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Using Math in Physics: *6. Reading the physics in a graph* FREE

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Using Math in Physics:

6. Reading the physics in a graph

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Learning to use math in physics involves combining ("blending") our everyday experiences and the conceptual ideas of physics with symbolic mathematical representations. Graphs are one of the best ways to learn to "build the blend." They are a mathematical representation that builds on visual recognition to create a bridge between words and equations. But students in introductory physics classes often see a graph as an endpoint—a task the teacher asks them to complete—rather than as a tool to help them make sense of a physical system. And most of the graph problems in traditional introductory physics texts simply ask students to extract a number from a graph. But if graphs are used appropriately, they can be a powerful tool in helping students learn to build the blend and develop their physical intuition and ability to think with math.

In this paper, I outline the issues involved in students' learning to use graphs effectively and present a variety of graphical problems to help them build the blend. This paper is one of a series of papers on using math in physics.[1] Math is used differently in physics than it is presented to students in math classes. Understanding these differences can help instructors understand student struggles and create more effective intuition-building curricula. The series identifies a number of distinct tools, often taken for granted by instructors, that students need to learn in order to "think with math."

In my text and class slides, when I use one of these tools, I note it with a small icon, reminding the students that we are using one of our general collection of methods for building physical knowledge with math. The icon I use for reading the physics in a graph is a screwdriver, shown at the top of this page, since it is one of the basic tools for taking something apart or putting it back together.

Since graphical problems take up a lot of space and my space here is limited, I've put many of the problems I want to share in the Supplementary Materials.[2] I encourage you to explore those as well as the ones given here.

Graphs are an important component of learning to build the blend

In professional practice, graphs are perhaps *the* most important way to code scientific information mathematically. As physicists, we might be prejudiced in favor of powerful sets of equations, such as Maxwell's equations, the Schrödinger and Dirac equations, or the Navier–Stokes equation; and advanced physics texts support this prejudice. But if you look at papers in the top journals that cover multiple sciences, such as *Science* or *Nature*, many of them display information as graphs. Very few use any equations.

But more than that, many students find it hard to get beyond the idea that science is about a list of memorized facts and algorithms (often inferred from their experience in earlier science classes). They find it hard to see math in science as about coding a coherent, sense-making understanding of the physical world and how it works. Graphs can help them learn to see math as sense-making. The reasons behind the success of this approach lie in the basic cognitive science of how we build complex concepts, and these ideas lead us to use graphs in specific blend-directed ways.

I developed many of the problems presented here from my experience as an instructor in classes designed to teach higher-order thinking skills in algebra-based physics classes with nonmajors (especially life-science students)[3,4] combined with theoretical perspectives drawn from the physics education research (PER) and cognitive sciences literatures cited below. I found that asking students to regularly translate between graphs, physics, and equations helped them become more comfortable with the idea of thinking mathematically.

In this paper, I focus on using graphs to help students "learn to build the blend of physics and math." I do not discuss the (huge) literature on the role of graphs in the laboratory and in the representation of experimental data.[5]

Complex concepts begin with physical experience

We create the world we live in from our sensory experience as infants—seeing, touching, moving. Cognitive scientists use the phrase *embodied cognition* to express the idea that we build our understanding of complex and abstract concepts beginning with physical experience[6] via metaphor[7] and blending—combining mental models to create something new.[8] (A concise description of blending and how it is used to create new ways of thinking about math in physics is given in earlier papers in this series and in Ref. 9.)

Since vision is the primary tool our brain uses to create the world we perceive,[10] and since graphs are a visual representation, they are an excellent place for students to learn to build the physics–math blend.

Graphs for the eye and graphs for the mind

Graphs build on the sense of space and location we develop as infants. The most straightforward symbolic representation of that sense is a *map*—a representation of actual physical space—a *graph for the eye*. I present two examples in Fig. 1.

On the left is a map with a walk from the College Park Metro Station to the Physics Building at the University of Maryland. If I did that walk and a drone watched me from above, the blue line would be the path I followed—a graph for the eye (or for the camera). On the right of Fig. 1 is a 3D map and the track of three random walkers diffusing in a gas—a graph for the eye in an important physical situation. Most high school and college students who have used a GPS to get

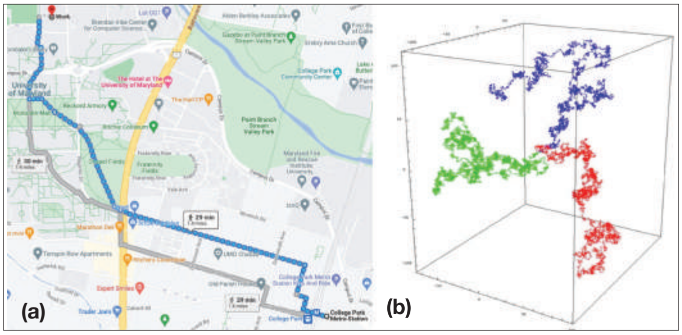


Fig. 1. (a) The track of a walk on a 2D map (Google Maps). (b) The track of three random walkers on a 3D map.

somewhere understand this concept.

