Mathematics **Brunswick School Department**
Calculus: Honors
Unit 4: Integration & Applications

Essential Understandings	- The concept of an "integral" as the operational inverse of a derivative and as a summation model is introduced using anti-derivatives. - Students find integration shortcuts for the basic functions. - Students find integrals of composite functions. - Students apply integrals to area & motion problems. - Students learn the Fundamental Theorem of Calculus. - Students perform numerical integration.
Essential Questions	- How do you find an integral using anti-derivatives? - What is an integral and how is it used in calculus? - How can an integral be used in the real world? - How do integrals apply to motion problems? - How do integrals apply to area problems?
Essential Knowledge	- An integral can be thought of as "undoing" a derivative problem. - Applying an integral to motion problems allows for quick computational methods to finding total distance traveled and the position shift of an object in motion. - Applying an integral can find areas under curves.
Vocabulary	- Terms: - area under a curve; average value; definite & indefinite integrals; Fundamental Theorem of Calculus; Rectangle approximation method for areas; mean value theorem for integrals; net vs total area; Riemann Sums; the Trapezoid & Simpson's rules
Essential Skills	- Find and evaluate an integral. - Apply integrals to various motion problems. - Apply integrals to various geometry problems.

Related Maine Learning Results	<u>Mathematics</u>
	A. Number
	  Real Number
	  A1. Students will know how to represent and use real numbers.
	    a. Use the concept of nth root.
	    b. Estimate the value(s) of roots and use technology to approximate them.
	    c. Compute using laws of exponents.
	    d. Multiply and divide numbers expressed in scientific notation.
	    e. Understand that some quadratic equations do not have real solutions and that there exist other number systems to allow for solutions to these equations.
	B. Data
	  Measurement and Approximation
	  B1. Students understand the relationship between precision and accuracy.
	    a. Express answers to a reasonable degree of precision in the context of a given problem.
	    b. Represent an approximate measurement using appropriate numbers of significant figures.
	    c. Know that most measurements are approximations and explain why it is useful to take the mean of repeated measurements.
	  Data Analysis
	  B2. Students understand correlation and cause and effect.
	    a. Recognize when correlation has been confused with cause and effect.
	    b. Create and interpret scatter plots and estimate correlation and lines of best fit.
	    c. Recognize positive and negative correlations based on data from a table or scatter plot.
	    d. Estimate the strength of correlation based upon a scatter plot.

Related Maine Learning Results	B3. Students understand and know how to describe distributions and find and use descriptive statistics for a set of data.
	   a. Find and apply range, quartiles, mean absolute deviation, and standard deviation (using technology) of a set of data.
	   b. Interpret, give examples of, and describe key differences among different types of distributions: uniform, normal, and skewed.
	   c. For the sample mean of normal distributions, use the standard deviation for a group of observations to establish 90%, 95%, or 99% confidence intervals.
	B4. Students understand that the purpose of random sampling is to reduce bias when creating a representative sample for a set of data.
	   a. Describe and account for the difference between sample statistics and statistics describing the distribution of the entire population.
	   b. Recognize that sample statistics produce estimates for the distribution of an entire population and recognize that larger sample sizes will produce more reliable estimates.
	   c. Apply methods of creating random samples and recognize possible sources of bias in samples.
	Probability
	B5. Students understand the relationship of probability to relative frequency and know how to find the probability of compound events.
	   a. Find the expected frequency of an event.
	   b. Find the expected value of events.
	   c. Find the probability of compound events including independent and dependent events.
	C. Geometry
	Geometric Figures
	C1. Students justify statements about polygons and solve problems.
	   a. Use the properties of triangles to prove theorems about figures and relationships among figures.
	   b. Solve for missing dimensions based on congruence and similarity.
	   c. Use the Pythagorean Theorem in situations where right triangles are created by adding segments to figures.
	   d. Use the distance formula.
	C2. Students justify statements about circles and solve problems.
	   a. Use the concepts of central and inscribed angles to solve problems and justify statements.
	   b. Use relationships among arc length and circumference, and areas of circles and sectors to solve problems and justify statements.

Related Maine Learning Results	C3. Students understand and use basic ideas of trigonometry.
	  a. Identify and find the value of trigonometric ratios for angles in right triangles.
	  b. Use trigonometry to solve for missing lengths in right triangles.
	  c. Use inverse trigonometric functions to find missing angles in right triangles.
	D. Algebra
	  Symbols and Expressions
	  D1. Students understand and use polynomials and expressions with rational exponents.
	    a. Simplify expressions including those with rational numbers.
	    b. Add, subtract, and multiply polynomials.
	    c. Factor the common term out of polynomial expressions.
	    d. Divide polynomials by (ax+b).
	  Equations and Inequalities
	  D2. Students solve families of equations and inequalities.
	    a. Solve systems of linear equations and inequalities in two unknowns and interpret their graphs.
	    b. Solve quadratic equations graphically, by factoring in cases where factoring is efficient, and by applying the quadratic formula.
	    c. Solve simple rational equations.
	    d. Solve absolute value equations and inequalities and interpret the results.
	    e. Apply the understanding that the solution(s) to equations of the form $f(x) = g(x)$ are x-value(s) of the point(s) of intersection of the graphs of $f(x)$ and $g(x)$ and common outputs in table of values.
	    f. Explain why the coordinates of the point of intersection of the lines represented by a system of equations is its solution and apply this understanding to solving problems.
	  D3. Students understand and apply ideas of logarithms.
	    a. Use and interpret logarithmic scales.
	    b. Solve equations in the form of $x + b^y$ using the equivalent form $y = \log_b x.$

Related Maine Learning Results	Functions and Relations
	D4.Students understand and interpret the characteristics of functions using graphs, tables, and algebraic techniques.
	   a. Recognize the graphs and sketch graphs of the basic functions.
	   b. Apply functions from these families to problem situations.
	   c. Use concepts such as domain, range, zeros, intercepts, and maximum and minimum values.
	   d. Use the concepts of average rate of change (table of values) and increasing and decreasing over intervals, and use these characteristics to compare functions.
	D5.Students express relationships recursively and use iterative methods to solve problems.
	   a. Express the (n+1)st term in terms of the nth term and describe relationships in terms of starting point and rule followed to transform one terms to the next.
	   b. Use technology to perform repeated calculations to develop solutions to real life problems involving linear, exponential, and other patterns of change.

Sample Lessons And Activities	▪ Students must find the area under a curve.
Sample Classroom Assessment Methods	▪ Homework ▪ Chapter quizzes and chapter exams: with and without graphing calculator – some to be done in-class, some to be done as take-home ▪ AP exam problems: multiple choice and free-response ▪ Student presentations to the class and to the teacher

Sample Resources	▪ Publications:
	   o Calculus: Graphical, Numerical, Algebraic, third edition
	▪ Other Resources:
	   o Various AP exam prep books
	   o Past AP exam questions: multiple choice and free-response
	   o Graphing calculator