# Grade 7
# Mathematics
# Unit 5: Angles and Circles

**Time Frame:** Approximately four weeks

## Unit Description

This unit provides practice in solving real-life and mathematical problems involving angles and circles. The relationships among radius, diameter, circumference, and area of a circle are examined. An understanding of angle relationships in triangles is developed.

## Student Understandings

Students can use facts about supplementary, complementary, vertical and adjacent angles to write and use them to solve simple equations for an unknown angle in a figure. They apply the formulas for circumference and area of a circle and can give an informal derivation of the relationship between the circumference and area of a circle.

## Guiding Questions

1. Can students identify supplementary, complementary, vertical and adjacent angles in a figure?
2. Can students write an algebraic equation and solve for an unknown angle in a figure?
3. Can students illustrate the relationships between a circle's circumference and area and the measures of its diameter and radius?
4. Can students identify and apply the angle-sum relationship for a triangle in problem-solving situations?

## Unit 5 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

| Grade-Level Expectations | |
|---|---|
| **GLE #** | **GLE Text and Benchmarks** |
| **Geometry** | |
| 24. | Identify and draw angles (using protractors), circles, diameters, radii, altitudes and 2-dimensional figures with given specifications (G-2-M) |
| 28. | Determine the radius, diameter, circumference, and area of a circle and apply these measures in real-life problems (G-5-M) (G-7-M) (M-6-M) |

| **CCSS for Mathematical Content** | |
|---|---|
| **CCSS#** | **CCSS Text** |
| **Geometry** | |
| 7.G.4 | Know the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle. |
| 7.G.5 | Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and use them to solve simple equations for an unknown angle in a figure. |

## Sample Activities

### Activity 1: Complementary, Supplementary, or Vertical? (CCSS: 7.G.5)

Materials list: paper, pencil, straightedge, protractor, patty paper or tracing paper

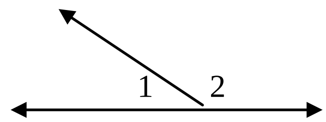
In this activity, students will investigate the properties of supplementary, complementary, and vertical angles and write an equation to solve for a missing angle. Before beginning the activity, students will assess their understanding of key terms used in this activity through a *vocabulary self-awareness* chart (view literacy strategy descriptions). Because students bring a range of word understandings to the learning of new concepts, it is important to assess students' vocabulary knowledge before interacting with the content. This awareness is valuable for students because it highlights their understanding of what they know, as well as what they still need to learn in order to fully comprehend the content. Have students copy the chart below in their notebook or *learning log* (view literacy strategy descriptions).

*Teacher note: make sure students leave plenty of room for examples and definitions since they will be adding to or modifying them throughout the activity.*

| Word | + | ✓ | - | Example or Illustration | Definition |
|---|---|---|---|---|---|
| Angle | | | | | |
| Adjacent angles | | | | | |
| Complementary angles | | | | | |
| Supplementary angles | | | | | |
| Vertical angles | | | | | |

Ask students to complete the chart before the activity begins by rating each vocabulary word according to their level of familiarity and understanding. A plus sign (+) indicates a high degree of comfort and knowledge, a check mark (✓) indicates uncertainty, and a minus sign (-) indicates the word is brand new to them. Also, ask students to try to supply a definition and/or example for each word. For words with check marks or minus signs, students may have to make guesses about definitions and examples. Over the course of the activity, allow time for students to revisit their self-awareness charts to add new information and update their growing knowledge about key vocabulary. *Students may want to add new knowledge with a different color or with ink to indicate how their understanding is changing. Make sure students keep the chart handy and remind them to add to their original entry throughout the activity.*

Draw the following pair of linear angles on the board and ask students to use a straightedge to draw a similar sketch labeling the left angle, “1” and the right angle, “2.” *Depending on the formative assessment resulting from the vocabulary self-awareness chart, a discussion of* ***angles*** *and* ***adjacent angles*** *may be appropriate, at this point.*




$m\angle 1 \approx$ _______
$m\angle 2 \approx$ _______

Students will use a protractor to measure each of the angles formed and write the angle measures to the side. *Use this time to review how to use this tool appropriately, if needed.* Next, students will draw a different pair of linear angles. Have them switch with two other people so that one student will measure ∠ 1 and record the measure and a different student will measure ∠ 2 and record the measure. When students have finished, make sure the original students get their sketch back. As a class, compile a list of students’ results. The list may look like the one below:

