8-1 Attributes of Polynomial Functions

TEKS FOCUS

Extends TEKS (2)(A) Graph the functions $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.

TEKS (1)(F) **Analyze** mathematical relationships to connect and communicate mathematical ideas.

Additional TEKS (1)(D)

VOCABULARY

- **Degree of a monomial** – the sum of the exponents of the variables
- **Degree of a polynomial** – for a polynomial in one variable, the greatest degree of the monomial terms
- **End behavior** – the direction of the graph to the far left and to the far right
- **Monomial** – a real number, a variable, or a product of a real number and one or more variables with whole-number exponents
- **Polynomial** – a monomial or a sum of monomials
- **Polynomial function** – A polynomial in the variable x defines a polynomial function of x.
- **Standard form of a polynomial function** – the polynomial function with the terms arranged by degree in descending numerical order
- **Turning point** – a point where the graph changes direction from upwards to downwards or from downwards to upwards

- **Analyze** – closely examine objects, ideas, or relationships to learn more about their nature

ESSENTIAL UNDERSTANDING

A polynomial function has distinguishing "behaviors." You can look at its algebraic form and know something about its graph. You can look at its graph and know something about its algebraic form.

take note Key Concept Standard Form of a Polynomial Function

The **standard form of a polynomial function** arranges the terms by degree in descending numerical order.

A polynomial function $P(x)$ in standard form is

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0,$$

where n is a nonnegative integer and $a_n, \ldots, a_0$ are real numbers.

$$P(x) = \underbrace{4x^3}_{\text{Cubic term}} + \underbrace{3x^2}_{\text{Quadratic term}} + \underbrace{5x}_{\text{Linear term}} - \underbrace{2}_{\text{Constant term}}$$

Key Concept Classifying Polynomials

You can classify a polynomial by its degree or by its number of terms. Polynomials of degrees zero through five have specific names, as shown in this table.

Degree	Name Using Degree	Polynomial Example	Number of Terms	Name Using Number of Terms
0	constant	5	1	monomial
1	linear	$x + 4$	2	binomial
2	quadratic	$4x^2$	1	monomial
3	cubic	$4x^3 - 2x^2 + x$	3	trinomial
4	quartic	$2x^4 + 5x^2$	2	binomial
5	quintic	$-x^5 + 4x^2 + 2x + 1$	4	polynomial of 4 terms

Key Concept Polynomial Functions

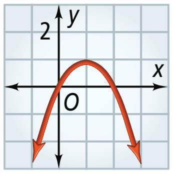
$y = 4x^4 + 6x^3 - x$

End Behavior: Up and Up

Turning Points: $(-1.07, -1.04)$, $(-0.27, 0.17)$, and $(0.22, -0.15)$

The function is decreasing when $x < -1.07$ and $-0.27 < x < 0.22$. The function increases when $-1.07 < x < -0.27$ and $x > 0.22$.

$y = -x^2 + 2x$

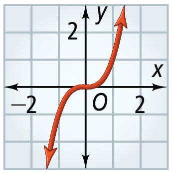


End Behavior: Down and Down

Turning Point: $(1, 1)$

The function is increasing when $x < 1$ and is decreasing when $x > 1$.

$y = x^3$

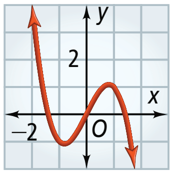


End Behavior: Down and Up

Zero turning points.

The function is increasing for all x.

$y = -x^3 + 2x$



End Behavior: Up and Down

Turning Points: $(-0.82, -1.09)$ and $(0.82, 1.09)$

The function is decreasing when $x < -0.82$ and when $x > 0.82$. The function is increasing when $-0.82 < x < 0.82$.

Key Concept Determining End Behavior

take note

You can determine the end behavior of a polynomial function of degree n from the leading term ax^n of the standard form.

End Behavior of a Polynomial Function With Leading Term ax^n

	n Even ($n \neq 0$)	n Odd
a Positive	Up and Up	Down and Up
a Negative	Down and Down	Up and Down

In general, the graph of a polynomial function of degree n ($n \geq 1$) has at most $n - 1$ turning points. The graph of a polynomial function of odd degree has an even number of turning points. The graph of a polynomial function of even degree has an odd number of turning points.

