Grade 8
Mathematics
Unit 2: Transformations and the Pythagorean Theorem

Time Frame: Approximately four weeks

Unit Description

The content of this unit focuses on the properties of transformations on the coordinate grid; the relationships among angles formed by parallel lines; surface area and volume of cylinders, spheres and cones; and applications of the Pythagorean theorem and its converse.

Student Understandings

Students can apply transformations and identify properties that remain the same as figures undergo transformations in the plane. Students can provide one justification of the Pythagorean theorem and its converse and apply both in real-life applications. Students can provide informal arguments to establish facts about congruency of triangles, similarity of triangles, and angles created when parallel lines are cut by a transversal.

Guiding Questions

1. Can students use transformations (reflections, translations, rotations, dilations) to match figures and note the properties of the figures that remain invariant under transformations?
2. Can students state and apply the Pythagorean theorem and its converse in finding the lengths of missing sides of right triangles, showing triangles are right respectively?
3. Can students demonstrate a conceptual and practical understanding of congruence, similarity and symmetry appropriately when discussing transformations of figures?

Unit 2 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

Grade-Level Expectations	
GLE #	**GLE Text and Benchmarks**
Geometry and Measurement	
25.	Predict, draw, and discuss the resulting changes in lengths, orientation, and angle measures, and coordinates when figures are translated, reflected across horizontal or vertical lines, and rotated on a grid. (G-3-M) (G-6-M)

26.	Predict, draw, and discuss the resulting changes in lengths, orientation, and angle measures that occur in figures under a similarity transformation (dilation) (G-3-M) (G-6-M)
31.	Use area to justify the Pythagorean theorem and apply the Pythagorean theorem and its converse in real-life problems (G-5-M) (G-7-M)
CCSS#	**CCSS Text**
8.G.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
8.G.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
Writing Standards for Literacy History/Social Studies, Science and Technical Subjects 6 – 12	
WHST.6-8.1b	Write arguments to support claims with clear reasons and relevant evidence. b. Support claim(s) with logical reasoning and demonstrate an understanding of the topic or text, using credible sources.

Sample Activities

Activity 1: Mathematical Reasoning (<u>WHST.6-8.1b</u>)

Materials List: pencil and paper

It is important in 8^{th} grade mathematics to use inductive thinking to generalize a pattern when observations are made as the students form conjectures and justify their mathematical reasoning. The students will make conjectures throughout the year. A conjecture is a statement that seems to hold true for more than one problem but has not been proven with formal mathematics. Begin today's lesson by displaying the following problem:

> Mrs. Brown wants to encourage her class to participate in the school fundraiser. She told the students that if they participate, she will treat them to ice cream after school. All of her students sold at least two items, so she had to treat all 22 students to ice cream. The students chose two or three scoops of ice cream in their cones. The ice cream shop charged by the scoop and Mrs. Brown needed to figure out how much to pay. She counted 57 scoops after all students had been served. How many students chose two scoops and how many chose three scoops?

Have students work with a shoulder partner and determine a method to solve the problem. *(Answer: 13 students had 3 scoops and 9 students had 2 scoops).* Explain to the students that it is important that they record their thinking as they solve this problem. Monitor the students as they think about the problem, asking questions for extension or guidance as needed. Once the students have had time to complete the problem, ask a pair of students to share their thinking about the problem. Explain to the students that throughout the year, it will be important to use mathematical language to justify their methods of solving problems. Make sure they understand that there may be more than one method of solving problems, and it is important to be able to explain methods to each other.

Post the next question and tell the students that they are to work with their partner, but each will turn in a complete explanation of the method used to solve the problem.

> Mr. White has a cycle shop. He is not very organized and when he takes the tires off bicycles or tricycles, he just puts them in a pile off to the side. Thursday he was very busy and when he got ready to put the bicycles and tricycles together again, he realized that he had a pile of 22 tires and 8 seats. He charges by the cycle and needs to know how many cycles he would be paid for on Thursday. Explain how you and your partner discussed the problem and what you did to find the number of cycles.

