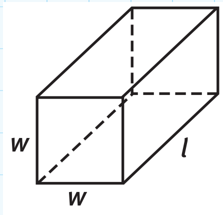
Introduction to Polynomials
Postal Service

ACTIVITY
4.1

SUGGESTED LEARNING STRATEGIES: Shared Reading, Create Representations, Think/Pair/Share

My Notes

The United States Postal Service will not accept rectangular packages if the perimeter of one end of the package plus the length of the package is greater than 130 in. Consider a rectangular package with square ends as shown in the figure.



1. Assume that the perimeter of one end of the package plus the length of the package equals the maximum 130 in. Complete the table with some possible measurements for the length and width of the package. Then find the corresponding volume of each package.

Width (in.)	Length (in.)	Volume (in.^{3})

2. Give an estimate for the largest possible volume of an acceptable United States Postal Service rectangular package with square ends.

3. Use the package described in Item 1.

a. Write an expression for l, the length of the package, in terms of w, the width of the square ends of the package.

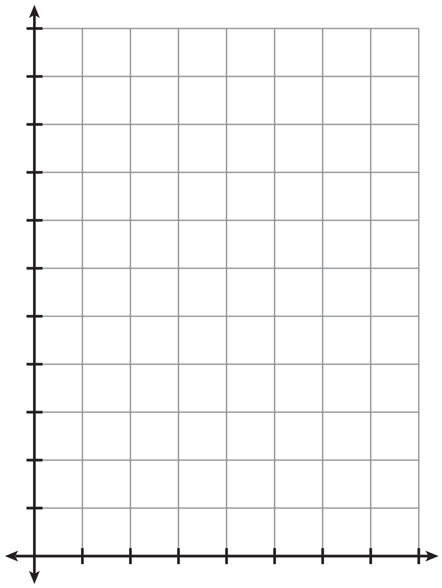
b. Write the volume of the package V as a function of w, the width of the square ends of the package.

SUGGESTED LEARNING STRATEGIES: Create Representations, Note-taking, Quickwrite

My Notes

4. Consider the smallest and largest possible values for *w* that makes sense for the function you wrote in Item 3b. Give the domain of the function as a model of the volume of the postal package.

5. Sketch a graph of the function in Item 3(b) over the domain that you found in Item 4. Include the scale on each axis.



TECHNOLOGY TIP

Graphing calculators will allow you to find the maximum and minimum of functions in the graphing window.

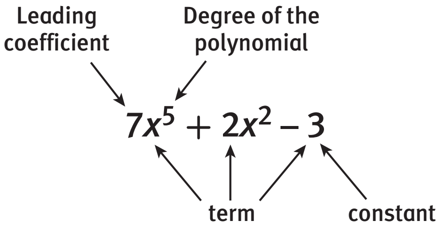
6. Use a graphing calculator to find the coordinates of the maximum point of the function that you graphed in Item 5.

7. What information do the coordinates of the maximum point of the function found in Item 6 provide with respect to an acceptable United States Postal Service package with square ends?

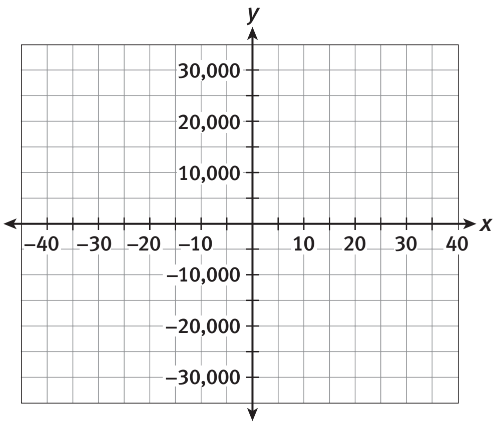
SUGGESTED LEARNING STRATEGIES: Marking the Text, Note-taking, Vocabulary Organizer, Interactive Word Wall, Create Representations

When using a function to model a given situation, such as the acceptable United States Postal Service package, you may be looking at only a portion of the entire domain of the function. Moving beyond the specific situation, you can examine the entire domain of the *polynomial function*.

A **polynomial function** in one variable is a function that can be written in the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$, where n is a nonnegative integer, the coefficients $a_0, a_1, \ldots a_n$ are real numbers, and $a_n \neq 0$. n is the **degree of the polynomial function**.



8. Write a polynomial function $f(x)$ defined over the set of real numbers such that it has the same function rule as $V(w)$ the rule you found in Item 3b. Sketch a graph of the function.