Problem 1

Classifying Polynomials

Think

How do you write a polynomial in standard form?
Combine like terms if possible. Then, write the terms with their degrees in descending order.

Write each polynomial in standard form. What is the classification of each polynomial by degree? By number of terms?

A $3x + 9x^2 + 5$

$9x^2 + 3x + 5$

The polynomial has degree 2 and 3 terms. It is a quadratic trinomial.

B $4x - 6x^2 + x^4 + 10x^2 - 12$

$x^4 + 4x^2 + 4x - 12$

The polynomial has degree 4 and 4 terms. It is a quartic polynomial of 4 terms.

Problem 2

Describing End Behavior of Polynomial Functions

Think

What do a and n represent?
a is the coefficient of the leading term. n is the exponent of the leading term.

Consider the leading term of each polynomial function. What is the end behavior of the graph? Check your answer with a graphing calculator.

A $y = 4x^3 - 3x$

The leading term is $4x^3$. Since n is odd and a is positive, the end behavior is down and up.

B $y = -2x^4 + 8x^3 - 8x^2 + 2$

The leading term is $-2x^4$. Since n is even and a is negative, the end behavior is down and down.

Problem 3

TEKS Process Standard (1)(F)

Graphing Cubic Functions

What is the graph of each cubic function? Describe the graph, including end behavior, turning points, and increasing/decreasing intervals.

Plan

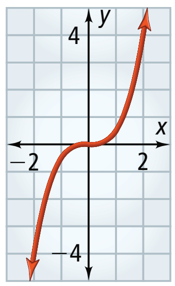
How can you graph a polynomial function? Make a table of values to help you sketch the middle part of the graph. Use what you know about end behavior to sketch the ends of the graph.

A $y = \frac{1}{2}x^3$

Step 1

x	y
−2	−4
−1	−0.5
0	0
1	0.5
2	4

Step 2



Step 3

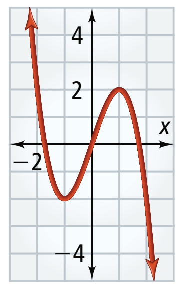
The end behavior is down and up. There are no turning points. The function is increasing for all x.

B $y = 3x - x^3$

Step 1

x	y
−2	2
−1	−2
0	0
1	2
2	−2

Step 2



Step 3

The end behavior is up and down with turning points at $(-1, -2)$ and $(1, 2)$. The function is decreasing when $x < -1$ and when $x > 1$. It is increasing when $-1 < x < 1$.

Problem 4

TEKS Process Standard (1)(D)

Using Differences to Determine Degree

What is the degree of the polynomial function for the data?

x	y
−3	−1
−2	−7
−1	−3
0	5
1	11
2	9
3	−7

With consecutive input values that differ by a constant, you can analyze the output differences to find the least-degree polynomial for the data.

If the first differences are constant, the function is linear. If the second differences (but not the first) are constant, the function is quadratic. If the third differences (but not the second) are constant, the function is cubic, and so on.

Think

How do you find the second differences? Subtract the consecutive first differences.

***y*-value**	−1	−7	−3	5	11	9	−7
1st difference	−6	4	8	6	−2	−16	
2nd difference	10	4	−2	−8	−14		
3rd difference	−6	−6	−6	−6			

The third differences are constant.

The degree of the polynomial function is 3.

PRACTICE and APPLICATION EXERCISES

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For additional support when completing your homework, go to PearsonTEXAS.com.

1. **Apply Mathematics (1)(A)** The data show the power generated by a wind turbine. The x column gives the wind speed in meters per second. The y column gives the power generated in kilowatts. What is the degree of the polynomial function that models the data?

x	y
5	10
6	17.28
7	27.44
8	40.96
9	58.32

Classify each polynomial by degree and by number of terms. Simplify first if necessary.