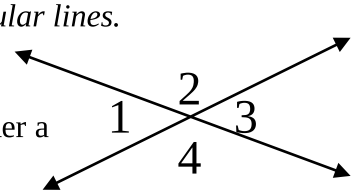
| | |
|---|---|
| $m\angle 1 \approx 58^\circ$ | $m\angle 2 \approx 122^\circ$ |
| $m\angle 1 \approx 30^\circ$ | $m\angle 2 \approx 155^\circ$ |
| $m\angle 1 \approx 140^\circ$ | $m\angle 2 \approx 40^\circ$ |
| $m\angle 1 \approx 37^\circ$ | $m\angle 2 \approx 145^\circ$ |

*The teacher’s list should be longer than this so that students have enough data to make a conjecture.*

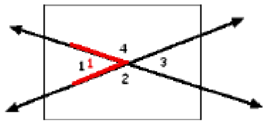
After the class has compiled a list big enough, ask students to make a conjecture about the measures of the two angles. The students should notice that the angle pairs sum to 180º. *Note: not all angle pairs will sum to exactly 180º, however, the sums should all be close enough to 180º for the students to make a proper conjecture.* Ask students why they think some of the angle pairs do not add up to the same number. Allow students to discuss some of the reasons they came up with. Reasons should include, but not be limited to, human error, thickness of the lines, and improper use of the protractor. Discuss the importance of accuracy and precision when measuring.

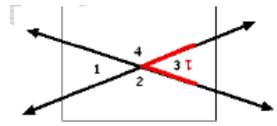
Ask students what pairs of angles that add up to 180º are called. Discuss what it means for two angles to be **supplementary**. Include examples of nonadjacent supplementary angles. Next, ask students if the same relationship would be true for pairs of angles that add up to 90º. Discuss what it means for two angles to be **complementary** and include examples of nonadjacent complementary angles. *Note: This may be a good time to have a discussion about parallel, perpendicular and intersecting lines. It should be noted that adjacent complementary angles are formed by perpendicular lines.*

Next, have students use a straightedge to draw two intersecting lines and label angles formed 1, 2, 3, 4 in either a clockwise or counterclockwise rotation.



Ask students to make a conjecture describing which angles are congruent. *Students should see that angles 1 and 3 are congruent and that angles 2 and 4 are congruent.* To verify this conjecture, students will trace angle 1 with patty paper and see if they can find other angles congruent to it. Repeat this procedure for all the angles. For example, students can see that the measure of angle 1 has to be the same as the measure of angle 3.



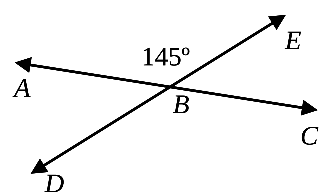




Ask students if they know the name of the non-adjacent angles formed when two lines intersect. Using *discussion* (view literacy strategy descriptions), specifically *Think-Pair-Square-Share,* students will describe how pairs of vertical angles are related in terms of their locations and what conjecture can be made about their measures. *Discussion* can improve learning and remembering when students participate in the dialog about concepts. Class discussion can be used to promote deeper processing of content and rehearsal of newly learned content. Ask students to think alone about how to define pairs of vertical angles in terms of their locations and what conjectures can be made about the measures of vertical angles. After students have thought about this for a short period of time, have them pair up with someone to share their thoughts. Then have pairs of students pair with other pairs, forming, in effect, small groups of four students. Monitor the brief discussions and elicit responses afterward. Be sure to encourage student pairs not to automatically adopt the ideas and solutions of their partners. These short-term discussion strategies work best when a diversity of perspectives are expressed. Allow students time after the discussion to record new learning on the *vocabulary self-awareness* chart. Remind students that they are allowed to use the chart as a reference when solving problems related to these terms.

**Activity 2: Find that Angle! (CCSS: 7.G.5)**

Materials list: pencil, paper, Find that Angle! BLM

In this activity, students will use facts about supplementary, complementary, vertical and adjacent angles to write and solve simple equations for an unknown angle. Sketch the following diagram on the board:




Students will use *SQPL* (view literacy strategy descriptions) to write questions that would need to be answered before they can find the missing measure of one of the angles. *SQPL* promotes purposeful learning by prompting students to ask and answer their own questions about content. Pair up students and based on the sketch, have them generate 2-3 questions that would need to be answered before finding the missing measure of ∠*ABD. Questions may include: What is the relationship between ∠ABD and ∠ABE? What is relationship between ∠DBC and ∠ABE? Can I use what I know about vertical angles to* *find the missing measure? Can I use what I know about supplementary angles to find the missing measure? How do I find the missing measure of angle if they are supplementary? What are the relationships between all of the angles shown?*

When all student pairs have thought of their questions, ask someone from each team to share questions with the whole class. As students ask their questions aloud, write them on the board. Similar questions will be asked by more than one pair. These should be starred or highlighted in some way. Once all questions have been shared, look over the student-generated list and decide whether teacher-generated questions need to be added. This may be necessary when students have failed to ask about important information they need to be sure to learn.

