3.3 Piecewise Functions
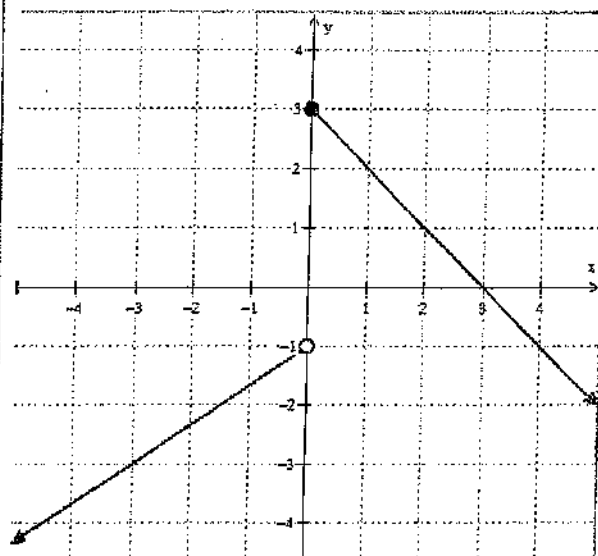
Piecewise Functions –

Algebraically

$$f(x) = \begin{cases} 2x + 8, & x \leq -2 \\ x^2 - 3, & -2 < x \leq 3 \\ \sqrt{x + 3}, & x > 3 \end{cases}$$

$f(-4) =$

$f(6) =$

$f(-2) =$

$f(0) =$

TRY IT!

$$f(x) = \begin{cases} 2x^3 - 1, & x < 1 \\ 3, & 1 \leq x < 5 \\ |x - 2|, & x \geq 5 \end{cases}$$
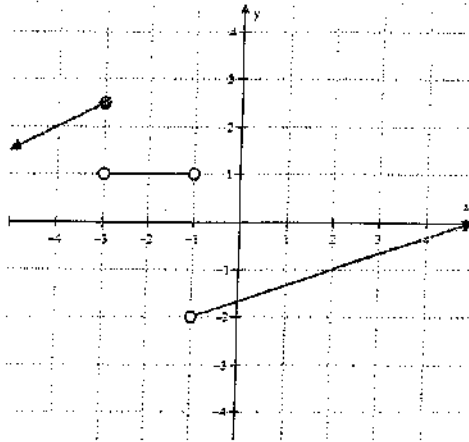
$f(8) =$

$f(0) =$

$f(4) =$

$f(5) =$

Graphically

$f(2) =$

$f(-3) =$

$f(-1) =$

$f(-4) =$

$f(0) =$

$f(-4) =$

$f(-1) =$

$f(3) =$

TRY IT!

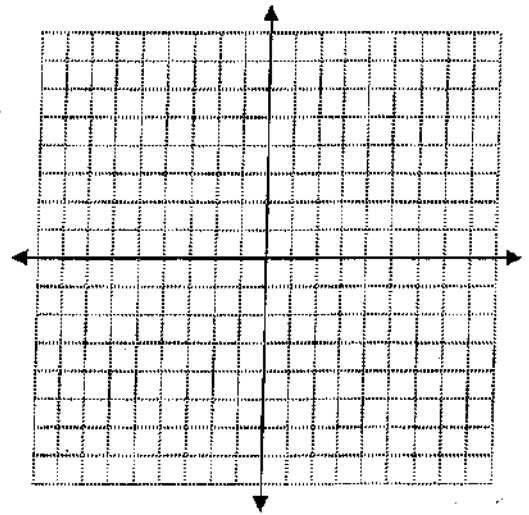$f(x) = \begin{cases} -2x + 1, & x < 0 \\ \frac{2}{3}x - 3, & x \geq 0 \end{cases}$
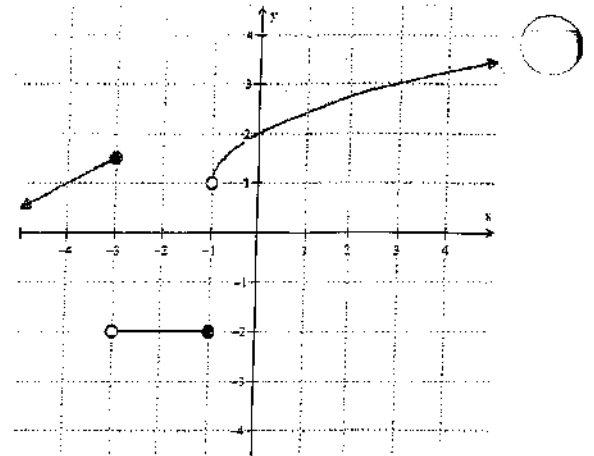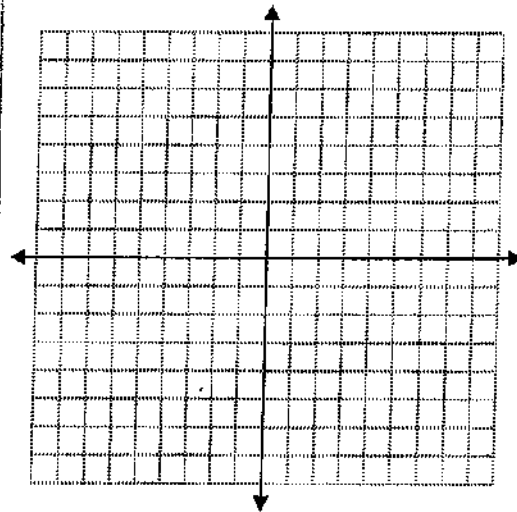
$f(x) = \begin{cases} 5, & x \leq 2 \\ 2x - 4, & x > 2 \end{cases}$

SUMMARY:

Now, summarize your notes here!

