Unit 2 Lesson 3 Part 1 Guided Notes

Probability Distributions

Before we get started, let's review some old vocab…

1. Variable – a characteristic or attribute that can assume different values.
   - Various letters of the alphabet, such as X, Y, Z, are used to represent variables.
   - Since the variables in this chapter are associated with probability, they are called **random variables**.
   - **Random variable – a variable whose values are determined by chance.**
2. Discrete variable – have finite number of possible values, or an infinite number of values that can be counted.
   - Whole numbers
3. Continuous variable – can assme all values in the interval between any two given values.
   - Fractions, decimals.

Discrete Probability Distribution - consists of the values a random variable can assume and the corresponding probabilities of the values. The probabilities are determined theoretically or by observation.

Theoretical vs. Observational

Theoretical

- Represent graphically the probability distribution for the sample space for tossing three coins.

Observational

- The baseball teams playing in the World Series play until one team wins four times. This means there can be anywhere from 4-7 games. Find the probability P(X) for each X, construct a probability distribution, and draw a graph for the data.

X	Number of games played
4	8
5	7
6	9
7	16

Two Requirements for a Probability Distribution

- The sum of the probabilities of all the events in the sample space must equal 1; that is,

  $\Sigma P(X) = 1$

- The probability of each event in the sample space must be between or equal to 0 and 1; that is,

  $0 \leq P(X) \leq 1$

Determine whether each is a probability distribution:

a)

X	0	5	10	15	20
P(X)	1/5	1/5	1/5	1/5	1/5

b)

X	1	2	3	4
P(X)	1/4	1/8	1/16	9/16

c)

X	0	2	4	6
P(X)	-1.0	1.5	0.3	0.2

d)

X	2	3	7
P(X)	0.5	0.3	0.4

Mean, Variance, Standard Deviation, Expected Values

- We are familiar with finding mean, variance, and standard deviation. However, for probability distributions they are computed differently than they are for samples.

Mean

- How would you compute the mean of the number of dots that show on top when a die is rolled? You could roll it, say, 10 times, recording the number of dots, and finding the mean; however the answer would only approximate the true mean.
- To get an exact answer, we would have to roll the die an *infinite number of times*. Since this is impossible, we cannot use the old way to compute the mean.
- Suppose two coins are tossed repeatedly, and the number of heads that occurred is recorded. What will be the mean of the number of heads?
    - What is the sample space?
    - What is the probability for each outcome in the sample space?
    - Probability of one head?
    - Probability of two heads?
    - Probability of no heads?
    - Hence, on average, you would expect the number of heads to be…

    So to find the mean for a probability distribution, you must multiply each possible outcome by its corresponding probability and find the sum of the products

- The mean of a random variable with a discrete probability distribution is

$$\mu = X_1 \times P(X_1) + X_2 \times P(X_2) + ... + X_n \times P(X_n)$$

$$\mu = \Sigma X \times P(X)$$

where $X_1, X_2, ..., X_n$ are the outcomes and $P(X_1), P(X_2), ...P(X_n)$ are the corresponding probabilities.

**Note: $\mu = \Sigma X \times P(X)$ means to sum the products.

Variance and Standard Deviation

- To find the **variance** for the random variable of a probability distribution, subtract the theoretical mean from the each outcome and square the difference. Then multiply each difference by its corresponding probability and add the products.

$$\sigma^2 = \Sigma[(X - \mu)^2 \times P(X)]$$

However, this is tedious, so here's a shortcut…

- Find the **variance** of a probability distribution by multiplying the square of each outcome by its corresponding probability, summing the products, and subtracting the square of the theoretical mean.

$$\sigma^2 = \Sigma[X^2 \times P(X)] - \mu^2$$

****This is the shortcut. Use this one!****

- As always, the standard deviation is just the square root of the variance.

Examples

1. Find the mean, variance, and standard deviation of the number of dots that appear when a die is tossed.

2. A box contains 5 balls. Two are numbered 3, one is numbered 4, and two are numbered 5. The balls are mixed and one is selected at random. After a ball is selected, it is recorded and replaced. If the experiment is repeated many times, find the mean, variance, and standard deviation of the numbers on the balls.

3. A talk radio station has four telephone lines. If the host is unable to talk (i.e. during a commercial) or is talking to a person, the other called are placed on hold. When all lines are in use, others who are trying to call in get a busy signal. The probability that 0, 1, 2, 3, or 4 people will get through is shown in the distribution. Find the mean, variance, and standard deviation for the distribution.

X	0	1	2	3	4
P(X)	0.18	0.34	0.23	0.21	0.04

Should the station have considered getting more phone lines installed?

Expected Value

- The expected value of a discrete random variable of a probability distribution is the theoretical average of the variable.

$$\mu = E(X) = \Sigma X \times P(X)$$

- The symbol E(X) is used for the expected value.
- The formula is the same as the theoretical mean.
- Expected value = Theoretical mean

Examples

1. One thousand tickets are sold at $1 each for a color television valued at $350. What is the expected value of the gain if you purchase one ticket?

2. One thousand tickets are sold at $1 each for four prizes of $100, $50, $25, and $10. After each prize drawing, the winning ticket is then returned to the pool of tickets. What is the expected value if you purchase one ticket?

3. A financial adviser suggests that his client select one of two types of bonds in which to invest $5,000. Bond X pays a return of 4% and has a default rate of 2%. Bond Y has a 2.5% return and a default rate of 1%. Find the expected rate of return and decide which bond would be a better investment. When the bond defaults, the investor loses all the investment.