After completion of this problem (*answer: 6 tricycles and 2 bicycles – 8 cycles),* discuss the methods used. Ask the students if they were able to solve this problem in the same way they solved the first problem. Let students know that as they explain their answers, using mathematical thinking and reasoning, they are justifying their methods using mathematical reasoning.

To make a conjecture, it is important to look for patterns that might help work other problems the same way. Put the third problem on the board:

> Farmer Brown's cows and ducks got out of the pen when his son left the gate open. The neighbor told Farmer Brown that he counted 52 legs. How many of Farmer Brown's ducks and how many cows escaped?

Ask the students if they can make a conjecture about how they might solve this problem before actually solving it. Given below are examples of how to think about 1 and 2 in the same manner, Problem 3 does not have the total number to work with, so the conjecture will not work the same way. Have students give their explanations to problem 3 as their exit ticket, which will give provide an opportunity to assess inductive reasoning so that throughout this unit, as they write conjectures, the students will better understand what is expected.

Problem 1: one method of solving would be to multiply 22 x 2 and get 44scoops, then subtract 57 – 44 to get 13 extra scoops, these being the three scoops, then subtracting 22-13 to determine 9 students got 2 scoops.

Problem 2: using the same method, 8 seats x 2 tires = 16. 22 – 16 = 6, so 6 were three-wheel tricycles, and 2 were bicycles

Problem 3: dividing by 4 legs, 52÷4=13 cows or 12 cows, 2 ducks, 11 cows, 4 ducks, etc. This problem has many solutions because there is not a total number of animals given.

These problems are examples of mathematical inductive reasoning problems.

Activity 2: Transformations! (GLE: 25; CCSS: 8.G.4)

Materials List: One-half Inch Grid BLM (six per group), Shapes BLM, Transformations BLM, Transformation Word Grid BLM, Transformation Review BLM, pencils, paper, scissors, ruler, large sheet of newsprint or chart paper, patty paper-optional

This activity was not changed because it already incorporates the CCSS.

Prior to the start of the lesson, make a transparency of the Shapes BLM or be prepared to show the BLM on a document camera or other display device.

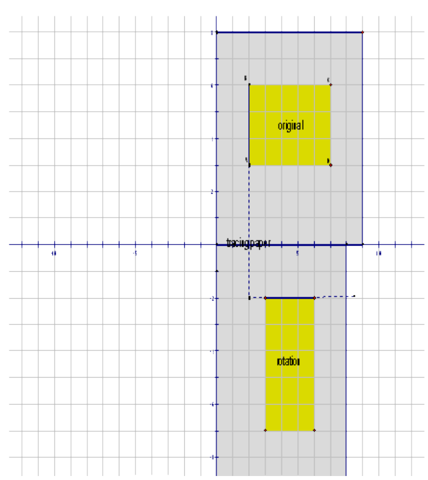
Have students work in pairs for this activity. Give each pair three copies of the One-half Inch Grid BLM. Tell students to draw the *x* and *y* axes in the center of each coordinate plane. Have students label the *origin*. Ask them to label both the *x*- and *y*-axes and locations of -7 to 7 on each axis. Review with students the naming of the quadrants with Quadrant I as the top right section of the coordinate plane. Moving counter-clockwise from Quadrant I, the other quadrants are numbered II, III, and IV. A short discussion about signs of the *x*- and *y*-coordinates of points in each quadrant should follow.

Distribute the shape cut-outs from the shape BLM. Make sure students can read each of the vertex labels. By cutting these out prior to class, students have time to begin the exploration of the transformations and their resulting changes in lengths, orientation and angle measures of each of the transformations. Ask students to justify using patty paper what changes have resulted to the length, orientation and angle measures as a result of the transformation (only the orientation will change with the reflection and rotation; there will be no change in length or angle measurements.) The shapes are congruent with the translation, reflection and rotation.

