3

Newtonian Mechanics

Seat belts and air bags save about 250,000 lives worldwide every year because they significantly reduce the risk of injury (belts alone by about 40% and belts with air bags by about 54%). How do seat belts and air bags provide this protection?

- How do seat belts and air bags save lives?
- If you stand on a bathroom scale in a moving elevator, does its reading change?
- Can a parachutist survive a fall if the parachute does not open?

IN THE LAST CHAPTER, we learned to *describe* motion—for example, to determine a car's acceleration when it stops abruptly during a collision. However, we did not discuss the causes of the acceleration. In this chapter, we will learn *why* an object has a particular acceleration. This knowledge will help us *explain* the motion of many objects: cars, car passengers, elevators, skydivers, and even rockets.

BE SURE YOU KNOW HOW TO:

- Draw a motion diagram for a moving object (Section 2.2).
- Determine the direction of acceleration using a motion diagram (Section 2.7).
- Add vectors graphically and determine their components (Sections 2.3, 2.4, and 2.6).

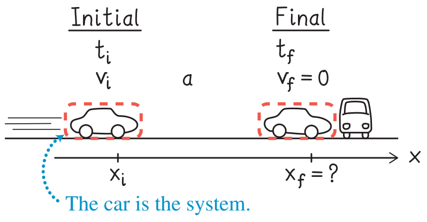
3.1 Describing and representing interactions

What causes objects to accelerate or maintain a constant velocity? Consider a simple experiment—standing on Rollerblades on a horizontal floor. No matter how hard you swing your arms or legs you cannot start moving by yourself; you need to either push off the floor or have someone push or pull you. Physicists say that the floor or the other person *interacts* with you, thus changing your motion. Objects can interact directly, when they touch each other, or at a distance, as when a magnet attracts or repels another magnet without touching it.

Choosing a system in a sketch of a process

We learned (in Chapter 2) that the first step in analyzing any process is sketching it. **Figure 3.1** shows a sketch of a car skidding to avoid a collision with a van. In this and later chapters we choose one particular object for detailed analysis. We call this object the **system**. All other objects that are not part of the system can interact with it (touch it, pull it, or push it) and are in the system's **environment**. Interactions between the system and objects in the environment are called **external interactions**.

FIGURE 3.1 A sketch of a car skidding to avoid a collision with a van.



The car is the system.

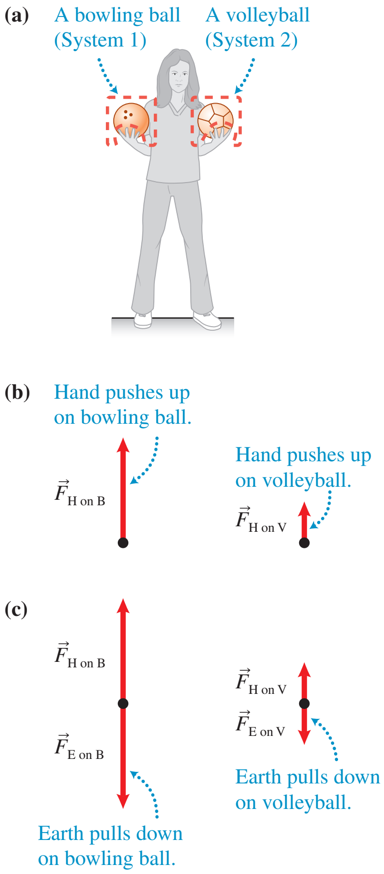
System A system is the object or group of objects that we choose to analyze. Everything outside that system is called its environment and consists of objects that might interact with the system (touch, push, or pull it) and affect its motion through external interactions.

On the sketch, we draw a boundary around the system to emphasize the system choice (shown by the red dashed line around the skidding car in Figure 3.1). Sometimes, a single system object has parts—like the wheels on the car and its axles. The parts interact with each other. Since both parts are in the system, these are called **internal interactions**. In this chapter we will model an object like a car as a point-like object and ignore such internal interactions.

Representing interactions

External interactions affect the motion or lack of motion of a system. Consider holding a bowling ball in one hand and a volleyball in the other. Each ball is considered as a system (**Figure 3.2a**). What objects interact with each ball? Your hand pushes up hard to keep the bowling ball steady and much less hard to keep the volleyball steady (Figure 3.2b). We use an arrow to represent the upward push exerted by each hand on one of the balls. Notice that the arrow for the interaction of the hand with the bowling ball is longer than that for the hand with the volleyball.

FIGURE 3.2 Representing external interactions (forces exerted on a system).



A bowling ball (System 1)

A volleyball (System 2)

Hand pushes up on bowling ball.

Hand pushes up on volleyball.

Earth pulls down on volleyball.

Earth pulls down on bowling ball.

The arrow represents a "force" that is exerted by the hand on the ball. **A force is a physical quantity that characterizes how hard and in what direction an external object pushes or pulls on the system.** The symbol for force is $\vec{F}$ with a subscript that identifies both the external object that exerts the force and the system on which the force is exerted. For example, the hand pushing on the bowling ball is represented as $\vec{F}_{\text{H on B}}$. The force that the hand exerts on the volleyball is $\vec{F}_{\text{H on V}}$. The arrow above the symbol indicates that force is a vector quantity with a magnitude *and* direction. The SI unit of a force is the *newton* (N). When you hold a 100-g ball, you exert an upward force that is a little less than 1.0 N.

Do any other objects exert forces on the balls? Intuitively, we know that something must be pulling down to balance the upward force your hands exert on the balls. The concept of **gravity** represents the interaction of planet Earth with the ball. Earth pulls downward on an object toward Earth's center. Because of this interaction we need to include a second arrow representing the force that Earth exerts on the ball $\vec{F}_{\text{E on B}}$ (see Figure 3.2c). Intuitively, we know that the two arrows for each ball should be of the same length, since the balls are not moving.

Do other objects besides the hands and Earth interact with the balls? Air surrounds everything close to Earth. Does it push down or up on the balls? Let's hypothesize that air pushes down. If so, then our hands have to push up harder on the balls to balance the combined effect of the downward push of the air and the downward pull of Earth. Let's test the hypothesis that air pushes down on the balls.

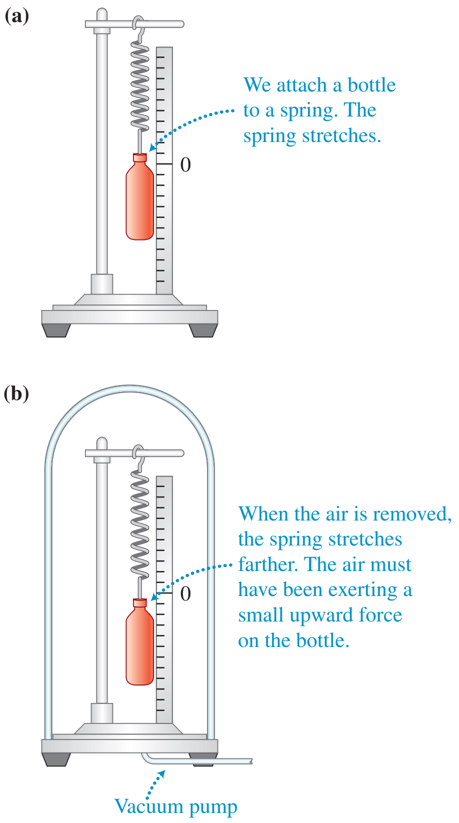
Testing a hypothesis

To test a hypothesis in science means to first accept it as a true statement (even if we disagree with it); then design an experiment whose outcome we can predict using this hypothesis (a testing experiment); then compare the outcome of the experiment and the prediction; and, finally, make a preliminary judgment about the hypothesis. If the outcome matches the prediction, we can say that the hypothesis has not been disproved by the experiment. When this happens, our confidence in the hypothesis increases. If the outcome and prediction are inconsistent, we need to reconsider the hypothesis and possibly reject it.

To test the hypothesis that air exerts a downward force on objects, we attach an empty closed water bottle to a spring and let it hang; the spring stretches (**Figure 3.3a**). Next we place the bottle and spring inside a large jar that is connected to a vacuum pump and pump the air out of the inside of the jar. We predict that *if* the air inside the jar pushes down on the bottle (the hypothesis), *then* when we pump the air out of the jar, it should be easier to support the bottle—the spring should stretch less (the prediction that follows from the hypothesis).

When we do the experiment, the outcome does not match the prediction—the spring actually stretches *slightly more* when the air is pumped out of the jar (Figure 3.3b). Evidently the air does not push down on the bottle; instead, it helps support the bottle by exerting an upward force on it. This outcome is surprising. If you study fluids, you will learn the mechanism by which air pushes up on objects.

FIGURE 3.3 A testing experiment to determine the effect of air on the bottle.



Reflect

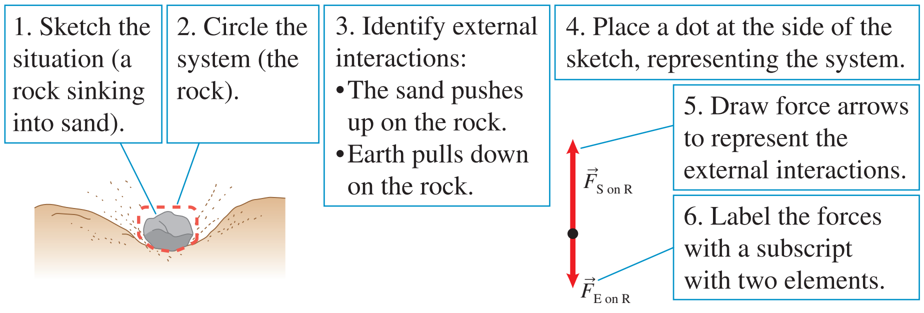
Let's reflect on what we have done here. We formulated an initial hypothesis—air *pushes down* on objects. Then we designed an experiment whose outcome we could predict using the hypothesis—the bottle on a spring in a vacuum jar. We used the hypothesis to make a prediction of the outcome of the testing experiment—the spring should stretch less in a vacuum. We then performed the experiment and found that something completely different happened. We revised our hypothesis—air *pushes up slightly* on objects. Note that air's upward push on the bottle is very small. Therefore, in many situations, and in all situations in this chapter, the effects of air can be ignored.

Drawing force diagrams

A **force diagram** (sometimes called a *free-body diagram*) represents the forces that objects in a system's environment exert on it (see Figure 3.2c). We represent the system by a dot to show that we model it as a point-like object. Arrows represent the forces. Unlike a motion diagram, a force diagram does not show us how a process changes with time; it shows us only the forces at a single instant. For processes in which no motion occurs, this makes no difference. But when motion does occur, we need to know if the force diagram is changing as the system moves.

Consider a rock dropped from above and sinking into sand, making a small crater. We construct a force diagram for the instant shortly after the rock touches the sand but before it completely stops moving.

PHYSICS TOOL BOX 3.1 Constructing a force diagram


1. Sketch the situation (a rock sinking into sand).
2. Circle the system (the rock).
3. Identify external interactions:
- The sand pushes up on the rock.
- Earth pulls down on the rock.
4. Place a dot at the side of the sketch, representing the system.
5. Draw force arrows to represent the external interactions.
6. Label the forces with a subscript with two elements.

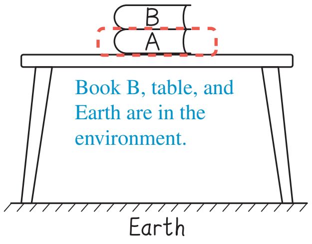
TIP Remember that on the force diagram, you only draw forces exerted *on* the system. Do not draw forces that the system exerts on other objects! For example, the rock exerts a force on the sand, but we do not include this force in the force diagram since the sand is not part of the system.

Notice that the upward-pointing arrow representing the force exerted by the sand on the rock is longer than the downward-pointing arrow representing the force exerted by Earth on the rock. The difference in lengths reflects the difference in the magnitudes of the forces.

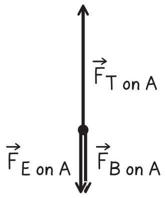
CONCEPTUAL EXERCISE 3.1 Force diagram for a book

Book A sits on a table with book B on top of it. Construct a force diagram for book A.

Sketch and translate We sketch the situation below. We choose book A as the system. Notice that the dashed line around book A passes between the table and book A, and between book B and book A. It's important to be precise in the way you draw this line so that the separation between the system and the environment is clear. In this example, Earth, the table, and book B are external environmental objects that exert forces on book A.

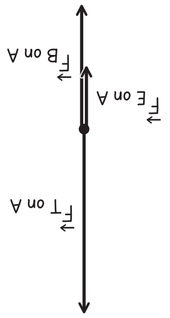


Simplify and diagram Draw a force diagram for book A, which is represented by a dot. Two objects in the environment touch book A. The table pushes up on the bottom surface of the book, exerting a force $\vec{F}_{\text{T on A}}$, and book B pushes down on the top surface of book A, exerting force $\vec{F}_{\text{B on A}}$. In addition, Earth exerts a downward force on book A $\vec{F}_{\text{E on A}}$. Intuitively, we feel that these forces should somehow cancel each other. The diagram shows this by carefully representing the length of the forces. Note that if you add the two downward forces, their combined length is the same as the upward force. Below we show how to find a resultant force vector when several forces are exerted on an object.



Try it yourself Construct a force diagram for book A after another book C is placed on top of book B.

Answer The same three objects interact with book A. Earth exerts the same downward force on book A ($\vec{F}_{\text{E on A}}$). C does not directly touch A and exerts no force on A. However, C does push down on B, so B exerts a greater force on A ($\vec{F}_{\text{B on A}}$). Because the downward force of B on A is greater, the table exerts a greater upward force on book A ($\vec{F}_{\text{T on A}}$).



Contact and noncontact forces

You can see in the above Conceptual Exercise that some of the forces exerted on book A are exerted by the objects directly touching the book—the table and book B. But Earth does not need to touch the book to exert a force on it. The force that Earth as a whole planet exerts on every object is an example of a gravitational force. Forces that require the interacting objects to be touching are called **contact forces**, while forces that do not require touching are called **noncontact forces**. The force a rope pulling on a crate exerts (sometimes called a *tension force*) is another example of a contact force; the force of a magnet on another magnet (a magnetic force) is another example of a noncontact force.

TIP When we write Earth with a capital E and no "the" before it, it means that we are talking about the whole planet, not the surface.

REVIEW QUESTION 3.1 How do we determine how many forces to draw on a force diagram?

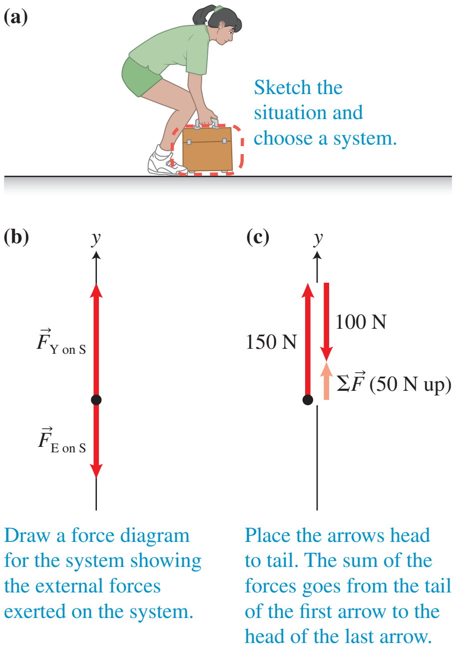
3.2 Adding and measuring forces

Usually, more than one environmental object exerts a force on a system. How can we add them to find the total force? In this chapter we will restrict our attention to forces that are exerted and point along one axis. Consider the process of lifting a suitcase.

Adding force vectors graphically

You lift a suitcase straight up (**Figure 3.4a**). Earth pulls down on the suitcase, exerting a force of magnitude $F_{\text{E on S}} = 100\text{ N}$, and you exert an upward force of magnitude $F_{\text{Y on S}} = 150\text{ N}$ (Figure 3.4b). What are the magnitude and direction of the total force exerted on the suitcase? The net effect of the two forces exerted along a vertical axis is the same as a 50-N force pointed straight up. Why? Remember that force is a vector. (We discussed the rules for vector addition in Chapter 2.) To add two vectors to find $\Sigma\vec{F} = \vec{F}_{\text{E on S}} + \vec{F}_{\text{Y on S}}$, we place them head to tail (see Figure 3.4c) and draw the vector that goes from the tail of the first vector to the head of the second vector. This new vector is the sum vector, or the resultant vector. In the case of forces it is called the **net force**.

FIGURE 3.4 The sum of the forces (the net force) exerted on the suitcase.



Sketch the situation and choose a system.

Draw a force diagram for the system showing the external forces exerted on the system.

Place the arrows head to tail. The sum of the forces goes from the tail of the first arrow to the head of the last arrow.

