Introduction to Integration by Substitution Name: Answers

AP Calculus

Many elementary functions (we could even say most elementary functions) do not have an elementary antiderivative. But we can find the antiderivative (indefinite integral) of some functions using a fun substitution strategy. Basically, you change tricky looking parts of a given integral problem into a single variable (most mathematicians use the letter u). Let's see how this method works.

Step one: Be sure this new idea applies, i.e. check to see whether the expression looks as though it could have arisen from a chain rule being applied.	$\int 2x(x^2-5)^{\frac{1}{3}}dx$
Step two: Off to the side of the original integration problem, write down what you are going to make *u* equal to. Often this may be the part of the expression that has the weird exponent.	Let u = x^2-5
Step three: Take the derivative of that u-equals equation with respect to x	u = x^2-5 $\frac{du}{dx}=2x$ (so if you cross multiply) du = 2xdx
Step four: Look at what you have as u- equals and its derivative and the original expression you were trying to integrate…those pieces should work in a way that now changes all the variables in the original to either a *u* or a *du*. Now it should be something you can integrate! Finally, convert it back to the original variables and you are done. Remember: You can always check by taking the derivative.	$\int 2x(x^2-5)^{\frac{1}{3}}dx$ $\int u^{\frac{1}{3}}du$ Which equals $\frac{u^{\frac{4}{3}}}{\frac{4}{3}}+c$ which we write as $\frac{3}{4}u^{\frac{4}{3}}+c$ And since we said u = x^2-5, we could say final answer is $\frac{3}{4}(x^2-5)^{\frac{4}{3}}+c$

Try this one on your own using the method described above:

$\int 3x^2(x^3+4)^5\,dx$

$u = x^3+4$

$\frac{du}{dx} = 3x^2$

$du = 3x^2dx$

$$\int u^5 du = \frac{1}{6}u^6 + C$$

$$= \boxed{\frac{1}{6}(x^3+4)^6 + C}$$

Use our new substitution method to integrate the following:

1) $\int (4x^3+2)(x^4+2x)^7\,dx$

$u = x^4+2x$

$\frac{du}{dx} = 4x^3+2$

$du = (4x^3+2)\,dx$

$\int u^7\,du = \frac{1}{8}u^8 + C$

$= \boxed{\frac{1}{8}(x^4+2x)^8 + C}$

Sometimes you need to do a little "tweaking" to use the substitution method. Try these.

2) $\int \frac{3x}{\sqrt{x^2-3}}\,dx = \int u^{-1/2} \cdot \frac{3}{2}\,du$

$u = x^2-3$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{3}{2}du = 3x\,dx$

$= \frac{3}{2}\int u^{-1/2}\,du$

$= \frac{3}{2} \cdot 2u^{1/2} + C$

$= \boxed{3(x^2-3)^{1/2} + C}$

3) $\int x(x+1)^9\,dx$

$u = x+1$

$\frac{du}{dx} = 1$

$du = dx$

$= \int (u-1)u^9\,du$

$= \int (u^{10}-u^9)\,du = \frac{1}{11}u^{11} - \frac{1}{10}u^{10} + C$

$= \boxed{\frac{1}{11}(x+1)^{11} - \frac{1}{10}(x+1)^{10} + C}$

What's problematic about using the substitution method to integrate this function?

4) $\int x\sqrt{2x^3-1}\,dx$

$u = 2x^3-1$

$\frac{du}{dx} = 6x^2$

You can't make it work.