M4P3 Script: Testing for Functionality

When someone types on a keyboard, the person typing presses a key and the corresponding letter appears on the screen. This, in essence, acts as what mathematicians like to call "functions." The person inputs which letter they want to appear by pressing a key and the computer outputs exactly one letter in return. Functions are very special in mathematics and today the lesson is all about recognizing them.

Testing For Functionality Using Domain and Range

A **function** is a relation that assigns each x-value exactly one y-value. This means that, for all elements in the domain, there can only be one output for any given input. Let's take a look at how the mapping diagram on this slide illustrates this concept.

At this point, every single element in the domain is mapped to exactly one element in the range. The arrows let us know which domain element goes with which range element. So, for instance, 1 goes to 6 and 2 goes to 8. If we were to write these as ordered pairs, the mapping diagram would look like this. At this point, this relation is a function because each element in the domain is mapped to exactly one element in the range. This is also evident in our list of points because there are no repeated x-values.

But, if we were to add another mapping, say from 5 to 6, this relation would no longer be a function. This would imply that if we could plug 5 in to an equation that we would get both 6 and 8 as a result, and that is impossible.

EX 1. Testing for Functionality

Let's look at three examples of this. We are asked to determine if each of the following represents a function or just a relation.

Example A gives us a mapping diagram. Again, the key here is to look for elements in the domain with more than one arrow. We do not see that here, and this means this is a function.

Example B gives us a table of x- and y-values. When you're looking at a table, you only need to look for repeated x-values. There are two -1's and this is all we need to say that this is just a relation and not a function.

Example C shows a set of friends and their favorite after-school activities. This would be the equivalent of someone asking each person what their favorite activities are. Pam's favorite activity is playing sports. But if we ask Jose this question, he comes up with two responses, which doesn't really make sense if we have restricted the question to asking for his absolute most favorite activity. Because Jose is mapped to two activities, this is just a relation and not a function.

The Vertical Line Test

Now, we move onto a more graphical approach to testing a relation's functionality. When looking at a graph to determine if a relation is a function, we use something called the Vertical Line Test. This test says that if a vertical line drawn through the graph of a relation touches it more than once, then the relation is not a function.

Our first graph is a circle. The nice thing about the Vertical Line Test is that you only have to find one vertical line that hits the graph at more than one spot to say it isn't a function. Once you find this one line, you can stop. If we draw a vertical line here, it hits the circle in two spots – this is all we need to say this graph is not a function.

The next example is the graph of a polynomial relation. Though it has many turning points, there is no place on this graph where a vertical line would cross more than once. Because of this, this graph is a function.

The last example is a linear graph. As before, there is no place where a vertical line would cross it more than once, so this is a function as well.

Learning Check

In this learning check, you will answer 3 true-or-false questions about functionality. If you answer incorrectly, please go back and review your notes.