3.3 Piecewise Functions

Piecewise Functions –

Algebraically

$$f(x) = \begin{cases} 2x + 8, & x \le -2 \\ x^2 - 3, & -2 < x \le 3 \\ \sqrt{x+3}, & x > 3 \end{cases}$$

$f(-4) =$	$f(6) =$	$f(-2) =$	$f(0) =$
$2(-4)+8$	$\sqrt{(6)+3}$	$2(-2)+8$	$(0)^2-3$
$-8+8$	$\sqrt{9}$	$-4+8$	-3
0	3	4	

TRY IT!

$$f(x) = \begin{cases} 2x^3 - 1, & x < 1 \\ 3, & 1 \le x < 5 \\ |x - 2|, & x \ge 5 \end{cases}$$

$f(8) =$	$f(0) =$	$f(4) =$	$f(5) =$
$|8-2|$	$2(0)^3-1$	3	$|5-2|$
$|6|$	-1		$|3|$
6			3

Graphically

$f(2) = -1$

$f(-3) = 2.5$

$f(-1) = \emptyset$

$f(-4) = 2$

$f(0) = 2$

$f(-4) = 1$

$f(-1) = -2$

$f(3) = 3$

TRY IT!

$f(x) = \begin{cases} -2x + 1, & x < 0 \\ \frac{2}{3}x - 3, & x \geq 0 \end{cases}$

$< \quad >$ open

$\leq \quad \geq$ closed

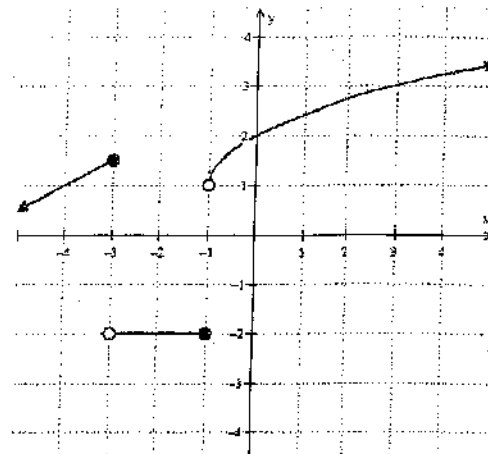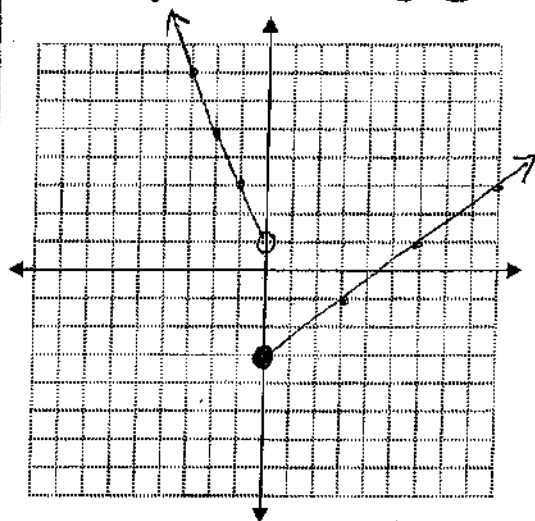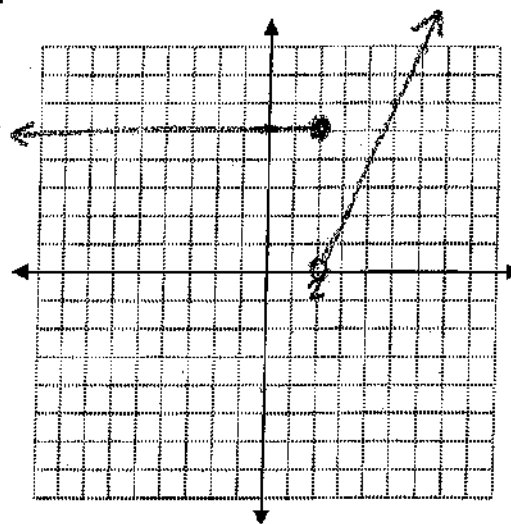$f(x) = \begin{cases} 5, & x \leq 2 \\ 2x - 4, & x > 2 \end{cases}$

SUMMARY:

Now, summarize your notes here!

Use the piecewise function to evaluate the following

1.

$$f(x) = \begin{cases} -2x^2 - 1, & x \le 2 \\ \dfrac{4}{5}x - 4, & x > 2 \end{cases}$$

a. $f(0) =$

b. $f(5) =$

c. $f(2) =$

d. $f(-3) =$

2.

$$f(x) = \begin{cases} x^3 - 7x, & x \le -3 \\ 8, & -3 < x \le 3 \\ \sqrt{2x + 3}, & x > 3 \end{cases}$$

a. $f(-5) =$

b. $f(11) =$

c. $f(0) =$

d. $f(3) =$

3.