Next, have students work with their partner to write a simple equation and then solve to find the measure of $\angle ABD$. *(*$x + 145^\circ = 180^\circ$*;* $x = 35^\circ$*)*. Tell them as they work through the problem to pay attention to information that helps answer the questions from the board. *They should be especially focused on material related to the questions that were starred or highlighted. These might be considered class consensus questions.*

Have a volunteer come to the overhead to explain how they found the missing angle measure. Ask students how they know the answer makes sense *(because if they add 145 and 35, they will get 180. If not, then there is an error in finding the missing measure).* Then have another volunteer describe which questions were answered by the students' solution path.

Distribute Find that Angle BLM. In the example problem, students will use a *process guide* (view literacy strategy descriptions) to enable them to progress independently through the steps required to apply previous learning about angles to a new situation. *Process guides* scaffold students' comprehension and are designed to stimulate thinking during or after their involvement in content instruction. Begin by explaining the guide's features, intent, and benefit to students. *Students will be able to use this guide as a reference with the remaining problems.*

Pair students with a partner to complete the process guide found at the top of the Find that Angle BLM. Monitor students' discussion and completion providing assistance where needed. Engage the whole class in discussion based on their responses to the guide, and use this feedback to provide additional explanation and to make any necessary modifications to the guide.

Students will work independently to complete the remaining problems on the BLM using the *process guide* as needed. Then students will exchange their papers with a partner to compare the responses and steps used to solve the problems. Discuss the responses to each problem as a whole class by inviting volunteers to come to the overhead to explain. *If a document camera is available, students can display their work and describe the steps they used to solve the problem.*

**Activity 3: What's Your Angle? (GLE: <u>24</u>)**

Materials List: What's Your Angle BLM, protractor, pencil

Give each student a copy of the What's Your Angle BLM. Individually, have students place three points on each circle to act as the vertices of a triangle, and then use the straight edge of a protractor to construct a triangle. Ask students to measure each angle of the triangle using the protractor, and record the measurements.

Have students share by comparing and discussing within their group the types of triangles drawn and the sum of angles of each of their triangles. Have students recognize that the sum of the angles in a triangle appears to be 180 degrees. Because of variances in measurements, this may not be obvious. This concept is revisited in the next activity.

Discuss the term altitude. Have the students draw an altitude in each of the triangles they drew inside the circles. Be sure to emphasize that the two perpendicular sides in right triangles are also altitudes.

For practice, have students draw triangles with specific side lengths and/or specific size angles. Give students the opportunity to draw 2-dimensional figures with specified angular measures.

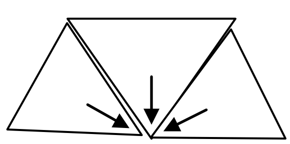
**Activity 4: Interior Angles of a Triangle (GLE: <u>24</u>)**

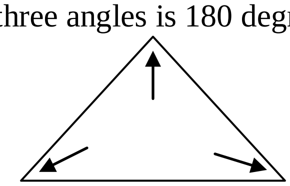
Materials List: construction paper, protractors, scissors, pencils, tape, math learning log

In this activity, the understanding that the sum of the interior angles of a triangle is 180 degrees is reinforced by using a method in which none of the individual angle measures is known. Since this process works for any triangle, it is considered a proof of the triangle sum theorem.

Divide students into teams of three students each. Give each student a sheet of construction paper and a protractor. Each group should also have at least one pair of scissors. Ask each student in the group to draw a different type triangle (acute, obtuse, right).

Have students cut out their triangles, label each angle with a letter (put the letter in the interior of the triangle near the vertex), and then tear off the angles of the triangle. Instruct students to place the angles adjacent to one another with vertices touching. Ask students how this proves that the sum of the three angles is 180 degrees.





Next, have each student draw a triangle on construction paper and label two of the angles with measures they find by using the protractor. The third angle should be marked with a question mark. On the back of the triangle, place the numerical value of the missing angle. Have teams exchange their triangles with another team, and then find the missing angle using the triangle sum theorem (rather than measuring the angle). They can quickly check each other's work by comparing their answers to the one written on the back of the triangle. Students should discuss if there are differences of opinion regarding the answers.

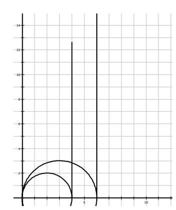
In their math *learning log* (view literacy strategy descriptions), have students describe how to find the sum of the interior angles of a triangle as well as any new understandings they now have about the angles in triangles. Afterward, they can share their log entry with a partner to check for accuracy and logic.

