CHAPTER 9

Quadratic Functions and Equations

FREE *Falling*

Physicists use quadratic equations to describe the motion of falling objects such as water over a waterfall.

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ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. factoring
2. quadratic
3. trinomial
4. x-intercept

A. the process of writing a number or an algebraic expression as a product

B. the x-coordinate of the point where the graph intersects the x-axis

C. a polynomial with three terms

D. a polynomial with degree 2

E. the first number of an ordered pair of numbers that describes the location of a point on the coordinate plane

Graph Functions

Graph each function for the given domain.

5. $y = -2x + 8$; D: $\{-4, -2, 0, 2, 4\}$
6. $y = (x + 1)^2$; D: $\{-3, -2, -1, 0, 1\}$
7. $y = x^2 + 3$; D: $\{-2, -1, 0, 1, 2\}$
8. $y = 2x^2$; D: all real numbers

Multiply Binomials

Find each product.

9. $(m + 2)(m + 5)$
10. $(y - 7)(y + 2)$
11. $(2a + 4)(5a + 6)$
12. $(x + 1)(x + 1)$
13. $(t + 5)(t + 5)$
14. $(3n - 8)(3n - 8)$

Factor Trinomials

Factor each polynomial completely.

15. $x^2 - 2x + 1$
16. $x^2 - x - 2$
17. $x^2 - 6x + 5$
18. $x^2 - x - 12$
19. $x^2 - 9x + 18$
20. $x^2 - 7x - 18$

Squares and Square Roots

Find the square root of each expression.

21. $\sqrt{36}$
22. $\sqrt{121}$
23. $-\sqrt{64}$
24. $\sqrt{16}\sqrt{81}$
25. $\sqrt{\frac{9}{25}}$
26. $-\sqrt{6(24)}$

Solve Multi-Step Equations

Solve each equation.

27. $3m + 5 = 11$
28. $3t + 4 = 10$
29. $5n + 13 = 28$
30. $2(k - 4) + k = 7$
31. $10 = \frac{r}{3} + 8$
32. $2(y - 6) = 8.6$

CHAPTER 9

Study Guide: Preview

Where You've Been

Previously, you

- identified and graphed linear functions.
- transformed linear functions.
- solved linear equations.
- factored quadratic polynomials, including perfect-square trinomials.

In This Chapter

You will study

- identifying and graphing quadratic functions.
- transforming quadratic equations.
- solving quadratic equations.
- using factoring to graph quadratic functions and solve quadratic equations.

Where You're Going

You can use the skills in this chapter

- to determine the maximum height of a ball thrown into the air.
- to graph higher-degree polynomials in future math classes, including Algebra 2.
- to solve problems about the height of launched or thrown objects in Physics.

Key Vocabulary/Vocabulario

axis of symmetry	eje de simetría
completing the square	completar el cuadrado
maximum	máximo
minimum	mínimo
parabola	parábola
quadratic equation	ecuación cuadrática
quadratic function	función cuadrática
vertex	vértice
zero of a function	cero de una función

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The value of a function is determined by its rule. The rule is an algebraic expression. What is true about the algebraic expression that determines a **quadratic function**?
2. The shape of a **parabola** is similar to the shape of an open parachute. Predict the shape of a *parabola*.
3. A **minimum** is a point on the graph of a curve with the least y-coordinate. How might a **maximum** be described?
4. An axis is an imaginary line. Use this information and your understanding of symmetry to define the term **axis of symmetry**.

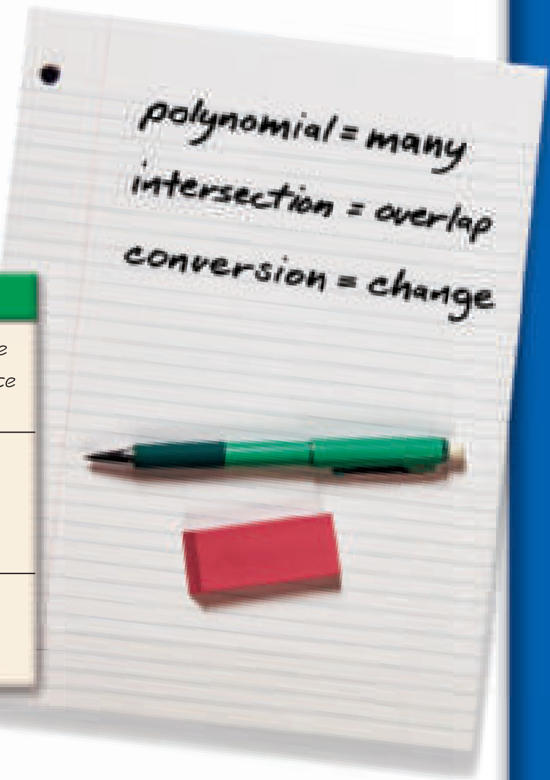
Reading and Writing Math

Study Strategy: Learn Vocabulary

Mathematics has a vocabulary all its own. Many new terms appear on the pages of your textbook. Learn these new terms as they are introduced. They will give you the necessary tools to understand new concepts.

Some tips to learning new vocabulary include:

- Look at the **context** in which a new word appears.
- Use **prefixes** or **suffixes** to figure out the word's meaning.
- Relate the new term to familiar **everyday words.** Keep in mind that a word's mathematical meaning may not exactly match its everyday meaning.



Vocabulary Word	Study Tip	Definition
Polynomial	*The prefix "poly-" means many.*	*One monomial or the sum or the difference of monomials*
Intersection	*Relate it to the meaning of the "intersection of two roads".*	*The overlapping region that shows the solution to a system of equations*
Conversion Factor	*Relate it to the word "convert", which means change or alter.*	*Used to convert a measurement to different units*

Try This

Complete the chart.

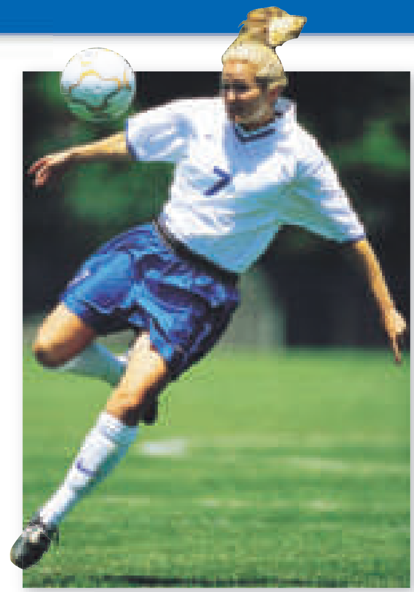
	Vocabulary Word	Study Tips	Definition
1.	Trinomial		
2.	Independent system		
3.	Variable		

Use the context of each sentence to define the underlined word. Then relate the word to everyday words.

4. If two linear equations in a system have the same graph, the graphs are called coincident lines, or simply the same line.

5. In the formula $d = rt$, d is isolated.

9-1 Identifying Quadratic Functions



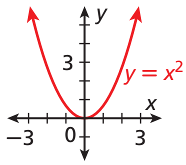
Objectives
Identify quadratic functions and determine whether they have a minimum or maximum.

Graph a quadratic function and give its domain and range.

Vocabulary
quadratic function
parabola
vertex
minimum
maximum

Why learn this?
The height of a soccer ball after it is kicked into the air can be described by a quadratic function. (See Exercise 51.)

The function $y = x^2$ is shown in the graph. Notice that the graph is not linear. This function is a *quadratic function*. A **quadratic function** is any function that can be written in the standard form $y = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. The function $y = x^2$ can be written as $y = 1x^2 + 0x + 0$, where $a = 1$, $b = 0$, and $c = 0$.



In Lesson 5-1, you identified linear functions by finding that a constant change in x corresponded to a constant change in y. The differences between y-values for a **constant change in x-values** are called ***first differences***.

Constant change in x-values: +1, +1, +1, +1

x	0	1	2	3	4
$y = x^2$	0	1	4	9	16

First differences: +1, +3, +5, +7

Second differences: +2, +2, +2

Notice that the quadratic function $y = x^2$ does not have constant first differences. It has constant ***second differences***. This is true for all quadratic functions.

EXAMPLE 1 Identifying Quadratic Functions

Tell whether each function is quadratic. Explain.

A

x	y
−4	8
−2	2
0	0
2	2
4	8

x: +2, +2, +2, +2
First differences: −6, −2, +2, +6
Second differences: +4, +4, +4

Since you are given a table of ordered pairs with a constant change in x-values, see if the second differences are constant.

Find the first differences, then find the second differences.

The function is quadratic. The second differences are constant.

B $y = -3x + 20$ *Since you are given an equation, use $y = ax^2 + bx + c$.*

This is not a quadratic function because the value of a is 0.

Caution!
Be sure there is a constant change in x-values before you try to find first or second differences.

Helpful Hint

Only *a* cannot equal 0. It is okay for the values of *b* and *c* to be 0.

Tell whether each function is quadratic. Explain.

C $y + 3x^2 = -4$

$$\begin{aligned} y + 3x^2 &= -4 \\ \underline{-3x^2} \quad & \underline{-3x^2} \\ y &= -3x^2 - 4 \end{aligned}$$

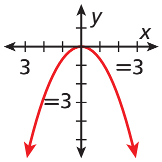
Try to write the function in the form $y = ax^2 + bx + c$ by solving for y. Subtract $3x^2$ from both sides.

This is a quadratic function because it can be written in the form $y = ax^2 + bx + c$ where $a = -3$, $b = 0$, and $c = -4$.

CHECK IT OUT! **Tell whether each function is quadratic. Explain.**

1a. $\{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$ **1b.** $y + x = 2x^2$

The graph of a quadratic function is a curve called a **parabola**. To graph a quadratic function, generate enough ordered pairs to see the shape of the parabola. Then connect the points with a smooth curve.



EXAMPLE 2 Graphing Quadratic Functions by Using a Table of Values

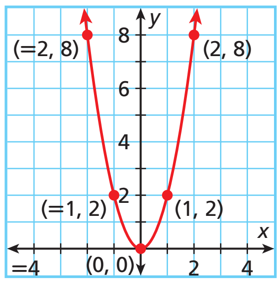
Use a table of values to graph each quadratic function.

A $y = 2x^2$

x	$y = 2x^2$
−2	8
−1	2
0	0
1	2
2	8

Make a table of values. Choose values of x and use them to find values of y.

Graph the points. Then connect the points with a smooth curve.

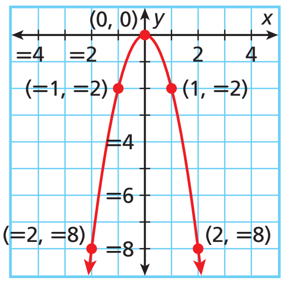


B $y = -2x^2$

x	$y = -2x^2$
−2	−8
−1	−2
0	0
1	−2
2	−8

Make a table of values. Choose values of x and use them to find values of y.

Graph the points. Then connect the points with a smooth curve.



CHECK IT OUT! **Use a table of values to graph each quadratic function.**

2a. $y = x^2 + 2$ **2b.** $y = -3x^2 + 1$

As shown in the graphs in Examples 2A and 2B, some parabolas open upward and some open downward. Notice that the only difference between the two equations is the value of *a*. When a quadratic function is written in the form $y = ax^2 + bx + c$, the value of *a* determines the direction a parabola opens.

- A parabola opens **upward** when $a > 0$.
- A parabola opens **downward** when $a < 0$.

EXAMPLE 3 Identifying the Direction of a Parabola

Tell whether the graph of each quadratic function opens upward or downward. Explain.

A $y = 4x^2$

$y = 4x^2$

$a = 4$ *Identify the value of a.*

Since $a > 0$, the parabola opens **upward**.

B $2x^2 + y = 5$

$$\begin{aligned} 2x^2 + y &= 5 \\ -2x^2 \qquad & \;\; -2x^2 \\ y &= -2x^2 + 5 \\ a &= -2 \end{aligned}$$

Write the function in the form $y = ax^2 + bx + c$ by solving for y. Subtract $2x^2$ from both sides.

Identify the value of a.

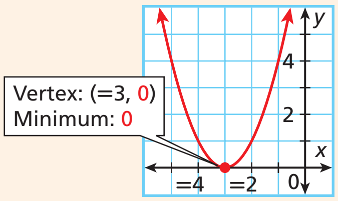
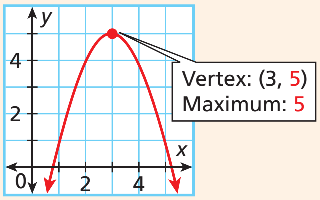
Since $a < 0$, the parabola opens **downward**.

CHECK IT OUT! **Tell whether the graph of each quadratic function opens upward or downward. Explain.**

3a. $f(x) = -4x^2 - x + 1$ **3b.** $y - 5x^2 = 2x - 6$

The highest or lowest point on a parabola is the **vertex**. If a parabola opens upward, the vertex is the lowest point. If a parabola opens downward, the vertex is the highest point.

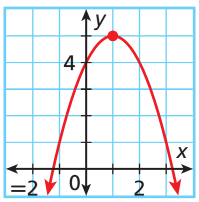
Know It! Note — Minimum and Maximum Values

WORDS	If $a > 0$, the parabola opens upward, and the y-value of the vertex is the **minimum** value of the function.	If $a < 0$, the parabola opens downward, and the y-value of the vertex is the **maximum** value of the function.
GRAPHS	$y = x^2 + 6x + 9$ 	$y = -x^2 + 6x - 4$ 

EXAMPLE 4 Identifying the Vertex and the Minimum or Maximum

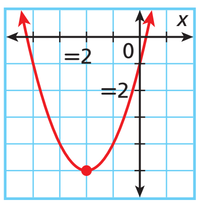
Identify the vertex of each parabola. Then give the minimum or maximum value of the function.

A



The vertex is $(1, 5)$, and the maximum is 5.

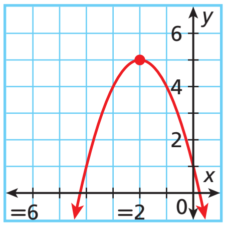
B



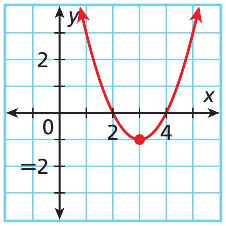
The vertex is $(-2, -5)$, and the minimum is -5.

CHECK IT OUT! **Identify the vertex of each parabola. Then give the minimum or maximum value of the function.**

4a.



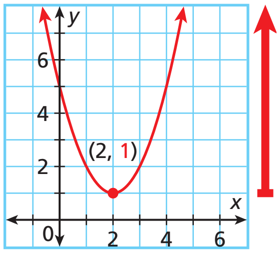
4b.



Caution!

You may not be able to see the entire graph, but that does not mean the graph stops. Remember that the arrows indicate that the graph continues.

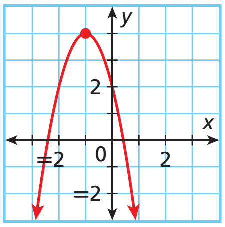
Unless a specific domain is given, you may assume that the domain of a quadratic function is all real numbers. You can find the range of a quadratic function by looking at its graph.



For the graph of $y = x^2 - 4x + 5$, the **range** begins at the minimum value of the function, where $y = \mathbf{1}$. All the y-values of the function are greater than or equal to 1. So the range is $\boldsymbol{y \geq 1}$.

EXAMPLE 5 Finding Domain and Range

Find the domain and range.



Step 1 The graph opens downward, so identify the maximum.

The vertex is $(-1, \mathbf{4})$, so the maximum is **4**.

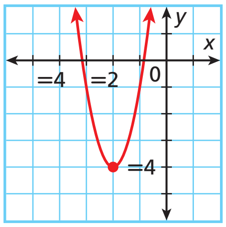
Step 2 Find the domain and range.

D: all real numbers

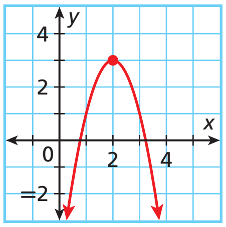
R: $y \leq \mathbf{4}$

CHECK IT OUT! **Find the domain and range.**

5a.



5b.

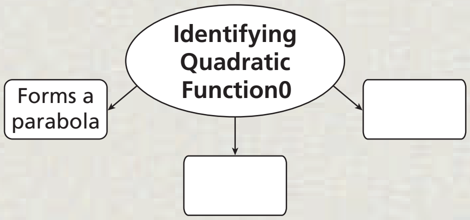


THINK AND DISCUSS

1. How can you identify a quadratic function from ordered pairs? from looking at the function rule?

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer below. In each box, describe a way of identifying quadratic functions.



9-1 Exercises

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GUIDED PRACTICE

1. **Vocabulary** The y-value of the vertex of a parabola that opens upward is the ___?___ value of the function. (*maximum* or *minimum*)

SEE EXAMPLE 1
p. 590

Tell whether each function is quadratic. Explain.

2. $y + 6x = -14$

3. $2x^2 + y = 3x - 1$

4.

x	−4	−3	−2	−1	0
y	39	18	3	−6	−9

5. $\{(-10, 15), (-9, 17), (-8, 19), (-7, 21), (-6, 23)\}$

SEE EXAMPLE 2
p. 591

Use a table of values to graph each quadratic function.

6. $y = 4x^2$

7. $y = \frac{1}{2}x^2$

8. $y = -x^2 + 1$

9. $y = -5x^2$

SEE EXAMPLE 3
p. 592

Tell whether the graph of each quadratic function opens upward or downward. Explain.

10. $y = -3x^2 + 4x$

11. $y = 1 - 2x + 6x^2$

12. $y + x^2 = -x - 2$

13. $y + 2 = x^2$

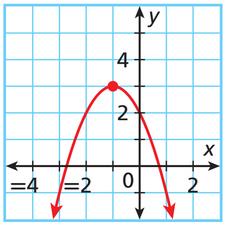
14. $y - 2x^2 = -3$

15. $y + 2 + 3x^2 = 1$

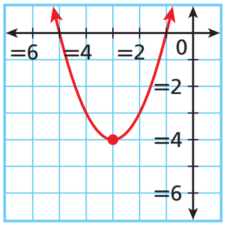
SEE EXAMPLE 4
p. 592

Identify the vertex of each parabola. Then give the minimum or maximum value of the function.

16.



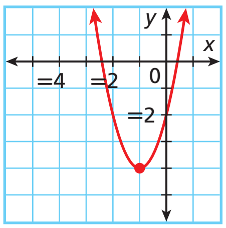
17.



SEE EXAMPLE 5
p. 593

Find the domain and range.

18.