To simplify operations with vectors, we will use components in our calculations. In this chapter we will only analyze situations when all forces are exerted along one axis (x or y) or when forces along one direction add to zero and all the remaining forces are along the other (perpendicular) direction. To find the component of a force along a particular direction, we first need to identify the positive direction, then compare the direction of the force with the identified positive direction. Forces that point in the positive direction of an axis have components equal to their magnitudes; forces that point in the negative direction have components equal to their magnitudes with a negative sign (see Sections 2.4 and 2.6 to review components of a vector). In our example of lifting a suitcase, the y-axis points up. The y-component of the force that you exert on the suitcase is positive and equal to $F_{\text{Y on S}\,y} = 150\text{ N}$, and the y-component of the force that Earth exerts on the suitcase is $F_{\text{E on S}\,y} = -100\text{ N}$. The sum of the y-components is $\Sigma F_y = 150\text{ N} + (-100) = 50\text{ N}$. This is the y-component of the sum of the forces if the y-axis points up. If the positive direction is down, the sum of the forces has a y-component $\Sigma F_y = -50\text{ N}$.

If several external objects in the environment exert forces on the system, we still use vector addition to find the *sum* of the forces exerted on the system:

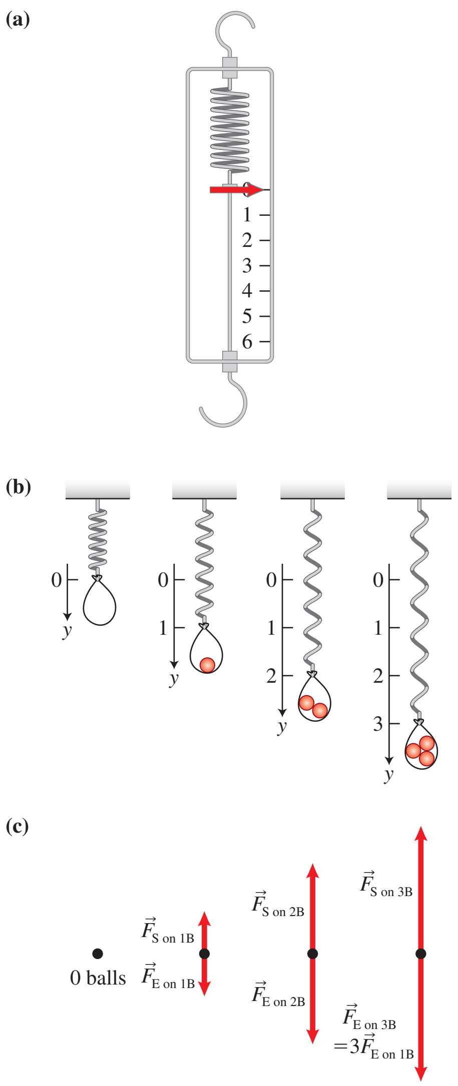
$$\Sigma\vec{F}_{\text{on S}} = \vec{F}_{1\text{ on S}} + \vec{F}_{2\text{ on S}} + \cdots + \vec{F}_{n\text{ on S}} \tag{3.1}$$

TIP The sum of the force vectors is not a new force being exerted. Rather, it is the combined effect of all the forces being exerted on the system. Because of this, the resultant vector should never be included in the force diagram for that system.

Measuring force magnitudes

The simplest device to measure forces is a spring scale (**Figure 3.5a**, on the next page); we will demonstrate this with a simple experiment. Imagine that a light plastic bag hangs from a light spring. The spring is not stretched. We then place one golf ball into the bag and observe that the spring stretches to a new length. We add a second ball and

FIGURE 3.5 Using a spring scale to measure forces.



observe that the spring stretches twice as far. We add a third ball and observe that the spring stretches three times as far. Figure 3.5b shows the four situations; for each one, notice the change in the length of the spring. You can see that the spring stretches the same amount for each ball, and that it stretches three times more for three balls than for one ball.

Next, we choose the bag with the golf balls as the system and analyze the forces exerted in each case. We assume that Earth exerts the same force on each ball $\vec{F}_{\text{E on 1B}}$ that is independent of the presence of other balls. Thus, the total force exerted by Earth on the three-ball system is three times greater than the force exerted on the one-ball system. Figure 3.5c shows force diagrams for each case. Because the bag does not fall, the spring in each case must exert a force on the system $\vec{F}_{\text{S on \#B}}$ that is equal in magnitude and opposite in direction to the force that Earth exerts on the system $\vec{F}_{\text{E on \#B}}$ so that the sum of the forces exerted on the system with a number (#) of golf balls is zero.

This experiment provides us with one method to measure an unknown force that an object exerts on a system. We calibrate a spring in terms of some standard force, such as Earth's pull on one or more golf balls. Then if some unknown force is exerted on a system, we can use the spring to exert a balancing force on that system. The unknown force is equal in magnitude and opposite in direction to the force exerted by the spring. In this case, we would be measuring force in units equal to Earth's pull on a golf ball. We could use any spring to balance a known standard force (1 N, or approximately the force that Earth exerts on a 100-g object) and then calibrate this spring in newtons by placing marks at equal distances as you pull on its end with increasing force. We thus build a spring scale—the simplest instrument to measure forces.

TIP In physics, force is a physical quantity that characterizes the direction and magnitude of an interaction between two objects. **For a force to exist, there must be two objects that interact,** just as a hug requires the interaction of two people. Force does not reside in an object. However, the idea that force resides in an object remains very strong; people say, "The truck's force caused a lot of damage to the telephone pole." We will be careful in this book to always identify the two interacting objects when speaking about any force. Remember, if you are considering a force that is exerted on a moving object but cannot find another object that interacts with it, then you are thinking of something else, not force.

REVIEW QUESTION 3.2 A book bag hanging from a spring scale is partially supported by a platform scale. The platform scale reads about 36 N and the spring scale reads about 28 N. Draw a sketch of the situation, construct a force diagram, and use them to find the magnitude of the force that Earth exerts on the bag.

3.3 Conceptual relationship between force and motion

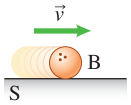
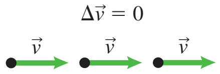
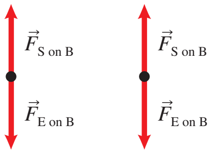
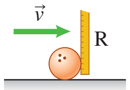
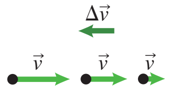
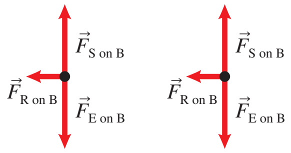
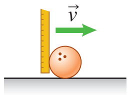
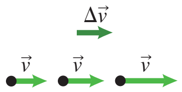
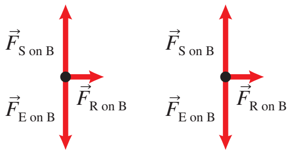
When we drew a force diagram for a ball held by a person, we intuitively drew the forces exerted on the ball as being equal in magnitude. What if the person catches a ball falling from above? Would she still need to exert a force on the ball that has a magnitude equal to that Earth exerts on the ball? In other words, is there a relationship between the forces that are exerted on an object and the way the object moves?

Observational Experiment **Table 3.1** helps us find out whether there is a pattern between the motion diagram and the force diagram for a moving object.

OBSERVATIONAL EXPERIMENT TABLE 3.1

How are motion and forces related?

VIDEO OET 3.1

Observational experiment	Analysis	
	Motion diagram	**Force diagrams for first and third positions**
Experiment 1. A bowling ball B rolls on a very hard, smooth surface S without slowing down. 		
Experiment 2. A ruler R lightly pushes the rolling bowling ball opposite the ball's direction of motion. The ball continues to move in the same direction, but slows down. 		
Experiment 3. A ruler R lightly pushes the rolling bowling ball in the direction of its motion. The ball speeds up. 		

Pattern

- In all the experiments, the vertical forces add to zero and cancel each other. We consider only forces exerted on the ball in the horizontal direction.
- In the first experiment, the sum of the forces exerted on the ball is zero; the ball's velocity remains constant.
- In the second and third experiments, when the ruler pushes the ball, the velocity change arrow ($\Delta\vec{v}$ arrow) points in the same direction as the sum of the forces.

Summary: The $\Delta\vec{v}$ arrow in all experiments is in the same direction as the sum of the forces. Notice that there is no pattern relating the *direction* of the velocity $\vec{v}$ to the direction of the sum of the forces. In Experiment 2, the velocity and the sum of the forces are in opposite directions, but in Experiment 3, they are in the same direction.

In each of the experiments in Table 3.1, the $\Delta\vec{v}$ arrow for a system and the sum of the forces $\Sigma\vec{F}$ that external objects exert on the system are in the same direction. In addition, we often observe that the $\vec{v}$ arrow for a system (the direction the object is moving) is in the same direction as the sum of the forces exerted on it. For example, a grocery cart moves in the direction the shopper pushes it and a soccer ball moves in the direction the player kicks it. We should test both ideas.

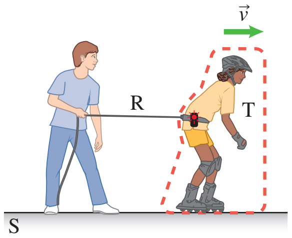
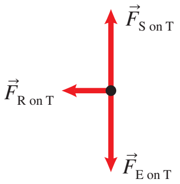
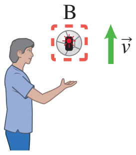
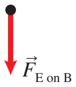
Testing possible relationships between force and motion

We have two possible ideas that relate motion and force:

1. An object's *velocity* $\vec{v}$ always points in the direction of the sum of the forces $\Sigma\vec{F}$ that other objects exert on it.
2. An object's *velocity change* $\Delta\vec{v}$ always points in the direction of the sum of the forces $\Sigma\vec{F}$ that other objects exert on it.

To test these two relationships, we use each to predict the outcome of the experiments in Testing Experiment Table 3.2. Then we perform the experiments and compare the outcomes with the predictions. From this comparison, we determine whether we can reject one or both of the relationships.

TESTING EXPERIMENT TABLE 3.2 Testing velocity and the sum of the forces $\Sigma\vec{F}$

Testing experiment	Prediction	Outcome
Experiment 1. Tanya, wearing a blinking LED on her belt, coasts to the right on Rollerblades. Her friend pulls her back lightly on a rope. If you took a long-exposure photo of Tanya moving, predict what the LED traces will look like after her friend starts pulling the rope.  	**Prediction based on idea 1:** The sum of the forces exerted on Tanya points to the left. Thus Tanya's velocity should immediately change from right to left. The LED traces should reverse direction the moment her friend starts pulling back.  **Prediction based on idea 2:** Because the sum of the forces points toward the left, the $\Delta\vec{v}$ arrow should point left. Tanya should continue moving to the right, slowing down until she stops. The traces should continue in the same direction as before, only they should get shorter and shorter.	Tanya slows down and eventually stops. The traces are shown below. 
Experiment 2. You throw a ball (with an attached LED) upward. If you took a long-exposure photo of the moving ball, predict what the LED traces will look like.  	**Prediction based on idea 1:** The only force exerted on the ball in flight points down. Thus the ball should immediately begin moving downward after you release it. The LED traces should not appear above the height at which the ball was released.  **Prediction based on idea 2:** Because the only force exerted on the ball in flight points down, the $\Delta\vec{v}$ arrow should point down, too. The ball should slow down until it stops, and then it will start speeding up moving down. The traces continue upward but will become shorter until the ball stops. Then they increase in length in the opposite direction.	The ball moves up at decreasing speed and then reverses direction and starts moving downward. The LED traces are shown at right. 

Conclusion

- All outcomes contradict the predictions based on idea 1—we can reject it.
- All outcomes are consistent with the predictions based on idea 2. This does not necessarily mean it is true, but it does mean our confidence in the idea increases.

Recall that the $\Delta\vec{v}$ arrow in a motion diagram is in the same direction as the object's acceleration $\vec{a}$. Thus, based on this idea and these testing experiments, we can now accept idea 2 with greater confidence.

Relating forces and motion The velocity change ($\Delta\vec{v}$) arrow in an object's motion diagram (and its acceleration $\vec{a}$) point in the same direction as the sum of the forces that other objects exert on it. If the sum of the forces points in the same direction as the object's velocity, the object speeds up; if the sum points in the opposite direction, it slows down. If the sum of the forces is zero, the object continues with no change in velocity.

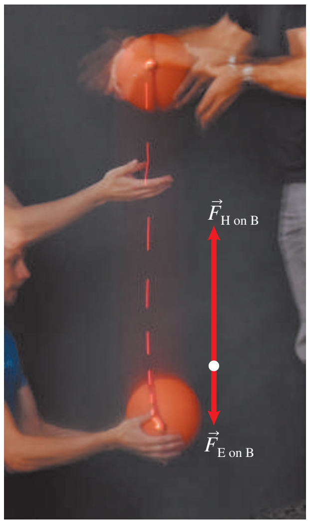


FIGURE 3.6 Force diagram for a ball being caught.

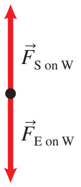
Reflect

How did we devise a relationship that allows us to explain why objects slow down, speed up, or continue at constant velocity? We first observed simple experiments and analyzed them with motion diagrams and force diagrams. We then tested two possible relationships between the objects' motion and the sum of all forces that other objects exerted on it. The above relationship emerged from this analysis and testing.

The relationship between forces and change in motion also applies to the situation of catching a dropped ball. **Figure 3.6** shows a ball with an attached blinking LED being dropped and caught and the corresponding force diagram for one instant during the catch.

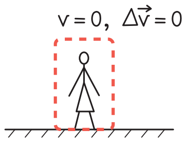
CONCEPTUAL EXERCISE 3.2 **Diagram Jeopardy**

The force diagram shown here describes the forces that external objects (the surface and Earth) exert on a woman (in this scenario, the force diagram does not change with time). Describe three different types of motion of the woman that are consistent with the force diagram.

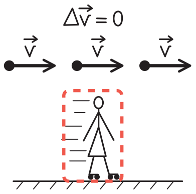


Sketch and translate Two equal-magnitude, oppositely directed forces are being exerted on the woman ($\Sigma\vec{F} = 0$). Thus, a motion diagram for the woman must have a zero velocity change ($\Delta\vec{v} = 0$).

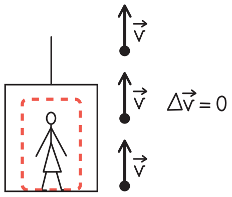
Simplify and diagram Three possible motions consistent with this idea are shown below.



1. She stands at rest on a horizontal surface.



2. She glides at constant velocity on Rollerblades on a smooth horizontal surface.



3. She stands on the floor of an elevator that moves up or down at constant velocity.

Note that in all three of the above, the velocity change arrow is zero. This is consistent with the sum of the forces being zero.

Try it yourself Suppose that the elevator described above was moving up at decreasing speed instead of at constant speed. How then would the force diagram be different?

Answer A velocity change ($\Delta\vec{v}$) arrow for the woman's motion would now point down, opposite the direction of her velocity. Thus, the sum of the forces $\Sigma\vec{F}$ that other objects exert on her must also point down. This means that the magnitude of the upward force $\vec{F}_{\text{S on W}}$ that the elevator floor (surface) exerts on her must now be less than the magnitude of the downward force $\vec{F}_{\text{E on W}}$ that Earth exerts on her ($F_{\text{S on W}} < F_{\text{E on W}}$).

REVIEW QUESTION 3.3 An elevator in a tall office building moves downward at constant speed. How does the magnitude of the upward force exerted by the cable on the elevator $\vec{F}_{\text{C on El}}$ compare to the magnitude of the downward force exerted by Earth on the elevator $\vec{F}_{\text{E on El}}$? Explain your reasoning.

3.4 Inertial reference frames and Newton's first law

Our description of the motion of an object depends on the observer's reference frame. However, in this chapter we have tacitly assumed that all observers were standing on Earth's surface. Are there any observers who will see a chosen object moving with changing velocity even though the sum of the forces exerted on the object appears to be zero?

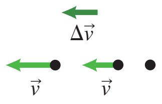
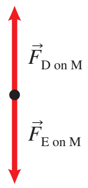
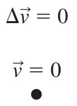
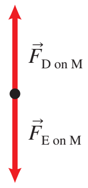
Inertial reference frames

In **Table 3.3**, we consider two different observers analyzing the same situation.

OBSERVATIONAL EXPERIMENT TABLE 3.3

Two observers watch the same coffee mug

VIDEO OET 3.3

Observational experiment	Analysis done by each observer
Experiment 1. Observer 1 is slouched down in the passenger seat of a car and cannot see outside the car. Suddenly, he observes a coffee mug (M) sliding toward him from the dashboard (D).	Observer 1 creates a motion diagram and a force diagram for the mug as he observes it. On the motion diagram, increasingly longer $\vec{v}$ arrows indicate that the mug's speed changes from zero to nonzero as seen by observer 1 even though no external object is exerting a force on it in that direction.  
Experiment 2. Observer 2 stands on the ground beside the car. She observes that the car starts moving forward at increasing speed and that the mug remains stationary with respect to her.	Observer 2 creates a motion diagram and force diagram for the mug as she observes it. There are no $\vec{v}$ or $\Delta\vec{v}$ arrows on the diagram, and the mug is at rest relative to her.  