But most of the graphs that we use in physics are not maps. They extend the concept of map metaphorically, creating *a graph for the mind*—one that needs an analysis or transformation to interpret.[11]

Students have many challenges with learning to use graphs effectively in physics

The skill of seeing what a graph has to tell you is as challenging to learn as the equation translation skills discussed in previous papers in this series. Learning to use graphs as effective tools in science requires four general skills:

- ***Constructing*** the representation and understanding how it codes mathematical information
- ***Interpreting*** what the information coded in the graph is telling you about the physical system
- ***Connecting*** the visual representation in the graph to equations describing the phenomena
- ***Knowing when and how*** to use graphs appropriately in physical problems

We need to understand the challenges students face learning these skills in order to find effective ways to scaffold problems to help them learn to build the physics/math blend.

Challenges in constructing graphs

Even in a purely mathematical context, constructing a graph requires learning many specific skills, including[12]

- creating the palette (axes, coordinates),
- placing and reading points on a graph as associated pairs,
- creating associated pairs from a function,
- identifying maxima, minima, zeros, crossing points, and other salient features.

By the time they've reached an AP physics or college physics class, most students have mastered these skills, at least in the context of creating a graph given a function or a set of pairs. But even in a math context, students may have difficulties that affect their ability to make physical sense with graphs.

Challenges in interpreting graphs

Student difficulties with reading graphs are the subject of extensive STEM education research.[13–15] Here are four errors documented in this literature that I have seen from my students in introductory physics classes that create barriers to their building the blend:

- ***Treating a graph as a picture***—A student may look at a rising velocity graph and say, "that rise means it's moving up" (or to the right, or anything about where it is in the image rather than a statement about its velocity). This is a "one-step-thinking" quick response that is quite natural,[16] especially if you aren't focused on the difference between "a graph for the eye" and "a graph for the mind."
- ***Confusing slope and height***—This is a common conceptual error that we also see when students work with symbolic representations: confusing a value, a change in a value, and a rate of change of that value.
- ***Confusing an interval and a point***—This is mostly relevant when students get confused trying to create a derivative at a point rather than realizing an interval is needed.
- ***Reading positive and negative areas under a curve***—When we are looking at a graph for constructing an integral, areas have to be treated as signed quantities. This can be challenging for many students as "area" intuitively seems like "a positive thing."

Challenges in connecting graphs and equations

A lot of the value of a graph in a physics class is helping students connect graphs to equations. If they learn to blend physical conceptual knowledge with a graph, that can help them bring equations into the blend.

Connecting graphs with equations is a translation skill between two very different kinds of symbolic representations. The difficulties students have with these are similar to those in connecting graphs to physical situations, for example, not being able to identify the y-intercept on a graph from an equation or confusing the slope and the value. Many of these are discussed in some of the previously cited references.[13–15]

Learning to make these connections can help students understand how equations code for physical meaning in complex situations such as traveling waves or parametric dependences.

Challenges in knowing when and how to use graphs

But just as being able to do symbolic math in the context of an abstract math class doesn't imply being able to use those skills in physics, being able to demonstrate graphical skills in math doesn't mean they can do them in physics. Many papers in the PER literature document student difficulties with interpreting the physical information in graphs in a physical context that are analogous to the ones found in math.[17]

Some of the difficulties that students have when using graphs in physics are simply translations of the difficulties seen with any graphs. But some arise because of the additional complexity of using math in physical situations. Because we are asking students to combine knowledge from two distinct domains, abstract math and conceptual physics, many of the difficulties students encounter are *epistemological*—errors about the nature of the knowledge they are learning.

The critical epistemological issue is often *framing*.[18] When you are given a problem to solve, you first must answer the questions, "What's going on here? Given the current situation (problem), what knowledge do I have that I can usefully apply?" This step is often not conscious but can be a response to expectations based on previous experiences in situations perceived as similar. Some framing errors I've often seen are as follows:

- Seeing graphs as solutions rather than sense-making tools
- Treating the graph as math rather than a way to express physical information
- Not seeing that multiple graphs are ways of highlighting different information about a physical situation

If asked to create a graph in a physics problem, students will often just sketch a pair of axes. Often, these will be unlabeled (or marked as "*x*" and "*y*" no matter what is actually being plotted) and have no tick marks to indicate scales on either axis.