$$f(x) = \begin{cases} \dfrac{3}{x+4}, & x < -5 \\ x^2 - 3x, & -5 < x \le 0 \\ x^4 - 7, & x > 0 \end{cases}$$

a. $f(-1) =$
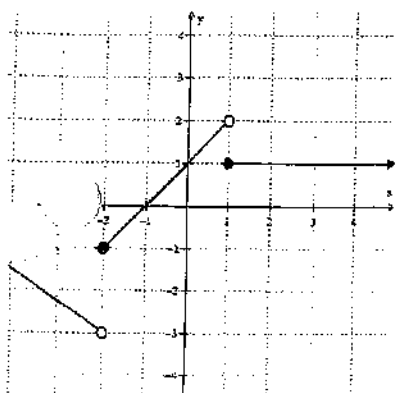
b. $f(4) =$

c. $f(-10) =$

d. $f(0) =$

4.

$$f(x) = \begin{cases} |2x + 7|, & x \le -4 \\ 1 + x^2, & -4 < x \le 1 \\ 6, & 1 < x < 3 \\ \dfrac{1}{3}x + 8, & x \ge 3 \end{cases}$$

a. $f(5) =$

b. $f(1) =$
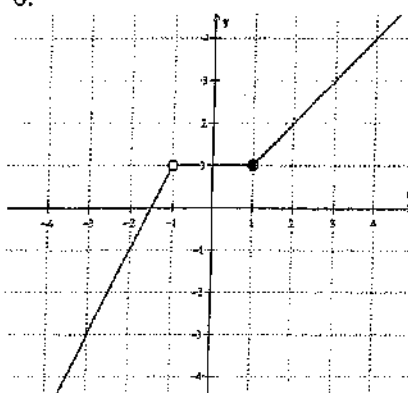
c. $f(-4) =$

d. $f(2) =$

5.

a. $f(-1) =$

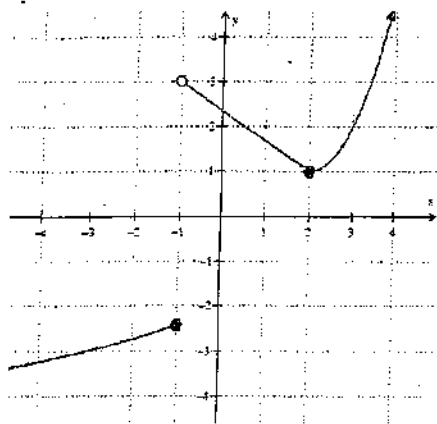b. $f(2) =$

c. $f(1) =$

d. $f(-2) =$

e. $f(0) =$

6.

a. $f(-3) =$

b. $f(4) =$

c. $f(1) =$

d. $f(-1) =$

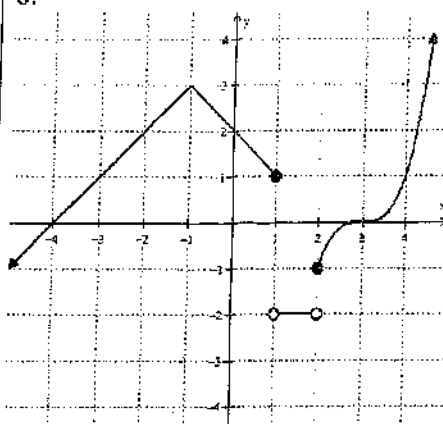e. $f(0) =$

7.

a. $f(3) =$

b. $f(-1) =$

c. $f(-3) =$

d. $f(2) =$

e. $f(0.5) =$

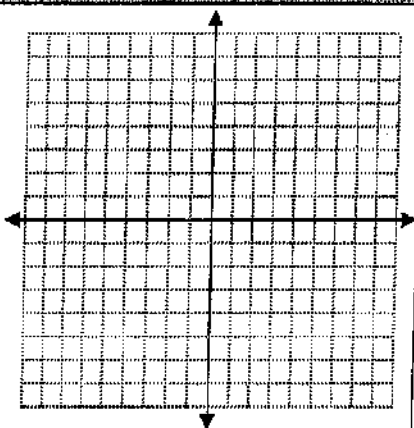**8.**

a. $f(-4) =$

b. $f(1) =$

c. $f(3) =$

d. $f(2) =$

e. $f(1.5) =$

Graph the following piecewise functions.

9.

$f(x) = \begin{cases} 2x + 3, & x \le 0 \\ \frac{1}{2}x - 1, & x > 0 \end{cases}$

10.

$f(x) = \begin{cases} -\frac{1}{3}x - 1, & x \le 3 \\ 2, & x > 3 \end{cases}$

11.

$f(x) = \begin{cases} 4 - x, & x < 2 \\ 2x - 6, & x \ge 2 \end{cases}$

12.
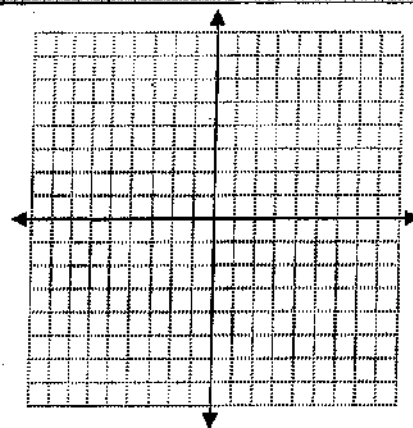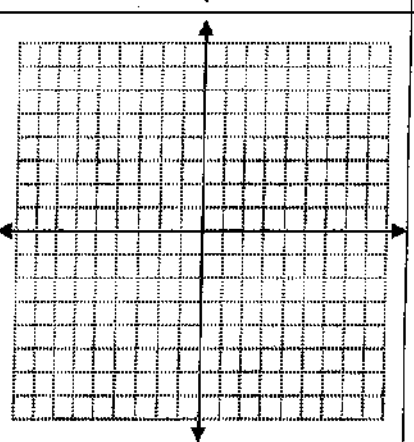
$f(x) = \begin{cases} \frac{2}{3}x + 3, & x \le 0 \\ 3, & 0 < x < 2 \\ -\frac{1}{2}x, & x \ge 2 \end{cases}$

