 6 x 7 layer and multiply that row 5 times.

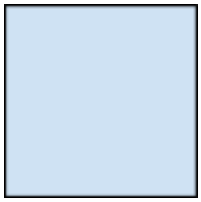
Finally, if students understanding that volume involves 3-dimensional figures, they should have no problem differentiating between area, perimeter, and volume. The only students that struggle with this distinction are students that were "taught" the formulas and told to apply them. Students who have had opportunities to physically interact with geometric figures will easily differentiate.

For more information and a video about these standards, click here.

5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.

5.G.4 Classify two-dimensional figures in a hierarchy based on their attributes.

In prior grades, students have investigated triangles, quadrilaterals, hexagons, pentagons, and circles. They should continue investigations of those shapes by classifying and creating hierarchies for each of the categories of those shapes.



This shape can be classified as a:

- Polygon (multiple sided shape)
- Quadrilateral (4 sided shape)
- Rectangle (all four angles are right angles)
- Parallelogram (two sets of parallel sides)
- Rhombus (two sets of parallel sides & all sides equal length)
- Square (all sides equal length & all right angles)

Different classifications of triangles based on side length include scalene, isosceles, and equilateral. Classifications based on angle measurements include acute, obtuse, and right. These classifications are not hierarchical, so there is *no* need to create a hierarchy of triangles.
Other shapes could have different classifications (e.g. regular hexagon (all equal sides) vs. irregular hexagon).

It is a difficult transition for students to understand how one shape can have many different classifications. It is cognitively demanding to shake up understanding in this way. We can get students to think critically about these shapes by playing games and offering challenges such as:

Sample Activity: True/False

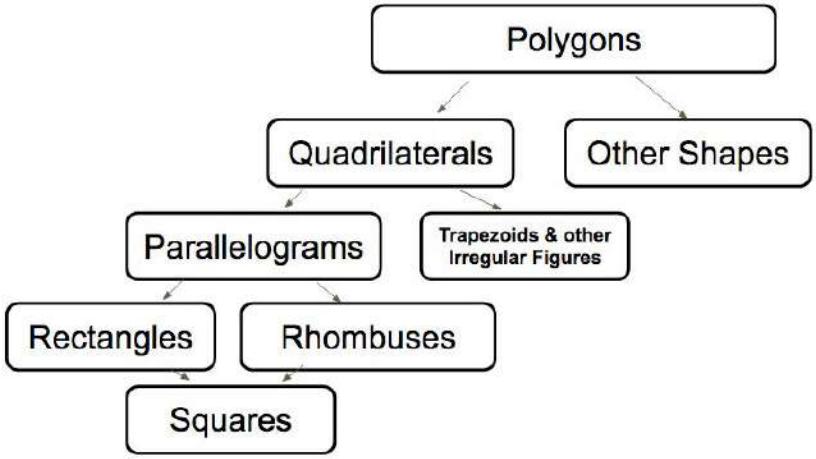
1.) A square is always a rectangle.
Answer: A square is a rectangle since a rectangle is defined as *any* four sided shape with four 90 degree angles. A square has four sides and four 90 degree angles, so a square is a rectangle.
2.) A rhombus is always a square.
Answer: A rhombus is not a square because a square *must* have four 90 degree angles but not all rhombuses have 90 degree angles.
3.) A rectangle is always a parallelogram.

Answer: A rectangle is always a parallelogram because a parallelogram is defined as a quadrilateral with two pairs of parallel sides. Rectangles have two pairs of parallel sides but, more specifically, have two pairs of equal sides, creating four 90 degree angles.

Sample Activity: Always/Sometimes/Never True

Teachers give a statement regarding the relationship between two shapes and students must determine if the statement is always true, sometimes true, or never true. Illustrative Mathematics has created a task with answers and can be found here.

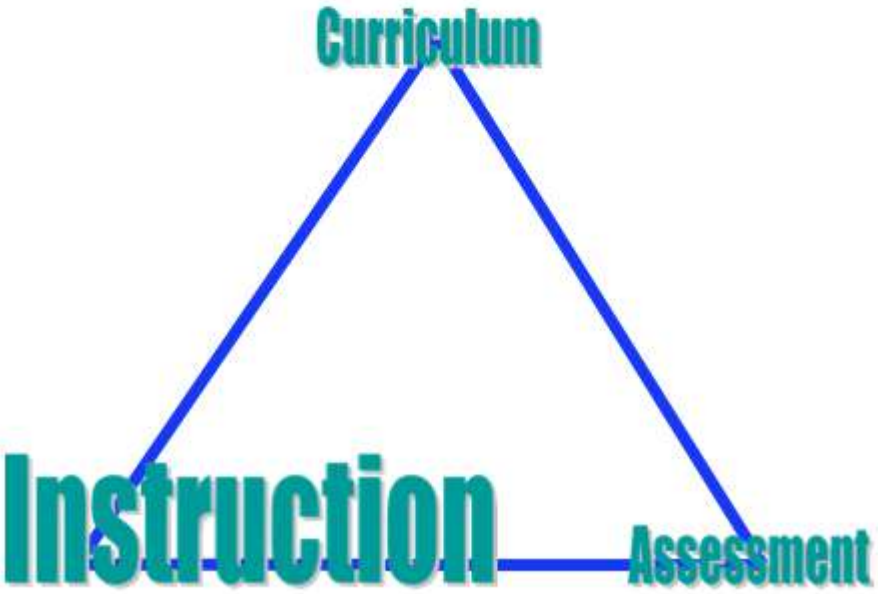
5.G.4 also calls for students to classify two dimensional shapes into a hierarchy based on their attributes. A sample hierarchy is shown below:



Not only should students place the shapes within the hierarchy, they must also justify their positions in the hierarchy. For example, students could say, “Parallelograms are under quadrilaterals because they are a specific type of quadrilateral. All parallelograms have 4 sides but they also have 2 pairs of parallel sides.”

A video for this standard can be found here.

Instruction



“I cannot teach you anything. I can only make you think.” Socrates

Marilyn Burns: 10 Big Math Ideas

Everyone's favorite math guru shares the top 10 ways you can enhance students' math learning, test scores, and skills

By Marilyn Burns | March 2005

1. Success comes from understanding.	Set the following expectation for your students: Do only what makes sense to you. Too often, students see math as a collection of steps and tricks that they must learn. And this misconception leads to common recurring errors-when subtracting, students will subtract the smaller from the larger rather than regrouping; or when dividing, they'll omit a zero and wind up with an answer that is ten times too small. In these instances, students arrive at answers that make no sense, and they rarely know why. Help students understand that they should always try to make sense of what they do in math. Always encourage them to explain the purpose for what they're doing, the logic of their procedures, and the reasonableness of their solutions.
2. Have students explain their reasoning.	During math lessons, probe children's thinking when they respond. Ask: Why do you think that? Why does that make sense? Convince us. Prove it. Does anyone have a different way to think about the problem? Does anyone have another explanation? When children are asked to explain their thinking, they are forced to organize their ideas. They have the opportunity to develop and extend their understanding. Teachers are accustomed to asking students to explain their thinking when their responses are incorrect. It's important, however, to ask children to explain their reasoning at all times.
3. Math class is a time for talk.	Communication is essential for learning. Having students work quietly-and by themselves-limits their learning opportunities. Interaction helps children clarify their ideas, get feedback for their thinking, and hear

	other points of view. Students can learn from one another as well as from their teachers. Make student talk a regular part of your lessons. Partner talk-sometimes called "turn and talk" or "think-pair-share"-encourages students to voice their ideas. Giving them a minute or so to talk with a neighbor also helps students get ready to contribute to a discussion. It's especially beneficial to students who are generally hesitant to share in front of the whole class.
4. Make writing a part of math learning.	Writing in math class best extends from children's talking. When partner talk, small- group interaction, or a whole-class discussion precedes a writing assignment, students have a chance to formulate their ideas before they're expected to write. Vary writing assignments. At the end of a lesson, students can write in their math journals or logs about what they learned and what questions they have. Or ask them to write about a particular math idea-"what I know about multiplication so far," or "what happens to the sums and products when adding even and odd numbers." When solving a problem, encourage students to record how they reasoned. Writing prompts on the board can help students get started writing. For example: Today I learned ..., I am still not sure about ..., I think the answer is ..., I think this because....
5. Present math activities in contexts.	Real-world contexts can give students access to otherwise abstract mathematical ideas. Contexts stimulate student interest and provide a purpose for learning. When connected to situations, mathematics comes alive. Contexts can draw on real-world examples. Contexts can also be created from imaginary situations, and children's books are ideal starting points for classroom math lessons. After reading Eric Carle's *Rooster's Off to See the World* (Simon & Schuster, 1991), for example, ask children if they can figure out how many animals went traveling. Or ask children to

	follow the calculations in Judith Viorst's *Alexander, Who Used to Be Rich Last Sunday* (Simon & Schuster, 1978), and figure out how Alexander spent his money.
6. Support learning with manipulatives.	Manipulative materials help make abstract mathematical ideas concrete. They give children the chance to grab onto mathematics ideas, turn them around, and view them in different ways. Manipulative materials can serve in several ways-to introduce concepts, to pose problems, and to use as tools to figure out solutions. It's important that manipulatives are not relegated to the early grades but are also available to older students.
7. Let your students push the curriculum.	Avoid having the curriculum push the children. Choose depth over breadth and avoid having your math program be a mile wide and an inch deep. As David Hawkins said in *The Having of Wonderful Ideas*, by Eleanor Duckworth (Teachers College Press, 1996), "You don't want to cover a subject; you want to uncover it." There are many pressures on teachers, and the school year passes very quickly, but students' understanding is key. Explore topics that interest the students more deeply, and take the time for side investigations that can extend lessons in different directions. One idea could be to invest your students in the standards and their proficiency in each. For example, provide a list (on a chart or poster) of every concept that will be taught and have students choose which concept they have the most trouble or are the most curious about. Students could have the opportunity to choose which concepts need to review, the ones they are least confident about, or ones they enjoy discussing.
8. The best activities meet the needs of all students.	Keep an eye out for instructional activities that are accessible to students with different levels of interest and experience. A wonderful quality of good children's books is that they delight adults as well. Of course, adults appreciate books for different reasons than children do, but enjoyment and learning can occur

	simultaneously at all levels. The same holds true for math. Look for activities that allow for students to seek their own level and that also lend themselves to extensions. For example, challenge children to find the sum of three consecutive numbers, such as 4 + 5 + 6. Ask them to do at least five different problems and see if they can discover how the sum relates to the addends. (The sum is always the middle number tripled.) Allowing the children to select their own numbers to add is a way for students to choose problems that are appropriate for them. Even those students who don't discover the relationship will benefit from the addition practice. Invite more able students to write about why they think the sum is always three times the middle number, or to investigate the sums of four consecutive numbers. The best way to know how to meet the needs of all students is to understand the curriculum and standards for the grade levels above and below. The question that must be asked is how can we stretch this down to the working level of my most struggling students and bring it up to meet the needs of the students exceeding grade level standards. Vertical collaboration is essential in this work.
9. Confusion is part of the process.	Remember that confusion and partial understanding are natural to the learning process. Don't expect all children to learn everything at the same time, and don't expect all children to get the same message from every lesson. Although we want all students to be successful, it's hard to reach every student with every lesson. Learning should be viewed as a long-range goal, not as a lesson objective. It's important that children do not feel deficient, hopeless, or excluded from learning mathematics. The classroom culture should reinforce the belief that errors are opportunities for learning and should support children taking risks without fear of failure or embarrassment.

10. Encourage different ways of thinking.	There's no one way to think about any mathematical problem. After children respond to a question (and, of course, have explained their thinking!), ask: Does anyone have a different idea? Keep asking until all children who volunteer have offered their ideas. By encouraging participation, you'll not only learn more about individual children's thinking, but you'll also send the message that there's more than one way to look at any problem or situation. That's when the potential for delight occurs.

About the Author

Marilyn Burns is the creator of Math Solutions, inservice workshops offered nationwide, and the author of numerous books and articles. She is author of the book *50 Problem- Solving Lessons, Grades 1-6*, distributed by Cuisenaire.

Getting Started: Creating a Math Rich Environment

"Classroom organization addresses all components of instruction – teaching strategies, student grouping assignments, and assessing." (Slavin, 1989)

The classroom **environment** should be **kid-friendly** and **safe**. Students are encouraged to (actively) participate in class, which includes verbally and physically.

The classroom should be organized where students have DAILY access to their **math tools** and **manipulatives**. The **"toolboxes"** should be readily available and easily accessible to everyone in the classroom.

The classroom should be arranged so that the students can easily transition from independent work to group/partner assignments (and vice versa) with minimal loss of instructional time. There should be space set up for whole group gathering for Number Talks, mini lessons, and discussions. Small group work spaces can include a table where students work with the teacher, as well as collaborative areas for students to work in groups. Centers/ Stations should also have predetermined areas and materials should be organized in advance of students working.

Building Your Instructional Tool Kit

To meet the diverse academic needs of the students in the class, learn as much about the students as possible. Use anecdotal data and testing data, as well as data from student interviews, surveys and observations. After getting a sense of "where" the students are academically, design lessons and investigations to challenge the students.

It is recommended to:

- design the "on-grade" level lesson first; then adapt that lesson to meet the needs of the students below grade level and above grade level

- provide varied learning experiences over several days, the varied learning experiences include opportunities for STUDENTS to use concrete objects, visual representations, technology, etc to learn the concept or skill through problem solving situations.

The goal is to address as many of the different learning styles and modalities as possible; so that all students have equal access to the curriculum.

Building a Community of Mathematicians

"In order to achieve a deep and true understanding of mathematics, children must first see themselves as becoming mathematicians."
(Wedekind, 2011)

The classroom culture begins with all students believing that they are mathematicians, and that they are part of a community. It's essential for students to learn what mathematicians do and how they work. In addition, students need to understand that mathematicians continue to grow and learn. A mathematical mindset and **high expectations** should be communicated and modeled for ALL students at all grade levels.

While classroom community can be established at any point in the school year, the best time to build it is at the very beginning of the school year. In the first days/weeks of a new school year, teachers and students are getting to know one another. This is the perfect time to establish norms and classroom expectations about doing mathematics.

Mini Lesson Ideas for Building Community:

- **Math is Everywhere**
- **We are ALL Mathematicians**
- **Mathematician Statements**
- **Routines for Math (Number Talks/Number Sense Routines, Transitions, etc)**
- **How to Work Together**
- **Organizing Toolkits**
- **How to Talk about Math**
- **How to Use Mistakes as Learning Opportunities**

It is important for teachers to establish this "risk free zone" so ALL students want to and will participate in the math investigations. All students have something to contribute to the learning and growth of their teacher and peers.

Vocabulary

Students should be introduced to the math vocabulary through active participation in the investigations. Introducing the vocabulary out of context or as an isolated word does not give students a reference point to connect the word to. Throughout the investigation, the teacher should use the correct math vocabulary. During the discussion portion of the lesson, the students should define the significant terms related to the lesson based on the teacher's questions and examples and based on their experiences during the investigation. It is recommended that the math term is to be recorded in the math notebook or journal with a teacher definition, a definition in the students' own words, and with a picture. The students should be encouraged daily to use the correct/appropriate math vocabulary in their discussions and writings.

Calculators

Calculators should be recognized as an instructional tool – similar to rulers and other manipulatives. As a result, students should have access to the calculators often (Van de Walle, 2000). In the primary grades (kindergarten – second), the calculator is for exploration. As the students show mastery of math facts and basic skills, the calculator should be used to reinforce the facts. In the upper elementary grades (third – fifth), the calculator should be used as another strategy to learn concepts through exploration (not rote calculations). Students should have many opportunities to use the calculator as another learning tool, especially in other content areas that rely on math (such as science) and for topics that require the application of basic skills (calculating the perimeter, area etc.). After students have shown understanding and mastery of an algorithm, the calculator should be used to check work.

Manipulatives

Manipulatives should be used everyday in the math classroom. They are necessary tools, especially for our visual and kinesthetic learners. Manipulatives make many concepts seem less abstract and confusing. Manipulatives allow students the opportunity to make changes within a problem to determine patterns and to draw conclusions. Students need opportunities to experience and manipulate tools that assist them in making sense of the math.

Initially, it is recommended to:

- give the students time to explore a particular manipulative before beginning math instruction with the manipulative
- introduce a new manipulative with an old concept, especially if the students have no prior experience with the manipulative

- model two to three simple problems with the manipulative, then allow the students opportunities to use the manipulative to complete additional problems based on the same concept or skill
- reconvene with the class to discuss what the students have learned and their results from using the manipulatives
- Encourage students to think of other manipulatives that could be used to address the same concept/skill.

Successful use of manipulatives in a classroom is a combination of the teachers' high expectations and challenging and engaging lessons in a "risk–free zone" of mutual respect for learning.

NOTE: Not all students will be comfortable using every manipulative. Some students will "get it" or will know the answer without using the manipulatives. Those students should still be encouraged to show or explain to the teacher how they arrived at the correct solution. The ultimate goal of using manipulatives is for the students to have conceptual understanding of a concept.

For a list of grade level appropriate manipulatives, please review the list at the end of this section.

Resources:

Math Strategies You Can Count On by Char Forsten

The First Days of School by Harry K. Wong

The Differentiated Math Classroom by Miki Murray

Elementary School Mathematics Teaching Developmentally by John A. Van De Walle

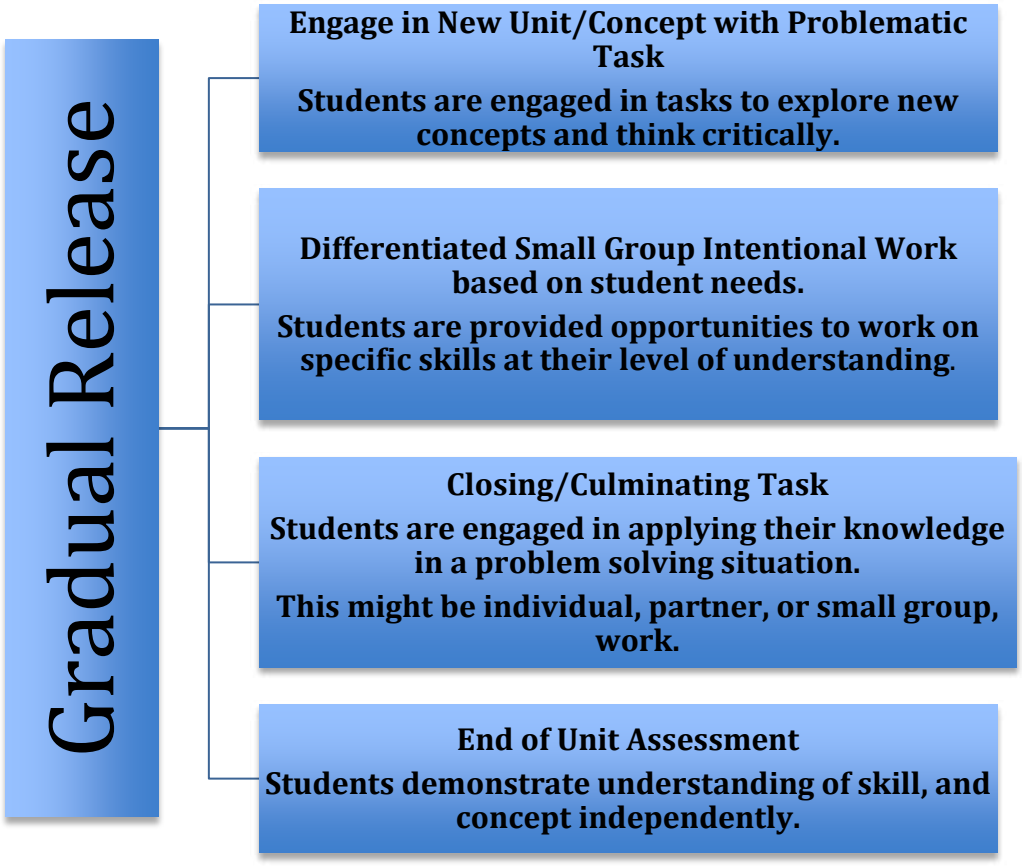
Teaching Student-Centered Mathematics by John A. Van de Walle and LouAnn H. Lovin

Gradual Release

The following visual shows how to plan instruction using a gradual release model **over the span of time it takes to teach a new concept (or unit).** While there is not a set amount of time for how long to spend on a new concept, teachers do need to assess students throughout to find out if students are grasping concepts, are ready to do meaningful independent practice, and if students need to have more time in small groups with the teacher.

When introducing a new concept it's important to ask open-ended questions to assess how students are using previous knowledge with a new situation or concept. From there, plan more differentiated lessons, either for small groups, independent practice, or scaffolded tasks.

Depending on the standard/concept to be learned, students might engage in several tasks before breaking up into differentiated small groups.



Differentiated Instruction for Math

What is Differentiated Instruction?

Differentiated instruction, also called differentiation, **is a process through which teachers enhance learning by matching student characteristics to instruction and assessment.** Differentiated instruction allows all students to access the same classroom curriculum by providing entry points, learning tasks, and outcomes that are tailored to students' needs (Hall, Strangman, & Meyer, 2003). Differentiated instruction is not a single strategy, but rather an approach to instruction that incorporates a variety of strategies.

Teachers can differentiate content, process, and/or product for students (Tomlinson, 1999). Differentiation of **content** refers to a change in the material being learned by a student. For example, if the classroom objective is for all students to subtract using renaming, some of the students may learn to subtract two-digit numbers, while others may learn to subtract larger numbers in the context of word problems.
Differentiation of **process** refers to the way in which a student accesses material. One student may explore a learning center, while another student collects information from the web.
Differentiation of **product** refers to the way in which a student shows what he or she has learned. For example, to demonstrate understanding of a geometric concept, one student may solve a problem set, while another builds a model.

When teachers differentiate, they do so in response to a student's readiness, interest, and/or learning profile. **Readiness** refers to the skill level and background knowledge of the child. **Interest** refers to topics that the student may want to explore or that will motivate the student. This can include interests relevant to the content area as well as outside interests of the student. Finally, a student's **learning profile** includes learning style (i.e., a visual, auditory, tactile, or kinesthetic learner), grouping preferences (i.e., individual, small group, or large group), and environmental preferences (i.e., lots of space or a quiet area to work). A teacher may differentiate based on any one of these factors or any combination of factors (Tomlinson, 1999).

Differentiation strategies support all students (gifted, varied socio-economic backgrounds, ethnic/racial groups,etc). The strategies provide all students the opportunity to learn the content (Dacey & Lynch, 2007)

How Is Differentiation Implemented?

Implementation looks different for each student and each assignment. Before beginning instruction, teachers should do three things:

1. Use diagnostic assessments to **determine student readiness**. These assessments can be formal or informal. Teachers can give 1-on-1 assessments, written pre-tests, question students about their background knowledge, or use KWL charts (charts that ask students to identify what they already **K**now, what they **W**ant to know, and what they have **L**earned about a topic).

2. **Determine student interest**. This can be done by using interest inventories and/or including students in the planning process. Teachers can ask students to tell them what specific interests they have in a particular topic, and then teachers can try to incorporate these interests into their lessons.

3. **Identify student learning styles and environmental preferences**. Learning styles can be measured using learning style inventories. Teachers can also get information about student learning styles by asking students how they learn best and by observing student activities. Identifying environmental preferences includes determining whether students work best in large or small groups and what environmental factors might contribute to or inhibit student learning. For example, a student might need to be free from distraction or have extra lighting while he or she works.

Teachers incorporate different instructional strategies based on the assessed needs of their students. Throughout a unit of study, teachers should assess students on a regular basis.

This assessment can be formal, but is often informal and can include taking anecdotal notes on student progress, examining students' work, and asking the student questions about his or her understanding of the topic. The results of the assessment could then be used to drive further instruction.

The curricular framework, in the previous section of this guide, can assist teachers in developing lessons to meet the needs of their students. The figure below provides an example of differentiating a concept based on the students' readiness levels.

Students in the first box who are having trouble adding and subtracting within 99 should practice generating strategies to add and subtract within 20. Students who demonstrate proficiency with addition and subtraction within 99 can move up to 999. Finally, students above grade level in working with place value concepts and properties of operations can begin to derive an algorithm for addition and subtraction.

What Does Differentiation Look Like for Math?

Math instruction can be differentiated to allow students to work on skills appropriate to their readiness level and to explore mathematics applications. The chart below offers a variety of strategies that can be used.

Strategy	Focus of Differentiation	Definition	Example
Tiered assignments	Readiness	Tiered assignments are designed to instruct students on essential skills that are provided at different levels of complexity, abstractness, and open-endedness. The curricular content and objective(s) are the same, but the process and/or product are varied according to the student's level of readiness.	In a unit on measurement, some students are taught basic measurement skills, including using a ruler to measure the length of objects. Other students can apply measurement skills to problems involving perimeter.
Compacting	Readiness	Compacting is the process of adjusting instruction to account for prior student mastery of learning objectives. Compacting involves a three-step process: (1) assess the student to determine his/her level of knowledge on the material to be studied and determine what he/she still needs to master; (2) create plans for what the student needs to know, and excuse the student from studying what he/she already knows; and (3) create plans for freed-up time to be spent in enriched or accelerated study.	A third grade class is learning to identify the parts of fractions. Diagnostics indicate that two students already know the parts of fractions. These students are excused from completing the identifying activities, and are taught to add and subtract fractions.
Interest Centers or Interest Groups	Readiness Interest	Interest centers (usually used with younger students) and interest groups (usually used with older students) are set up so that learning experiences are directed toward a specific	Interest Centers - Centers can focus on specific math skills, such as addition, and provide activities that are high interest, such as counting jellybeans or adding the number of eyes on two aliens.

Strategy	Focus of Differentiation	Definition	Example
		learner interest. Allowing students to choose a topic can be motivating to them.	Interest Groups - Students can work in small groups to research a math topic of interest, such as how geometry applies to architecture or how math is used in art.
Flexible Grouping*	Readiness Interest Learning Profile	Students work as part of many different groups depending on the task and/or content. Sometimes students are placed in groups based on readiness, other times they are placed based on interest and/or learning profile. Groups can either be assigned by the teacher or chosen by the students. Students can be assigned purposefully to a group or assigned randomly. This strategy allows students to work with a wide variety of peers and keeps them from being labeled as advanced or struggling.	The teacher may assign groups based on readiness for direct instruction on algebraic concepts, and allow students to choose their own groups for projects that investigate famous mathematicians.
Learning Contracts	Readiness Learning Profile	Learning contracts begin with an agreement between the teacher and the student. The teacher specifies the necessary skills expected to be learned by the student and the required components of the assignment, while the student identifies methods for completing the tasks. This strategy (1) allows students to work at an appropriate pace; (2) can target learning styles; and (3) helps students work independently, learn planning skills, and eliminate unnecessary skill practice.	A student decides to follow a football team over a two- month period and make inferences about players' performances based on their scoring patterns and physical characteristics. The student, with the teacher's guidance, develops a plan for collecting and analyzing the data and conducting research about football. The student decides to create a PowerPoint presentation to present his or her findings to the class.

Choice Boards	Readiness Interest Learning Profile	Choice boards are organizers that contain a variety of activities. Students can choose one or several activities to complete as they learn a skill or develop a product. Choice boards can be organized so that students are required to choose options that focus on several different skills.	Students are given a choice board that contains a list of possible activities they can complete to learn about volume. For example, students can choose to complete an inquiry lesson where they measure volume using various containers, use a textbook to read about measuring volume, or watch a video in which the steps are explained. The activities are based on the following learning styles: visual, auditory, kinesthetic, and tactile. Students must complete two activities from the board and must choose these activities from two different learning styles.

* More information about grouping strategies can be found in *Strategies to Improve Access to the General Education Curriculum.* Available at http://www.k8accesscenter.org/training_resources/curricular_materials.asp

References

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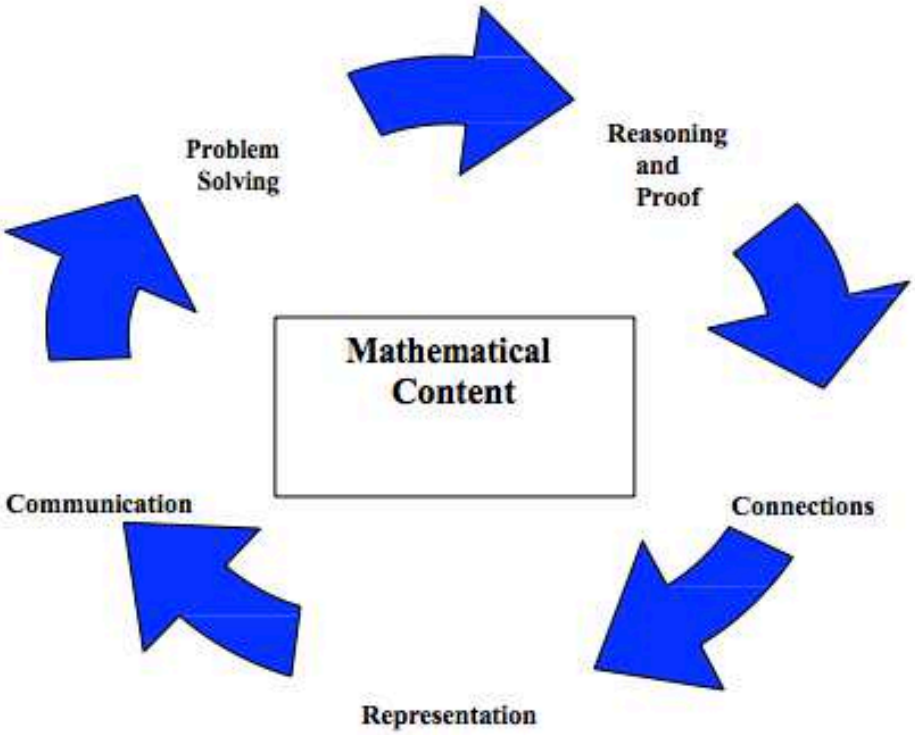
Hall, T., Strangman, N., & Meyer, A. (2003). *Differentiated instruction and implications for UDL implementation.* National Center on Accessing the General Curriculum. Retrieved July 9, 2004 from: http://www.k8accesscenter.org/training_resources/udl/diffinstruction.asp

Tomlinson, C.A. (1999). *How to differentiate instruction in mixed-ability classrooms.* Alexandria, VA: ASCD.

http://www.cast.org/ncac/index.cfm?i=2876 – This site contains an article by Tracy Hall at the National Center for Accessing the General Curriculum. The article discusses differentiation as it applies to the general education classroom.

http://members.shaw.ca/priscillatheroux/differentiatingstrategies.html - The Enhancing Learning with Technology site provides explanations for various differentiation strategies.