But if they are to begin to build the blend, they need to frame the task of creating a graph as making a mathematical model of a physical situation, as discussed in Fig. 1 of paper 4 in this series, "Toy Models." This requires tying the explicit graph-creation steps given above to physical meaning, asking questions such as, "What does the origin mean physically? What are appropriate units and scales for each axis?"

I note that some educational researchers conceive of the difficulty of using math in science as an issue of "transfer"—moving skills learned in one domain into another. In my experience, both as a practicing physicist and as a physics education researcher, this feels misleading. What happens when someone builds good physical intuition with math is not simply a "transfer" of skills. Rather, it is building something new from the joining of distinct mental spaces. Physical concepts are merged with mathematical symbolic representations to create a richer and more powerful mental structure.[19] This is precisely what Fauconnier and Turner[8,20] describe in their *blending* model of building new concepts that I rely on in this series.

Carefully designed tasks can help students build the blend

These challenges suggest that, as with equations, it is not enough to assume "They can do it because they learned it in math." To use graphs effectively in physics, students need to do tasks that help them learn to make the appropriate math–physics connections—to read the physics in a graph. This includes a number of diverse skills, including

- interpreting the physics coded in a single graph,
- seeing that multiple graphs can highlight different aspects of a physical situation,
- checking for consistency among different graphs for a single situation,
- seeing graphs as an intermediary between physical concepts and equations.

As with any complex skill, a good way to help students is to scaffold by beginning with a simple example and then increasing the level of complexity a step at a time.

Reading the physics in a graph

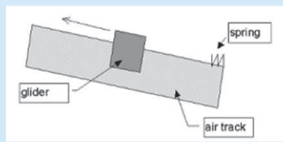
Interpreting the graphs of a single situation

A good way to start helping students build the blend using graphs is to ask them to interpret the physical correlates of various salient points on a graph for the mind—one that requires some mental processing that asks them to connect the physics and the math. Kinematics offers lots of opportunities for problems of this type. It's a good idea to start with graphs of position vs. time since that requires fewer levels of inference. The problem *Dancer position graphs* in the Supplementary Materials[2] is an example. The *PhET Moving Man* simulation[21] is an excellent way to generate such problems.

Once you have worked a few of those in class and for homework, you might try something like the problem shown in Fig. 2. I regularly assigned this for homework and, in Course Center hours, I observed it as being highly effective in getting students to struggle appropriately and work out the correct answers together.

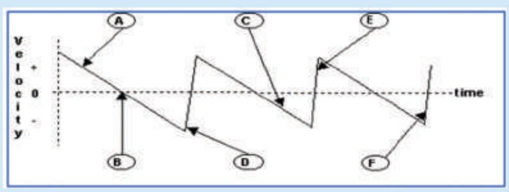
A nice next step is to ask about forces in an acceleration graph. An example of this is *The Juggler* in the Supplementary Materials.[2] This problem is *much* more challenging (and in-

A cart is moving on an air track as shown in the figure on the right. The track has a spring at one end and has its other end raised. The cart is started sliding up the track by pressing it against the spring and releasing it.



The clock is started just as the cart leaves the spring. Take the direction the cart is moving in initially to be the positive x direction and take the bottom of the spring to be the origin. Friction can be ignored.

The graph below shows a plot of the cart's velocity as a function of time.



For the physical situations described below, identify which of the letters corresponds to the situation described.

1. The cart is at its highest point on the track.
2. The cart is instantaneously not moving.
3. The cart is in contact with the spring.
4. The cart is moving down the track toward the origin.
5. The cart has acceleration of zero.

Fig. 2. Interpreting the physics in a single graph.

teresting to discuss in class) since it requires not only translations between kinematic variables, but also bringing physical principles to bear (Newton's second law and knowledge about the gravitational force).