**Activity 5: *π!* (CCSS: 7.G.4)**

Materials List: 3-5 different sized lids per group (place a mark on the edge of each lid with a permanent marker to indicate a *starting point*), 2 sheets of cm grid paper per group, tape, rulers, pencils, Around the Lid BLM, calculator, *Circumference and the Dragon of Pi (optional)*

The purpose of this activity is to have students investigate or discover π. Have students work in pairs for this activity. Distribute three to five different-sized lids for each group, two sheets of cm grid paper, and rulers. Have students tape the two sheets of grid paper together. ($8\frac{1}{2}$ in edge to $8\frac{1}{2}$ inch edge to make the paper 22" long). Draw an *x*-axis along the bottom and a *y*-axis along the left edge to represent quadrant I of a coordinate plane.

Instruct students to place each lid on the *x*-axis so that the diameter of the lid rests along the *x*-axis, and the outside edge of the lid rests on the *origin.* Ask students to mark the endpoint of the diameter and then draw a line through that point which is perpendicular to the *x*-axis. (Note: Depending on the length of the diameter, this may or may not be one of the vertical lines on the grid.) Have students trace around the lid showing the semi-circle along the *x*-axis.



Next, instruct students to place the point marked as the *starting point* on the *x*-axis at the point where the end of the diameter of the lid is marked. Have the students roll the circle along the perpendicular line that they drew earlier by first placing the *starting point* at the intersection of the *x*-axis and the perpendicular line. When the *starting point* rotates back to the vertical line, have students mark this point.

Discuss the fact that this one rotation is called the *circumference* of the circle. Have the students find the lengths of diameter (marked on the *x*-axis) and circumference (marked on the perpendicular line) by reading the units on the grid and record their answers on Around the Lid BLM. Repeat the process with all lids.

| Lid Number | Diameter | Circumference | Ratio $\frac{C}{d}$ | Decimal Value |
|---|---|---|---|---|
| | | | | |
| | | | | |

Allow students to use a calculator to compute the ratio of the circumference to the diameter as a decimal correct to five places and place their answers in a table. Once students have completed the table on the Around the Lid BLM, ask them to record three observations for the data in the chart.

Lead a discussion concerning the students' findings and the relationship between the circumference and diameter of a circle. Have students see that the pattern is multiplicative and that the decimal ratios are close to the value of $\pi$. Make sure that students understand that the relationship between the circumference and diameter indicates that the circumference is a little more than three times the length of the diameter. After the discussion, have the students calculate the average ratio for all five.

For a closure to this activity, read the book *Circumference and the Dragon of Pi* to the class to reinforce the concepts of pi, circumference, diameter, and radius. An alternative method is to begin the activity, read up through page 13, do the activity, and then finish reading the book.

**Activity 6: Pricing Pizza (CCSS: 7.G.4)**

Materials List: Pricing Pizza BLM, centimeter grid paper, compass, string, 1-inch grid paper (optional for below level students) and pencil

This activity asks students to think about how a pizza is priced relative to its diameter, radius, circumference, and area. Students are asked to find these measures for three different circular pizzas and to decide which measures are most closely related to price. The purpose of the activity is to encourage students to think about measuring circles, not to introduce the formulas. They should use counting and estimating strategies to find area and circumference. As students work through the activity, they should be looking for connections between a circle's diameter, radius, area and circumference. Students should also search for clues that tell when each of these measurements gives useful information about a circular object in a given situation.

Prior to starting the activity, make sure students understand *circumference, radius, and diameter.* Students will complete vocabulary cards to demonstrate their understanding of these terms and will continue to use the card to add new knowledge and examples.

Begin the activity by discussing with students that many pizza restaurants sell small, medium, and large pizzas—usually measured by the diameter of a circular pie. Of course, the prices are different for the three sizes. Ask students if they think a large pizza is usually the best buy and why. Distribute Pricing Pizza BLM and centimeter grid paper.

Read the problem with the class so that they understand the context in which they will be investigating. Have students use centimeter grid paper and a compass to construct models of the three pizzas in the problem. Use the scale 1 in = 1 cm. *Note: Below level students that have trouble with this scaled model can use large sheets of inch grid paper or tape together several sheets to make circles the actual size of the pizzas.*

Have students work in pairs or small groups to find the radius, circumference, and area of each circle. Make sure students are recording their findings on the BLM and can explain how they arrived at their answers.

