Grade 8
Mathematics
Unit 6: Functions, Growth and Patterns Part 2

Time Frame: Approximately three weeks

Unit Description

This unit examines the nature of changes to the input variables in function settings through the use of tables and sequences. There is emphasis on recognizing and differentiating between linear and exponential change and developing the expression for the *n*th term for a given arithmetic or geometric sequence.

Student Understandings

Students recognize the nature of linear functions and develop rules to represent these functions. Students will be able to represent function graphs with tables, words, and coordinates. They will communicate the information represented by linear function graphs. Students will switch between tables, graphs, equations and verbal representations communicating information gathered from each representation. They can analyze a graph and describe a functional relationship.

Guiding Questions

1. Can students differentiate between linear and non linear growth patterns and discuss each verbally, numerically, graphically, and symbolically?
2. Can students represent function graphs with tables, words and coordinates?
3. Can students communicate information represented by linear graphs, equations and tables?
4. Can students communicate verbally a functional relationship represented graphically?

Unit 6 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

Grade-Level Expectations	
GLE #	**GLE Text and Benchmarks**
13.	Switch between functions represented as tables, equations, graphs, and verbal representations, with and without technology (A-3-M) (P-2-M) (A-4-M)
46.	Distinguish between and explain when real-life numerical patterns are linear/arithmetic (i.e., grows by addition) or exponential/geometric (i.e., grows by multiplication) (P-1-M)

CCSS#	CCSS Text
8.EE.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
8.EE.8	Analyze and solve pairs of simultaneous equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically and estimate solutions by graphing the equations. Solve simple cases by inspection. c. Solve real-world and mathematical problems leading to two linear equations in two variables.
8.F.3	Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not a straight line.*
8.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation models, and in terms of its graph or table of values.
8.F.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Sample Activities:

Activity 1: Make Up a Rule (GLE: 13; CCSS: 8.F.4)

Materials List: Practice with Rules BLM, paper, pencil

This activity has not changed because it already incorporates the CCSS.

Have students work in pairs to generate at least six sets of four ordered pairs that may or may not be a function. Tell students that they must create at least three sets that are functions. Tell them not to mark these as functions or non-functions because they will exchange papers with a partner. Have student pairs exchange their papers with their partner; each student should determine whether the ordered pairs will be considered functions or not. If the ordered pairs are a function, have students determine the rule or equation that describes the function. Students should explain to their partners the procedure used to find the next term, as well as write the equation. These functions can be arithmetic or geometric patterns. The challenge is to generate the rules for the functions and determine if the function is arithmetic or geometric. Give students time to determine the procedures followed to find the next term in the sequences, write the sequence and rule (equation) on their paper, and pass the paper back to the writer. The writer should verify whether the rule (equation) describes the function and if the correct responses were given for non-functions. Have students graph at least two of their functions, write the rule for the function and write the ordered pair in a table on the graph. These can be posted or turned in for teacher use in formative assessment.

Distribute the Practice with Rules BLM and give students time to complete the sequence and write the rules. During discussion of the BLM, challenge the students to explain why they know if the sequence of numbers is linear or not.

Activity 2: Real Rules (GLEs: 46; CCSS: 8.F.5)

Materials List: Real Rules Car Mileage Chart BLM , newsprint, markers, Real Situations with Sequences BLM, paper, pencil, graph paper

This activity has not changed because it already incorporates the CCSS.

Distribute Real Rules Car Mileage Chart BLM. Discuss what information students can gather from the chart. Model how to use the information from the charts and develop a rule.

Examples:

If Bill drives a Honda CRZ 2door, the chart shows 37 miles per gallon. Using the number of gallons and the cost per gallon would give the ordered pairs (1, \$3.65) (2, \$7.30) (3, 10.95), etc. The equation to represent the relationship of the cost per gallon and the number of gallons purchased would be $y = 3.65x$, where x is the number of gallons and y is the cost of the gasoline.