2. $a^2 + a^3 - 4a^4$

3. 7

4. $2x(3x)$

5. $(2a - 5)(a^2 - 1)$

6. $(-8d^3 - 7) + (-d^3 - 6)$

7. $b(b - 3)^2$

8. Copy and complete the table, which shows the first and second differences in y-values for consecutive x-values for a polynomial function of degree 2.

x	y	1st diff.	2nd diff.
−3	14	−8	2
−2	6	■	2
−1	■	−4	2
0	−4	−2	2
1	■	0	2
2	−6	■	
3	■		

9. **Use Multiple Representations to Communicate Mathematical Ideas (1)(D)** The outputs for a certain function are 1, 2, 4, 8, 16, 32, and so on.

a. Find the first differences of this function.

b. Find the second differences of this function.

c. Find the tenth difference of this function.

d. Can you find a polynomial function that matches the original outputs? Explain your reasoning.

10. **Connect Mathematical Ideas (1)(F)** A cubic polynomial function f has leading coefficient 2 and constant term 7. If $f(1) = 7$ and $f(2) = 9$, what is $f(-2)$? Explain how you found your answer.

Describe the shape of the graph of each cubic function including end behavior, turning points, and increasing/decreasing intervals.

11. $y = 3x^3 - x - 3$

12. $y = -9x^3 - 2x^2 + 5x + 3$

13. $y = 10x^3 + 9$

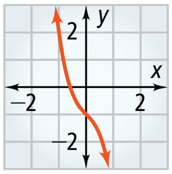
14. $y = 3x^3$

15. $y = -4x^3 - 5x^2$

16. $y = 8x^3$

Determine the sign of the leading coefficient and the least possible degree of the polynomial function for each graph.

17.



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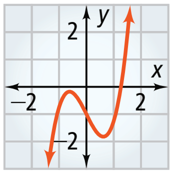

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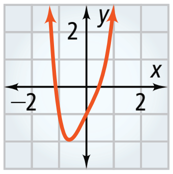

Determine the end behavior of the graph of each polynomial function.

20. $y = -7x^3 + 8x^2 + x$

21. $y = -3x + 6x^2 - 1$

22. $y = 1 - 4x - 6x^3 - 15x^6$

23. $y = 8x^{11} - 2x^9 + 3x^6 + 4$

24. $y = -x^5 - 15x^7 - 4x^9$

25. $y = -3 - 6x^5 - 9x^8$

26. **Connect Mathematical Ideas (1)(F)** Write an equation for a polynomial function that has three turning points and end behavior up and up.

27. Show that the third differences of a polynomial function of degree 3 are nonzero and constant. First, use $f(x) = x^3 - 3x^2 - 2x - 6$. Then show third differences are nonzero and constant for $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$.

28. Suppose that a function pairs elements from set A with elements from set B. A function is called *onto* if it pairs every element in B with at least one element in A. For each type of polynomial function, and for each set B, determine whether the function is *always, sometimes,* or *never* onto.

a. linear; $B =$ all real numbers

b. quadratic; $B =$ all real numbers

c. quadratic; $B =$ all real numbers greater than or equal to 4

d. cubic; $B =$ all real numbers

29. Make a table of second differences for each polynomial function. Using your tables, make a conjecture about the second differences of quadratic functions.

a. $y = 2x^2$ **b.** $y = 7x^2$ **c.** $y = 7x^2 + 1$ **d.** $y = 7x^2 + 3x + 1$

Determine the degree of the polynomial function with the given data.

30.

x	−2	−1	0	1	2
y	16	7	2	1	4

31.

x	−2	−1	0	1	2
y	−15	−9	−9	−9	−3

TEXAS Test Practice

32. Which expression is a cubic polynomial?

A. x^3 **B.** $3x + 3$ **C.** $2x^2 + 3x - 1$ **D.** $3x$

33. Which equation has $-3 \pm 5i$ as its solutions?

F. $x^2 + 6x = -34$ **G.** $x^2 + 6x = -14$ **H.** $x^2 + 3x = 4$ **J.** $x^2 + 3x = 2$

34. What is the discriminant of $qx^2 + rx + s = 0$?

A. qrs **B.** $q^2 - 4rs$ **C.** $r^2 - 4qs$ **D.** $s^2 - 4qr$

35. What is a simpler form of $x^2(3x^2 - 2x) - 3x^4$? Classify the polynomial by degree and by number of terms.