ALGEBRA SKILLZ
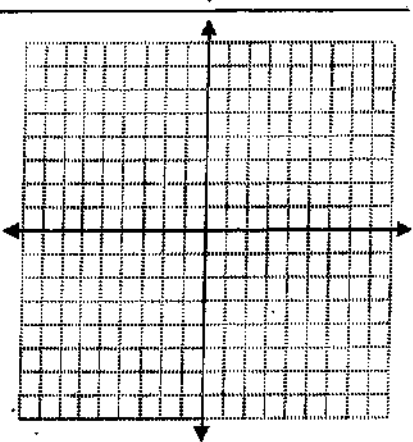
GRAPH	SIMPLIFY	SOLVE

GRAPH
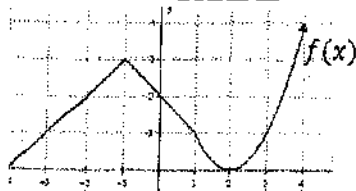
a. $f(-1) =$

b. y-intercept $=$

c. $f(x) = 1$ when $x =$

SIMPLIFY

Simplify the radical.

a. $\sqrt{24}$

b. $4\sqrt{40}$

SOLVE

Solve for x.

a. $15 = \frac{5}{x} + 4$

FACTOR

b. $x^2 - 12x + 35$

3.3 Piecewise Functions

Use the piecewise function to evaluate the following.

1.

$$f(x) = \begin{cases} -2x^2 - 1, & x \le 2 \\ \dfrac{4}{5}x - 4, & x > 2 \end{cases}$$

a. $f(0) =$

$-2(0)^2 - 1$

-1

b. $f(5) =$

$\frac{4}{5}(5) - 4$

0

c. $f(2) =$

$-2(2)^2 - 1$

$-2 \cdot 4 - 1$

-9

d. $f(-3) =$

$-2(-3)^2 - 1$

$-2 \cdot 9 - 1$

-19

2.

$$f(x) = \begin{cases} x^3 - 7x, & x \le -3 \\ 8, & -3 < x \le 3 \\ \sqrt{2x + 3}, & x > 3 \end{cases}$$

a. $f(-5) =$

$(-5)^3 - 7(-5)$

-90

b. $f(11) =$

$\sqrt{2(11) + 3} = \sqrt{25}$

5

c. $f(0) =$

8

d. $f(3) =$

8

3.

$$f(x) = \begin{cases} \dfrac{3}{x + 4}, & x < -5 \\ x^2 - 3x, & -5 < x \le 0 \\ x^4 - 7, & x > 0 \end{cases}$$

a. $f(-1) =$

$(-1)^2 - 3(-1)$

4

b. $f(4) =$

$(4)^4 - 7$

249

c. $f(-10) =$

$\dfrac{3}{(-10) + 4} = \dfrac{3}{-6}$

$-\frac{1}{2}$

d. $f(0) =$

$(0)^2 - 3(0)$

$0 - 0$

0

4.

$$f(x) = \begin{cases} |2x + 7|, & x \le -4 \\ 1 + x^2, & -4 < x \le 1 \\ 6, & 1 < x < 3 \\ \dfrac{1}{3}x + 8, & x \ge 3 \end{cases}$$

a. $f(5) =$

$\frac{1}{3}(5) + 8$

$29/3$ or $9.\overline{6}$

b. $f(1) =$

$1 + (1)^2 = 2$

c. $f(-4) =$

$|2(-4) + 7|$

$|-8 + 7| = |-1| = 1$

d. $f(2) =$

6

5.

a. $f(-1) = 0$

b. $f(2) = 1$

c. $f(1) = 1$

d. $f(-2) = -1$

- $f(0)$ 1

6.

a. $f(-3) = -3$

b. $f(4) = 4$

c. $f(1) = 1$

d. $f(-1) = \cancel{0}$

- $f(0) -$ 1

7.

a. $f(3) = 2$

b. $f(-1) = -2.5$

c. $f(-3) = -3$

d. $f(2) = 1$

e. $f(0.5) = 2$

8.

a. $f(-4) = 0$

b. $f(1) = 1$

c. $f(3) = 0$

d. $f(2) = -1$

e. $f(1.5) = -2$

Graph the following piecewise functions.

9.

$$f(x) = \begin{cases} 2x + 3, & x \le 0 \\ \frac{1}{2}x - 1, & x > 0 \end{cases}$$

10.