Pattern

Observer 1: The forces exerted on the mug by Earth and by the dashboard surface add to zero. But the velocity of the mug increases as it slides off the dashboard. This is inconsistent with the rule relating the sum of the forces and the change in velocity.
Observer 2: The forces exerted on the mug by Earth and by the dashboard surface add to zero. Thus the velocity of the mug should not change, and it does not. This is consistent with the rule relating the sum of the forces and the change in velocity.

VIDEO 3.1

Observer 2 in Table 3.3 can account for what is happening using the rule relating the sum of the forces and changing velocity, but observer 1 cannot. For observer 1, the mug's velocity changes for no apparent reason.

Similarly, in the video in the margin, you see the balls fly inside the box for no reason. For the observer inside the box, Newton's laws cannot explain their behavior. Can you think of some other observers who would be able to explain the balls' motion?

It appears that the applicability of the relationship between the force and motion diagram depends on the reference frame of the observer. Observers who *can* explain the behavior of the mug (observer 2) and the balls by using the relationship between the sum of the forces and changing velocity are said to be observers in **inertial reference frames**. Those who *cannot* explain the behavior of the mug (observer 1) and the balls using this relationship are said to be observers in **noninertial reference frames**. Any observer who accelerates with respect to Earth is a noninertial reference frame observer.

Observers in inertial reference frames can explain the changes in velocity of objects by considering the forces exerted on them by other objects. Observers in noninertial reference frames cannot. From now on, we will always analyze phenomena from the point of view of observers in inertial reference frames. This idea is summarized by Isaac Newton's first law.

Newton's first law of motion For an observer in an inertial reference frame, when no other objects exert forces on a system or when the forces exerted on the system add to zero, the system continues moving at constant velocity (including remaining at rest).

Isaac Newton. Isaac Newton (1643–1727) invented differential and integral calculus, formulated the law of universal gravitation, developed a new theory of light, and put together the ideas for his three laws of motion.

TIP It is important to note that in physics some mathematical terms and symbols have several different meanings. Zero could mean a balance or an absence. For example, when we say that two forces add to zero, we mean that they balance the effects of each other; when we say an object moves with zero acceleration, we mean that the acceleration is absent. A negative sign in front of a quantity could mean direction or subtraction. Every time you meet a zero or a negative sign, stop and determine how it is being used in that equation.

Physicists have analyzed the motion of thousands of objects from the point of view of observers in inertial reference frames and found no contradictions to the rule. Newton's first law of motion limits the reference frames with respect to which the other laws that you will learn in this chapter are valid—these other laws work only for the observers in inertial reference frames. In this and following chapters we will assume that observers of events are not accelerating.

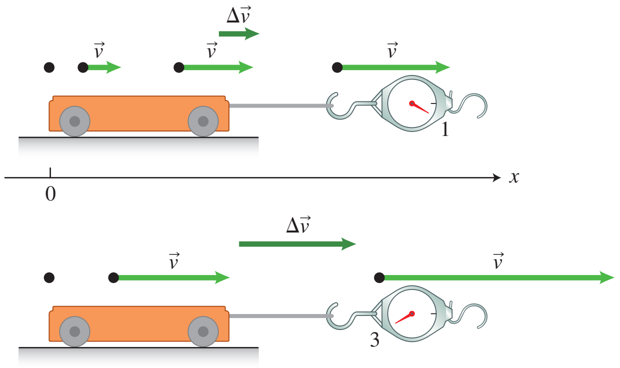
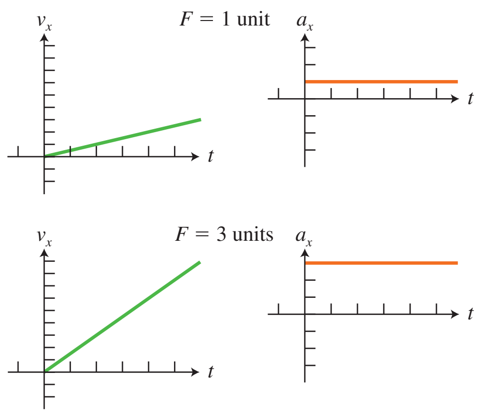
REVIEW QUESTION 3.4 What is the main difference between inertial and noninertial reference frames? Give an example.

3.5 Newton's second law

Our conceptual analyses in Section 3.3 allowed us to deduce a qualitative relationship between forces and changes in an object's motion. In this section we will learn how to predict the magnitude of an object's acceleration if we know the forces exerted on it. The experiments in Observational Experiment **Table 3.4** will help us construct this quantitative relationship.

OBSERVATIONAL EXPERIMENT TABLE 3.4 Forces and resulting acceleration

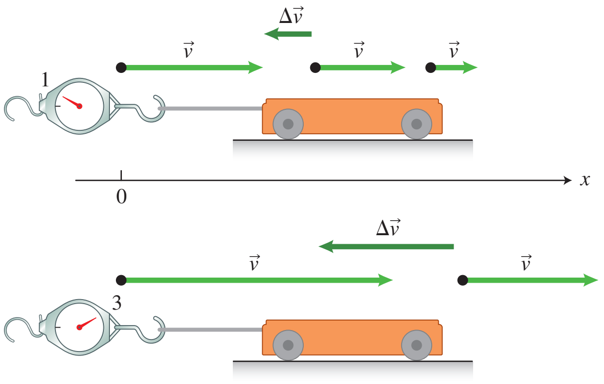
VIDEO OET 3.4

Observational experiment	Analysis
Experiment 1. A cart starts at rest on a smooth horizontal track. A spring scale continuously exerts one unit of force in the positive direction. The experiment is repeated four more times. Each time, the force probe exerts one additional unit of force on the cart (up to three units). We record the value of the force, and use a motion detector on the track to record the cart's speed and acceleration. 	Using this information, we create velocity-versus-time and acceleration-versus-time graphs for two of the five different magnitudes of force. Note that the greater the force, the greater the acceleration. 

(CONTINUED)

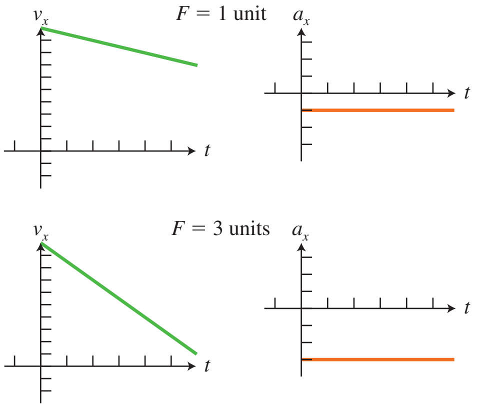
Observational experiment

Experiment 2. We repeat the same five experiments, only this time the cart is moving in the positive direction, and the probe pulls back on the cart in the negative direction so that the cart slows down.



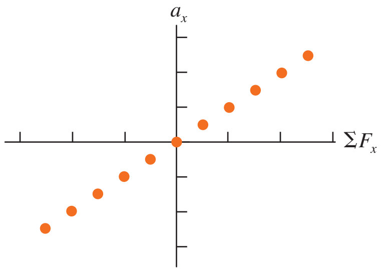
Analysis

We create velocity-versus-time and acceleration-versus-time graphs for the cart when forces of two different magnitudes oppose the cart's motion.



Pattern

- When the sum of the forces exerted on the cart is constant, its acceleration is constant—the cart's speed increases at a constant rate.
- When we plot acceleration versus force using the five positive and five negative values of the force, we obtain the graph at the right. The eleventh point is (0,0), which we know from previous experiments.
- The acceleration is directly proportional to the force exerted by the spring scale (in this case, it is the sum of all forces) and points in the direction of the force.



The outcome of these experiments expressed mathematically is as follows:

$$\vec{a} \propto \Sigma\vec{F} \tag{3.2}$$

where $\Sigma\vec{F}$ is the sum of all the forces that other objects exert on the system (not an additional force), and $\vec{a}$ is the system's acceleration. The symbol $\propto$ means "is proportional to." In other words, if the sum of the forces doubles, then the acceleration doubles. When the sum of the forces is zero, the acceleration is zero. When the sum of the forces exerted on an object is constant, the object's resulting acceleration (not velocity) is constant.

Mass, another physical quantity

Do other physical quantities affect acceleration? You know from experience that it is easy to pull an empty cardboard box across the floor, but it is much harder when it is full of books. The amount of matter being pulled must affect the acceleration.

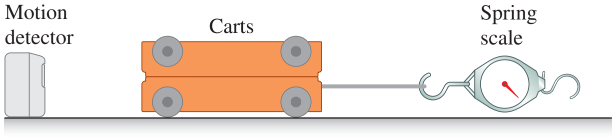
Let's perform another experiment to find the quantitative effect of the amount of *matter* being pulled. We use a spring scale to pull one cart, then two carts stacked on top of each other, and then three and four carts on top of each other. In each case, the spring scale exerts the same force on the carts, regardless of how many carts are being pulled. The experiment is summarized in Observational Experiment **Table 3.5**.

OBSERVATIONAL EXPERIMENT TABLE 3.5 Amount of matter and acceleration

Observational experiment

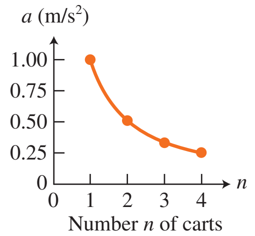
We pull the indicated number of stacked carts using an identical pulling force and measure the acceleration with a motion detector.

Number n of carts	Acceleration (m/s^2)
1	1.00
2	0.49
3	0.34
4	0.25

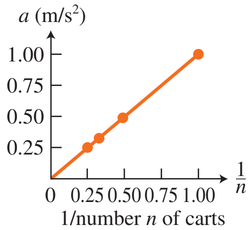


Analysis

We graph the acceleration versus number of carts for constant pulling force.



From the graph, we see that increasing the number of carts decreases the acceleration. To check whether this relationship is inversely proportional, we plot a versus $\frac{1}{n}$.



Pattern

Since the graph a versus $\frac{1}{n}$ is a straight line, we conclude that a is inversely proportional to n, which we write as $a \propto \frac{1}{n}$.

From the pattern observed in Table 3.5, we conclude that *the greater the amount of matter being pulled, the smaller the object's acceleration when the same force is exerted on it.* This property of an object, which affects its acceleration, is called **mass**.

To measure the mass of an object quantitatively, we first define a standard unit of mass. The choice for the unit of mass is arbitrary, but after the unit has been chosen, the masses of all other objects can be determined from this unit. The SI standard of mass is the kilogram (kg). A quart of milk has a mass of about 1 kg. Suppose, for example, that you exert a constant pulling force on a 1.0-kg object (and that all other forces exerted on this object add to zero), and you measure its acceleration. You then exert the same pulling force on another object of unknown mass. Your measurement indicates that it has half the acceleration of the standard 1.0-kg object. Thus, its mass is twice the standard mass (2.0 kg). This method is not practical for everyday use. Later we will learn another method of measuring the mass of an object, a method that is simple enough to use in everyday life.

Our experiments indicate that when the same force is exerted on two objects, the one with the greater mass will have a smaller acceleration. Mathematically:

$$a \propto \frac{1}{m} \qquad (3.3)$$

Mass Mass m characterizes the amount of matter in an object. When the same unbalanced force is exerted on two objects, the object with greater mass has a smaller acceleration. The SI unit of mass is the kilogram (kg). Mass is a scalar quantity, and masses add as scalars.

Sum of the forces, mass and acceleration

We have found that the acceleration $\vec{a}$ of a system is proportional to the vector sum of the forces $\Sigma\vec{F}$ exerted on it by other objects [Eq. (3.2)] and inversely proportional to the mass m of the system [Eq. (3.3)]. We can combine these two proportionalities into a single equation on the next page.

$$\vec{a}_{\text{S}} \propto \frac{\Sigma \vec{F}_{\text{on S}}}{m_{\text{S}}} \qquad (3.4)$$

Rearrange the above to get $m_{\text{S}}\vec{a}_{\text{S}} \propto \Sigma \vec{F}_{\text{on S}}$. We can turn this into an equation if we choose the unit of force to be $\text{kg} \cdot \text{m/s}^2$. Because force is such a ubiquitous quantity, physicists have given the force unit a special name called a **newton** (N). A force of 1 newton (1 N) causes an object with a mass of 1 kg to accelerate at $1\ \text{m/s}^2$.

$$1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2 \qquad (3.5)$$

TIP A force of one pound can also be defined (1 lb equals 4.45 N). When the sum of the forces exerted on an object of a unit of mass (called a slug in the Imperial system of units) is equal to 1 lb, the acceleration of the object is 1 foot/s^2. A slug is a relatively massive unit: 1 slug = 14.95 kg.

Equation (3.4), rewritten with an equality sign instead of a proportional sign, is called **Newton's second law**.

Newton's second law The acceleration $\vec{a}_{\text{S}}$ of a system is proportional to the vector sum of all forces being exerted on the system and inversely proportional to the mass m of the system:

$$\vec{a}_{\text{S}} = \frac{\Sigma \vec{F}_{\text{on S}}}{m_{\text{S}}} = \frac{\vec{F}_{1\ \text{on S}} + \vec{F}_{2\ \text{on S}} + \cdots}{m_{\text{S}}} \qquad (3.6)$$

Here the subscripts 1 and 2 stand for the objects exerting forces on the system.

The acceleration of the system points in the same direction as the vector sum of the forces.

TIP Notice that the "vector sum of the forces" mentioned in the definition at right does not mean the sum of their magnitudes. Vectors are not added as numbers; their directions affect the magnitude of the vector sum.

Does this new equation make sense? For example, does it work in extreme cases? First, imagine an object with an infinitely large mass. According to the law, it will have zero acceleration for any process in which the sum of the forces exerted on it is finite:

$$\vec{a}_{\text{S}} = \frac{\Sigma \vec{F}_{\text{on S}}}{\infty} = 0$$

This seems reasonable, as an infinitely massive object would not change motion due to finite forces exerted on it. On the other hand, an object with a zero mass will have an infinitely large acceleration when a finite magnitude force is exerted on it:

$$\vec{a}_{\text{S}} = \frac{\Sigma \vec{F}_{\text{on S}}}{0} = \infty$$

Both extreme cases make sense. Newton's second law is a so-called *cause-effect relationship*. The right side of the equation (the sum of the forces being exerted divided by the mass of the system) is the cause of the effect (the acceleration) on the left side.

$$\text{Effect} \rightarrow \vec{a}_{\text{S}} = \frac{\Sigma \vec{F}_{\text{on S}}}{m_{\text{S}}} \leftarrow \text{Cause}$$

TIP In the cause-effect relationship, the quantities on the right side of the equation can be varied independently. In our case, these are the sum of the forces and the mass. Each can be changed independently of the other to affect acceleration. In the operational definition, the quantities on the right side of the equation cannot be varied independently. The velocity change depends on the time interval during which it occurs.

On the other hand, $\vec{a} = \Delta\vec{v}/\Delta t$ is called an *operational definition* of acceleration. It tells us how to determine the quantity acceleration but does not tell us *why* it has a particular value. For example, suppose that an elevator's speed changes from 2 m/s to 5 m/s in 3 s as it moves vertically along a straight line in the positive y-direction. The elevator's acceleration (using the definition of acceleration) is

$$a_y = \frac{5\ \text{m/s} - 2\ \text{m/s}}{3\ \text{s}} = +1\ \text{m/s}^2$$

This operational definition does not tell you the reason for the acceleration. If you know that the mass of the elevator is 500 kg and that Earth exerts a 5000-N downward force on the accelerating elevator and the cable exerts a 5500-N upward force on it, then using the cause-effect relationship of Newton's second law:

$$\frac{5500\ \text{N} + (-5000\ \text{N})}{500\ \text{kg}} = +1\ \text{m/s}^2$$

Thus, you obtain the same number using two different methods—one from kinematics (the part of physics that *describe*s motion) and the other from dynamics (the part of physics that *explains* motion).

Force components used for forces along one axis

You can use the component form of Newton's second law for the *x*- and *y*-directions instead of the vector equation [Eq. (3.6)]:

$$a_{\text{S}x} = \frac{\Sigma F_{\text{on S}\,x}}{m_\text{S}} \tag{3.7x}$$

and

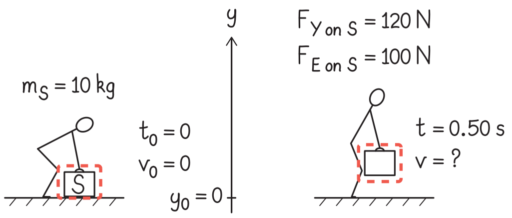
$$a_{\text{S}y} = \frac{\Sigma F_{\text{on S}\,y}}{m_\text{S}} \tag{3.7y}$$

The next example shows you how to apply Newton's second law in component form to solve problems.