Process Standards



The process standards must be integrated daily into mathematics instruction. They should never be taught as a separate stand-alone unit. Students should learn the mathematical content standards ***through…not in addition to…***the process standards.

Therefore, math content should be the result of well-designed problem situations that require students to reason mathematically, communicate with one another, use connections and representations to support their efforts, and justify their reasoning through valid arguments.

Process Standard: Problem Solving

Instructional programs from pre-kindergarten through grade 12 should enable all students to--

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

NCTM (2000)

What does it mean to teach math through problem solving?

The National Council of Teachers of Mathematics states in the *Principles and Standards for School Mathematics* (2000) that **understanding** must be the goal for all of the mathematics we teach. **It is important to realize that understanding cannot be taught directly.** Teaching through problem solving is an approach that allows students to learn mathematics with understanding as they use various solution methods to solve problems. "*Most, if not all, important mathematics concepts and procedures can best be taught through problem solving* (Van de Walle, page 11, 2006). Students should have frequent opportunities to formulate, grapple with, and solve problems that require a significant amount of effort and should be encouraged to reflect on their thinking (NCTM, 2000, pg. 51).

The emphasis in problem solving has shifted from teaching problem solving to teaching *through* problem solving. The focus is on teaching mathematical topics through problem-solving contexts and inquiry-oriented environments which are characterized by the teacher "helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, conjecturing, exploring, testing, and verifying (Lester et al., 1994). This is in opposition to the approach in which the skills in the chapter are taught directly and then the students are expected to apply those skills as they complete the few word problems at the end of the chapter. Teaching through problem solving reverses the process so that math skills and concepts become the valuable outcome of the problem-solving experience as opposed to a prerequisite for solving the problem.

Why should I teach math through problem solving?

"Children need to see math as a conceptual, growth subject that they should think about and make sense of." (Boaler, pg. 34, 2016)

There are many reasons for making the curriculum switch to teaching through problem solving. Although it may be more difficult initially, the reward is that your students will learn the math you teach with understanding and will see that math is a subject that does make sense. Other benefits include:

- Problem solving focuses students' attention on ideas and sense-making. This means they must reflect on the math; therefore, new ideas are more likely to be integrated with existing ones. This focus improves understanding.
- Problem solving helps students realize that they are capable of doing mathematics. When students are given problems to solve and know that you expect a solution, you are saying to them "I believe you can do this." Self-esteem is enhanced every time they solve a problem.
- Problem solving provides ongoing assessment data. When teachers listen to their students discuss ideas and defend solutions, a steady stream of valuable assessment information can be attained.
- Problem solving tasks are easier to differentiate. Since students are allowed to choose strategies they want to use, they can use methods to solve the problem that make sense to them. Their understanding is also increased as they hear others share their solution strategies.
- Problem solving naturally engages students in all five of the process standards: problem solving, reasoning and proof, communication, connections and representation.
- Problem solving helps students learn how to construct their own methods to solve problems and apply these methods to new problem-solving situations. This allows them to solve a variety of problems without having to memorize different procedures for each new problem (Hiebert, 1996).
- Problem solving is engaging. For this reason, students are more likely to actively participate resulting in fewer discipline problems (Van de Walle, 2000).

What are the Characteristics of a Problem-Solving Approach

- Interactions between students/students and teacher/students
- Mathematical dialogue and consensus between students

- Teachers providing just enough information to establish background/intent of the problem, and students clarifying, interpreting, and attempting to construct one or more solution processes (Cobb et al., 1991)
- Teachers accepting right/wrong answers in a non-evaluative way
- Teachers guiding, coaching, asking insightful questions and sharing in the process of solving problems (Lester et al., 1994)
- Teachers knowing when it is appropriate to intervene, and when to step back and let students make their own way
- **When using a problem-solving approach to teaching mathematics, the remaining process standards of communication, connections, representation, and reasoning and proof can be integrated into classroom instruction**

How do I plan problem-based lessons?

A problem-based approach should be used in the classroom every day. In classrooms where both traditional skill-based teaching and teaching through problems are used, children become confused about when they are supposed to use their own strategies for figuring out a problem and when they are supposed to use the "teacher's approach" (Mokros, Russell, and Economopoulous, 1995).

Step 1 – Decide on the math concept or standard that you plan to teach.

Step 2: Think about your students. What do they already know and understand about this topic? Is there some background information they need before being able to solve the problem?

Step 3: Decide on a task. Keep it simple! Good tasks need not be elaborate. Pictures, videos, and word problems offer students the opportunity to notice and wonder about concepts/ideas. **The solution involves children doing the mathematics**.

Step 4: Predict what will happen. Think about the strategies that the students might use, but be prepared to see the students using strategies that you never considered.

Step 5: Plan the "mini lesson." This is the part of the lesson where you introduce the problem to be solved and review any background concepts or math vocabulary that is important for solving the problem. It is important to refrain from "teaching" the students how to go about solving the problem.

Step 6: Plan the small group portion of the lesson. What will they do? How can you facilitate their efforts without telling them “how” to solve the problem? What kinds of questions might you ask? How will you decide which students will share in the discussion?

Step 7: Plan the discussion of the lesson. This is the part of the lesson in which the teacher intentionally asks students to share with the class their findings and the strategies they used to solve the problem. **This is a very important part of the lesson and should never be omitted** (Van de Walle, 2000). **In order for students to retain their learning, they need to talk about it and summarize their learning each day.**

Math Workshop

Problem-solving lessons should be implemented using the Math Workshop framework. Math Workshop begins with a mini lesson, progresses to work time, which can be small group work, partner work, or meaningful independent work. The workshop ends with a whole group discussion. **A MINIMUM OF 60 MINUTES SHOULD BE DEVOTED TO MATH WORKSHOP DAILY.**

What does a problem-solving lesson look like?

Mini Lesson

The teacher sets a purpose for the day's lesson and explicitly connects the day's lesson to previous learning to help students make connections. Next, the teacher poses the problem to be investigated while children are encouraged to ask questions about the task. This might include "Notice and Wonder", asking about vocabulary, or asking for more information about the task. Explain the task to them if necessary but do not *tell* students *how* to solve it. Review any background math concepts or math vocabulary. Do not directly teach the new concepts. These will emerge through the process. Posting a picture, a video, a contextual word problem or a piece of children's literature are all possible ways to introduce an investigation.

Work Time

Students should work in small groups, partners, or independently to solve the problem. It depends on where in a unit of study the class is. If it's still early in a concept, small groups and pairs are recommended. However students do need to be gradually released to apply their knowledge in meaningful independent practice prior to a concept being assessed. A variety of tools (concrete materials, technology, etc) should be available for them to use. The teacher's role is to monitor student progress and encourage them to look for multiple solution strategies. They should be encouraged to use whatever strategies make sense to them in solving the problem. If a student is stuck, facilitate by asking questions that can help them get started. The teacher should also differentiate instruction by providing different types or levels of investigations or problems to solve including through teacher led small groups. Numberless word problems are an effective way to differentiate tasks. It is during work time that the teacher **intentionally** chooses which students/groups will share during the discussion time.

This work time ***could also*** be a time for differentiated games, activities, and small group remediation/extension. Teachers use data to assign games and activities that are appropriate for each group based on students' working level.

Whole Class Discussion

This is the most important and most neglected phase of math workshop. If the students were working on a task, the teacher has circulated to strategically show a progression of ideas from least to most complex. Ask the groups to share their solutions, strategies and explain their thinking. Students should be asked to explain how they arrived at their solutions whether those solutions are correct or incorrect. Many times, when students have an incorrect solution, the process of talking about how they arrived at the solution will cause them to realize their error without having to be told by the teacher. An additional bonus in having the students share their strategies is that others see strategies that they had not considered and thus gain a wider repertoire of problem-solving strategies.

In most cases, students will make the main teaching points for you during the whole class discussion since they have "constructed" this new knowledge by being actively engaged in "using" the mathematics to solve the problem. Nevertheless, **it is important to summarize the discussion for everyone, emphasizing and explaining key points. Make sure you relate the new math concepts back to the task the children have been working on so that the discussion remains meaningful.**

If the work time consisted of a variety of games and activities, the teacher should have students debrief their activity and describe the learning that occurred. This provides the teacher to have some information about what the students learned and holds the students accountable for some kind of work output. This can be a time to highlight some new thinking that was displayed in small groups from which other students can benefit.

Catch and Release Model of Discussion

Another model for discussion throughout the workshop time is Catch and Release. In this model, while the students are working, the teacher might observe common misconceptions or common findings and stop students where they are, ask, "What are you noticing?" have some discussion, then release students to continue working. Sometimes this method helps alleviate time constraints, however it is still important to have students **summarize** their learning at the end of the lesson.

Another way to use the catch and release model would be to break up the task into parts and do a mini-debrief after each part. A whole group debrief at the end of the task with the intent of summing up the learning is still necessary, but breaking it into pieces could make the task easier to digest.

The chart below shows a daily flow of Math Workshop including approximately how much time should be devoted to each component.

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Small- and large-group discussion should make-up the majority of the lesson time. Therefore, there will be a much greater percentage of student talk than teacher talk.

Grouping Strategies for Problem Solving

It is important to consider the ways that we group students for problem-solving investigations. It is also important to consider our purpose when grouping students. **Flexible groupings** should always be used so that students are provided opportunities to work at their independent level as well as be challenged. When introducing a new concept, heterogeneous (mixed ability levels) groupings are appropriate. When differentiating for student current levels of understanding, homogenous (same ability level) groupings are more appropriate. How can we make sure that all students remain interested, challenged, and supported? Here are some ways of grouping that have proven successful:

- Group students strategically by ability levels. This will provide the group with a range of math knowledge for problem solving. A good time to use this grouping pattern is when you believe that students, through their work together, can raise their level of performance.
- Randomly select students to work together for short-term projects. Invite students to create a group by lining up according to size and then count off for grouping. Another way to group is to form groups based on attributes such as those who are wearing shoes that tie, those who are wearing Velcro closings, people with brown eyes, people with blue eye, etc. You can also challenge students to make up their next way to group.
- Grouping by interest is another successful grouping strategy. Often students are motivated by particular interests. When you put those students together, they often perform beyond your expectations. They may be more willing to persevere with problem solving.
- Invite students to help you develop a list of grouping ideas to use throughout the year. Developing many different grouping practices helps students understand that all members of the classroom can work with one another.

Taking the time to build these values and routines in the classroom will pay off. The classroom will be perceived as a safe place for taking the necessary risks for learning, as well as for performing optimally during assessment time (Kallick & Brewer, 1997)

What kinds of problem-solving tasks should we use to teach math concepts?

- **The problem-solving task must be tiered, meaning students of different ability levels have an entry point into the task.** They should have the appropriate ideas to solve the problem and yet still find it challenging and interesting.

- **The task must require students to reflect and communicate about the mathematics embedded in the problem.** Reflection and communication are the processes through which understanding develops. The tasks provide the context in which students can reflect on and communicate about mathematics (Hiebert, 1996).

- **The task must allow students the opportunity to use tools.** Tools are learning supports and do not only include physical materials such as manipulatives. Other tools that should be encouraged are skills that have been previously acquired (background knowledge), written symbols, pictures, and verbal language.

- **The problem-solving task should leave behind important mathematical residue** (Hiebert, 1996). The “residue” that results should be new strategies for solving problems and a deeper understanding of the math concepts that were embedded in the task. Solving the problem must require the use of mathematical ideas and must be based on sound and significant mathematics (NCTM, 2000).

- **The task should be based on the knowledge of the range of ways that diverse students learn mathematics.** Teachers must display sensitivity to, and draw on, students' diverse background experiences and dispositions when designing tasks for students.

- **The task should be engaging.** The problem must be of interest to the students so that they have a desire to solve the problem.

- **The problem-solving task must help develop students’ understandings of the math standards.** It is critical that teachers have a standards-based focus when designing mathematical tasks. The goal is for the students to develop a solid understanding of the math standard(s) through the process of solving problems.

- **The problem-solving task must stimulate students to make connections, reason mathematically, create mathematical representations, and promote communication about mathematics.** These process standards are easily integrated into instruction with a problem-based focus.

How to Create Good Problem-Solving Tasks

A good problem can be used as the basis for an entire lesson. There are many resource books and websites that contain good problems for students to solve, but it is also possible to make up your own good questions. There are two helpful approaches that can be used in creating good problems for students to solve.

Method 1: Making a Standard Question Better: Take a standard question from a workbook, textbook, released assessment, or other resources and open it up to require deeper thinking and possibly include multiple answers.

Example:
(Before): The math team went to the aquarium to do research. Each team member paid $12 for the trip. There were 25 team members on the trip. What was the total amount the team members paid? (Source: Released NC EOG (4th Grade)

(After): The math team went to the aquarium to do research. Each team member paid $12 for the trip. There were 25 team members on the trip. They will also stop for lunch on the way back to school. If the lunch costs $6 per person, how much more would the total cost be with lunch than without lunch (if they bring their own lunch)? Next week, they are bringing twice as many students on the same field trip. How much more will the total cost be? How do you know?

Method 2: Working Backwards: Take a standard or a learning target and think about ways it is used in a realistic situation. Think of all of the different ways it can be interpreted and solved. If there are multiple ways to represent the problem, the task is open. If there are few ways, the task is closed. Aim for open tasks.

Example:
Skill: Thinking of fractions as division.

Task: At Parent Night, the organizer is trying to determine how many sandwiches to give to each table. Unfortunately, there aren't the same amount of people at each table. The following chart shows the number of people at each table:

Table Number	Number of People
1	4
2	6
3	5
4	8

The organizer ordered 18 sandwiches to split between the tables. Each table can only get whole sandwiches (no splitting between tables). How many sandwiches would you give to each table to make it as fair as possible? If you gave that amount out, how much of each sandwich would each person at the table get?

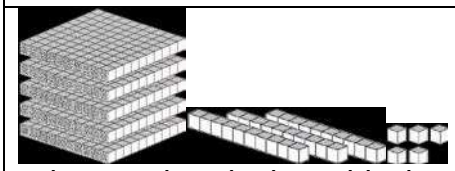
Two Types of Problem-Solving

Grades K-2

Closed Problems	Open Problems
 What number do these blocks represent? a) 125 b) 25 c) 15 d) 51	Using base 10 blocks, how many different ways can you show the number 25? Which way uses the most number of blocks and which way uses the fewest number of blocks?
Lisa has 2 dimes, 4 nickels and 5 pennies in her pocket. How much money does she have in all?	Lisa has coins in her purse that have a total value of 45 cents. What coins might she have? What coins might she have if she has exactly 11 coins in
I bought a pencil for 45 cents. I gave the cashier 50 cents. How much change did I get?	I bought something and got 5 cents change. How much did it cost and how much money did I give to pay for it?
There are 19 children in Mrs. Johnson's class. Is this an even number or an odd number?	When the children in a class each got a partner, there was one child left over. How many children might be in the
What is the length of your desk?	How many objects can you find that are longer than 1 foot but shorter than 2 feet in length?
John ate 4 apples. Janice ate 8 apples. How many apples did they eat altogether?	John and Janice ate 12 apples. How many apples might they each have eaten?
There are 6 tables in the classroom. Four students sit at each table. How many students are in the classroom?	A class has 24 students. They sit at tables in the classroom. Each table has the same amount of students. How many tables might there be in the classroom and how many
Jimmy has 14 marbles. Logan has 10 marbles. How many marbles do they have altogether?	Jimmy and Logan are playing marbles. They have 24 marbles between the two of them. Jimmy has more marbles than Logan. How many marbles might each of them
A basketball player scored 5 points in her first games and 4 points in her second game. How many points did she score in both games?	A basketball player scored 9 points in two games. What might her scores in each of the games be?
6 + 5 + 2 = __	? + ? + ? = 13. What might the missing numbers be?

Two Types of Problem-Solving

Grades 3-5

Closed Problems	Open Problems
I bought lunch and received 3 quarters, 2 dimes, and 1 nickel in change. How much change did I receive in all?	I bought my lunch and received change of $1.00 using quarters, dimes, and nickels. How might the change have looked?
Round 348 to the nearest tens.	A number has been rounded to 350. What might the number be?
Write >, <, or = 1.5__2 1.9____2 1____1.3	I am thinking of some decimal numbers between 1 and 2. What might they be? Give at least 15 answers.
 What number do these blocks represent? e) 125 f) 25 g) 15 h) 525	Using base 10 blocks, how many different ways can you show the number 535? Which way uses the most number of blocks and which way uses the fewest number of blocks?
Which of the following numbers are divisible by 3? 6, 28, 18, 12	What do you know and what can you find out about the multiples of 3…..3, 6, 9, 12, 15, 18, 21…?
34 + 36 + 38 = __	Three consecutive even numbers add up to a number between 100 and 200. What might the numbers be?
23 + 9 = ___	Make up some different ways to add 9 to 23 in your head. In how many ways can you do it?
100 ÷ 20 = __	The answer to a division question is 5. What might the question be?
Eighty-four children were divided into 4 equal teams. How many children were in each team?	Eighty-four children in four grades are arranged into teams with the same number on each team. How many teams are there and how many children might there be on each team?

Is 370 divisible by ten?	What could you add to 361 to make it divisible by 10?
Find the area of the rectangle. 	I am thinking of a shape with an area of thirty square tiles. What might the shape look like?

Sullivan, P. & Lilburn, P. (2002). *Good questions for math teaching: Why ask them and what to ask (K-6)*. California: Math Solutions Publication

Math Workshop Lesson Plan Template

(also located in Resource Section)

LESSON TITLE:
MATERIALS NEEDED:
ENDURING UNDERSTANDING: **ESSENTIAL QUESTIONS:**
CONTENT STANDARDS ADDRESSED: *(Process standards are embedded)*
MINI LESSON: **What information/background knowledge will be presented to students?**
WORK TIME: (Small group OR Partner OR Independent) **What will students do?** **Plans for Differentiation:**
WHOLE GROUP DISCUSSION QUESTIONS: **How will we summarize the learning?** Teacher Reflection Questions: What do we want students to know and be able to do? How will we know that they have it? What will we do if they don't get it? What will we do if they do get it?

Process Standard: Communication

Instructional programs from prekindergarten through grade 12 should enable all students to--

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

NCTM (2000)

Talking to Learn Mathematics

The process standard of communication should include oral language as well as written language. This section focuses on how "talking mathematics" is an important vehicle for helping children make sense of math and develop true mathematical understanding.

There are two important components to teaching students to communicate about math;

1) Students need to be explicitly taught **HOW** to communicate with others respectfully.

2) Students need to be provided language and precise math vocabulary in order to communicate their ideas clearly with others.

How do conversations help students develop strong mathematical ideas?

- When students talk about the mathematics they are doing, they are able to share and compare their methods and solution strategies thus gaining a wide repertoire of useful strategies.
- Math conversations help students learn new, possibly more effective, approaches by hearing others.
- Conversations allow students to examine their own concepts and ideas in light of others' questions and/or counter assertions.
- Conversations help clarify confusion as students comment on each other's methods and ask each other questions.
- Talking mathematics allows students to articulate and refine their ideas.
- Discussing ideas and strategies with classmates results in the creation of new mathematical ideas and theories.
- As students say things out loud, they can "hear" their errors.
- Talking supports the "construction" of new mathematical understanding.

What does a math conversation look/sound like?

- Students sharing observations, strategies and personal stories connected to the math experience.
- Students seeking clarification by asking questions.
- Students speculating and posing new questions.
- Students brainstorming, estimating and thinking hypothetically.
- Because students are “thinking out loud” the talk is characterized by “rough-draft” talk - hesitant, pauses, false starts, trial and error, etc.
- Talk is more student-directed.
- There is a greater percentage of student talk than teacher talk. The teacher trusts that students are capable of coming up with new ideas and making connections; therefore, he/she doesn’t do all of the thinking for the students.
- There is a constant revision of ideas
- Kids listening to one another and valuing others’ comments.
- Students justifying their strategies and solutions.
- Students constantly searching for patterns to use in analyzing mathematical situations.
- Students agreeing, disagreeing, challenging one another’s ideas.
- Students clarifying and revising thoughts, strategies, ideas, and solutions.
- Students building off one another’s ideas.

Teacher’s role:

- Provide worthwhile mathematical problems; ones that challenge each students’ thinking.
- Be curious and wonder about mathematics.
- Ask open-ended questions.
- Model patterns of discourse, i.e. making an argument, asking questions (see below for examples).
- Respect and value all contributions.
- Incorporate small group work regularly.
- Model active listening.
- Have students discuss in small groups before whole group convenes.
- Allow more wait time; allow time for everyone to think.
- Share your own thinking as you work through problems.
- Think aloud yourself.

- Pose questions and problems.
- Allow informal language; don't expect polished speech.
- Be a co-participant in the conversation.
- Use mathematical vocabulary but don't demand that students use the vocabulary when sharing.
- Struggle for solutions with the students.
- Participate without dominating. Know when to intervene.
- Don't interpret everything the students say and repeat to the group (if every comment is filtered through the teacher, there is little chance that the students will develop a conversation among themselves).
- Do math yourself; participate in a mathematical culture.

Questions to Encourage Math Talk

These questions and prompts not only encourage students to talk mathematically, but they are instrumental in getting students to reason mathematically and prove their assertions, make connections, and use representations as they work together to solve problems.

· How might you start solving the problem?

· What problem solving strategy might you use?

· Do you agree with Susie's explanation?

· Explain why you think your answer is reasonable.

· Who can explain what Jonah said using different words?

· Did anyone think about the problem in a different way?

· Does anyone have any questions they want to ask Jose about his solution?

· What do you notice about…?

· What do you find interesting?

· So… let's see…I wonder what would happen if...?

· Would anyone like to add to what JaNita just said?

- Would you explain that in a different way?
- Will someone say what John just said in a different way?
- Will you explain that again so that everyone can hear?
- Will you say a little more about that?
- Do you think there are cases where that wouldn't work?
- Is Kaia's idea very different from your idea?
- Does that seem right to you, Lisa? How could you convince yourself?
- Does this remind you of any other mathematical investigations you've done?
- Would it help you to try to solve a simpler problem?
- What can you tell me about…?
- Do you agree with what Yoshi said?
- How did you work it out?
- Does that answer make sense?
- Talk about why your answer makes sense.
- What strategy did you use?
- How is Joe's strategy different/the same as John's strategy?
- Do you see any patterns?
- Could you do it another way?
- Can you convince each other that you have found all of the possibilities?

The above questions are specific for providing students opportunities to share their thinking and strategies.

Sentence Starters to Help Students Communicate

- I think the answer is_____ because_____.
- I know that _____ because _____.
- I don't know _____ but I do know _____.
- This reminds me of _____ so _____.
- I figured it out by _____.
- If _____ then _____.
- I'm wondering about _____.
- I'm not sure about _____.
- I agree with _____ because _____.
- I disagree with _____ because _____.
- I can prove that _____.
- My answer is reasonable because _____.
- My strategy was _____.

Resources:

Chapin, S. et al. (2013) *Classroom Discussions in Math.* Sausalito, CA: Scholastic.

Corwin, R. (1996). *Talking mathematics: Supporting children's voices*. Portsmouth, NH: Heinemann.

Frailey, K. (2002). *Talking to learn: The potential of exploratory conversations in helping children learn mathematics*. Unpublished Doctoral Dissertation.

Process Standard: Communication Writing to Learn Mathematics

Writing in math class supports learning because it requires students to organize, clarify, and reflect on their ideas – all useful processes for making sense of mathematics. In addition, writing can be useful for assessment, providing insight into students' understandings and misconceptions about the content they are studying. Writing in math class isn't meant to produce a polished product, but rather to provide a way for students to reflect on their own learning and to explore, extend, and cement their ideas about the math they are learning. Teachers should pay attention to what the students write, not how they write it (Burns, 2007). It is important to make sure students understand that they are writing to support their learning and not to create a perfect piece.

It is helpful for students to keep a math journal for their writing. Listed below are some suggestions for incorporating writing in math instruction:

- Write about the strategy or strategies that you used to solve the problem.
- Write about how you know the answer that your group came up with is the correct answer.
- Write about what you learned today during math workshop.
- Write about what you are unsure about or confused by? Write about any new questions you have. Did solving this problem make you wonder about anything?
- Write about what was easy for you and what was difficult for you?
- Draw a picture to show how you solved the problem.
- Write about the strategy that you used and convince me that it was the best strategy for you.
- Write a story about what (addition, subtraction, division, fractions, patterns, etc.) means to you. Why is it important? When do you use it?
- Write about how (addition and subtraction, multiplication and division, fractions and decimals) are alike and different.
- Write about two shapes that we have been learning about. How are they alike and how are they different?
- Write about how you would explain to a Martian how to measure the perimeter of the classroom. Use this idea for any new math content being taught.

- Write a letter to the principal explaining to him/her what it means to be a good problem solver.
- Write a class book modeled after a piece of children's literature incorporating a mathematical concept. For example, after reading *The Doorbell Rang* and figuring out how many cookies each child should get, students can write their own division stories using new characters, setting, and context.

Students do not have to write about every math problem/activity they do in class. Incorporate writing in math once or twice a week to help students ponder new mathematical ideas and reflect on their new mathematical learning. Don't forget to include these pieces as valuable assessment information!

Process Standard: Reasoning and Proof

Instructional programs from prekindergarten through grade 12 should enable all students to--

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Reasoning is central to making sense of and learning mathematics with understanding.

What does reasoning and proof look like in the classroom?

- Making discoveries or drawing conclusions as a result of thinking, explaining or justifying an idea.
- Forming conclusions, inferences, and judgments.
- Being expected to provide justifications and explanations. This holds children accountable for the assertions they make and the solutions they offer.
- Explaining how and why they solved the problem in the way that they did.
- Finding and using patterns to analyze mathematical situations. Recognizing patterns is the key to the understanding of mathematical concepts.
- Defending ideas, strategies, and solutions, correct and incorrect.
- Making conjectures and supporting them by gathering evidence and building valid mathematical arguments.
- Examining, exploring, thinking about, and discussion a variety of mathematical possibilities.
- Developed through consistent use in many contexts.

Questions teachers can use to encourage reasoning and proof in the classroom:

- How did you get your answer?
- Tell me how you thought about that.
- Can you solve the problem in another way?
- Why does your solution work?
- Do you think that strategy will always work?
- What discoveries did you make?
- Did you notice any patterns?

Through the use of reasoning, students see that math makes sense (NCTM, 2000).

Process Standard: Connections

Instructional programs from prekindergarten through grade 12 should enable all students to--

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

NCTM (2000)

What does it mean to make connections and why is it important?

- Making connections involves a rich interplay among mathematical topics, between mathematics and other subjects, and between mathematics and their own interests.
- Students should be encouraged to connect mathematical concepts to their daily lives.
- Connections should be explored and capitalized on in helping students make sense of the mathematics being examined.
- Students should connect existing knowledge and background experiences to make sense of new mathematical ideas.
- Connections should be woven into daily practice.
- Teachers need to help students be conscious and aware of the connections they make.

Questions teachers can use to encourage students to make connections:

- Does this remind you of anything we've done before?
- Can someone think of a time when you've needed to….(measure, add, subtract, etc.)?
- How is this idea related to…(addition, subtraction, multiplication, etc.)?
- When might a scientist need to use what we're learning today?
- How is this important to you in your everyday lives?
- Can you use what we learned about addition to help you solve this new problem?

- This reminds me of the problem we solved last week. What patterns did we discover when solving that problem and how can you use that same pattern to help you solve this new problem?

Process Standard: Representation

Instructional programs from prekindergarten through grade 12 should enable all students to--

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

NCTM (2000)

What is Representation and why is it important?

- Representations include models, manipulatives, drawings, pictures, equations, diagrams, tables, charts, graphs, symbols, mental images, words, and ideas.
- Representations are necessary to students' understanding of mathematical concepts and relationships.
- Representations allow students to communicate mathematical approaches, arguments, and understanding to themselves and to others.
- Representations allow students to recognize connections among related concepts and apply mathematics to realistic problems.
- Students should represent their mathematical ideas in ways that make sense to them, even if those representations are not conventional.
- Students should also learn conventional forms of representation in ways that facilitate their learning of mathematics and their communication with others about mathematical ideas.

Questions to encourage representation:

- Can you illustrate what you're saying using your Base 10 blocks (or whatever manipulative being used)?
- How can you organize your results?
- Can you display your data in a graph?
- Can you organize your results in a T-chart?
- Will you show me your strategy using a different manipulative?
- Will you show me how you arrived at your solution?
- Will you draw a picture to show your findings?

- Describe the strategy that you used to find the answer by writing it down in your math journal.
- Show me several different ways that you can present your findings to the class.

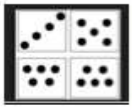
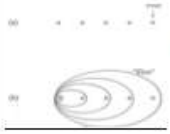
Number Sense

"With strong number sense, children become more apt to attempt problems and make sense of mathematics. It is the key to understanding all math."
(Shumway, pg. 8, 2011)

Number sense is a construct that relates to having an intuitive feel for number size and combinations as well as the ability to flexibly work with numbers in problem situations in order to make sound decisions and reasonable judgments.

Over 90 percent of the computation done outside the classroom is done without pencil and paper, using mental computation, estimation, or a calculator.
Cawelti, Gordon. *Handbook of Research on Improving Student Achievement,* Third Edition. Educational Research Service, copyright 2004

Number Sense Trajectory

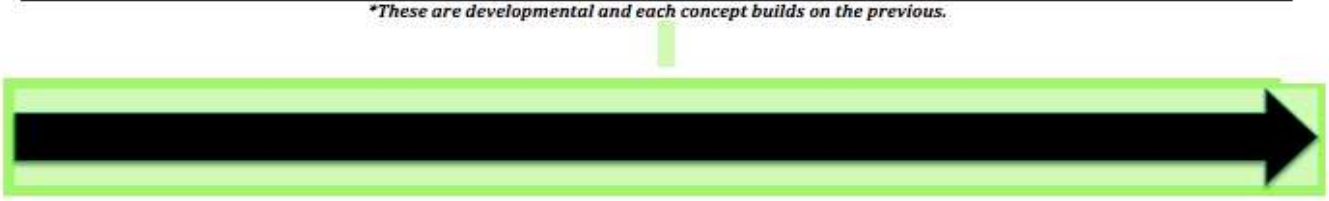
Subitizing	Magnitude	Counting	One-to-One Correspondence	Cardinality	Hierarchical Inclusion	Conservation
The ability to see small amounts without counting. Perceptual: Quantities less than 5. Conceptual: Larger quantities that can be seen as groupings.	The ability to tell which of two sets has more without counting	The ability to say the rote count sequence. Meaning is given to the sequence with 1:1 correspondence	Saying one number name for each object counted.	When a set of objects is counted, the last number said tells how many in all.	Smaller numbers are nested inside of bigger numbers. Idea of one more and one less	The number of objects remains the same when they are rearranged spatially.
Example	**Example**	**Example**	**Example**	**Example**	**Example**	**Example**
		"1,2,3,4,5,6,7,8,9,10"		Student is asked," How many (apples) did you count? "5" If student recounts from one, this is not cardinality.		