On the board or overhead projector, record the measurements that students found for the radius, circumference, and area of each circle. Discuss how they found the measurements and whether any seem unreasonable. Because students are counting and estimating, circumferences and areas will vary but should be within a reasonable range. Ask students if they see any patterns in the measurements that might help to predict circumference and area. Do not give the formulas now unless students see the relationships among diameter, radius, circumference and area and can explain why they make sense. Discuss the answers to part B. Many students will say that the diameter is most closely related to price because as it changes 3 inches, the price changes by $3.00.

**Activity 7: Covering a Circle (CCSS: 7.G.4)**

Materials List: Graphic Organizer BLM ,Covering a Circle BLM

In this activity, students will find the area of a given circle. The purpose is not for them to find or use a formula but to see the need for a shortcut—a formula—for finding the area of a circle.

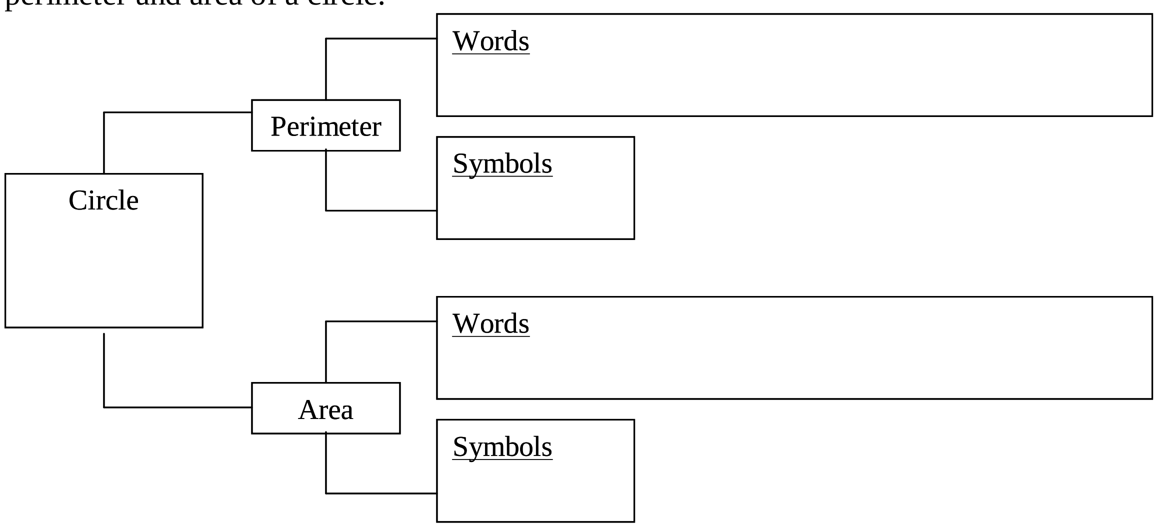
A review of how one finds the area of other shapes might help students begin to think about strategies for finding the area of a circle. Students will use a *graphic organizer* (view literacy strategy descriptions), to record the name of the shape and the rules for finding its perimeter and area. *Graphic organizers* are effective in enabling students to assimilate new information by organizing it in visual and logical ways. Organizing previously learned information about rectangles, squares, parallelograms, and triangles will enable students to assimilate new information about circles. It is not important for students to present the rules as they are written here. What is important is that they have a correct formula that makes sense to them. The goal is not only to have efficient methods of finding area and perimeter, but to be flexible enough to realize that their method and another's method may sound different but accomplish the same thing.

Next, students will explore how to find the area of a circle using what they know about finding the area of other polygons. Distribute Covering a Circle BLM and have them work with a partner to experiment in finding an easier way to find the area of a circle. Encourage them to think about different methods and strategies. Check that they are prepared to share their answers and strategies. If students are struggling, ask questions that will help them cut down on the work by using the symmetry of the circle.

Have students share their measures and their methods. Some strategies students may report include:

- *Find the area of the largest rectangle made from whole squares that could fit inside the circle. Then count and add the whole squares and parts of squares that were not inside the rectangle.*
- *Find the area of half of the circle and double it.*
- *Find the area of a quarter of the circle and multiply by 4.*
- *Surround the circle with a large square and find the area of it. Then find the area of the region outside the circle but inside the large square by counting the grid squares and parts of grid squares. Subtract this area from the area of the large square to get the area of the circle.*

Students will revisit their graphic organizer and add a new “branch” to the back that includes circles. They should include and/or modify their previous explanation of the perimeter and area of a circle.