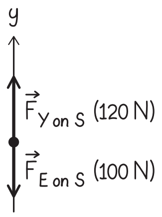
EXAMPLE 3.3 Lifting a suitcase

Earth exerts an approximately 100-N force on a 10-kg suitcase. Suppose you exert an upward 120-N force on the suitcase. If the suitcase starts at rest, how fast is it traveling after lifting for 0.50 s?

Sketch and translate First, we sketch the initial and final states of the process, choosing the suitcase as the system. The sketch helps us visualize the process and also brings together all the known information, letting our brains focus on other aspects of solving the problem. One common aspect of problems like this is the use of a two-step strategy. Here, we use Newton's second law to determine the acceleration of the suitcase and then use kinematics to determine the suitcase's speed after lifting for 0.50 s.



Simplify and diagram Next, we construct a force diagram for the suitcase while being lifted. The *y*-components of the forces exerted on the suitcase are your upward pull on the suitcase $F_{\text{Y on S}\,y} = +F_{\text{Y on S}} = +120\ \text{N}$ and Earth's downward pull on the suitcase $F_{\text{E on S}\,y} = -F_{\text{E on S}} = -100\ \text{N}$. Because the upward force is larger, the suitcase will have an upward acceleration $\vec{a}$.



Represent mathematically Since all the forces are along the *y*-axis, we apply the *y*-component form of Newton's second law to determine the suitcase's acceleration (notice how the subscripts in the equation below change from step to step):

$$a_{\text{S}y} = \frac{\Sigma F_{\text{on S}\,y}}{m_\text{S}} = \frac{F_{\text{Y on S}\,y} + F_{\text{E on S}\,y}}{m_\text{S}}$$

$$= \frac{(+F_{\text{Y on S}}) + (-F_{\text{E on S}})}{m_\text{S}} = \frac{F_{\text{Y on S}} - F_{\text{E on S}}}{m_\text{S}}$$

After using Newton's second law to determine the acceleration of the suitcase, we then use kinematics to determine the suitcase's speed after traveling upward for 0.50 s:

$$v_y = v_{0y} + a_y t$$

The initial velocity is $v_{0y} = 0$.

Solve and evaluate Now substitute the known information in the Newton's second law *y*-component equation above to find the acceleration of the suitcase:

$$a_{\text{S}y} = \frac{F_{\text{Y on S}} - F_{\text{E on S}}}{m_\text{S}} = \frac{120\ \text{N} - 100\ \text{N}}{10\ \text{kg}} = +2.0\ \text{m/s}^2$$

Insert this and other known information into the kinematics equation to find the vertical velocity of the suitcase after lifting for 0.50 s:

$$v_y = v_{0y} + a_y t = 0 + (+2.0\ \text{m/s}^2)(0.50\ \text{s}) = +1.0\ \text{m/s}$$

The unit for velocity is correct and the magnitude is reasonable.

Try it yourself How far up did you pull the suitcase during this 0.50 s?

Answer The average speed while lifting it was $(0 + 1.0\ \text{m/s})/2 = 0.50\ \text{m/s}$. Thus you lifted the suitcase $y - y_0 = (0.50\ \text{m/s})(0.50\ \text{s}) = 0.25\ \text{m}$.

REVIEW QUESTION 3.5 Your friend says that $m\vec{a}$ is a force exerted on an object and it should be represented on the force diagram. Do you agree or disagree with him? Explain your answer.

3.6 Gravitational force law

In the last example, we were given the mass of a suitcase (10 kg) and the approximate magnitude of the gravitational force that Earth exerts on it (100 N). Is it possible to determine the magnitude of this force by just knowing the mass of the suitcase?

Imagine that we evacuate all the air from a 3.0-m-long Plexiglas tube, place a motion sensor at the top, and drop objects of various sizes, shapes, and compositions through the tube (as was done with a feather and an apple in Figure 2.26). The measurements taken by the motion sensor reveal that all objects fall straight down with the same acceleration, approximately 9.8 m/s^2. Earth is the only object that exerts the force on the falling object $\vec{F}_{\text{E on O}}$ during the entire flight. If we choose the positive y-axis pointing down and apply the y-component form of Newton's second law, we get

$$a_{\text{O}y} = \frac{1}{m_{\text{O}}} F_{\text{E on O}\,y} = \frac{1}{m_{\text{O}}}(+F_{\text{E on O}}) = +\frac{F_{\text{E on O}}}{m_{\text{O}}}$$

Every object dropped in our experiment had the same free-fall acceleration, $g = 9.8\ \text{m/s}^2$, even those with very different masses (such as a ping-pong ball and a lead ball). Thus, the gravitational force that Earth exerts on each object must be proportional to its mass so that the mass cancels when we calculate the acceleration. Earth must exert a force on a 10-kg object that is 10 times greater than that on a 1-kg object:

$$a_{\text{O}y} = \frac{F_{\text{E on O}}}{m_{\text{O}}} = g$$

This reasoning leaves just one possibility for the magnitude of the force that Earth exerts on an object:

$$F_{\text{E on O}} = m_{\text{O}}g = m_{\text{O}}(9.8\ \text{m/s}^2)$$

The ratio of the force and the mass is a constant for all objects—the so-called gravitational coefficient g, already familiar to us as free-fall acceleration.

Gravitational force The magnitude of the gravitational force that Earth exerts on any object $F_{\text{E on O}}$ when on or near its surface equals the product of the object's mass m and the coefficient g:

$$F_{\text{E on O}} = m_{\text{O}}g \qquad (3.8)$$

where $g = 9.8\ \text{m/s}^2 = 9.8\ \text{N/kg}$ on or near Earth's surface. This force points toward the center of Earth.

TIP Equation (3.8) is a cause-effect relation because m_{O} and g can be varied independently.

The value of the free-fall acceleration g in the above gravitational force relation [Eq. (3.8)] does not mean that the object is actually falling. The g is just used to determine the magnitude of the gravitational force exerted on the object by Earth whether the object is falling, sitting at rest on a table, or moving down a waterslide. To avoid confusion, we will use $g = 9.8\ \text{N/kg}$ rather than $9.8\ \text{m/s}^2$ when calculating the gravitational force. Sometimes, when we do not need high precision in the calculation, we will even use 10 N/kg.

We will learn in the chapter on circular motion (Chapter 5) that the gravitational coefficient g at a particular point depends on the mass of Earth and on how far this point is from the center of Earth. On Mars or the Moon, the gravitational coefficient depends on the mass of Mars or the Moon, respectively. The gravitational coefficient is 1.6 N/kg on the Moon and 3.7 N/kg on Mars. You could throw a ball upward higher on the Moon since g is smaller there, resulting in a smaller force exerted downward on the ball.

TIP The *weight* of an object on a planet is the force that the planet exerts on the object. We will not use the term "weight of an object" because it implies that weight is a property of the object rather than an interaction between two objects.

REVIEW QUESTION 3.6 Newton's second law says that the acceleration of an object is inversely proportional to its mass. However, the acceleration with which all objects fall in the absence of air is the same. How can this be?

3.7 Skills for applying Newton's second law for one-dimensional processes

In this section we will develop a strategy that can be used whenever a process involves force and motion. We will introduce the strategy by applying it to the 2006 skydive of diving champion Michael Holmes. After more than 7000 successful jumps, Holmes jumped from an airplane 3700 m above Lake Taupo in New Zealand. His main parachute failed to open, and his backup chute became tangled in its cords. The partially opened backup parachute slowed his descent to about 36 m/s (80 mi/h) as he reached a 2-m-high thicket of wild shrubbery. Holmes survived.

TIP In all problems, unless specifically noted, we will assume that the observer is on the ground and is not accelerating.

PROBLEM-SOLVING STRATEGY 3.1 Applying Newton's laws for one-dimensional processes

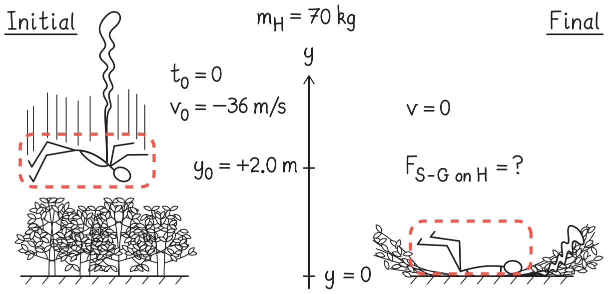
EXAMPLE 3.4 **Holmes's skydive**

Michael Holmes (70 kg) was moving downward at 36 m/s (80 mi/h) and was stopped by 2.0-m-high shrubbery and the ground. Estimate the average force exerted by the shrubbery and ground on his body while stopping his fall.

Sketch and translate

- Sketch the process.
- Choose the system.
- Choose a coordinate system.
- Label the sketch with everything you know about the situation.
- Identify the unknown that you need to find. Label it with a question mark on the sketch.

We sketch the process, choosing Holmes as the system (H). We want to know the average force that the shrubbery and ground (S-G) exert on him from when he first touches the shrubbery to the instant when he stops. We choose the y-axis pointing up and the origin at the ground where Holmes comes to rest. We use kinematics to find his acceleration while stopping and Newton's second law to find the average force that the shrubbery and ground exerted on him while stopping him.



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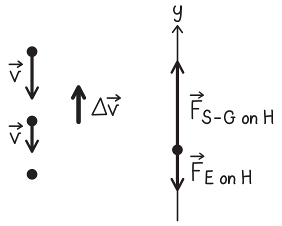
Simplify and diagram

- Make appropriate simplifying assumptions about the process. For example, can you neglect the size of the system? Can you assume that forces or acceleration is constant?
- Then represent the process with a motion diagram and/or force diagram(s). Make sure the diagrams are consistent with each other.



We model Holmes as a point-like object and assume that the forces being exerted on him are constant so that they lead to a constant acceleration. A motion diagram for his motion while stopping is shown along with the corresponding force diagram. To draw the force diagram we first identify the objects interacting with Holmes as he slows down: the shrubbery and ground (combined as one interaction) and Earth. The shrubbery and ground exert an upward force $\vec{F}_{\text{S-G on H}}$ on Holmes. Earth exerts a downward gravitational force $\vec{F}_{\text{E on H}}$. The force diagram is the same for all points of the motion diagram because the acceleration is constant. On the force diagram the arrow for $\vec{F}_{\text{S-G on H}}$ must be longer to match the motion diagram, which shows the velocity change arrow pointing up.

Represent mathematically

- Convert these qualitative representations into quantitative mathematical descriptions of the situation using kinematics equations and Newton's second law for motion along the axis. Determine the signs for the force components in the equations. Add the force components (with either positive or negative signs) to find the sum of the forces.

The *y*-component of Holmes's average acceleration is

$$a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)}$$

The *y*-component of Newton's second law with the positive *y*-direction up is

$$a_y = \frac{\Sigma F_{\text{on H}\,y}}{m_{\text{H}}}$$

The *y*-component of the force exerted by the shrubbery-ground on Holmes is $F_{\text{S-G on H}\,y} = +F_{\text{S-G on H}}$, and the *y*-component of the force exerted by Earth is $F_{\text{E on H}\,y} = -F_{\text{E on H}} = -m_{\text{H}}g$. Therefore,

$$a_y = \frac{F_{\text{S-G on H}\,y} + F_{\text{E on H}\,y}}{m_{\text{H}}} = \frac{(+F_{\text{S-G on H}}) + (-F_{\text{E on H}})}{m_{\text{H}}} = \frac{+F_{\text{S-G on H}} - m_{\text{H}}g}{m_{\text{H}}}$$

$$\Rightarrow F_{\text{S-G on H}} = m_{\text{H}}a_y + m_{\text{H}}g$$

Solve and evaluate

- Substitute the known values into the mathematical expressions and solve for the unknowns.
- Finally, evaluate your work to see if it is reasonable (check units, limiting cases, and whether the answer has a reasonable magnitude). Check whether all representations—mathematical, pictorial, and graphical—are consistent with each other.

Holmes's average acceleration was

$$a_y = \frac{0^2 - (-36\text{ m/s})^2}{2(0 - 2.0\text{ m})} = +324\text{ m/s}^2$$

Holmes's initial velocity is negative, since he is moving in the negative direction. His initial position is +2.0 m at the top of the shrubbery, and his final position is zero at the ground. His velocity in the negative direction is decreasing, which means the velocity change and the acceleration both point in the opposite direction (positive). The average magnitude of the force exerted by the shrubbery and ground on Holmes is

$$F_{\text{S-G on H}} = m_{\text{H}}a_y + m_{\text{H}}g = (70\text{ kg})(324\text{ m/s}^2) + (70\text{ kg})(9.8\text{ N/kg})$$
$$= 22{,}680\text{ kg}\cdot\text{m/s}^2 + 686\text{ N} = 23{,}366\text{ N} = 23{,}000\text{ N}$$

The force has a magnitude greater than the force exerted by Earth—thus the results are consistent with the force diagram and motion diagram. The magnitude is huge and the units are correct. A limiting case for zero acceleration gives us a correct prediction—the force exerted on Holmes by the shrubbery and ground equals the force exerted by Earth.

Try it yourself What process involving forces can be described by the equation $50\text{ kg} \times 2\text{ m/s}^2 = 50\text{ kg} \times 9.8\text{ N/kg} + (-390\text{ N})$?

Answer

A 50-kg person is landing on a thick cushion that exerts a 390-N upward force on her but cannot slow her down. She continues to accelerate down, but her acceleration is less than g (only 2 m/s^2). The vertical axis points down.

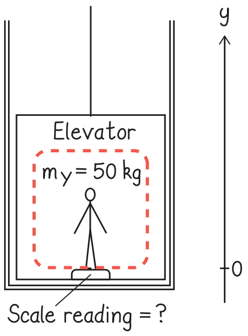
An elevator ride standing on a bathroom scale

In Example 3.5, we consider a much less dangerous process, one you could try the next time you ride an elevator. When you stand on a bathroom scale, the scale reading indicates how hard you are pushing on the scale. The force that it exerts on you balances the downward force that Earth exerts on you, resulting in your zero acceleration. What will the scale read if you stand on it in a moving elevator?

EXAMPLE 3.5 Elevator ride

You stand on a bathroom scale in an elevator as it makes a trip from the first floor to the tenth floor of a hotel. Your mass is 50 kg. When you stand on the scale in the stationary elevator, it reads 490 N (110 lb). What will the scale read (a) early in the trip while the elevator's upward acceleration is 1.0 m/s^2, (b) while the elevator moves up at a constant speed of 4.0 m/s, and (c) when the elevator slows to a stop with a downward acceleration of 1.0 m/s^2 magnitude?

Sketch and translate We sketch the situation as shown at right, choosing you as the system. The coordinate axis points upward with its origin at the first floor of the elevator shaft. Your mass is $m_Y = 50$ kg, the magnitude of the force that Earth exerts on you is $F_{\text{E on Y}} = m_Y g = 490$ N, and your acceleration is (a) $a_y = +1.0 \text{ m/s}^2$ (the upward velocity is increasing); (b) $a_y = 0$ (v is a constant 4.0 m/s upward); and (c) $a_y = -1.0 \text{ m/s}^2$ (the upward velocity is decreasing, so the acceleration points in the opposite, negative direction).



Simplify and diagram We model you as a point-like object and represent you as a dot in both the motion and force diagrams, shown for each part of the trip in Figures a, b, and c. On the diagrams, E represents Earth, Y is you, and S is the scale. The magnitude of the downward force that Earth exerts does not change (it equals $m_Y \vec{g}$, and neither m_Y nor $\vec{g}$ changes). Notice that the force diagrams and motion diagrams are consistent with each other for each part of the trip. The length of the arrows representing the force that the scale exerts on you changes from one case to the next so that the sum of the forces points in the same direction as your velocity change arrow.

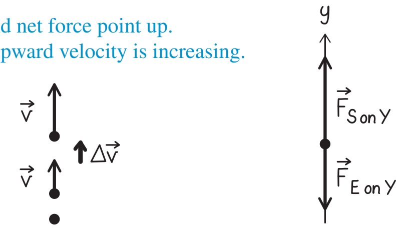
Represent mathematically The motion and the forces are entirely along the vertical y-axis. Thus, we use the vertical y-component form of Newton's second law [Eq. (3.7y)] to analyze the process. There are two forces exerted on you (the system) so there will be two vertical y-component forces on the right side of the equation: the y-component of the force that Earth exerts on you, $F_{\text{E on Y}\,y} = -m_Y g$, and the y-component of the force that the scale exerts on you, $F_{\text{S on Y}\,y} = +F_{\text{S on Y}}$:

$$a_{Yy} = \frac{\Sigma F_y}{m_Y} = \frac{F_{\text{E on Y}\,y} + F_{\text{S on Y}\,y}}{m_Y} = \frac{-m_Y g \; + F_{\text{S on Y}}}{m_Y}$$

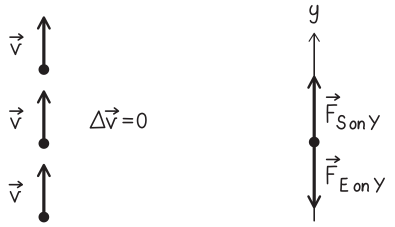
Multiplying both sides by m_Y, we get $a_{Yy} m_Y = -m_Y g \; + F_{\text{S on Y}}$. We can now move $-m_Y g$ to the left side: $m_Y a_{Yy} + m_Y g = F_{\text{S on Y}}$, or

$$F_{\text{S on Y}} = m_Y a_{Yy} + m_Y g = m_Y a_{Yy} + \; 490 \text{ N}$$

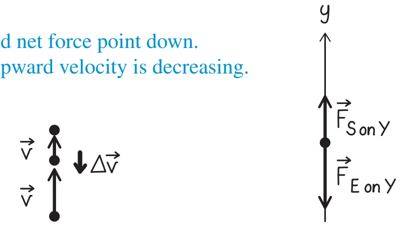
(a) $\Delta\vec{v}$ and net force point up.
The upward velocity is increasing.