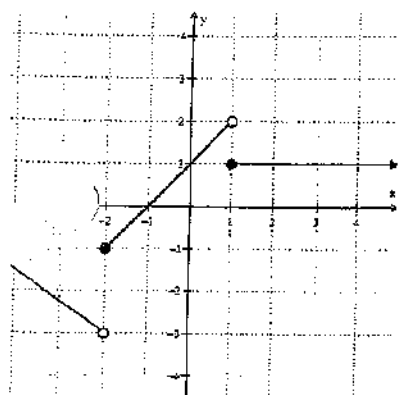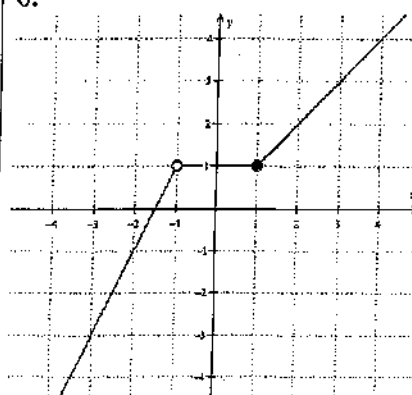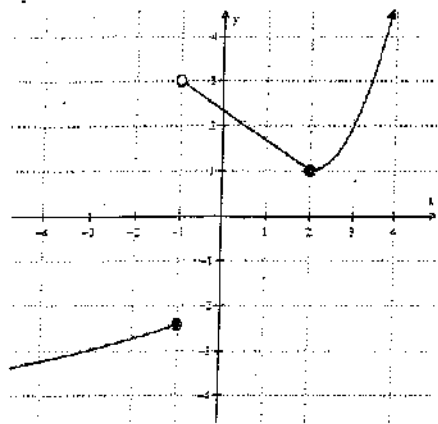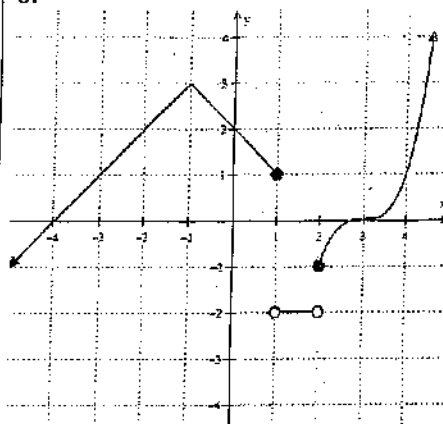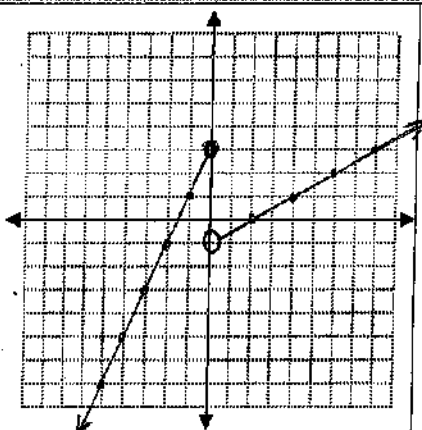$$f(x) = \begin{cases} -\frac{1}{3}x - 1, & x \le 3 \\ 2, & x > 3 \end{cases}$$

11.

$-|x+4|$

$$f(x) = \begin{cases} 4 - x, & x < 2 \\ 2x - 6, & x \ge 2 \end{cases}$$

12.
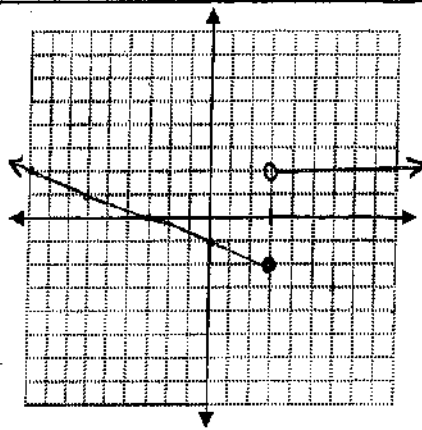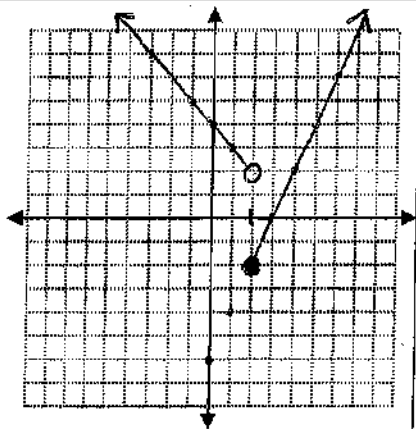
$$f(x) = \begin{cases} \frac{2}{3}x + 3, & x \le 0 \\ 3, & 0 < x < 2 \\ -\frac{1}{2}x, & x \ge 2 \end{cases}$$
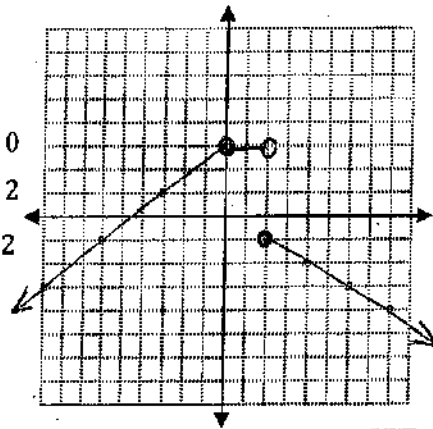
ALGEBRA SKILLZ

GRAPH
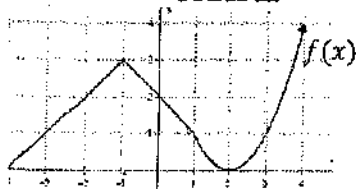
a. $f(-1) = 3$

b. y-intercept $= (0, 2)$

c. $f(x) = 1$ when $x = 1$

d. x-intercept(s) $= (2, 0)$

SIMPLIFY

Simplify the radical.

a. $\sqrt{24}$

$\sqrt{4}\sqrt{6} = 2\sqrt{6}$

b. $4\sqrt{40}$

$4 \cdot 2\sqrt{10}$

$\sqrt{4}\sqrt{10}$

$8\sqrt{10}$

SOLVE

Solve for x.