**Activity 8: “Squaring” a Circle (CCSS: 7.G.4)**

Materials List: Circles and Radius Squares BLM, Grid Paper BLM, lids from Activity 5

In this activity, students make squares with sides the same length as the radius of a circle and then determine how many of these “radius squares” are needed to cover the circle. They will easily see that four is too many and three are too few. The goal of the activity is to help students to discover the formula for finding the area of a circle and to understand why it makes sense.

Distribute Circles and Radius Squares BLM and Grid Paper BLM to students. For each circle, have students cut out several copies of the radius squares it takes to cover the circle. Then students will find out how many radius squares it takes to cover the circle cutting the radius squares into parts if needed. Students will record their data in a table with these column headings:

| Circle | Radius of circle | Area of radius square | Area of circle | Number of radius squares needed |
|---|---|---|---|---|

Next, students will draw a couple of their own circles on centimeter grid paper. They can use the circles from objects in Activity 5. Students will make radius squares for each circle, and find out how many radius squares it takes to cover each circle. Students will add this data to their table. Then have students compare answers with a partner. *Students should see that for each of the three circles, four radius squares are too many to cover the circle and three radius squares are too few.*

| **Circle** | **Radius of circle** | **Area of radius square** | **Area of circle** | **Number of radius squares needed** |
|---|---|---|---|---|
| 1 | 6 units | 36 square units | About 113 square units | A bit more than 3 |
| 2 | 4 units | 16 square units | About 50.3 square units | A bit more than 3 |
| 3 | 3 units | 9 square units | About 28.3 square units | A bit more than 3 |

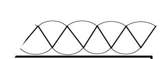
Have students share their strategies for finding the number of radius squares needed to cover a circle. Then ask students to look at the table and identify patterns they notice. *Students should be able to determine that it takes just over three radius squares to cover a circle. The answer, of course, is that it takes π squares to cover a circle. Help them to formulate a way to describe finding the area of a circle:*

1. *The area of a circle is a little more than 3 times the area of a square that has the circle's radius as its side length.*
2. *The area of the radius square is found by multiplying the length by its width, which is radius times the radius, r x r or $r^2$.*
3. *The area of a circle is thus (a little bit more than 3) x r x r or (a little bit more than 3) x $r^2$*

**Activity 9: Understanding Area of a Circle (CCSS: 7.G.4)**
*This activity did not change because it already addresses the CCSS.*

Materials List: paper plates (not Styrofoam), ruler or straight edge, scissors, pencil, paper, tape

Have the students cut a paper plate into eight sectors and position the sectors of the circle as shown to the right.



Lead students to understand that the shape formed is beginning to look like a parallelogram. You may want to have the students cut each of the eight pieces in half and use the resulting sixteen pieces to reform the parallelogram shape. The smaller the pieces, the more the shape will look like a parallelogram. Ask students leading questions so that they understand 1) the base of the rectangular shape is $\frac{1}{2}$ of the circumference and 2) the height of the parallelogram is the radius of the circle. Develop the formula for the area of a circle by starting with the formula for the area of a parallelogram, $A = bh$, and substituting $\frac{1}{2}C$ for *b* and *r* for the *h*. This results in the following:

$A = bh = \frac{1}{2}C(r) = \frac{1}{2}\pi d(r) = \frac{1}{2}\pi(2r)(r) = \pi r^2$, the formula for finding the area of a circle.

Provide students with practice in using the formula.

**Activity 10: Real Life Measures (GLE: <u>28</u>)**

Materials List: a round/circular object for each pair, measuring tapes, paper, pencil, Circles in Real Life BLM

Have a class discussion of the need to use correct terminology and how to measure circular objects. Ask, "What is someone referring to when he/she speaks of 22-inch rims on a vehicle? A 45-inch round table? Where on each of these objects would you measure to verify the information stated?" (*diameter*)

Ask students to work in groups of four for this activity. Give each pair of students a round or circular object and a tape measure. Have the students measure the diameter, radius, and circumference of their object. Record these measurements. Trade objects with the other pair in the group, measure, and record the measurements. Have the groups study and discuss the relationships between the radius and diameter measurements and between the diameter and the circumference measurements. Ask, "Do your measurements have the same relationships that are found in the circles in the previous activity? Why or Why not?"

Have each pair of students use their measurements to find the area of its object, then trade objects with another pair and find the area of the new object. Groups can compare their answers and check for accuracy and logic.

Provide real-life problems in which students find circumference, area, diameter or radius of given objects (e.g., car or truck tires, bicycle tires, gasoline drums, circular pools, silos, tree trunks, water tanks). Additional practice can be found on Circles in Real Life BLM.

