① $f'(x) = 12x^2 - 6x + 1$, $f(1) = 5$, $f(0) =$

$f(x) = 4x^3 - 3x^2 + x + C$

at $(1,5)$: $5 = 4 - 3 + 1 + C$

$\boxed{C = 3}$

so $f(x) = 4x^3 - 3x^2 + x + 3$

so $\boxed{f(0) = 3}$ $\boxed{B}$

② $g'(x) = \dfrac{5x^2 + 4x + 5}{\sqrt{x}}$

$g'(x) = \dfrac{5x^2}{x^{1/2}} + \dfrac{4x}{x^{1/2}} + \dfrac{5}{x^{1/2}}$

$g'(x) = 5x^{3/2} + 4x^{1/2} + 5x^{-1/2}$

$g(x) = 5\left(\frac{2}{5}\right)x^{5/2} + 4\left(\frac{2}{3}\right)x^{3/2} + 5(2)x^{1/2} + C$

$g(x) = 2x^{5/2} + \frac{8}{3}x^{3/2} + 10x^{1/2} + C$

factor out least powers

$\boxed{g(x) = 2x^{1/2}\left(x^2 + \frac{4}{3}x + 5\right) + C}$ $\boxed{B}$

③ $f''(t) = 2(3t+1)$, $f'(1) = 3$, $f(1) = 5$

$f''(t) = 6t + 2$

$f'(t) = 3t^2 + 2t + C$

for $f'(1) = 3$: $3 = 3 + 2 + C$ $\boxed{C = -2}$

so $f'(t) = 3t^2 + 2t - 2$

$f(t) = t^3 + t^2 - 2t + C$

for $f(1) = 5$: $5 = 1 + 1 - 2 + C$ $\boxed{C = 5}$

so $\boxed{f(t) = t^3 + t^2 - 2t + 5}$ $\boxed{E}$

④ Antiderivatives of $f(x) = \sin x \cos x$.

Plan: take derivative of each I, II, and III

I. $F_1(x) = \frac{1}{2}(\sin x)^2$, $F_1'(x) = (\sin x)' \cdot \cos x$ ✓

II. $F_2(x) = -\frac{1}{4}\cos 2x = -\frac{1}{4}\underbrace{(\cos^2 x - \sin^2 x)}_{}$ Double-Angle

$F_2'(x) = -\frac{1}{4}(2\cos x(-\sin x) - 2\sin x \cos x)$

$= -\frac{1}{4}(-4\sin x \cos x)$

$= \sin x \cos x$ ✓

III. $F_3(x) = -\frac{1}{2}(\cos x)^2$, $F_3'(x) = -(\cos x)'(-\sin x)$

$= \sin x \cos x$ ✓

So $\boxed{D}$ $\boxed{\text{I, II, \& III}}$

⑤ $a(t) = 8 - 8t$, $v(0) = 12$

$v(t) = 8t - 4t^2 + C$

for $v(0) = 12$: $12 = C$

so $v(t) = 8t - 4t^2 + 12$

$x(t) = \text{position} = -\frac{4}{3}t^3 + 4t^2 + 12t + C$

Maximize $x(t)$

so $x'(t) = v(t) = -4t^2 + 8t + 12 = 0$

$-4(t^2 - 2t - 3) = 0$

$-4(t-3)(t+1) = 0$

So $t = 3$ or $\cancel{t = -1}$

since $x''(3) = a(3) = 8 - 8(3) < 0$,

$t = 3$ maximizes $x(t)$

so $\boxed{3 \text{ seconds}}$ $\boxed{C}$

⑥ (a) $\int (\sqrt{x^3} + 2x + 1)\,dx$

$= \int (x^{3/2} + 2x + 1)\,dx$

$= \boxed{\frac{2}{5}x^{5/2} + x^2 + x + C}$

(b) $\int \left(\dfrac{x^3 + 2x - 3}{x^4}\right) dx$

$= \int \left(\dfrac{x^3}{x^4} + \dfrac{2x}{x^4} - \dfrac{3}{x^4}\right) dx$

$= \int \left(\frac{1}{x} + 2x^{-3} - 3x^{-4}\right) dx$

$= \boxed{\ln|x| - x^{-2} + x^{-3} + C}$

(6) (c) $\int (2t^2-1)^2 dt$
$= \int (4t^4 - 4t^2 + 1)\,dt$
$= \boxed{\frac{4}{5}t^5 - \frac{4}{3}t^3 + t + C}$

(d) $\int (\theta^2 + \sec^2\theta - \csc\theta\cot\theta)\,d\theta$
$= \boxed{\frac{1}{3}\theta^3 + \tan\theta + \csc\theta + C}$

(e) $\int \left(\frac{\cos x}{1-\cos^2 x}\right)dx$
$= \int \left(\frac{\cos x}{\sin^2 x}\right)dx$
$= \int \left(\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}\right)dx$
$= \int (\csc x \cot x)\,dx$
$= \boxed{-\csc x + C}$

(f) $\int (\cos x + 3^x)\,dx$
$= \boxed{\sin x + \frac{1}{\ln 3}\cdot 3^x + C}$

(7) (a) $f'(x) = 4x,\ f(0) = 6$
$f(x) = 2x^2 + C$
for $f(0)=6$: $6 = 2(0^2) + C$
$C = 6$
so $\boxed{f(x) = 2x^2 + 6}$

(b) $h'(t) = 8t^3 + 5,\ h(1) = -4$
$h(t) = 2t^4 + 5t + C$
for $h(1) = -4$: $-4 = 2 + 5 + C$
$C = -11$
so $\boxed{h(t) = 2t^4 + 5t - 11}$

(c) $f''(x) = 2,\ f'(2) = 5,\ f(2) = 10$
$f'(x) = 2x + C,\ 5 = 2(2) + C$
$C = 1$
so $f'(x) = 2x + 1$
$f(x) = x^2 + x + C,\ 10 = 2^2 + 2 + C$
$C = 4$
so $\boxed{f(x) = x^2 + x + 4}$

(d) $f''(x) = x^{-3/2},\ f'(4) = 2,\ f(0) = 0$
$f'(x) = -2x^{-1/2} + C;\ 2 = \frac{-2}{\sqrt{4}} + C,\ \boxed{C = 3}$
so $f'(x) = -2x^{-1/2} + 3$
$f(x) = -4x^{1/2} + 3x + C;\ 0 = 0 + 0 + C,\ \boxed{C = 0}$
so $\boxed{f(x) = -4\sqrt{x} + 3x}$

(e) $f''(x) = \sin x,\ f'(0) = 1,\ f(0) = 6$
$f'(x) = -\cos x + C,\ 1 = -\cos(0) + C$
$C = 2$
so $f'(x) = -\cos x + 2$

$f(x) = -\sin x + 2x + C,\ 6 = -\sin(0) + 2(0) + C$
$C = 6$
so $\boxed{f(x) = -\sin x + 2x + 6}$