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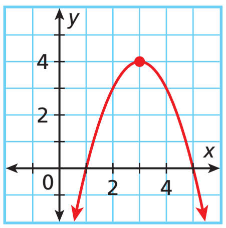

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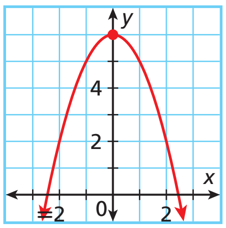

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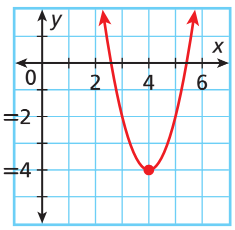


PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
22–25	1
26–29	2
30–32	3
33–34	4
35–38	5

Extra Practice

Skills Practice p. S20
Application Practice p. S36

Tell whether each function is quadratic. Explain.

22.

x	-2	-1	0	1	2
y	-1	0	4	9	15

23. $-3x^2 + x = y - 11$

24. $\{(0, -3), (1, -2), (2, 1), (3, 6), (4, 13)\}$

25. $y = \frac{2}{3}x - \frac{4}{9} + \frac{1}{6}x^2$

Use a table of values to graph each quadratic function.

26. $y = x^2 - 5$

27. $y = -\frac{1}{2}x^2$

28. $y = -2x^2 + 2$

29. $y = 3x^2 - 2$

Tell whether the graph of each quadratic function opens upward or downward. Explain.

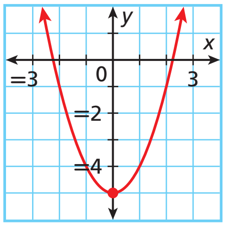
30. $y = 7x^2 - 4x$

31. $x - 3x^2 + y = 5$

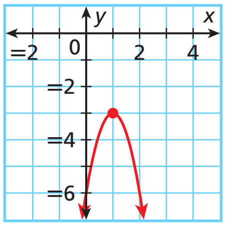
32. $y = -\frac{2}{3}x^2$

Identify the vertex of each parabola. Then give the minimum or maximum value of the function.

33.

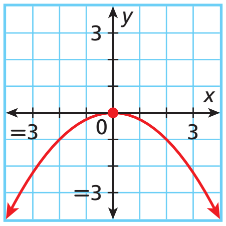


34.



Find the domain and range.

35.



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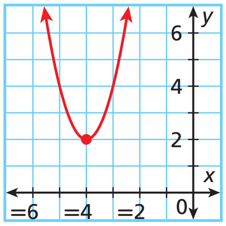

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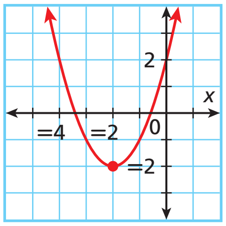
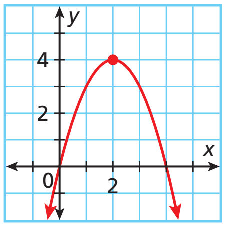


38.



Tell whether each statement is sometimes, always, or never true.

39. The graph of a quadratic function is a straight line.

40. The range of a quadratic function is the set of all real numbers.

41. The highest power in a quadratic function is 2.

42. The graph of a quadratic function contains the point $(0, 0)$.

43. The vertex of a parabola occurs at the minimum value of the function.

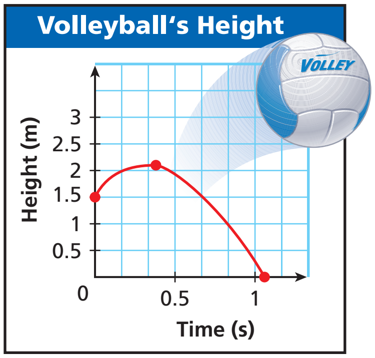
44. The graph of a quadratic function that has a minimum opens upward.

Tell whether each function is quadratic. If it is, write the function in standard form. If not, explain why not.

45. $y = 3x - 1$ **46.** $y = 2x^2 - 5 + 3x$ **47.** $y = (x + 1)^2$

48. $y = 5 - (x - 1)^2$ **49.** $y = 3x^2 - 9$ **50.** $y = (x + 1)^3 - x^2$

51. Estimation **The graph shows the approximate height y in meters of a volleyball x seconds after it is served.**



a. Estimate the time it takes for the volleyball to reach its greatest height.

b. Estimate the greatest height that the volleyball reaches.

c. Critical Thinking If the domain of a quadratic function is all real numbers, why is the domain of this function limited to nonnegative numbers?

52. Sports The height in feet of a soccer ball x seconds after it is kicked into the air is modeled by the function $y = 48x - 16x^2$.

a. Graph the function.

b. In this situation, what values make sense for the domain?

c. Does the soccer ball ever reach a height of 50 ft? How do you know?

Tell whether each function is linear, quadratic, or neither.

53. $y = \frac{1}{2}x - x^2$ **54.** $y = \frac{1}{2}x - 3$ **55.** $y + 3 = -x^2$ **56.** $y - 2x^2 = 0$

57. $y = \frac{1}{2}x(x^2)$ **58.** $y = \frac{3}{x^2}$ **59.** $y = \frac{3}{2}x$ **60.** $x^2 + 2x + 1 = y$

61. Marine Biology A scientist records the motion of a dolphin as it jumps from the water. The function $h(t) = -16t^2 + 32t$ models the dolphin's height in feet above the water after t seconds.

a. Graph the function.

b. What domain makes sense for this situation?

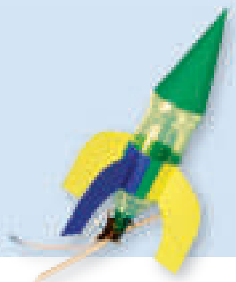
c. What is the dolphin's maximum height above the water?

d. How long is the dolphin out of the water?

62. Write About It Explain how to tell the difference between a linear function and a quadratic function when given each of the following:

a. ordered pairs **b.** the function rule **c.** the graph

MULTI-STEP TEST PREP



63. This problem will prepare you for the Multi-Step Test Prep on page 620.

A rocket team is using simulation software to create and study water bottle rockets. The team begins by simulating the launch of a rocket without a parachute. The table gives data for one rocket design.

a. Show that the data represent a quadratic function.

b. Graph the function.

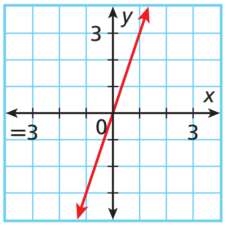
c. The acceleration due to gravity is 9.8 m/s^2. How is this number related to the data for this water bottle rocket?

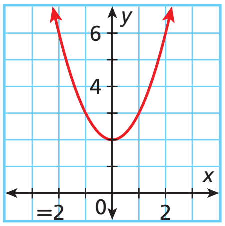
Time (s)	Height (m)
0	0
1	34.3
2	58.8
3	73.5
4	78.4
5	73.5
6	58.8
7	34.3
8	0

64. **Critical Thinking** Given the function $-3 - y = x^2 + x$, why is it incorrect to state that the parabola opens upward and has a minimum?

TEST PREP

65. Which of the following is the graph of a quadratic function?

Ⓐ


Ⓒ


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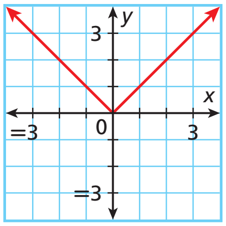

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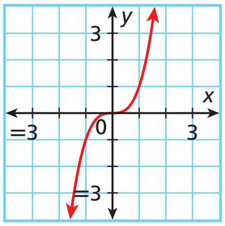


66. Which of the following quadratic functions has a maximum?

Ⓕ $2x^2 - y = 3x - 2$

Ⓖ $y = x^2 + 4x + 16$

Ⓗ $y - x^2 + 6 = 9x$

Ⓙ $y + 3x^2 = 9$

67. **Short Response** Is the function $f(x) = 5 - 2x^2 + 3x$ quadratic? Explain your answer by using two different methods of identification.

CHALLENGE AND EXTEND

68. **Multi-Step** A rectangular picture measuring 6 in. by 10 in. is surrounded by a frame with uniform width x. Write a quadratic function to show the combined area of the picture and frame.

69. **Graphing Calculator** Use a graphing calculator to find the domain and range of the quadratic functions $y = x^2 - 4$ and $y = -(x + 2)^2$.

SPIRAL REVIEW

Write each number as a power of the given base. *(Lesson 1-4)*

70. 10,000; base 10

71. 16; base -2

72. $\frac{8}{27}$; base $\frac{2}{3}$

73. A map shows a scale of 1 inch:3 miles. On the map, the distance from Lin's home to the park is $14\frac{1}{4}$ inches. What is the actual distance? *(Lesson 2-6)*

Write a function to describe the situation. Find a reasonable domain and range for the function. *(Lesson 4-3)*

74. Camp Wildwood has collected $400 in registration fees. It can enroll another 3 campers for $25 each.

75. Sal works between 30 and 35 hours per week. He earns $9 per hour.

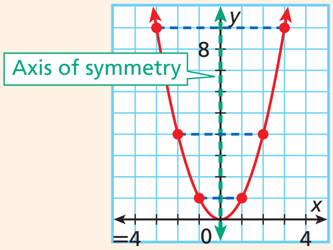
9-2 Algebra LAB

Use with Lesson 9-2

Explore the Axis of Symmetry

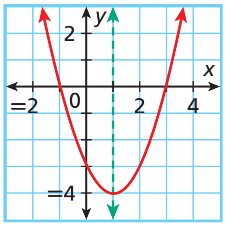
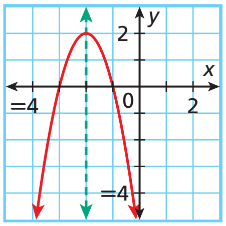
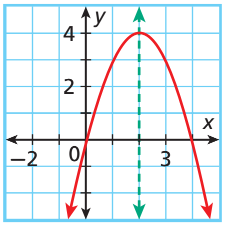
Every graph of a quadratic function is a parabola that is symmetric about a vertical line through its vertex called the *axis of symmetry*.

There is a relationship between a and b in the quadratic function and the equation of the axis of symmetry.



Activity

1 Complete the table.

Function	$y = 1x^2 - 2x - 3$	$y = -2x^2 - 8x - 6$	$y = -1x^2 + 4x$
Graph			
a	1		
b	−2		
$\frac{b}{a}$			
Axis of Symmetry (from graph)	$x = 1$		

2 Compare the axis of symmetry with $\frac{b}{a}$ in your chart. What can you multiply $\frac{b}{a}$ by to get the number in the equation of the axis of symmetry? (*Hint:* Write and solve an equation to find the value.) Check your answer for each function.

3 Use your answer from Problem 2 to complete the equation of the axis of symmetry of a quadratic function. $x =$ ___?___

Try This

For the graph of each quadratic function, find the equation of the axis of symmetry.

1. $y = 2x^2 + 12x - 7$ **2.** $y = 4x^2 + 8x - 12$ **3.** $y = 5x^2 - 20x + 10$

4. $y = -3x^2 + 9x + 1$ **5.** $y = x^2 - 7$ **6.** $y = 3x^2 + x + 4$

9-2 Characteristics of Quadratic Functions

Objectives

Find the zeros of a quadratic function from its graph.

Find the axis of symmetry and the vertex of a parabola.

Vocabulary

zero of a function
axis of symmetry

Who uses this?

Engineers can use characteristics of quadratic functions to find the height of the arch supports of bridges. (See Example 5.)

Recall that an x-intercept of a function is a value of x when $y = 0$. A **zero of a function** is an x-value that makes the function equal to 0. So a zero of a function is the same as an x-intercept of a function. Since a graph intersects the x-axis at the point or points containing an x-intercept these intersections are also at the zeros of the function. A quadratic function may have one, two, or no zeros.

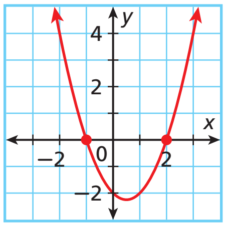
Helpful Hint

Notice that if a parabola has only one zero, the zero is the x-coordinate of the vertex.

EXAMPLE 1 Finding Zeros of Quadratic Functions From Graphs

Find the zeros of each quadratic function from its graph. Check your answer.

A $y = x^2 - x - 2$



The zeros appear to be −1 and 2.

Check

$y = x^2 - x - 2$

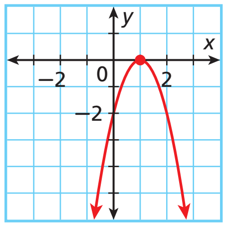
$y = (-1)^2 - (-1) - 2$

$= 1 + 1 - 2 = 0$ ✓

$y = 2^2 - 2 - 2$

$= 4 - 2 - 2 = 0$ ✓

B $y = -2x^2 + 4x - 2$



The only zero appears to be 1.

Check

$y = -2x^2 + 4x - 2$

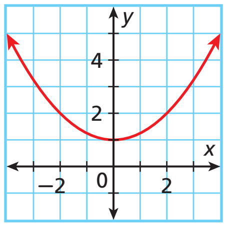
$y = -2(1)^2 + 4(1) - 2$

$= -2(1) + 4 - 2$

$= -2 + 4 - 2$

$= 0$ ✓

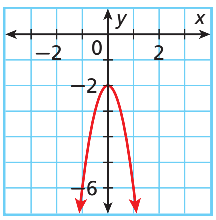
C $y = \frac{1}{4}x^2 + 1$



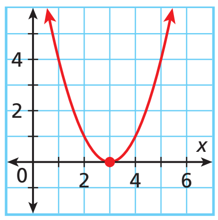
The graph does not cross the x-axis, so there are no zeros of this function.

CHECK IT OUT! **Find the zeros of each quadratic function from its graph. Check your answer.**

1a. $y = -4x^2 - 2$



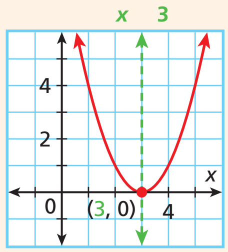
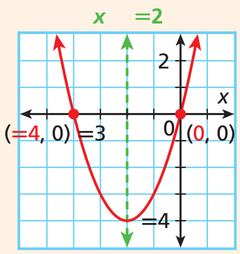
1b. $y = x^2 - 6x + 9$



A vertical line that divides a parabola into two symmetrical halves is the **axis of symmetry**. The axis of symmetry always passes through the vertex of the parabola. You can use the zeros to find the axis of symmetry.

Know It! Note

Finding the Axis of Symmetry by Using Zeros

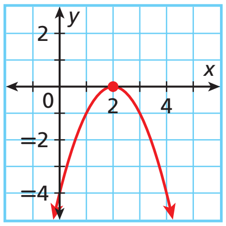
WORDS	NUMBERS	GRAPH
One Zero		
If a function has one zero, use the *x*-coordinate of the vertex to find the axis of symmetry.	Vertex: $(3, 0)$ Axis of symmetry: $x = 3$	
Two Zeros		
If a function has two zeros, use the average of the two zeros to find the axis of symmetry.	$\frac{-4+0}{2} = \frac{-4}{2} = -2$ Axis of symmetry: $x = -2$	

EXAMPLE **2**

Finding the Axis of Symmetry by Using Zeros

Find the axis of symmetry of each parabola.

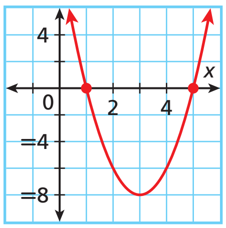
A



$(2, 0)$ *Identify the x-coordinate of the vertex.*

The axis of symmetry is $x = 2$.

B



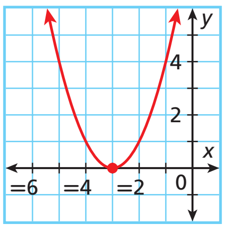
$\frac{1+5}{2} = \frac{6}{2} = 3$ *Find the average of the zeros.*

The axis of symmetry is $x = 3$.

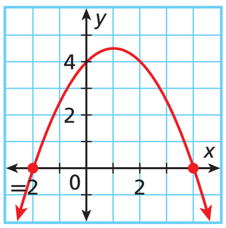
CHECK IT OUT!

Find the axis of symmetry of each parabola.

2a.



2b.



If a function has no zeros or they are difficult to identify from a graph, you can use a formula to find the axis of symmetry. The formula works for all quadratic functions.

Know it! Note

Finding the Axis of Symmetry by Using the Formula

FORMULA	EXAMPLE
For a quadratic function $y = ax^2 + bx + c$, the axis of symmetry is the vertical line $x = -\frac{b}{2a}$.	$y = 2x^2 + 4x + 5$ $x = -\frac{b}{2a}$ $= -\frac{4}{2(2)} = -1$ The axis of symmetry is $x = -1$.

EXAMPLE 3 Finding the Axis of Symmetry by Using the Formula

Find the axis of symmetry of the graph of $y = x^2 + 3x + 4$.

Step 1 Find the values of a and b.

$y = 1x^2 + 3x + 4$

$a = 1,\ b = 3$

Step 2 Use the formula $x = -\frac{b}{2a}$.

$x = -\frac{3}{2(1)} = -\frac{3}{2} = -1.5$

The axis of symmetry is $x = -1.5$.

CHECK IT OUT! **3.** Find the axis of symmetry of the graph of $y = 2x^2 + x + 3$.

Once you have found the axis of symmetry, you can use it to identify the vertex.

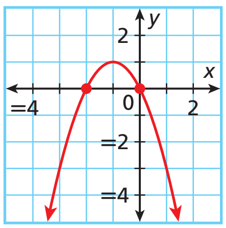
Know it! Note

Finding the Vertex of a Parabola	
Step 1	To find the x-coordinate of the vertex, find the axis of symmetry by using zeros or the formula.
Step 2	To find the corresponding y-coordinate, substitute the x-coordinate of the vertex into the function.
Step 3	Write the vertex as an ordered pair.

EXAMPLE 4 Finding the Vertex of a Parabola

Find the vertex.

A $y = -x^2 - 2x$



Step 1 Find the x-coordinate.

The zeros are -2 and 0.

$x = \frac{-2 + 0}{2} = \frac{-2}{2} = -1$

Step 2 Find the corresponding y-coordinate.

$y = -x^2 - 2x$ *Use the function rule.*

$= -(-1)^2 - 2(-1) = 1$ *Substitute −1 for x.*

Step 3 Write the ordered pair.

$(-1, 1)$

The vertex is $(-1, 1)$.

Caution!

In Example 4A Step 2, use the order of operations to simplify the function.

$-(-1)^2 = -(1) = -1$

Find the vertex.

B $y = 5x^2 - 10x + 3$

Step 1 Find the x-coordinate.

$a = 5$, $b = -10$ — *Identify a and b.*

$$x = -\frac{b}{2a}$$

$$= -\frac{-10}{2(5)} = -\frac{-10}{10} = 1$$ — *Substitute 5 for a and −10 for b.*

The x-coordinate of the vertex is 1.

Step 2 Find the corresponding y-coordinate.

$y = 5x^2 - 10x + 3$ — *Use the function rule.*

$= 5(1)^2 - 10(1) + 3$ — *Substitute 1 for x.*

$= 5 - 10 + 3$

$= -2$

Step 3 Write the ordered pair.