**Activity 11: Circumference and Area (GLEs: <u>24</u>, 28)**

Materials List: rulers, pencils, compasses, Circumference and Area BLM, computer with internet access (optional)

Break the students into groups of 4. Give each group rulers and three radii measurements. Have the students draw three different circles with the given radii. Give $\frac{1}{3}$ of the groups the measurements 4 cm, 8 cm, 16 cm, another third the measurements 2cm, 4cm, 8cm, and the last third of the groups the measurements 3cm, 6cm, 12cm.

Have students complete the Circumference and Area BLM chart with given radii, diameter, circumference, and area and answer the questions that follow.

Discuss as a class the three different sets of numbers used and each group's observations and reasoning.

*An example of the chart with some answers is given.*

| Radius | Area | Diameter | Circumference |
|---|---|---|---|
| **2 cm** | 12.56cm² | 4cm | 12.56cm |
| **4 cm** | 50.24cm² | 8 cm | 25.12cm |
| **8 cm** | 200.96cm² | 16 cm | 50.24cm |

After discussing each group's observations and reasoning, have students complete this statement: "If the radius of a circle tripled, then …" Be sure to have students justify their reasoning.

An interactive math lesson for calculating the circumference of a circle can be found at http://www.aaamath.com/geo612-circumference-circle.html.
An interactive math lesson to teach the finding the area of a circle can be found at http://www.aaamath.com/geo612-area-circle.html.

**Activity 12: Replacing Trees (CCSS: <u>7.G.4</u>)**

Materials List: Replacing Trees BLM

Begin by explaining to students that they will have a chance to use their knowledge of areas of circles to investigate a real-world problem. Present the following words to students: city, tree, law, replacement, diameter, circumference, and area. Students will use these words to write a *lesson impression* (view literacy strategy descriptions) text that will enable them to make a guess as to what the activity will involve. *Lesson impressions* create situational interest in the content to be covered by capitalizing on students' curiosity. By asking students to form a written impression of the topic to be discussed, they become eager to discover how closely their impression text matches the actual content. Have students write a short paragraph using the given words in the context they think will be used for the activity. When students finish their impression texts, invite volunteers to read what they have written to the class. Anticipation is heightened when several students share their different impressions, leaving students to wonder whose is the closest to the actual content.

**Example of Lesson Impression for a real-world problem**

**Impression Words:** ***city, tree, law, replacement, diameter, circumference, and area***

**Impression Text:** *My dad was telling me this morning about a new* ***law*** *he read about in the newspaper. The* ***city*** *is planning on building a new park with a walking track around the* ***circumference*** *of the park. There are a lot of trees in the park and the law says the city will have to pay a* ***replacement*** *cost for every* ***tree*** *that is cut down. The cost is figured using the* ***area*** *of the park and the* ***diameter*** *of each tree that is cut down. I'm glad that the city cares about the trees and is trying to protect them by passing this law.*

Distribute Replacing Trees BLM and read the problem with the class. Make sure they understand the idea of a cross-section of a tree trunk and the two replacement rules. Have students work on the problem and follow up in groups of three or four. Remind students they must offer an explanation for their choice and give mathematical reasons why they chose one rule over the other. For struggling students, have them trace the diagram from the BLM onto grid paper and draw a picture of what it would mean to follow each of the rules.

Have groups share their answers and strategies. Challenge them to offer better explanations than simply, "If you want more trees in your neighborhood, the area rule is better." Have them talk about the fact that, in the given example, the area rule will provide four times as many new trees as the diameter rule. Some students may talk about the cost of the trees and say that it is not fair to make someone come up with the money necessary to buy all the trees needed to satisfy the area model.

Ask students to go back to their impression text and compare their text with the actual information presented. Have students describe the similarities and differences in their *learning logs* (view literacy strategy descriptions), and exchange with a partner to see whose impression text came closest to the actual context of the problem.

## Sample Assessments

### General Assessments

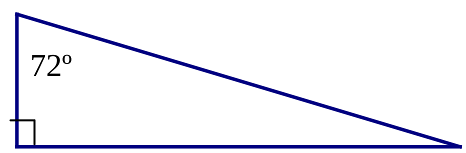
- Determine student understanding as the student engages in the various activities.

