Larson, Edwards Calculus of a Single Variable, 6e Early Transcendentals

10.3 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises

Finding a Derivative In Exercises 1–4, find dy/dx.

1. $x = t^2,\ y = 7 - 6t$
2. $x = \sqrt[3]{t},\ y = 4 - t$
3. $x = \sin^2\theta,\ y = \cos^2\theta$
4. $x = 2e^{\theta},\ y = e^{-\theta/2}$

Finding Slope and Concavity In Exercises 5–14, find dy/dx and d^2y/dx^2, and find the slope and concavity (if possible) at the given value of the parameter.

Parametric Equations	Parameter
5. $x = 4t,\ y = 3t - 2$	$t = 3$
6. $x = \sqrt{t},\ y = 3t - 1$	$t = 1$
7. $x = t + 1,\ y = t^2 + 3t$	$t = -1$
8. $x = t^2 + 5t + 4,\ y = 4t$	$t = 0$
9. $x = 4\cos\theta,\ y = 4\sin\theta$	$\theta = \frac{\pi}{4}$
10. $x = \cos\theta,\ y = 3\sin\theta$	$\theta = 0$
11. $x = 2 + \sec\theta,\ y = 1 + 2\tan\theta$	$\theta = \frac{\pi}{6}$
12. $x = \sqrt{t},\ y = \sqrt{t - 1}$	$t = 2$
13. $x = \cos^3\theta,\ y = \sin^3\theta$	$\theta = \frac{\pi}{4}$
14. $x = \theta - \sin\theta,\ y = 1 - \cos\theta$	$\theta = \pi$

Finding Equations of Tangent Lines In Exercises 15–18, find an equation of the tangent line at each given point on the curve.

15. $x = 2\cot\theta,\quad y = 2\sin^2\theta,$
$\left(-\frac{2}{\sqrt{3}}, \frac{3}{2}\right), (0, 2), \left(2\sqrt{3}, \frac{1}{2}\right)$

16. $x = 2 - 3\cos\theta,\quad y = 3 + 2\sin\theta,$
$(-1, 3), (2, 5), \left(\frac{4 + 3\sqrt{3}}{2}, 2\right)$

17. $x = t^2 - 4,\quad y = t^2 - 2t,\quad (0, 0), (-3, -1), (-3, 3)$

18. $x = t^4 + 2,\quad y = t^3 + t,\quad (2, 0), (3, -2), (18, 10)$

Finding an Equation of a Tangent Line In Exercises 19–22, (a) use a graphing utility to graph the curve represented by the parametric equations, (b) use a graphing utility to find dx/dt, dy/dt, and dy/dx at the given value of the parameter, (c) find an equation of the tangent line to the curve at the given value of the parameter, and (d) use a graphing utility to graph the curve and the tangent line from part (c).

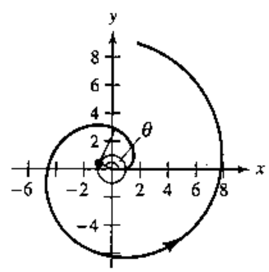
Parametric Equations	Parameter
19. $x = 6t,\ y = t^2 + 4$	$t = 1$
20. $x = t - 2,\ y = \frac{1}{t} + 3$	$t = 1$
21. $x = t^2 - t + 2,\ y = t^3 - 3t$	$t = -1$
22. $x = 3t - t^2,\quad y = 2t^{3/2}$	$t = \frac{1}{4}$

Finding Equations of Tangent Lines In Exercises 23–26, find the equations of the tangent lines at the point where the curve crosses itself.

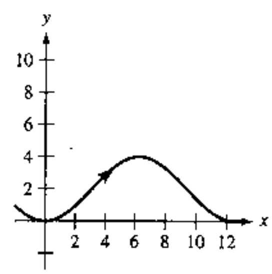
23. $x = 2\sin 2t,\quad y = 3\sin t$
24. $x = 2 - \pi\cos t,\quad y = 2t - \pi\sin t$
25. $x = t^2 - t,\quad y = t^3 - 3t - 1$
26. $x = t^3 - 6t,\quad y = t^2$

Horizontal and Vertical Tangency In Exercises 27 and 28, find all points (if any) of horizontal and vertical tangency to the portion of the curve shown.