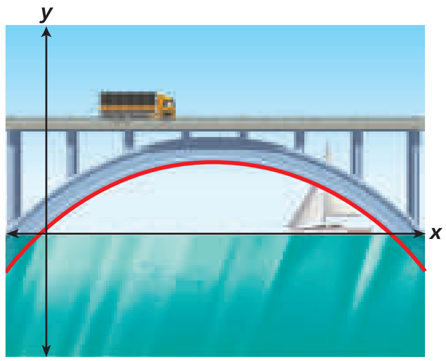
The vertex is $(1, -2)$.

CHECK IT OUT!

4. Find the vertex of the graph of $y = x^2 - 4x - 10$.

EXAMPLE 5 Architecture Application

The height above water level of a curved arch support for a bridge can be modeled by $f(x) = -0.007x^2 + 0.84x + 0.8$, where x is the distance in feet from where the arch support enters the water. Can a sailboat that is 24 feet tall pass under the bridge? Explain.



The vertex represents the highest point of the arch support.

Step 1 Find the x-coordinate.

$a = -0.007$, $b = 0.84$ — *Identify a and b.*

$$x = -\frac{b}{2a}$$

$$= -\frac{0.84}{2(-0.007)} = 60$$ — *Substitute −0.007 for a and 0.84 for b.*

Step 2 Find the corresponding y-coordinate.

$f(x) = -0.007x^2 + 0.84x + 0.8$ — *Use the function rule.*

$= -0.007(60)^2 + 0.84(60) + 0.8$ — *Substitute 60 for x.*

$= 26$

Since the height of the arch support is 26 feet, the sailboat can pass under the bridge.

CHECK IT OUT!

5. The height of a small rise in a roller coaster track is modeled by $f(x) = -0.07x^2 + 0.42x + 6.37$, where x is the distance in feet from a support pole at ground level. Find the height of the rise.

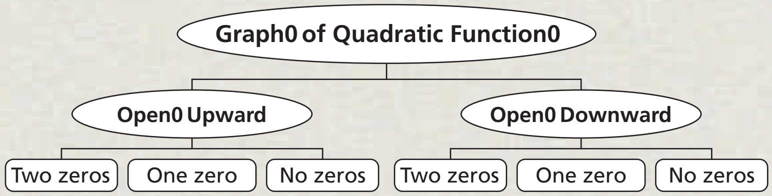
THINK AND DISCUSS

1. How do you find the zeros of a function from its graph?

2. Describe how to find the axis of symmetry of a quadratic function if its graph does not cross the x-axis

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, sketch a graph that fits the given description.



9-2 Exercises

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Homework Help Online
KEYWORD: MA7 9-2
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

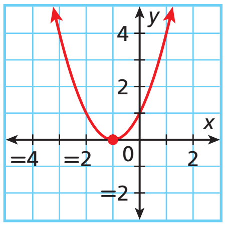
Vocabulary **Apply the vocabulary from this lesson to answer each question.**

1. Why is the *zero of a function* the same as an x-intercept of a function?

2. Where is the *axis of symmetry* of a parabola located?

SEE EXAMPLE 1
p. 599

Find the zeros of each quadratic function from its graph. Check your answer.

3.


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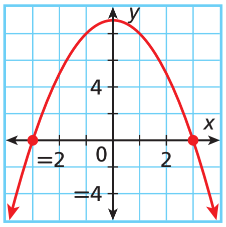

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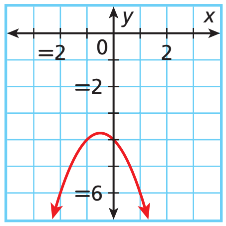
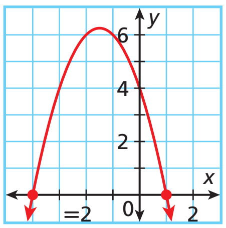


SEE EXAMPLE 2
p. 600

Find the axis of symmetry of each parabola.

6.


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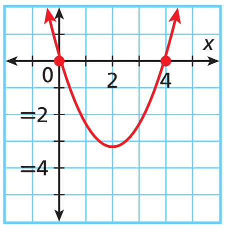

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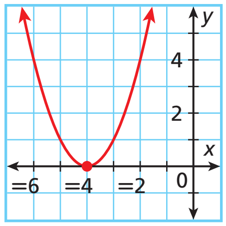


SEE EXAMPLE 3
p. 601

For each quadratic function, find the axis of symmetry of its graph.

9. $y = x^2 + 4x - 7$

10. $y = 3x^2 - 18x + 1$

11. $y = 2x^2 + 3x - 4$

12. $y = -3x^2 + x + 5$

SEE EXAMPLE 4
p. 601

Find the vertex of each parabola.

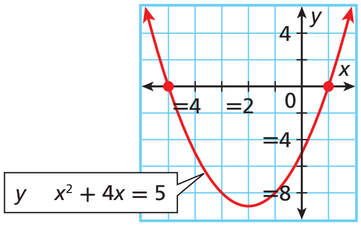
13. $y = -5x^2 + 10x + 3$

14. $y = x^2 + 4x - 7$

15. $y = \frac{1}{2}x^2 + 2x$

16. $y = -x^2 + 6x + 1$

17.



SEE EXAMPLE 5
p. 602

18. Archery The height in feet above the ground of an arrow after it is shot can be modeled by $y = -16t^2 + 63t + 4$. Can the arrow pass over a tree that is 68 feet tall? Explain.

PRACTICE AND PROBLEM SOLVING

Independent Practice

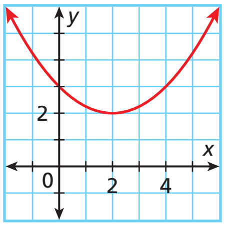
For Exercises	See Example
19–21	1
22–24	2
25–28	3
29–33	4
34	5

Extra Practice

Skills Practice p. S20
Application Practice p. S36

Find the zeros of each quadratic function from its graph. Check your answer.

19.



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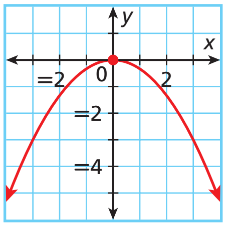
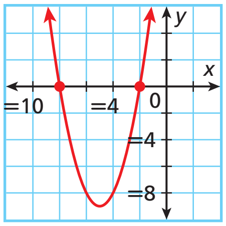
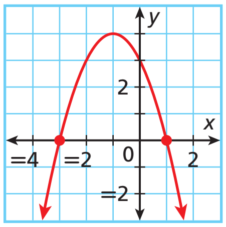


21.



Find the axis of symmetry of each parabola.

22.



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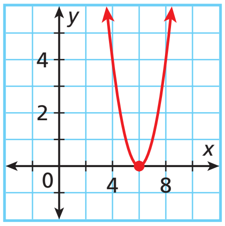

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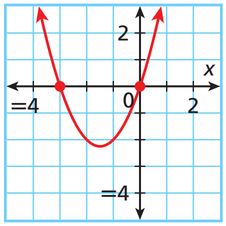
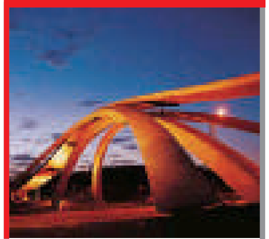


LINK

Engineering



A bridge designed by Leonardo da Vinci in 1502 was completed in Ås, Norway, in 2001. The original design was to span the Bosporus River and link Europe with Asia.

For each quadratic function, find the axis of symmetry of its graph.

25. $y = x^2 + x + 2$

26. $y = 3x^2 - 2x - 6$

27. $y = \frac{1}{2}x^2 - 5x + 4$

28. $y = -2x^2 + \frac{1}{3}x - \frac{3}{4}$

Find the vertex of each parabola.

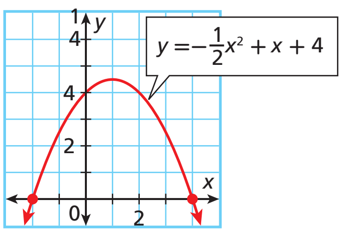
29. $y = x^2 + 7x$

30. $y = -x^2 + 8x + 16$

31. $y = -2x^2 - 8x - 3$

32. $y = -x^2 + \frac{1}{2}x + 2$

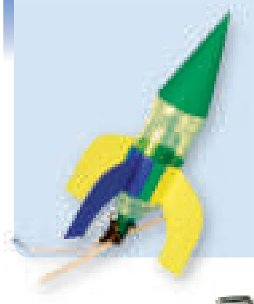
33.



34. Engineering The height in feet of the curved arch support for a pedestrian bridge over a creek can be modeled by $f(x) = -6.28x^2 + 4.5x$, where x is the distance in feet from where the arch support enters the water. If there is a flood that raises the level of the creek by 5.5 feet, will the top of the arch support be above the water? Explain.

35. Critical Thinking What conclusion can be drawn about the axis of symmetry of any quadratic function for which $b = 0$?

MULTI-STEP TEST PREP



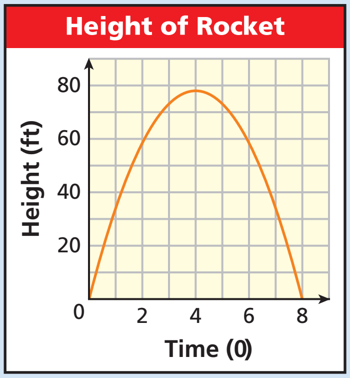
36. This problem will prepare you for the Multi-Step Test Prep on page 620.

a. Use the graph of the height of a water bottle rocket to estimate the coordinates of the parabola's vertex.

b. What does the vertex represent?

c. Find the zeros of the function. What do they represent?

d. Find the axis of symmetry. How is it related to the vertex and the zeros?



Graphing Calculator **Tell how many zeros each quadratic function has.**

37. $y = 8x^2 - 4x + 2$

38. $0 = y + 16x^2$

39. $\frac{1}{4}x^2 - 7x - 12 = y - 4$

40. **Write About It** If you are given the axis of symmetry of a quadratic function and know that the function has two zeros, how would you describe the location of the two zeros?

TEST PREP

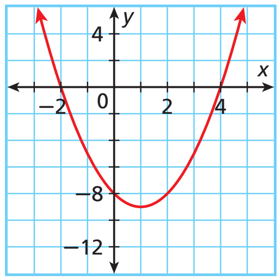
41. Which function has the zeros shown in the graph?

Ⓐ $y = x^2 + 2x + 8$

Ⓑ $y = x^2 - 2x - 8$

Ⓒ $y = x^2 + 2x - 8$

Ⓓ $y = 2x^2 - 2x + 8$



42. Which of the following functions has a graph with an axis of symmetry of $x = -\frac{1}{2}$?

Ⓕ $y = 2x^2 - 2x + 5$

Ⓖ $2x + 5 = 2x^2 - y$

Ⓗ $2x^2 + y = 2x + 5$

Ⓙ $2x - y = 5 - 2x^2$

43. **Gridded Response** For the graph of $f(y) = -3 + 20x - 5x^2$, what is the x-coordinate of its vertex?

CHALLENGE AND EXTEND

44. Describe the domain and range of a quadratic function that has exactly one zero and whose graph opens downward.

45. **Graphing Calculator** The height in feet of a parabolic bridge support is modeled by $f(x) = -0.01x^2 + 20$, where $y = -5$ represents ground level and the x-axis represents the middle of the bridge. Find the height and the width of the bridge support.

SPIRAL REVIEW

46. The value of y varies directly with x, and $y = -4$ when $x = 2$. Find y when $x = 6$. *(Lesson 5-5)*

Write each equation in slope-intercept form. *(Lesson 5-6)*

47. $2x + y = 3$

48. $4y = 12x - 8$

49. $10 - 5y = 20x$

Tell whether each function is quadratic. Explain. *(Lesson 9-1)*

50. $y = 5x - 7$

51. $x^2 - 5x = 2 + y$

52. $y = -x^2 - 6x$

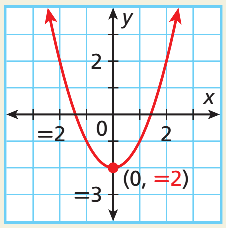
9-3 Graphing Quadratic Functions

Objective
Graph a quadratic function in the form $y = ax^2 + bx + c$.

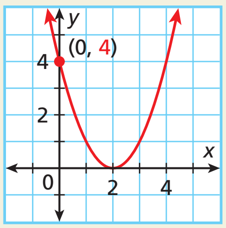
Why use this?
Graphs of quadratic functions can help you determine how high an object is tossed or kicked. (See Exercise 14.)

Recall that a y-intercept is the y-coordinate of the point where a graph intersects the y-axis. The x-coordinate of this point is always 0. For a quadratic function written in the form $y = ax^2 + bx + c$, when $x = 0$, $y = c$. So the y-intercept of a quadratic function is c.

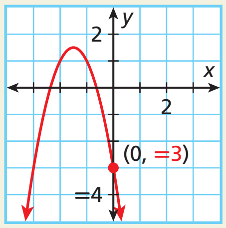
$y = x^2 - 2$
$y = x^2 + (-2)$



$y = x^2 - 4x + 4$



$y = -2x^2 - 6x - 3$
$y = -2x^2 - 6x + (-3)$



In the previous lesson, you found the axis of symmetry and vertex of a parabola. You can use these characteristics, the y-intercept, and symmetry to graph a quadratic function.

EXAMPLE 1 Graphing a Quadratic Function

Graph $y = x^2 - 4x - 5$.

Step 1 Find the axis of symmetry.

$x = -\frac{-4}{2(1)}$ — *Use $x = -\frac{b}{2a}$. Substitute 1 for a and −4 for b.*

$= 2$ — *Simplify.*

The axis of symmetry is $x = 2$.

Step 2 Find the vertex.

$y = x^2 - 4x - 5$

$= 2^2 - 4(2) - 5$ — *The x-coordinate of the vertex is 2. Substitute 2 for x.*

$= 4 - 8 - 5$ — *Simplify.*

$= -9$ — *The y-coordinate is −9.*

The vertex is $(2, -9)$.

Step 3 Find the y-intercept.

$y = x^2 - 4x - 5$

$y = x^2 - 4x + (-5)$ — *Identify c.*

The y-intercept is -5; the graph passes through $(0, -5)$.

Step 4 Find two more points on the same side of the axis of symmetry as the point containing the y-intercept.

Since the axis of symmetry is $x = 2$, choose x-values less than 2.

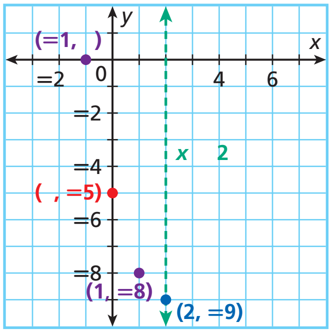
Let $x = 1$.		Let $x = -1$.
$y = 1^2 - 4(1) - 5$	*Substitute x-coordinates.*	$y = -1^2 - 4(-1) - 5$
$= 1 - 4 - 5$	*Simplify.*	$= 1 + 4 - 5$
$= -8$		$= 0$

Two other points are $(1, -8)$ and $(-1, 0)$.

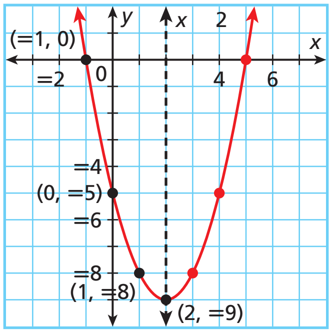
Helpful Hint

Because a parabola is symmetrical, each point is the same number of units away from the axis of symmetry as its reflected point.

Step 5 Graph the **axis of symmetry**, the **vertex**, the point containing the **y-intercept**, and **two other points**.



Step 6 **Reflect** the points across the axis of symmetry. Connect the points with a smooth curve.



CHECK IT OUT! **Graph each quadratic function.**

1a. $y = 2x^2 + 6x + 2$ **1b.** $y + 6x = x^2 + 9$

EXAMPLE 2 ***Problem-Solving Application***

PROBLEM SOLVING

The height in feet of a football that is kicked can be modeled by the function $f(x) = -16x^2 + 64x$, where x is the time in seconds after it is kicked. Find the football's maximum height and the time it takes the football to reach this height. Then find how long the football is in the air.

1 Understand the Problem

The **answer** includes three parts: the maximum height, the time to reach the maximum height, and the time to reach the ground.

List the important information:

- The function $f(x) = -16x^2 + 64x$ models the approximate height of the football after x seconds.

Remember!

The vertex is the highest or lowest point on a parabola. Therefore, in the example, it gives the maximum height of the football.

2 Make a Plan

Find the vertex of the graph because the maximum height of the football and the time it takes to reach it are the coordinates of the vertex. The football will hit the ground when its height is 0, so find the zeros of the function. You can do this by graphing.

3 Solve

Step 1 Find the axis of symmetry.

$x = -\frac{64}{2(-16)}$ *Use $x = -\frac{b}{2a}$. Substitute −16 for a and 64 for b.*

$= -\frac{64}{-32} = 2$ *Simplify.*

The axis of symmetry is $x = 2$.

Step 2 Find the vertex.

$y = -16x^2 + 64x$

$= -16(2)^2 + 64(2)$ *The x-coordinate of the vertex is 2. Substitute 2 for x.*

$= -16(4) + 128$ *Simplify.*

$= -64 + 128$

$= 64$ *The y-coordinate is 64.*

The vertex is $(2, 64)$.

Step 3 Find the y-intercept.

$y = -16x^2 + 64x + 0$ *Identify c.*

The y-intercept is 0; the graph passes through $(0, 0)$.

Step 4 Find another point on the same side of the axis of symmetry as the point containing the y-intercept.

Since the axis of symmetry is $x = 2$, choose an x-value that is less than 2.

Let $x = 1$.

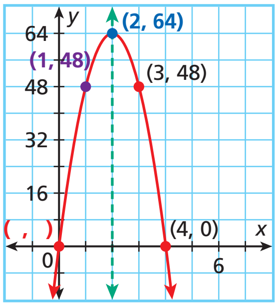
$y = -16(1)^2 + 64(1)$ *Substitute 1 for x.*

$= -16 + 64$ *Simplify.*

$= 48$

Another point is $(1, 48)$.

Step 5 Graph the **axis of symmetry**, the **vertex**, the point containing the **y-intercept**, and the **other point**. Then **reflect** the points across the axis of symmetry. Connect the points with a smooth curve.



The vertex is $(2, 64)$. So at 2 seconds, the football has reached its maximum height of 64 feet. The graph shows the zeros of the function are 0 and 4. At 0 seconds the football has not yet been kicked, and at 4 seconds it reaches the ground. The football is in the air for 4 seconds.

4 Look Back

Check by substituting $(2, 64)$ and $(4, 0)$ into the function.

$64 = -16(2)^2 + 64(2)$ $\quad$ $0 = -16(4)^2 + 64(4)$

$64 = -64 + 128$ $\quad$ $0 = -256 + 256$

$64 = 64$ ✓ $\quad$ $0 = 0$ ✓

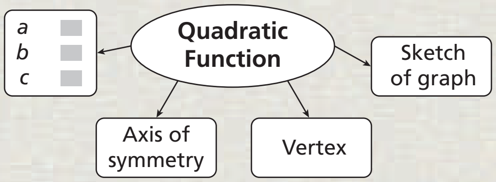
CHECK IT OUT!