Post a large sheet of newsprint on the wall for the new vocabulary used. As each new geometry term is discussed, have a student add the word to the *word wall poster. The first words might be translation, rotation, reflection, orientation and congruent.*

Distribute Transformations BLM

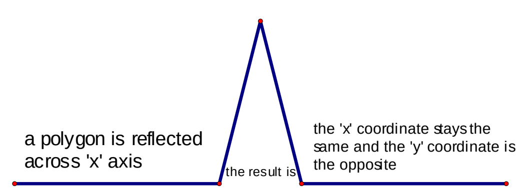
- Have students position the rectangle on the coordinate plane so that vertices A and B lie on the coordinates given in the table on the Transformations BLM. Record the coordinates of all four vertices of the rectangle in its original position in Column 1 of the table.
- Have students *translate* the rectangle up (or down) and right (or left), making sure to move the rectangle to the position that is given for vertex B, and then record the new coordinates for other vertices in Column 2. Make sure that the students understand that A′ represents the image of point A, B′ means the image of point B, etc.



- Have students return the rectangle to its original location and record coordinates of each vertex after a 90° clockwise *rotation* around the origin. If students have difficulties visualizing what to do, first have them use a small piece of tape to hold the rectangle on the grid. Students can then place a sheet of patty paper or a transparency sheet over the grid and trace the *x* and *y* axes and the rectangle, being sure to label the coordinates on the copy. While holding the transparency/patty paper at the origin with their pencil points, students can rotate the copy until the *y*-axis ends up on the *x*-axis. This will result in a 90 degree rotation. Have students discuss the new coordinates and identify the quadrant in which the rotated rectangle lies. Students should record the new coordinates in Column 3.
- Have students return the rectangle to its original location and then perform a *reflection* of the rectangle across the y-axis. Be sure to discuss *line of symmetry* as the rectangle is reflected. Model lifting the rectangle from the plane and flipping the rectangle over the *y*-axis, if needed. Have students record coordinates of the four vertices in Column 4.
- Have students complete the same actions using their trapezoid, right triangle, and isosceles triangle, recording all of the new coordinates on the chart. Remind them always to return their shapes to the original position before making a *transformation.*

After the class has had time to complete the transformations of all four shapes, have the groups make some conjectures about how they might be able to determine the positions of polygons after a transformation from the information in the chart. Distribute the Transformation *Word Grid* (view literacy strategy descriptions) for use with this discussion. Students should check each of the true changes for the reflection, rotation and translation. Students should keep their *word grid* for use with dilation activity. Have the groups share their conjectures with the class by using *professor know-it-all* (view literacy strategy descriptions). The group that is sharing conjectures will be selected by the teacher; therefore, all groups should be ready to go first. The group will go to the front of the class, and using their conjectures, justify their thinking and answer questions from the class about one of their conjectures. *Professor know-it-all* enlists the group or pair of students together to become the "teacher" as the class asks questions so that they can better understand the content. With this activity, the teacher will select a second group to share another conjecture and continue until all conjectures and thinking are clearly understood by the class.

Have students use the Transformation Review BLM as a *graphic organizer* (view literacy strategy descriptions) to guide them as they review the results of the different transformations. *Graphic organizers* are an aid for students to organize the content so that they are able to make connections. The *graphic organizer* is used to have the students state the "rule" that they have determined through the activity. Go through the example on the BLM as a class (at right). Allow students to discuss the answer with a partner. Students should write that the result of reflecting a polygon across the *y*-axis is that the *x*-coordinates are opposites of the originals and the *y*-coordinates stay the same. The BLM gives them either the initial position with the transformation used or the result of a transformation, and students will complete the statement by

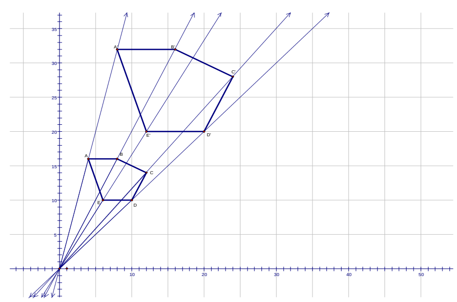


either giving the result of the transformation or the transformation used to give the result. This can be used as a summative assessment for activity 1.

Activity 3: Dilations (GLEs: 26; CCSS: 8.G.4)

Materials List: Transformation Dilations BLM, Quadrant I Grid BLM, protractor, pencil, paper, ruler

This activity was not changed because it already incorporates the CCSS.