My Notes

ACADEMIC VOCABULARY

polynomial function

MATH TERMS

Polynomial functions are named by their **degree**. Here is a list of some common polynomial functions.

Degree	Name
0	Constant
1	Linear
2	Quadratic
3	Cubic
4	Quartic

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Interactive Word Wall, Think/Pair/Share, Create Representations, Discussion Group

My Notes

MATH TERMS

A function value $f(a)$ is called a **relative maximum** of f if there is an interval around a where for any x in that interval $f(a) \geq f(x)$. A function value $f(a)$ is called a **relative minimum** of f if there is an interval around a where for any x in that interval $f(a) \leq f(x)$.

ACADEMIC VOCABULARY

end behavior

MATH TIP

Recall that the phrase "approaches positive infinity ∞" means "increases without bound," and that "approaches negative infinity $-\infty$" means "decreases without bound."

9. Name any **relative maximum** values and **relative minimum** values of the function $f(x)$ in Item 8.

10. Name any x- or y-intercepts of the function $f(x) = -4x^3 + 130x^2$.

When looking at the **end behavior** of a graph, you determine what happens to the graph on the extreme right and left ends of the x-axis. That is, you look to see what happens to y as x approaches $-\infty$ and ∞.

11. Examine the end behavior of $f(x) = -4x^3 + 130x^2$.

a. As x goes to ∞, what behavior does the function have?

b. How is the function behaving as x approaches $-\infty$?

12. Examine the end behavior of $f(x) = 3x^2 - 6$.

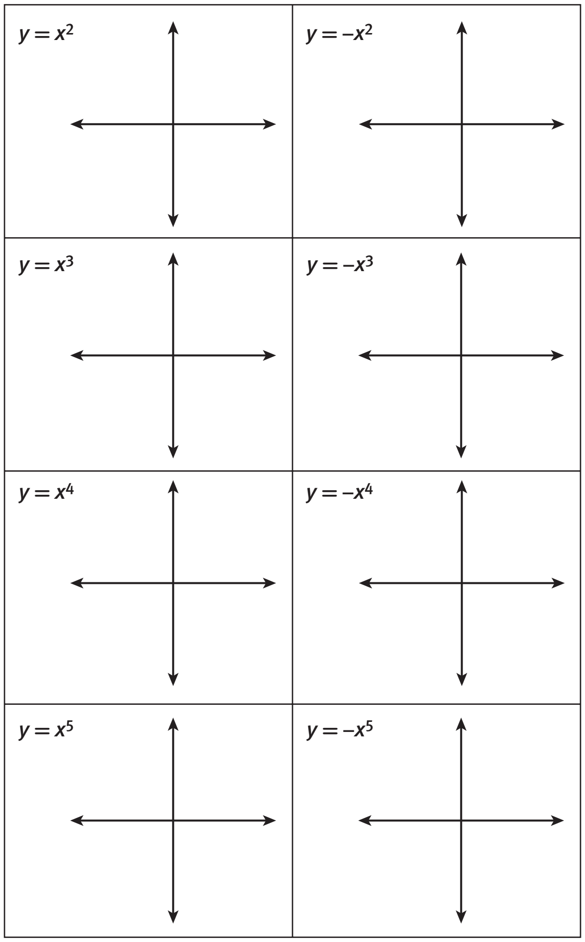
a. As x goes to ∞, what behavior does the function have?

b. How is the function behaving as x approaches $-\infty$?

SUGGESTED LEARNING STRATEGIES: Create Representations, Think/Pair/Share

My Notes

13. Use a graphing calculator to examine the *end behavior* of polynomial functions in general. Sketch each given function on the axes below.



SUGGESTED LEARNING STRATEGIES: Quickwrite, Group Presentation

My Notes

14. Make a conjecture about the end behavior of polynomial functions. Explain your reasoning.

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

For Items 1–4, decide if each function is a polynomial. If it is, write the function in standard form, and then state the degree and leading coefficient.

1. $f(x) = 5x - x^3 + 3x^5 - 2$

2. $f(x) = -\frac{2}{3}x^3 - 8x^4 - 2x + 7$

3. $f(x) = 4^x + 2x^2 + x + 5$

4. $f(x) = -5x^3 + x^6 + \frac{2}{x}$

5. Given $f(x) = 3x^3 + 5x^2 + 4x + 3$, find $f(3)$.

Describe the end behavior of each function.

6. $f(x) = x^6 - 2x^3 + 3x^2 + 2$

7. $f(x) = -x^3 + 7x^2 - 11$

8. MATHEMATICAL REFLECTION Which new concept in this investigation has been easiest for you to understand? Which one has been most difficult?