2. As Molly dives into her pool, her height in feet above the water can be modeled by the function $f(x) = -16x^2 + 24x$, where x is the time in seconds after she begins diving. Find the maximum height of her dive and the time it takes Molly to reach this height. Then find how long it takes her to reach the pool.

THINK AND DISCUSS

1. Explain how to find the y-intercept of a quadratic function that is written in the form $ax^2 - y = bx + c$.
2. Explain how to graph a quadratic function.
3. What do you think the vertex and zeros of the function will tell you for the situation in the Check It Out for Example 2?
4. **GET ORGANIZED** Copy and complete the graphic organizer using your own quadratic function.

Know it! Note



9-3 Exercises

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KEYWORD: MA7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1 p. 606

Graph each quadratic function.

1. $y = x^2 - 2x - 3$
2. $-y - 3x^2 = -3$
3. $y = 2x^2 + 2x - 4$
4. $y = x^2 + 4x - 8$
5. $y + x^2 + 5x + 2 = 0$
6. $y = 4x^2 + 2$

SEE EXAMPLE 2 p. 607

7. **Multi-Step** The height in feet of a golf ball that is hit from the ground can be modeled by the function $f(x) = -16x^2 + 96x$, where x is the time in seconds after the ball is hit. Find the ball's maximum height and the time it takes the ball to reach this height. Then find how long the ball is in the air.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
8–13	1
14	2

Extra Practice

Skills Practice p. S20
Application Practice p. S36

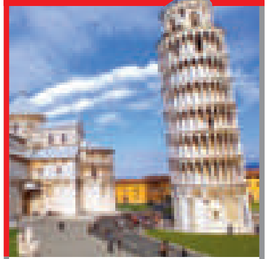
Graph each quadratic function.

8. $y = -4x^2 + 12x - 5$
9. $y = 3x^2 + 12x + 9$
10. $y - 7x^2 - 14x = 3$
11. $y = -x^2 + 2x$
12. $y - 1 = 4x^2 + 8x$
13. $y = -2x^2 - 3x + 4$
14. **Multi-Step** A juggler tosses a ring into the air. The height of the ring in feet above the juggler's hands can be modeled by the function $f(x) = -16x^2 + 16x$, where x is the time in seconds after the ring is tossed. Find the ring's maximum height above the juggler's hands and the time it takes the ring to reach this height. Then find how long the ring is in the air.

For each quadratic function, find the axis of symmetry and the vertex of its graph.

15. $y = x^2 - 8x$
16. $y = -x^2 + 6x - 4$
17. $y = 4 - 3x^2$
18. $y = -2x^2 - 4$
19. $y = -x^2 - x - 4$
20. $y = x^2 + 8x + 16$

Travel



Building began on the Tower of Pisa, located in Pisa, Italy, in 1173. The tower started leaning after the third story was added. At the fifth story, attempts were made to correct the leaning. The tower was finally complete in 1350.

Graph each quadratic function. On your graph, label the coordinates of the vertex. Draw and label the axis of symmetry.

21. $y = -x^2$ **22.** $y = -x^2 + 4x$ **23.** $y = x^2 - 6x + 4$

24. $y = x^2 - x$ **25.** $y = 3x^2 - 4$ **26.** $y = -2x^2 - 16x - 25$

27. **Travel** While on a vacation in Italy, Rudy visited the Leaning Tower of Pisa. When he leaned over the railing to look down from the tower, his sunglasses fell off. The height in meters of the sunglasses as they fell can be approximated by the function $y = -5x^2 + 50$, where x is the time in seconds.

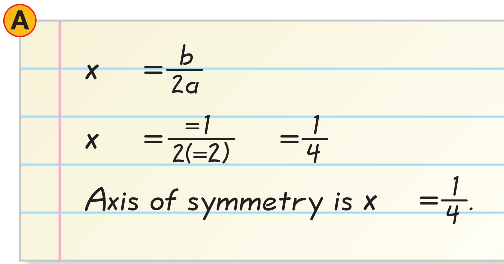
a. Graph the function. (*Hint:* Use a graphing calculator.)

b. What is a reasonable domain and range?

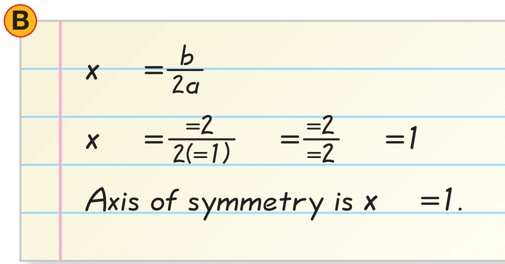
c. How long did it take for the glasses to reach the ground?

28. **ERROR ANALYSIS** Two students found the equation of the axis of symmetry for the graph of $f(x) = -x^2 - 2x + 1$. Who is incorrect? Explain the error.

A

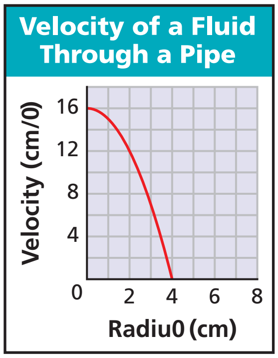


B



29. **Critical Thinking** The point $(5, 4)$ lies on the graph of a quadratic function whose axis of symmetry is $x = 2$. Find another point on the graph. Explain how you found the point.

Engineering **Use the graph for Exercises 30–32. The velocity v in centimeters per second of a fluid flowing in a pipe varies according to the radius r of the pipe.**



30. Find the radius of the pipe when the velocity is 7 cm/s.

31. Find the velocity of the fluid when the radius is 2 cm.

32. What is a reasonable domain for this function? Explain.

33. **Critical Thinking** The graph of a quadratic function has the vertex $(0, 5)$. One point on the graph is $(1, 6)$. Find another point on the graph. Explain how you found the point.

34. **Write About It** Explain how the vertex and the range can help you graph a quadratic function.

MULTI-STEP TEST PREP

35. This problem will prepare you for the Multi-Step Test Prep on page 620. A water bottle rocket is shot upward with an initial velocity of $v_i = 45$ ft/s from the roof of a school, which is at h_i, 50 ft above the ground. The equation $h = -\frac{1}{2}at^2 + v_i t + h_i$ models the rocket's height as a function of time. The acceleration due to gravity a is 32 ft/s^2.

a. Write the equation for height as a function of time for this situation.

b. Find the vertex of this parabola.

c. Sketch the graph of this parabola and label the vertex.

d. What do the coordinates of the vertex represent in terms of time and height?

36. Copy and complete the table for each function.

Function	Graph Opens	Axis of Symmetry	Vertex	Zeros	Domain and Range
$y = x^2 + 4$		$x =$	$(\ , \ )$		D: R:
$y = -x^2 + 4$		$x =$	$(\ , \ )$		D: R:
$y + 8 - x^2 = -2x$		$x =$	$(\ , \ )$		D: R:

TEST PREP

37. Which is the axis of symmetry for the graph of $f(x) = 6 - 5x + \frac{1}{2}x^2$?

Ⓐ $x = 5$ Ⓑ $x = \frac{1}{20}$ Ⓒ $x = -5$ Ⓓ $x = -\frac{1}{20}$

38. What are the coordinates of the vertex for the graph of $f(x) = x^2 - 5x + 6$?

Ⓕ $\left(-\frac{5}{2}, -\frac{1}{4}\right)$ Ⓖ $\left(-\frac{5}{2}, \frac{1}{4}\right)$ Ⓗ $\left(\frac{5}{2}, \frac{1}{4}\right)$ Ⓙ $\left(\frac{5}{2}, -\frac{1}{4}\right)$

39. Which function's graph has an axis of symmetry of $x = 1$ and a vertex of $(1, 8)$?

Ⓐ $y = -x^2 + x + 8$ Ⓒ $y = 2x^2 - 4x - 8$
Ⓑ $y = x^2 + 8x + 1$ Ⓓ $y = -3x^2 + 6x + 5$

40. **Short Response** Graph $y = x^2 + 3x + 2$. What are the zeros, the axis of symmetry, and the coordinates of the vertex? Show your work.

CHALLENGE AND EXTEND

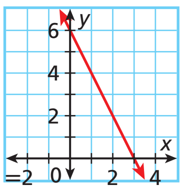
41. The graph of a quadratic function has its vertex at $(1, -4)$ and one zero of the function is 3. Find the other zero. Explain how you found the other zero.

42. The x-intercepts of a quadratic function are 3 and -3. The y-intercept is 6. What are the coordinates of the vertex? Does the function have a maximum or a minimum? Explain.

SPIRAL REVIEW

Find the x- and y-intercepts. *(Lesson 5-2)*

43.



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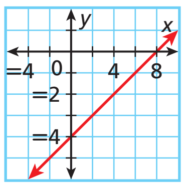

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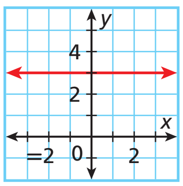

Solve each system by using any method. *(Lesson 6-3)*

46. $\begin{cases} 3x - y = 2 \\ x + 4y = 18 \end{cases}$

47. $\begin{cases} 2x + 3y = 3 \\ 4x - y = 13 \end{cases}$

48. $\begin{cases} -2x + 3y = 12 \\ 6x + y = 4 \end{cases}$

For each function, find the vertex of its graph. *(Lesson 9-2)*

49. $y = x^2 + 2x - 15$

50. $y = -3x^2 + 12x - 4$

51. $y = -2x - x^2 + 3$

9-4 Technology LAB

Use with Lesson 9-4

The Family of Quadratic Functions

In Chapter 5, you learned that functions whose graphs share the same basic characteristics form a *family of functions*. All quadratic functions form a family because their graphs are all parabolas. You can use a graphing calculator to explore the family of quadratic functions.

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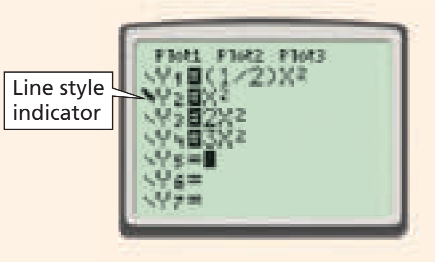
Activity

Describe how the value of a affects the graph of $y = ax^2$.

1. Press Y=. Enter $\mathbf{Y_1}$ through $\mathbf{Y_4}$ as shown.

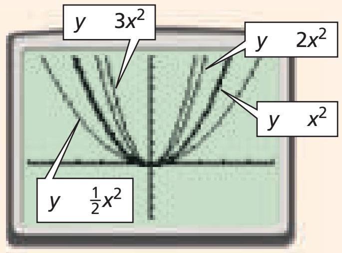
   Notice that $\mathbf{Y_2}$ represents the parent function $y = x^2$. To make it stand out from the other functions, change its line style. When you enter $\mathbf{Y_2}$, move the cursor to the line style indicator by pressing ◀. Then press ENTER to cycle through the choices until a thicker line appears.



2. Press GRAPH.

   Keep in mind the values of a as the functions are graphed. The graphing calculator will graph the functions in order.

   Notice that the graph of $y = \frac{1}{2}x^2$ is wider than the graph of the parent function. The graphs of $y = 2x^2$ and $y = 3x^2$ are narrower than the graph of the parent function.



Try This

1. How would the graph of $y = 6x^2$ compare with the graph of the parent function?
2. How would the graph of $y = \frac{1}{5}x^2$ compare with the graph of the parent function?
3. **Make a Conjecture** Make a conjecture about the effect of a on the graph of $y = ax^2$.

Consider the graphs of $y = -\frac{1}{2}x^2$, $y = -x^2$, $y = -2x^2$, and $y = -3x^2$.

4. Describe the differences in the graphs.
5. How would the graph of $y = -8x^2$ compare with the graph of $y = -x^2$?
6. How do these results affect your conjecture from Problem 3?

Consider the graphs of $y = x^2 - 1$, $y = x^2$, $y = x^2 + 2$, and $y = x^2 + 4$.

7. Describe the differences in the graphs.
8. How would the graph of $y = x^2 - 7$ compare with the graph of the parent function?
9. **Make a Conjecture** Make a conjecture about the effect of c on the graph of $y = x^2 + c$.

9-4 Transforming Quadratic Functions

Objective
Graph and transform quadratic functions.

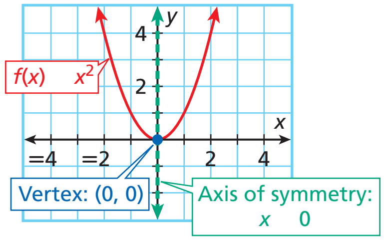
Why learn this?
You can compare how long it takes raindrops to reach the ground from different heights. (See Exercise 18.)

Remember!
You saw in Lesson 5-9 that the graphs of all linear functions are transformations of the linear parent function, $y = x$.

The quadratic parent function is $f(x) = x^2$. The graph of all other quadratic functions are transformations of the graph of $f(x) = x^2$.

For the parent function $f(x) = x^2$:

- The axis of symmetry is $x = 0$, or the y-axis.
- The vertex is $(0, 0)$.
- The function has only one zero, 0.



Compare the coefficients in the following functions.

$$f(x) = x^2 \qquad g(x) = \frac{1}{2}x^2$$

$$h(x) = -3x^2$$

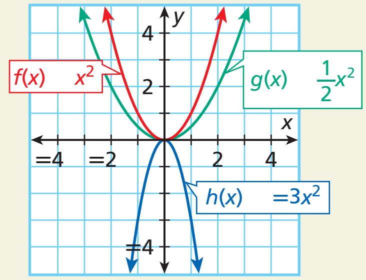
$$f(x) = 1x^2 + 0x + 0$$

$$g(x) = \frac{1}{2}x^2 + 0x + 0$$

$$h(x) = -3x^2 + 0x + 0$$

Same	Different
• $b = 0$ • $c = 0$	• Value of a

Compare the graphs of the same functions.

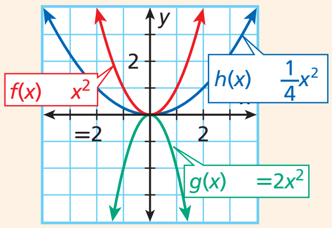


Same	Different
• Axis of symmetry is $x = 0$. • Vertex is $(0, 0)$.	• Widths of parabolas

The value of a in a quadratic function determines not only the direction a parabola opens, but also the width of the parabola.

Know it! Note — Width of a Parabola

WORDS	EXAMPLES
The graph of $f(x) = ax^2$ is **narrower** than the graph of $f(x) = x^2$ if $\lvert a \rvert > 1$ and **wider** if $\lvert a \rvert < 1$.	Compare the graphs of $g(x)$ and $h(x)$ with the graph of $f(x)$. $\lvert -2 \rvert$? 1 $\qquad$ $\left\lvert \frac{1}{4} \right\rvert$? 1 $2 > 1$ $\qquad$ $\frac{1}{4} < 1$ **narrower** $\qquad$ **wider**



EXAMPLE 1 Comparing Widths of Parabolas

Order the functions from narrowest graph to widest.

A $f(x) = -2x^2, g(x) = \frac{1}{3}x^2, h(x) = 4x^2$

Step 1 Find $|a|$ for each function.

$|-2| = 2 \qquad \left|\frac{1}{3}\right| = \frac{1}{3} \qquad |4| = 4$

Step 2 Order the functions.

$h(x) = 4x^2$

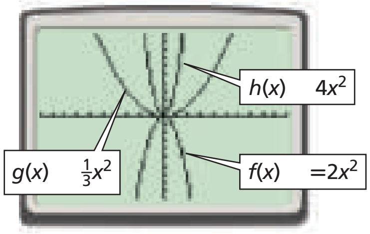
$f(x) = -2x^2$

$g(x) = \frac{1}{3}x^2$

The function with the narrowest graph has the greatest $|a|$.

Check Use a graphing calculator to compare the graphs.

$h(x) = 4x^2$ has the narrowest graph, and $g(x) = \frac{1}{3}x^2$ has the widest graph. ✓



B $f(x) = 2x^2, g(x) = -2x^2$

Step 1 Find $|a|$ for each function.

$|2| = 2 \qquad |-2| = 2$

Step 2 Order the functions from narrowest graph to widest.

Since the absolute values are equal, the graphs are the same width.

CHECK IT OUT! **Order the functions from narrowest graph to widest.**

1a. $f(x) = -x^2, g(x) = \frac{2}{3}x^2$

1b. $f(x) = -4x^2, g(x) = 6x^2, h(x) = 0.2x^2$

Compare the coefficients in the following functions.

$f(x) = x^2 \qquad g(x) = x^2 - 4$

$h(x) = x^2 + 3$

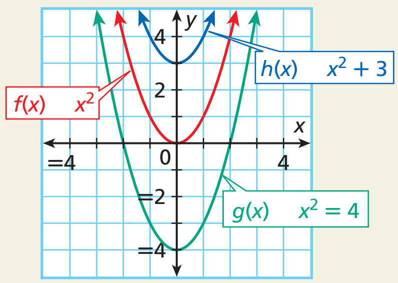
$f(x) = 1x^2 + 0x + 0$

$g(x) = 1x^2 + 0x + -4$

$h(x) = 1x^2 + 0x + 3$

Same	Different
• $a = 1$ • $b = 0$	• Value of c

Compare the graphs of the same functions.



Same	Different
• Axis of symmetry is $x = 0$. • Width of parabola	• Vertex of parabola

The value of c makes these graphs look different. The value of c in a quadratic function determines not only the value of the y-intercept but also a vertical translation of the graph of $f(x) = ax^2$ up or down the y-axis.

Vertical Translations of a Parabola

The graph of the function $f(x) = x^2 + c$ is the graph of $f(x) = x^2$ translated vertically.

- If $c > 0$, the graph of $f(x) = x^2$ is translated c units **up**.
- If $c < 0$, the graph of $f(x) = x^2$ is translated c units **down**.

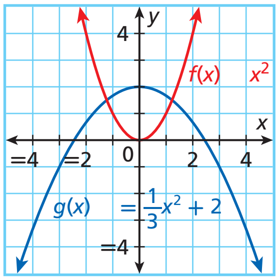
EXAMPLE 2

Comparing Graphs of Quadratic Functions

Compare the graph of each function with the graph of $f(x) = x^2$.

A $g(x) = -\frac{1}{3}x^2 + 2$

Method 1 Compare the graphs.



- The graph of $g(x) = -\frac{1}{3}x^2 + 2$ is **wider** than the graph of $f(x) = x^2$.
- The graph of $g(x) = -\frac{1}{3}x^2 + 2$ opens **downward**, and the graph of $f(x) = x^2$ opens **upward**.
- The axis of symmetry is the same.
- The vertex of $f(x) = x^2$ is $(0, 0)$.
  The vertex of $g(x) = -\frac{1}{3}x^2 + 2$ is translated **2 units up** to $(0, 2)$.