Discuss dilations as another transformation. Ask if anyone has an idea about what a dilation might be. Students will relate to the eye doctor dilating their eyes, but very few of them relate a dilation to being an enlargement or a reduction.

Provide students with copies of the Quadrant I Grid BLM and the Dilations BLM. Have students plot the vertices of the polygon given on the Dilations BLM on a coordinate grid and then connect the points to form the polygon. Have students find the measure of each angle and the distance from vertex to vertex (i.e., length) for each side.

(*Teacher Note: Have students use rulers to measure lengths of sides which are not vertical or horizontal*.)

Next, have students use a ruler and draw a dotted line from the origin and extend the line through Vertex A of the polygon. Continue to do this by drawing lines from the origin through each of the other four vertices (see diagram). Tell students that the number each of the coordinates in the ordered pair is multiplied by is called the *scale factor.* As other figures are dilated, have students give the *scale factor.*

Instruct the students to follow the steps on the Dilations BLM and then discuss their conjectures about dilations and their effect upon angle measures, side lengths, and coordinates of the original figure. Make sure the students understand that the dilation is different from the reflections, translations and rotations, because it is the only one that produces similar figures – the other transformations produce congruent figures.

As a real-life connection, lead a discussion about when dilations are used in everyday life: using a projector to show an image to an entire class, enlarging a picture from the image stored in a digital camera, projecting a video on large screens at sporting events, or making a scale drawing of a large object.

Activity 4: Similarity (CCSS: 8.G.4)

Materials list: Similarity BLM, Grid BLM, pencil, paper

This activity has not changed because it already incorporates the CCSS.

Begin the lesson with a *SQPL* (view literacy strategy descriptions) statement similar to "A two-dimensional shape cannot be transformed into a different shape." In pairs the students will generate 2-3 questions that they would like answered throughout the day's lesson. Students will share these questions as the teacher writes the student questions on a chart or the board. Have students determine if any of the questions could be combined because they are asking the same question. The question that needs to be answered is whether translations, rotations, dilations and reflections change the congruence or similarity of figures and if this does not come up, add your own question.

Distribute Similarity BLM. Tell the students that as they work through the activity, they should be able to answer their questions. Give students time to work in pairs and complete the activity. Have students share their conjecture about the effects of these transformations on the congruency and similarity of figures.

Ask the students to look over their questions from the beginning *SQPL* and go through the answers to their questions based on today's exploration. Discuss the questions written at the beginning of the class and have the students share their answers to each.

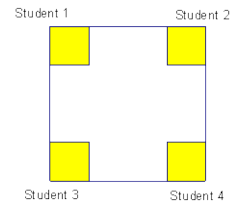
Activity 5: Developing the Theorem (GLE: 31)

Materials list: grid paper, straight edge, scissors, paper, pencil

Have students draw a right triangle on grid paper with the two perpendicular sides having lengths of 3 and 4 units. Have students draw a square using one of the legs of the triangle as the side of the square (i.e., draw a 3 x 3 square). Repeat using the other leg as a side of a square (i.e., draw a 4 x 4 square). Have students find the area of each square. Ask students to determine a method for finding the area of the square of the hypotenuse of their right triangles and to note how the areas of the three squares relate to one another. (Some students may remember the Pythagorean Theorem from previous years and use that information to determine the length of the hypotenuse. Others may compare the length of the hypotenuse to the units on the grid paper. The process used is not important, but all students should eventually see that the hypotenuse length is 5 and the area of the corresponding square is 25 square units.) Have students show that the sum of the areas of the two smaller squares is the same as the area of the square formed by the hypotenuse by cutting and rearranging the small squares inside the larger squares. Many texts and websites show how to do this. Two websites which use animations to develop the Pythagorean theorem are http://www.nadn.navy.mil/MathDept/mdm/pyth.html and http://www.pbs.org/wgbh/nova/proof/puzzle/theorem.html.

Have students practice finding side lengths of various right triangles using the Pythagorean Theorem.

