#### **DO NOW**

The composition of the 108<sup>th</sup> Congress is 51 Republicans, 48 Democrats, and 1 Independent. A committee on aid to higher education is formed with 3 Senators to be chosen at random to head the committee. Find the probability that the group of 3 consists of...

- a) All Republicans
- b)All Democrats

c) One Democrat, one Republican, and one Independent





#### DISCRETE PROBABILITY DISTRIBUTIONS

Objectives:

- Construct a probability distribution for a random variable.
- Find the mean, variance, standard deviation, and expected value for a discrete random variable.
- Find the exact probability for X successes in n trials of a binomial experiment.
- Find the mean, variance, standard deviation for the variable of a binomial distribution.
- Find the probabilities for outcomes of variables using the Poisson distribution.



#### **PROBABILITY DISTRIBUTIONS**

Before we get started, let's review some old vocab...

- Variable a characteristic or attribute that can assume different values.
  - a) Various letters of the alphabet, such as X, Y, Z, are used to represent variables.
  - b) Since the variables in this chapter are associated with probability, they are called **random variables**.
  - c) Random variable a variable whose values are determined by chance.
- 2. <u>Discrete variable</u> have finite number of possible values, or an infinite number of values that can be counted.
  - a) Whole numbers
- 3. <u>Continuous variable</u> can assme all values in the interval between any two given values.
  - a) Fractions, decimals.



## **DISCRETE OR CONTINUOUS?**

The outcomes when a die is rolled

A person's weight

 The temperature throughout 4 hours

The outcomes when you flip a coin



#### **PROBABILITY DISTRIBUTION**

Discrete Probability Distribution – consists of the values a random variable can assume and the corresponding probabilities of the values. The probabilities are determined theoretically or by observation.



#### THEORETICAL VS. OBSERVATIONAL

#### Theoretical

 Represent graphically the probability distribution for the sample space for tossing three coins.



#### THEORETICAL VS. OBSERVATIONAL

#### Observational

The baseball teams playing in the World Series play until one team wins four times. This means there can be anywhere from 4-7 games. Find the probability P(X) for each X, construct a probability distribution, and draw a graph for the data.

X	Number of
	games played
4	8
5	7
6	9
7	16



# TWO REQUIREMENTS FOR A PROBABILITY DISTRIBUTION

 The sum of the probabilities of all the events in the sample space must equal 1; that is,

 $\Sigma P(X) = 1$ 

 The probability of each event in the sample space must be between or equal to 0 and 1; that is,

 $0 \leq P(X) \leq 1$ 



#### DETERMINE WHETHER EACH IS A PROBABILITY DISTRIBUTION

a)	X	0	5	10	15	20
	P(X)	1/5	1/5	1/5	1/5	1/5
b)	x	1	2	3	4	
	P(X)	1/4	1/8	1/16	9/16	
c)	x	0	2	4	6	
	P(X)	-1.0	1.5	0.3	0.2	
d)	X	2	3	7		
	P(X)	0.5	0.3	0.4		



# **DO NOW**

Suppose that 7 people enter a swim meet. Assuming that there are no ties, in how many ways could the gold, silver, and bronze medals be awards?



#### MEAN, VARIANCE, STDEV, EXPECTED VALUES

•We are familiar with finding mean, variance, and standard deviation. However, for probability distributions they are computed differently than they are for samples.



#### MEAN

- How would you compute the mean of the number of dots that show on top when a die is rolled? You could roll it, say, 10 times, recording the number of dots, and finding the mean; however the answer would only approximate the true mean.
- To get an exact answer, we would have to roll the die an infinite number of times. Since this is impossible, we cannot use the old way to compute the mean.



#### MEAN

- Suppose two coins are tossed repeatedly, and the number of heads that occurred is recorded. What will be the mean of the number of heads?
  - What is the sample space?
  - What is the probability for each outcome in the sample space?
  - Probability of one head?
  - Probability of two heads?
  - Probability of no heads?



Hence, on average, you would expect the number of heads to be...

$$\frac{1}{4} \cdot 2 + \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 0 = 1$$

That is, if it were possible to toss the coins many times or an infinite number of times, the *average* number of heads would be 1.

So to find the mean for a probability distribution, you must multiply each possible outcome by its corresponding probability and find the sum of the products



#### MEAN

The mean of a random variable with a discrete probability distribution is

#### $\mu = X_1 \times P(X_1) + X_2 \times P(X_2) + \ldots + X_n \times P(X_n)$

#### $\mu = \Sigma X \times P(X)$

where  $X_1, X_2, ..., X_n$  are the outcomes and  $P(X_1), P(X_2), ..., P(X_n)$  are the corresponding probabilities.

\*\*Note:  $\mu = \Sigma X \times P(X)$  means to sum the products.



# VARIANCE AND STANDARD DEVIATION

• To find the <u>variance</u> for the random variable of a probability distribution, subtract the theoretical mean from the each outcome and square the difference. Then multiply each difference by its corresponding probability and add the products.

$$\sigma^2 = \Sigma[(X - \mu)^2 \times P(X)]$$

However, this is tedious, so here's a shortcut...



## VARIANCE AND STANDARD DEVIATION

Find the <u>variance</u> of a probability distribution by multiplying the square of each outcome by its corresponding probability, summing the products, and subtracting the square of the theoretical mean.