Helpful Hint

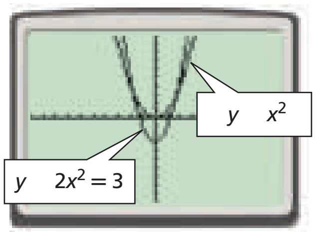
When comparing graphs, it is helpful to draw them on the same coordinate plane.

B $g(x) = 2x^2 - 3$

Method 2 Use the functions.

- Since $|2| > |1|$, the graph of $g(x) = 2x^2 - 3$ is **narrower** than the graph of $f(x) = x^2$.
- Since $-\frac{b}{2a} = 0$ for both functions, the axis of symmetry is the same.
- The vertex of $f(x) = x^2$ is $(0, 0)$. The vertex of $g(x) = 2x^2 - 3$ is translated **3 units down** to $(0, -3)$.

Check Use a graph to verify all comparisons.



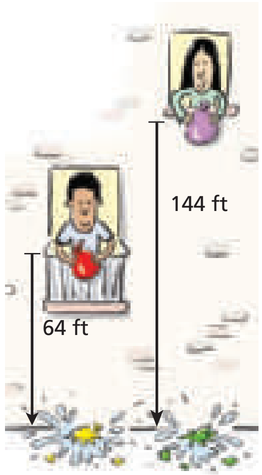
CHECK IT OUT! **Compare the graph of each function with the graph of $f(x) = x^2$.**

2a. $g(x) = -x^2 - 4$ **2b.** $g(x) = 3x^2 + 9$ **2c.** $g(x) = \frac{1}{2}x^2 + 2$

The quadratic function $h(t) = -16t^2 + c$ can be used to approximate the height h in feet above the ground of a falling object t seconds after it is dropped from a height of c feet. This model is used only to approximate the height of falling objects because it does not account for air resistance, wind, and other real-world factors.

EXAMPLE 3 *Physics Application*

Two identical water balloons are dropped from different heights as shown in the diagram.



a. Write the two height functions and compare their graphs.

Step 1 Write the height functions.
The y-intercept c represents the original height.

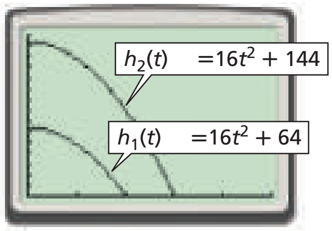
$h_1(t) = -16t^2 + 64$ *Dropped from 64 feet*

$h_2(t) = -16t^2 + 144$ *Dropped from 144 feet*

Caution!

Remember that the graphs shown here represent the *height* of the objects over time, not the *paths* of the objects.

Step 2 Use a graphing calculator.
Since time and height cannot be negative, set the window for nonnegative values.



The graph of h_2 is a vertical translation of the graph of h_1. Since the balloon in h_2 is dropped from 80 feet higher than the one in h_1, the y-intercept of h_2 is 80 units higher.

b. Use the graphs to tell when each water balloon reaches the ground.

The zeros of each function are when the water balloons reach the ground.

The water balloon dropped from 64 feet reaches the ground in 2 seconds. The water balloon dropped from 144 feet reaches the ground in 3 seconds.

Check These answers seem reasonable because the water balloon dropped from a greater height should take longer to reach the ground.

CHECK IT OUT!

3. Two tennis balls are dropped, one from a height of 16 feet and the other from a height of 100 feet.

a. Write the two height functions and compare their graphs.

b. Use the graphs to tell when each tennis ball reaches the ground.

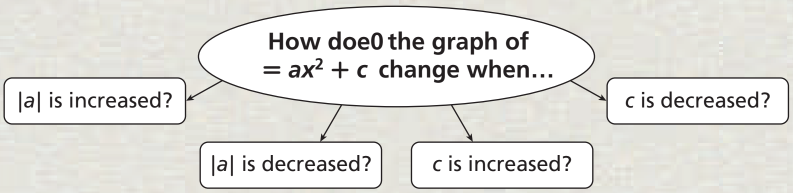
THINK AND DISCUSS

1. Describe how the graph of $y = x^2 + c$ differs from the graph of $y = x^2$ when the value of c is positive and when the value of c is negative.

2. Tell how to determine whether a graph of a function is wider or narrower than the graph of $f(x) = x^2$.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer by explaining how each change affects the graph $y = ax^2 + c$.



9-4 Exercises

GUIDED PRACTICE

SEE EXAMPLE 1
p. 614

Order the functions from narrowest graph to widest.

1. $f(x) = 3x^2,\ g(x) = 2x^2$

2. $f(x) = 5x^2,\ g(x) = -5x^2$

3. $f(x) = \frac{3}{4}x^2,\ g(x) = -2x^2,$
   $h(x) = -8x^2$

4. $f(x) = x^2,\ g(x) = -\frac{4}{5}x^2,$
   $h(x) = 3x^2$

SEE EXAMPLE 2
p. 615

Compare the graph of each function with the graph of $f(x) = x^2$.

5. $g(x) = x^2 + 6$

6. $g(x) = -2x^2 + 5$

7. $g(x) = \frac{1}{3}x^2$

8. $g(x) = -\frac{1}{4}x^2 - 2$

SEE EXAMPLE 3
p. 616

9. **Multi-Step** Two baseballs are dropped, one from a height of 16 feet and the other from a height of 256 feet.
   a. Write the two height functions and compare their graphs.
   b. Use the graphs to tell when each baseball reaches the ground.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
10–13	1
14–17	2
18	3

Extra Practice

Skills Practice p. S20
Application Practice p. S36

Order the functions from narrowest graph to widest.

10. $f(x) = x^2,\ g(x) = 4x^2$

11. $f(x) = -2x^2,\ g(x) = \frac{1}{2}x^2$

12. $f(x) = -x^2,\ g(x) = -\frac{5}{8}x^2,\ h(x) = \frac{1}{2}x^2$

13. $f(x) = -5x^2,\ g(x) = -\frac{3}{8}x^2,\ h(x) = 3x^2$

Compare the graph of each function with the graph of $f(x) = x^2$.

14. $g(x) = \frac{1}{2}x^2 - 10$

15. $g(x) = -4x^2 - 2$

16. $g(x) = \frac{2}{3}x^2 - 9$

17. $g(x) = -\frac{1}{5}x^2 + 1$

18. **Multi-Step** A raindrop falls from a cloud at an altitude of 10,000 ft. Another raindrop falls from a cloud at an altitude of 14,400 ft.
   a. Write the two height functions and compare their graphs.
   b. Use the graphs to tell when each raindrop reaches the ground.

Tell whether each statement is sometimes, always, or never true.

19. The graphs of $f(x) = ax^2$ and $g(x) = -ax^2$ have the same width.

20. The function $f(x) = ax^2 + c$ has three zeros.

21. The graph of $y = ax^2 + 1$ has its vertex at the origin.

22. The graph of $y = -x^2 + c$ intersects the x-axis.

23. **Data Collection** Use a graphing calculator and a motion detector to graph the height of a falling object over time.
   a. Find a function to model the height of the object while it is in motion.
   b. **Critical Thinking** Explain why the value of a in your function is not -16.

Write a function to describe each of the following.

24. The graph of $f(x) = x^2 + 10$ is translated 10 units down.

25. The graph of $f(x) = 3x^2 - 2$ is translated 4 units down.

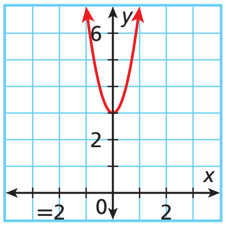
26. The graph of $f(x) = 0.5x^2$ is narrowed.

27. The graph of $f(x) = -5x^2$ is narrowed and translated 2 units up.

28. The graph of $f(x) = x^2 - 7$ is widened and has no x-intercept.

Match each function to its graph.

29. $f(x) = 4x^2 - 3$ **30.** $f(x) = \frac{1}{4}x^2 - 3$ **31.** $f(x) = 4x^2 + 3$ **32.** $f(x) = -\frac{1}{4}x^2 - 3$

A.


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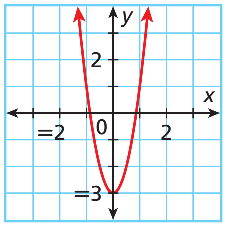
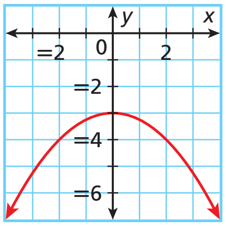
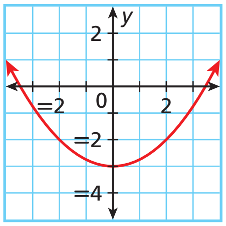


C.


D.


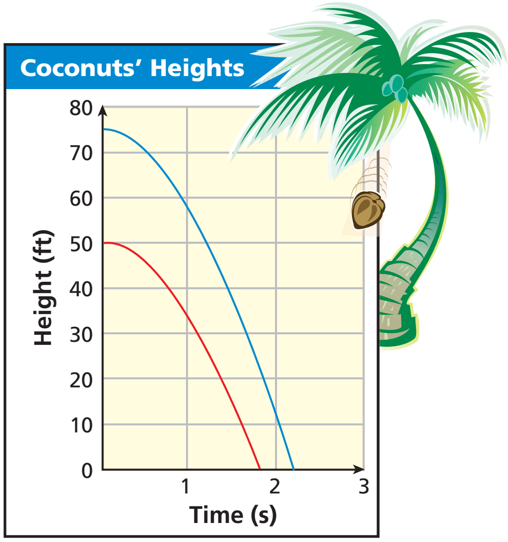
33. **Critical Thinking** For what values of a and c will $f(x) = ax^2 + c$ have one zero?

34. **Physics** The graph compares the heights of two identical coconuts that fell from different trees.

a. What are the starting heights of each coconut?

b. What is a possible function for the blue graph?

c. Estimate the time for each coconut to reach the ground.



35. Give an example of a quadratic function for each description.

a. Its graph opens upward.

b. Its graph has the same width as in part **a**, but the graph opens downward.

c. Its graph is narrower than the graph in part **a.**

36. **Critical Thinking** Describe how the effect that the value of c has on the graph of $y = x^2 + c$ is similar to the effect that the value of b has on the graph of $y = x + b$.

37. **Write About It** Explain how you know, without graphing, what the graph of $f(x) = \frac{1}{10}x^2 - 5$ looks like.

38. This problem will prepare you for the Multi-Step Test Prep on page 620.

a. Use a graphing calculator to graph $y = (x - 3)^2$. Compare this graph to the graph of $y = x^2$. How does this differ from $y = x^2 - 3$?

b. The equation $h = -16(x - 2)^2 + 64$ describes the height in feet of a water bottle rocket as a function of time. What is the highest point that the rocket will reach? When will it return to the ground?

c. How can the vertex be located from the equation? from the graph?

TEST PREP

39. Which function's graph is the result of shifting the graph of $f(x) = -x^2 - 4$ 3 units down?

Ⓐ $g(x) = -x^2 - 1$

Ⓑ $g(x) = -\frac{1}{3}x^2 - 4$

Ⓒ $g(x) = -4x^2 - 4$

Ⓓ $g(x) = -x^2 - 7$

40. Which of the following is true when the graph of $f(x) = x^2 + 4$ is transformed into the graph of $g(x) = 2x^2 + 4$?

Ⓕ The new function has more zeroes than the old function.

Ⓖ Both functions have the same vertex.

Ⓗ The function is translated up.

Ⓙ The axis of symmetry changes.

41. **Gridded Response** For what value of c will $f(x) = x^2 + c$ have one zero?

CHALLENGE AND EXTEND

42. **Graphing Calculator** Graph the functions $f(x) = (x+1)^2$, $g(x) = (x+4)^2$, $h(x) = (x-2)^2$, and $k(x) = (x-5)^2$. Make a conjecture about the result of transforming the graph of $f(x) = x^2$ into the graph of $f(x) = (x-h)^2$.

43. Using the function $f(x) = x^2$, write each new function:

a. The graph is translated 7 units down.

b. The graph is reflected across the x-axis and translated 2 units up.

c. Each y-value is halved, and then the graph is translated 1 unit up.

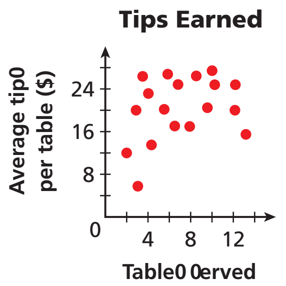
SPIRAL REVIEW

44. Justify each step. *(Lesson 1-7)*

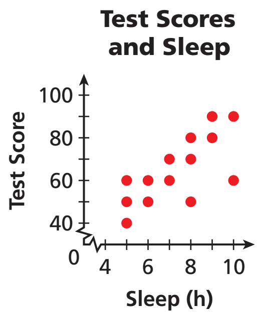
Procedure	Justification
$5x - 2(4 - x)$	
$5x - 2(4 - x) = 5x - 8 + 2x$	**a.** ____?____
$= 5x + 2x - 8$	**b.** ____?____
$= (5x + 2x) - 8$	**c.** ____?____
$= 7x - 8$	**d.** ____?____

Describe the correlation illustrated by each scatter plot. *(Lesson 4-5)*

45.



46.



Graph each quadratic function. *(Lesson 9-3)*

47. $y = 2x^2 - 1$

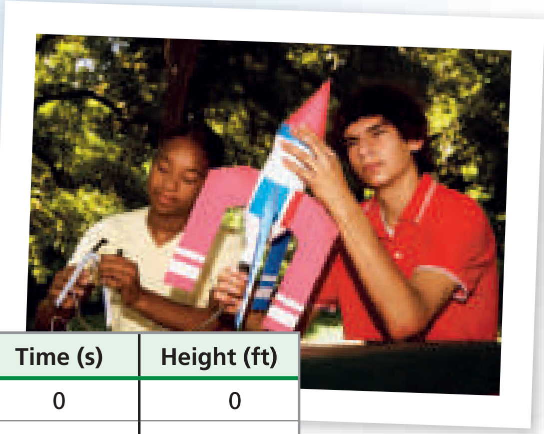
48. $y = x^2 - 2x - 2$

49. $y = -3x^2 - x + 6$

MULTI-STEP TEST PREP

Quadratic Functions

The Sky's the Limit The Physics Club is using computer simulation software to design a water bottle rocket that doesn't have a parachute. The data for their current design are shown in the table.



Time (s)	Height (ft)
0	0
1	80
2	128
3	144
4	128
5	80

1. Tell whether the data satisfy a quadratic function.
2. Graph the function from Problem 1.
3. Find and label the zeros, axis of symmetry, and vertex.
4. Explain what the x- and y-coordinates of the vertex represent in the context of the problem.
5. Estimate how many seconds it will take the rocket to reach 110 feet. Explain.

READY TO GO ON?

Quiz for Lessons 9-1 Through 9-4

9-1 Identifying Quadratic Functions

Tell whether each function is quadratic. Explain.

1. $y + 2x^2 = 3x$ **2.** $x^2 + y = 4 + x^2$ **3.** $(-2, 12)(-1, 3)(0, 0)(1, 3)$

Tell whether the graph of each quadratic function opens upward or downward and whether the parabola has a maximum or a minimum.

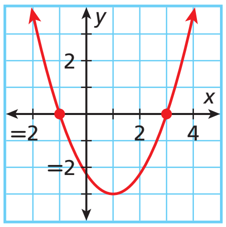
4. $y = -x^2 - 7x + 18$ **5.** $y - 2x^2 = 4x + 3$ **6.** $f(x) = 5x - 0.5x^2$

7. Graph the function $y = \frac{1}{2}x^2 - 2$ and give the domain and range.

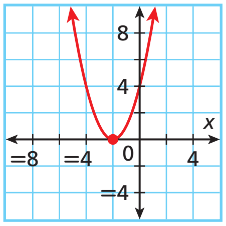
9-2 Characteristics of Quadratic Functions

Find the zeros of each function from its graph. Then find its the axis of symmetry.

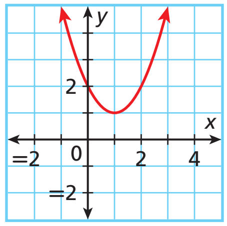
8.



9.



10.



Find the vertex of each parabola.

11. $y = x^2 + 6x + 2$ **12.** $y = 3 + 4x - 2x^2$ **13.** $y = 3x^2 + 12x - 12$

14. The height in feet of the curved roof of an aircraft hangar can be modeled by $y = -0.02x^2 + 1.6x$, where x is the distance in feet from one wall at ground level. How tall is the hangar?

9-3 Graphing Quadratic Functions

Graph each quadratic function.

15. $y = x^2 + 3x + 9$ **16.** $y = x^2 - 2x - 15$ **17.** $y = x^2 - 2x - 8$

18. $y = 2x^2 - 6$ **19.** $y = 4x^2 + 8x - 2$ **20.** $y = 2x^2 + 10x + 1$

9-4 Transforming Quadratic Functions

Compare the graph of each function with the graph of $f(x) = x^2$.

21. $g(x) = x^2 - 2$ **22.** $g(x) = \frac{2}{3}x^2$ **23.** $g(x) = 5x^2 + 3$ **24.** $g(x) = -x^2 + 4$

25. The pilot of a hot-air balloon drops a sandbag onto a target from a height of 196 feet. Later, he drops an identical sandbag from a height of 676 feet.

a. Write the two height functions and compare their graphs. Use $h(t) = -16t^2 + c$, where c is the height of the balloon.

b. Use the graphs to tell when each sandbag will reach the ground.

9-5 Solving Quadratic Equations by Graphing

Objective
Solve quadratic equations by graphing.

Vocabulary
quadratic equation

Who uses this?
Dolphin trainers can use solutions of quadratic equations to plan the choreography for their shows. (See Example 2.)

Every quadratic function has a related *quadratic equation*. A **quadratic equation** is an equation that can be written in the standard form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$.

When writing a quadratic function as its related quadratic equation, you replace y with 0. So $y = 0$.

$$y = ax^2 + bx + c$$
$$0 = ax^2 + bx + c$$
$$ax^2 + bx + c = 0$$

One way to solve a quadratic equation in standard form is to graph the related function and find the x-values where $y = 0$. In other words, find the zeros of the related function. Recall that a quadratic function may have two, one, or no zeros.

Know it! Note

Solving Quadratic Equations by Graphing
Step 1 Write the related function.
Step 2 Graph the related function.
Step 3 Find the zeros of the related function.

EXAMPLE 1 **Solving Quadratic Equations by Graphing**

Solve each equation by graphing the related function.

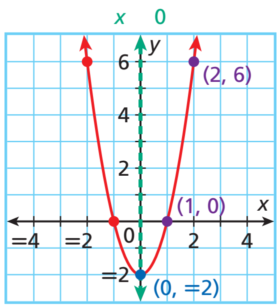
A $2x^2 - 2 = 0$

Step 1 Write the related function.