a. $15 = \frac{5}{x} + 4$

$11 = \frac{5}{x}$ $11x = 5$ $x = 5/11$

FACTOR

b. $x^2 - 12x + 35$

$(x - 7)(x - 5)$

Use the piecewise function to evaluate the following.

$$f(x) = \begin{cases} \dfrac{3}{x-2}, & x < -3 \\ 2x^2 - 3x, & -3 < x \le 6 \\ 8, & x > 6 \end{cases}$$

a. $f(-1) =$

b. $f(-4) =$

c. $f(9) =$

d. $f(6) =$

2. Graph the following piecewise function.

$$f(x) = \begin{cases} -\dfrac{1}{3}x - 2, & x \le 0 \\ \dfrac{1}{2}x + 1, & x > 0 \end{cases}$$

3. **NUMERICALLY** Use the piecewise function to fill in the table.
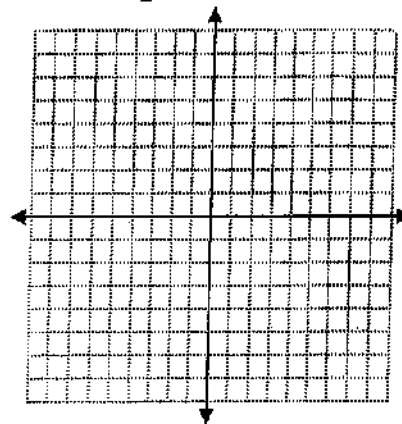
$$f(x) = \begin{cases} -x + 4, & x \le 0 \\ -3x + 18, & x > 0 \end{cases}$$

x	f(x)
-2	
0	
1	
	-12
	9

4. **GRAPHICALLY** Sully's blood pressure changes throughout the school day. Sketch a graph of his blood pressure over time. LABEL THE GRAPH! Let x stand for the time since 0800, so 1000 would be $x = 2$, 1200 would be $x = 4$, etc...

Sully's Day

- Sully's blood pressure starts at 90 and rises 5 points every hour for the first 4 hours.

- Sully chills out for lunch from 12-1 and maintains a cool 110 blood pressure.

- Last period of the day hits from 1-3 and Sully's blood pressure rises from 110 at 10 points per hour.

- School ends and Sully's blood pressure starts dropping 2 points per hour until his 8 o'clock bedtime.

5. ALGEBRAICALLY Use the picture of the piecewise function to answer the following.

GRAPH	Equation of the pieces	Domain for the pieces	Piecewise function
			$f(x) = \begin{cases} \\ \\ \end{cases}$

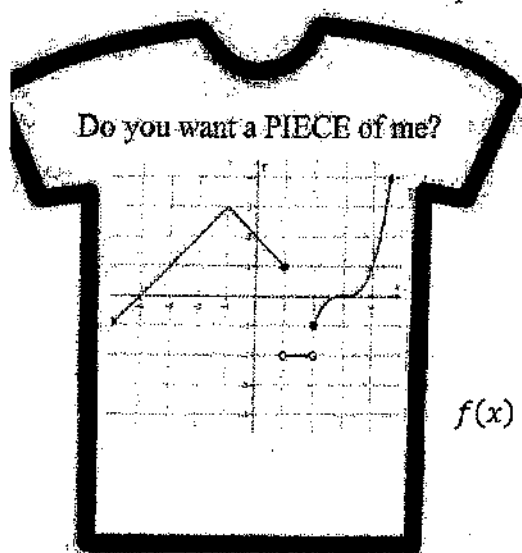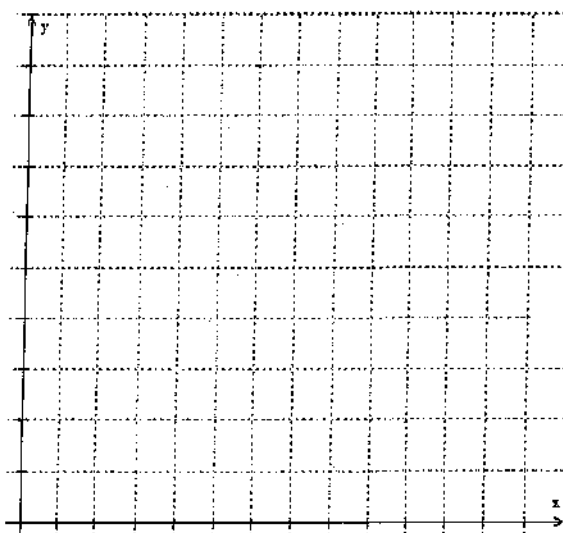**6. VERBALLY** Mr. Brust wants to make t-shirts for his Algebra 2 students (shown below). Custom Ink will make the shirts for the following cost. Write a piecewise function to represent individual cost of a t-shirt as function of the number of shirts made. Graph it!   Label the graph!

Do you want a PIECE of me?

```
0-20 shirts = $25 each
21-30 shirts = $20 each
31-50 shirts = $15 each
51+ shirts = $10 each
```

$f(x) = \begin{cases} \\ \\ \\ \end{cases}$

7. SAT PREP Below are sample SAT questions. The SAT is the main standardized test that colleges look at for admission. One is multiple choices; the other is free response where you must grid in your answer. Blow it up.

MULITPLE CHOICE

A regulation for riding a certain amusement park ride requires that a child be between 30 inches and 50 inches tall. Which of the following inequalities can be used to determine whether or not the child's height h satisfies the regulation for this ride?

