The Binomial Distribution

- Many types of probability problems have only two outcomes or can be reduced to two outcomes.
  - Coin toss
  - Winning or losing a game
  - Baby being male or female
  - True/false questions
  - Effective or ineffective medical treatment
  - Normal or abnormal blood pressure
  - Correct or incorrect answer
- A **binomial experiment** is a probability experiment that satisfies the following four requirements:
  1. There must be a fixed number of trials.
  2. Each trial can only have two outcomes, or outcomes that can be reduced to two outcomes. These outcomes can be considered as success or failure.
  3. The outcomes of each trial must be independent of one another.
  4. The probability of a success must remain the same for each trial.
- The outcomes of a binomial experiment and the corresponding probabilities of these outcomes are called a **binomial distribution**.

Notation for Binomial Distribution

P(S)	Probability of Success
P(F)	Probability of Failure
P	Numerical Probability of Success
Q	Numerical Probability of Failure
N	Number of Trials
X	Number of Successes in n Trials

Binomial Probability Formula

- In a binomial experiment, the probability of exactly X successes in n trials is

$$P(X) = \frac{n!}{(n-X)!X!} \cdot p^X \cdot q^{n-X}$$

Examples

1. A coin is tossed 3 times. Find the probability of getting exactly 2 heads.

2. A survey found that one out of five Americans say he or she has visited a doctor in any given month. If 10 people are selected at random, find the probability that exactly 3 will have visited a doctor last month.

3. A survey from Teenage Research Unlimited found that 30% of teenage consumers receive their spending money from part-time jobs. If 5 teenagers are selected at random, find the probability that at least 3 of them will have part-time jobs.

4. *Public Opinion* reported that 5% of Americans are afraid of being alone in a house at night. If a random sample of 20 Americans is selected, find these probabilities by using the binomial table.

    a) There are exactly 5 people in the sample who are afraid of being alone at night.

    b) There are at most 3 people in the sample who are afraid of being alone at night.

    c) There are at least 3 people in the sample who are afraid of being alone at night.

Mean, Variance, Standard Deviation

- Mean:
  - $\mu = n \cdot p$
- Variance:
  - $\sigma^2 = n \cdot p \cdot q$
- Standard Deviation:
  - $\sigma = \sqrt{n \cdot p \cdot q}$

Examples

1. A coin it tossed 4 times. Find the mean, variance, and standard deviation of the number of heads that will be obtained.

2. A die is rolled 360 times. Find the mean, variance, and standard deviation of the number of 4s that will be rolled.

3. The *Statistics Bulletin* published by Metropolitan Life Insurance Co. reported that 2% of all American births result in twins. If a random sample of 8,000 births is taken, find the mean, variance, and standard deviation of the number of births that would result in twins.