**These are developmental and each concept builds on the previous.*

Number Relationships Trajectory

Part/Whole Relationships	Compensation	Unitizing
The understanding that a given number has different parts. Composing and decomposing numbers, understanding that 6 is made up of 1 &5, 3&3, 2&4.	The ability to see the parts of a whole and are able to compensate and work flexibly with numbers.	Once students begin to gain a solid understanding of the preceding number sense, then they can construct unitizing as they work with larger numbers. With unitizing students understand that numbers take on different meanings not just as counting.
Example: In our classroom there are 8 students wearing sneakers. Some of the students are wearing Nike and some of the students are wearing Addidas. How many students are wearing Nike and how many could be wearing Addidas?	**Example:** If I know that 5+1=6, then I also know that 4+2=6 because 4 is one less than 5 and 2 is one more than 1.	**Example:** 10 ones can be thought of as a bundle or group of ten. 10 tens can be thought of as a hundred. OR Depending on its place, 2 can represent just two, or it can represent 2 tens, 2 hundred, etc.

These are developmental and each concept builds on the previous.



Shumway, J. *Number Sense Routines Building Numerical Literacy Every Day in Grades K-3.*Stenhouse, 2011

**For more explanation about number sense progression, see Young Mathematicians at Work: Constructing Number Sense, Addition, and Subtraction by Catherine Twomey Fosnot and Maarten Dolk (2001a) as well as the work of Douglas Clements (1999, 2007, 2008; Sarama and Clements 2009) of the University of Buffalo.*

Ideas to Help Students Build Number Sense and Progress

Help students develop a deeper sense of cardinality; give them counting activities with concrete objects. The students should progress to creating sets of counters that matches a set on a card.

Address “more, same, less” relationships, students should be asked the following questions equally during activities - “which set is more” and then asked “which set is less”. Students should also be given opportunities to label sets (concretely and pictorially) with less, same and more cards.

- Teach numeral writing and recognition with similar numbers together so that the students can identify their similarities and differences. The calculator is a good tool for numeral recognition where student can find and press the correct number (especially numbers they can relate to-- ages, number of brothers and sisters, number of windows in classroom, etc). (**Zaner Bloser)**
- Practice frequent short drills (using movement and in rhythm) of counting on and backwards to improve oral counting. Using the calculator is also helpful because the students get to see the numbers as they say them with the beat.
- Utilize concrete objects when counting on and counting back. Hide some objects under a cup or piece of paper. The students identify how many are hidden and then begin counting on to determine the total represented.
- Continue building number sense by giving students opportunities to learn and understand relationships between numbers using patterns. Expose the students to common number patterns (such as dice or dominos) and ask the students to make the patterns on construction paper. Introduce different patterns for the same number as the students begin to learn the patterns.
- Assist students with “one and two more/less” relationship. Ask them to find the number that is one less (more) or two more (less) than the number represented on the dot plate or domino. Students can also take turns reading/stating the resulting number sentence. The calculator can also be used to review the relationship of one or two more and less.
- Allow students to practice showing the relationship of numbers to 5 and 10, use a ten frame (see teacher resource section of the guide) and ask the students to represent the number in the frame. Students can also practice more and less relationships using the frames. The calculator can also be used to help students understand the relationship between numbers and 5 or 10 by pressing f.J5 or 10 -:.
- Focus on a particular number throughout the investigation to help students develop part-part-whole relationships. Students use different materials and formats to create the number. It is important that students say or “read” the parts aloud and/or draw or write them down on some form of recording sheet.

- Introduce missing-part relationships by focusing on a particular number as well. A portion of the designated amount of materials is hidden (under a cup or piece of paper), and the students determine the hidden amount.
- Extend the four number relationships (visual/spatial relationships, more/less, anchors/benchmarks of 5 and 10, and part-part-whole) to numbers between 10 and
  20. It is recommended to continue to use concrete objects and visual representations to show the relationships as was done with numbers up to 10.
- Associate the numbers with images when addressing doubles (6 + 6 = 12). The students should draw pictures or make posters for each double. The calculator can be used to assist students with identifying doubles. The other relationships (more/less, part-part-whole, anchors with 5 and 10) should also be integrated into the study of doubles.
- Connect numbers with objects/situations that students can relate to, especially for estimation.

Teachers are encouraged to use the following prompts to help young children begin to understand estimation:

- More or less than_____? Will it be more or less than 10 footprints?
- Closer to____or to_____? Will the apple weigh closer to 10 cubes or closer 30 cubes?
- Less than___, between _ and__, or more than_____? Are there less than 20, between 20 and 50, or more than 50 cubes in the Unifix bar?
- About_____? Use one of these numbers: 5, 10, 15, 20, 25, 30, 35, 40, … About how many footprints?

Other activities for relating numbers to the students' world include:

- Write a number on the board for the students. Include a unit (dollars, hours, cars, meters, minutes, etc) and ask the students to state what they think of when you say the number with the unit. Change the units and ask the students for their thoughts again. During another time keep the unit the same and change the number.
- Pick any number, large or small, and a unit with which the students are familiar. Then make up a series of questions to determine if it is reasonable. *Could the teacher be 15 feet tall? Could your living room be 15 feet wide? Can a man jump 15 feet high?*
- Pick any number (such as seven) and have groups of children find ways to tell about that number.

- Graph situations that students can connect to, for example their favorite color. Once a graph is made, it is very important to take a few minutes to ask as many number questions as is appropriate for the graph and to encourage student to make up questions about the graph, as well. The graphs focus attention on counts of realistic things, an important connection. Equally important, graphs clearly exhibit comparisons between and among numbers that are rarely made when only one number or quantity is considered at a time.
- Develop “one more than/one less than” relationships to larger numbers using the base-ten-frames (or a similar tool). This activity can help the students develop their mental math abilities. Part-part-whole concepts using tools as the ten frames can also help students develop their mental math abilities. The students extend the strategies learned for single digit numbers to double digit numbers.

Strategies for Helping Children Master the Basic Facts

"All children are able to master the basic facts – including children with learning disabilities. Children simply need to construct efficient mental tools that will help them." (Van de Walle, 94, 2006).

"An efficient strategy is one that can be done mentally and quickly. Counting is not efficient. If drill is undertaken when counting is the only strategy available, all you get is faster counting." (Van de Walle, 95, 2006).

Strategies *must* be debriefed – Provide opportunities for students to discover and develop strategies as they solve story/word problems or as they investigate a category of facts you present. When a student suggests a new strategy make sure everyone else in the room understands how it is used. Don't be tempted to just tell them the strategy to use. Instead, continue to discuss strategies invented by the class and plan lessons that encourage them to invent and *practice* strategy use. Have students discuss ways that *they* can use to think of facts easily. Create a poster of strategies that students develop. Have the students create names for the strategies that make sense to them.

Practice Strategy Selection or Strategy Retrieval – After children have worked on two or three strategies, provide an opportunity or problem that allows them to select which strategy would be appropriate to solve certain facts.

Children should NEVER be drilled on the basic facts apart from a focus on the strategy used. Doing this focuses on memorization as opposed to a strategic approach based on number relationships and sense-making.

There are 100 basic arithmetic facts, zero through nine. That can be reduced by half if students understand the commutative property. Still, that is a lot for students to memorize by rote. Below are some effective strategies that students can learn that will facilitate more successful retrieval of the basic facts. Try to avoid teaching these strategies directly. Providing well-planned opportunities for students to explore and discuss the number relationships evident in the facts will go a long way in ensuring that students will understand the strategies and use them instead of the non-efficient method of counting.

Strategies for Addition Facts

One-More-Than and Two-More-Than Facts

Students can find sums like 5 + 1 and 6 + 2 by counting on. With practice, they will begin to be able to do this mentally without having to count up 1 or 2. This strategy allows them to check off 36 of the math facts to be learned.

Facts with Zero – Nineteen facts have zero as one of the addends. Word problems involving zero will help students see that not all answers to addition problems are bigger. Soon they will realize that the sum is always the other number. 8 + 0 = 8, 0 + 4 = 4

Doubles – There are 10 doubles facts from 0 to 9. A good strategy is to have students draw pictures they can use to remember the doubles. Some suggestions that teachers have used include: 3 is the bug double (3 + 3 = 6 legs); 4 is the spider double (4 + 4 = 8
legs); 5 is the hand double (5 + 5 = 10 fingers); 6 is the egg carton double (6
+ 6 = 12
eggs); 7 is the calendar double (7 + 7 = 14 days); 8 is the crayon box double (8 + 8 = 16 crayons); 9 is the eighteen-wheeler double (9 + 9 = 18 wheels). Post pictures of these in the classroom. Students will begin to develop mental images of these and will most likely remember them. Challenge students to come up with their own examples, as well.

Near-Doubles – These are also called the “doubles-plus-one” facts. There are 18 of these. The strategy is to double the smaller number and add 1. These should be taught after students have an understanding of the doubles facts. To introduce this strategy, you can write ten near-doubles facts on the board and allow them to solve the problems and discuss in groups their ideas for “good” methods to use. Some students may double the smaller addend and add 1, while others may double the larger addend and subtract 1. If no one uses a near double strategy, write the corresponding doubles fact and ask them to consider how they could use that to help.

Make-Ten Facts – These facts all have at least one addend of 8 or 9. One strategy for solving these facts is to build onto the 8 or 9 up to 10 and then add on the rest. For 4 + 8, start with 8, then 2 more makes 10, and that leaves 2 more for 12. An activity to help with this is to give students two ten-frames, have them model each addend, and then decide on the easiest way to show the total. They should see that moving counters into the frame showing either 8 or 9 to fill that one up is a sensible choice – then they would just add the ones remaining to 10. Students need plenty of

time to investigate with ten frames. They also need to be able to explain what they are doing as they use the ten frames to practice the make-ten facts.

Doubles Plus Two – The preceding strategies cover all but 12 facts – 6 if you consider the commutative property. Of those 6, 3 of them can be remembered using the doubles plus two strategy. These are 3 + 5, 4 + 6, and 5 + 7. Students can double the smaller number and add 2. Students may also discover that you can take 1 from the larger addend and give it to the smaller. With this idea, 4 + 6 could be transformed into 5 + 5. The strategy involves doubling the number *in between*.

Make-Ten Extended – Three of the 6 remaining facts have 7 as an addend. Through the use of ten-frames, students can build onto 7 up to ten and then add the rest; therefore, 7 + 4 could be thought of as 10 + 1 = 11. Again, ten-frames are critical in helping students discover this strategy.

Strategies for Subtraction Facts

Subtraction as Think-Addition – This strategy encourages students to think "What goes with this part to make the total?" This think-addition strategy makes use of the known addition facts. For example, when given 9 – 4, children should be able to think spontaneously, "Four and *what* makes nine?" Typically, students rely on holding up 9 fingers and putting down 4. Or, they might count up from 4 or back from nine, using their fingers. Counting in this way is not an efficient strategy. Help students understand this strategy by using word problems that sound like addition but have a missing addend. *Logan had 5 Webkinz, Gran gave him some more. Then he had 12 Webkinz. How many Webkinz did Gran give Logan?* Students must have mastery of addition facts to be successful with this strategy. All of the facts can be learned using think-addition; however, there are several other strategies that students can use. These are more sophisticated strategies and should not be required of all students.

Build up through 10 – This includes all facts where the part or the subtracted number is either 8 or 9.

Example 14 – 8
Start with 8
How much to 10? (2)
How much more to 14? (4)
So 14 minus 8 is (6).

Back Down through 10 – Take 15 – 6, start with the total of 15 and take off 5, that takes you down to 10, then take off 1 more to get to 9. For 14 - 6, start with the total of 14, take off 4 to get to 10, then take off 2 more to get 8.

Strategies for Multiplication Facts

Doubles – Facts that have 2 as a factor are the same as the addition doubles and should already be known by students who know their addition facts. Students should see that 2 x 7 or 7 x 2 can be thought of as 7 + 7.

Fives Facts – Facts with 5 as the first or second factor are fairly easy for children to remember because they should be familiar with counting by 5's.

Zeros and Ones – Thirty-six facts have at least one factor that is either 0 or 1. These facts seem easy to adults, but children sometimes get confused as to why 6 + 0 stays the same, but 6 x 0 is always zero and 1 + 4 is a one-more idea and 1 x 4 stays the same. It is helpful to use story problems to help students develop the concepts behind these facts. Simply telling students that any number multiplied by zero is zero is not enough to develop a true understanding of that concept.

Nifty Nines – Looking at patterns in the nines facts can make the nines fairly easy to learn. There are 2 patterns that students can discover. The first is that the tens digit of the product is always one less than the "other" factor (the one other than 9). For example, 4 x 9 is going to be thirty-something (3__) because 3 is one less than 4. The second pattern is that the sum of the two digits in the product is always 9. These two ideas can be used together to get any nine fact quickly. For 7 x 9, 1 less than 7 is 6, 6 and 3 make 9, so the answer is 63. Children are not likely to "invent" this strategy so teachers should write the nines table on the board and encourage students to find as many patterns as they can. They can also look at the 9's row and column on a multiplication chart (3.ATO.9) and discuss the patterns they see. Since the conceptual basis for this strategy will not be readily apparent to the students, they should be given the opportunity to see that the rules work because of an interesting pattern that occurs in our number system. **Teaching the students how to use this strategy will be confusing to the students unless they are encouraged to discover the patterns themselves.**

Distributive Property - This approach could be known as the "friendly fact" approach. This fact uses the distributive property to use known multiplication facts to find unknown facts. For example, if a student does not know 8 x 7, he/she could use 8 x (5 + 2) = 8 x 5 + 8 x 2 or 7 x (10 - 2) = 7 x 10 - 7 x 2. This approach builds flexibility for more complicated multiplication later.

Strategies for Division Facts

Think-Multiplication - Mastery of multiplication facts and connections between multiplication and division are the key elements of division fact mastery. For example, for 36 ÷ 9, students should think, “nine times what is thirty-six?” If students know their multiplication facts well, 42 ÷ 6 becomes closely tied to 6 x 7 and 24 ÷6 becomes closely tied to 6 x 4, etc. Word problems continue to be a key method for creating this connection.

Near facts – Divisions that do not come out evenly are more common in real situations than divisions without remainders. A useful strategy for determining 60 ÷ 8, most people run through some of the multiplication facts in their heads – 8 times 6 (too low), 8 x 7 (close), 8 x 8 (too high) so it must be 7. That is 56 and 4 more. This process can and should be drilled.

The above information was adapted from *Teaching Student-Centered Mathematics* by John A. Van de Walle

What about timed tests?

Teachers who use timed tests believe that the tests help children learn the basic facts. This makes no instructional sense. Children who perform well under time pressure display their skills. Children who have difficulty with skills, or who work more slowly, run the risk of reinforcing wrong learning under pressure. In addition, children can become fearful and negative toward their math learning (Burns, 2007, pg. 192).

Using timed tests to help students learn the math facts:

- Does not measure children’s understanding.
- Focuses on memorization, not on appropriate strategy use. When students are under pressure to complete a list of facts in a short amount of time, they will not focus on choosing and using the strategies they have learned.
- Doesn’t ensure that students will be able to use the facts in problem-solving situations.
- Conveys that memorizing is what mathematics is all about, not thinking and reasoning to figure out answers.
- Has been contributed to the development of math anxiety.

Note: It is acceptable to have students practice their math facts at home, keeping in mind the emphasis must remain on strategy use. Educating parents about the strategies that students are learning cannot be overemphasized. A homework assignment might include a list of math facts for students to complete and should also include an area for students to describe the strategy they used to solve the fact.

Manipulatives

Manipulatives should be used everyday in the math classroom. They are necessary tools, especially for our visual and kinesthetic learners. Manipulatives make many concepts seem less abstract and confusing. Manipulatives allow students the opportunity to make changes within a problem to determine patterns and to draw conclusions. Students need opportunities to experience and manipulate tools that assist them in making sense of the math.

To supply the elementary mathematics classroom, the following manipulatives are recommended:

Grades K-2

Base 10 sets
Basic Balance/Scales Bears or other counters Calculators
Cuisenaire Rods Dice
Geared Clocks Geoboards Hundreds Charts
Inchworm Rulers Set Jumbo Foam Dice Linking Cubes
Measurement Devices/Containers (for length, volume, capacity, and equivalencies)
Money Collection Pattern Blocks
Plastic Chips or other counters Rulers
Square Tiles Thermometers
Two-color bean counters Unifix Cubes
Wooden Cubes
Wooden Geometric Solids

Grades 3-5

Calculators Cuisenaire Rods Decimal Squares (4-5) Dice
Fraction Circles Set Fraction Squares Set Fraction Tower Set
Geoboards
Geometric solids (wooden) Folding Geometric Shapes (4-5) Measurement Devices/Containers (length, volume, capacity, and equivalencies)
Pattern Blocks
Precision Balance with Weights Protractors (4-5)
Rulers Square Tiles
Square (wooden) Cubes Tangrams Thermometers
Unifix Cubes

Linking Mathematics and Children's Literature

Benefits of the Literature Connection

Linking mathematics instruction to children's literature has become increasingly popular in recent years for a variety of reasons. The math - literature connection motivates students, generates interest in math, helps students connect mathematical ideas to their personal experiences, accommodates children with different learning styles, inspires mathematical investigations, promotes mathematical reasoning, helps bring meaning to abstract math concepts, places math ideas in a cultural context, and provides a context for using mathematics to solve problems.

Ways to use Children's Literature in Teaching Mathematics

Many children's books are explicitly about mathematics, such as books about counting or shapes while other books have mathematics embedded within a larger context. These books are generally not perceived as "math books," but mathematics appears as a natural element within stories, problems, personal vignettes, or cultural events. Welchman- Tischler (1992) has classified the ways to use such books as follows:

1. To provide a context or model for an activity or investigation with mathematical content.

2. To inspire a creative mathematics experience for children.

3. To pose an interesting problem.

4. To prepare for a mathematics concept or skill.

5. To develop or explain a mathematics concept or skill.

6. To review a mathematics concept or skill.

Though any given book could likely be used in multiple ways, the common element in these various approaches is the intent to use literature to provide vicarious mathematical experiences based on real problems or situations of interest to teachers and students.
A list of math stories can be found in the Resources section.

Technology and Mathematics

Canvas

Canvas is a learning management system used in Rock Hill Schools to provide teachers, parents, and students a common platform on which to post assignments, documents, videos, and any other resource of interest for stakeholders. Many teachers put homework, assignments, and assessments on Canvas. We do not recommend that teachers use Canvas for math assessments since the assessment and analysis of student work is more important than the final answer. Canvas can and should be used to allow students to communicate, collaborate, and discuss issues with one another using a virtual platform.

Rock Hill Schools has 4 Instructional Technology Specialists who provide professional development on how to use Canvas to extend and organize learning.

iPads/Devices

Many upper elementary classrooms have access to iPads and other devices. These devices should not be used for skill-and-drill practice, but to extend meaningful learning. Apps used for fact practice should:

- Not have a time pressure
- Have conceptual basis for the operations
- Handle incorrect answers appropriately and constructively.

Source: https://tjzager.wordpress.com/2016/01/05/my-criteria-for-fact-based-apps/

iPads can also be used to record student strategies and thinking. Apps like the Showme app and Educreations allow the opportunity to record their verbal explanations as they write on a digital sketch pad. This provides the teacher an opportunity to assess and evaluate students without having to spend valuable class time meeting with them 1 on 1.

Promethean and Math Instruction

All classrooms in Rock Hill Schools have and use Activclassrooms. Teachers must understand the power to improve math instruction through the proper use of Promethean Boards. Here are some basic flipchart fundamentals as found on Promethean Planet.

1. Use proven planning strategies.

When constructing a flipchart lesson for use on the Activboard, begin by using the same lesson planning strategies you would typically use when building a lesson that is not being delivered on an interactive whiteboard. Establish lesson/activity objectives, expected outcomes, attainment targets being addressed and description of specific material to be covered during the lesson. A good lesson is a good lesson, regardless of the vehicle used for delivery, and it always begins with the aforementioned core elements.

2. Maximize student participation.

When considering the role of the student in the lesson, attempt to create multiple opportunities for interaction, response and feedback. You want the students actively participating in the lesson, not just serving as a passive audience to a presentation. Try to “mix up” the types of interaction as well. Students can interact verbally, come up to the board individually, work from their seats and reflect their device on the board, plug their device into the board using VGA adapter, or participate as a group using Activotes. A great way to keep students on task is to keep the Activotes out all the time and ask students to agree or disagree with what an individual has contributed to the discussion or lesson.

3. With experience comes confidence.

It's often daunting when you see other people's advanced flipcharts and you wonder 'how will I ever achieve that?' The answer is, in time you will! Any beginner can advance their own Activboard skills by experimenting with the software and starting with the more basic tools, such as rub and reveal.

Here's an easy three-point guide to this simple, but effective, technique:

- Type some text on the page.
- Write over the text with a pen.
- Then, in the lesson, you can use the eraser tool to erase the pen, 'revealing' the text beneath...

4. Take advantage of what you already have.

The new and improved resource library features more than 15,000 teaching resources including images, backgrounds, lessons, sounds, shapes, lines, grids, annotations, and flash activities...all searchable by keyword.

It also contains lesson-building templates for creating whole-group assessment pages including voting buttons, question page layouts and backgrounds. Before spending hours looking for resources elsewhere, have a look in the resource library first!

There are also thousands of pre-made flipcharts, weblinks and resource packs ready for download in the Resource Section on Promethean Planet. Planet offers teachers a place to upload their flipchart lessons and share them with others around the country. Search for ones you want and download them to use with your students. Once you download the lesson, it's yours to modify, add to, and use as you wish!

5. Sharpen your skills.

Once you are familiar with Activstudio or Activprimary, you'll be ready to move on to more advanced skills and techniques. Planet's Activtips section offers quick tips for integrating tools and techniques into various curriculum areas. Inspired by user suggestions and questions, the Activtips section has something for everyone.

The Forum and Blog is another fantastic area for sharpening your skills. With users just like you looking to sharpen their skills, there's always someone ready and willing to share an idea and offer a helping hand.

6. Share ideas and lessons with others.

Perhaps the greatest resource for creating top notch flipcharts is your colleagues. When logged in to the district network all district teachers, via our network servers, have access to a district wide 'drop box' where teachers can place flipchart lessons for mass consumption. You will find an icon on your desktop labeled Promethean Flipcharts. Divide and conquer is their motto and teachers are able to share the load when it comes to lesson planning. In doing so, they also tend to learn new techniques, master new Activsoftware tools and develop consistent, proven methods for lesson construction and delivery.

The Promethean Planet online resource library is a great place to post lessons as well, particularly because they are aligned to curriculum topics and student age ranges for you.

And don't forget... The Planet forum is a superb place to start sharing ideas and lesson plans with other Activboard users from across the globe. Source: www.prometheanplanet.com

Glossary of Terms

Algorithm - A specific set of instructions for carrying out a procedure or solving a problem.

Array - An arrangement (usually rectangular) of objects or numbers

Base - the face (of a polyhedron) or segment (of a polygon) that is being used as the reference to measure the height (altitude) of the polygon or polyhedron.

Benchmarks - Important units used as a referent for estimation. Benchmark numbers for fractions could be 0, Yz , l, l1/2, and so forth. Benchmark for measurements could be multiples of standard units. Benchmarks for whole numbers could be multiples of l0, l00, l000, and so forth.

Box Plot (Box-and-Whisker Plot) - A representation of data with a rectangular box extending from the lower quartile to the upper quartile of the data and two lines extending from the ends of the box to the extreme values of the data.

Cardinality of a Set - When counting a set, the last number word used (how many in a set)

Composing/Decomposing a Number - A strategy used to reinforce number sense. Involves conceptualizing a number as being made up of two or more parts: putting the parts together to make a number is *composing* a number; breaking a number into two or more parts is *decomposing* the number.

Compute Fluently - Use efficient and accurate methods for computing.

Cone - a solid with a circular face and a point that is not in the same plane as the face

Congruent - having the same shape and same size

Conjecture – informed guessing

Cylinder - a solid with two congruent circular faces that are parallel and connected by a curved surface

Dot Plot (Line Plot) - A representation of data made by making a horizontal line and placing an "x" or "dot" above the corresponding value on the line for every data element

Edge - A line segment where two faces of a polyhedron meet.

Faces - The flat polygonal regions of a polyhedron.

Geometric Pattern - A pattern involving geometric shapes so that students see a pattern that involves square or triangular numbers; the pattern typically involves multiplication facts

Histogram - A special type of bar graph that displays the frequency of data as rectangles with areas proportionate to the corresponding frequencies. Each bar has the same width. The width of the bar represents a range of values along the horizontal axis.

Inverse Relationship Between Operations - The inverse of a mathematical operation undoes the operation. For example, subtraction undoes addition.

Line Graph - In a line graph, points representing two related pieces of data are plotted and then connected by a line.

Line Plot (Dot Plot) – A representation of data made by making a horizontal line and placing an "x" or "dot" above the corresponding value on the line for every data element

Mean - (a measure of central tendency) also known as the average, the sum of the values in a data set divided by the number of items in the data set

Median - (a measure of central tendency) the middle value of an ordered set of values

Mode - (a measure of central tendency) the score or data value in a set that occurs the most often

Models - Concrete, pictorial, symbolic, verbal, and algorithmic representations.

Nets - A two-dimensional fold-up model of a polyhedron.

Networks – A graph or directed graph together with a function that assigns a positive real number to each edge.

Perfect Square - The product of an integer multiplied by itself. For example, 4 is a perfect square because 2 X 2 = 4.

Plane (common notion) - a two dimensional surface that extends infinitely in all directions

Polygon - A closed plane figure with *n* sides. The sides of a polygon are line segments.

Polygonal Regions - Flat surfaces enclosed by polygons.

Polyhedron - A closed three-dimensional object whose surfaces are formed by polygonal regions (e.g. prism, pyramid, octahedron).

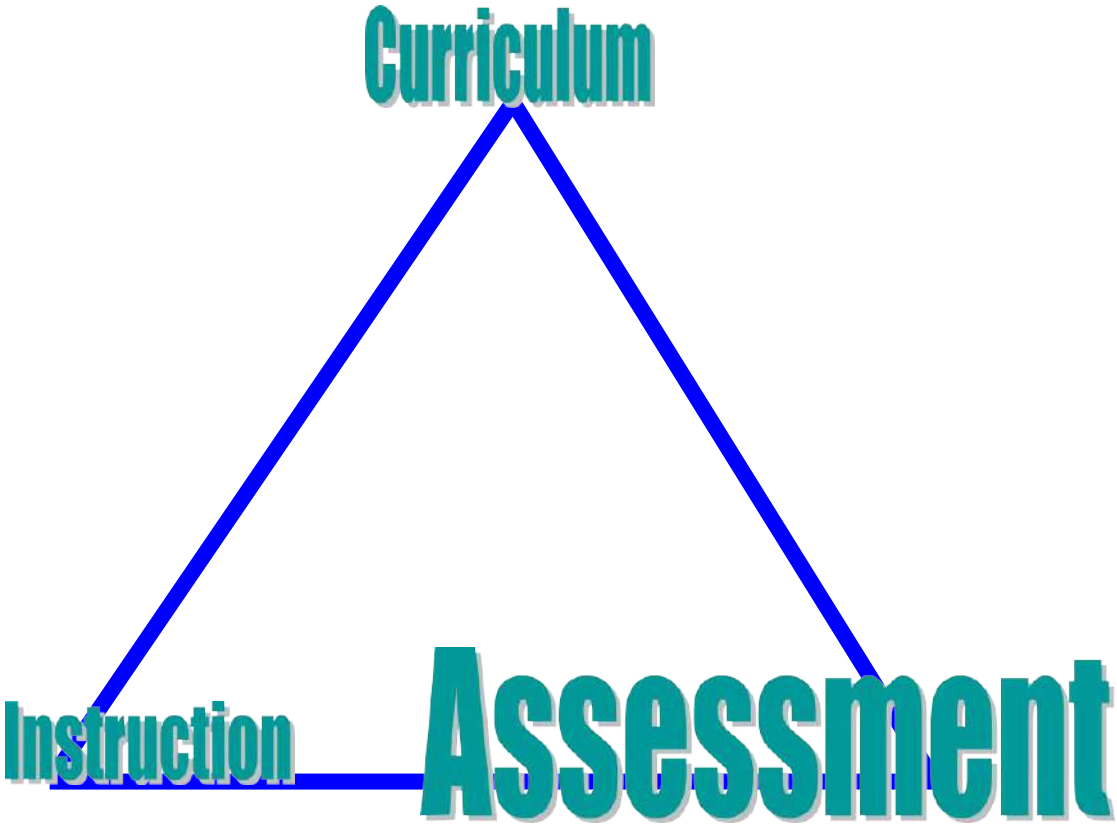
Prism - A polyhedron with two congruent, parallel bases that are polygons, and all remaining faces parallelograms.

Pyramid - A polyhedron with a polygon for a base and all other sides being triangles with one common vertex.

Similar - when two figures have the same shape and corresponding sides are proportional and corresponding angles are congruent.