(A) $|h - 10| < 50$
(B) $|h - 20| < 40$
(C) $|h - 30| < 20$
(D) $|h - 40| < 10$
(E) $|h - 45| < 5$

GRID IN

If $x < 0 < y$, find the value of $x + y$ given:

$2|x - 9| = 24$
$|xy| = 15$

1. Use the piecewise function to evaluate the following.

$$f(x) = \begin{cases} \dfrac{3}{x-2}, & x < -3 \\ 2x^2 - 3x, & -3 < x \le 6 \\ 8, & x > 6 \end{cases}$$

a. $f(-1) =$

$2(-1)^2 - 3(-1)$

$2 \cdot 1 + 3 = \boxed{5}$

b. $f(-4) =$

$\dfrac{2}{(-4)-2} = \dfrac{2}{-6} = \boxed{-1/3}$
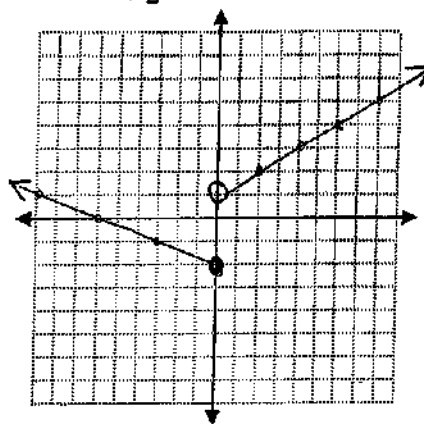
c. $f(9) =$

$\boxed{8}$

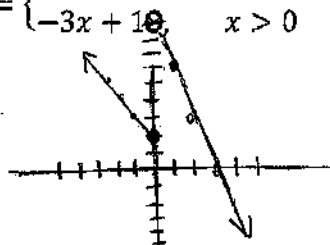d. $f(6) =$

$2(6)^2 - 3(6)$

$72 - 18 = \boxed{54}$

2. Graph the following piecewise function.

$$f(x) = \begin{cases} -\dfrac{1}{3}x - 2, & x \le 0 \\ \dfrac{1}{2}x + 1, & x > 0 \end{cases}$$

3. NUMERICALLY Use the piecewise function to fill in the table.

$$f(x) = \begin{cases} -x + 4, & x \le 0 \\ -3x + 18, & x > 0 \end{cases}$$

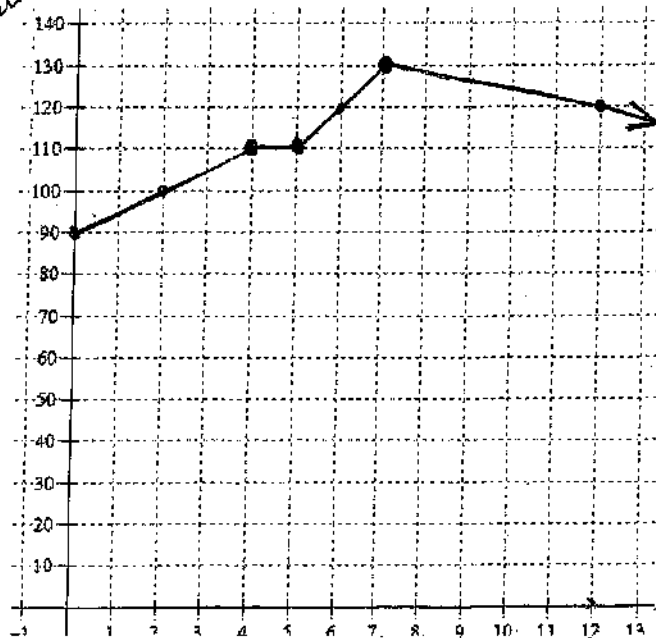| x	f(x)	
-2	6	$-(-2)+4$
0	4	$-(0)+4$
1	15	$-3(1)+18$
10	-12	
-5	9	

4. GRAPHICALLY Sully's blood pressure changes throughout the school day. Sketch a graph of his blood pressure over time. LABEL THE GRAPH! Let x stand for the time since 0800, so 1000 would be $x = 2$, 1200 would be $x = 4$, etc...

Sully's Day

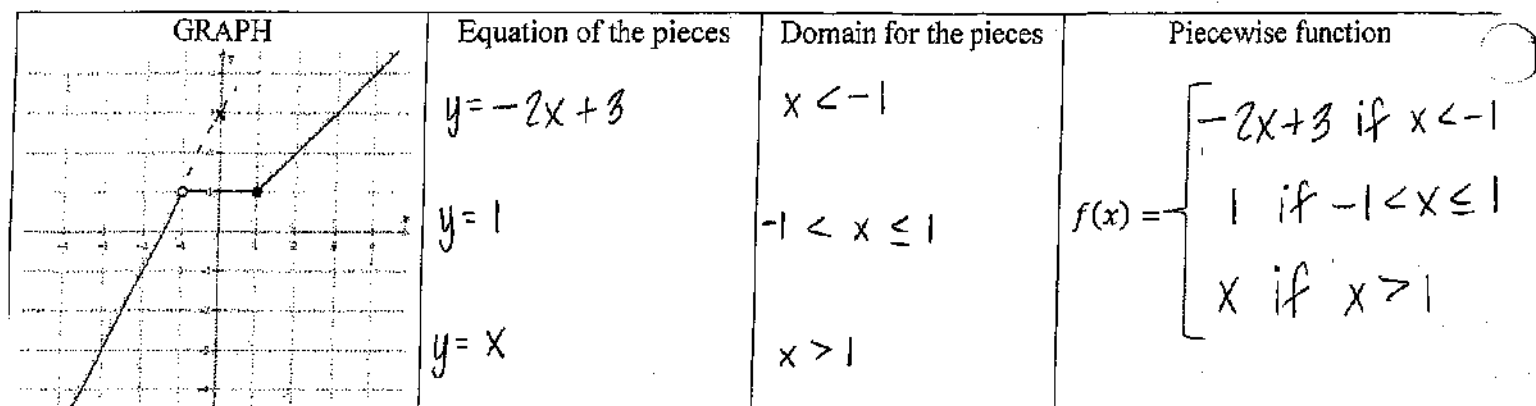- Sully's blood pressure starts at 90 and rises 5 points every hour for the first 4 hours.