If Joe is driving a Hyundai Sonata on the highway and Bill is driving the Honda CRZ 2-door on the highway, use the chart to develop a rule that would express the difference in miles traveled per gallon of gasoline *for every gallon of gasoline; Bill's car will travel 9 miles more than Joe's.* Question: If Joe and Bill each buy 20 gallons of gasoline, how many more miles will Joe be able to travel? Explain your mathematical rule. (Bill's mileage b = *9 miles further than Joe for each gallon purchased (20 gallons purchased) and write the equation b = 9(20)*
b= 180 miles further with 20 gallons of gasoline than Joe

Distribute newsprint and explain to the students that they will use the Real Rules Car Mileage Chart BLM to create a linear representation of the mileage differences of the two vehicles showing gallons of gas and miles traveled. Students should determine the equation for the mileage graphed and then describe the rate of change and how this relates to the slope. Have students create questions that could be answered once the rule for their line has been determined using the n^{th} term in their sequence. Instruct students to work in pairs and discuss their graphs and questions before beginning *professor know-it- all* (view literacy strategy descriptions). Remember, when one of the pairs of students is *professor know- it-all*, the selection of him/her is random not voluntary. This assures that all students are actively involved in understanding their rule and sequence. A site that gives city/hwy mileage and average cost of fuel is available at http://www.fueleconomy.gov/feg/best-worst.shtml

Distribute Real Situations with Sequences BLM, and give students time to work through each of these relationships prior to class discussion.

Activity 3: Name That Term! (GLEs: 46; CCSS: 8.F.4)

Materials List: Name That Term BLM, pencil, paper

This activity has not changed because it already incorporates the CCSS.

Provide students with Name That Term BLM and have them explore the patterns, then answer the questions, either independently or with a partner. This activity works best if students are not working in groups of four. Have students determine whether the sequence is arithmetic (linear) or geometric (nonlinear) and justify their choice using a table, graph or explanation.

Have students develop at least two patterns and write a real-life situation that could be illustrated with the sequence. Students should exchange patterns with another pair of students or another individual and discuss the sequences, justifying their table and graph relationships to the patterns.

Activity 4: How Much Do I Get? (GLE: 13, 46; CCSS: 8.F.4)

Materials List: paper, pencil, graph paper, colored pencils

This activity has not changed because it already incorporates the CCSS.

Pose the following to the students and have them explore which is the better salary option.

> *You have been hired to do yard work for the summer, and you will be paid every day for 15 days. But first you have to choose your salary option as (1) get paid $10 the 1st day, $11 the 2nd day, $12 the 3rd day, and so on, or (2) get paid $.01 the first day, $.02 the 2nd day, $.04 the 3rd day, $.08 the 4th day, and so on.*

Have students create a table or chart of their values and a rule that explains the relationship in the chart. Have students decide which type of sequence each of the salary options illustrates and generate the 15 terms in each sequence. Next, have students determine the amount they would get paid at the end of the 15 days to determine the best salary option.

Have the students graph each of these salary options on the same graph in different colors and make observations about the relationship of the two options. Lead the class in a discussion about how the information in the chart, graph, or rule all relate to the situation. Have students explain in their mathematics *learning logs* (view literacy strategy descriptions) whether either of these situations could appropriately start at 0.

Solutions: The first situation increases by $1 a day and on day 15 would be paid $24, while the second situation starts at $.01 and this amount doubles every day so that on the 15th day the person would be paid $163.84.

Activity 5: Generally Speaking (GLE: 46; CCSS: 8.F.4)

Materials List: Generally Speaking BLM, paper, pencil

This activity has not changed because it already incorporates the CCSS.

Provide students with Generally Speaking BLM. This is a variation of the *word grid* (view literacy strategy descriptions) strategy to help students with algebraic vocabulary. The top of the BLM has the students exploring sequences, and the *word grid* at the bottom connects the information from the explorations to the terms, linear and nonlinear, arithmetic and geometric. Understanding the algebraic language is very important to the students as they work to master the abstract thinking involved in algebra. The modified *word grid* is placed at the bottom of the exploration page. The student is given a number pattern, and from the pattern, the student should complete the top chart explaining the pattern of growth: (a) the procedure followed to find the next term in the sequence and (b) an equation to describe the growth.