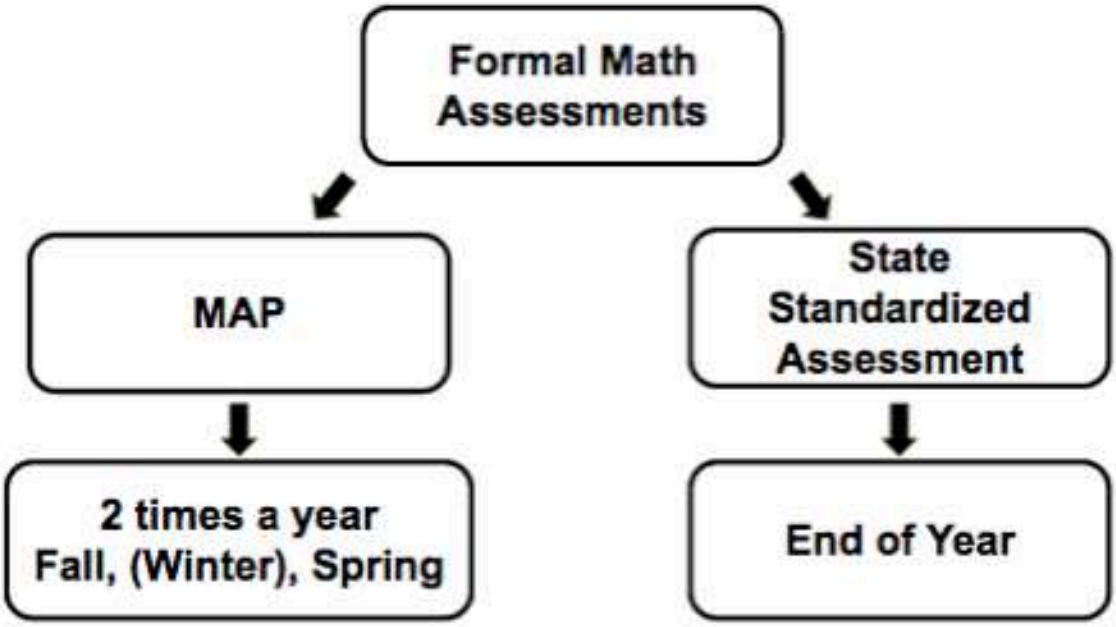
Vertex (of a polyhedron) - a point where three or more edges of a polyhedron meet

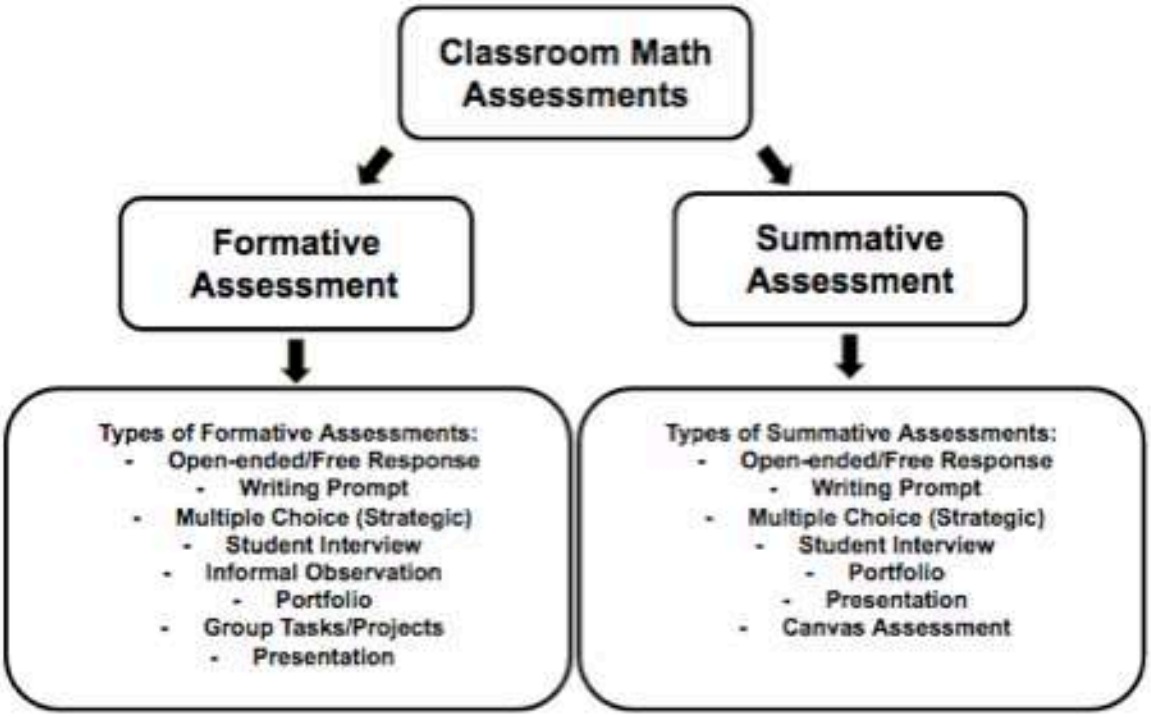
Assessment



"And while assignments and quizzes are important, merely checking whether or not answers are correct is insufficient. Assessments must also uncover what students understand and provide insights into how they think and reason. Key to assessing students' math learning is to delve into how students arrive at answers."
Marilyn Burns

Rock Hill Schools Math Assessment Program





Assessment

Mathematical assessments should:

- Be an integral part of instruction and should enhance student learning.

  Assessments that enhance mathematics learning become a routine part of instruction.

  Such assessments incorporate activities that are consistent with, and sometimes the same as, the activities used in instruction.

- Promote equity.

  In an equitable assessment, each student has an opportunity to demonstrate his/her understanding of mathematics.

  Assessments should allow for multiple approaches because different students show what they know and can do in different ways.

- Be a tool for monitoring student progress and evaluating student achievement.

  As teachers monitor student progress and achievement, they should focus on student understanding as well as procedural skills.

  As students monitor their own progress, the teacher's feedback on the assessments should be constructive and focused to help the students understand his/her mistakes and ways to improve.

  Assessments of isolated facts and skills should not be emphasized above assessments of conceptual mathematical understandings.

- Be a valuable tool for making instructional decisions

  When teachers use assessment to make instructional decisions they can make instruction more responsive to students' needs.

  Teachers should engage in ongoing analysis of teaching and learning by observing, listening to, and gathering information about students to assess what they are learning and the effects of the instructional tasks presented (NCTM).

Types of Assessments

Summative Assessments - Summative assessments are final assessments given periodically to determine at a particular point and time what students know and do not know. Examples of summative assessments:

State Standardized Assessment

District benchmark or interim assessment - MAP

Performance-based assessment - A well-defined task is identified and students are asked to create, produce, or do something, often in settings that involve real-world application of knowledge and skills.

Post- Test/ Unit Test

Formative Assessments - Formative assessments are part of the instructional process that provides the information needed to adjust teaching and learning while it is happening. Formative assessment is "for" learning, it informs both teachers and students about student understanding. Examples of formative assessments:

Pre-tests, quizzes, and post-tests

Math journals and portfolios

Academic prompts

Observations/ Anecdotal records

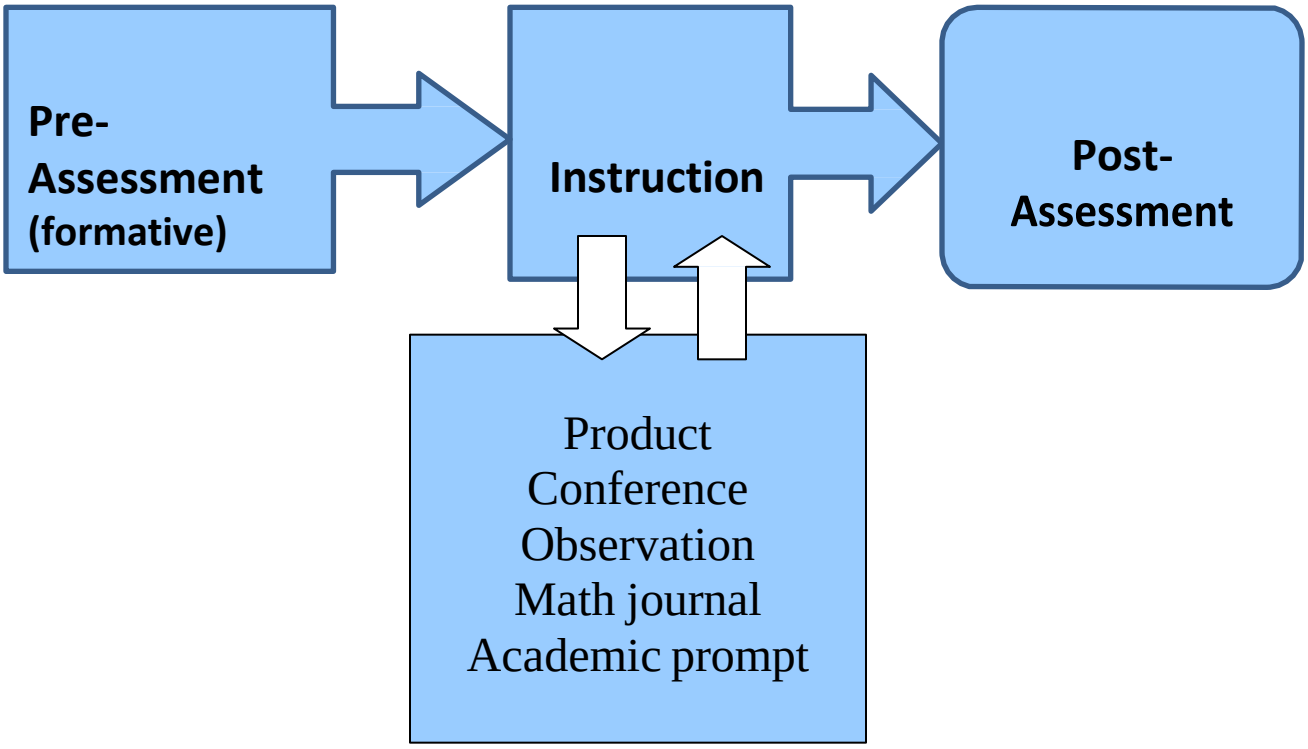
Conferences/Interviews

Exit Slips/Tickets to Transition (Leave)

Balancing Assessment

In a balanced assessment program of summative and formative assessments, teachers are able to gather information that is an integral part of the learning process for students and teachers. To better understand what students have learned, teachers need to consider information from tests, products created, observational notes, student led conferences, and communication among students and teachers. The more teachers know about individual students as they engage in the learning process, the better they can adjust instruction to ensure that all students continue to achieve progressively toward the learning goal.

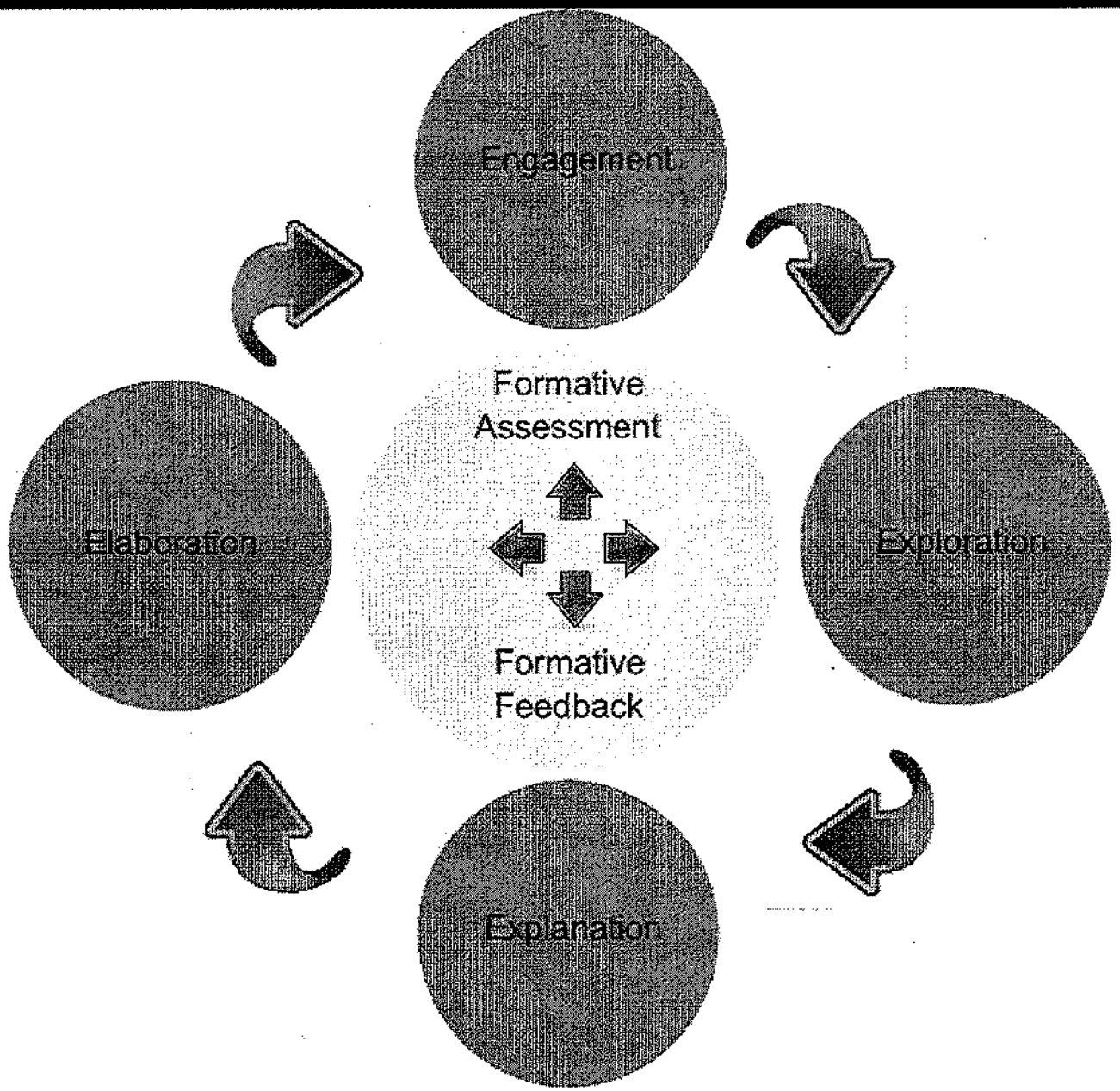
Making Formative Assessments and Summative Assessment Seamless



To design instruction to meet the needs of students, instruction and assessment should be intertwined in the unit's lessons. Formative assessments should be embedded to inform instruction.

The teacher administers a pre-test to determine what the students know about the concept. After giving the pre-test, several formative assessments should be presented or incorporated into the teaching. The formative assessments give the teacher immediate feedback about the students' understanding of the concept. The teacher is able to monitor and adjust instruction, so students can receive remediation, if necessary, or can be further challenged. The goal is for students to have opportunities to practice and master concepts and learning objectives before being given the post test.

The forms (observation, product, conference, etc.) of assessment can be used as formative or summative assessment tools. The design of the assessment determines the type of feedback the teacher will receive.


Engagement
Exploration
Explanation
Elaboration
Formative
Assessment
Formative
Feedback

Common Formative Assessments

What Are Common Formative Assessments?

- Periodic assessments collaboratively designed by grade-level teams of teachers.
- Designed as matching pre- and post-assessments to ensure same-assessment comparisons of student growth.
- Similar in design and format to district and state assessments.
- Should be a blend of item types, including selected-response (multiple choice, true/false, matching), and constructed-response (short- or extended).
- Student results should be analyzed to guide instructional planning and delivery.

Guidelines for Designing Common Formative Assessments:

- Identify standard/indicators for your grade level.
- Create no more than one to five learning targets for that assessment.
- "Unwrap" the standards for the concepts students need to know and be able to do.
- From those "unwrapped"' standards, determine Big Ideas that represent the integrated understanding students need to gain.
- Determine the level of rigor and the level of understanding (using the indicators' verb) to assess the concepts.
- Collaboratively design common formative pre- and post-assessments - aligned to one another - that assess students' understanding of the concepts, skills, and Big Ideas from the "unwrapped" standards.
- If the concept is new learning for your grade level, the pre-test should include prerequisite skills.
- Include both selected-response and constructed-response items.
- Guarantee that each target receives enough of a sampling to certify learning (generally five to ten questions per target area).
- Review items to determine if student assessment results will provide *evidence of proficiency* regarding the standards in focus; modify items as needed.

Benefits of Using Common Formative Assessments:

- Regular and timely feedback regarding student attainment of most critical standards. This allows teachers to modify instruction to better meet the diverse learning needs of all students
- Multiple-measure assessments that allow students to demonstrate their understanding in a variety of formats
- Ongoing collaboration opportunities for grade-level, course, and department teachers
- Consistent expectations within a grade level, course, and department regarding standards, instruction, and assessment priorities
- Agreed-upon criteria for proficiency to be met within each individual classroom, grade level, school, and district
- Deliberate alignment of classroom, school, district, and state assessments to better prepare students for success on state assessments

- Results that have predictive value as to how students are likely to do on each succeeding assessment, in time to make instructional modifications
- *Source:* Larry Ainsworth & Donald Viegut, *Common Formative Assessments: How to Connect Standards-based Instruction and Assessment* (Corwin Press, 2006)

Building and Using Formative Assessments

What should teachers do with the information collected from assessments?
After teachers have planned and gathered information collected from pre-assessments and /or other forms of data, the information should be interpreted to determine how the data is to be utilized to enhance student learning. In essence, teachers should be using several sources from which to evaluate student achievement.

Teachers should provide immediate feedback by:

- Differentiating instruction or presenting instruction in a new way
- Planning instruction that is challenging and engaging using the standards through enrichment and performance-based problems or tasks
- Focusing on concepts/ skills that students are having difficulty with
- Collaborating with other teachers who might experience similar challenges
- Continuing to monitor student progress
- Praising students for current accomplishments

Key points for pre- and post assessments:

- Pre- and Post assessments must assess the same objectives.
- Pre- and Post assessments should include strategies which will clearly identify students' strengths and weaknesses (from computation to problem solving and from concrete to abstract).
- Pre- and Post assessment methods should be congruent with the learning objectives.
- Pre- assessments should be designed to give immediate feedback.
- Pre- assessments should be analyzed for driving instruction, but not graded and recorded in the gradebook.
- Pre- assessments should give students a snapshot of their ability to apply certain skills or concepts. (Reassure students that they may not know all of the answers.)
- Pre- assessments should be a working document for students.
- Post- assessments should be graded; however, if the student did not show mastery on the post- assessment, students should have on-going opportunities to meet the targeted learning goal.

(paraphrased from) http://www-personal.umich.edu/~espring/bestPractices/pretest.html and http://www.coedu.usf.edu/ctc/4.HTM

Summative and Formative Assessments can be either formal or informal.

Formal assessments

- Have data which support the conclusions made from the test. These types of tests are usually referred to as standardized measures.
- Tests that have been tried before on students and have statistics which support the conclusion such as the student is performing below average for his age.
- The data is mathematically computed and summarized. Scores such as percentiles, stanines, or standard scores are most commonly given from this type of assessment.
- Formal or standardized measures should be used to assess overall achievement, to compare a student's performance with others at their age or grade, or to identify comparable strengths and weaknesses with peers.
- Most formal assessments are also **summative** in nature.

Informal assessments

- Assessments that are not data driven but rather content and performance driven.
- Informal assessments should be used to inform instruction.
- The most effective teaching is based on identifying performance objectives, instructing according to these objectives, and then assessing these performance objectives.
- Are sometimes referred to as criterion-referenced measures or performance-based measures.

http://content.scholastic.com/browse/article.jsp?id=4452

http://www.associatedcontent.com/article/918027/how_to_compare_formal_and_informal.html?cat=4

Establishing a Baseline for Math Instruction at the Beginning of the School Year

A pre-assessment should be used to guide instruction and to identify what students know and what they need to know.

Below are a range of data sources and pre-assessment tools that should be used to establish a baseline for math instruction at the beginning of the school year.

Permanent Records

- Report cards
- Test score data

Measure of Academic Progress (MAP) – Grades K-8

- MAP data indicate which students have met the benchmark for the grade and which students have not yet learned the grade-level material.

Formal Pre-assessment

- Student Interviews (1:1)
- Pre-tests

Informal Pre-Assessment

- Inventories and Surveys
- Anecdotal Records
- Checklists

Measure of Academic Progress (MAP)

MAP is:

- A computerized adaptive assessment that measures the students' ability levels in the five strands of mathematics.
- Administered to students in kindergarten through grade 8

Information MAP data provides for teachers:

- MAP data indicates which students are meeting their projected average yearly growth goals, and which students are not meeting growth goals.
- Defines flexible groups for instruction.
- Guides differentiated instruction.
- Links test results to skills and concepts included in state standards.
- MAP tests provide highly accurate results that can be used to:
    - Identify the skills and concepts individual students have learned.
    - Diagnose instructional needs.
    - Monitor academic growth over time.
    - Make data-driven decisions at the classroom, school, and district levels.
    - Place new students into appropriate instructional programs.

How the data informs the teacher's instruction:

- It provides useful information about where a student is learning and guides differentiated instruction.
- It provides information regarding a student's strengths and areas for improvement.
- Student growth and achievement status are both reported, so that teachers can make informed decisions about remediation and enrichment opportunities.

Information obtained from MAP data

MAP data can give you an indication of math levels and ranges that exist within your class and the number of students that fall within each of those math levels.

There are many reports that can be generated by classroom teachers to be used for instructional guidance. Various reports give you an indication of math levels and ranges that exist within your class and the number of students that fall within each of those math levels. There are also reports that can be generated to share student progress with individual students and parents.
For a complete list of reports available and their purpose:

https://teach.mapnwea.org/assist/help_map/ApplicationHelp.htm#UsingTestResults/MAPReportsFinder.htm%3FTocPath%3D____3

Log on to the NWEA website teach.map.nwea.org

Authentic Assessment: A New Approach to Assessment

With a new approach to assessment there is a shift in content, learning, teaching, evaluation/ assessment, and expectation.

Towards	Away From
Content: Rich variety of mathematical topics and problem situations	***Content***: Just arithmetic
Learning: Investigating problems	***Learning***: Memorizing and repeating
Instruction: Questioning and listening	***Instruction***: Telling
Evaluation: Several sources judged/ evaluated by teacher	***Evaluation***: Single test judged externally
Expectation: Concepts and procedures to solve problems	***Expectation***: Mastering isolated concepts and procedures

Major shifts in assessment practices:

Towards	Away From
Assessing students' fullest mathematical ability	Assessing only students knowledge of specific facts and isolated skills
Comparing students' performance with established criteria	Comparing students' performance with that of other students
Giving support to teachers and credence to their informed judgment	Designing "teacher-proof" / ready-made assessment system
Making the assessment process public, participatory, and dynamic (performances, exhibitions)	Making the assessment process secret, exclusive, and fixed
Giving students multiple opportunities to demonstrate their fullest mathematical ability	Restricting students to a single way of demonstrating their mathematic knowledge
Developing a shared vision of what to assess and how to do it	Developing assessment by oneself
Using assessment results to ensure that all students have the opportunity to achieve their potential	Using assessment to filter and select students out of the opportunity to learn mathematics
Aligning assessment with curriculum and instruction	Treating assessment as independent of curriculum or instruction
Basing inferences on multiple sources of evidence	Basing inferences on a single source of evidence
Viewing students as active participants in the assessment process	Viewing students as the objects of assessment
Regarding assessment as continual and recursive	Regarding assessment as sporadic and conclusive

Holding all concerned with mathematical learning accountable for assessment results	Holding only a few accountable for assessment results

Authentic Assessment in the Problem-Solving Classroom

Assessment in the problem-solving classroom provides students with the opportunity to express their learning through many modalities and resources.

These are some ways in which students can be assessed in the problem-solving classroom:

Anecdotal records
Checklists
Interviews/Conferences
Inventories/Surveys
Portfolios
Rubrics
Math Journals

Anecdotal Records – anecdotal records are brief or simple positive notes written about the student's interaction with the teacher, other students, the environment, and/or materials. Anecdotal records can be instrumental in capturing observations about what concepts the student understands and whether scaffolding or enrichment may be beneficial. These notes are usually informal and based on direct observation. Anecdotal notes are used to capture the richness of the learning experiences, provide written documentation in a portfolio, or guide further instruction or curriculum planning.

http://www.ncrel.org/sdrs/areas/issues/methods/instrctn/in5lk37.htm
www.fcs.utah.edu/info/cfdc/2610/anecdotal_records.doc

Checklists – checklists are a fast way to document whether students mastered a requirement for a lesson or unit. The advantages of the checklists are they are easy to develop and they clearly identify the learning expectation(s). The disadvantage is the teacher has limited feedback on the student's method or strategy.

Possible procedures for making/using a checklist:

- Write the students' names down the left-hand side of the paper.
- Write the expected behaviors, skills, or processes to be observed along the top of the paper.
- Write entries or note behaviors in the appropriate cells.
- Identify your key of symbols that represent the quality of the work/performance observed.

Sample Checklist:
(taken from Mathematics Assessment a Practical Handbook for Grades K – 2 by NCTM)

Week of: April 1	Rote Counting	Counts on	Counts back	Uses 1 to 1 correspondence
Jonathan	+ (80+)	__ (inconsistent)	+	+
Thomasina	- (to 19)	__	__	+

Interviews/Conferences – interviews and conferences are typically formal, planned conversations with students. They commonly focus on a preset of skills or topics and the questions are determined ahead of time.

Possible procedure for developing interview or conference questions:

- Plan a list of questions ahead of time. Have some questions for all students and a few for a select group of students.
- Be accommodating, and follow each student's lead.
- Pay attention to what the students are saying. Ask for clarification if needed.
- Give the students manipulatives, in case they cannot clearly express their understanding with words.
- Reword questions, if necessary. Do not show signs of frustration.
- Keep notes for future reference.
- Avoid questions that give you right answers (to definitions or procedures). Ask questions that give you an idea of how the students are thinking.

Sample Interview/Conference Prompts:

Tell me more about that.
Can you show me another way?
Help me understand.
Why did you …?
How did you know what to do next?
What else do you know about …?
What were you thinking when you …?

(taken from Mathematics Assessment a Practical Handbook for Grades 3 - 5 by NCTM)

Inventories/Surveys – inventories and surveys can give teachers information about a student's attitude about learning math. Inventories/surveys may be given at the beginning of the year or beginning of a unit and then given other times throughout the year to determine if attitudes have changed or are different for various topics.

Possible procedure for developing inventories or surveys:

- Identify what you want to know.
- Plan a list of questions ahead of time.
- The questions maybe read to younger students or typed on a form for older students.

Sample Inventory/Survey Prompts:
Indicate --- yes, almost always – sometimes – no, not really

I like using manipulatives to help me with my math.
I like learning about new things in math.
What I like best in math is ….
I listen to directions and I follow directions

(taken from Mathematics Assessment a Practical Handbook for Grades 3 - 5 by NCTM)

Sample Inventory/Survey Prompts:
Mark the face that matches your face.

This is how I feel about …

Math class…….…… ☺ ☹ **…..**

Multiplication …… ☺ ☹ **…..**

Writing about math …… ☺ ☹ **…..**

Participating in math class …… ☺ ☹ **…..**

Working with a group …… ☺ ☹ **…..**

(taken from Mathematics Assessment a Practical Handbook for Grades K – 2 by NCTM)

Portfolios – portfolios are a compilation of student work. This work should be authentic and created by the student (no commercially produced worksheet). The pieces maybe chosen by the student or chosen by the teacher; nevertheless, the students should know the reason for collecting their work. Student portfolios can also be digital portfolios, such as a blog or the Portfolio feature in Canvas.

Possible reasons for collecting student work:

- To display or praise work the students like the most or considers outstanding
- To demonstrate the students' development and understanding of a concept
- To illustrate a representative sample of the students' most prized work and work that shows their progress

Sample Portfolio Entries: The students should include an explanation with work.

Work that showed what the students have learned
Tasks that were really new and hard
Work the students did with a group
Ideas from their journals
Work that demonstrates specific concepts addressed in the current unit

Rubrics – a rubric is a set of criteria to evaluate an assignment and indicate the level of completion. Rubrics can be broad or detailed, and they can be holistic or analytic. Broad rubrics identify the levels of learning for any assignment or problem. The detailed rubric has some parts that are like the broad rubric, but will also include certain expectations for a particular task. A holistic rubric evaluates the student's performance based on the whole task where one score is assigned. An analytic rubric evaluates and gives each part of the task a score. The scores are then totaled to represent the entire task.

Possible procedures for developing a rubric:

- Clear explanations of what is considered superb work and what is required as proof of learning
- A set of behavior criteria that explains the lowest level of performance
- A set of behavior criteria that is unmistakably higher than what a typical student would perform
- A description(s) of the behavior that represents the middle level of learning

Sample Rubrics:

A Holistic Rubric
(taken from Mathematics Assessment a Practical Handbook for Grades 3 - 5 by NCTM)

4	Fully accomplishes the purpose of the task. Shows a good understanding and use of the main ideas of the problem. Communicates thinking clearly, using writing, calculations, diagrams and charts, or other representations.
3	Substantially accomplishes the purpose of the task. Shows a reasonable understanding and use of the main ideas of the problem. Communicates thinking fairly well, but may use only one representation.
2	Partially accomplishes the purpose of the task. Shows partial but limited grasp of the main mathematical ideas. Recorded work may be incomplete, misdirected, or not clearly presented.
1	Shows little or no progress in accomplishing the purpose of the task. Shows little understanding of the main mathematical ideas. Work is almost or completely impossible to decipher.

A Broad Analytic Rubric
(taken from Mathematics Assessment a Practical Handbook for Grades K – 2 by NCTM)

Understanding the problem	0	Complete misunderstanding of the problem
	1	Part of the problem misunderstood or misinterpreted
	2	Complete understanding of the problem
Planning a solution	0	No attempt, or totally inappropriate plan
	1	Partially correct plan based on correct interpretation of part of the problem
	2	Plan that could have led to a correct solution if implemented properly
Getting an answer	0	No answer, or wrong answer based on an inappropriate plan
	1	Copying error; computational error; partial answer
	2	Correct answer and correct label for the answer

Math Journals – a math journal is a journal entry in which the student writes about the experience received from a specific math investigation or problem solving activity. Journal writing is a very valuable assessment technique because it provides an opportunity for students to think through and then communicate through writing what was done and what was required to solve the specific math problem. Math journaling also provides an opportunity for students to reflect and self-assess what they have learned.