$(x = 4 - 5)$

- Sully chills out for lunch from 12-1 and maintains a cool 110 blood pressure.

$(x = 5 - 7)$

- Last period of the day hits from 1-3 and Sully's blood pressure rises from 110 at 10 points per hour.

- School ends and Sully's blood pressure starts dropping 2 points per hour until his 8 o'clock bedtime.

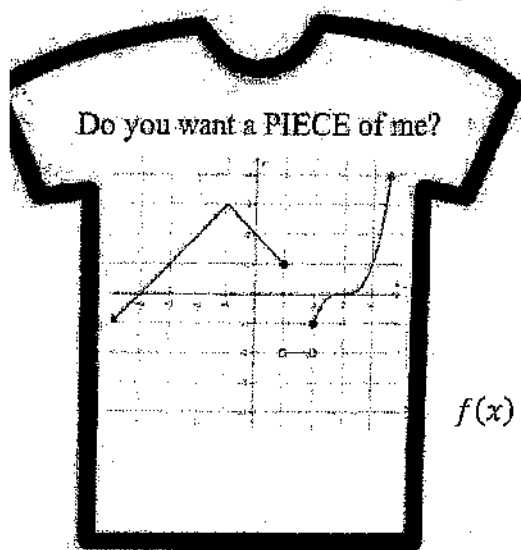## 5. ALGEBRAICALLY Use the picture of the piecewise function to answer the following.

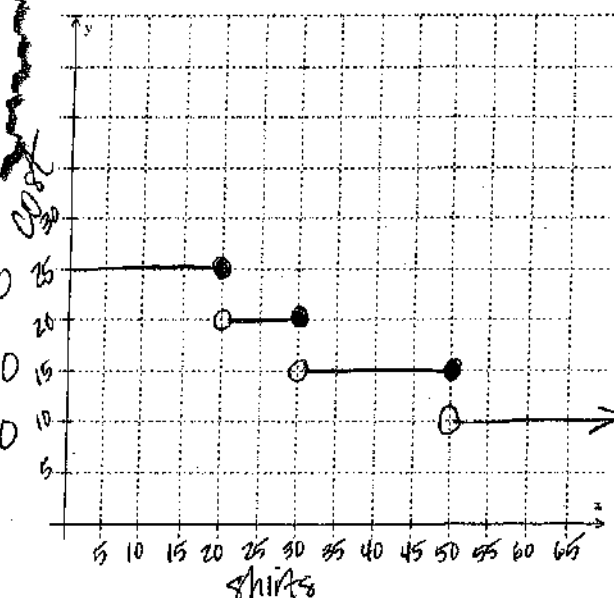| GRAPH	Equation of the pieces	Domain for the pieces	Piecewise function
	$y = -2x + 3$	$x < -1$	
	$y = 1$	$-1 < x \leq 1$	$f(x) = \begin{cases} -2x+3 & \text{if } x < -1 \\ 1 & \text{if } -1 < x \leq 1 \\ x & \text{if } x > 1 \end{cases}$
	$y = x$	$x > 1$	

6. VERBALLY Mr. Brust wants to make t-shirts for his Algebra 2 students (shown below). Custom Ink will make the shirts for the following cost. Write a piecewise function to represent individual cost of a t-shirt as function of the number of shirts made. Graph it!

Do you want a PIECE of me?

```
0-20 shirts = $25 each
21-30 shirts = $20 each
31-50 shirts = $15 each
51+ shirts = $10 each
```

Label the graph!

$$f(x) = \begin{cases} 25 & 0 < x \leq 20 \\ 20 & 21 \leq x \leq 30 \\ 15 & 31 \leq x \leq 50 \\ 10 & x \geq 51 \end{cases}$$

shirts

7. SAT PREP Below are sample SAT questions. The SAT is the main standardized test that colleges look at for admission. One is multiple choices; the other is free response where you must grid in your answer. Blow it up.

MULITPLE CHOICE

A regulation for riding a certain amusement park ride requires that a child be between 30 inches and 50 inches tall. Which of the following inequalities can be used to determine whether or not the child's height h satisfies the regulation for this ride?

$h - 40 < 10$ & $h - 40 > -10$
$h < 50$ & $h > 30$

(A) $|h - 10| < 50$
(B) $|h - 20| < 40$
(C) $|h - 30| < 20$
(D) $|h - 40| < 10$
(E) $|h - 45| < 5$

$h - 45 < 5$ & $h - 45 > -5$
$h < 50$ & $h > 40$

GRID IN

If $x < 0 < y$, find the value of $x + y$ given: $-3 + 5 \neq 2$

$2|x - 9| = 24$
$|xy| = 15$

$2|x-9| = \frac{24}{2}$

$|x - 9| = 12$

$x - 9 = 12$ & $x - 9 = -12$
$x = 21$ & $x = -3$ ✓

$|-3y| = 15$
$-3y = 15$ $-3y = -15$