Explain to the students that they will explore the number pattern to determine the pattern. For example, the pattern 2, 4, 6, 8. . . increases by 2 each time and can be generated using the equation $y = 2x$ which creates a line with a slope of 2. Since this pattern is integers, only integers are used to create the points and form the equation for a line. Remember that the linear equation is formed by including many additional points that are not a part of the sequence of integers.

When the top chart is completed, discuss the sequences by having various students explain the different patterns and how the *word grid* helps them better describe these patterns.

To complete the *word grid*, the students should complete the bottom grid on the BLM. Explain that in the top row, the headings describe what to look for. The columns are labeled, “linear,” “nonlinear,” “arithmetic,” and “geometric.” Explain that they have explored the sequences in the top chart and are going back to the example of $y = 2$. The students would check linear (because the slope is always up 2, right 1 or down 2, left 1. When the equation is linear, the pattern is an arithmetic sequence. If the pattern is “arithmetic,” the students will check the cell to indicate this. The students will have two checks beside each of the sequences explored in the top chart. Ensure that there are no questions regarding how to complete the *word grid*. Students should determine whether the sequence is linear or non linear, and arithmetic or geometric.

Once the students have completed the BLM, explain to them that this *word grid* is to help them become more confident with the terms linear and nonlinear, and arithmetic and geometric sequences. Ask students to explain observations that can be made about these terms using the *word grid*. *The word grid should help students visualize that linear sequences are arithmetic and non-linear sequences are not arithmetic.*

Divide the sequences among pairs of students and instruct them to describe a real-life situation that matches each sequence. Many of these sequences will work with money situations, which are sometimes easier for students.

Next, provide students with a word description of a sequence and then have them write the n^{th} term as an equation. *Examples of word descriptions might be as follows: a) Pat had $4 on June 1 and each month he saved $25.* $y = 25x + 4$; *b) Mary makes $6.75 an hour for babysitting.* $y = \$6.75x$

Activity 6: Describing situations with graphs (CCSS: 8.F.5)

Materials: Describing Situations with Graphs BLM, pencil, paper

This activity begins with a *process guide* (view literacy strategy descriptions). The strategy has been modified to give struggling readers a method of making connections between the words and the graphs. Other learners will get the opportunity to construct deeper meaning of the connections. Tell the students that they will be given a graph and a situation and that as a class they will discuss the situation that is represented by the graph. The statements below the graph will be answered through the discussion, and the students will be reminded to look at these statements and justify how the statement can be proved from the graph. Distribute page 1 of the Describing Situations with Graphs BLM. Have students follow directions and monitor to make sure the struggling readers are making the necessary connections.

Distribute the page 2 of the BLM and have students work in pairs to complete the two situations. Discuss results as a class using *professor know-it-all* (view literacy strategy descriptions). Remember that this is random to encourage all students to make sure they understand the content.

Activity 7: Are You Sure? (GLE: 46; CCSS: 8.F.4)

Materials List: Are You Sure? BLM, transparency of Are You Sure? Directions BLM, newsprint, markers, paper, pencils

This activity was changed minimally to incorporate CCSS 8.F.4.

Before class, make copies of Are You Sure? BLM and cut apart the cards. Distribute one or two number sequences to each pair of students.

Make a transparency from Are You Sure? Directions BLM and place it on the overhead. Have the students follow the directions for the sequences they were given. Give them time to follow the instructions. *FYI: For differentiation purposes, 8, 9 are easier, 1, 2, 4, 5, 7 are medium, and 3, 6 and 10 are more difficult.*

Have students write their questions related to their sequence on newsprint, including some questions in which students are to find the value if the term or arrangement number is known and others in which the term or arrangement number is to be calculated. Example: For the sequence 8, 10, 12, a student determines the rule to be $y = 2x + 6$ and writes one question: What will be the value of the 50th term in this pattern? Another question that could be asked: Which term in the sequence will have a value of 26? Have student pairs present their pattern and pose their questions to other groups or to the entire class. Encourage class discussion as different questions are posed.