Possible procedure for using a math journal:

- Should be written in at the end of the exercise
- Should be written in when introducing a new concept
- Should contain specific details regarding areas of difficulty and areas of success or to determine growth in problem solving
- Should not take more than 5-7 minutes

Sample Math Journal Prompts:

If I missed_____ I would have to_____.
Tips I would give a friend to solve this problem are.....
Could you have found the answer by doing something different? What?
Was this hard or easy? Why?
I knew my answer was incorrect when ….
I found my mistake when …
My answer makes sense because…
What other strategies could you use to solve this problem?
Were you frustrated with this problem? Why or why not?
What decisions had to be made when solving this problem?
Is math your favorite subject? Why or why not?
I still wonder …

http://math.about.com/library/weekly/aa123001a.html

Creating Assessment Tasks:

Assessment tasks should shift the focus from correct answer tasks to tasks that require an explanation. One way to do this is when the assessment asks for specific answers or skills, ask students to give explanations instead.

For example, instead of asking students the answers to 6 + 6 and 6 + 7, ask them to write in their math journals about how knowing the answer for one problem helps with the other.

Instead of giving a routine word problem, provide the answer and ask for a justification.

Instead of having students do an entire page of adding fractions with common denominators give them the problem 1/4 + 3/2 and ask, "Why is a common denominator necessary?" or "Is there a strategy for getting the solution without using common denominators?" Have them use their tools to support their written explanations.

Sample Assessment Tasks:

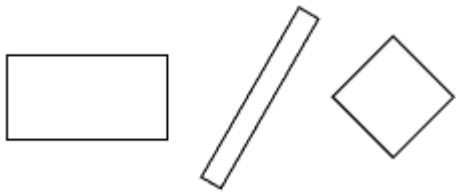
Logan has 34 marbles, Jennifer has 27 marbles, and Chris has 23 marbles. Write and solve as many problems as you can that use this information and represent adding two-digit numbers.

Dave says 13 ÷4 is 3 ¼. Martha says it's 3 R 1. Zach says they are both wrong. He thinks it's 3.25. Why are all three of them correct? Describe a situation where Dave's answer makes the most sense. Describe one where Zach's answer is reasonable.

How many different ways can you make 27 cents, using pennies, nickels, dimes, or quarters? Did you find all the ways? How do you know?

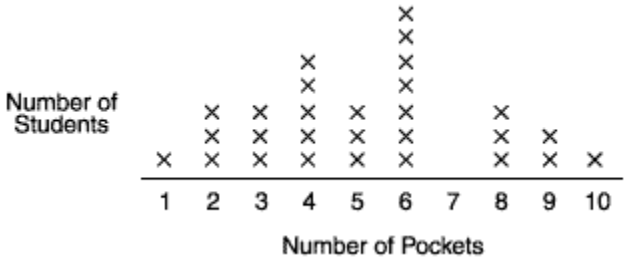
Make a triangle with one right angle and two sides of equal length. Can you make more than one triangle with this set of properties? If so, what is the relationship of the triangles to one another?

Before the lesson:
Tell me everything you can about these shapes:



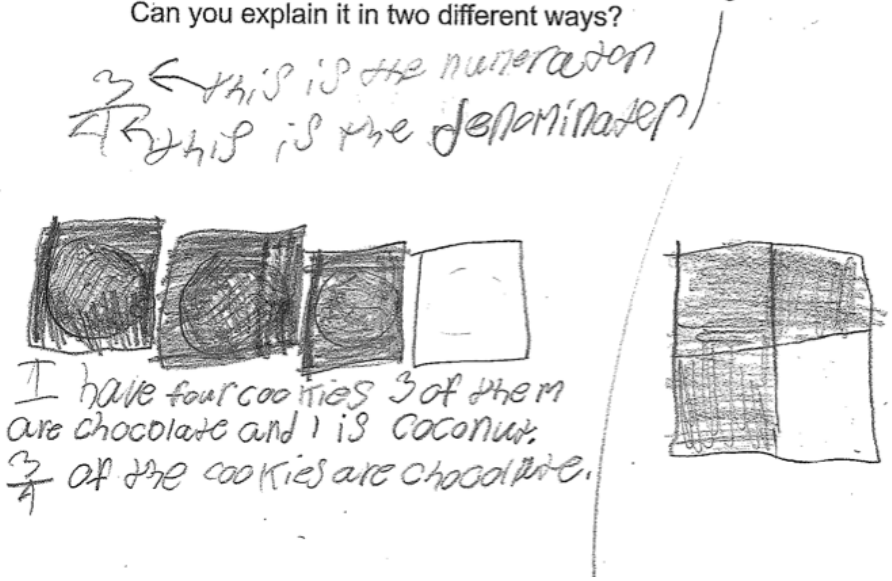
After the lesson: Have the students create a rubric of criteria. Ask the same question as above.

Here is a graph. What does it tell you?



Sample Tasks with Student Responses:

In the fraction ¾, how would you explain the meaning of the top number to a third-grader? How would you explain the meaning of the bottom number? Can you explain it in two different ways?

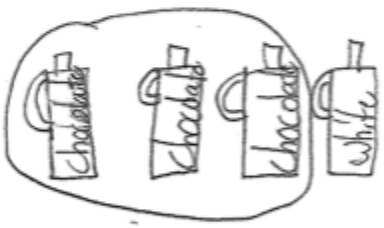


In the fraction ¾, how would you explain the meaning of the top number to a third-grader? How would you explain the meaning of the bottom number? Can you explain it in two different ways?

3→ 25+25+25=75
4→ 25+25+25+25=100

75=3 Quarters
100=4 Quarters

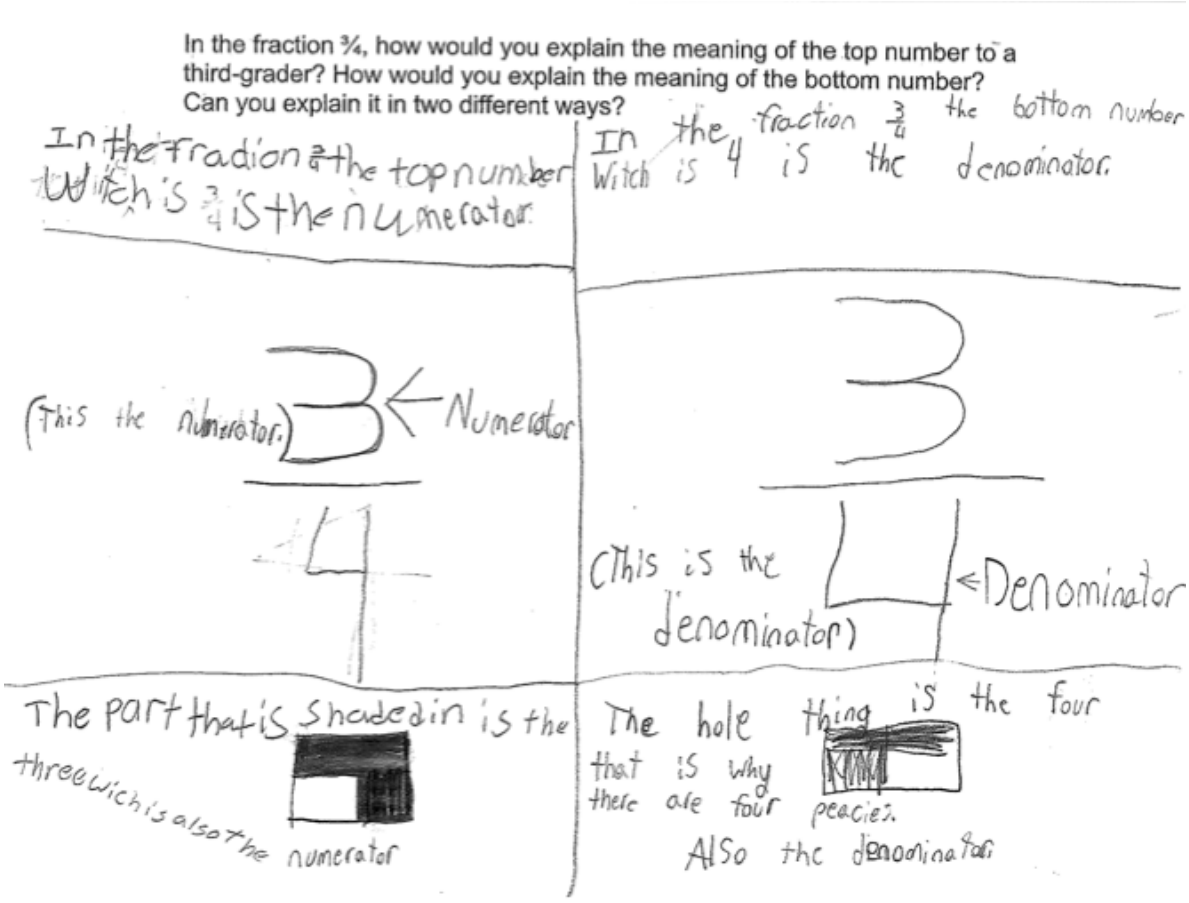
It's like quarters

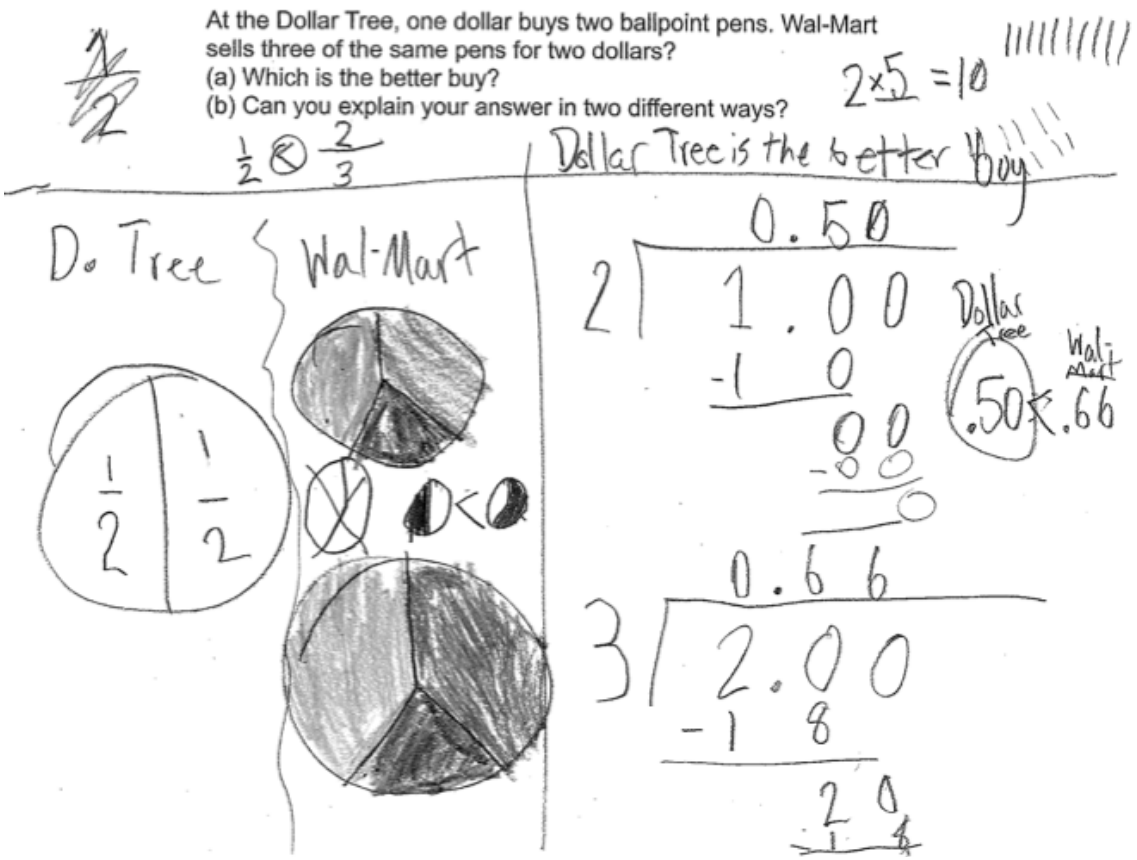


3 out of four milk are chocolate

It's like milk

In the fraction ¾, how would you explain the meaning of the top number to a third-grader? How would you explain the meaning of the bottom number? Can you explain it in two different ways?




At the Dollar Tree, one dollar buys two ballpoint pens. Wal-Mart sells three of the same pens for two dollars?
(a) Which is the better buy?
(b) Can you explain your answer in two different ways?
2×5 =10
1/2 ⊗ 2/3
Dollar Tree is the better buy
D. Tree
Wal-Mart
1/2
1/2
0.50
2 1.00
-1 0
00
-00
0
Dollar Tree
Wal-Mart
.50 < .66
0.66
3 2.00
-1 8
20
-18
2

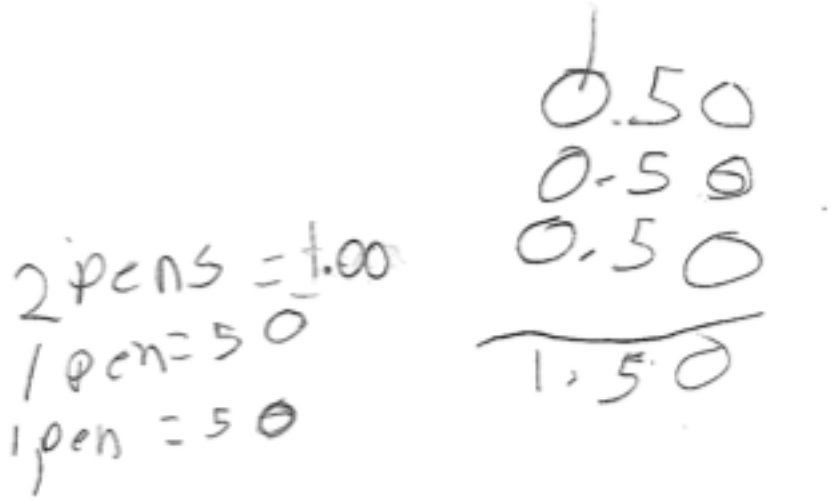
At the Dollar Tree, one dollar buys two ballpoint pens. Wal-Mart sells three of the same pens for two dollars?
(a) Which is the better buy?
(b) Can you explain your answer in two different ways?

Dollar Tree — 1.00 $\frac{1}{2}$ of 1.00 is .50 for each ballpoint

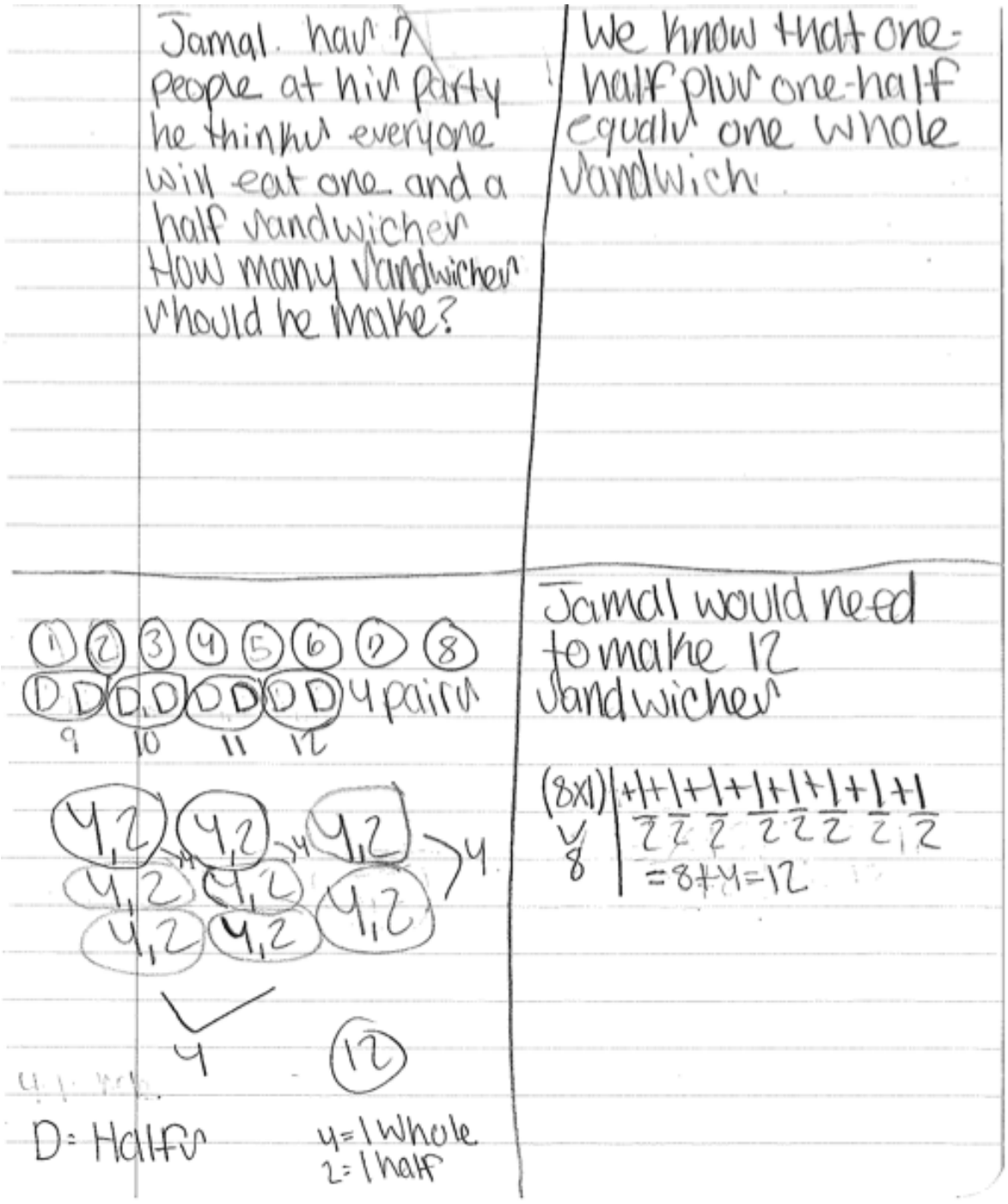
Walmart — 2.00 $\frac{1}{3}$ of 2.00 is about 66¢ for each ball point

Dollar is a better buy because 50¢ is less than 66¢

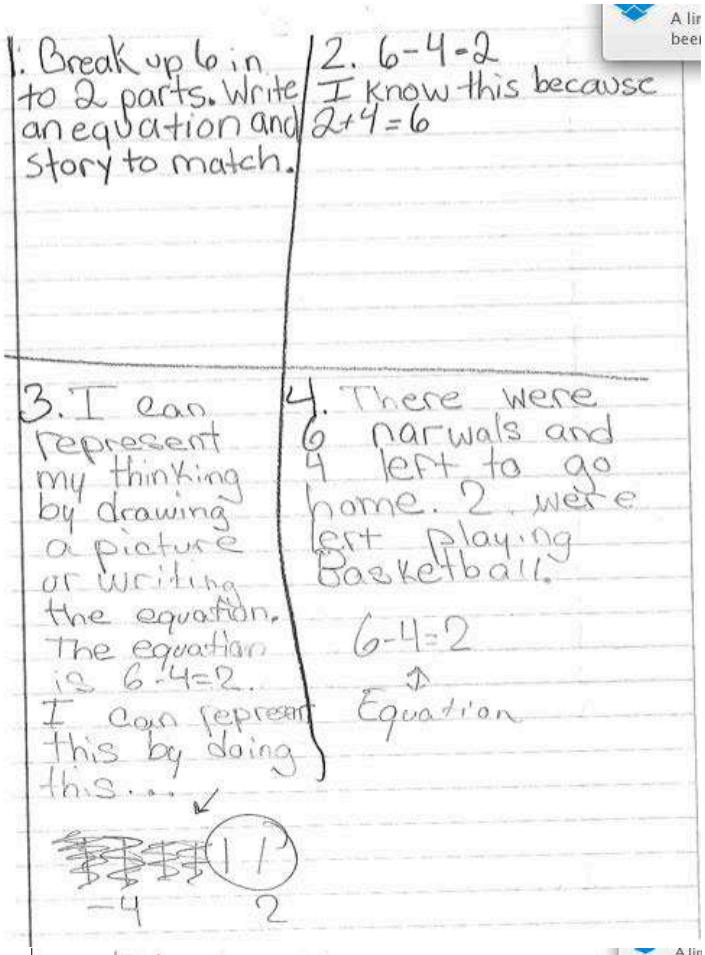
At the Dollar Tree, one dollar buys two ballpoint pens. Wal-Mart sells three of the same pens for two dollars?
(a) Which is the better buy?
(b) Can you explain your answer in two different ways?

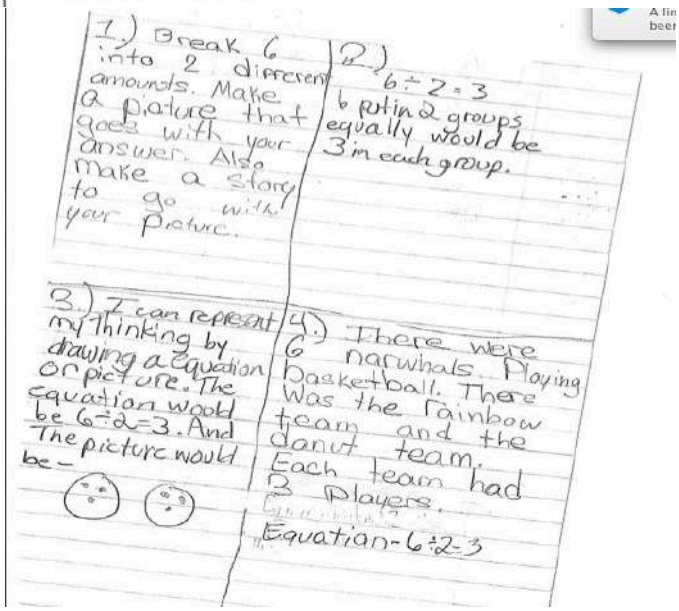


Jamal invited seven of his friends to lunch on Saturday. He thinks that each of the eight people (his seven guests and himself) will eat one and a half sandwiches. How many sandwiches should he make? Be able to explain your solution using two different strategies.



Think about the number 6 broken into 2 different amounts. Draw a picture to show a way that 6 things can be in 2 parts. Think up a story to go with your picture.





The assessment tools on the following pages will assist your grade level in developing common formative assessments. Each teacher should review his/her assessment using the Reflecting on Assessment tool on the next page. After the individual review, the grade level teachers should have collegial dialogue on how to best assess students to meet the specific learning goals.

What Makes a Quality Assessment?

The following questions can be asked in the assessment design process to ensure high quality assessments:

- Are the concepts and skills worth mastering? (Essential learnings)
- Are assessment questions, prompts, tasks aligned to the exact learning targets?
- Will this assessment give me an accurate picture of student understanding right now?
- Do the assessment items require thought and application of knowledge? (Rigor)
- Are there multiple right answers for assessment items?
- Does the assessment appeal to varied student learning styles?
- Does this assessment lend itself to provide quality feedback to students on how they can improve?
- Have we created a rubric or performance criteria informing students about their proficiency on this assessment?

Template for Creating a Common Assessment

Overview: What unit is this? What essential understanding that is being addressed?
Pacing: When in the unit will this assessment be used?
Purpose: What is the purpose of this assessment?
Content for Assessment: Standards being addressed:
Content for Assessment: Standards being addressed:
Content for Assessment: Standards being addressed:
Anticipated Number and Types of Items on Assessment: **___ Fill in the Blank** **____ Label diagram/chart** **___ Multiple Choice** **____ Short Answer** **___ Matching** **____ Other**

Adapted from Building Common Assessments Workshop with Cassandra Erkens, Solution Tree

Reflecting on Assessment (Form A)

Answer the following questions about your classroom assessment you are analyzing.

1. How many questions are on the assessment? _____

2. Are the questions short answer, multiple choice, or more open-ended which require explanation? Are there any problem solving type questions on the assessment?

   _____

3. Reviewing the questions on this assessment, can the students know the content and get the questions wrong? Can the students not know the content and get the questions right? _____

4. What patterns do you see in students' errors? (Did students compute incorrectly? Did they comprehend the questions incorrectly? Carelessness?)

   _____

   Do you and your grade level agree that the assessment is developmentally appropriate for your students? Why or why not?

   _____

5. Is this test teacher created, or textbook generated? (Textbook generated includes copy and pasted) _____

6. What information/knowledge did you gain **about your students** by looking more in depth at this assessment? Be specific: Do you need to re-teach any concepts? Do you need to challenge students more? Were ALL students successful in demonstrating their knowledge? What evidence does this test give you?_____

7. What information did you gain about your assessment of students? Is it authentic? Does it give them opportunities to explain their thinking? Does it apply to real-world situations? Does it give every student the opportunity to be successful? Does it communicate that you have high expectations of ALL learners?

   _____

Reflecting on Assessment (Form B)

Answer the following questions about your classroom assessment you are analyzing.

1. What specific skills or concepts are you observing/discussing with the student?

   _____

2. How did the students demonstrate their knowledge of a particular skill or concept? How did you document the students' knowledge?

   _____

3. How many students in YOUR class met mastery of the skills/concepts?_____

4. Reviewing the questions you asked or the expected behaviors you observed, can the students know the content and not be able to demonstrate the skills? Can the students not know the content and demonstrate the skills?

   _____

5. What patterns do you see in students' errors/challenges with performing specific skills/concepts? (Did students compute incorrectly? Comprehend the questions incorrectly?)

   _____

   Do you and your grade level agree that the assessment is developmentally appropriate for your students? Why or why not?

   _____

6. Is this assessment teacher created, or textbook generated? (Textbook generated includes copy and pasted) _____

7. What information/knowledge did you gain **about your students** by looking more in depth at this assessment? Be specific: Do you need to re-teach any concepts? Do you need to challenge students more? Were ALL students successful in demonstrating their knowledge? What evidence does this test give you?

   _____

8. What information did you gain about your assessment of students? Is it authentic? Does it give them opportunities to explain their thinking? Does it apply to real-world situations? Does it give every student the opportunity to be successful? Does it communicate that you have high expectations of ALL learners?

   _____

Resources:

Burns, M. (2007). About teaching mathematics: A K-8 resource. Third edition. Sausalito, CA: Math Solutions Publications.

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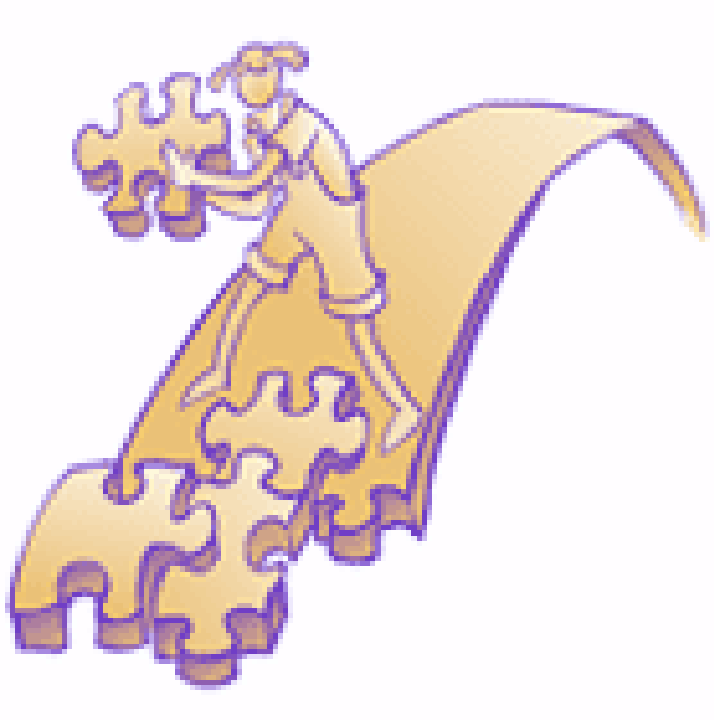
Van de Walle. (2006). Elementary and middle school mathematics: Teaching developmentally. White Plains, NY: Allyn & Bacon

Teacher Resources



Classroom organization addresses all components of instruction – teaching strategies, student grouping assignments, and assessing (Slavin, 1989).

Math Workshop Lesson Plan Template

LESSON TITLE:

MATERIALS NEEDED:

ENDURING UNDERSTANDING:

ESSENTIAL QUESTIONS:

CONTENT STANDARDS ADDRESSED: *(Process standards are embedded)*

MINI LESSON:
What information/background knowledge will be presented to students?

WORK TIME: (Small group OR Partner OR Independent)
What will students do?

Plans for Differentiation:

WHOLE GROUP DISCUSSION QUESTIONS:
How will we summarize the learning?

Teacher Reflection Questions:
What do we want students to know and be able to do?
How will we know that they have it?
What will we do if they don't get it?
What will we do if they do get it?

Let's Go Visiting

SUGGESTED MATERIALS: *Let's Go Visiting* by Sue Williams, Linking Cubes, Promethean Board, Hundreds chart

PROCESS INDICATORS ADDRESSED:
Problem solving, Reasoning and Proof, Connections, Communication ,Representation

CONTENT INDICATORS ADDRESSED:
K.NS.1 Count forward by ones and tens to 100.
K.NS.2 Count forward by ones beginning from any number less than 100.

MINI LESSON

- Set the stage for the problem-solving situation that will follow the story by asking them to think about how many animals the child in the book sees each day.
- Read the story and allow the students to comment about the book.
- Read the book again and allow students to write the numerals on the board for the quantity of animals on each page.
- On the 2 red calves' page, ask if they know how many animals they saw in the first 2 days. Show the cubes. Write this on board. Do same for three kittens' page. If kids are ready, write it in an addition problem.
- After reading the story, present the problem to be solved: How many animals did the child visit in all of the days?
- Allow kids to estimate and write estimates on board.
- Show the students the page where the child is sleeping with all the animals and ask them if they think this is all the animals. Let them try to count the animals. Ask if there is a way that they can be sure that this is all the animals that the child visited in all six days.
- Tell the kids that they are now going to go into small groups to figure out how many animals the child visited in all six days.

SMALL GROUP WORK

- Students will work in small groups to solve the problem. The students will use the cubes and to represent the number of animals the child sees each day. Circulate and assist kids by asking questions to probe their thinking. Be careful not to direct them toward a particular strategy. The goal is that the students work together to come up with a strategy that they understand and that makes sense to them. If a group solves the problem quickly and with ease, encourage them to use another strategy to determine the total number of cubes.

Plans for Differentiation: Allowing students to choose their own strategies for counting the cubes will allow them to solve the problem at the level of difficulty that they are comfortable with. Some kids will count by 1's, others who are more advanced might count by 5's or 10's and others may even use counting on or an adding strategy. Provide extra assistance for students who are having trouble getting started and/or counting the cubes.