Have students brainstorm predictions as to whether the Pythagorean Theorem will work when finding side lengths of triangles that do not have a right angle. Divide the students into groups of four to use the *discussion strategy* (view literacy strategy descriptions), *Think Pair Square Share.* Students will be given think-alone time to explore the question before pairing up with a partner to share their thinking (students1 and 2; students 3 and 4). Once these two have shared, have them share with the other pair in their group. Through these discussions, teach students to question each other's ideas and practice justifying their conjectures. Have groups of four come to a consensus with one idea that they wish to share with the class during class discussion. As a class, discuss conjecture(s) that students have made about the results of their explorations. Write conjectures on the board as the class validates student conjectures.



Activity 6: The Theorem (GLE: 31)

Materials list: The Theorem BLM, pencils, paper, calculators, graph paper

Provide students with the side lengths of several right triangles missing the length of one of the sides. Discuss the use of the formula as it applies to the missing lengths in the triangles. Extend this activity to include real-life situations that require students to find the length of one of the sides of a right triangle with situations by distributing The Theorem BLM. Have students verify their solutions to the BLM by comparing answers with another student and discussing any results that differ.

Activity 7: The Converse of the Pythagorean Theorem (GLE: 31)

Materials list: Converse of Pythagorean Theorem BLM, pencil, paper, ruler, calculator, Squares Cut Outs BLM (cut them out prior to lesson), spaghetti

Distribute square cut-outs to each pair of students. Have students restate what was learned about the Pythagorean Theorem. Tell them that with the squares, they are to build triangles and write the dimensions of the triangles. Give groups time to find at least 5 different triangles *(They are not looking for right triangles, just triangles.* Have students share the triangles that were formed and the dimensions of these triangles.

Discuss these triangles, and ask if anyone can think of some way to use the Pythagorean Theorem to prove whether these triangles are right triangles. *Use the converse of the Pythagorean Theorem and if these measurements do not form a true equation, the triangle is NOT a right triangle and vice versa.*

Distribute Converse of Pythagorean Theorem BLM for independent practice. When complete discuss student responses.

Activity 8: RAFTing with Pythagoras (GLE: 31)

Materials: pencil, paper

Once the students have acquired this new content information and concepts on the Pythagorean Theorem, *RAFT writing* (view literacy strategy descriptions) is suited to help them rework, apply and extend their understandings. The students will project themselves into the role of the sides of a triangle as they discuss what they have learned. Each of the letters of RAFT stands for the following:

R (Role of the writer) the student will write from the point of view of the sides of a triangle
A (Audience) the student will write to Pythagoras as if they are the sides of a triangle
F (Form of the writing, letter, song, etc.) this can be student choice or it can be assigned
T (Topic) the student will tell Pythagoras what they think the sides of a triangle think about the Pythagorean theorem.

Give students time to create their writing and then encourage them to read their *RAFT writing* to the class and discuss student understandings of how the Pythagorean Theorem and side lengths of triangles are related.

Example:
Dear Pythagoras,

> *We have wanted for so long to be appreciated for our length. When you manipulated us and formed the right triangle, we were proud, but when you formed squares from our lengths, we were overwhelmed! Then to think! You used these squares to find any missing lengths. When you told people that the squares of two legs of the right triangle would be equal to the hypotenuse, we could not have been happier.*

Thank you,
Sides of a triangle

Activity 9: Pythagorean Theorem and Distance (CCSS: 8.G.8)

Materials: Distance BLM, grid paper, pencil

Teacher Note: The students will determine how to use the Pythagorean theorem to find the distance between two points on the coordinate plane by using coordinates of vertices of an associated right triangle.