- Create extensions to an activity by increasing the difficulty or by asking "what if" questions.
- Encourage the student to create his/her own questions to evaluate his/her understanding of circle concepts.
- Have the student create a portfolio containing samples to demonstrate understanding of one of the concepts in the unit. The following is an example:

  Anne called The Pizza Man to order a 12-inch pizza for her and her best friend. When the pizza delivery boy came to the door with her pizza, he had two boxes. He told Anne that they had run out of twelve-inch pizza boxes and the owner had told him to deliver two, six-inch pizzas instead. He told Anne that it was the same amount of pizza and each of them would have her own pizza. Anne disagreed with the pizza boy and convinced him that she was correct. Explain how you think Anne might have proven this to the pizza boy. Use diagrams and /or mathematics in your proof.
- Have the student trace, measure and record the diameter of three circular objects and then find the circumference and area of each.
- Have the student complete journal writings using such topics as:
  - Explain the difference between circumference and perimeter.
  - Form a conjecture about the relationship between the shape of rectangles and their areas and perimeters.
- Have the student use the given measurements of a circle to find the other measurements. Example: A dinner plate has a diameter of about 9 inches. Find its circumference and area.

**Activity-Specific Assessments**

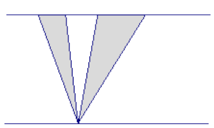
- <u>Activity 3</u>: Jordan rides his skateboard over a ramp shown below. What is the measure of the angle that the ramp makes with the ground?



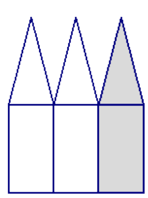

*Solution: The angle measures 18º*

- <u>Activity 4</u>: The student will find the sum of the measures of the angles of each shaded region and explain his/her reasoning.

a)



b)



*Solutions: a) 360º b) 540º*

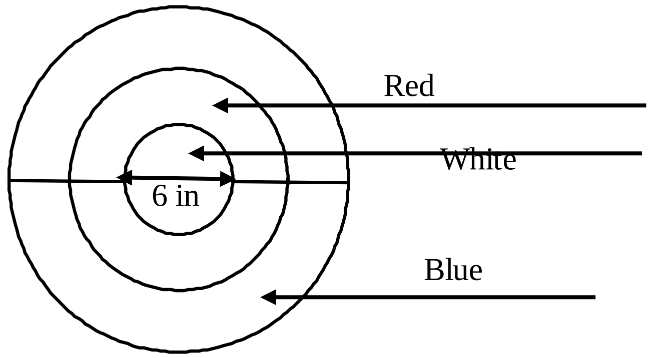
- Activity 6: The student can demonstrate an understanding of radius, circumference and diameter.
  Which measurement of a circular pizza—diameter, radius, circumference or area—best indicates its size?
  *Solutions: Pizza sizes are usually given by diameter, so the logical measure that best indicates its size is area. However, the radius and circumference are also defensible answers because as any of the four measures of a circle increase or decrease, so do the other three. Therefore, since all four measures are related, any of them are technically plausible as indicators of the size of the pizza.*

- Activity 10: The student can describe the connection of area and circumference to real-life.
  Some everyday circular objects are commonly described by giving their radius or diameter. In the examples below, explain what useful information (if any) you would get from calculating the area or circumference of the circle.
  - A 3.5-inch diameter computer disk
    *Solution: the area of a computer disk tells you something about the storage space on the disk.*
  - A 21-inch diameter bicycle wheel
    *Solution: the circumference of a bicycle wheel tells you how far the bike travels in one revolution.*
  - A 12-inch water pipe
    *Solution: the diameter and related cross-sectional area of a water pipe will tell you how much water can flow through the pipe.*
  - A lawn sprinkler that sprays a 15-meter radius section of lawn
    *Solution: the area of a lawn sprinkler's spray would let you estimate how much of your lawn will get watered at each location the device is used and will allow you to estimate how long it will take to water your lawn.*

- Activity 11: The student can correctly find radius, circumference and diameter.
  A large burner on a standard electric stove is about 8 inches in diameter.
  a) What are the radius, circumference, and area of the burner? *Solution: radius = 4 in., circumference = about 25 in., area = about 50 sq. in.*
  b) How would the area and circumference of a smaller 4-inch diameter burner compare to the area and circumference of the 8-inch burner?
  *Solution: the circumference will be about half—about 12.5 inches. The area, however, will be only one fourth as much—about 12.5 sq. in.*

- Activity 12: The student will work the following problem correctly:
  Randy's dad cuts circular targets for archery practice. He asked Randy to help him by cutting one circle with a diameter of 12 inches out of red plastic and a second circle with a diameter of 18 inches out of blue plastic. He wanted a white center bull's eye circle cut with a diameter of 6 inches. Randy was to glue these circles together into a target that looked like the

picture. Determine the area of each color on the target, and explain your method of solving the problem.




*Solution: The white bull's eye has an area of 28.26 sq in*
*The red area that is showing has an area of 84.78 sq in*
*(Whole red circle area of 113.04 – white circle area of 28.26)*
*The blue area that is showing has an area of 141.3 sq in*
*(Whole blue area circle of 254.34 – whole red circle of 113.04)*