27. Involute of a circle:
$x = \cos\theta + \theta\sin\theta$
$y = \sin\theta - \theta\cos\theta$



28. $x = 2\theta$
$y = 2(1 - \cos\theta)$

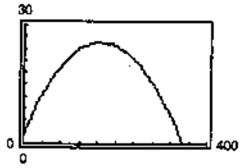


Horizontal and Vertical Tangency In Exercises 29–38, find all points (if any) of horizontal and vertical tangency to the curve. Use a graphing utility to confirm your results.

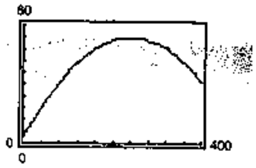
29. $x = 4 - t,\quad y = t^2$
30. $x = t + 1,\quad y = t^2 + 3t$
31. $x = t + 4,\quad y = t^3 - 3t$
32. $x = t^2 - t + 2,\quad y = t^3 - 3t$
33. $x = 3\cos\theta,\quad y = 3\sin\theta$
34. $x = \cos\theta,\quad y = 2\sin 2\theta$
35. $x = 5 + 3\cos\theta,\quad y = -2 + \sin\theta$
36. $x = 4\cos^2\theta,\quad y = 2\sin\theta$
37. $x = \sec\theta,\quad y = \tan\theta$
38. $x = \cos^2\theta,\quad y = \cos\theta$

Determining Concavity In Exercises 39–44, determine the open t-intervals on which the curve is concave downward or concave upward.

39. $x = 3t^2,\quad y = t^3 - t$
40. $x = 2 + t^2,\quad y = t^2 + t^3$
41. $x = 2t + \ln t,\quad y = 2t - \ln t$
42. $x = t^2,\quad y = \ln t$
43. $x = \sin t,\quad y = \cos t,\quad 0 < t < \pi$
44. $x = 4\cos t,\quad y = 2\sin t,\quad 0 < t < 2\pi$

69. d; (4, 0) is on the graph. **71.** b; (1, 0) is on the graph.
73. $x = a\theta - b\sin\theta$; $y = a - b\cos\theta$
75. False. The graph of the parametric equations is the portion of the line $y = x$ when $x \ge 0$.
77. True
79. (a) $x = \left(\frac{440}{3}\cos\theta\right)t$; $y = 3 + \left(\frac{440}{3}\sin\theta\right)t - 16t^2$
(b)


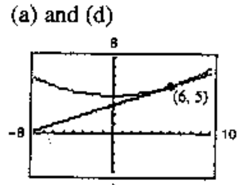
Not a home run

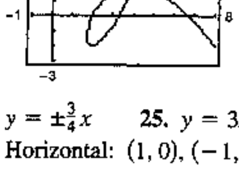
(c)


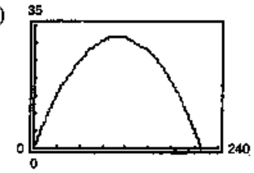
Home run

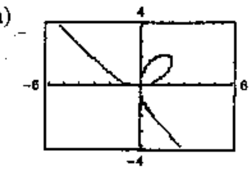
(d) 19.4°

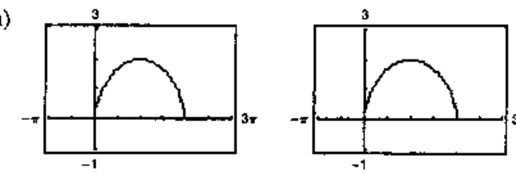
Section 10.3 *(page 711)*

1. $-3/t$ **3.** -1
5. $\dfrac{dy}{dx} = \dfrac{3}{4}, \dfrac{d^2y}{dx^2} = 0$; Neither concave upward nor concave downward
7. $dy/dx = 2t + 3$, $d^2y/dx^2 = 2$
At $t = -1$, $dy/dx = 1$, $d^2y/dx^2 = 2$; Concave upward
9. $dy/dx = -\cot\theta$, $d^2y/dx^2 = -(\csc\theta)^3/4$
At $\theta = \pi/4$, $dy/dx = -1$, $d^2y/dx^2 = -\sqrt{2}/2$;
Concave downward
11. $dy/dx = 2\csc\theta$, $d^2y/dx^2 = -2\cot^3\theta$
At $\theta = \pi/6$, $dy/dx = 4$, $d^2y/dx^2 = -6\sqrt{3}$;
Concave downward
13. $dy/dx = -\tan\theta$, $d^2y/dx^2 = \sec^4\theta\csc\theta/3$
At $\theta = \pi/4$, $dy/dx = -1$, $d^2y/dx^2 = 4\sqrt{2}/3$;
Concave upward
15. $\left(-2/\sqrt{3}, 3/2\right)$: $3\sqrt{3}x - 8y + 18 = 0$
$(0, 2)$: $y - 2 = 0$
$\left(2\sqrt{3}, 1/2\right)$: $\sqrt{3}x + 8y - 10 = 0$
17. $(0, 0)$: $2y - x = 0$
$(-3, -1)$: $y + 1 = 0$
$(-3, 3)$: $2x - y + 9 = 0$
19. (a) and (d)