(b) $\Delta\vec{v}$ and net force are zero.
The velocity is constant.



(c) $\Delta\vec{v}$ and net force point down.
The upward velocity is decreasing.



Remember that $m_Y g = 490$ N is the magnitude of the force that Earth exerts on you. The expression for $F_{\text{S on Y}}$ gives the magnitude of the force that the scale exerts on you.

Solve and evaluate We can now use the last equation to predict the scale reading for the three parts of the trip.

(a) Early in the trip, the elevator is speeding up, and its acceleration is $a_{Yy} = +1.0 \text{ m/s}^2$. During that time interval, the force exerted by the scale on you should be

$$\begin{aligned} F_{\text{S on Y}} &= m_Y \, a_{Yy} + 490 \text{ N} \\ &= (50 \text{ kg})(+1.0 \text{ m/s}^2) + 490 \text{ N} = 540 \text{ N} \end{aligned}$$

(b) In the middle of the trip, when the elevator moves at constant velocity, your acceleration is zero and the scale should read

$$\begin{aligned} F_{\text{S on Y}} &= m_Y \, a_{Yy} + 490 \text{ N} \\ &= (50 \text{ kg})(0 \text{ m/s}^2) + 490 \text{ N} = 490 \text{ N} \end{aligned}$$

(CONTINUED)

(c) When the elevator is slowing down near the end of the trip, its acceleration points downward and is $a_y = -1.0\ \text{m/s}^2$. Then the force exerted by the scale on you should be

$$\begin{aligned} F_{\text{S on Y}} &= m_{\text{Y}}\, a_{\text{Y}y} + 490\ \text{N} \\ &= (50\ \text{kg})(-1.0\ \text{m/s}^2) + 490\ \text{N} = 440\ \text{N} \end{aligned}$$

When the elevator is at rest or moving at constant speed, the scale reading equals the magnitude of the force that Earth exerts on you. When the elevator accelerates upward, the scale reads more. When it accelerates downward, even if you are moving upward, the scale reads less. What is also important is that the motion and force diagrams in Figures a, b, and c are consistent with each other and the force diagrams are consistent with the predicted scale readings—an important consistency check of the motion diagrams, force diagrams, and math.

Try it yourself You are standing on the scale in the elevator. The scale first reads 490 N, then 440 N, then 490 N again, and finally 540 N. What could possibly be happening to the elevator?

Answer The elevator is at rest, then moves down with downward acceleration of 1 m/s^2, then moves at constant velocity, and finally starts to slow down with acceleration of 1 m/s^2 pointing upward.

It might not seem very important to know what a scale reads in an elevator as it moves, but if we consider the cable supporting the elevator, the value of such calculations becomes apparent. We deduce that the force that the cable exerts on the elevator when the elevator is accelerating upward is greater than that when the elevator is at rest or moving with constant velocity. This means that the cable must be strong enough to withstand a magnitude of force of at least $mg + ma$, where m is the mass of the elevator and its maximum load, and a is the magnitude of the elevator's maximum vertical acceleration.

REVIEW QUESTION 3.7 Three friends argue about the type of information a bathroom scale reports: Eugenia says that it reads the force that Earth exerts on a person, Alan says that it reads the sum of the forces exerted on the person by Earth and the scale, and Mike says that the scale reads the force that the person exerts on the scale. Who do you think is correct? Why?

3.8 Forces come in pairs: Newton's third law

So far, we have analyzed a system's acceleration due to the forces exerted on it by external objects. What effect does the system have on these external objects? To help answer this question, we observe the interaction of two objects and analyze what happens to each of them. Suppose, while wearing Rollerblades, you push abruptly on a wheeled cart that is loaded with a heavy box. If you and the cart are on a hard smooth floor, the cart starts to move (it accelerates), and you also start to move and accelerate in the opposite direction. Evidently, you exerted a force on the cart and the cart exerted a force on you. Since the accelerations were in opposite directions, the forces must point in opposite directions. Let's consider more quantitatively the effect of such a mutual interaction between two objects.

Consider the experiments in Observational Experiment **Table 3.6**. Two dynamics carts roll freely on a smooth track before colliding. We mount force probes on each cart in order to measure the forces that each cart exerts on the other while colliding. A motion sensor on each end of the track records the initial velocity of each cart before the collision.

OBSERVATIONAL EXPERIMENT TABLE 3.6 Forces that two dynamics carts exert on each other

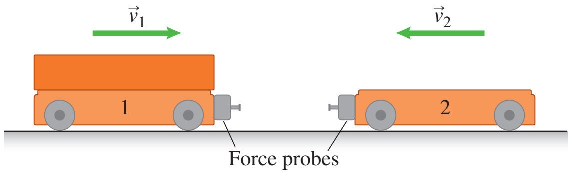
VIDEO
OET 3.6

Observational experiment

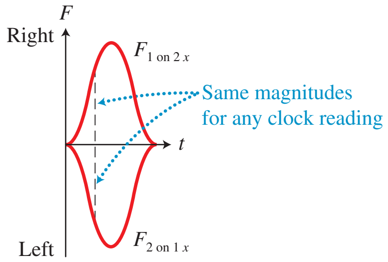
Analysis

Experiment 1. Two carts of different masses move toward each other on a level track. A motion detector indicates their speed before the collision, and force probes record the forces exerted by each cart on the other. Before the collision:

$$m_1 = 1.0\text{ kg},\ v_{1x} = +2\text{ m/s}$$
$$m_2 = 0.5\text{ kg},\ v_{2x} = -2\text{ m/s}$$

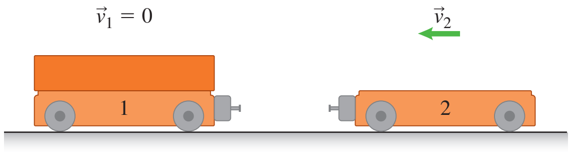


Because both carts changed velocities due to the collision, they must have exerted forces on each other. The computer recordings from the force probes show that the forces that the carts exert on each other vary with time and at each time have the same magnitude and point in opposite directions. Cart 1 exerts a force on cart 2 toward the right, and cart 2 exerts a force on cart 1 toward the left.

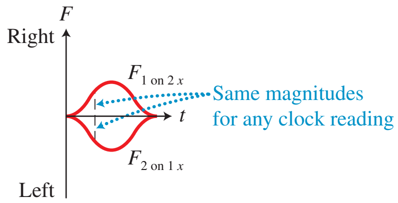


Experiment 2. Cart masses and velocities before collision:

$$m_1 = 1.0\text{ kg},\ v_{1x} = 0\text{ m/s (at rest)}$$
$$m_2 = 0.5\text{ kg},\ v_{2x} = -1\text{ m/s}$$

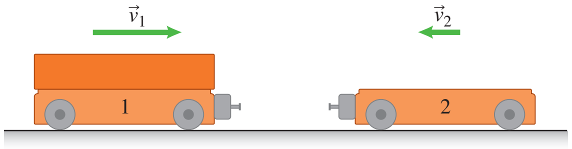


Although the forces that the carts exert on each other are smaller than in the first experiment, the magnitudes of the forces at each time are still the same.

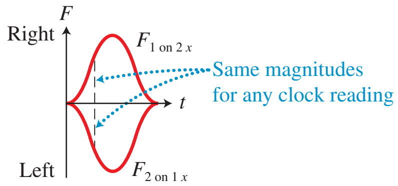


Experiment 3. Cart masses and velocities before collision:

$$m_1 = 1.0\text{ kg},\ v_{1x} = +2\text{ m/s}$$
$$m_2 = 0.5\text{ kg},\ v_{2x} = -1\text{ m/s}$$



The same analysis applies.



Pattern

In each experiment, independent of the masses and velocities of the carts before the collisions, at every instant during the collision the force that cart 1 exerted on cart 2 $\vec{F}_{1\text{ on }2}$ had the same magnitude as but pointed in the opposite direction from the force that cart 2 exerted on cart 1 $\vec{F}_{2\text{ on }1}$.

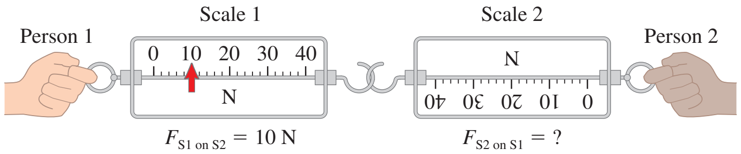
The cart collisions in Table 3.6 indicate that the force that one cart exerts on another is equal in magnitude and opposite in direction to the force that the second cart exerts on the first.

$$\vec{F}_{1\text{ on }2} = -\vec{F}_{2\text{ on }1}$$

Will the pattern that we found allow us to correctly predict the results of a new experiment?

TESTING EXPERIMENT TABLE 3.7 Testing the relationship between the forces that interacting objects exert on each other

VIDEO
TET 3.7

Testing experiment	Prediction	Outcome
Two people are holding spring scales that are hooked together. The scales remain horizontal during the experiment. Person 1 pulls her scale so that scale 1 reads 10 N, while person 2 just holds scale 2. Predict the reading of scale 2.  Predict the reading of scale 1 when person 2 pulls scale 2 exerting an arbitrary force.	If the pattern relating the two forces that interacting objects exert on each other (they have the same magnitudes and opposite directions) is correct, and scale 1 pulls on scale 2 exerting a force of 10 N, then scale 2 should pull on scale 1 in the opposite direction exerting a force of 10 N. Both scales will have the same reading. The same is true for the readings of the scales when scale 2 pulls on scale 1.	The experiments show that no matter who pulls on a scale, the scales have the same reading.

Conclusion

The outcome supports the pattern in the forces that two objects exert on each other during the interaction. These forces are of the same magnitude and opposite in direction.

To be convinced of the validity of this conclusion, we need many more testing experiments. So far, physicists have found no experiments involving the dynamics of everyday processes in which interacting objects exert forces on each other that are not of equal magnitude and oppositely directed. This relationship between the forces is called **Newton's third law**.

Newton's third law When two objects interact, object 1 exerts a force on object 2. Object 2 in turn exerts an equal-magnitude, oppositely directed force on object 1:

$$\vec{F}_{1\text{ on }2} = -\vec{F}_{2\text{ on }1} \tag{3.9}$$

Note that these forces are exerted on different objects and *cannot* be added to find the sum of the forces exerted on one object.

It seems counterintuitive that two interacting objects always exert forces of the same magnitude on each other. Imagine a game of ping-pong. A paddle hits the ball and the ball flies rapidly toward the other side of the table. However, the paddle moves forward with little change in motion. How is it possible that the light ball exerted a force on the paddle of the same magnitude as the force the paddle exerted on the ball?

To resolve this apparent contradiction, think about the masses of the interacting objects and their corresponding accelerations. If the forces are the same, the object with larger mass has a smaller magnitude acceleration than the object with smaller mass:

$$a_{\text{paddle}} = \frac{F_{\text{ball on paddle}}}{m_{\text{paddle}}} \quad \text{and} \quad a_{\text{ball}} = \frac{F_{\text{paddle on ball}}}{m_{\text{ball}}}$$

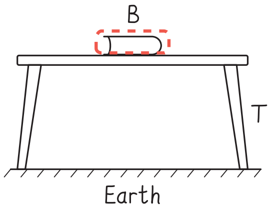
Because the mass of the ball is so small, the same force leads to a large change in velocity. The paddle's mass, on the other hand, is much larger. Thus, the same magnitude force leads to an almost zero velocity change. We observe the velocity change and incorrectly associate that alone with the force exerted on the object.

TIP Remember that the forces in Newton's third law are exerted on two different objects. This means that the two forces will never show up on the same force diagram, and they should not be added together to find the sum of the forces. You have to choose the system and consider only the forces exerted on *it*!

CONCEPTUAL EXERCISE 3.6 A book on a table

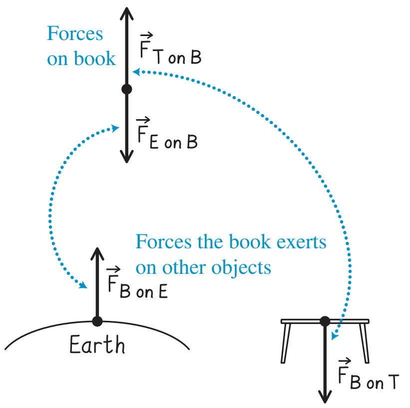
A book sits on a tabletop. Identify the forces exerted on the book by other objects. Then, for each of these forces, identify the force that the book exerts on another object. Explain why the book is not accelerating.

Sketch and translate Draw a sketch of the situation and choose the book as the system.



Simplify and diagram Assume that the tabletop is perfectly horizontal and model the book as a point-like object. A force diagram for the book is shown at right. Earth exerts a downward gravitational force on the book $\vec{F}_{\text{E on B}}$, and the table exerts an upward force on the book $\vec{F}_{\text{T on B}}$. Newton's second law explains why the book is not accelerating: the forces exerted on it by other objects are balanced and add to zero.

The subscripts on each force identify the two objects involved in the interaction. The Newton's third law pair force will have its subscripts reversed. For example, Earth exerts a downward gravitational force on the book ($\vec{F}_{\text{E on B}}$). According to Newton's third law, the book must exert an equal-magnitude upward gravitational force on Earth ($\vec{F}_{\text{B on E}} = -\vec{F}_{\text{E on B}}$), as shown at right. The table exerts an upward contact force on the book ($\vec{F}_{\text{T on B}}$), so the book must exert an equal-magnitude downward contact force on the table ($\vec{F}_{\text{B on T}} = -\vec{F}_{\text{T on B}}$).



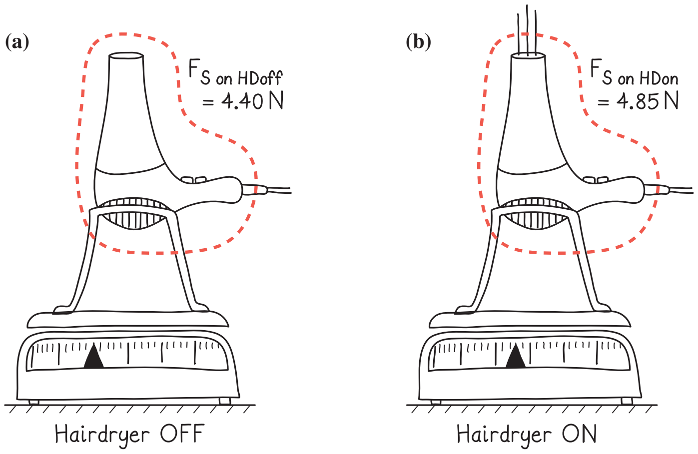
Try it yourself A horse pulls on a sled that is stuck in snow and not moving. Your friend Chris says this happens because the horse exerts on the sled the same magnitude force that the sled exerts on the horse. Since the sum of the forces is zero, there is no acceleration. What is wrong with Chris's reasoning?

Answer Chris added the forces exerted on two different objects and did not consider all forces exerted on the sled. If you choose the sled as the system, then the horse pulls forward on the sled, and the snow exerts a backward, resistive force. If these two horizontal forces happen to be of the same magnitude, they add to zero, and the sled does not accelerate horizontally. If, on the other hand, we choose the horse as the system, the ground exerts a forward force on the horse's hooves (since the horse is exerting a force backward on the ground), and the sled pulls back on the horse. If those forces have the same magnitude, the net horizontal force is again zero, and the horse does not accelerate.

EXAMPLE 3.7 Hairdryer on a scale

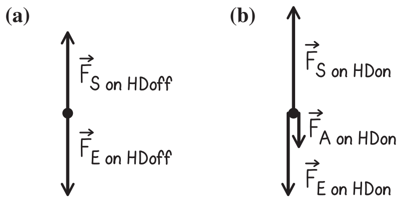
Hairdryers contain a small propeller that pushes air away from the dryer through a nozzle. You place a hairdryer on a scale with the nozzle pointing up, and it reads 0.99 lb. When you turn the hairdryer on, so that the hairdryer is pushing the air upward, the reading of the scale increases to 1.09 lb. Explain the change in the reading qualitatively and quantitatively.