WHOLE GROUP DISCUSSION

Reconvene the class and allow each group to share the strategy (s) they used to count the cubes. Show their representations on the Promethean board. Write the numerals as they count. Let the kids discuss which strategies worked best for them. Ask the kids to summarize what they felt they learned in today's lesson.

Quack and Count

SUGGESTED MATERIALS:
Linking cubes, square tiles, other various counters, *Quack and Count* by Keith Baker

PROCESS INDICATORS INTEGRATED:
Problem solving, Connections, Communication, Representation, Reasoning and Proof

CONTENT INDICATORS ADDRESSED:
1.ATO.6 Demonstrate: a. addition and subtraction through 20. B. fluency with addition and related subtraction facts through 10.

MINI LESSON
Read the story. Ask students to talk about the math they see in the book. Ask them to talk about how many different ways the author made 7. List the ways as they share. Ask students to look at the list and make observations. Ask them if there is a way to use a mathematical symbol to show what the author is doing (i.e., one plus 6 is 1 + 6). Pose the question – What if there were 8 ducks? I wonder how many different ways the author could have made 8. Let students predict. Do 8 together as a whole group. Challenge students to work together to come up with all the ways to make 9, 10, and 11.

Example of ways to make 8:
7 + 1
6 + 2
5 + 3
4 + 4
3 + 5
2 + 6
1 + 7

Note: The book does not use zero as a way to make the sum of 7. Some students may choose to use zero. This should be accepted. Help students see how the patterns are different is zero is used.

SMALL GROUP WORK
Students work together to solve the problem of how many different ways to make 8, 9, 10, and 11. The teacher should encourage them to generate strategies to solve the problem that makes the most sense to them.

Plans for Differentiation:
Students might work with numbers less than 7. Some students may need the entire small group time to find the ways to do just one number while some students may go beyond the numbers asked of them. Allowing students to choose the tools and strategies that work best for them will allow all students to solve the problem in the way that makes the most sense for them.

WHOLE GROUP DISCUSSION
Students share their strategies and solutions with the whole group. Ask students to "prove" that they got all of the possibilities. How did they determine that they had all of the possibilities? List all possibilities on the board. Ask them to look for patterns. Finally, draw a T-chart on the board. As they notice patterns, ask them to predict what the next number would be. Students will begin to notice that the number of ways to make any given sum is one less than the sum. Ask them if they can predict the number of ways to make the sum of 25, 50, 100, etc. using this pattern.

Sum	Number of Ways to Make the Sum
7	6
8	7
9	8
10	9
11	10
12	11

Measurement Scavenger Hunt

SUGGESTED MATERIALS

For each group of 4:

rulers, yardsticks, square tiles, centimeter cubes, picture of football field, Karate belt or other item of similar length, children's book on measurement (see suggestions below)

PROCESS INDICATORS ADDRESSED:

Problem solving, communication, representation, reasoning and proof, connections

CONTENT INDICATORS ADDRESSED:

2.MDA.1 Select and use appropriate tools (e.g. rulers, yardsticks, meter sticks, measuring tapes) to measure the length of an object.

MINI LESSON

- Read a children's book about measurement such as *How Big is a Foot* by Rolf Myller, *Inch by Inch* by Leo Lionni, or *Measuring Penny* by Loreen Leedy.
- Show the class pairs of straight objects such as a pencil and a football field (show a picture of this), the teacher and a crayon, or the height of a book and a Karate belt. For each pair, ask: Which is longer? Which is shorter? How can we tell? What are some things that you could use to measure these items? Allow students to search through their toolboxes and share items that they might use to measure each item. Avoid telling them directly which tool is the most efficient to use. The goal is to have them discuss, investigate, and discover as they engage in a small group scavenger hunt.
- Before beginning the scavenger hunt, briefly discuss each tool. Ask students if they can name each tool and tell the length of each tool. For example, a ruler is a foot long, a yardstick is a yard long, a square tile is an inch long, and the centimeter cubes are a centimeter long.
- Tell the class that they are going to work in their groups to go on a measurement scavenger hunt. Tell them that they may use any of the tools in their toolboxes to measure the items that they choose. Encourage them to talk together and agree upon a tool to use before measuring each item. Encourage them to use a variety of measurement tools.

SMALL GROUP WORK

Students should work in groups of three to four to locate and measure objects decided on in advance by the teacher. They should be encouraged to use their own strategies to decide what and how to measure. Answer questions as necessary but be careful not to "tell" the students how to measure each item or what tool to use. The goal is to tell them just enough so that they can figure out the most efficient measurement tool to use for each item. Through the process of engaging in the investigation, students will begin to discover that some tools and units (inches, centimeters, or feet) are more efficient than others when measuring certain things.

Plans for Differentiation: Some students might choose to use square tiles if they haven't had a lot of experience with using rulers or yardsticks. It's important not to discourage this. Students should be allowed to use the tool that is easiest for them.

WHOLE GROUP DISCUSSION

After the students have been given sufficient time to conduct the investigation, reconvene the group to share the objects chosen, tools used to measure, and measurements discovered. Ask students to talk about which tools were easier for them to use to measure each object. Through the follow-up discussion, students should begin to see that certain tools are better to use to measure certain objects. For example, we wouldn't want to use centimeter cubes or even a ruler to measure something as tall as a basketball goal. Be sure to relate these newly constructed ideas back to the ideas shared during the mini lesson. It is important to remember that the goal of this lesson is not **exact** measurement. Instead, students are being given the opportunity to build understanding of appropriate tools for linear measurement while at the same time, beginning to practice using those tools to measure.

How Old are You?

SUGGESTED MATERIALS: assorted counters, Unifix/Linking Cubes, square tiles, Base 10 Blocks, Cuisenaire Rods

PROCESS INDICATORS ADDRESSED:
Problem solving, Connections, Communication, Representation, Reasoning and Proof

CONTENT INDICATORS ADDRESSED:
3.NSBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10-9, using knowledge of place value and properties of operations. *Note, this is an extension problem beyond just multiples of 10.

MINI LESSON

Present the problem to be solved:
Today is Juan's birthday and he is 8 years old. How many months old is he? How many months old will he be when he turns 9?
Discuss any background knowledge needed to solve the problem. For example, students need to remember how many days in a year before they can generate strategies to solve the problem.
Explain that students are to divide into small groups and use strategies of their choosing to solve the problem.

SMALL GROUP WORK

Students should continue to work in groups of 3-4 Students are encouraged to use their own strategies to solve the problem. As the students explore using manipulatives and other problem-solving strategies of their choosing, the teacher will facilitate by asking individual groups:

- What are some ways that your group has discovered?
- What strategies did your group use?
- Can you think of another strategy that would work?
- Could you show your strategy using a picture?
-

Plans for Differentiation: Allowing students to choose the problem-solving strategy to use instead of dictating how they should solve the problem will also allow for differentiation. The problem could be made more difficult by having the students find out how many weeks or days old Juan is. Some students may need the entire block of small group time just to solve the first part of the problem.

WHOLE GROUP DISCUSSION

After the students have been given sufficient time to generate a variety of strategies, reconvene the group to share strategies and solutions. Allow each group the opportunity to share their findings as well as new learning and connections made. Ask students if they can come up with a pictorial representation for their concrete manipulative representations. Allow them to draw their representations on the Promethean Board.

Toothpick Triangles

SUGGESTED MATERIALS: toothpicks, paper, pencils

MATHEMATICAL PROCESSES: The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

CONTENT INDICATORS ADDRESSED:
4.ATO.5 Generate a number or shape pattern that follows a given rule and determine a tem that appears later in the sequence.

MINI LESSON

- How many toothpicks does it take to make one triangle?
- Is there a way to make 2 triangles with 5 toothpicks?
- Is there a way to use 7 toothpicks to make 3 triangles?

Present the problem to be solved:
If it takes 3 toothpicks to make 1 triangle, 5 toothpicks to make 2 triangles and 7 toothpicks to make 3 triangles, how many toothpicks does it take to make a row of 10 triangles? 25? 50? 100?

SMALL GROUP WORK
Students work in groups of 2 – 4 to solve the problem. Students are encouraged to use their own strategies to solve the problem. Some may choose to draw pictures and some may choose to use the toothpicks.

Plans for Differentiation: Some children might actually need to lay out the entire row of triangles to solve the problem. Some may see the pattern immediately. Groups that finish quickly should be encouraged to find a rule for finding out how many toothpicks it would take to make any number of triangles. Challenge them to solve the same problem with another shape such as a square or a triangle.

WHOLE GROUP DISCUSSION
After the students have been given sufficient time to conduct the investigation, reconvene the group to share strategies and solutions. Allow each group the opportunity to share their findings as well as new learning and connections made. After students share strategies and solutions, show them how to organize their findings in a t-chart.

Triangles	Toothpicks
1	3
2	5
3	7
4	9

Then, ask the students to look for patterns in the chart. There are patterns in the toothpick column (increasing by 2 in each row), but the patterns between the triangle and toothpick column is what they need to see to figure out the solution for a greater number of triangles without having to actually make all of the triangles.

Encourage them to think of a rule for the chart. A rule for the chart would be to multiply the number of triangles by 2 and add 1. For example, to make 4 triangles, it would take 9 toothpicks because 4 x 2 + 1 = 9. Challenge students to figure out why this works. Another way to write this using symbols would be:

$$\triangle \times 2 + 1 = T$$

Fractions – Unlike Denominators

SUGGESTED MATERIALS: fraction tiles, fraction bars, fraction pieces, color tiles, pattern blocks, grid paper, and colored pencils.

PROCESS Standard:
The student will understand and utilize the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation.

CONTENT INDICATORS ADDRESSED:
5.NSF.1 Add and subtract fractions with unlike denominators (including mixed numbers) using a variety of models, including an area model and number line. .

MINI LESSON

Introduce and discuss the details of the first problem together. Have the students identify the key components of the problem. Have the students recognize that this is an addition problem and the fractions have different denominators.

SMALL GROUP WORK

Students will work together in groups of 3-4 to solve the second and third problems. They should be encouraged to use "tools" from their toolboxes to determine the solution for each problem. The teacher should monitor progress as students work in their groups. Encourage students to explain how they know their solutions are accurate.

Maria plants vegetables in 2/3 of her garden and fruit in 1/6 of her garden. How much of the garden has been planted? **2/3 + 1/6 = 4/6 + 1/6 = 5/6**

It took Kayla ¼ of an hour to clean her room on Saturday. Robin cleaned her room in 3/8 of an hour. How much time did they spend cleaning their rooms? **1/4 + 3/8 = 2/8 + 3/8 = 5/8**

Samantha wants to make a Pirate ship banner for the Fall Festival. Two-fifths of a yard of red cloth was used to make the banner, and half of a yard of black cloth was used. How much cloth did she use?
2/5 + 1/2 = 4/10 + 5/10 = 9/10

Plans for Differentiation: This is an introductory lesson, so differentiation will be the tools the students choose to use. You could also differentiate based on the students' ability to add like denominator fractions.

WHOLE GROUP DISCUSSION

Students should share the different strategies they chose. As they share, teachers can ask the following questions when appropriate:
For addition, what do we have to do when the fractions have unlike denominators?
Did one problem challenge you more than another problem?
What previous math concepts can you use to add fractions with unlike denominators?
How can you convince me that your strategy worked?

MATH STUDENT RUBRIC			
Level	**Understanding**	**Strategies, Reasoning, Procedures**	**Communication**
Novice	I do not understand the problem.	I am not sure how to do it.	I have no explanation. I'm not sure how to draw the problem, or which numbers to use.
Apprentice	I think I understand the problem.	I got started. I'm still thinking. I have part of the answer. It would help me to work with somebody.	I can explain some of what I did. I tried to use pictures, numbers, graphs, or words. My answer might be right.
Practitioner	I got it! I understand the problem.	I used a plan to solve the problem.	I can tell you or show you how I got the answer. I used mathematical terms and models, pictures, graphs, or numbers to explain how I did the problem. You can see how I did it.
Expert	I got it! I can prove to you that I'm right. I know I'm right.	I can show you several plans to solve this problem. I can prove to you that my answer is right.	I very clearly laid out step by step how I solved the problem. I used words, pictures, numbers, models, or graphs to show exactly how I did the problem, and proved that my answer is right. I can even show you another way to do the problem.

How to Assess Problem-Solving Skills in Math by Bena Kallick and Ross Brewer, Scholastic Professional Books, 1997

EXEMPLARS RUBRIC

Level	Understanding	Strategies, Reasoning, Procedures	Communication
Novice	• There is no solution, or the solution has no relationship to the task. • Inappropriate concepts or procedures are used.	• No evidence of a strategy or procedure, or uses a strategy that does not help solve the problem. • No evidence of mathematical reasoning. • There were so many errors in mathematical procedure that the problem could not be solved.	• There is no explanation of the solution, the explanation cannot be understood or it is unrelated to the problem. • There is no use or inappropriate use of mathematical representations (e.g., figures, diagrams, graphs, tables, etc.) • There is no use, or mostly inappropriate use, of mathematical terminology and notation.
Apprentice	• The solution is not complete, indicating that parts of the problem are not understood.	• Uses a strategy that is partially useful, leading some way toward a solution, but not to a full solution of the problem. • Some evidence of mathematical reasoning. • Could not completely carry out mathematical procedures.	• There is an incomplete explanation; it may not be clearly presented. • There is some use of appropriate mathematical representation. • There is some use of mathematical terminology and notation appropriate to the problem.
Practitioner	• The solution shows that the student has a broad understanding of the problem and the major concepts necessary for its solution.	• Uses a strategy that leads to a solution of the problem. • Uses effective mathematical reasoning. • Mathematical procedures used appropriately.	• There is a clear explanation. • There is appropriate use of accurate mathematical representation. • There is effective use of mathematical terminology and notation.
Expert	• The solution shows a deep understanding of the problem, including the ability to identify the appropriate mathematical concepts and the information necessary for its solution.	• Uses a very efficient and sophisticated strategy leading directly to a solution. • Employs refined and complex reasoning. • Applies procedures accurately to correctly solve the problem and verify the results.	• There is a clear, effective explanation detailing how the problem is solved. All of the steps are included so that the reader does not need to infer how and why decisions were made. • Mathematical representation is actively used as a means of communicating ideas related to the solution of the problem. • There is precise and appropriate use of mathematical terminology and notation.

How to Assess Problem-Solving Skills in Math by Bena Kallick and Ross Brewer, Scholastic Professional Books, 1997

PROCESS STANDARD PROMPTS

Connections Prompts

Does this remind you of anything we've done before?
Can someone think of a time when you've needed to…(measure, add, subtract, etc.)?
How is this idea related to…(addition, subtraction, multiplication, etc.)?
When might a (scientist, chef, doctor, architect, etc.) need to use what we're learning today?
How is what we're learning today important to you in your everyday life?
Would it help you to try to solve a simpler problem?
Can you use what we learned about addition to help you solve this new problem?
This reminds me of the problem we solved last week. What patterns did we discover when solving that problem and how can you use those same patterns to help you solve this new problem?

Reasoning and Proof Prompts

Explain why you think your answer is reasonable.
Explain why your answer makes sense?
Do you agree with Susie's explanation?
How did you get your answer?
Tell me how you thought about that.
Can you solve the problem in another way?
How can you be sure that you've found all the solutions?
Why does your solution work?
Is Kaia's idea very different from yours?
Does that seem right to you, Lisa? How can you convince yourself.
How is Karen's strategy the same as/different from Brandy's strategy?
Do you think your strategy will always work?
What discoveries did you make?
Did you notice any patterns?

Communication Prompts

Oral

What strategy might you use to solve the problem?
Talk with your group to come up with a plan to solve the problem.
Talk about what tools you might need to help you solve the problem.
Do you see a pattern that might help?
Who can explain what Rahimme said using different words?
Did anyone think about the problem in a different way?
Does anyone have any questions they want to ask Jose about his solution?
What do you notice about?
What do you find interesting?
I wonder what would happen if…?
Would anyone like to add to what JaNita just said?
Please explain that in a different way.
Will you say a little more about that?
Do you agree with what Yoshi just said?

Written

Write about the strategy that you used to solve the problem.
Write about how you know the answer is correct.
Write about what you discovered during this investigation.
Write about what you are confused about.
Write down any new questions you have.

Representation Prompts

Can you illustrate what you're saying using your Base 10 blocks (or whatever manipulative being used)?
How can you organize your results?
Can you display your data in a graph or chart?
Can you organize your results in a T-chart (function table)?
Show me your strategy using a different manipulative.
Show me how you arrived at your solution.
Can you draw a picture to show your findings?
Describe the strategy that you used to find the answer by writing it down in your math journal.
Show me several different ways that you can present your findings to the class.

PROCESS STANDARD: PROBLEM SOLVING

Characteristics of a Problem-Solving Approach

Continuous interaction/discussion between students/students and teacher/students.
Hands-on active learning
Students using manipulatives and other tools to find solutions.
Teachers providing just enough information to establish background/intent of problem but not "telling" students how to solve the problem.
Teachers guiding, coaching, asking insightful questions and sharing in the process of solving the problem.
Teachers knowing when to intervene, and when to step back.
Students solving open-ended problems.
Students reasoning, communicating, making connections, and representing mathematical ideas.

How to Plan Problem-Based Lessons

Step 1 – Decide on the math indicator.
Step 2: Think about your students. What do they already know and understand about this topic? Is there some background information they need before being able to solve the problem?
Step 3: Decide on a task. Keep it simple! Good tasks need not be elaborate. Often a simple story problem is all that is necessary as long as the solution involves children in the intended mathematics.
Step 4: Predict what will happen. Think about the strategies that the students might use, but be prepared to see the students using strategies that you never considered.
Step 5: Plan the "mini lesson." This is the part of the lesson where you introduce the problem to be solved and review any background concepts or math vocabulary that is important for solving the problem. It is important to refrain from "teaching" the students how to go about solving the problem.
Step 6: Plan the small group portion of the lesson. What will they do? How can you facilitate their efforts without telling them "how" to solve the problem? How will you differentiate? What kinds of questions might you ask?
Step 7: Plan the after portion of the lesson. This is the part of the lesson in which students share with the class their findings and the strategies they used to solve the problem. This is a very important part of the lesson and should never be omitted.

How to Create Good Tasks

A good problem can be used as the basis for an entire lesson. There are two helpful approaches that can be used in creating good problems for students to solve.
Method 1: Working Backward
Step 1: Identify a topic (indicator).
Step 2: Think of a closed question and write down the answer.
Step 3: Make up a question that includes (or addresses) the answer.
For example:
Step 1: The indicator to be taught is finding the mean of a set of data.
Step 2: The closed question might be *The children in the Williams family are aged 3, 8, 9, 10, and 15*. What is the mean of their ages? The answer is 9.
Step 3: The good question could be There are five children in a family. Their average age is 9. How old might the children be?
Method 2: Adapting a Standard Question
Step 1: Identify a topic.
Step 2: Think of a standard question.
Step 3: Adapt it to make a good question.
For example:
Step 1: The topic for tomorrow is measuring length using nonstandard units.
Step 2: A typical exercise might be *What is the length of your table measured in orange Cuisenaire Rods*?
Step 3: The good question could be *Can you find objects in the classroom that are 10 Cuisenaire Rods long?*

Problem Solving Steps (not always linear)

1. Read the problem.
2. Circle/highlight the important facts.
3. What are you supposed to find out?
4. Create a plan to solve the problem.
5. Are there any "tricky" parts to the problem?
6. Which math strategies/tools will you use?
7. Work together to implement the strategies.
8. Use tools (pictures, writing, manipulatives, talking).
9. Monitor your progress. Are the strategies working?
10. Look for patterns that will help you solve the problem.
11. Look at your solution and decide if it is reasonable.
12. Solve the problem using a different strategy to verify your solutions.
13. Be prepared to defend your strategies/solutions.

Example of a Mathematical Conversation

In this scenario, a second-grade teacher posed the problem of figuring out her weight based on the weight of her four-month-old baby. It is clear to see how the students' understanding of how to solve the problem is clarified as a result of hearing and building on one another's ideas and theories. The teacher's role was one of allowing the students to share, questioning and responding when necessary, accepting all responses correct or incorrect, and finally challenging the students to justify their strategies and solutions by working in groups and using tools to help them make sense of the problem.

Teacher: *Can you believe my baby weighs 12 pounds now? He's growing so fast! And I'm exactly 10 times as heavy as my baby.*

Brandy: *(excitedly) Can I do that problem?*

Shaquasia: *I figured it out!*

Tonya: *All ya gotta do is add 12 plus 10.*

William: *Twelve times ten.*

Larry: *Yeah, twelve times ten.*

Teacher: *Twelve times ten?*

Demetrius: *120.*

Teacher: *Will you talk why you would say twelve times ten and how you figured 120?*

Demetrius: *Because I know 10 times 10 is – um – 10 times 10 is a hundred and I added 12 and that's – um – a hundred times twelve equals twenty.*

William: *I like…took 12…took 10, I mean, and said 10, 20, 30, 40, 50, 60, 70, 80, 90, 100…then I went 110…and 120…and I came up with one hundred and twenty…cause I added ten…twelve times.*

Demetrius: *That's what I did!*

Megan: *I figured out twelve times ten and then I like…I know twelve times ten….and then I did…and then I figured out I can do twelve….ten times…I mean…ten…twelve times…and then I said 10, 20, 30, 40, 50, 60, 70, 80, 90, 100…and then I said…um…I need twelve…so I counted two more and I said that equals 120…so you weigh 120 pounds.*

Teacher: *Okay, Larry how did you do it?*

Larry: *I figured out…I figured out 20…and then I figured a one and then I put the one in front of the 20 and I got 120. That's how I got it.*

Teacher: *Will you explain that again?*

Larry: *Well, somehow I got 20 and then I put a one…and then put the one in front.*

Teacher: *Explain the 20. How did you get that?*

Larry: *I have 12…and then I got 20…uh…(puzzled expression).*

Demetrius: *That's how I got 120. I knew ten times ten was 100…that's 100…and I needed 2 more so I added ten times two….and that equals 20.*

Teacher: *Okay, we have some great ideas and strategies here. What I want you to do now is to go back to your tables and work with your groups to prove to me that I weigh 120 pounds. You can use manipulatives, drawings, or equations to convince me that your solution is accurate. I need to be able to see that you understand the strategy that you use.*

99 Chart

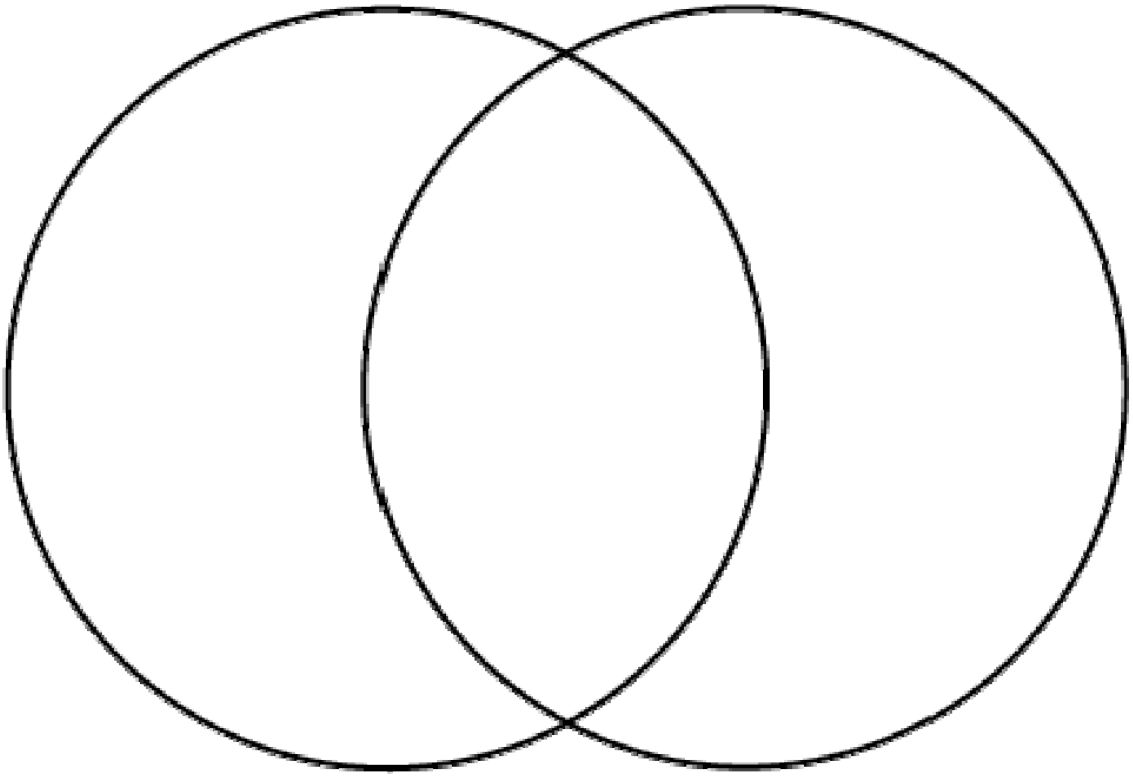
0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

100 Chart

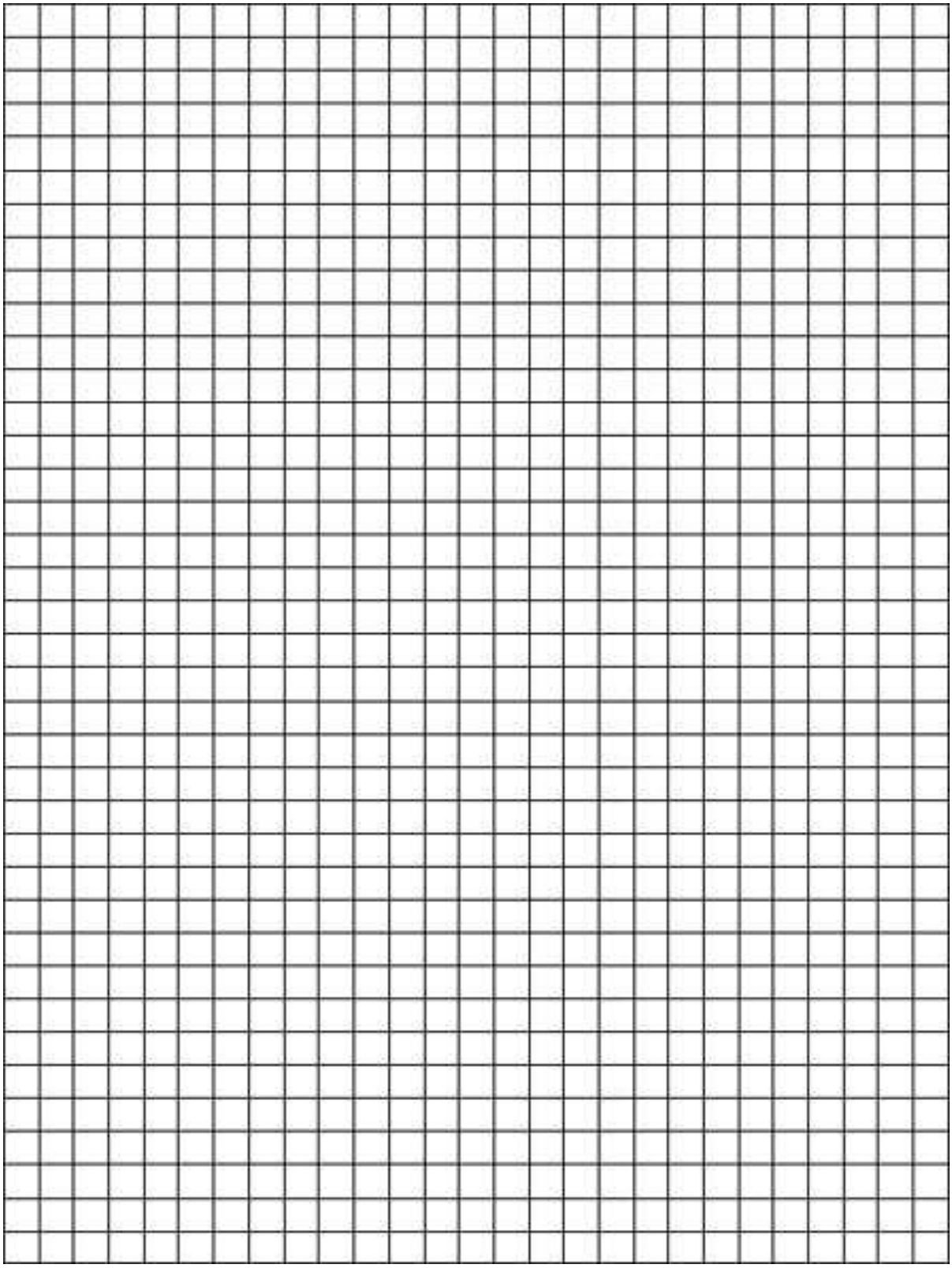
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

VENN DIAGRAM

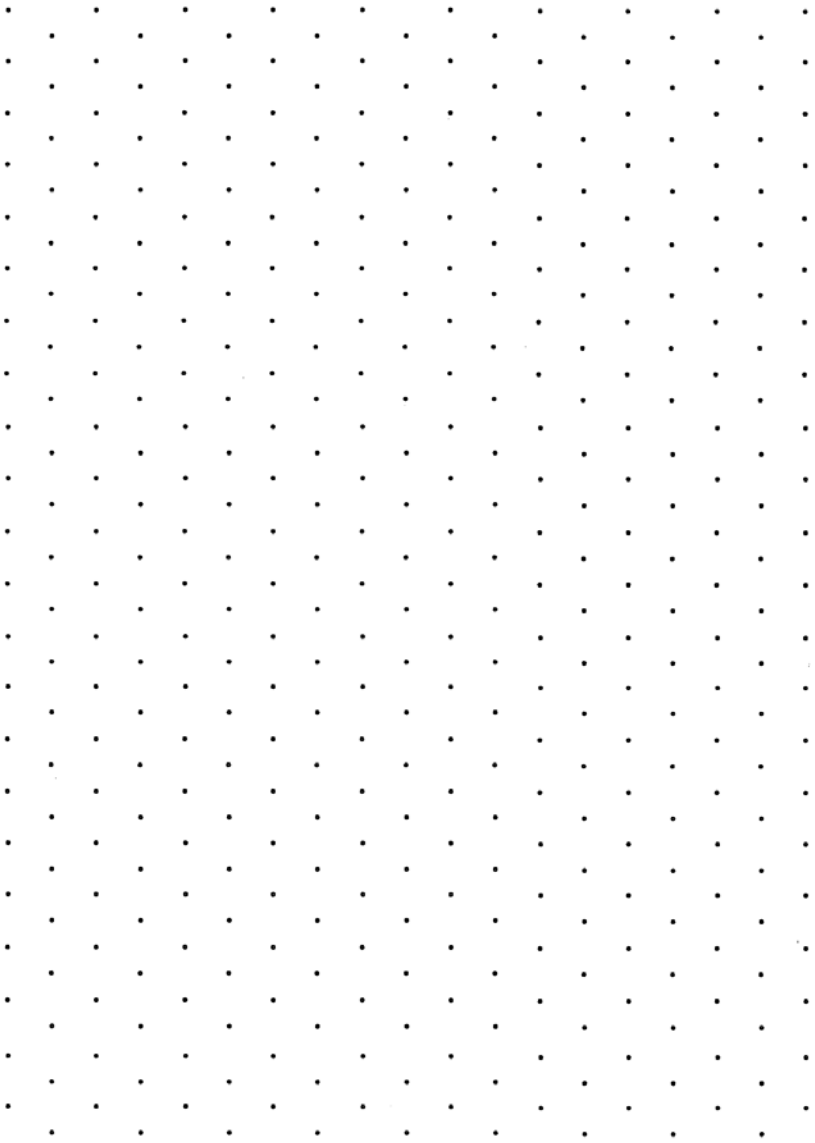
Title:



What information did you learn or what conclusions can be drawn from having completed this Venn diagram?