Post an equation on the board in slope/*y*-intercept form such as $y = 3x + 5$. Challenge pairs of students to determine a sequence of numbers that could be represented by this equation. Once the students have determined a sequence, have them write the ordered pairs to represent the first three values in their sequence. Discuss these results as a class. Ask the students to determine the "*y*" value for the 10th term in their sequence and relate this to the slope/*y*-intercept formula. Ask the pairs of students to think about how the *y*-intercept in the equation is represented in the sequence.
. . . 8, 11, 14, 17, 20 . . . is a sequence that can be represented with this equation using positive integers. The ordered pairs would be (1, 8,)(2,11)(3,14)(4,17)(5,20).
. . . 2, -1, -4, -7, -11 . . . is a sequence that can be represented with this equation using negative integers. The ordered pairs would be (1, 2)(2, -1)(3,-4)(4,-7)(5,-11).
The two sequences both represent the equation and the difference of +3 between each of the y-values which is represented by the slope of 3 in the equation. The y-intercept is represented when the students find the value of the term in the sequence where x has a value of 0. (0, 5)

Give students two or three more equations in the same form and have them determine sequences, write the ordered pairs, and explain how the sequences relate to the equation.

Activity 8: Student Council Dilemma (GLE: 13; CCSS: 8.F.4)

Materials: grid paper, paper, pencil

Provide the students with the following problems.

> The Student Council wants to build a wheelchair ramp for one of the grandparent volunteers at school. They called the city and found that the distance from the bottom of the ramp to the base of the building should be fifteen feet for every one foot it goes up. In other words, to rise one foot, the ramp should be fifteen feet long. The student council wants to make a table of values for ramps that have to rise 1 foot, 2 feet, 3 feet or 4 feet. Make the table and write an equation in

slope/*y*-intercept form that can be used to solve the problem. Graph the equation. Show the slope on the graph of the equation.

My family had to call a plumber on Saturday because the washing machine was flooding the house. The plumber came to the house and said that his charge was $54 to come to the house and his hourly fee was $70 on a weekday but on Saturday he charges time and a half so the fee is $105/hour. Write an equation in slope/*y*-intercept form that represents this situation, make a table of values, and graph the situation. Show how you can determine slope from the graph.

Give students time to work on these problems and then use *professor know-it-all* (view literacy strategy descriptions) to have students prepare a presentation of how to explain these situation. Remember that this strategy is not a volunteer presentation but randomly selected by the teacher. Student groups should present their information to the class, and all students should be given the opportunity to ask questions.

Have students turn to their shoulder partner and discuss the information that was shared by the presenters. Groups should come up with one statement to share with the class about what they learned from the problem and/or the presenters. Have groups share and post statements for the class to reflect upon at the beginning of class the next day.

Activity 9: Comparing Slopes and *y*-Intercepts (CCSS: 8.EE.5)

Materials: Comparing Slopes and *y*-Intercept BLM, grid paper, pencil, paper

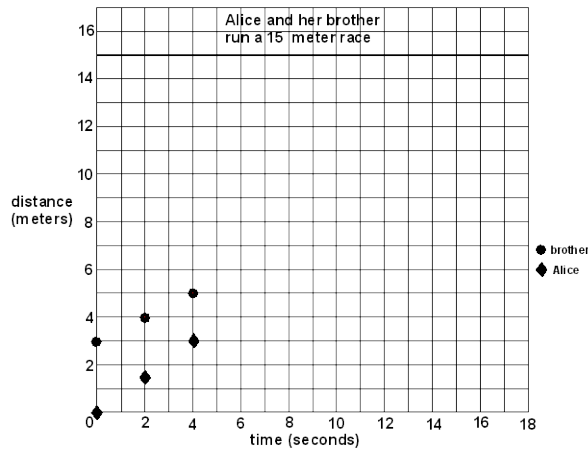
Begin the activity by distributing grid paper and having the students plot the line that passes through the points (3, 4) and (4, 6). Discuss the slope of the line formed and have a student justify the slope of 2. Have the students determine the equation for the line using the slope-intercept form of an equation ($y = 2x - 2$). Working in pairs, challenge the students to determine an equation for a line that will be parallel to the line on the graph. Circulate as the students work to determine a solution asking questions that will spur thinking if students are struggling with a method. As students begin to find an equation for a line parallel, have student pairs go to the board and explain their solution and justify the mathematics used to find their solution. *The students may or may not discover the concept that equations of lines with the same slope are parallel. This activity gives more opportunity to discover this, so do not point it out at this time.*