$2x^2 - 2 = y$, or $y = 2x^2 + 0x - 2$

Step 2 Graph the function.

- The axis of symmetry is $x = 0$.
- The vertex is $(0, -2)$.
- Two other points are $(1, 0)$ and $(2, 6)$.
- Graph the points and reflect them across the axis of symmetry.



Step 3 Find the zeros.

The zeros appear to be −1 and 1.

Check

$2x^2 - 2$	0
$2(-1)^2 - 2$	0
$2(1) - 2$	0
$2 - 2$	0
0	0 ✓

Substitute −1 and 1 for x in the quadratic equation.

$2x^2 - 2$	0
$2(1)^2 - 2$	0
$2(1) - 2$	0
$2 - 2$	0
0	0 ✓

Solve each equation by graphing the related function.

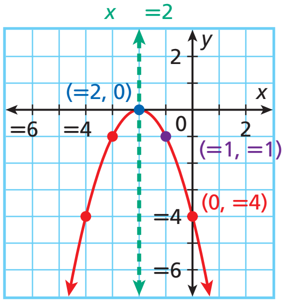
B $-x^2 - 4x - 4 = 0$

Step 1 Write the related function.

$y = -x^2 - 4x - 4$

Step 2 Graph the function.

- The axis of symmetry is $x = -2$.
- The vertex is $(-2, 0)$.
- The y-intercept is -4.
- Another point is $(-1, -1)$.
- Graph the points and reflect them across the axis of symmetry.



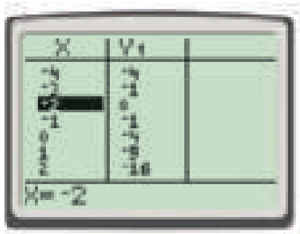
Step 3 Find the zeros.

The only zero appears to be -2.

Check $y = -x^2 - 4x - 4$

0	$-(-2)^2 - 4(-2) - 4$
0	$-(4) + 8 - 4$
0	$-4 + 4$
0	0 ✓

You can also confirm the solution by using the **Table** function. Enter the equation and press **2nd** **GRAPH** (TABLE). When $y = 0$, $x = -2$. The x-intercept is -2.



C $x^2 + 5 = 4x$

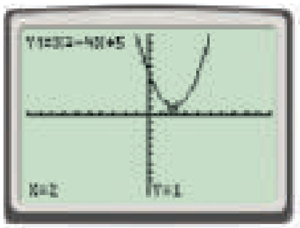
Step 1 Write the related function.

$x^2 - 4x + 5 = 0$

$y = x^2 - 4x + 5$

Step 2 Graph the function.

Use a graphing calculator.

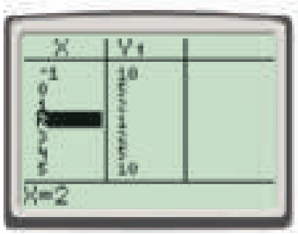


Step 3 Find the zeros.

The function appears to have no zeros.

The equation has no real-number solutions.

Check reasonableness Use the table function.



There are no zeros in the Y1 column. Also, the signs of the values in this column do not change. The function appears to have no zeros.

CHECK IT OUT! **Solve each equation by graphing the related function.**

1a. $x^2 - 8x - 16 = 2x^2$

1b. $6x + 10 = -x^2$

1c. $-x^2 + 4 = 0$

EXAMPLE 2 ***Aquatics Application***

A dolphin jumps out of the water. The quadratic function $y = -16x^2 + 20x$ models the dolphin's height above the water after x seconds. About how long is the dolphin out of the water?

When the dolphin leaves the water, its height is 0, and when the dolphin reenters the water, its height is 0. So solve $0 = -16x^2 + 20x$ to find the times when the dolphin leaves and reenters the water.

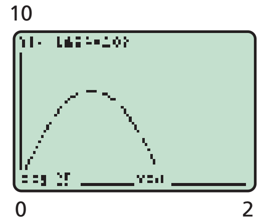
Step 1 Write the related function.

$0 = -16x^2 + 20x$

$y = -16x^2 + 20x$

Step 2 Graph the function.

Use a graphing calculator.



Step 3 Use TRACE to estimate the zeros.

The zeros appear to be 0 and 1.25.

The dolphin leaves the water at 0 seconds and reenters the water at 1.25 seconds.

The dolphin is out of the water for about 1.25 seconds.

Check $0 = -16x^2 + 20x$

0	$-16(1.25)^2 + 20(1.25)$
0	$-16(1.5625) + 25$
0	$-25 + 25$
0	0 ✓

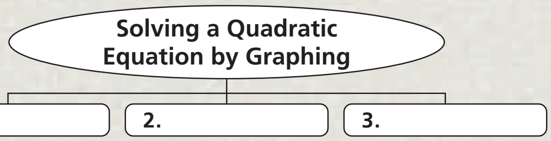
Substitute 1.25 for x in the quadratic equation.

2. **What if...?** Another dolphin jumps out of the water. The quadratic function $y = -16x^2 + 32x$ models the dolphin's height above the water after x seconds. About how long is the dolphin out of the water?

THINK AND DISCUSS

1. Describe the graph of a quadratic function whose related quadratic equation has only one solution.
2. Describe the graph of a quadratic function whose related quadratic equation has no real solutions.
3. Describe the graph of a quadratic function whose related quadratic equation has two solutions.

4. **GET ORGANIZED** Copy and complete the graphic organizer. In each of the boxes, write the steps for solving quadratic equations by graphing.



9-5 Exercises

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GUIDED PRACTICE

1. **Vocabulary** Write two words related to the graph of the related function that can be used to find the solution of a *quadratic equation*.

SEE EXAMPLE 1 p. 622

Solve each equation by graphing the related function.

2. $x^2 - 4 = 0$
3. $x^2 = 16$
4. $-2x^2 - 6 = 0$
5. $-x^2 + 12x - 36 = 0$
6. $-x^2 = -9$
7. $2x^2 = 3x^2 - 2x - 8$
8. $x^2 - 6x + 9 = 0$
9. $8x = -4x^2 - 4$
10. $x^2 + 5x + 4 = 0$
11. $x^2 + 2 = 0$
12. $x^2 - 6x = 7$
13. $x^2 + 5x = -8$

SEE EXAMPLE 2 p. 624

14. **Sports** A baseball coach uses a pitching machine to simulate pop flies during practice. The baseball is shot out of the pitching machine with a velocity of 80 feet per second. The quadratic function $y = -16x^2 + 80x$ models the height of the baseball after x seconds. How long is the baseball in the air?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
15–23	1
24	2

Extra Practice
Skills Practice p. S21
Application Practice p. S36

Solve each equation by graphing the related function.

15. $-x^2 + 16 = 0$
16. $3x^2 = -7$
17. $5x^2 - 12x + 10 = x^2 + 10x$
18. $x^2 + 10x + 25 = 0$
19. $-4x^2 - 24x = 36$
20. $-9x^2 + 10x - 9 = -8x$
21. $-x^2 - 1 = 0$
22. $3x^2 - 27 = 0$
23. $4x^2 - 4x + 5 = 2x^2$

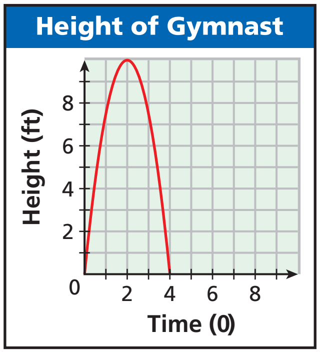
24. **Geography** Yosemite Falls in California is made of three smaller falls. The upper fall drops 1450 feet. The height h in feet of a water droplet falling from the upper fall to the next fall is modeled by $h(t) = -16t^2 + 1450$, where t is the time in seconds after the initial fall. Estimate the time it takes for the droplet to reach the next cascade.

Tell whether each statement is always, sometimes, or never true.

25. If the graph of a quadratic function has its vertex at the origin, then the related quadratic equation has exactly one solution.
26. If the graph of a quadratic function opens upward, then the related quadratic equation has two solutions.
27. If the graph of a quadratic function has its vertex on the x-axis, then the related quadratic equation has exactly one solution.
28. If the graph of a quadratic function has its vertex in the first quadrant, then the related quadratic equation has two solutions.
29. A quadratic equation in the form $ax^2 - c = 0$, where $a < 0$ and $c > 0$, has two solutions.

30. **Graphing Calculator** A fireworks shell is fired from a mortar. Its height is modeled by the function $h(t) = -16(t - 7)^2 + 784$, where t is the time in seconds and h is the height in feet.
    a. Graph the function.
    b. If the shell is supposed to explode at its maximum height, at what height should it explode?
    c. If the shell does not explode, how long will it take to return to the ground?

31. **Athletics** The graph shows the height y in feet of a gymnast jumping off a vault after x seconds.



a. How long does the gymnast stay in the air?

b. What is the maximum height that the gymnast reaches?

c. Explain why the function $y = -5x^2 + 10x$ cannot accurately model the gymnast's motion.

32. **Graphing Calculator** Use a graphing calculator to solve the equation $x^2 = x + 12$ by graphing $y_1 = x^2$ and $y_2 = x + 12$ and finding the x-coordinates of the points of intersection. (*Hint:* Find the points of intersection by using the 2nd CALC TRACE function after graphing.)

LINK Biology

Some species of kangaroos are able to jump 30 feet in distance and 6 feet in height.

33. **Biology** The quadratic function $y = -5x^2 + 7x$ approximates the height y of a kangaroo x seconds after it has jumped. About how long does it take the kangaroo to return to the ground?

For Exercises 34–36, use the table to determine the solutions of the related quadratic equation.

34.

x	y
−2	−1
−1	0
0	−1
1	−4
2	−9

35.

x	y
−2	−6
−1	0
0	2
1	0
2	−6

36.

x	y
−2	6
−1	3
0	2
1	3
2	6

37. **Geometry** The hypotenuse of a right triangle is 4 cm longer than one leg and 8 cm longer than the other leg. Let x represent the length of the hypotenuse.

a. Write an expression for the length of each leg in terms of x?

b. Use the Pythagorean Theorem to write an equation that can be solved for x.

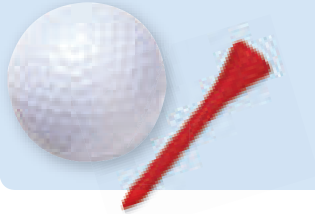
c. Find the solutions of your equation from part **b.**

d. **Critical Thinking** What do the solutions of your equation represent? Are both solutions reasonable? Explain.

38. **Write About It** Explain how to find solutions of a quadratic equation by analyzing a table of values.

39. **Critical Thinking** Explain why a quadratic equation in the form $ax^2 - c = 0$, where $a > 0$ and $c > 0$, will always have two solutions. Explain why a quadratic equation in the form $ax^2 + c = 0$, where $a > 0$ and $c > 0$, will never have any real-number solutions.

MULTI-STEP TEST PREP



40. This problem will prepare you for the Multi-Step Test Prep on page 660.
The quadratic equation $0 = -16t^2 + 80t$ gives the time t in seconds when a golf ball is at height 0 feet.

a. How long is the golf ball in the air?

b. What is the maximum height of the golf ball?

c. After how many seconds is the ball at its maximum height?

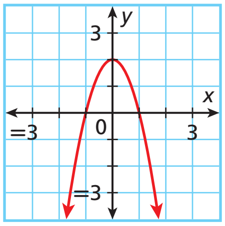
d. What is the height of the ball after 3.5 seconds? Is there another time when the ball reaches that height? Explain.

TEST PREP

41. Use the graph to find the number of solutions of $-2x^2 + 2 = 0$.

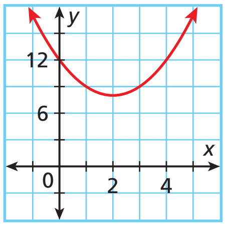
(A) 0 (C) 2

(B) 1 (D) 3

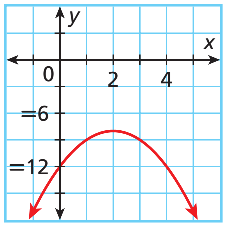


42. Which graph could be used to find the solutions of $x^2 = -4x + 12$?

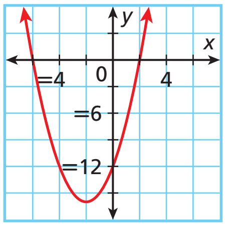
(F)



(H)



(G)



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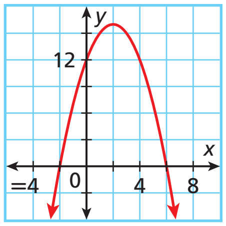

43. **Short Response** Find the solutions of $2x^2 + x - 1 = 0$ by graphing. Explain how the graph of the related function shows the solutions of the equation.

CHALLENGE AND EXTEND

Graphing Calculator **Use a graphing calculator to find the solutions of each quadratic equation.**

44. $\frac{5}{16}x + \frac{1}{4}x^2 = \frac{3}{5}$

45. $1200x^2 - 650x - 100 = -200x - 175$

46. $\frac{1}{5}x + \frac{3}{4}x^2 = \frac{7}{12}$

47. $400x^2 - 100 = -300x + 456$

SPIRAL REVIEW

Write an equation in point-slope form for the line with the given slope that contains the given point. *(Lesson 5–7)*

48. $\text{slope} = \frac{1}{2}; (2, 3)$

49. $\text{slope} = -3; (-2, 4)$

50. $\text{slope} = 0; (2, 1)$

Simplify. *(Lesson 7–4)*

51. $\frac{3^4}{3}$

52. $\frac{5^2 \cdot 2^4}{5 \cdot 2^2}$

53. $\frac{(x^4)^5}{(x^3)^3}$

54. $\left(\frac{x^3}{y^2}\right)^{-3}$

55. $\left(\frac{a^2b^3}{ab^2}\right)^3$

56. $\left(\frac{4s}{3t}\right)^{-2}$

57. $\left(\frac{2}{3}\right)^{-3} \cdot \left(\frac{a^3}{b}\right)^{-2}$

58. $\left(\frac{-k^2}{5k^3}\right)^{-3}$

Compare the graph of each function with the graph of $f(x) = x^2$. *(Lesson 9–4)*

59. $g(x) = 3x^2$

60. $g(x) = x^2 - 8$

61. $g(x) = \frac{3}{4}x^2 + 2$

9-5 Technology LAB

Use with Lesson 9-5

Explore Roots, Zeros, and *x*-Intercepts

The solutions, or *roots*, of a quadratic equation are the x-intercepts, or zeros, of the related quadratic function. You can use tables or graphs on a graphing calculator to understand the connections between zeros, roots, and x-intercepts.

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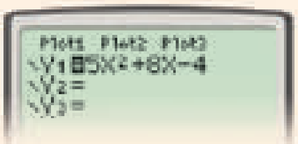
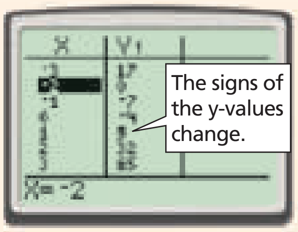
Activity 1

Solve $5x^2 + 8x - 4 = 0$ by using a table.



1. Enter the related function in Y_1.
2. Press 2nd GRAPH (TABLE) to use the **TABLE** function.
3. Scroll through the values by using ▲ and ▼. Look for values of 0 in the Y_1 column. The corresponding x-value is a zero of the function. There is one zero at -2.

   Also look for places where the signs of nonzero y-values change. There is a zero between the corresponding x-values. So there is another zero somewhere between 0 and 1.



4. To get a better estimate of the zero, change the table settings. Press 2nd WINDOW (TBLSET) to view the **TABLE SETUP** screen. Set **TblStart = 0** and the step value **△Tbl = .1**. Press 2nd GRAPH (TABLE) to see the table again. The table will show you more x-values between 0 and 1.

5. Scroll through the values by using ▲ and ▼. The second zero is at 0.4.

   The zeros of the function, -2 and 0.4, are the solutions, or roots, of the equation $5x^2 + 8x - 4 = 0$. Check the solutions algebraically.

Check

$5x^2 + 8x - 4$	$= 0$
$5(-2)^2 + 8(-2) - 4$	0
$5(4) - 16 - 4$	0
$20 - 16 - 4$	0
0	0 ✓

$5x^2 + 8x - 4$	$= 0$
$5(0.4)^2 + 8(0.4) - 4$	0
$5(0.16) + 3.2 - 4$	0
$0.8 + 3.2 - 4$	0
0	0 ✓

Try This

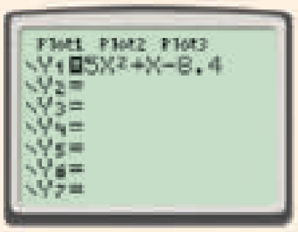
Solve each equation by using a table.

1. $x^2 - 4x - 5 = 0$
2. $x^2 - x - 6 = 0$
3. $2x^2 + x - 1 = 0$
4. $5x^2 - 6x - 8 = 0$
5. **Critical Thinking** How would you find the zero of a function that showed a sign change in the y-values between the x-values 1.2 and 1.3?
6. **Make a Conjecture** If you scrolled up and down the list and found only positive values, what might you conclude?

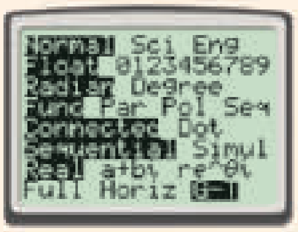
Activity 2

Solve $5x^2 + x - 8.4 = 0$ by using a table and a graph.

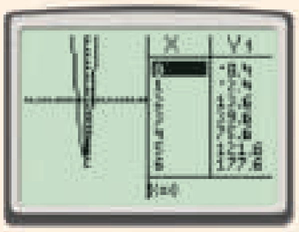
1. Enter the related function in $\mathbf{Y_1}$.



2. To view both the table and the graph at the same time, set your calculator to the Graph-Table mode. Press MODE and select **G-T**.



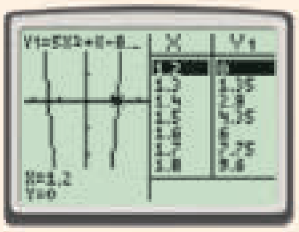
3. Press GRAPH. You should see the graph and the table. Notice that the function appears to have one negative zero and one positive zero near the *y*-axis.



4. To get a closer view of the graph, press ZOOM and select **4:ZDecimal**.

5. Press TRACE. Use ◄ to scroll to find the negative zero. The graph and the table show that the zero is -1.4.

6. Use ► to scroll and find the positive zero. The graph and the table show that the zero is 1.2.



The solutions are -1.4 and 1.2. Check the solutions algebraically.

$5x^2 + x - 8.4$	$= 0$
$5(-1.4)^2 + (-1.4) - 8.4$	0
$5(1.96) - 1.4 - 8.4$	0
$9.8 - 1.4 - 8.4$	0
0	0 ✓

$5x^2 + x - 8.4$	$= 0$
$5(1.2)^2 + (1.2) - 8.4$	0
$5(1.44) + 1.2 - 8.4$	0
$7.2 + 1.2 - 8.4$	0
0	0 ✓

Try This

Solve each equation by using a table and a graph.