#### $\sigma^2 = \Sigma[X^2 \times P(X)] - \mu^2$

\*\*\*\*This is the shortcut. Use this one!\*\*\*\*

 As always, the standard deviation is just the square root of the variance.



- 1. Find the mean, variance, and standard deviation of the number of dots that appear when a die is tossed.
- 2. A box contains 5 balls. Two are numbered 3, one is numbered 4, and two are numbered 5. The balls are mixed and one is selected at random. After a ball is selected, it is recorded and replaced. If the experiment is repeated many times, find the mean, variance, and standard deviation of the numbers on the balls.



3. A talk radio station has four telephone lines. If the host is unable to talk (i.e. during a commercial) or is talking to a person, the other called are placed on hold. When all lines are in use, others who are trying to call in get a busy signal. The probability that 0, 1, 2, 3, or 4 people will get through is shown in the distribution. Find the mean, variance, and standard deviation for the distribution.

X	0	1	2	3	4
P(X)	0.18	0.34	0.23	0.21	0.04

Should the station have considered getting more phone lines installed?



#### EXPECTED VALUE

 The expected value of a discrete random variable of a probability distribution is the theoretical average of the variable.

$$\mu = E(X) = \Sigma X \times P(X)$$

- The symbol E(X) is used for the expected value.
- The formula is the same as the theoretical mean.
- Expected value = Theoretical mean



 One thousand tickets are sold at \$1 each for a color television valued at \$350. What is the expected value of the gain if you purchase one ticket?

 One thousand tickets are sold at \$1 each for four prizes of \$100, \$50, \$25, and \$10. After each prize drawing, the winning ticket is then returned to the pool of tickets. What is the expected value if you purchase one ticket?



3. A financial adviser suggests that his client select one of two types of bonds in which to invest \$5,000. Bond X pays a return of 4% and has a default rate of 2%. Bond Y has a 2.5% return and a default rate of 1%. Find the expected rate of return and decide which bond would be a better investment. When the bond defaults, the investor loses all the investment.



# **DO NOW**

•500 tickets are sold at \$1 each for an iPad valued at \$400. What is the expected value of the gain if you purchase one ticket?



### THE BINOMIAL DISTRIBUTION

- Many types of probability problems have only two outcomes or can be reduced to two outcomes.
  - Coin toss
  - Winning or losing a game
  - Baby being male or female
  - True/false questions
  - Effective or ineffective medical treatment
  - Normal or abnormal blood pressure
  - Correct or incorrect answer



#### THE BINOMIAL DISTRIBUTION

- A <u>binomial experiment</u> is a probability experiment that satisfies the following four requirements:
  - 1. There must be a fixed number of trials.
  - 2. Each trial can only have two outcomes, or outcomes that can be reduced to two outcomes. These outcomes can be considered as success or failure.
  - 3. The outcomes of each trial must be independent of one another.
  - 4. The probability of a success must remain the same for each trial.
- The outcomes of a binomial experiment and the corresponding probabilities of these outcomes are called a <u>binomial distribution</u>.



#### NOTATION FOR BINOMIAL DISTRIBUTION

- P(S) Probability of Success
- P(F) Probability of Failure
  - p Numerical Probability of Success
  - q Numerical Probability of Failure

P(S) = p and P(F) = 1 - p = q

- n Number of Trials
- X Number of Successes in n Trials

\*\*\*Note that  $0 \le X \le n$  and X = 0, 1, 2, 3, ..., n.



#### BINOMIAL PROBABILITY FORMULA

 In a binomial experiment, the probability of exactly X successes in n trials is

$$P(X) = \frac{n!}{(n-X)! X!} \times p^X \times q^{n-X}$$



# **DO NOW**

A package contains 12 resistors, 3 of which are defective. If 4 are selected, find the probability of getting 1 defective resistor.



1. A coin is tossed 3 times. Find the probability of getting exactly 2 heads.

2. A survey found that one out of five Americans say he or she has visited a doctor in any given month. If 10 people are selected at random, find the probability that exactly 3 will have visited a doctor last month.

3. A survey from Teenage Research Unlimited found that 30% of teenage consumers receive their spending money from part-time jobs. If 5 teenagers are selected at random, find the probability that at least 3 of them will have part-time jobs.



- 4. Public Opinion reported that 5% of Americans are afraid of being alone in a house at night. If a random sample of 20 Americans is selected, find these probabilities by using the binomial table.
  - a) There are exactly 5 people in the sample who are afraid of being alone at night.
  - b) There are at most 3 people in the sample who are afraid of being alone at night.
  - c) There are at least 3 people in the sample who are afraid of being alone at night.



# MEAN, VARIANCE, STDEV

- Mean:

   *µ* = n × p
- •Variance: • $\sigma^2 = n \times p \times q$
- •Standard Deviation: • $\sigma = sqrt(n \times p \times q)$



1. A coin it tossed 4 times. Find the mean, variance, and standard deviation of the number of heads that will be obtained.

2. A die is rolled 360 times. Find the mean, variance, and standard deviation of the number of 4s that will be rolled.

3. The *Statistics Bulletin* published by Metropolitan Life Insurance Co. reported that 2% of all American births result in twins. If a random sample of 8,000 births is taken, find the mean, variance, and standard deviation of the number of births that would result in twins.



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