Sketch and translate We sketch the process (a) before the hairdryer is turned on and (b) after it is turned on. The system is the hairdryer (with the propeller). We point the positive y-axis down. We know that the force that the scale exerts on the hairdryer when it is off is $F_{\text{S on HDoff}} = 0.99\text{ lb} = 4.40\text{ N}$. When the hairdryer is on, the force the scale exerts on it is $F_{\text{S on HDon}} = 1.09\text{ lb} = 4.85\text{ N}$. We need to qualitatively explain the increase of the reading and then explain the quantitative increase of 0.45 N.



(CONTINUED)

Simplify and diagram After the hairdryer is turned on, the scale moves downward a little, but very quickly the situation stabilizes. Thus we will assume that the turned-on hairdryer is not moving with respect to the scale; the scale is stationary after it reaches the reading of 4.85 N. Therefore, the acceleration of the system is zero. There are two objects that interact with the dryer: Earth and the scale. We draw a force diagram for the system (a) when the hairdryer is off and (b) when it is on. The force that Earth exerts on the dryer is the same in both experiments. If diagram (a) is correct, this means that diagram (b) needs an additional downward force of 0.45 N to explain the increase in the reading. What object is exerting this force? Remember that when the dryer is on, the propeller inside is rotating. It pushes the air up, exerting a force $F_{\text{HDon on A}}$; therefore, according to Newton's third law, the air should push the propeller and consequently the whole hairdryer down. Accordingly, we add another force to the diagram in (b), $F_{\text{A on HDon}}$, so that the sum of the forces is still zero. We can now determine the magnitude of this force using the reading of the scale.



Represent mathematically Using the force diagram in (b) we can write

$$a_y = \frac{\Sigma F_y}{m_{\text{HD}}} = \frac{F_{\text{E on HDon } y} + F_{\text{S on HDon } y} + F_{\text{A on HDon } y}}{m_{\text{HD}}}$$

$$= \frac{F_{\text{E on HDon}} + (-F_{\text{S on HDon}}) + F_{\text{A on HDon}}}{m_{\text{HD}}} = 0$$

$$\Rightarrow F_{\text{A on HDon}} = F_{\text{S on HDon}} + (-F_{\text{E on HDon}})$$

Solve and evaluate Using the last equation from the previous step, we find the force that the air exerts on the hairdryer:

$$F_{\text{A on HDon}} = F_{\text{S on HDon}} + (-F_{\text{E on HDon}}) = 4.85\text{ N} - 4.45\text{ N} = 0.45\text{ N}$$

The result makes sense. The magnitude is smaller than the force Earth exerts on the hairdryer but not insignificant. Such an increase in supporting force can explain an observation you may have made while watering plants with a hose. If the flow of water is strong, you can feel the expelled water pushing back on the hose. The same principle explains the behavior of a rotating sprinkler.

Try it yourself An 80-kg basketball player pushes off a gym floor exerting a force that is 4 times greater than the force Earth exerts on him and that the push lasts 0.10 s (unrealistically short). Estimate how fast he is moving when he leaves the floor?

Answer

About 3 m/s. He would move faster if he took longer to push off while exerting the same force.

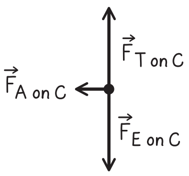
EXAMPLE 3.8 Learning to linearize data

Alex is investigating the motion of a battery-powered fan attached to a low-friction cart (a fan cart) that is moving on a horizontal track. As the fan blades rotate, they exert a force on the air, and the air exerts an equal and opposite force on the blades, making the cart move (an analogy for the fan cart is a hairdryer on wheels). Using a motion detector, Alex finds that the cart moves with constant acceleration. He also measures how the acceleration of the cart a_x depends on the mass of the objects that he adds to the cart (m_{added}). His measurements are shown in the table below.

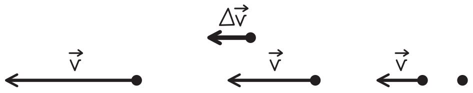
m_{added} (kg)	a_x (m/s^2)
0.10	0.25
0.20	0.21
0.30	0.19
0.40	0.17
0.50	0.15

(a) Draw a force diagram for the cart and use it to explain why the cart moves at constant acceleration for fixed added mass. (b) Two physical quantities that are not listed in the table affect the motion of the cart. Determine these two quantities using the data above. (Hint: Rearrange the mathematical expression for the acceleration of the cart to obtain linear dependence on added mass and then plot the graph using the data in the table.)

Sketch and translate The cart is moving to the left on a horizontal surface. The acceleration of the cart is also to the left due to the fan pushing air to the right (the fan works in a similar way to the hairdryer that we investigated in the previous example). We know the masses of the additional objects on the cart and the respective accelerations. We also know that the acceleration of any system is affected by the sum of the forces exerted on it and the system's mass. Because the mass of the cart (together with the fan) contributes to the system's mass, we hypothesize that the quantities that we need to determine are the sum of the forces and the mass of the cart.



Simplify and diagram We assume that the fan cart can be modeled as a point-like object and that the track is smooth, so we do not need to worry about friction. The only objects interacting with the cart (C) are Earth (E), the track (T), and the air (A). The motion diagram and the force diagram for the cart are shown in the figures below and at right. From the force diagram, we see that the forces exerted on the cart by Earth and the track add to zero, and the only force that is causing the acceleration of the cart to the left is the force exerted by the air.



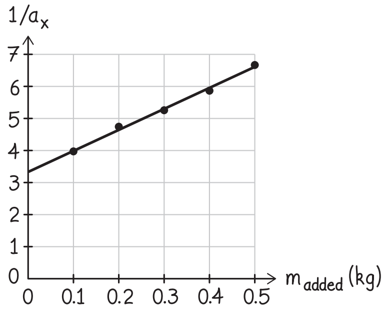
Represent mathematically We write Newton's second law in component form along the axis of motion:

$$a_x = \frac{F_{\text{A on C}}}{m_{\text{added}} + m_{\text{C}}}$$

Since m_{added} is changed in each experiment (it's an independent variable), it is useful to rearrange the equation so that it is a linear function with linear dependence on m_{added}:

$$\frac{1}{a_x} = \frac{m_{\text{added}} + m_{\text{C}}}{F_{\text{A on C}}} = \frac{m_{\text{added}}}{F_{\text{A on C}}} + \frac{m_{\text{C}}}{F_{\text{A on C}}}$$

and to plot $1/a_x$ against m_{added} (see graph at right). The function for $1/a_x$ is a linear function of the form $mx + b$; thus $m = 1/F_{\text{A on C}}$ is the slope of the graph and the intercept is $b = m_{\text{C}}/F_{\text{A on C}}$. (Do not confuse m for the slope with the symbol m for mass.)



Solve and evaluate The intercept of the graph with the vertical axis is 1 divided by the acceleration the cart would have had if the added mass was zero. Thus $m_{\text{C}}/F_{\text{A on C}} = 3.35 \text{ kg/N}$.

We can find the slope of the graph as Δy divided by Δx. For example, when $\Delta y = 6 - 3.35$ and $\Delta x = 0.4 - 0$, this ratio is $2.65/0.4 = 6.63$. The slope of the graph is thus

$$\frac{1}{F_{\text{A on C}}} = \frac{\Delta y}{\Delta x} = 6.63 \frac{1}{\text{N}}$$

Therefore, $F_{\text{A on C}} = 0.15 \text{ N}$ and $m_{\text{C}} = 0.51 \text{ kg}$.

Try it yourself How would the graph change (consider intercept and slope) if you placed a second fan on the same cart and repeat the experiment?

Answer The slope of the graph is inversely proportional to the force exerted by the air on the cart; therefore the slope will be cut in half. The intercept is inversely proportional to the force that the air exerts on the cart and directly proportional to the mass of the cart and fans, therefore there is not enough data to determine it. But if we assume that the mass of the fan is much smaller than the mass of the cart, then the intercept will be between the original one and half of its value.

REVIEW QUESTION 3.8 Is the following sentence true? When you hold a heavy object in your hands, you exert the same magnitude force on the object as the object exerts on you but in the opposite direction, and because these forces add to zero, the object stays at rest.

3.9 Seat belts and air bags

At the beginning of the chapter we posed a question about air bags. How do they save lives? We now have all the physics needed to investigate this question. Consider **Figure 3.7**. An air bag is like a balloon that is packed in a small box in the steering wheel or the passenger side dashboard. Air bags are designed to deploy when a car has a negative acceleration of magnitude $10g\ [10(9.8 \text{ m/s}^2) = 98 \text{ m/s}^2 \approx 100 \text{ m/s}^2]$ or more. When a car has such a rapid decrease in speed, the bag inflates with nitrogen gas in about 0.04 s and forms a cushion for the occupant's chest and head. The bag has two important effects:

1. It spreads the force that stops the person over a larger area of the body.
2. It increases the stopping distance, and consequently the stopping time interval, thus reducing the average force stopping the occupant.

Why is spreading the stopping force over a larger area of the body an advantage? If a person uses only a seat belt, his head is not belted to the seat and tends to continue moving forward during a collision, even though his chest and waist are restrained. To stop the head without an air bag, the neck must exert considerable force on the head. This can cause a dangerous stretching of the spinal cord and muscles of the neck, a phenomenon known as "whiplash." The air bag exerts a more uniform force across the upper body and head and helps all parts to stop together.

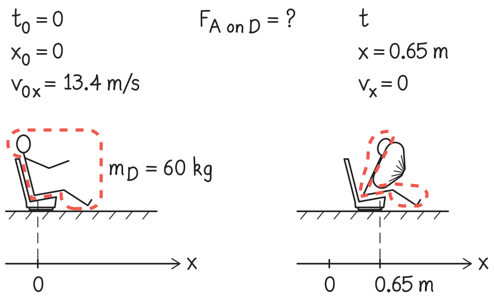
FIGURE 3.7 An air bag stops a crash test dummy during a collision.

How does the air bag increase the stopping distance? Suppose a test car is traveling at a constant speed of 13.4 m/s (30 mi/h) until it collides head-on with a concrete wall. The front of the car crumples about 0.65 m. A crash test dummy is rigidly attached to the car's seat and is further protected by the rapidly inflating air bag. The dummy also travels about 0.65 m before coming to rest. Without an air bag or a seat belt, the dummy would continue to move forward at the initial velocity of the car. It would then crash into the steering wheel or windshield of the stopped car and stop in a distance much less than 0.65 m—like flying into a rigid wall. The smaller the stopping distance, the greater the acceleration, and therefore the greater the force that is exerted on the dummy. Let's estimate the average force exerted by the air bag on the body during a collision.

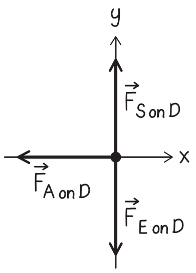
EXAMPLE 3.9 Force exerted by air bag on driver during collision

A 60-kg crash test dummy moving at 13.4 m/s (30 mi/h) stops during a collision in a distance of 0.65 m. Estimate the magnitude of the average force that the air bag and seat belt exert on the dummy.

Sketch and translate We sketch and label the situation as shown below, choosing the crash test dummy as the system. The positive *x*-direction is in the direction of motion, and the origin is at the position of the dummy at the start of the collision.



Simplify and diagram We model the dummy D as a point-like object and assume that the primary force exerted on the dummy while stopping is due to the air bag and seat belt's $\vec{F}_{\text{A on D}}$, shown in the force diagram. We can ignore the downward gravitational force that Earth exerts on the dummy $\vec{F}_{\text{E on D}}$ and the upward force that the car seat exerts on the dummy $\vec{F}_{\text{S on D}}$ since they add to zero and do not contribute to the acceleration.



Represent mathematically To determine the dummy's acceleration, we use kinematics:

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)}$$

Once we have the dummy's acceleration, we apply the *x*-component form of Newton's second law to find the force exerted by the air bag and seat belts on the dummy:

$$a_x = \frac{F_{\text{A on D}\,x}}{m_\text{D}} = \frac{-F_{\text{A on D}}}{m_\text{D}} = -\frac{F_{\text{A on D}}}{m_\text{D}}$$

$$\Rightarrow F_{\text{A on D}} = -m_\text{D} a_x$$

Solve and evaluate We know that $v_{0x} = +13.4$ m/s and $v_x = 0$ (the dummy has stopped). The initial position of the dummy is $x_0 = 0$ and the final position is $x = 0.65$ m. The acceleration of the dummy while in contact with the air bag and seat belt is

$$a_x = \frac{0^2 - (13.4 \text{ m/s})^2}{2(0.65 \text{ m} - 0 \text{ m})} = -138 \text{ m/s}^2$$

Thus, the magnitude of the average force exerted by the air bag and seat belt on the dummy is

$$F_{\text{A on D}} = -(60 \text{ kg})(-138 \text{ m/s}^2) = 8300 \text{ N}$$

This force $[8300 \text{ N}(1 \text{ lb}/4.45 \text{ N}) = 1900 \text{ lb}]$ is almost 1 ton. Is this estimate reasonable? The magnitude is large, but experiments with crash test dummies in the real world are consistent with a force this large in magnitude, a very survivable collision.

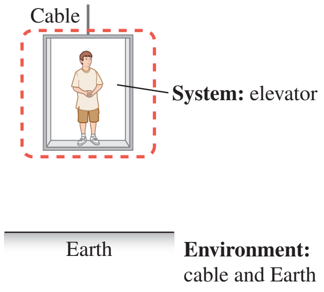
Try it yourself Find the acceleration of the dummy and the magnitude of the average force needed to stop the dummy if it is not belted, has no air bag, and stops in 0.1 m when hitting a hard surface.

Answer -900 m/s^2 and 54,000 N.

REVIEW QUESTION 3.9 Explain how an air bag and seat belt reduce the force exerted on the driver of a car during a collision.

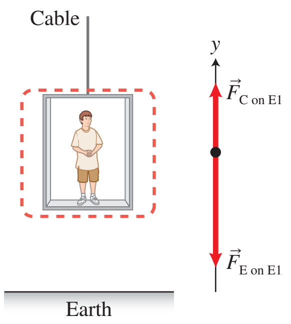
Summary

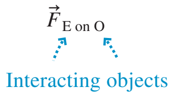
A **system** is circled in a sketch of a process. Other objects that interact with the system are called the **environment**. (Sections 3.1 and 3.5)



The **force** that one object exerts on another characterizes an interaction between the two objects (a pull or a push). The unit of force is the newton (N); $1\ \text{N} = (1\ \text{kg})(1\ \text{m/s}^2)$. (Sections 3.1 and 3.5)

A **force diagram** represents the forces that external objects exert on the system. The arrows in the diagram point in the directions of the forces, and their lengths indicate the relative magnitudes of the forces. (Sections 3.1 and 3.2)

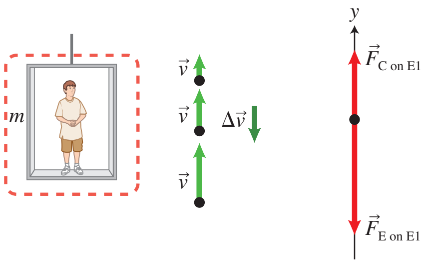




A force is denoted by the symbol $\vec{F}$ with two subscripts indicating the object that is exerting the force and the system object.

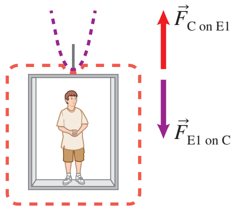
Newton's first law of motion If no other objects exert forces on a system or if the forces exerted on the system add to zero, then the system continues moving at constant velocity (as seen by observers in inertial reference frames). (Section 3.4)

Newton's second law The acceleration a_S of a system is proportional to the sum of the forces that other objects exert on the system and inversely proportional to its mass m. (Section 3.5)



$$\vec{a}_S = \frac{\Sigma \vec{F}_{\text{on S}}}{m_S} = \frac{\vec{F}_{1\text{ on S}} + \vec{F}_{2\text{ on S}} + \cdots}{m_S} \qquad \text{Eq. (3.6)}$$

Newton's third law Two objects exert equal-magnitude and opposite direction forces of the same type on each other. (Section 3.8)



$$\vec{F}_{1\text{ on }2} = -\vec{F}_{2\text{ on }1} \qquad \text{Eq. (3.9)}$$

The **gravitational force** $\vec{F}_{\text{E on O}}$ that Earth exerts on an object of mass m when on or near Earth's surface depends on the gravitational coefficient g of Earth. If on or near another planet or the Moon, the gravitational coefficient near those objects is different. (Section 3.6)

Magnitude

$$F_{\text{E on O}} = m_{\text{O}}g \qquad \text{Eq. (3.8)}$$

where

$$g = 9.8\ \text{m/s}^2 = 9.8\ \text{N/kg}$$

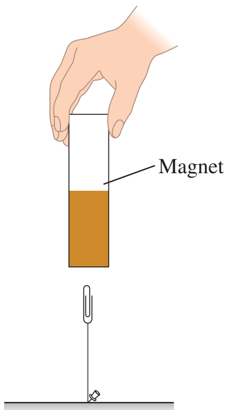
when the object is on or near Earth's surface.