7. $2x^2 - x - 3 = 0$ **8.** $5x^2 + 13x + 6 = 0$ **9.** $10x^2 - 3x - 4$ **10.** $x^2 - 2x - 0.96 = 0$

11. Critical Thinking Suppose that when you graphed a quadratic function, you could see only one side of the graph and one zero. What methods would you use to try to find the other zero?

9-6 Solving Quadratic Equations by Factoring

Objective
Solve quadratic equations by factoring.

Who uses this?
In order to determine how many seconds she will be in the air, a high diver can use a quadratic equation. (See Example 3.)

You have solved quadratic equations by graphing. Another method used to solve quadratic equations is to factor and use the Zero Product Property.

Know it! Note

Zero Product Property

For all real numbers *a* and *b*,

WORDS	NUMBERS	ALGEBRA
If the product of two quantities equals zero, at least one of the quantities equals zero.	$3(0) = 0$ $0(4) = 0$	If $ab = 0$, then $a = 0$ or $b = 0$.

EXAMPLE 1 **Using the Zero Product Property**

Use the Zero Product Property to solve each equation. Check your answer.

A $(x - 3)(x + 7) = 0$

$x - 3 = 0$ or $x + 7 = 0$ — *Use the Zero Product Property.*

$x = 3$ or $x = -7$ — *Solve each equation.*

The solutions are 3 and −7.

Check

$$\begin{array}{r|l} (x-3)(x+7) & 0 \\ \hline (3-3)(3+7) & 0 \\ (0)(10) & 0 \\ 0 & 0\ \checkmark \end{array}$$

Substitute each solution for x into the original equation.

$$\begin{array}{r|l} (x-3)(x+7) & 0 \\ \hline (-7-3)(-7+7) & 0 \\ (-10)(0) & 0 \\ 0 & 0\ \checkmark \end{array}$$

B $(x)(x - 5) = 0$

$x = 0$ or $x - 5 = 0$ — *Use the Zero Product Property.*

$x = 5$ — *Solve the second equation.*

The solutions are 0 and 5.

Check

$$\begin{array}{r|l} (x)(x-5) & 0 \\ \hline (0)(0-5) & 0 \\ (0)(-5) & 0 \\ 0 & 0\ \checkmark \end{array}$$

Substitute each solution for x into the original equation.

$$\begin{array}{r|l} (x)(x-5) & 0 \\ \hline (5)(5-5) & 0 \\ (5)(0) & 0 \\ 0 & 0\ \checkmark \end{array}$$

CHECK IT OUT!

Use the Zero Product Property to solve each equation. Check your answer.

1a. $(x)(x + 4) = 0$

1b. $(x + 4)(x - 3) = 0$

If a quadratic equation is written in standard form, $ax^2 + bx + c = 0$, then to solve the equation, you may need to factor before using the Zero Product Property.

> **Helpful Hint**
>
> To review factoring techniques, see Lessons 8-3 through 8-5.

EXAMPLE 2 Solving Quadratic Equations by Factoring

Solve each quadratic equation by factoring.

A $x^2 + 7x + 10 = 0$

$(x + 5)(x + 2) = 0$	*Factor the trinomial.*
$x + 5 = 0$ or $x + 2 = 0$	*Use the Zero Product Property.*
$x = -5$ or $x = -2$	*Solve each equation.*

The solutions are -5 and -2.

Check

$$\begin{array}{r|l} x^2 + 7x + 10 & 0 \\ \hline (-5)^2 + 7(-5) + 10 & 0 \\ 25 - 35 + 10 & 0 \\ 0 & 0 \checkmark \end{array} \qquad \begin{array}{r|l} x^2 + 7x + 10 & 0 \\ \hline (-2)^2 + 7(-2) + 10 & 0 \\ 4 - 14 + 10 & 0 \\ 0 & 0 \checkmark \end{array}$$

B $x^2 + 2x = 8$

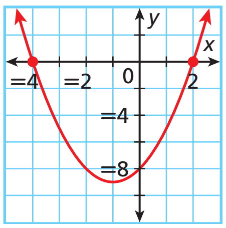
$$\begin{array}{rcr} x^2 + 2x & = & 8 \\ -8 & & -8 \\ \hline x^2 + 2x - 8 & = & 0 \end{array}$$

The equation must be written in standard form. So subtract 8 from both sides.

$(x + 4)(x - 2) = 0$	*Factor the trinomial.*
$x + 4 = 0$ or $x - 2 = 0$	*Use the Zero Product Property.*
$x = -4$ or $x = 2$	*Solve each equation.*

The solutions are -4 and 2.

Check Graph the related quadratic function. The zeros of the related function should be the same as the solutions from factoring.



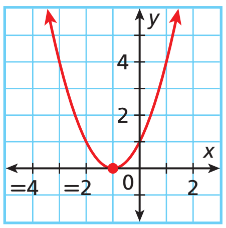
The graph of $y = x^2 + 2x - 8$ shows two zeros appear to be -4 and 2, the same as the solutions from factoring. ✓

C $x^2 + 2x + 1 = 0$

$(x + 1)(x + 1) = 0$	*Factor the trinomial.*
$x + 1 = 0$ or $x + 1 = 0$	*Use the Zero Product property.*
$x = -1$ or $x = -1$	*Solve each equation.*

Both factors result in the same solution, so there is one solution, -1.

Check Graph the related quadratic function.



The graph of $y = x^2 + 2x + 1$ shows that one zero appears to be -1, the same as the solution from factoring. ✓

Solve each quadratic equation by factoring.

D $-2x^2 = 18 - 12x$

$-2x^2 + 12x - 18 = 0$	*Write the equation in standard form.*
$-2(x^2 - 6x + 9) = 0$	*Factor out the GCF, −2.*
$-2(x-3)(x-3) = 0$	*Factor the trinomial.*
$-2 \neq 0$ or $x - 3 = 0$	*Use the Zero Product Property. −2 cannot equal 0.*
$x = 3$	*Solve the remaining equation.*

The only solution is 3.

Check

$-2x^2$	$= 18 - 12x$	
$-2(3)^2$	$18 - 12(3)$	*Substitute 3 into the original equation.*
-18	$18 - 36$	
-18	-18 ✓	

Helpful Hint

$(x - 3)(x - 3)$ is a perfect square. Since both factors are the same, you solve only one of them.

CHECK IT OUT! **Solve each quadratic equation by factoring. Check your answer.**

2a. $x^2 - 6x + 9 = 0$

2b. $x^2 + 4x = 5$

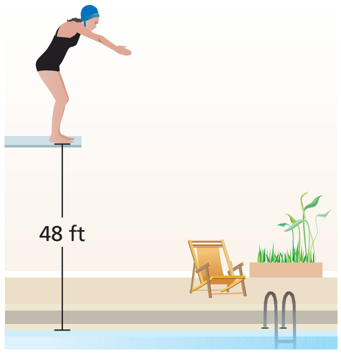
2c. $30x = -9x^2 - 25$

2d. $3x^2 - 4x + 1 = 0$

EXAMPLE 3

Sports Application

The height of a diver above the water during a dive can be modeled by $h = -16t^2 + 8t + 48$, where h is height in feet and t is time in seconds. Find the time it takes for the diver to reach the water.



$h = -16t^2 + 8t + 48$

$0 = -16t^2 + 8t + 48$	*The diver reaches the water when h = 0.*
$0 = -8(2t^2 - t - 6)$	*Factor out the GCF, −8.*
$0 = -8(2t + 3)(t - 2)$	*Factor the trinomial.*
$-8 \neq 0$, $2t + 3 = 0$ or $t - 2 = 0$	*Use the Zero Product Property.*
$2t = -3$ or $t = 2$	*Solve each equation.*
$t = -\frac{3}{2}$ ✗	*Since time cannot be negative, $-\frac{3}{2}$ does not make sense in this situation.*

It takes the diver 2 seconds to reach the water.

Check

0	$= -16t^2 + 8t + 48$	
0	$-16(2)^2 + 8(2) + 48$	*Substitute 2 into the original equation.*
0	$-64 + 16 + 48$	
0	0 ✓	

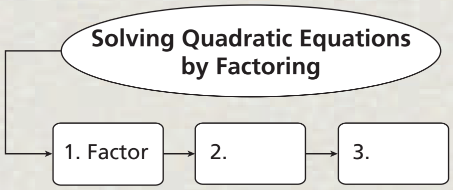
CHECK IT OUT! **3. What if...?** The equation for the height above the water for another diver can be modeled by $h = -16t^2 + 8t + 24$. Find the time it takes this diver to reach the water.

THINK AND DISCUSS

1. Explain two ways to solve $x^2 + x - 6 = 0$. How are these two methods similar?

2. For the quadratic equation $0 = (x + 2)(x - 6)$, what are the x-intercepts of the related function?

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write a step used to solve a quadratic equation by factoring.



9-6 Exercises

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GUIDED PRACTICE

SEE EXAMPLE 1 p. 630

Use the Zero Product Property to solve each equation. Check your answer.

1. $(x + 2)(x - 8) = 0$
2. $(x - 6)(x - 5) = 0$
3. $(x + 7)(x + 9) = 0$
4. $(x)(x - 1) = 0$
5. $(x)(x + 11) = 0$
6. $(3x + 2)(4x - 1) = 0$

SEE EXAMPLE 2 p. 631

Solve each quadratic equation by factoring. Check your answer.

7. $x^2 + 4x - 12 = 0$
8. $x^2 - 8x - 9 = 0$
9. $x^2 - 5x + 6 = 0$
10. $x^2 - 3x = 10$
11. $x^2 + 10x = -16$
12. $x^2 + 2x = 15$
13. $x^2 - 8x + 16 = 0$
14. $-3x^2 = 18x + 27$
15. $x^2 + 36 = 12x$
16. $x^2 + 14x + 49 = 0$
17. $x^2 - 16x + 64 = 0$
18. $2x^2 + 6x = -18$

SEE EXAMPLE 3 p. 632

19. **Games** A group of friends tries to keep a beanbag from touching the ground without using their hands. Once the beanbag has been kicked, its height can be modeled by $h = -16t^2 + 14t + 2$, where h is the height in feet above the ground and t is the time in seconds. Find the time it takes the beanbag to reach the ground.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
20–25	1
26–31	2
32	3

Extra Practice
Skills Practice p. S21
Application Practice p. S36

Use the Zero Product Property to solve each equation. Check your answer.

20. $(x - 8)(x + 6) = 0$
21. $(x + 4)(x + 7) = 0$
22. $(x - 2)(x - 5) = 0$
23. $(x - 9)(x) = 0$
24. $(x)(x + 25) = 0$
25. $(2x + 1)(3x - 1) = 0$

Solve each quadratic equation by factoring. Check your answer.

26. $x^2 + 8x + 15 = 0$
27. $x^2 - 2x - 8 = 0$
28. $x^2 - 4x + 3 = 0$
29. $x^2 + 10x + 25 = 0$
30. $x^2 - x = 12$
31. $-x^2 = 4x + 4$

32. **Multi-Step** The height of a flare can be approximated by the function $h = -16t^2 + 95t + 6$, where h is the height in feet and t is the time in seconds. Find the time it takes the flare to hit the ground.

Determine the number of solutions of each equation.

33. $(x + 8)(x + 8) = 0$
34. $(x - 3)(x + 3) = 0$
35. $(x + 7)^2 = 0$
36. $3x^2 + 12x + 9 = 0$
37. $x^2 + 12x + 40 = 4$
38. $(x - 2)^2 = 9$
39. **ERROR ANALYSIS** Which solution is incorrect? Explain the error.

A

$x^2 + x = 2 \quad 0$

$(x = 1)(x + 2) \quad 0$

$x \quad 1$ or $x \quad =2$

B

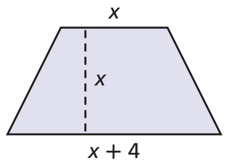
$x^2 + x = 2 \quad 0$

$(x = 1)(x + 2) \quad 0$

$x \quad =1$ or $x \quad 2$

40. **Number Theory** Write an equation that could be used to find two consecutive even integers whose product is 24. Let x represent the first integer. Solve the equation and give the two integers.

41. **Geometry** The photo shows a traditional thatched house as found in Santana, Madeira in Portugal. The front of the house is in the shape of a triangle. Suppose the base of the triangle is 1 m less than its height and the area of the triangle is 15 m^2. Find the height of the triangle. (*Hint:* Use $A = \frac{1}{2}bh$.)
42. **Multi-Step** The length of a rectangle is 1 ft less than 3 times the width. The area is 310 ft^2. Find the dimensions of the rectangle.
43. **Physics** The height of a fireworks rocket in meters can be approximated by $h = -5t^2 + 30t$, where h is the height in meters and t is time in seconds. Find the time it takes the rocket to reach the ground after it has been launched.



44. **Geometry** One base of a trapezoid is the same length as the height of the trapezoid. The other base is 4 cm more than the height. The area of the trapezoid is 48 cm^2. Find the length of the shorter base. (*Hint:* Use $A = \frac{1}{2}h(b_1 + b_2)$.)
45. **Critical Thinking** Can you solve $(x - 2)(x + 3) = 5$ by solving $x - 2 = 5$ and $x + 3 = 5$? Why or why not?
46. **Write About It** Explain why you set each factor equal to zero when solving a quadratic equation by factoring.

MULTI-STEP TEST PREP

47. This problem will prepare you for the Multi-Step Test Prep on page 660.

A tee box is 48 feet above its fairway. Starting with an initial elevation of 48 ft at the tee box and an initial velocity of 32 ft/s, the quadratic equation $0 = -16t^2 + 32t + 48$ gives the time t in seconds when a golf ball is at height 0 feet on the fairway.

a. Solve the quadratic equation by factoring to see how long the ball is in the air.

b. What is the height of the ball at 1 second?

c. Is the ball at its maximum height at 1 second? Explain.

TEST PREP

48. What are the solutions to $(x - 1)(2x + 5) = 0$?

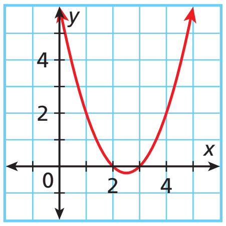
Ⓐ -1 and $\frac{5}{2}$

Ⓑ -1 and $\frac{2}{5}$

Ⓒ 1 and $-\frac{5}{2}$

Ⓓ 1 and $-\frac{2}{5}$

49. Which graph could be used to solve the quadratic equation $x^2 - 5x + 6 = 0$?

Ⓕ


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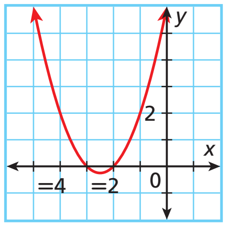

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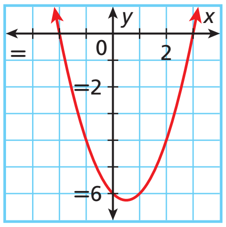

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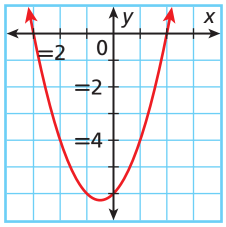


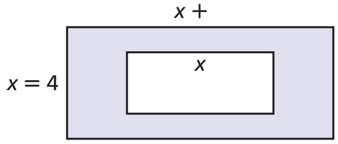
CHALLENGE AND EXTEND

Solve each quadratic equation by factoring.

50. $6x^2 + 11x = 10$ **51.** $0.2x^2 + 1 = -1.2x$ **52.** $\frac{1}{3}x^2 = 2x - 3$

53. $75x - 45 = -30x^2$ **54.** $x^2 = -4(2x + 3)$ **55.** $\frac{x(x - 3)}{2} = 5$

Geometry **Use the diagram for Exercises 56–58.**



56. Write a polynomial to represent the area of the larger rectangle.

57. Write a polynomial to represent the area of the smaller rectangle.

58. Write a polynomial to represent the area of the shaded region, then solve for x given that the area of the shaded region is 48 square units.

SPIRAL REVIEW

Write an algebraic expression for each situation. *(Lesson 1-1)*

59. Veronica's height, if she is 4 inches shorter than her friend, and her friend is f inches tall

60. The number of tires needed to build m minivans, including 1 spare tire for each minivan.

Find each square root. *(Lesson 1-5)*

61. $\sqrt{121}$ **62.** $-\sqrt{64}$ **63.** $-\sqrt{100}$ **64.** $\sqrt{225}$

Solve each equation by graphing the related function. *(Lesson 9-5)*

65. $x^2 - 49 = 0$ **66.** $x^2 = x + 12$ **67.** $-x^2 + 8x = 15$

9-7 Solving Quadratic Equations by Using Square Roots

Objective

Solve quadratic equations by using square roots.

Why learn this?

Square roots can be used to find how much fencing is needed for a pen at a zoo. (See Example 4.)

Some quadratic equations cannot be easily solved by factoring. Square roots can be used to solve some of these quadratic equations. Recall from Lesson 1–5 that every positive real number has two square roots, one positive and one negative.

$$3(3) = 3^2 = 9 \rightarrow \sqrt{9} = 3 \quad \leftarrow \text{Positive square root of 9}$$

$$(-3)(-3) = (-3)^2 = 9 \rightarrow -\sqrt{9} = -3 \quad \leftarrow \text{Negative square root of 9}$$

Reading Math

The expression ± 3 is read "plus or minus three."

When you take the square root of a positive real number and the sign of the square root is not indicated, you must find both the positive and negative square root. This is indicated by $\pm\sqrt{\ }$.

$$\pm\sqrt{9} = \pm 3 \quad \leftarrow \text{Positive and negative square roots of 9}$$

Know it! Note

Square-Root Property

WORDS	NUMBERS	ALGEBRA
To solve a quadratic equation in the form $x^2 = a$, take the square root of both sides.	$x^2 = 15$ $\sqrt{x^2} = \pm\sqrt{15}$ $x = \pm\sqrt{15}$	If $x^2 = a$ and a is a positive real number, then $x = \pm\sqrt{a}$.

EXAMPLE 1 Using Square Roots to Solve $x^2 = a$

Solve using square roots.

A $x^2 = 16$

$x = \pm\sqrt{16}$ — *Solve for x by taking the square root of both sides. Use ± to show both square roots.*

$x = \pm 4$

The solutions are 4 and −4.

Check

x^2	$= 16$
$(4)^2$	16
16	16 ✓

Substitute 4 and −4 into the original equation.

x^2	$= 16$
$(-4)^2$	16
16	16 ✓

Solve using square roots.

B $x^2 = -4$

$x = \pm\sqrt{-4}$ *There is no real number whose square is negative.*

There is no real solution.

CHECK IT OUT! **Solve using square roots. Check your answer.**

1a. $x^2 = 121$ **1b.** $x^2 = 0$ **1c.** $x^2 = -16$

If a quadratic equation is not written in the form $x^2 = a$, use inverse operations to isolate x^2 before taking the square root of both sides.