(b) At $t = 1$, $dx/dt = 6$, $dy/dt = 2$, and $dy/dx = 1/3$.
(c) $y = \frac{1}{3}x + 3$
21. (a) and (d)


(b) At $t = -1$, $dx/dt = -3$, $dy/dt = 0$, and $dy/dx = 0$.
(c) $y = 2$
23. $y = \pm\frac{3}{4}x$ **25.** $y = 3x - 5$ and $y = 1$
27. Horizontal: $(1, 0)$, $(-1, \pi)$, $(1, -2\pi)$
Vertical: $(\pi/2, 1)$, $(-3\pi/2, -1)$, $(5\pi/2, 1)$
29. Horizontal: $(4, 0)$
Vertical: None
31. Horizontal: $(5, -2)$, $(3, 2)$
Vertical: None
33. Horizontal: $(0, 3)$, $(0, -3)$
Vertical: $(3, 0)$, $(-3, 0)$
35. Horizontal: $(5, -1)$, $(5, -3)$
Vertical: $(8, -2)$, $(2, -2)$
37. Horizontal: None
Vertical: $(1, 0)$, $(-1, 0)$
39. Concave downward: $-\infty < t < 0$
Concave upward: $0 < t < \infty$
41. Concave upward: $t > 0$
43. Concave downward: $0 < t < \pi/2$
Concave upward: $\pi/2 < t < \pi$
45. $4\sqrt{13} \approx 14.422$ **47.** $\sqrt{2}(1 - e^{-\pi/2}) \approx 1.12$
49. $\frac{1}{12}\left[\ln\left(\sqrt{37} + 6\right) + 6\sqrt{37}\right] \approx 3.249$ **51.** $6a$ **53.** $8a$
55. (a)


(b) 219.2 ft
(c) 230.8 ft
57. (a)


(b) $(0, 0)$, $\left(4\sqrt[3]{2}/3, 4\sqrt[3]{4}/3\right)$
(c) About 6.557
59. (a)


(b) The average speed of the particle on the second path is twice the average speed of the particle on the first path.
(c) 4π
61. $S = 2\pi\displaystyle\int_0^4 \sqrt{10}(t + 2)\,dt = 32\pi\sqrt{10} \approx 317.907$
63. $S = 2\pi\displaystyle\int_0^{\pi/2} \left(\sin\theta\cos\theta\sqrt{4\cos^2\theta + 1}\right)d\theta$
$= \dfrac{\left(5\sqrt{5} - 1\right)\pi}{6}$
≈ 5.330
65. (a) $27\pi\sqrt{13}$ (b) $18\pi\sqrt{13}$ **67.** 50π **69.** $12\pi a^2/5$
71. See Theorem 10.7, Parametric Form of the Derivative, on page 706.
73. 6
75. (a) $S = 2\pi\displaystyle\int_a^b g(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
(b) $S = 2\pi\displaystyle\int_a^b f(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
77. Proof **79.** $3\pi/2$ **81.** d **82.** b **83.** f **84.** c
85. a **86.** e **87.** $\left(\frac{3}{4}, \frac{8}{5}\right)$ **89.** 288π
91. (a) $dy/dx = \sin\theta/(1 - \cos\theta)$; $d^2y/dx^2 = -1/[a(\cos\theta - 1)^2]$
(b) $y = \left(2 + \sqrt{3}\right)\left[x - a\left(\pi/6 - \frac{1}{2}\right)\right] + a\left(1 - \sqrt{3}/2\right)$
(c) $(a(2n + 1)\pi, 2a)$
(d) Concave downward on $(0, 2\pi)$, $(2\pi, 4\pi)$, etc.
(e) $s = 8a$
93. Proof