Distribute Comparing Slopes and *y*-Intercept BLM and have the student pairs work together to complete the graphs and questions of each of the sets of equations. When the students complete the BLM, use the *discussion* (view literacy strategy descriptions) strategy Think Pair Square Share. Have each student pair get with another student pair and develop at least two conjectures from the information gathered by graphing these equations and answering the questions. These conjectures are important. To ensure that all groups understand this, have groups of four choose one of their conjectures to share with the class. Write each group's conjecture on the board to give validity to their thinking. Make sure that the students have developed the following two conjectures 1) linear equations with the same slope form lines that are parallel, and 2) linear equations with the same *y*-intercept will cross the *y*-axis at the same point.

As closure, ask the students to take their graphs of the original equation out and check to see that the second equation they wrote to be parallel to the original equation has the same slope. As an exit ticket, have students write an equation that will have a negative slope and the same *y*-intercept as the original equation.

Activity 10: Comparing Functions? (CCSS: 8.EE.5)

Materials: Comparing Functions? BLM, paper, pencil



Begin the lesson by showing the graph (at the right) and the statement: "Alice gave her brother too much of a head start."

The strategy *SQPL* (view literacy strategy descriptions) has been used in previous units to encourage thinking about the content prior to developing the content in the classroom. In this activity, *SQPL* will be used to spur the students' thinking about the possibilities of what the graph of Alice's speed will show. Generating questions about the statement will give students the opportunity to develop a conceptual understanding of what the graph illustrates.

Have the students get with a shoulder partner and generate 2-3 questions that need to be answered before determining if the statement is true. After the students have developed these questions, have each pair of students share their question with the class until all are posted on the board. Some possible questions might be: *Why did Alice's brother start at 3 meters? How fast was Alice running? How fast was Alice's brother running?*

Next, tell the students that Alice's speed can be represented by the equation $y = 0.75x$. Give the groups time to use what they know about functions and equations to determine the answer to the question. Challenge the groups to determine answers to each of the questions that were posed during the introduction of the activity.

A possible answer: Alice can still win the race even though she gave her brother a 3 meter head start if their speed stays constant.

As a class, discuss the results of the students' investigations. Review the list of questions generated at the beginning of the class with the SQPL to ensure that all questions have been answered. Distribute Who Wins? BLM and have the students work independently or with a partner to answer the questions. Have students choose one of the problems on the BLM to explain in their math *learning logs* (view literacy strategy descriptions). These explanations can be a good formative assessment piece.

2013-14
Activity 11: Systems of Equations (CCSS: 8.EE.8)

Materials List: paper, pencil, Graphing Systems of Equations BLM, graphing calculator (optional)

Use the Graphing Systems of Equations BLM to work through this activity with students. Have students read the scenario on the BLM to visualize two people walking in the same direction at different rates, with the faster walker starting out behind the slower walker. At some point, the faster walker will overtake the slower walker.

Suppose that Sam is the slower walker and James is the faster walker. Sam starts his walk and is walking at a rate of 1.5 mph, and one hour later James starts his walk and is walking at a rate of 2.5 miles per hour.

Ask the students how to use graphs to determine where and when James will overtake Sam. Review with the students the distance = rate × time relationship and guide them to the establishment of an equation for both Sam and James (*Sam's equation should be* $d = 1.5t$*, and James' equation should be* $d = 2.5(t-1)$). Have students graph each equation and find the point of intersection (*2.5, 3.75*).

Lead the students to the discovery that two and one-half hours after Sam started, James would overtake him. They both would have walked 3.75 miles. Show the students that the goal of the process is to find a solution that makes each equation true, and that the solution to the system of equations is the point where the two lines intersect (in this case, (2.5, 3.75). Lead students to write a definition of a system of equations.

