AP Calculus BC

1) Given $f(x) = \sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$, find f'(x) and $\int f(x) dx$ and the interval of convergence for all three power series.

2) Generate a power series for $\sin\left(\frac{1}{2}x\right)$, centered at 0, including the general term in two different ways.

3) Using their Taylor series, show that a) $\frac{d}{dx}(e^x) = e^x$ and b) if $F(x) = \int e^x dx$ with F(0) = 1, then $F(x) = e^x$.

Generate power series for the following, centered at 0, using the table. Include the general term.

$$4) \ f(x) = \cos(\sqrt{2x})$$

$$5) f(x) = \sin x^3$$

6)
$$f(x) = e^{-x}$$

$$7) \ f(x) = x^2 e^x$$

$$8) \ f(x) = \frac{\tan^{-1} x}{x}$$

9.
$$f(x) = x \cos(x^{3/2})$$