Distribute grid paper to students. Instruct the students to plot (4, 5) and (-5, 2) on grid paper. Tell them that today they will find the distance between two points on the coordinate grid. Next, have the students draw a vertical and horizontal line through each of the points. Have them extend the lines until they cross to show where the lines meet. The students should find that the lines meet at (4, 2) and at (-5,5) forming a rectangle. Ask the students if they can determine how to find the distance between point (4, 5) and (-5, 2) using the lines that they have sketched. Have students label the distance between (4,5) and (-5,2) as *d*. Label the legs of the right triangle *a* and *b*. The length of the longer leg will be the difference in the x coordinates 4 – (-5), or 9 units. The length of the shorter leg will be the difference in the y coordinates 5 – 2 or 3 units. Ask the students why they took the difference of the x coordinates to find the horizontal length to assess the understanding that the *x*-coordinates are horizontal locations and the *y*-coordinates are vertical locations. At this point the students can apply the Pythagorean Theorem to find the length of the hypotenuse of the right triangle, therefore, finding the distance between the two points.

$$d^2 = 9^2+3^2$$
$$d^2 = 81 + 9$$
$$d^2 = 90$$
$$\sqrt{d^2} = \sqrt{90}$$

$$d \approx 9.49$$

> Students have not learned to simplify radicals, but they have learned to determine whether the square root of 90 is closer to the square root of 81 or 100. Using a calculator, they can find the approximate decimal representation of the distance between the points.

The distance between the two points is about 9.49 units. Distribute Distance BLM and give students time to complete the problems and discuss solutions with a shoulder partner.

Using the strategy *professor know-it-all* (view literacy strategy descriptions), have shoulder partners prepare to present one of their solutions to the class and answer questions that students may have. All students should prepare, but random groups will be selected to present their information and answer questions, while the other students hold the professor accountable for the accuracy and logic of their solutions.

Activity 10: Building Dilemma (GLE: 31)

Materials: Building BLM, paper, pencil

Explain to the students that they will use what they have learned about the Pythagorean theorem and surface area to complete the following situation (on the BLM). They will work in student teams to design the house and explain their thinking.

> In Louisiana, one builder has decided that to prevent dangerous hurricane winds from damaging homes it would be best to build homes that have a triangular shape as shown on the BLM. The inside walls should be 14 feet high but the overall design must be built with the extreme angle of the roof approximately 60° from the ground as the base. Determine the lowest possible height of the house to accommodate the 14 foot walls. The home buyer wants at least 2000 square feet of living area. Determine the length of the

house and the square area of the roof. If roofing costs are $25/square foot, how much will the roof cost?

Sample Assessments

Performance assessments can be used to ascertain student achievement. For example:

General Assessments

- Provide the student with paper and the scale of 0.25 inches to represent 2 feet. The student will 1) draw a model of a rectangular swimming pool measuring 16 feet by 36 feet; 2) draw a 2 foot by 6 foot diving board so that it bisects one of the short ends of the pool; 3) find the perimeter and area of the pool; and 4) put a walk around the perimeter of the pool with a width of 4 feet and find the area and the outer perimeter of the walk.
- Provide the student with a sketch of a baseball diamond showing that there are 90 feet between the bases. The student will prepare a presentation explaining how to determine the distance the catcher must throw the baseball to the 2nd baseman if he needs to get the runner on second base out.
- Provide the student with several right triangles that have a missing side measure. The student will find the lengths of the missing sides.
- Distribute a piece of grid paper which shows a polygon and a transformation of the polygon (the second polygon). The students will determine a transformation(s) that would produce the second polygon.
- Whenever possible, create extensions to an activity by increasing the difficulty or by asking “what if” questions.
- Have students produce a portfolio containing samples of experiments and activities.
- Have students create a scale drawing. A rubric that assesses the appropriateness of the scale factor, as well as the accuracy of the drawing, will be used to determine student understanding.

Activity-Specific Assessments

- <u>Activity 2</u>: Have students write a justification as to why and give the new coordinates of a triangle that are reflected and translated.

- <u>Activity 6</u>: Give students situations such as these to have them solve to practice real-life Pythagorean Theorem problems.

    a) Washington, DC, is 494 miles east of Indianapolis, Indiana. Birmingham, Alabama, is 433 miles south of Indianapolis. Determine the distance from Birmingham to Washington D.C.

    b) The ladder of a water slide is 8 ft. high, and the length of the slide is 17 ft. Determine the length of the horizontal base of the slide. Justify all of your thinking using valid mathematical reasoning.

- Activity 7: Provide the student with a list of number triples that represent the side lengths *for triangles. Challenge students to determine which triples represent the side lengths of* a right triangle.