Continue using the BLM to present real-life examples to show when a system of equations might have no solution or many solutions. Give the students a number of problems involving 2 × 2 systems of equations, and have them use a graphing calculator to solve them graphically. Emphasize that the solution of a system is the point(s) where the graphs intersect and that the point(s) is (are) the common solution(s) to both equations. Therefore, solutions to systems of equations are written as ordered pairs.

Using an algebra textbook as a reference, provide opportunities for students to practice solving systems of equations by graphing. Include systems with one solution, no solutions, and an infinite number of solutions.

2013-14
Activity 12: Substitution (CCSS: 8.EE.8)

Materials List: paper, pencil, graph paper, calculator

Begin by reviewing the process for solving systems of equations graphically. Inform the students that it is not always easy to find a good graphing window that allows the determination of points of intersection from observation. Show them an example of a system that is difficult to solve by graphing. Explain that there are other methods of finding solutions to systems and that one such method is called the substitution method. The following example might prove useful in modeling the substitution method.

> Alan Wise runs a red light while driving at 80 kilometers per hour. His action is witnessed by a deputy sheriff, who is 0.6 kilometer behind him when he ran the light. The deputy is traveling at 100 kilometers per hour. If Alan will be out of the deputy's jurisdiction in another 5 kilometers, will he be caught?

Lead the students through the process of determining the system of equations that might assist in finding the solution to the problem. Use the relationship distance = rate × time, where time is given in hours and distance is how far he is from the traffic light in kilometers. Monitor the students as they work with a partner to determine an equation that represents Alan's speed which would be $d = 80t$ and an equation to represent the deputy's speed as $d = 100t - 0.6$. The right member of the deputy's equation can be substituted for the left member of Alan's equation which results in the equation $100t - 0.6 = 80t$. Solve the equation for *t*, and a solution of 0.03 would be determined. Substituting back into either or both of the equations, the value of *d* will be found to be 2.4 kilometers. The point common to both lines and thus the solution is (0.03, 2.4). Because the 2.4 kilometers is less than 5, Alan is within the deputy's jurisdiction and will get a ticket.
Have students use *split-page notetaking* (view literacy strategy descriptions) as the students work through the process of substituting to solve a system of equations. They should perform the calculations on the left side of the page and write the steps that they follow on the right side of the page. A sample of what *split-page notetaking* might look like in this situation follows.

$2x + y = 10$ $5x - y = 18$	Solve one equation for either *x* or *y*.
$2x + y = 10$ $-2x \quad -2x$ $y = 10 - 2x$	Substitute that equation into the other equation for the solved variable
$5x - (10 - 2x) = 18$	Solve for the remaining variable
$5x - 10 + 2x = 18$ $7x - 10 = 18$ $+10 \quad +10$ $7x = 28$ $x = 4$	Simplification of left side Substitute your answer for the variable in either of the original equations

Work with students individually and in small groups to ensure mastery of the process. Demonstrate for students how they can review their notes by covering information in one column and using the information in the other to try to recall the covered information. Students can quiz each other over the content of the split-page notes in preparation for quizzes and other class activity.

Sample Assessments

General Assessments

- Provide the student with a table of values and a graph and have the students determine the equation for each.
- Provide students with at least 2 situations and have the students determine the differences in the graphs of the situations, including the slope.
- Have the student determine whether a specific number is a term in a sequence whose n^{th} term is given. For example, is 24 is a term in the sequence whose n^{th} term is $a_n = 5 + 2(n - 1)$.
- Whenever possible, create extensions to an activity by increasing the difficulty or by asking "what if" questions.
- Have the student create portfolios containing samples of their experiments and activities.

Activity-Specific Assessments

- Activity 3: The student will sketch a pattern to represent $y = 3x + 1$, graph the equation and make a table of values to represent the equation.

- Activity 4: The student will write an equation to represent a linear function and one that is not linear.

- Activity 9: The student will be given 5 - 6 equations and determine the slope and *y*-intercept of each.