Isometric Dot Paper



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Math Websites

http://ed.sc.gov/instruction/standards-learning/mathematics/
South Carolina State Department of Education (math content pages)

http://nctm.org --- National Council of Teachers of Mathematics is "THE VOICE" on best practice for mathematics. You can become a member to access materials, or there are also tons of resources free to anyone.

http://www.khake.com/page47.html --- Math Education Web Resources--- lesson plans

http://www.amathsdictionary.com --- teacher and student friendly dictionary of math terms

http://www.funbrain.com/ --- math games

http://www.math.com/ --- math lessons and practice problems

http://www.mathforum.org/ --- teacher

resources http://www.purplemath.com/ --- math assistance

http://www.coolmath4kids.com/ --- math games

http://mathcounts.org/ --- math counts (problem of the

week) http://www.learner.org/teacherslab/math/geometry/ --- math

activities http://arcytech.org/java/integers/ --- math

lessons http://nlvm.usu.edu/en/nav/vlibrary.html --- virtual

manipulatives

http://www.shodor.org/interactivate/activities/index.html --- math activities

http://illuminations.nctm.org/ --- (NCTM affiliate) math activities

http://school.discovery.com/teachingtools/teachingtools.html --- lesson plans and student activities

http://www.pbs.org/teachersource/math.htm --- lesson plans and student activities

http://www.goenc.com/ --- math and science activities

http://www.mste.uiuc.edu/resources.php --- math activities

http://www.mathprojects.com/ --- lesson plans and

activities http://msteacher.org/math.aspx --- math activities

www.donorschoose.org --- obtain resources and materials

www.e2c2.com/fileupload.asp --- For learning profiles to differentiate instruction, download the file entitled "Profile Assessments for Cards."

http://education.ti.com --- Texas Instrument website

http://mathworld.wolfram.com/ --- math resources and links

http://www.free.ed.gov/ ---- FREE, the website that makes teaching resources from federal agencies easier to find

http://www.stat.uiuc.edu/courses/statl00/cuwu/ --- statistics (guess the correlation coefficient for a scatterplot)

http://www.mathcats.com/mathtoolbox/index.html --- everyday items as

manipulatives http://www.learner.org/teacherslab/math/geometry/ --- space and shape in

Geometry http://www.figurethis.org/ --- family outreach

http://atschool.eduweb.co.uk/toftwood/resources.html --- Interactive Games for Primary Grades

http://www.cut-the-knot.org - Interactive Mathematics Miscellany and Puzzles

http://www.joma.org/ --- The Journal of Online Mathematics and its Applications

http://enchantedmind.com/html/puzzles/interactive_puzzles/creative_java_puzzles.html --- Math Puzzles

http://www.bbc.co.uk/education/mathsfile/gameswheel.html --- The Maths Files Game Show

http://teacherlink.org/content/math/interactive/flash/top.html --- Teacherlink: Interactive Mathematics Projects using Macromedia Flash

http://www.math.uah.edu/stat/ --- virtual laboratories in probabilities and statistics

http://gfletchy.com --- Georgia website with 3-act tasks and other activities

http://youcubed.org --- Interactive games and activities

http://www.dpi.state.nc.us/accountability/testing/releasedforms --- Released NC state assessments
http://wodb.ca --- “Which One Doesn’t Belong” activities

http://www.101qs.com --- Activities and rigorous questions

http://engageny.com --- Modules & Units for each standard and skill

http://standardsexplained.wordpress.com --- Standards videos and other resources

https://www.illustrativemathematics.org --- Interactive and engaging tasks for each grade level

http://www.abcya.com/base_ten.htm --- Interactive Base Ten Blocks

https://mathreasoninginventory.com/ --- Marilyn Burns’ diagnostic assessment tools

https://nrich.maths.org/4348 --- Virtual cuisenaire rods

https://insidemathematics.org --- This website has tools for teachers, videos of actual lessons in classrooms, as well as tons of information for teachers as well as lesson ideas.

k-5mathteachingresources.com--- The lessons are aligned with our state standards, as well as there are great resources for basic number sense building activities.

Glossary of Terms

Algorithm - A specific set of instructions for carrying out a procedure or solving a problem.

Array - An arrangement (usually rectangular) of objects or numbers

Base - the face (of a polyhedron) or segment (of a polygon) that is being used as the reference to measure the height (altitude) of the polygon or polyhedron.

Benchmarks - Important units used as a referent for estimation. Benchmark numbers for fractions could be 0, Yz , l, $l^{l}/_{2}$, and so forth. Benchmark for measurements could be multiples of standard units. Benchmarks for whole numbers could be multiples of l0, l00, l000, and so forth.

Box Plot (Box-and-Whisker Plot) - A representation of data with a rectangular box extending from the lower quartile to the upper quartile of the data and two lines extending from the ends of the box to the extreme values of the data.

Cardinality of a Set - When counting a set, the last number word used (how many in a set)

Composing/Decomposing a Number - A strategy used to reinforce number sense. Involves conceptualizing a number as being made up of two or more parts: putting the parts together to make a number us *composing* a number; breaking a number into two or more parts is *decomposing* the number.

Compute Fluently - Use efficient and accurate methods for computing.

Cone - a solid with a circular face and a point that is not in the same plane as the face

Congruent - having the same shape and same size

Conjecture – informed guessing

Cylinder - a solid with two congruent circular faces that are parallel and connected by a curved surface

Dot Plot (Line Plot) - A representation of data made by making a horizontal line and placing an "x" or "dot" above the corresponding value on the line for every data element

Edge - A line segment where two faces of a polyhedron meet.

Faces - The flat polygonal regions of a polyhedron.

Geometric Pattern - A pattern involving geometric shapes so that students see a pattern that involves square or triangular numbers; the pattern typically involves multiplication facts

Histogram - A special type of bar graph that displays the frequency of data as rectangles with areas proportionate to the corresponding frequencies. Each bar has the same width. The width of the bar represents a range of values along the horizontal axis.

Inverse Relationship Between Operations - The inverse of a mathematical operation undoes the operation. For example, subtraction undoes addition.

Line Graph - In a line graph, points representing two related pieces of data are plotted and then connected by a line.

Line Plot (Dot Plot) – A representation of data made by making a horizontal line and placing an "x" or "dot" above the corresponding value on the line for every data element

Mean - (a measure of central tendency) also known as the average, the sum of the values in a data set divided by the number of items in the data set

Median - (a measure of central tendency) the middle value of an ordered set of values

Mode - (a measure of central tendency) the score or data value in a set that occurs the most often

Models - Concrete, pictorial, symbolic, verbal, and algorithmic representations.

Nets - A two-dimensional fold-up model of a polyhedron.

Networks – A graph or directed graph together with a function that assigns a positive real number to each edge.

Perfect Square - The product of an integer multiplied by itself. For example, 4 is a perfect square because 2 X 2 = 4.

Plane (common notion) - a two dimensional surface that extends infinitely in all directions

Polygon - A closed plane figure with *n* sides. The sides of a polygon are line segments.

Polygonal Regions - Flat surfaces enclosed by polygons.

Polyhedron - A closed three-dimensional object whose surfaces are formed by polygonal regions (e.g. prism, pyramid, octahedron).

Prism - A polyhedron with two congruent, parallel bases that are polygons, and all remaining faces parallelograms.

Pyramid - A polyhedron with a polygon for a base and all other sides being triangles with one common vertex.

Similar - when two figures have the same shape and corresponding sides and angles are proportional

Vertex (of a polyhedron) - a point where three or more edges of a polyhedron meet

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http://www.cast.org/ncac/index.cfm?i=2876 - This site contains an article by Tracy Hall at the National Center for Accessing the General Curriculum. The article discusses differentiation as it applies to the general education classroom.

http://members.shaw.ca/priscillatheroux/differentiatingstrategies.html - The Enhancing Learning with Technology site provides explanations for various differentiation strategies.

Integrating Children's Literature and Math

NUMBERS AND OPERATIONS

COUNTING AND NUMBER SENSE		
Title	**Author**	**Annotations/Lesson Ideas**
Annie's One to Ten	Annie Owen	The illustrations in this book show all the different combinations of objects that can be grouped together to equal 10. Children can write addition and subtraction number sentences for each page. In groups, have children list all the different ways they can think of to make 10.
Anno's Counting Book	Mitsumasa Anno	Provide counters in two different colors and suggest to the children that they use one color to count the girls and the other for the boys. Have them put out counters to represent the little people in the house on the first page. How many people are there altogether? Continue in this way throughout the book.
Arctic Fives Arrive	Elinor J. Pinczes	In this book, groups of five animals keep arriving on an iceberg to view the northern lights. After viewing the northern lights, the animal groups leave the ice in reverse order. Great for reinforcing skip counting by 5's.
"Band-Aids" *in Where the Sidewalk Ends*	Shel Silverstein	Ask students to count all of the Band-Aids and determine the total number. Challenge them to figure out how many Band-Aids there would be if they added the "box full of thirty-five more."
The Baseball Counting Book	Barbara Barbieri and Brian Shaw	On each page, starting with zero and ending with twenty, children see how each number corresponds to baseball.
Bat Jamboree	Kathi Appelt	Students can count, add, and subtract the bats in the jamboree.
The Cheerios Counting Book	Barbara Barbieri McGrath	Students can estimate and count Cheerios.
City by Numbers	Stephen T. Johnson	Students can search for the numbers 0-21 hidden in the urban landscape of New York City.
Counting Crocodiles	Judy Sierra and Will Hillenbrand	A clever monkey uses her ability to count to outwit the hungry crocodiles that stand between her and a banana tree on another island across the sea. Students can count the crocodiles in the story.

Counting on Frank	Rod Clement	One of the facts shared in the book is that only ten humpback whales would fit in the narrator's house. Ask students to estimate how many humpback whales would fit in their houses.
The Crayon Counting Book	Pam Munoz Ryan and Jerry Pallotta	Students can count and sort colors and investigate odd and even numbers.
Emily's First 100 Days of School	Rosemary Wells	On the 100th day of school, the children share something related to 100. One child brings 100 pieces of candy corn; another runs 100 yards. Emily worries about not having 100 things but realizes that she has written a letter to her family about what she has learned and has included 100 kisses (X's).
Even Steven and Odd Todd	Kathryn Cristaldi and Henry Morehouse	Even Steven, who likes everything to come in even numbers, is upset with the arrival of Cousin Odd Todd. An activities and games section is included.
Feast for 10	Cathryn Falwell	Numbers from one to ten are used to tell how members of a family shop work together to prepare a meal.
Frogs Jump: A Counting Book	Alan Brooks	This counting book gives children practice in matching number symbols to equivalent numbers of animals as well as providing practice in adding and subtracting the number 1.
From One to One Hundred	Teri Sloat	People and animals introduce the numbers one through ten and then count by tens up to 100.
The Grapes of Math	Greg Tang	Illustrated riddles introduce strategies for solving a variety of math problems using visual clues.
Great Estimations	Bruce Goldstone	This book will show students how to train their eyes and their minds to make really great estimations—by making estimating into a game.
Harriet Goes to the Circus	Betsy Maestro and Giulio Maestro	Harriet wakes up early to go the circus because she wants to be first in line. This story is useful for teaching ordinal numbers.
How Many Seeds in a Pumpkin?	Margaret Mcnamara	This book can be used around Halloween or Thanksgiving. Students can predict, estimate, and then count seeds in a pumpkin.
How Many Snails? A Counting Book	Paul Giganti and Donald Crews	This story provides the opportunity for students to count groups of objects and also to sort things by characteristics. On the last page, the author wonders how many stars there are. This is a good opportunity to talk about large numbers and even introduce the concept of infinity.

The Hundred Penny Box	Sharon Bell Mathis	Michael loves his great-great-aunt Dew, even if she can't always remember his name. He especially loves to spend time with her and her beloved hundred penny box, listening to stories about each of the hundred years of her life.
The Icky Bug Counting Book	Jerry Pallotta and Ralph Masiello	Of particular interest in this book is that the numbers extend to 26, introducing twenty-six different insects. The extensions suggest ideas for class-made books.
The King's Commissioners	Aileen Friedman	Have students identify the many ways to count the King's Commissioners in the book. Encourage them to comment on the various counting methods used to get to 47. Discuss whether or not the ways presented made it easier or harder to find the total number of commissioners.
Let's Go Visiting	Sue Williams	Students can figure out the total number of animals that the little boy saw.
The M&M's Counting Book	Barbara Barbieri McGrath	Give students a small bag of M&M's and have them estimate and count how many are in the bag. Show a large bag of M&M's and have them use their findings to estimate how many are in the bag. Children can also sort the candy by color and count the number of each.
Marvelous Math	Lee Bennett Hopkins	Marvelous Math is a look at the sometimes surprising ways math is part of our daily's life. The poems cover a wide range of topics from multiplication, division, and fractions to time, counting and measurement.
Math Curse	Jon Scieszka	When the teacher tells her class that they can think of almost everything as a math problem, one student acquires a math anxiety which becomes a real curse. Helps students see the prevalence of math in their everyday lives.
Math for All Seasons	Greg Tang	This book encourages students to find the sum without counting one by one. Challenges students to solve problems by looking for patterns, symmetries, and familiar number combinations.
Moira's Birthday	Robert Munsch	Moira's parents are expecting six of her friends to come to her birthday party but 200 people show up. Moira orders 200 pizzas and 200 birthday cakes, but the pizza parlor and bakery can only order 10 right away. Ask students to talk about whether she needed to order that much food. How many pizzas would be needed for each guest to have 2 pieces? What other information is needed? Students could also create birthday graphs after reading this book.

More or Less	Stuart J. Murphy	Eddie has a booth at the school fair, guessing people's ages. He hasn't guessed wrong yet, but if he does, he gets dunked. This book gives students the opportunity to compare whole numbers and understanding what's more and what's less.
More than One	Miriam Schlein	In this book, readers learn that one can be more than one, as when "one pair of shoes is two shoes." Throughout the book, this idea is expanded upon to show that the number one can represent other numbers.
Mouse Count	Ellen Stoll Walsh	Ten mice outsmart a hungry snake. Helpful for counting 1 through 10.
My Even Day	Doris Fisher	This book continues where "One Odd Day" left off. The boy wakes up to discover that life is back on an "even" keel. Mom has two heads; he sees eight beds; he's served four flapjacks; he's got two left shoes; all this before he even gets to school!
The Napping House	Audrey Wood	Students can figure out the total number of feet in the story. Challenge them to figure out the total number of toes, fingers, or fingers and toes.
Numbers	Henry Arthur Pluckrose	This book helps students see that numbers are all around and are used everyday.
One Duck Stuck	Phyllis Root	Challenge the students to figure out how many animals came to help the one duck stuck in the muck.
One Gorilla	Atsuko Morozumi	The first page says, “Here is a list of things I love.” Have students figure out how many things the author loved altogether. Allow them to use a variety of strategies to solve the problem.
One Hundred is a Family	Pam Ryan	Groups making up many different kinds of "families" introduce the numbers from one to ten and then by tens to one hundred.
One Hundred Ways to Get to 100	Jerry Pallotta	Teaches counting by ones, twos, fours, tens, twenties, and so on...
1 Hunter	Pat Hutchins	Challenge students to figure out the total number of animals seen by the hunter.
One is a Snail, Ten is a Crab	April Pulley Sayre and Jeff Sayre	In this story, the authors count the number of feet different animals have. The book begins with a snail's one foot, and then moves on to 2 for humans, 4 for dogs, 6 for insects, 8 for spiders, and 10 for crabs. Each odd number is represented by the even-numbered animal plus one snail. After 10, the numbers go by 10s to 100, with the number shown in two ways, for example: "70 is seven crabs- or ten insects and a crab. 80 is eight crabs - or ten spiders.”

One Lonely Sea Horse	Saxton Freymann and Joost Elffers	Students can figure out the total number of sea creatures that the lonely sea horse finds while looking for friends.
One Odd Day	Doris Fisher and Dani Sneed	One morning, a boy awakens to find that everything is literally odd. The numbers on his clock read 1, 3, 5, 7, 9, and 11; his shirt has three sleeves and his dog has five legs. At school, the calendar only has odd numbers. His mother tells him to grab seven bananas before catching bus number nine. Students can search for all the odd numbers hidden throughout the pages of this vivid book.
One Riddle, One Answer	Lauren Thompson	After hearing the story, students can first try to solve Aziza's riddle themselves, then they can write their own number riddles for their peers to solve.
One Watermelon Seed	Celia Barker Lottridge	This is a counting book that not only gives children the opportunity to count from 1 to 10, but also from 10 to 100.
Only One	Marc Harshman	"There may be 500 seeds, but there is only one pumpkin." After reading this story, give students the opportunity to discuss things that come in groups of various sizes. They could write their own class book modeled after this one.
A Place for Zero	Angeline Sparagna LoPresti	As Zero searches to find his place, he learns of his additive and multiplicative identities, and then he establishes place value.
Popcorn	Frank Asch	Animated story available online at frankasch.com. Have kids estimate how many kernels of unpopped and popped corn would fit into various containers. Pose the question: "Sam's friends brought 12 cans of popcorn to the party. The popcorn cans held 3 ounces of unpopped corn each. How many ounces of unpopped corn did the guests bring altogether?"
Roar! A Noisy Counting Book	Pamela Duncan Edwards and Henry Cole	A lion cub meets one red monkey, two pink flamingos, three orange warthogs, and so on but his loud roar keeps scaring them away. Finally, he meets the right playmates: nine other roaring cubs. Students can figure out the total number of animals the lion cub meets. For an extra challenge, they can figure out the number of feet on all the animals the lion cub meets.
Rock It, Sock it, Number Line	Bill Martin, Jr.	Introduces the numbers one through ten as vegetables and numbers dance together at the king's and queen's garden party before jumping into the soup to be eaten by a crowned boy and girl.
Six-Dinner Sid	Inga Moore	Students can investigate how many dinners Sid ate in one week when he lived on Aristotle Street.

Stay in Line	Teddy Slater	Simple math concepts are woven into a story about twelve kids and their trip to the zoo. Students can investigate how many different ways a dozen children can be grouped. What if there were two dozen children.
Ten Black Dots	Donald Crews	Challenge students to figure out how many black dots are on all of the pages altogether. Students can also create their own number books similar to this one.
Ten Little Rabbits	Virginia Grossman	Have students figure out how many rabbits there are altogether.
Two Ways to Count to Ten	Ruby Dee	Challenge kids to investigate ways to count to other numbers. How many ways are there to count to 12, 24, etc.? Why are there only two ways to count to 11, 13, 15, etc.? Great springboard for discussion on prime numbers, factors, and patterns.
When Sheep Cannot Sleep: The Counting Book	Satoshi Kitamura	When Wooly the sheep suffers from insomnia, he goes for a walk and gets into just about everything. Each illustration features objects for children to count.
LARGE NUMBERS		
Title	**Author**	**Annotations/Lesson Ideas**
Can You Count to a Googol?	Robert Wells	Introduces the concepts of very large numbers up to a googol.
G is for Googol	David M. Schwartz	Explains the meaning of mathematical terms which begin with the different letters of the alphabet from abacus, binary, and cubit to zillion.
A Grain of Rice	Helena Clare Pittman	Challenge students to figure out how many grains of rice Rani received in all. Calculators will be helpful. The answer is 1,073,741,823 – more than 1 billion grains of rice. Have students discuss how the number increased so quickly in a short amount of time.
How Much is a Million	David M. Schwartz	Have students actually make one million. Have them each draw 100 stars and hang the sheets of paper on the wall. This will take a long time so they may wish to enlist other classes to help.
If you Hopped Like a Frog	David M. Schwartz	Did you know that a frog can jump 29 times its body length? Or that an ant can lift an object 50 times its weight? Introduces the concept of ratio by comparing what humans would be able to do if they had bodies like different animals.
A Million Dots	Andrew Clements and Mike Reed	This book helps children see what a million actually looks like.

A Million Fish…More or Less	Patricia C. McKissack	This story can lead to a discussion of large numbers and an activity in which students can investigate dividing numbers in half. Pose the problem: "If Hugh Thomas had five hundred thousand fish and Mosley the raccoon took half, how much would he have left?"
Millions of Cats	Wanda Gag	Have students come up with a plan to figure out how much space one million cats would occupy.
On Beyond a Million	David Schwartz	Explores counting by powers of ten all the way up to the largest numbers that exist.
ADDITION/SUBTRACTION		
Title	**Author**	**Annotations/Lesson Ideas**
Annie's One to Ten	Annie Owen	Have students write addition and subtraction number sentences for the items on each page.
Bennie's Pennies	Pat Brisson	Benny sets off in the morning with five shiny new pennies to spend and eventually buys something for his mother, brother, sister, dog, and cat. Give students 5 pennies and let them "spend" their pennies as the story is read. This book helps students understand the concept of subtraction.
Centipede's 100 Shoes	Tony Ross	Ask students to figure out if the author figured correctly and if the little centipede really did sell all one hundred shoes and forty-two socks.
Counting Wildflowers	Bruce McMillan	This book uses beautiful photographs of wildflowers to illustrate the concepts of addition and subtraction up to 20.
Domino Addition	Lynette Long, Ph.D.	Have students place beans out to match the number of dots on each half of the domino. Then they can total the beans and write the complete addition number sentence for each example. Students can play "Domino War" similar to the game of War played with playing cards. The game can be varied by multiplying the dots on each side of the domino played.
Five Little Monkeys Jumping on the Bed	Eileen Christelow	This book is great for counting and subtraction. Give students monkey cutouts and let them act out the story as it is read.
How Many Feet in the Bed	Diane Johnston Hamm	Questions to investigate: How many feet were in the bed? How many feet are in the class? Introduce the possibility that a pet dog or cat could also be in the bed. If there were 10 feet in the bed, for example, there could be 5 people, or 3 people and one cat, or 1 person, 1 cat and 1 dog. Have the children figure different possibilities for 12 feet in the bed.

Mission Addition	Loreen Leedy	Miss Prime and her animal students explore addition by finding many examples in the world around them. Challenge students to find examples of addition around the classroom, school and at home.
The $1.00 Word Riddle Book	Marilyn Burns	This book serves as a springboard for providing students useful practice with addition as they first figure the values of their names and then search for one-dollar words.
One Less Fish	Kim Michelle Toft and Allan Sheather	Give students 12 goldfish crackers and have them eat one fish cracker each time one fish disappears. Write the subtraction problem on the board. Have students create their own fish subtraction stories.
1, 2, 3 to the Zoo	Eric Carle	Each car on the train has one more zoo animal than the one before, from the first car with an elephant to the last with ten birds.
Quack and Count	Keith Baker	In this counting and adding book, a family of ducklings shows sums that add up to seven. After examining all the ways to make 7, students can be challenged to find ways to make 8, 9, 10, etc.
Subtraction Action	Loreen Leedy	Miss Prime and her class learn how to use subtraction to solve problems as they get ready for the school fair. Each problem is first presented as a word problem such as, "If we have ten tickets to the fair, and our class uses six of them, how many tickets will be left over?" The problem is then written out as a subtraction equation and completed systematically.
10 for Dinner	Jo Ellen Bogart	It's Margo's birthday and she invites 10 friends to the party. Children can think about combinations of numbers that add up to 10. The children can add birthday gifts, party food, etc. Students can write a class book modeled after this one.
Ten Friends	Bruce Goldstone	Challenge students to write as many different number sentences as they can that show different combinations of numbers that add to ten. This book is available at Scholastic.com.
Ten, Nine, Eight	Molly Bang	In this story, a young girl and her father count their ways through nightly rituals of bath, stuffed animals, dressing, and good night kisses. Students can count objects as the book is read. This book also helps children begin to understand the concept of subtraction.
Ten Sly Piranhas: A Counting Story in Reverse	William Wise	Ten sly piranhas are swimming in a river, but one at a time they disappear, until there is only one left. Great springboard for introducing subtraction.

Ten Terrible Dinosaurs	Paul Strickland	A group of rollicking dinosaurs counts down from ten to one. Great for introducing subtraction.
12 Ways to Get to 11	Eve Merriam	Combinations of items add up to 11. This is a good springboard for discussion of different combinations of numbers. Have their students come up with their own "number stories." This book provides a good alternative way of thinking about addition. Students can make a book modeled after this one, but use a number other than 11.
Multiplication/Division		
Title	**Author**	**Annotations/Lesson Ideas**
Amanda Bean's Amazing Dream	Cindy Neuschwander	After reading the story, revisit each illustration and talk with students about different ways to count the objects. Have students write their own multiplication stories. Additional lesson ideas and investigations are provided at the end of the book.
Anno's Mysterious Multiplying Jar	Mitsumasa Anno	Have students use calculators to verify the number of mountains, kingdoms, villages, etc. there are altogether.
"Beasts of Burden" *in The Man Who Counted: A Collection of Mathematical Adventures*	Malba Tahan	In this story, the narrator and Beremiz, wise mathematician, can't figure out how to divide an inheritance of thirty-five camels between three brothers. Have students work together to come up with a possible solution that Beremiz might use to divide the camels.
Bunches and Bunches of Bunnies	Louise Mathews	The book shows 81 bunnies in one classroom at school. Have them investigate how these bunnies could be split equally into smaller classes. Other problems to pose include "There are 25 bunnies shown at the ball. Four are in the band. If the rest are dancing, how many are dancing?" "At the beach, eight bunnies find shells. If each of these bunnies finds four shells, how many seashells are found altogether by these bunnies?" "When all the bunnies gathered together at the family reunion, there are 144 bunnies. One pound of rabbit pellets will feed approximately three bunnies each day. How many pounds of rabbit pellets would be necessary to feed all 144 bunnies?"
The Doorbell Rang	Pat Hutchins	If the children split all of Grandma's cookies among themselves, how many cookies would they get apiece? If each child in the story had one cup of milk with the cookies, how many quarts or gallons of milk would they need?

Each Orange Has Eight Slices	Paul Giganti and Donald Crews	This book provides an opportunity for students to think about multiplication in a way that makes sense. Students can write their own multiplication books similar to this one.
The 512 Ants on Sullivan Street	Carol A. Losi	Ants at a picnic keep doubling until they steal all the food. If the pattern continues, how many ants would be needed for the next three food items? Find the total number of ants each time a new food is added to the story.
The Great Divide	Dayle Ann Dodds	This book demonstrates the basic principle of division. Eighty contestants start out on bikes; they come to a fork in the path and half go left. The other 40 keep going by boat, until they reach a whirlpool where half of them are again knocked out of the race. This continues until only five contestants are left, leaving a tricky problem to be solved.
The King's Chessboard	David Birch	Challenge students to figure out how many grains of rice the wise man would get after all 64 squares were covered. Have kids investigate what is happening to the numbers each day.
Minnie's Diner: A Multiplying Menu	Dayle Ann Dodds	Ask students to talk about how many specials would be on the tray if there was a Grandpa McFay that came in at the end of the story. How would the story be different if there were more brothers?
One Hundred Hungry Ants	Elinor J. Pinczes	Talk about the different ways the littlest ant arranged his friends. Choose a new number (12, 24, 60, etc.) of ants and have students think about all the ways they could arrange themselves into rectangles to get to the picnic. Have them investigate what would happen if there were 29, 57, 105, etc. ants.
One of Each	Mary Ann Hoberman	In the story, Oliver solved the problem of not having enough fruit for his new friends by cutting slices of fruit and sharing them. This story is a good springboard for division.
A Remainder of One	Elinor J. Pinczes	Have students figure out how many ways 72 bugs could line up in equal lines? Try this with other numbers, including odd numbers. This is a good illustration of factors and prime numbers.
Sea Squares	Joy N. Hulme	Use tiles to interpret the pairs of numbers geometrically. Demonstrate with the page that shows 4 seals with 4 flippers each. Show the children how to arrange 16 tiles into a square array and tell them that 16 is therefore called a square number. Have the children use square tiles to check that the larger of each of the other pairs of numbers is also a square number.