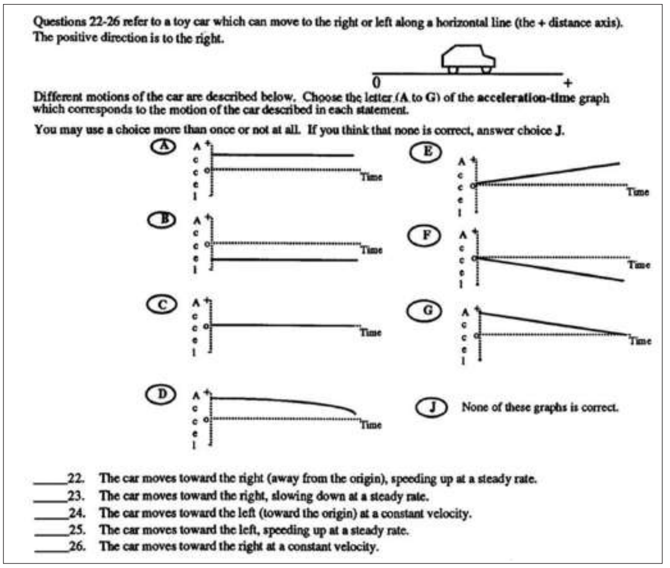
A more direct but equally challenging example asks students to pick the graph that describes the force/stretch characteristics of *Realistic springs* in the Supplementary Materials.[2]

Matching multiple graphs to multiple situations

Since variation is a critical part of learning to make sense of something,[22] a good next step is to vary the physical situation and ask students to choose how the graph changes.

Thornton and Sokoloff introduced one way to do this to great advantage in their groundbreaking concept tests.[23] A number of graphs of a single variable are given, and the student's task is to match the graphs to a list of physical situations described in words.

Figure 3 shows one from the Force and Motion Conceptual Evaluation (FMCE) that I consider one of the best prototypes of this type of problem. It's considerably simpler than *The Juggler* since it only requires translation from velocity to acceleration and doesn't require integrating concepts about force.


Questions 22-26 refer to a toy car which can move to the right or left along a horizontal line (the + distance axis). The positive direction is to the right.

Different motions of the car are described below. Choose the letter (A to G) of the **acceleration-time** graph which corresponds to the motion of the car described in each statement.

You may use a choice more than once or not at all. If you think that none is correct, answer choice J.

J None of these graphs is correct.

_____22. The car moves toward the right (away from the origin), speeding up at a steady rate.
_____23. The car moves toward the right, slowing down at a steady rate.
_____24. The car moves toward the left (toward the origin) at a constant velocity.
_____25. The car moves toward the left, speeding up at a steady rate.
_____26. The car moves toward the right at a constant velocity.

Fig. 3. Matching situations to a set of graphs. Reproduced from Thornton and Sokoloff, FMCE, used with permission.

Learning how multiple graphs provide multiple perspectives

The next step is to use multiple graphs to describe a single physical situation. This is an outstanding example of the deep epistemological idea that in science we don't just have answers, we have a network of supporting fundamental principles that let us get to an answer in multiple different ways. If you've done position, velocity, and acceleration graphs in your study of kinematics, you've already begun helping your students see this.

But in physics, we may have a dozen different ways of looking at a physical situation! In a situation with motion, we

The position of a mass hanging from a spring is measured by a sonic ranger sitting 25 cm below its equilibrium position. The mass is started oscillating. Later, the sonic ranger begins to take data.

Below are shown time graphs associated with the motion of the mass. Graph A shows the mass's position as measured by the ranger.

For each of the following physical quantities, which graph could represent that quantity for this situation? If none are possible, answer N.

1. velocity of the mass
2. net force on the mass
3. force exerted by the spring on the mass
4. kinetic energy of the mass
5. potential energy of the spring
6. gravitational potential energy of the mass

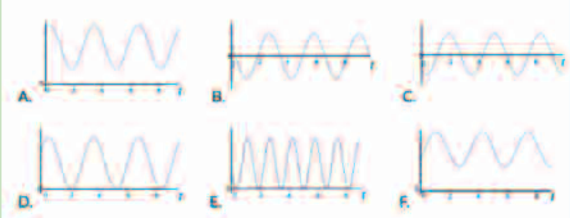


Fig. 4. Matching multiple variables to graphs of a single situation.

can look at position, velocity, acceleration, multiple kinds of forces, momentum, and energy—total, kinetic, and potential. Figure 4 shows an example that requires students to think about how each different variable says something about a physical situation.

To do a problem like this successfully, students have to be able to do the equivalent of "running a mental video of what's happening" and recognize what happens to each variable as the system evolves, a useful skill in developing an understanding of mechanism.[24]

I've included some additional examples in the Supplementary Materials.[2]

Considering multiple situations and multiple graphs

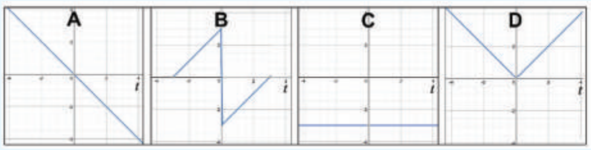
Finally, we can get seriously challenging, asking students to do everything at once—matching graphs to multiple situations and multiple variables. Since this requires managing many concepts at once, it's a good idea to start with simple graphs. The problem in Fig. 5 shows an example where the graphs are all straight lines—but you have to interpret what's happening physically in each case in terms of many variables.

