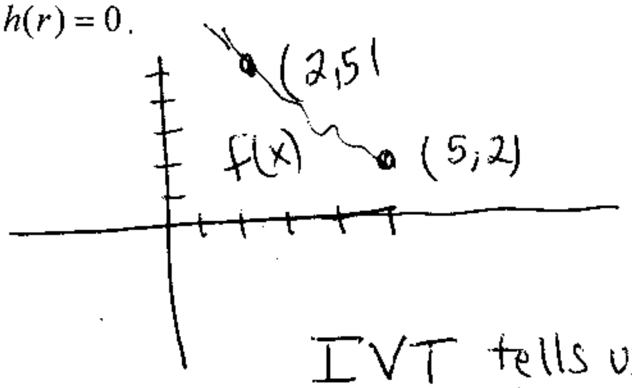
**Intermediate Value Theorem** **Name:**

*AP Calculus*

1) Let $f$ be a twice differentiable function (which means what it sounds like it means) such that $f(2)=5$ and $f(5)=2$. Let $h(x)=f(x)-x$. Explain why there must be a value $r$ for $2<r<5$ such that $h(r)=0$.

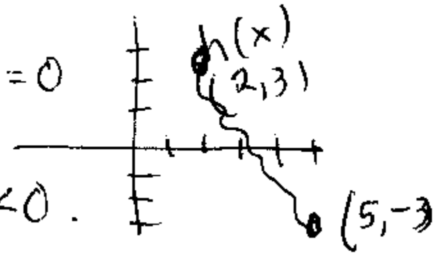



$h(x) = f(x) - x$

$h(2) = 5-2 = 3$

$h(5) = 2-5 = -3$

IVT tells us that $h(r)=0$ for some r in $(2,5)$ since $h(2)>0$ and $h(5)<0$.

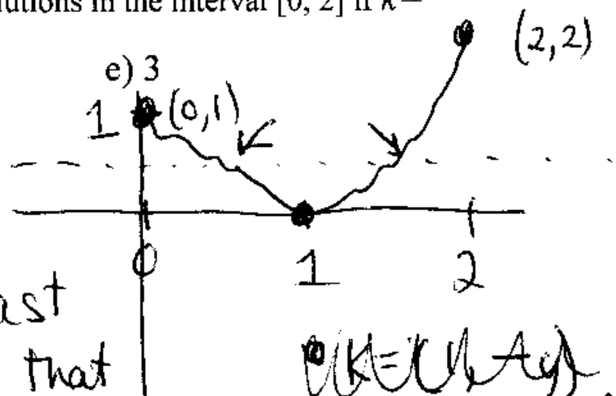



| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | 1 | k | 2 |

2) The function $f$ is continuous on the closed interval [0, 2] and has values that are given in the table above. The equation $f(x)=\frac{1}{2}$ must have at least two solutions in the interval [0, 2] if $k=$

$\boxed{\text{a) } 0}$ b) ½ c) 1 d) 2 e) 3

Explain your answer:




K must be ~~between~~ less than ½ to ensure at least two solutions. The IVT tells us that we'll have $f(c)=\frac{1}{2}$ for some value of c between f(0) and f(k) and between f(k) and f(2) since $f(0)=1$ and $f(k)=0$ and $f(2)=2$

3) Let $f$ be a function that is differentiable on the open interval (1, 10). If $f(2)=-5$, $f(5)=5$, *and* $f(9)=-5$ which of the following MUST be true:

I. $f$ has at least 2 zeros. ✓

II. The graph of $f$ has at least one horizontal tangent. ✓

III. For some $c$, $2<c<5$, $f(c)=3$. ✓