Questions

Multiple Choice Questions

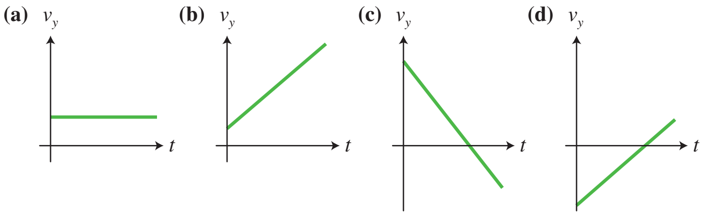
1. An upward-moving elevator slows to a stop as it approaches the top floor. Which answer below best describes the relative magnitudes of the upward force that the cable exerts on the elevator $\vec{F}_{\text{C on El}}$ and the downward gravitational force that Earth exerts on the elevator $\vec{F}_{\text{E on El}}$?
   (a) $F_{\text{C on El}} > F_{\text{E on El}}$
   (b) $F_{\text{C on El}} = F_{\text{E on El}}$
   (c) $F_{\text{C on El}} < F_{\text{E on El}}$
   (d) Not enough information is given to answer the question.
2. You apply the brakes of your car abruptly and your book starts sliding off the front seat. Three observers explain this differently. Observer A says that the book continued moving and the car accelerated from underneath it. Observer B says that the car pushed forward on the book. Observer C says that she must be in a noninertial reference frame because the book started moving without any extra objects interacting with it. Which of the observers is correct?
   (a) A
   (b) B
   (c) C
   (d) A and C
   (e) All of the observers
3. Which of the statements below explains why a child lurches forward in a stroller when you abruptly stop the stroller?
   (a) The child does not lurch forward but instead continues her motion.
   (b) Your pull on the stroller causes the child to move in the opposite direction.
   (c) Newton's third law
4. Which observers can explain the phenomenon of whiplash, which occurs when a car stops abruptly, using Newton's laws?
   (a) The driver of the car
   (b) A passenger in the car
   (c) An observer on the sidewalk beside the car and road
5. Which vector quantities describing a moving object are always in the same direction?
   (a) Velocity and acceleration
   (b) Velocity and the sum of the forces
   (c) Acceleration and the sum of the forces
   (d) Acceleration and force
   (e) Both b and c are correct.
6. You have probably observed that magnets attract objects made of certain metals, such as steel. You tie a steel paperclip to a tabletop with string and bring a strong magnet above it so that the paperclip is positioned as shown in **Figure Q3.6**. Which answer correctly compares the magnitude of the force exerted by the magnet on the paperclip to the magnitude of the force exerted by the paperclip on the magnet?
   (a) $F_{\text{magnet on clip}} > F_{\text{clip on magnet}}$
   (b) $F_{\text{magnet on clip}} < F_{\text{clip on magnet}}$
   (c) $F_{\text{magnet on clip}} = F_{\text{clip on magnet}}$

FIGURE Q3.6



7. Which of the following velocity-versus-time graphs represent the motion of the object for which $\Sigma F_y > 0$? Choose all answers that are correct.

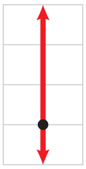
FIGURE Q3.7

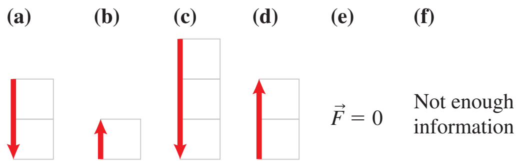


8. A book sits on a tabletop. What force is the Newton's third law pair to the force that Earth exerts on the book? Choose the correct answer with the best explanation.
   (a) The force that the table exerts on the book because it is equal and opposite in direction to the force that Earth exerts on the book
   (b) The force that the table exerts on the book because the table and the book are touching each other
   (c) The force that the table exerts on the book because it describes the same interaction
   (d) The force that the book exerts on Earth because it describes the same interaction
   (e) The force that the book exerts on Earth because it is equal and opposite in direction to the force that Earth exerts on the book
9. A spaceship moves in outer space. What happens to its motion if there are no external forces exerted on it? If there is a constant force exerted on it in the direction of its motion? If something exerts a force opposite its motion?
   (a) It keeps moving; it speeds up with constant acceleration; it slows down with constant acceleration.
   (b) It slows down; it moves with constant velocity; it slows down.
   (c) It slows down; it moves with constant velocity; it stops instantly.
10. A 0.10-kg apple falls on Earth, whose mass is about 6×10^{24} kg. Which is true of the gravitational force that Earth exerts on the apple?
    (a) It is bigger than the force that the apple exerts on Earth by almost 25 orders of magnitude.
    (b) It is the same magnitude.
    (c) We do not know the magnitude of the force the apple exerts on Earth.
11. A man stands on a scale and holds a heavy object in his hands. What happens to the scale reading if the man quickly lifts the object upward and then stops lifting it?
    (a) The reading increases, returns briefly to the reading when standing stationary, then decreases.
    (b) The reading decreases, returns briefly to the reading when standing stationary, then increases.
    (c) Nothing, since the mass of the person with the object remains the same. Thus the reading does not change.
12. You stand on a bathroom scale in a moving elevator. What happens to the scale reading if the cable holding the elevator suddenly breaks?
    (a) The reading will increase.
    (b) The reading will not change.
    (c) The reading will decrease a little.
    (d) The reading will drop to 0 instantly.
13. A person pushes a 10-kg crate, exerting a 200-N force on it, but the crate's acceleration is only $5\ \text{m/s}^2$. Explain.
    (a) The crate pushes back on the person, thus the total force is reduced.
    (b) There are other forces exerted on the crate so that the total force is reduced.
    (c) Not enough information is given to answer the question.
14. Two small balls of the same material, one of mass m and the other of mass $2m$, are dropped simultaneously from the Leaning Tower of Pisa. On which ball does Earth exert a bigger force?
    (a) On the $2m$ ball
    (b) On the m ball
    (c) Earth exerts the same force on both balls because they fall with the same acceleration.
15. A box full of lead and a box of the same size full of feathers are floating inside a spaceship that has left the solar system. Choose equipment that you can use to compare their masses.
    (a) A balance scale
    (b) A digital scale
    (c) A watch with a second hand and a meter stick

16. **Figure Q3.16** shows an unlabeled force diagram for a moving object that is missing one force. The length of the sides of the square grid corresponds to a force magnitude of 1 N. Which additional force has to be exerted on the object so that the object (I) moves at a constant speed, (II) accelerates downward, (III) accelerates upward, and (IV) moves down? For each case, choose all answers from (a) to (f) that are correct.

FIGURE Q3.16



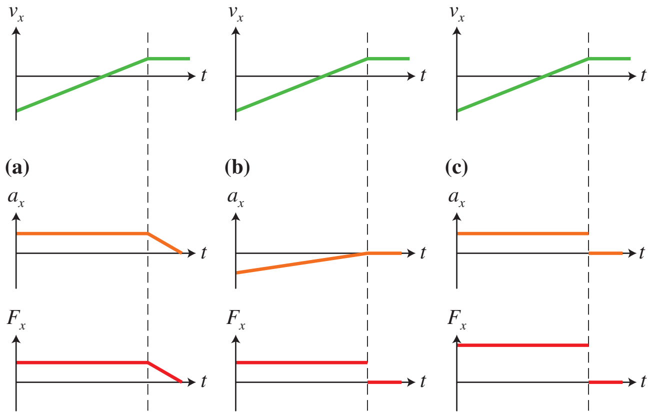


17. A person jumps from a wall and lands stiff-legged. Which statement best explains why the person is less likely to be injured when landing on soft sand than on concrete?
    - (a) The concrete can exert a greater force than the sand.
    - (b) The person sinks into the sand, increasing the stopping distance.
    - (c) The upward acceleration of the person in the sand is less than on concrete, thus the force that the sand exerts on the person is less.
    - (d) b and c
    - (e) a, b, and c
18. A 3000-kg spaceship is moving away from a space station at a constant speed of 3 m/s. The astronaut in the spaceship decides to return to the space station by switching on engines that expel fuel so that the sum of the forces exerted on the spaceship by the expelled fuel points toward the space station. What is the magnitude of the minimum force needed to bring the spaceship back to the space station?
    - (a) 9000 N
    - (b) 1000 N
    - (c) Any force larger than zero
    - (d) The spaceship will keep moving away from the space station no matter how large the force.
    - (e) Not enough information is given to answer the question.

Conceptual Questions

19. **Figure Q3.19** is a velocity-versus-time graph for the vertical motion of an object. Choose the correct combination (a, b, or c) of an acceleration-versus-time graph and a force-versus-time graph for the object.

FIGURE Q3.19



20. Explain the purpose of crumple zones, that is, the front of a car that collapses during a collision.
21. Explain why when landing on a firm surface after a fall you should not land with stiff legs.
22. A small car bumps into a large truck. Compare the forces that the truck exerts on the car and the car exerts on the truck if before the collision (a) the truck was stationary and the car was moving; (b) the car and the truck were moving in opposite directions; (c) the car and the truck were moving in the same direction.
23. You are pulling a sled. Compare the forces that you exert on the sled and the sled exerts on you if you (a) move at constant velocity; (b) speed up; (c) slow down.
24. You stand on a bathroom scale in a moving elevator. The elevator is moving up at increasing speed. The acceleration is constant. Draw three consecutive force diagrams for you.
25. You are holding a 100-g apple. (a) What is the force that you exert on the apple? (b) What is the force that the apple exerts on you? Support your answer with a motion diagram and a force diagram.
26. You throw a 100-g apple upward. (a) While the apple is still in your hand (we’ll call this period “the throw”), is the force that you exert on the apple more than, less than, or the same as the force that you exert on the apple when you are holding it at rest? (b) Support your answer with a motion diagram for the throw and a force diagram for one instant during the throw. (c) Is the force that the apple exerts on you at this instant more than, less than, or the same as the force that you exert on the apple when you are holding it at rest? Explain your answer.
27. After having been thrown upward, a 100-g apple falls back into your hand and you catch it. (a) While you are catching it, is the force that you exert on the apple more than, less than, or the same as the force that you exert on the apple when you are holding it at rest? (b) Support your answer with a motion diagram and a force diagram for one instant of the catch. (c) Is the force that the apple exerts on you at this instant more than, less than, or the same as the force that you exert on the apple when you are holding it at rest? Explain your answer.

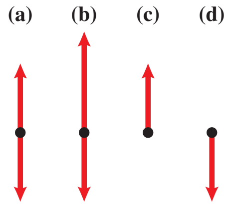
Problems

Below, **BIO** indicates a problem with a biological or medical focus. Problems labeled **EST** ask you to estimate the answer to a quantitative problem rather than derive a specific answer. Asterisks indicate the level of difficulty of the problem. Problems with no * are considered to be the least difficult. A single * marks moderately difficult problems. Two ** indicate more difficult problems.

3.1 and 3.2 Describing and representing interactions and Adding and measuring forces

1. * In **Figure P3.1** you see unlabeled force diagrams for balls in different situations. Match the diagrams with the following descriptions. (1) A ball is moving upward after it leaves your hand. (2) You hold a ball in your hand. (3) A ball is falling down. (4) You are throwing a ball (still in your hand) straight up. (5) You are lifting a ball at a constant pace. Explain your choices. Label the forces on the diagrams.

FIGURE P3.1

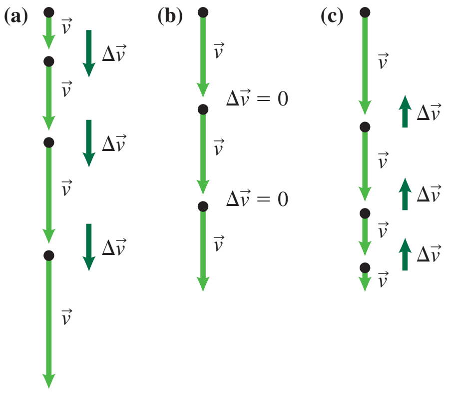


2. Draw a force diagram (a) for a bag hanging at rest from a spring; (b) for the same bag sitting on a table; and (c) for the same bag that you start to lift so it moves up faster and faster.
3. For each of the following situations, draw the forces exerted on the moving object and identify the other object causing each force. (a) You pull a wagon along a level floor using a rope oriented horizontally. (b) A bus moving on a horizontal road slows down in order to stop. (c) You lift your overnight bag into the overhead compartment on an airplane.
4. You hang a book bag on a spring scale and place the bag on a platform scale so that the platform scale reads 25.7 N and the spring scale reads 17.6 N. (a) Draw a force diagram for the book bag to represent the situation. (b) What is the magnitude of the force that Earth exerts on the bag?

3.3 Conceptual relationship between force and motion

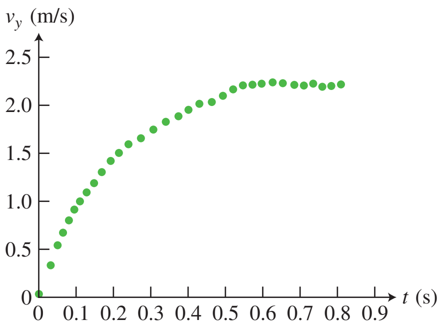
5. A block of dry ice slides at constant velocity along a smooth, horizontal surface. (a) Construct a motion diagram. (b) Draw position- and velocity-versus-time graphs. (c) Construct a force diagram for the block for three instants represented by dots on the motion diagram. Are the diagrams consistent with each other?
6. * You throw a ball upward. (a) Draw a motion diagram and two force diagrams for the ball on its way up and another motion diagram and two force diagrams for the ball on its way down. (b) Represent the motion of the ball with a position-versus-time graph and velocity-versus-time graph.
7. A string pulls horizontally on a cart so that it moves at increasing speed along a smooth horizontal surface. When the cart is moving medium-fast, the pulling is stopped abruptly. (a) Describe in words what happens to the cart's motion when the pulling stops. (b) Illustrate your description with motion diagrams, force diagrams, and position-versus-time and velocity-versus-time graphs. Indicate on the graphs when the pulling stopped. What assumptions did you make?
8. * Solving the previous problem, your friend says that after the string stops pulling, the cart starts slowing down. (a) Explain why your friend would think this way. (b) Do you agree with him? Explain your opinion. (c) Explain how you can design an experiment to test his idea.
9. * A string pulls horizontally on a cart so that it moves at increasing speed along a smooth horizontal surface. When the cart is moving medium-fast, the magnitude of the pulling force is reduced to half its former magnitude. (a) Describe what happens to the cart's motion after the reduction in the string pulling. (b) Illustrate your description with motion diagrams, force diagrams, and position-versus-time and velocity-versus-time graphs.
10. A block of dry ice slides at a constant velocity on a smooth horizontal surface. A second block of dry ice slides twice as fast on the same surface (at a higher constant velocity). Compare the resultant forces exerted on each block. Explain your reasoning.
11. Three motion diagrams for a moving elevator are shown in **Figure P3.11**. Construct two force diagrams (for two consecutive moments) for the elevator for *each* motion diagram. Be sure that the lengths of the force arrows are the appropriate relative lengths and that there is consistency between the force diagrams and the motion diagrams. What assumptions did you make?

FIGURE P3.11



12. * A student holds a thin aluminum pie pan horizontally 2 m above the ground and releases it. Using a motion detector, she obtains the graph shown in **Figure P3.12**. Based on her measurements, (a) draw force diagrams for the pie pan at times 0.05 s, 0.3 s, and 0.7 s, and (b) estimate the distance that the pan travels once it reaches constant speed.

FIGURE P3.12



13. * Figures P3.11a, b, and c show three motion diagrams for an elevator moving downward. (a) For each diagram, say everything you can about the elevator's motion. (b) Draw a force diagram for each motion diagram. (c) Could you draw a different motion diagram for each force diagram? Explain how that is possible.

3.4 Inertial reference frames and Newton's first law

14. * A train traveling from New York to Philadelphia is passing a station. A ball is sitting on the floor of the train not moving with respect to the train. (a) Draw a force diagram and a motion diagram for the ball as seen by the observers on the train and on the platform. (b) The ball now starts accelerating forward relative to the floor. Draw force and motion diagrams for the ball as seen by the observers on the train and on the platform. Which of the observers can use Newton's first law to explain the ball's acceleration? Explain.
15. * Explain the phenomenon of whiplash from two points of view: that of an observer on the ground and an observer in the car.

3.5 Newton's second law

16. * An astronaut exerts a 100-N force pushing a beam into place on the International Space Station. The beam accelerates at $0.10\ \text{m/s}^2$. Determine the mass of the beam. What is the percent uncertainty in your answer?

17. Four people participate in a rope competition. Two of them pull the rope right, exerting forces of magnitude 330 N and 380 N. The other two pull left, exerting forces of magnitude 300 N and 400 N. What is the sum of the forces exerted on the rope?

18. * **Shot put throw** During a practice shot put throw, the 7.0-kg shot left world champion C. J. Hunter's hand at speed 13 m/s. While making the throw, his hand pushed the shot a distance of 1.7 m. Describe all the physical quantities you can determine using this information. Describe the assumptions you need to make to determine them.