Graphs can help bridge concepts and equations

In math classes, graphs are typically associated with abstract symbolic equations. As we begin to use graphs to help students learn to interpret what's happening physically in a graph, graphs connect what's happening physically and how it's represented in equations.

One of the most important reasons for using symbolic equations rather than putting numbers in right away is that you can explore how a physical phenomenon depends on the parameters in the problem. Online graphing tools such as the Desmos Graphing Calculator[25] can be tremendously useful

An object's motion is restricted to one dimension along a position axis. In the figure below are shown four possible graphs describing the motion. In each case, the horizontal axis represents time, but the vertical axis can represent position (P), velocity (V), or acceleration (A).

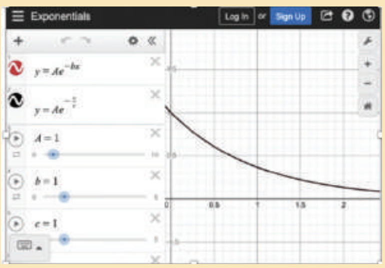


For each of the motions described below in words, select which graph or graphs could describe the motion if the vertical axis were chosen to be an appropriate variable, P, V, or A.

1. A ball rolling up to a wall and bouncing back.
2. A dropped superball falling on a cement floor.
3. A ball thrown upward.

Fig. 5. Matching multiple variables for multiple situations.

Explore what changing the parameters, A, b, and c, does to the exponential function using the Desmos Graphing Calculator. Then answer the questions below:



1. A radioactive element used as a tracer to detect tumors is created in a cyclotron. If the number of atoms of the element created is N_0, after a time t, some of the atoms will have decayed. The total number left at that time will be $N = N_0 e^{-at}$. It takes 2 h to deliver the element from the cyclotron to the facility where the test is done. If you want to choose a tracer that will have more available when the test is performed, what value of a should you be looking for?

   A. A large value of a
   B. A small value of a
   C. It doesn't matter

2. The electric potential near a charge in an ionic fluid is given by $V = V_0 e^{-r/\lambda}$, where r is the distance from the charge and λ is the Debye length. At a given distance from the charge, if the Debye length is increased, what happens to the potential?

   A. It increases
   B. It stays the same
   C. It decreases
   D. There is not enough information to decide

Fig. 6. Using graphs to make sense of parametric dependences.

in this exploration. Problems such as the one in Fig. 6 can help students see how valuable it can be to represent physical meaning mathematically, either through graphs or symbolic equations. We often put physical meaning to a parameter but sometimes to its reciprocal. The problems in Fig. 6 start with a common mathematical form of an equation and bridge to a different and physically relevant parameter. I give more examples in the Supplementary Materials.[2]

A more complex and physical problem (also authentic for life-science students!) is *Opening an ion channel—perhaps*. The problem *Shifting graphs* can prepare students to make physical sense of the challenging traveling wave equation, $y(x, t) = A \sin(kx - \omega t)$. Fitting graphs to experimental data in lab by trying different equations and varying parameters can also be helpful.

Instructional suggestions

Problems of the types shown here are easy to modify slightly so that quiz and exam problems look like class and homework problems but have to be done by using similar reasoning rather than simply looking for memorized answers.

The problems here are presented as multiple-choice or matching questions that are easily autograded. Note that in some of these problems, the correct choice is "None of these graphs work." Including some of these questions (and having "none" be the right answer) is useful in helping students see that they need to reason through the graph–physics connection in each case and not just focus on the answer.

For each of these types of questions, it's also valuable to have the students generate their own graphs in situations like those described above that can be done in (hand-graded) homework or in-class group work. I've included some examples in the Supplementary Materials[2] with answer keys for the multiple-choice questions. Solutions with explanations are available to instructors in *The Living Physics Portal*.[26]

The kind of working with graphs described here can be of particular value to life-science students in algebra-based physics classes. For these students, problems with biological or chemical context can be particularly motivating—feel "authentic."[27] Two such graph problems are included at the end of the Supplementary Materials.[2] You can find many more authentic blend-building graph problems in the NEXUS/Physics problem collection in ComPADRE[28] and *The Living Physics Portal*.[26] You can build your own using the marvelous online resources available at the University of Colorado's PhET simulations,[29] Andrew Duffy's simulation page,[30] and the AAPT's ComPADRE website.[31]

Acknowledgments

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