17 Kings and 42 Elephants	Margaret Mahy	Explore the concept of division with remainders by having students solve the problem of how 17 kings could share the work of caring for 42 elephants. Other math questions – How many footprints did they all make in the jungle? How many ears and tails did they have altogether?
10 Bears in My Bed	Stan Mack	If ten bears are in the bed, how many feet are in the bed? How many toes?
2 x 2 = Boo!	Loreen Leedy	Students can create scenes like those in the book and create multiplication problems to go with their scenes. For example, they could create three bats with two eyes each and ask how many eyes do the bats have?
What Comes in 2's, 3's, & 4's?	Suzanne Aker	Great springboard for introducing multiplication.
Fractions/Decimals/Percents		
Title	**Author**	**Annotations/Lesson Ideas**
Apple Fractions	Jerry Pallotta and Rob Bolster	The authors use a variety of different apples to teach kids all about fractions. Students can figure out how to share a certain number of apples among the whole class and tell what fraction of the apple each child would get to eat.
Eating Fractions	Bruce McMillan	This book shows various foods as wholes and then as halves, thirds, fourths, etc.
Fraction Action	Loreen Leedy	In this book, the hippo teacher teaches her students about halves, thirds and fourths, and about sets, fair shares, and fractions in money.
Fraction Fun	David Adler	This book introduces four fraction activities.
Full House: An Invitation to Fractions	Dayle Ann Dodds	Readers will be inspired to do the math and discover that one delicious cake divided by five hungry guests and one hospitable hostess equals a perfect midnight snack at the Strawberry Inn. Great introduction to fractions.
The Hershey's Milk Chocolate Bar Fractions Book	Jerry Pallotta	This book uses chocolate bars to help students understand the concept of fractions.
Inchworm and a Half	Elinor J. Pinczes	What's a fraction? A puzzled inchworm enlists the aid of 1/2-inch, 1/4-inch, and 1/4-inch worms in her quest to measure all the vegetables in their garden.
My Half Day	Doris Fisher	The main character awakens to discover that his hair is half long and half buzzed and then discovers his milk is one-third gooey paste. This book helps students learn about a variety of fractions as the main character makes a trip to Camp Fraction.

Piece = Part = Portion: Fractions = Decimals = Percents	Scott Gifford	This book introduces the idea that fractions, decimals, and percents are different ways of saying the same thing.
The Wishing Club: A Story about Fractions	Donna Jo Napoli	Petey wishes on the first star he sees, but instead of the dollar he hopes for, he gets only a quarter. The next night his little brother Joey wishes for a cookie - and gets just half of one. Try as they might, the children each wind up with a fraction of what they wished for.

ALGEBRA		
Title	**Author**	**Annotations/Lesson Ideas**
Anno's Magic Seeds	Misumasa Anno	Students can search for patterns to determine how many seeds Jack gets over ten years and then use their patterns to think about how to predict the number of seeds after longer periods of time.
The Little Scarecrow Boy	Margaret Wise Brown	This story is full of patterns and repetitions in threes and sixes.
Pattern Bugs	Trudy Harris	By using the sounds in the poems, the details of the pictures, and the various blocks of color that frame the initial spread, children can find and identify repetitive patterns. Each picture contains six patterns that match that picture's border.
Pattern Fish	Trudy Harris	Brightly colored fish inhabit a world of patterns, beginning with the simplest AB pattern and growing increasingly complex. Upon closer inspection, the patterns can be seen throughout, as pictures both express and reinforce the pattern of the words. For example, the ABB pattern introduced by an eel is echoed not only on the creature's body (stripe-dot-dot), but also in the bubbles that issue from its mouth (large-small-small), the underwater reeds (short-tall-tall) and plants (curve-star-star), and borders of the page (yellow-red-red).
Patterns are Everywhere!	Dominick and Alan Adunagow	This book helps children find patterns in their everyday lives. After reading the story, have students look for patterns in the classroom or in the school as they walk to lunch, art, etc.
Patterns Everywhere	Julie Dalton	This book helps children find patterns all around the world and in their own backyards. After reading the story, have students look for patterns in the classroom or in the school.

Patterns in Peru	Cindy Neuschwander	The characters in this story must use their understanding of patterns and sequences to locate the lost city—and the way back.
Rabbits Rabbits Everywhere: A Fibonacci Tale	Ann McCallum	Students can investigate the patterns in the story as the rabbits multiply.
Rooster's off to see the World	Eric Carle	Show students the pattern of pictures of the animals that appear on the inside front and back covers. Ask the students to figure out how many animals altogether set out to see the world.
Sort it Out	Barbara Mariconda	Children who are studying patterns and sorting will find this book interesting, especially because the packrat has so many ways to sort objects in addition to using color, size, and shape.
The Talking Cloth	Rhonda Mitchell	Many patterned designs decorate the "Talking Cloth" as well as the pages of the book.
Two of Everything	Lily Toy Hong	Challenge the students to solve the problem of how much money the couple would have at the end of ten days if they threw in one nickel each day. How many nickels would they have at the end of 10 days, 25 days, 100 days? Encourage them to create a T-chart and look for patterns. This is also a great book to use to introduce students to the concept of doubles.

MEASUREMENT

LENGTH AND WEIGHT		
Title	**Author**	**Annotations/Lesson Ideas**
Counting on Frank	Rod Clement	One of the facts shared in the book is that a gum tree grows about six and one-half feet every year. Have students figure out how tall they would be if they grew at the same rate and make a representation of how tall they would be.
Flat Stanley	Jeff Brown	A large bulletin board fell on Stanley and flattened him. When he went to the doctor, the nurse took his measurements and discovered he was 4 feet tall, 1 foot wide and ½ inch thick. Construct Stanley according to his new proportions.
Hershey's Milk Chocolate: Weights and Measures	Jerry Pallotta	Introduces various measures of weight, size, and volume using Hershey's brand candies and other products. Students can weigh and measure Hershey chocolate bars.

How Big is A Foot?	Rolf Myller	Have students write a letter to the apprentice offering him advice. This book is helpful for helping students realize the need for standard units of measurement.
How Tall, How Short, How Faraway	David A. Adler	Introduces several measuring systems such as the Egyptian system, the inch-pound system, and the metric system.
Inch by Inch	Leo Lionni	Use as a springboard for investigating measurement. Students can estimate and then measure things in the room using one-inch tiles or rulers. Encourage students to discuss why it is more difficult to measure taller, wider, or longer items using the one-inch tiles or rulers. Have them consider better tools to use to measure the longer items.
Is a Blue Whale the Biggest Thing There Is?	Robert E. Wells	Pose the question: "How many fourth graders do you think it would take to equal the blue whale's length of 100 feet?"
Inchworm and a Half	Elinor J. Pinczes	Several small worms use their varying lengths to measure the vegetables in a garden.
Jim and the Beanstalk	Raymond Briggs	Show the page that shows the giant sitting at his table, reading a little book. "On this page it looks as if the giant's thumb is about the size of the book. If his thumb is the size of a book, how long is his whole hand?" Have student's estimate the length of the giant's hand. Then pose the problem: "If the giant's hand is about thirty inches long, how tall is the giant?"
Length	Henry Arthur Pluckrose	Photographs and text introduce the concept of length and how to measure it.
Measuring Penny	Loreen Leedy	Lisa learns about the mathematics of measuring by measuring her dog Penny with all sorts of units, including pounds, inches, dog biscuits, and cotton swabs.
Millions to Measure	David Schwartz and Steven Kellogg	Marvelosissimo the Magician explains the development of standard units of measure, and shows the simplicity of calculating length, height, weight, and volume using the metric system.
"One Inch Tall" in Where the Sidewalk Ends	Shel Silverstein	Great poem to use to help students understand the concept of one inch. This poem could be a springboard for having students generate common referents for an inch.
Spaghetti and Meatballs for All	Cindy Neuschwander	Have students re-create the book by using square tiles to represent the tables. Have them determine the perimeter and area of the different table arrangements.

Twelve Snails to One Lizard, a Tale of Mischief and Measurement	Susan Hightower	Milo is desperate to measure a log that he will use to patch a hole in his dam. Milo and Bubba try to measure the log, using one animal after another as a measuring tool.
Weight	Henry Arthur Pluckrose	Photographs and text introduce the concept of weight and how to measure it.
Who Sank the Boat?	Pamela Allen	The reader is invited to guess who causes the boat to sink when five animal friends of varying sizes decide to go for a row. Great introduction to the concept of weight.
Zachary Zormer: Shape Transformer	Joanne Anderson	This book is a good introduction to the concepts of area, perimeter, length, and width. Each Friday, Ms. Merkle has her students share objects as part of a mathematical show-and-tell. Zachary has a tendency to forget and must use his ingenuity to meet the assignment's requirements. One week, he uses a piece of paper to create a Moebius Strip, which serves as something to measure. The following Friday, he uses a scrap of paper to make an expanding frame for his lesson on perimeters. At the end of the book, each of Zachary's three transformations is explained in well-detailed steps so children can try them on their own.
VOLUME/CAPACITY		
Title	**Author**	**Annotations/Lesson Ideas**
Cook-a-Doodle-Doo!	Janet Stevens and Susan Stevens Crummel	This book will help students discover that learning to measure is an important part of cooking and baking.
A House for Birdie	Stuart J. Murphy	This story about helping out a friend explains the math concept of capacity -- what will fit in a container of a particular shape and size.
Me and the Measure of Things	Joan Sweeney	Introduces young readers to the units of measure. What's the difference between a cup and an ounce? What gets measured in bushels and when do you use a scale? Illustrations teach children the differences between wet and dry measurements, weight, size, and length.
Pigs in the Pantry: Fun with Math & Cooking	Amy Axelrod	Measurement concepts are explored as Mr. Pig and the piglets try to cook Mrs. Pig's favorite dish to cheer her up when she's sick. Includes a recipe for chili.

TIME AND MONEY		
Title	**Author**	**Annotations/Lesson Ideas**
Alexander, Who Used to Be Rich Last Sunday	Judith Viorst	Read the book in its entirety. Then read it again stopping each time Alexander spends some of his dollar and ask students to figure out how much money he has left. Let them use play or real money if possible.
All in a Day	Mistumasa Anno	Illustrations show the similarities and differences in children and their activities in eight different parts of the world throughout one 24-hour day.
Annabelle Swift, Kindergartner	Amy Schwartz	Have students use information in the book to figure out how many cartons of milk Annabelle's class purchased. Students can also figure out how much it would cost to buy milk for their own class. For a measurement activity, they could figure out how many pints, quarts, and/or gallons of milk that their whole class would drink if everyone purchased and drank all of their milk at lunch.
Arthur's Funny Money	Lillian Hoban	Arthur decides to charge $.25 to wash a bicycle. Ask students to figure out how many bicycles he will have to wash before he has $5.25
Bats Around the Clock	Kathi Appelt	Click Dark, the bat-version host of American Batstand, leads the 12-hour dance program. A rhyming verse teaches children how to tell time while dancing to the oldies.
The Big Buck Adventure	Shelley Gill	Rhyming account of a little girl's quandary as she tries to decide what she can get with her dollar in a candy shop, toy store, deli, and pet department.
Bunny Money	Rosemary Wells	Have students use play money to count and spend money along with the two main characters.
Caps for Sale	Esphyr Slobodkina	Have students figure out how much would it cost to buy all of the caps.
A Chair for My Mother	Vera B. Williams	A child, her waitress mother, and her grandmother save dimes to buy a comfortable armchair after all their furniture is lost in a fire.
Chicken Soup with Rice	Maurice Sendak	Great resource for teaching the months of the year. Students can create a class months of the year book.
Clocks and More Clocks	Pat Hutchins	When the hall clock reads twenty minutes past four, the attic clock reads twenty-three minutes past four, the kitchen clock reads twenty-five minutes past four, and the bedroom clock reads twenty-six minutes past four, what should Mr. Higgins do? He can't tell which of his clocks tells the right time. Have children discuss what the problem might be.

The Coin Counting Book	Rozanne Lanczak Williams	This book gives children the opportunity to count and add while learning the names and denominations of all of the U.S. coins. What do you get when you add five pennies together? What coin combinations add up to a quarter?
Cookie's Week	Cindy Ward	Cookie gets into mischief each day of the week. This book can help young children learn the days of the week.
A Dollar for a Penny	Julie Glass	Penny sets up a lemonade stand to earn money for her mother's birthday card.
Five Minutes Peace	Jill Murphy	Mrs. Large just wants five minutes peace. She heads for the kitchen and finds exactly three minutes and 45 seconds of peace.
The Go Around Dollar	Barbara Johnston Adams	A story describing how a single dollar changes hands, accompanied by facts about the one-dollar bill.
The Grouchy Ladybug	Eric Carle	The grouchy ladybug begins the day at five o'clock in the morning and ends it around six o'clock at night. How many hours was he flying around?
How Do You Know What Time it Is?	Robert Wells	This book introduces the concept of time and how we measure it.
If You Made a Million	David Schwartz	The book shows four different combinations of coins that are equivalent to a one dollar bill. Have the students list as many combinations as they can of coins and bills that are equivalent to a five dollar bill.
Jelly Beans for Sale	Bruce McMillan	This book introduces pennies, nickels, dimes, and quarters and the value of each; that is, each cent is worth one jelly bean. The author represents several different amounts of money with many combinations of coins. Available at Scholastic.com
The Money Tree	Sarah Stuart	Miss McGillicuddy notices a strange tree in her yard. Month by month, as the seasons change, it grows, faster than any normal plant, into a money tree.
Monster Math School Time	Grace MacCarone	From the time they get up at seven in the morning until they go to bed at eight o'clock at night, monsters spend a busy day, especially at school. Activities for students are included.
Monster Money Book	Loreen Leedy	The members of the Monster Club discuss money and how to manage it.
My Grandmother's Clock	Geraldine McCaughrean	A child, wondering why Grandma doesn't have the grandfather clock in her house repaired, learns that there are many ways to measure time.

My Rows and Piles of Coins	Tololwa M. Mollel	A Tanzanian boy saves his coins to buy a bicycle so that he can help his parents carry goods to market, but then he discovers that in spite of all he has saved, he still does not have enough money.
On the Day You were Born	Debra Frasier	Students can investigate the number of days that have passed since they were born.
Once Upon a Dime	Nancy Kelly Allen	Farmer Worth discovers that a special tree on his farm produces different denominations of money, depending on what animal fertilizer he uses.
Only Six More Days	Marisabina Russo	Students can use calendars and counters or Snap Cubes to figure out how many more days until their birthdays.
P. Bear's New Year's Party	Paul Owen Lewis	The first guest arrives at one o'clock and at two o'clock, two more guests arrive. Every hour after that, the number of guests that arrives matches the hour on the clock. Challenge kids to answer the question on that last page of the book, "How many guests came to the party?" This gives students the opportunity to practice counting and addition while reviewing time.
Pigs Go to Market: Fun with Math and Shopping	Amy Axelrod	The Pigs can't wait to throw their annual Halloween party. But when Grandpa and Grandma Pig eat all of the candy, the Pigs have to make a last minute trip to the market, where Mrs. Pig wins a free shopping spree. This book provides practice with the math skills of price and quantity.
Pigs on a Blanket: Fun with Math and Time	Amy Axelrod	Great story for teaching elapsed time. Students can figure out how long it actually took the pigs to drive to the beach and how much longer the drive than was than it should have been.
Pigs Will Be Pigs: Fun with Math and Money	Amy Axelrod	Provides an excellent way to talk about how decimals are used to represent money numerically. Also provides a context that focuses on the values of coins and bills and provides practice with adding money.
Quarter from the Tooth Fairy	Caren Holtzman	A boy has trouble deciding how to spend the quarter he gets from the Tooth Fairy. He learns about money and the different coin combinations that make up a quarter.
"Smart" *in Where the Sidewalk Ends*	Shel Silverstein	Have the students write a letter to the little boy offering him advice. Did the boy get a good deal like he thought he did?
The Story of Money	Betsy Maestro	The author gives a history of money, beginning with the barter system in ancient times, to the first use of coins and paper money, to the development of modern monetary systems.

Telling Time	Jules Older	This book explains the concept of time, from seconds to hours on both analog and digital clocks and from years to millennia on the calendar. A poem at the end reminds children how many seconds are in a minute, how many minutes are in an hour, and so on.
365 Penguins	Jean-Luc Fromental	On the first day of the new year, the mailman brings a surprise - a penguin! One by one, day by day, penguins fill the house. As they arrive, readers must recall the number of days in each month. By the end of February, they are calculating the number of penguins in all. Then Father decides to organize them, first into four groups of 15, later in boxes by the dozen, and, finally, into a cubic formation.
Time	Henry Arthur Pluckrose	Time is explained in a historical, future and present sense. Readers are given examples of each type of time being described. Time is also explained in the realm of seasons, day and night, and calendars, along with telling time on a clock. Both an analog clock and a digital clock are explained as well as how to read them.
Time to…	Bruce McMillan	This book introduces children to the concept of telling time. Photos follow a little boy's daily activities as each double-page spread advances the clock by one hour. This not only familiarizes children with different kinds of clocks, but also introduces the concept of A.M. and P.M.
Today is Monday	Eric Carle	Each day of the week brings a new food, until on Sunday all the world's children can come and eat it up.
The Toothpaste Millionaire	Jean Merrill	Have students find out the cost of toothpaste in a local store and calculate the cost per ounce.
26 Letters and 99 Cents	Tana Hoban	Color photographs of letters, numbers, coins, and common objects introduce the alphabet, coinage, and the counting system.
The Very Hungry Caterpillar	Eric Carle	Challenge students to figure out how many things the caterpillar ate in one week. They could also create a book entitled "The Very _____ _____ following the model using days of the week for each page.
What Time is It?	Sheila Keenan	A child is so excited about her Grandpa's visit that she counts the hours until he arrives. Challenge students to figure out how many hours until some event is to occur in the classroom (recess, lunch, end of school day, pizza party, etc.).

What Time is It, Mr. Crocodile?	Judy Sierra	There is a clock face on every double page spread so children can enjoy the countdown as well as reinforcing telling time for those just learning the skill.
America's Champion Swimmer: Gertrude Ederle	David Adler	Students can choose a sport to investigate with an emphasis on measurement. Have them use newspapers, the Internet, magazines, and other available sources to find out what aspects of measurement are used in that sport. For example, in football both time and length are important and the game is timed in minutes and seconds.

GEOMETRY

TITLE	AUTHOR	COMMENT
Bear in a Square	Stella Blackstone	This book is a good introduction to two-dimensional shapes. Bear looks for shapes around him.
A Cloak for the Dreamer	Aileen Friedman	Have students write letters to the character Misha explaining the problem with the cloak and offering solutions to the problem. Students can also make cloak patterns of their own.
Color Farm	Lois Ehlert	The rooster, dog, sheep, cow, pig, and other animals on a farm are made up of colorful shapes such as square, circle, rectangle, and triangle.
Cubes, Cones, Cylinders and Spheres	Tana Hoban	Photographs of all kinds of familiar objects depict a variety of shapes, including cubes, cones, and spheres. Follow up by having students find examples of these three-dimensional shapes in the classroom or at home.
Ed Emberley's Picture Pie	Ed Emberley	Shows how to cut a basic circle into arcs and curves and use the pieces to draw birds, animals, snowmen, fish, and many other objects and designs. This book could also be used to help students understand the concept of fractions. Teachers can have students make pictures and tell what fraction of the circles they used to create their designs.
Ed Emberley's Picture Pie 2	Ed Emberley	Shows how circles, squares, and triangles can be used to create all kinds of pictures. Students can use shapes to create their own pictures after enjoying this book.
Grandfather Tang's Story	Ann Tompert	Allow students to make their own tangrams and make pictures with their tangrams.
The Greedy Triangle	Marilyn Burns	Good book to use as a springboard for exploring shapes.
Look at Annette	Marion Walter	Have students use mirrors and die cuts of letters to see which of the letters of the alphabet have symmetry. Which letters have one line of symmetry and which

		ones have more than one line of symmetry? Can they find words that have symmetry (WOW, MOM, etc.)
Mummy Math: An Adventure in Geometry	Cindy Neuschwander	Stuck inside a pyramid with only each other, their dog Riley, and geometric hieroglyphics to help them find their way, the twins must use their math knowledge to solve the riddles on the walls and locate the burial chamber.
Reflections	Ann Jonas	Chronicles a child's busy day by the sea, in a forest, and then home with the hope of going to a carnival or concert. The illustrations change when the book is turned upside down. This book can be used to introduce reflections.
Sam Johnson and the Blue Ribbon Quilt	Lisa Campbell Ernst	After reading the book and looking at the various quilt patterns, students explore ideas in geometry as they work together to make paper quilts.
The Secret Birthday Message	Eric Carle	Students can find all the shapes in the pictures throughout the book.
Shape	Henry Arthur Pluckrose	Photographs of familiar objects introduce basic shapes of squares, circles, rectangles and triangles.
The Shape of Things	Dandi Daley Mackall	Great springboard to introduce shapes-circles, squares, rectangles, triangles to young children.
Geometry The Village of Round and Square Houses	Ann Grifalconi	Students can draw the shapes that make up the structure of their own homes and write the names of the shapes. Students can create three-dimensional structures and make their own villages.
Shape Up! Fun with Triangles and Other Polygons	David A. Adler	The author uses cheese slices, pretzel sticks, a slice of bread, graph paper, a pencil, and more to introduce various polygons.
Shapes, Shapes, Shapes	Tana Hoban	Talk about each photograph with the students, asking them to identify as many shapes as they can. After examining the photographs in the book, have the children look for and describe the shapes they see in the classroom, around the school, at home, or on walks in the neighborhood.
So Many Circles, So Many Squares	Tana Hoban	The geometric concepts of circles and squares are shown in photographs of wheels, signs, pots, and other familiar objects.
The Tangram Magician	Lisa Campbell Ernst and Lee Ernst	Have students experiment with making different shapes with the tangram pieces. Invite children to create some of the tangram figures that are illustrated in the book as well as some of their own.

Three Pigs, One Wolf, and Seven Magic Shapes	Grace Maccarone	In this story, the pigs meet magic animals that give them seven magic shapes to help them find their fortune. These shapes form a tangram. Notes at the end of the book include a cut-out tangram and suggestions for creating other shapes with this new tool.
The Warlord's Puzzle	Virginia Pilegard	After hearing the story, students use their own set of tangrams and explore making different polygons.
When a Line Bends…A Shape Begins	Rhonda Gowler Greene	Ask students to explain the author's meaning when she wrote, "When a line bends….A shape begins." Have students find triangles, circles, squares, octagons, diamonds, rectangles, etc. throughout the book.

DATA ANALYSIS AND PROBABILITY

TITLE	AUTHOR	ANNOTATIONS/LESSON IDEAS
Caps for Sale	Esphyr Slobodkina	Have students investigate how many different arrangements of caps he could wear if he always wore the checkered one on his head and kept the same color hats together?
Chrysanthemum	Kevin Henkes	Students can create graphs using data of how many letters are in their names.
Graphs	Bonnie Bader	A boring family reunion provides data for charting graphs for a homework assignment.
Jesse Bear, What will You Wear	Nancy White Carlstrom	Use Teddy Bear die cuts and cutouts of shirts and pants to figure out how many different combinations of outfits Jesse could wear. Challenge them to add hats to the combinations. Children will be surprised by how many more combinations are available when three articles of clothing are used. See http://illuminations.nctm.org/ActivityDetail.aspx?ID=3 for an interactive activity (Bobbie Bear) that can be done using the Promethean board.
No Fair!	Caren Holtzman	Two children play several games of chance trying to figure out what is mathematically fair. Challenge students to create games that are mathematically fair.
It's Probably Penny	Loreen Leedy	Lisa's class is learning about probability. For part of her homework, she has to think of an event that will happen, one that might happen, and one that can't happen.
A Three Hat Day	Laura Geringe	Suppose R.R. Pottle wanted to cheer himself up on another day and put on the same three hats before

		going for his walk. But suppose that he decided to put on the hats in a different order? And then the next day he put the same three hats on in a different order again. How many days could R.R. Pottle wear those same three hats if each day he put them on in a different order?
A Very Improbable Story: A Math Adventure	Edward Einhorn	Waking up one morning and finding a talking cat on his head, Ethan is informed that the cat will not leave until he has won a game of probability. Challenge students to think of events that are likely, equally likely, unlikely, certain, or impossible.
The Best Vacation	Stuart J. Murphy	Have students select a survey question and collect data by interviewing fellow students. As they complete their surveys, have them select a way to represent the data and share it with the class.

59 writing prompts for math teachers

Prompts for math journals (assessing disposition)

- My best experience with math was when_____.
- My worst experience with math was when_____.
- I love math because...
- I hate math because_____.
- I want to become better at math so that I_____.
- Is math your favorite subject? Why or why not?
- What did you like most about your math class last year? What did you like the least?
- People who are good at math_____.
- One mathematics activity I really enjoy is _____ because _____.
- When I study for a test, I_____.
- When it comes to math, I find it difficult to_____.
- When I hear someone say math is fun, I_____.
- When I see a word problem, the first thing I do is _____. Then I _____.
- Does mathematics or math class scare you in any way?
- Draw a picture of a mathematician and describe what a mathematician does.
- Explain how you feel about mathematics now as compared to before you took this class.

Prompts for learning logs (assessing learning)

- My three personal goals for this term are_____.
- Explain everything you know about _____.
- What is a _____? Write all you can about _____.
- Find something that you learned today that is similar to something you already knew. Write about these similarities.
- How would you use what you learned today in your life?
- The main idea today's lesson was_____.
- Write a definition in your own words of a _____.
- What is the most significant thing you learned this week?
- What questions are still unanswered at the end of this week?
- Last week in math I learned _____.
- My favorite part of math last week was _____.

- The hardest part of math last week was _____.
- This is how I used math this week (outside of school)_____.
- Describe any discoveries you made about mathematics today, this week/month/year.
- Give two examples of situations in which you have used, seen or can find the concept of _____. OUTSIDE of this classroom.
- I want to learn more about _____.
- I wish I knew more about_____.
- I need more help understanding _____.
- My math grade now is _____ because_____.
- What was your grade on the exam/quiz/project? If you were not satisfied with your score, what can you do to improve? If you were happy with your score, what did you do well?
- Write instructions for a (insert grade level here) grader to follow when (adding fractions, finding percentages, calculating averages, etc.)
- How would you describe a _____?
- The difference between_____ and _____ is_____.
- Compare and contrast the terms _____.
- What patterns do you notice in _____.
- You know several ways to_____(solve an equation, add fractions, etc.) Which method is your favorite? Why?
- Make a list of objects or figures in the room which have _____. How can you tell?
- Write a letter to your teacher explaining what you understand about _____, and what is still giving you trouble.
- Write and solve a word problem whose solution involves _____.
- Describe practical uses for _____.
- Write___ possible test questions for this unit.
- Write a letter to a student who will be taking this class next year, giving some advice about this class.

Prompts to use for problem-solving (assessing process)

- The most important part of solving a problem is_____.
- Describe the process you undertook to solve this problem. (Problem needs to be provided.)

- I knew I was right when_____.
- Tips I would give a friend to solve this problem are_____._____._____.
- Could you have found the answer by doing something different? What?
- What strategy did you use to solve this problem and why?
- Was this problem difficult or easy? Why?
- Were you frustrated with this problem? Why or why not?
- Where else could you use this type of problem solving?
- What would happen if you missed a step? Why?
- What other strategies could you use to solve this problem?

Number Sense Routine Planning Template

Name of Routine:	**Problem:** _____ **Anticipated Responses:**
Mathematical Goal:	**What is the math?** **Where are my students? What are the entry points for my students?**
Recording	**Will Students Record? If so, how?** **What is important about my recording?**
Questions/Math Talk Accountability	**What questions will I ask?** **What are some follow up questions?** **What talking strategies will I use:** **Revoicing, Restating, Agree/Disagree, Adding on, Wait Time**
Follow Up/Next Steps	**Anchor Chart? Formative Assessment? Exit Slip? Journal?** **What do I notice the students noticing?** **How do I move forward?**

Stage of Numeracy	Definition
Subitizing	ability to see small amounts without counting
Magnitude	ability to tell which of two sets has more without counting
Count Sequence	ability to say the counting sequence, which happens before one-to-one correspondence is fully developed.
One-to-One Correspondence	is saying one number for each object counted.
Cardinality	is when you count a group of objects, and the last number you say tells how many in all. Students who don't have this, recount objects when asked "how many?"
Hierarchical Inclusion	understanding that smaller numbers are part of bigger numbers. Idea of one more and one less
Part/Whole Relationships	understanding different ways to make up a given number, composing and decomposing numbers, understanding that 6 is made up of 1 &5, 3&3, 2&4.
Compensation	see the parts of a whole and are able to compensate, so if I know that 5+1=6, then I also know that 4+2=6 because 4 is one less than 5 and 2 is one more than 1.
Unitizing	Once students begin to gain a solid understanding of the preceding number sense, then they can construct unitizing as they work with larger numbers. Place value

Observable Student Behavior	Stage of Numeracy
While playing a game during math, Deborah rolls the dice and automatically says the number without counting.	Subitizing
When snack is passed out, a student states, "Hey, you got more goldfish than I do." OR "Hey, I got less"	Magnitude
Jennifer recites, 1,2,3,4,5,6,7,8,9,10 for her teacher as she skims over objects.	Count Sequence
While solving a problem, Susan counts out objects, as she counts them she says one number name for each object counted.	One-to-One Correspondence
During morning message, Christi asks students to count all the lower case letters in the message. Christi then asks, "How many lowercase letters are in our message?" Natalee raises her hand and states, "23 lowercase letters."	Cardinality
While working on their tens-frames, Jill tells students to place 2 counters on the frame. Next she tells them to put 3 counters on the tens frame. Dawn keeps her two counters, and just adds one more when the teacher says to put 3 counters.	Hierarchical Inclusion
Ashley is working on a problem which asks her to show all the different ways to make the number 9. Ashley uses 0+9=9, 4+5=9, 3+6=9, and 8+1=9, 4+4+1=9	Part/Whole Relationships
Rachel is practicing her addition. She previously solved 3+2=5, now she's trying to solve 3+3=6. She told her teacher, Renee "If 3+2=5, then if I just add 1 more that will be 6, because 3 is one more than 2 and 6 is one more than 5."	Compensation
Stephanie, Kimberly, Amy, and Chris are working as a group on the following problem: " The recycling club would like to take bags of recycling to the Recycling Center. The Center will only take bags that hold 10 bottles or 100 cans. If your group collects 38 bottles and 234 cans, how many bags of each will you take to the Recycling Center? Stephanie says they need to draw 38 bottles and circle groups of 10. Amy agrees with her. Chris says they can just count by 10's and 100's and see what's left over. Kimberly says they need 38 bags for the bottles and 234 bags for the cans.	Unitizing

Problem:	Strategy I used:
Solution	Other Strategies Observed

Name _____
Date: _____

Mathematics Problem Solving Board

What is the question asking me to do?	What data do I have? (What data is not necessary and/or What data do I need?)	☑ the Strategies ☐ Draw a Picture or Diagram ☐ Look for a Pattern ☐ Guess and Check (Estimate) ☐ Act it Out (Using Manipulatives) ☐ Make a Table/List ☐ Work Backwards
Plan (use the back if more room is necessary)	Picture/Diagram/Table/Rule/Graph/Thinking Map	
Final Answer (use a complete sentence)	Check your answer. Is it reasonable?	