19. * You know the sum of the forces $\Sigma\vec{F}$ exerted on an object of mass m during Δt seconds. The object is at rest at the beginning of the time interval. List three physical quantities that you can determine about that object's motion using this information. Then explain how you will determine them.

20. * You record the displacement of an object as a constant force is exerted on it. (a) If the time interval during which the force is exerted doubles, how does the object's displacement change? Indicate all the assumptions that you made. (b) Explain how your answer changes if one of the assumptions is not valid.

21. * **Equation Jeopardy 1** Invent a problem for which the following equation can be a solution:

$$(40\ \text{kg})a_x = 200\ \text{N} - 40\ \text{N}$$

3.6 and 3.7 Gravitational force law and Skills for applying Newton's second law for one-dimensional processes

22. * **Equation Jeopardy 2** Describe in words a problem for which the following equation is a solution and draw a force diagram that is consistent with the equation (specify the direction of the axis):

$$3.0\ \text{m/s}^2 \times 3.0\ \text{kg} = +29.4\ \text{N} - F_{\text{R on O}}$$

23. **Equation Jeopardy 3** Describe in words a problem for which the following equation is a solution and draw a force diagram that is consistent with the equation (specify the direction of the axis):

$$\frac{0.8\,\text{m/s} - 1.2\,\text{m/s}}{1.6\,\text{s}} = \frac{\Sigma F_x}{50\,\text{kg}}$$

24. * **Equation Jeopardy 4** Describe in words a problem for which the following equation is a solution and draw a force diagram that is consistent with the equation (specify the direction of the axis):

$$2.0\ \text{m/s}^2 = \frac{196\ \text{N} - F_{\text{P on O}}}{20\ \text{kg}}$$

25. * **Spider-Man** Spider-Man holds the bottom of an elevator with one hand. With his other hand, he holds a spider cord attached to a 50-kg box of explosives at the bottom of the cord. Determine the force that the cord exerts on the box if (a) the elevator is at rest; (b) the elevator accelerates up at $2.0\ \text{m/s}^2$; (c) the upward-moving elevator's speed decreases at a rate of $2.0\ \text{m/s}^2$; and (d) the elevator falls freely.

26. ** Matt is wearing Rollerblades. Beth pushes him along a hallway with a large spring, keeping the spring compressed and consequently the force that the spring exerts on Matt constant at all times. They conduct several experiments in which Matt starts from rest and travels 12.0 m while carrying objects of different mass in his backpack, recording the time interval for each trip. Their data are shown in the table at right. (a) Draw a force diagram for Matt and use it to explain why he is moving with a constant acceleration. (b) Two physical quantities that are not listed in the table also affect Matt's motion. Determine these two quantities using the data above. (Hint: This problem requires linearization. See Example 3.8 for help.)

Added mass (kg)	Time interval (s)
0	19.0
3.0	19.4
6.0	19.9
9.0	20.3
12.0	20.8
15.0	21.2

27. * **Stuntwoman** The downward acceleration of a 60-kg stuntwoman near the end of a fall from a very high building is $7.0\ \text{m/s}^2$. What resistive force does the air exert on her body at that point?

28. **EST** Estimate the average force that a baseball pitcher's hand exerts on a 0.145-kg baseball as he throws a 40 m/s (90 mi/h) pitch. Indicate all of the assumptions you made.

29. * **Super Hornet jet takeoff** A 2.1×10^4-kg F-18 Super Hornet jet airplane (see Figure P3.29) goes from zero to 265 km/h in 90 m during takeoff from the flight deck of the USS Nimitz aircraft carrier. What physical quantities can you determine using this information? Make a list and determine the values of three of them.

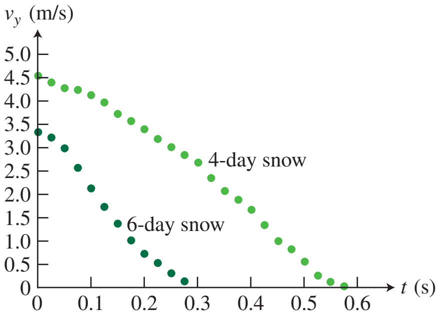
FIGURE P3.29

30. **Lunar Lander** The Lunar Lander of mass 2.0×10^4 kg made the last 150 m of its trip to the Moon's surface in 120 s, descending at approximately constant speed. The *Handbook of Lunar Pilots* indicates that the gravitational constant on the Moon is 1.633 N/kg. Using these quantities, what can you learn about the Lunar Lander's motion?

31. Aisha throws a 0.3-kg ball upward. Frances, standing on a balcony above Aisha, catches the ball by exerting a 1-N downward force on the ball. (a) Draw a motion diagram and a force diagram for the ball during the time interval when Frances is catching it. (b) Determine the acceleration of the ball.

32. * Students Lucia, Isabel, and Austin are investigating how snow stops a dropped 500-g lemon juice bottle. In particular, they are interested in how the force exerted by the snow depends on the age of the snow. They take high-speed videos of the bottle while it sinks into the snow, taking their first set of measurements 4 days after fresh snowfall and the second set of measurements 2 days later. After analyzing the videos frame by frame (see photo), they plot a graph that shows how the velocity of the bottle from the moment the bottle touches the snow changes for both types of snow (Figure P3.32).

FIGURE P3.32



They each explain their results as follows:

Lucia: The 6-day snow exerts a larger force on the bottle because it stops the bottle in a shorter time.

Isabel: The time taken to stop the bottle does not say much about the force. The 6-day snow exerts a larger force on the bottle because the slope of its $v_y(t)$ graph is steeper.

Austin: We cannot compare the forces exerted by the snow because the initial velocities are different. We need to repeat the experiments and make sure we always drop the bottle from the same height.

Explain how each student reached her/his conclusion and decide who (if anyone) is correct. Indicate any assumptions that you have made.

33. * **Astronaut** Karen Nyberg, a 60-kg astronaut, sits on a bathroom scale in a rocket that is taking off vertically with an acceleration of $3g$. What does the scale read?
34. * A 0.10-kg apple falls off a tree branch that is 2.0 m above the grass. The apple sinks 0.060 m into the grass while stopping. Determine the force that the grass exerts on the apple while stopping it. Indicate any assumptions you made.
35. ** An 80-kg fireman slides 5.0 m down a fire pole. He holds the pole, which exerts a 500-N steady resistive force on the fireman. At the bottom he slows to a stop in 0.40 m by bending his knees. What can you determine using this information? Determine it.

3.8 Forces come in pairs: Newton's third law

36. * Earth exerts a 1.0-N gravitational force on an apple as it falls toward the ground. (a) What force does the apple exert on Earth? (b) Compare the accelerations of the apple and Earth due to these forces. The mass of the apple is about 100 g and the mass of Earth is about 6×10^{24} kg.
37. * You push a bowling ball down the lane toward the pins. Draw force diagrams for the ball (a) just before you let it go; (b) when the ball is rolling (for two clock readings); (c) as the ball is hitting a bowling pin. (d) For each force exerted on the ball in parts (a)–(c), draw the Newton's third law force beside the force diagram, and indicate the object on which these third law forces are exerted.
38. * EST (a) A 50-kg skater initially at rest throws a 4-kg medicine ball horizontally. Describe what happens to the skater and to the ball. (b) Estimate the acceleration of the ball during the throw and of the skater using a reasonable value for the force that a skater can exert on the medicine ball. (c) The skater moving to the right catches the ball moving to the left. After the catch, both objects move to the right. Draw force diagrams for the skater and for the ball while the ball is being caught.
39. ** EST Basketball player LeBron James can jump vertically over 0.9 m. Estimate the force that he exerts on the surface of the basketball court as he jumps. (a) Compare this force with the force that the surface exerts on James. Describe all assumptions used in your estimate and state how each assumption affects the result. (b) Repeat the problem looking at the time interval when he is landing back on the floor.
40. * EST The Scottish Tug of War Association contests involve eight-person teams pulling on a rope in opposite directions. Estimate the magnitude of the force that the rope exerts on each team. Indicate any assumptions you made and include a force diagram for a short section of the rope.
41. Consider the experiment described in Question 3.6 (Figure Q3.6). (a) Draw force diagrams for the magnet and for the paperclip. (b) Which of the forces that you have drawn are pairs according to Newton's third law? Assume all objects are at rest. (c) If the mass of the magnet is 0.300 kg and the force exerted by the hand on the magnet is 3.18 N, what is the magnitude of the force exerted by the paperclip on the magnet? (d) Can you determine the mass of the paperclip based on these data? Explain.
42. * EST A friend drops a 0.625-kg basketball from 2 m above you. (a) Estimate the smallest force that the basketball exerts on your hands while you are catching it. (b) How does it compare to the force that you have to exert on the ball to catch it? Use force diagrams to support your answer.

3.9 Seat belts and air bags

43. **Car safety** The National Transportation Safety Board indicates that a person in a car crash has a reasonable chance of survival if his or her acceleration is less than $300\ \text{m/s}^2$. (a) What magnitude force would cause this acceleration in such a collision? (b) What stopping distance is needed if the initial speed before the collision is 20 m/s (72 km/h, or 45 mi/h)? (c) Indicate any assumptions you made.
44. * A 70-kg person in a moving car stops during a car collision in a distance of 0.60 m. The stopping force that the air bag exerts on the person is 8000 N. Name at least three physical quantities describing the person's motion that you can determine using this information, and then determine them.

General Problems

45. * BIO EST **Left ventricle pumping** The lower left chamber of the heart (the left ventricle) pumps blood into the aorta. According to biophysical studies, a left ventricular contraction lasts about 0.20 s and pumps 88 g of blood. This blood starts at rest and after 0.20 s is moving through the aorta at about 2 m/s. (a) Estimate the force exerted on the blood by the left ventricle. (b) What is the percent uncertainty in your answer? (c) What assumptions did you make? Did the assumptions increase or decrease the calculated value of the force compared to the actual value?
46. ** EST **Acorn hits deck** You are sitting on a deck of your house surrounded by oak trees. You hear the sound of an acorn hitting the deck. You wonder if an acorn will do much damage if instead of the deck it hits your head. Make appropriate estimations and assumptions and provide a reasonable answer.
47. ** EST **Olympic dive** During a practice dive, Olympic diver Fu Mingxia reached a maximum height of 5.0 m above the water. She came to rest 0.40 s after hitting the water. Estimate the average force that the water exerted on her while stopping her.
48. * Komila knows that an egg breaks if it falls onto a concrete floor from a height of 0.4 m. She finds, however, that an egg does not break when it falls from the same height onto a floor that is covered with a 2.0-cm-thick carpet. Try to explain the outcome of her experiments. Draw a force diagram for the egg just before it stops. Based on your explanation, predict what the thickness of the carpet should be for an egg to survive a drop from a height of 1.0 m. Indicate any assumptions you have made.
49. ** EST You are doing squats on a bathroom scale. You decide to push off the scale and jump up. Estimate the reading as you push off and as you land. Indicate any assumptions you made.
50. ** EST Estimate the horizontal speed of the runner shown in **Figure P3.50** at the instant she leaves contact with the starting blocks. Indicate any assumptions you made.

FIGURE P3.50

51. ** EST Estimate the maximum acceleration of Earth if all people got together and jumped up simultaneously.
52. ** EST Estimate how much Earth would move during the jump described in Problem 51.

Reading Passage Problems

Col. John Stapp crash tests From 1946 through 1958, Col. John Stapp headed the U.S. Air Force Aero Medical Laboratory's studies of the human body's ability to tolerate high accelerations during plane crashes. Conventional wisdom at the time indicated that a plane's negative acceleration should not exceed $180\ \text{m/s}^2$ (18 times gravitational acceleration, or $18g$). Stapp and his colleagues built a 700-kg "Gee Whiz" rocket sled, track, and stopping pistons to measure human tolerance to high acceleration. Starting in June 1949, Stapp and other live subjects rode the sled. In one of Stapp's rides, the sled started at rest and 360 m later was traveling at speed 67 m/s when its braking system was applied, stopping the sled in 6.0 m. He had demonstrated that $18g$ was not a limit for human deceleration.

53. In an early practice run while the rocket sled was stopping, a passenger dummy broke its restraining device and the window of the rocket sled and stopped after skidding down the track. What physics principle best explains this outcome?
    (a) Newton's first law
    (b) Newton's second law
    (c) Newton's third law
    (d) The first and second laws
    (e) All three laws

54. What is Stapp's 67 m/s speed in miles per hour?
(a) 30 mi/h (b) 40 mi/h (c) 100 mi/h
(d) 120 mi/h (e) 150 mi/h

55. What is the magnitude of the acceleration of Stapp and his sled as their speed increased from zero to 67 m/s?
(a) 5 $\mathrm{m/s^2}$ (b) 6 $\mathrm{m/s^2}$ (c) 10 $\mathrm{m/s^2}$
(d) 12 $\mathrm{m/s^2}$ (e) 14 $\mathrm{m/s^2}$

56. What is the magnitude of the acceleration of Stapp and his sled as their speed decreased from 67 m/s to zero?
(a) 12g (b) 19g (c) 26g
(d) 38g (e) 48g

57. What is the average force exerted by the restraining system on 80-kg Stapp while his speed decreased from 67 m/s to zero in a distance of 6.0 m?
(a) 10,000 N (b) 20,000 N (c) 30,000 N
(d) 40,000 N (e) 50,000 N

58. What is the time interval for Stapp and his sled to stop as their speed decreased from 67 m/s to zero?
(a) 0.09 s (b) 0.18 s (c) 0.34 s
(d) 5.4 s (e) 10.8 s

Using proportions A proportion is defined as an equality between two ratios; for instance, $a/b = c/d$. Proportions can be used to determine the expected change in one quantity when another quantity changes. Suppose, for example, that the speed of a car doubles. By what factor does the stopping distance of the car change? Proportions can also be used to answer everyday questions, such as whether a large container or a small container of a product is a better buy on a cost-per-unit-mass basis.

Suppose that a small pizza costs a certain amount. How much should a larger pizza of the same thickness cost? If the cost depends on the amount of ingredients used, then the cost should increase in proportion to the pizza's area and not in proportion to its diameter:

$$\text{Cost} = k(\text{Area}) = k(\pi r^2) \qquad (3.10)$$

where r is the radius of the pizza and k is a constant that depends on the price of the ingredients per unit area. If the area of the pizza doubles, the cost should double, but k remains unchanged.

Let us rearrange Eq. (3.10) so the two variable quantities (cost and radius) are on the right side of the equation and the constants are on the left:

$$k\pi = \frac{\text{Cost}}{r^2}$$

This equation should apply to any size pizza. If r increases, the cost should increase so that the ratio Cost/r^2 remains constant. Thus, we can write a proportion for pizzas of different sizes:

$$k\pi = \frac{\text{Cost}}{r^2} = \frac{\text{Cost}'}{r'^2}$$

For example, if a 3.5-in.-radius pizza costs \$4.00, then a 5.0-in. radius pizza should cost

$$\text{Cost}' = \frac{r'^2}{r^2}\text{Cost} = \frac{(5.0\text{ in})^2}{(3.5\text{ in})^2}(\$4.00) = \$8.20$$

This process can be used for most equations relating two quantities that change while all other quantities remain constant.

59. The downward distance d that an object falls in a time interval t if starting at rest is $d = \frac{1}{2}at^2$. On the Moon, a rock falls 10.0 m in 3.50 s. How far will the object fall in 5.00 s, assuming the same acceleration?
(a) 14.3 m (b) 20.4 m (c) 4.90 m
(d) 7.00 m (e) 10.0 m

60. The downward distance d that an object falls in a time interval t if starting at rest is $d = \frac{1}{2}at^2$. On the Moon, a rock falls 10.0 m in 3.50 s. What time interval t is needed for it to fall 15.0 m, assuming the same acceleration?
(a) 2.33 s (b) 2.86 s (c) 3.50 s
(d) 4.29 s (e) 5.25 s

61. A car's braking distance d (the distance it travels if rolling to a stop after the brakes are applied) depends on its initial speed v_0, the maximum friction force $\vec{f}_{\text{R on C}}$ exerted by the road on the car, and the car's mass m according to the equation

$$\frac{2f_{s\,\max}}{m}d = v_0^2$$

Suppose the braking distance for a particular car and road surface is 26 m when the initial speed is 18 m/s. What is the braking distance when traveling at 27 m/s?
(a) 59 m (b) 39 m (c) 26 m
(d) 17 m (e) 12 m

62. You decide to open a pizza parlor. The ingredients require that you charge \$4.50 for a 7.0-in.-diameter pizza. How large should you make a pizza whose price is \$10.00, assuming the cost is based entirely on the cost of ingredients?
(a) 1.4 in. (b) 3.1 in. (c) 7.0 in.
(d) 10 in. (e) 16 in.

63. A circular wool quilt of 1.2 m diameter costs \$200. What should the price of a 1.6-m-diameter quilt be if it is to have the same cost per unit area?
(a) \$110 (b) \$150 (c) \$270 (d) \$360