EXAMPLE 2 Using Square Roots to Solve Quadratic Equations

Helpful Hint

The square root of 0 is neither positive nor negative. It is only 0.

Solve using square roots.

A $x^2 + 5 = 5$

$x^2 + 5 = 5$

$\underline{-5 \quad -5}$ *Subtract 5 from both sides.*

$x^2 = 0$

$x = \pm\sqrt{0} = 0$ *Take the square root of both sides.*

The solution is 0.

B $4x^2 - 25 = 0$

$4x^2 - 25 = 0$

$\underline{+25 \quad +25}$ *Add 25 to both sides.*

$\frac{4x^2}{4} = \frac{25}{4}$ *Divide by 4 on both sides.*

$x^2 = \frac{25}{4}$

$x = \pm\sqrt{\frac{25}{4}} = \pm\frac{5}{2}$ *Take the square root of both sides. Use ± to show both square roots.*

The solutions are $\frac{5}{2}$ and $-\frac{5}{2}$.

Check

$4x^2 - 25 = 0$	
$4\left(\frac{5}{2}\right)^2 - 25$	0
$4\left(\frac{25}{4}\right) - 25$	0
$25 - 25$	0 ✓

$4x^2 - 25 = 0$	
$4\left(-\frac{5}{2}\right)^2 - 25$	0
$4\left(\frac{25}{4}\right) - 25$	0
$25 - 25$	0 ✓

CHECK IT OUT! **Solve by using square roots. Check your answer.**

2a. $100x^2 + 49 = 0$ **2b.** $36x^2 = 1$

When solving quadratic equations by using square roots, you may need to find the square root of a number that is not a perfect square. In this case, the answer is an irrational number. You can approximate the solutions.

EXAMPLE 3 Approximating Solutions

Solve. Round to the nearest hundredth.

A $x^2 = 10$

$x = \pm\sqrt{10}$ *Take the square root of both sides.*

$x \approx \pm 3.16$ *Evaluate $\sqrt{10}$ on a calculator.*

The approximate solutions are 3.16 and −3.16.

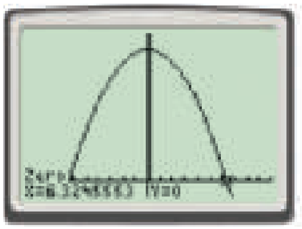
Solve. Round to the nearest hundredth.

B $0 = -2x^2 + 80$

$$0 = -2x^2 + 80$$
$$\underline{-80} \quad \underline{-80} \qquad \textit{Subtract 80 from both sides.}$$
$$\frac{-80}{-2} = \frac{-2x^2}{-2} \qquad \textit{Divide by } -2 \textit{ on both sides.}$$
$$40 = x^2$$
$$\pm\sqrt{40} = x \qquad \textit{Take the square root of both sides.}$$
$$x \approx \pm 6.32 \qquad \textit{Evaluate } \sqrt{40} \textit{ on a calculator.}$$

The approximate solutions are 6.32 and −6.32.

Check Use a graphing calculator to support your answer.



Use the zero function.
The approximate solutions are 6.32 and −6.32. ✓

CHECK IT OUT!

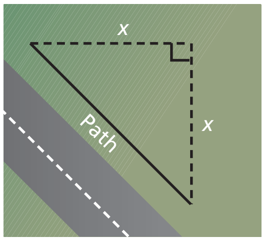
Solve. Round to the nearest hundredth.

3a. $0 = 90 - x^2$ **3b.** $2x^2 - 64 = 0$ **3c.** $x^2 + 45 = 0$

EXAMPLE 4

Consumer Application

A zookeeper is buying fencing to enclose a pen at the zoo. The pen is an isosceles right triangle. There is already a fence on the side that borders a path. The area of the pen will be 4500 square feet. The zookeeper can buy the fencing in whole feet only. How many feet of fencing should he buy?



Let x represent the length of one of the sides.

$$\frac{1}{2}bh = A \qquad \textit{Use the formula for area of a triangle.}$$
$$\frac{1}{2}x(x) = 4500 \qquad \textit{Substitute x for both b and h and 4500 for A.}$$
$$(2)\frac{1}{2}x^2 = 4500(2) \qquad \textit{Simplify. Multiply both sides by 2.}$$
$$x = \pm\sqrt{9000} \qquad \textit{Take the square root of both sides.}$$
$$x \approx \pm 94.9 \qquad \textit{Evaluate } \sqrt{9000} \textit{ on a calculator.}$$

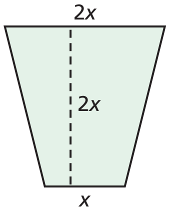
Negative numbers are not reasonable for length, so $x \approx 94.9$ is the only solution that makes sense. Therefore, the zookeeper needs 95 + 95, or 190, feet of fencing.

Remember!

An isosceles triangle has at least two sides of the same length.

CHECK IT OUT!

4. A house is on a lot that is shaped like a trapezoid. The solid lines show the boundaries, where x represents the width of the front yard. Find the width of the front yard, given that the area is 6000 square feet. Round to the nearest foot. (*Hint:* Use $A = \frac{1}{2}h(b_1 + b_2)$.)

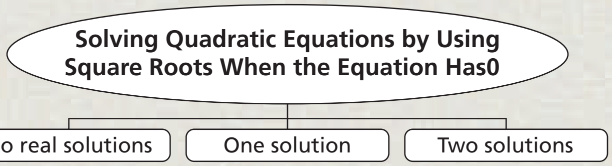


THINK AND DISCUSS

1. Explain why there are no solutions to the quadratic equation $x^2 = -9$.

2. Describe how to estimate the solutions of $4 = x^2 - 16$. What are the approximate solutions?

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of a quadratic equation with the given number of solutions. Solve each equation.



9-7 Exercises

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GUIDED PRACTICE

SEE EXAMPLE 1 p. 636

Solve using square roots. Check your answer.

1. $x^2 = 225$ **2.** $x^2 = 49$ **3.** $x^2 = -100$

4. $x^2 = 400$ **5.** $-25 = x^2$ **6.** $36 = x^2$

SEE EXAMPLE 2 p. 637

7. $3x^2 - 75 = 0$ **8.** $0 = 81x^2 - 25$ **9.** $49x^2 + 64 = 0$

10. $16x^2 + 10 = 131$ **11.** $0 = 4x^2 - 16$ **12.** $100x^2 + 26 = 10$

SEE EXAMPLE 3 p. 637

Solve. Round to the nearest hundredth.

13. $3x^2 = 81$ **14.** $0 = x^2 - 60$ **15.** $100 - 5x^2 = 0$

SEE EXAMPLE 4 p. 638

16. Geometry The length of a rectangle is 3 times its width. The area of the rectangle is 170 square meters. Find the width. Round to the nearest tenth of a meter. (*Hint:* Use $A = bh$.)

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
17–22	1
23–28	2
29–34	3
35	4

Extra Practice
Skills Practice p. S21
Application Practice p. S36

Solve using square roots. Check your answer.

17. $x^2 = 169$ **18.** $x^2 = 25$ **19.** $x^2 = -36$

20. $x^2 = 10{,}000$ **21.** $-121 = x^2$ **22.** $625 = x^2$

23. $4 - 81x^2 = 0$ **24.** $-4x^2 - 49 = 0$ **25.** $64x^2 - 5 = 20$

26. $9x^2 + 9 = 25$ **27.** $49x^2 + 1 = 170$ **28.** $81x^2 + 17 = 81$

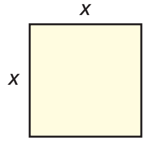
Solve. Round to the nearest hundredth.

29. $4x^2 = 88$ **30.** $x^2 - 29 = 0$ **31.** $x^2 + 40 = 144$

32. $3x^2 - 84 = 0$ **33.** $50 - x^2 = 0$ **34.** $2x^2 - 10 = 64$

35. **Entertainment** For a scene in a movie, a sack of money is dropped from the roof of a 600 ft skyscraper. The height of the sack above the ground is given by $h = -16t^2 + 600$, where t is the time in seconds. How long will it take the sack to reach the ground? Round to the nearest tenth of a second.

36. **Geometry** The area of a square is 196 m^2. Find the dimensions of the square.

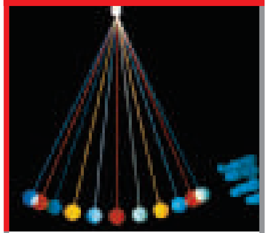


37. **Number Theory** If $a = 2b$ and $2ab = 36$, find all possible solutions for a and b.

38. **Geometry** The geometric mean of two positive numbers a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$. Find the geometric mean of 2 and 18.

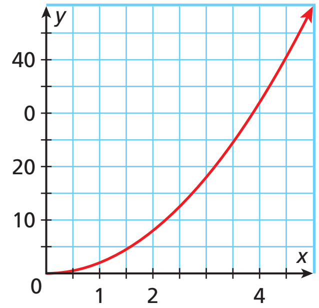
LINK

Physics



The first pendulum clock was invented by Christian Huygens, a Dutch physicist and mathematician, around 1656. Early pendulum clocks swung about 50° to the left and right. Modern pendulum clocks swing only 10° to 15°.

39. **Estimation** The area y of any rectangle with side length x and one side twice as long as the other is represented by $y = 2x^2$. Use the graph to estimate the dimensions of such a rectangle whose area is 35 square feet.



40. **Physics** The period of a pendulum is the amount of time it takes to swing back and forth one time. The relationship between the length of the pendulum L in inches and the length of the period t in seconds can be approximated by $L = 9.78t^2$. Find the period of a pendulum whose length is 60 inches. Round to the nearest tenth of a second.

41. **ERROR ANALYSIS** Which solution is incorrect? Explain the error.

A

$x^2 + 100 \quad 0$
$x^2 \quad 100$
$x \quad 10$ or $x \quad = 10$

B

$x^2 + 100 \quad 0$
$x^2 \quad = 100$
no solution

Determine whether each statement is always, sometimes, or never true.

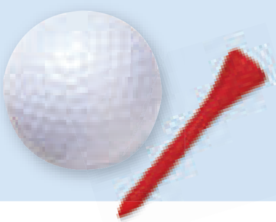
42. There are two solutions to $x^2 = n$ when n is positive.

43. If n is a rational number, then the solutions to $x^2 = n$ are rational numbers.

44. **Multi-Step** The height in feet of a soccer ball kicked upward from the ground with initial velocity 60 feet per second is modeled by $h = -16t^2 + 60t$, where t is the time in seconds. Find the time it takes for the ball to return to the ground. Round to the nearest tenth of a second.

45. **Critical Thinking** For the equation $x^2 = a$, describe the values of a that will result in each of the following.
   a. two solutions **b.** one solution **c.** no solution

MULTI-STEP TEST PREP



46. This problem will prepare you for the Multi-Step Test Prep on page 660.

The equation $d = 16t^2$ describes the distance d in feet that a golf ball falls in relation to the number of seconds t that it falls.

a. How many seconds will it take a golf ball to drop to the ground from a height of 4 feet?
b. Make a table and graph the related function.
c. How far will the golf ball drop in 1 second?
d. How many seconds will it take the golf ball to drop 64 feet?

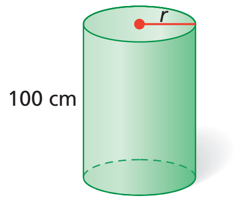
For the quadratic equation $x^2 + a = 0$, determine whether each value of a will result in two rational solutions. Explain.

47. $-\frac{1}{2}$ **48.** $\frac{1}{2}$ **49.** $-\frac{1}{4}$ **50.** $\frac{1}{4}$

51. Write About It Explain why the quadratic equation $x^2 + 4 = 0$ has no solutions but the quadratic equation $x^2 - 4 = 0$ has two solutions.

Test Prep

52. The formula for finding the approximate volume of a cylinder is $V = 3.14r^2h$, where r is the radius and h is the height. The height of a cylinder is 100 cm, and the approximate volume is 1256 cm^3. Find the radius of the cylinder.



Ⓐ 400 cm
Ⓑ 20 cm
Ⓒ 4 cm
Ⓓ 2 cm

53. Which best describes the positive solution of $\frac{1}{2}x^2 = 20$?

Ⓕ Between 4 and 5
Ⓖ Between 5 and 6
Ⓗ Between 6 and 7
Ⓙ Between 7 and 8

54. Which best describes the solutions of $81x^2 - 169 = 0$?

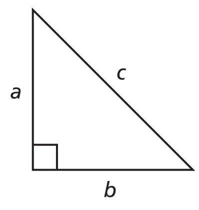
Ⓐ Two rational solutions
Ⓑ Two irrational solutions
Ⓒ No solution
Ⓓ One solution

CHALLENGE AND EXTEND

Find the solutions of each equation without using a calculator.

55. $288x^2 - 19 = -1$ **56.** $-75x^2 = -48$ **57.** $x^2 = \frac{128}{242}$

58. Geometry The Pythagorean Theorem states that $a^2 + b^2 = c^2$ if a and b represent the lengths of the legs of a right triangle and c represents the length of the hypotenuse.

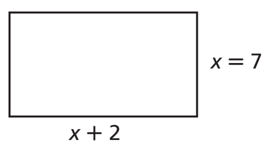


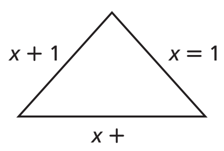
a. Find the length of the hypotenuse if the lengths of the legs are 9 cm and 12 cm.

b. Find the length of each leg of an isosceles right triangle whose hypotenuse is 10 cm. Round to the nearest tenth of a centimeter.

SPIRAL REVIEW

59. The figures shown have the same perimeter. What is the value of x? *(Lesson 2-4)*





60. Identify which of the following lines are parallel:

$y = -2x + 3$, $2x - y = 8$, $6x - 2y = 10$, and $y + 4 = 2(3 - x)$. *(Lesson 5-8)*

Solve each quadratic equation by factoring. Check your answer. *(Lesson 9-6)*

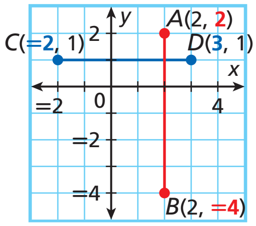
61. $x^2 - 6x + 8 = 0$ **62.** $x^2 + 5x - 6 = 0$ **63.** $x^2 - 5x = 14$

The Distance Formula

Connecting Algebra to **Geometry**

See Skills Bank page S68

You can find the length of a vertical or horizontal line segment in the coordinate plane by subtracting coordinates.

WORDS	NUMBERS	ALGEBRA
The length of a vertical line segment is the absolute value of the difference between the *y*-coordinates of the endpoints.	$AB = \lvert 2-(-4) \rvert = \lvert 6 \rvert = 6$ 	The distance between $P(x_1, y_1)$ and $Q(x_1, y_2)$ is $\lvert y_2 - y_1 \rvert$.
The length of a horizontal line segment is the absolute value of the difference between the *x*-coordinates of the endpoints.	$CD = \lvert -2-3 \rvert = \lvert -5 \rvert = 5$	The distance between $P(x_1, y_1)$ and $Q(x_2, y_1)$ is $\lvert x_2 - x_1 \rvert$.

Example 1

Find the length of the line segment that connects $S(-4.5, 7.1)$ and $T(-4.5, 0.3)$.

The *x*-coordinates are the same, so this is a vertical line segment. Subtract the *y*-coordinates and find the absolute value of the difference.

$\lvert y_2 - y_1 \rvert$	*Formula for the length of a vertical line segment*
$\lvert 0.3 - 7.1 \rvert$	*Substitute.*
$\lvert -6.8 \rvert = 6.8$	*Subtract and find the absolute value.*

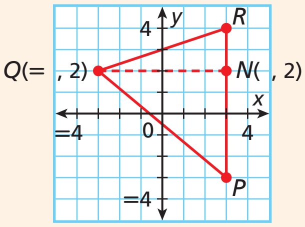
Try This

Find the length of the line segment that connects each pair of points.

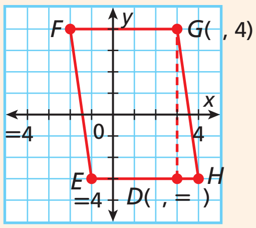
1. $X(-1, 3)$ and $Y(4, 3)$
2. $M(5, -2)$ and $N(5, -8)$
3. $C(3, -1)$ and $D(3, 5)$
4. $P(14, -5)$ and $Q(25, -5)$
5. $A(-6, 0.5)$ and $B(-6, -4.3)$
6. $E(1.4, -0.7)$ and $F(3.8, -0.7)$

Find the length of each segment.

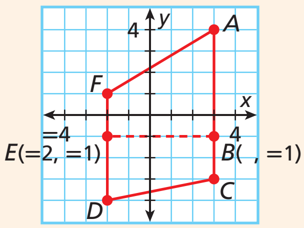
7. the altitude of $\triangle PQR$



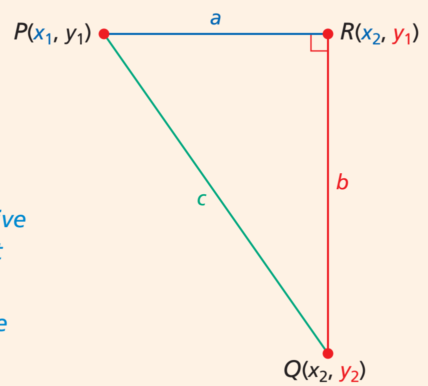
8. the height of parallelogram *EFGH*



9. the height of trapezoid *ACDF*



To find the length of a line segment that is not vertical or horizontal, such as *PQ*, think of it as the hypotenuse of a right triangle. Then you can use the Pythagorean Theorem.



$c^2 = a^2 + b^2$	*Pythagorean Theorem*
$(PQ)^2 = (PR)^2 + (QR)^2$	*Substitute.*
$PQ = \sqrt{(PR)^2 + (QR)^2}$	*Solve for PQ. Use the positive square root to represent distance.*
$= \sqrt{\underbrace{(x_2 - x_1)^2}_{\text{Horizontal segment}} + \underbrace{(y_2 - y_1)^2}_{\text{Vertical segment}}}$	*Use the Formula to find the length of each segment.*

This is an example of the Distance Formula.

The Distance Formula

The distance between points *P* and *Q* with coordinates $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Example 2

Find the length of the line segment that connects $H(20, -11)$ and $K(-4, 18)$.

Use the Distance Formula. Let $(20, -11)$ be (x_1, y_1) and $(-4, 18)$ be (x_2, y_2).

$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	*Write the formula.*
$= \sqrt{(-4 - 20)^2 + [18 - (-11)]^2}$	*Substitute.*
$= \sqrt{(-24)^2 + (29)^2}$	*Find the differences.*
$= \sqrt{576 + 841}$	*Square the differences.*
$= \sqrt{1417} \approx 37.64$	*Use a calculator to find the square root.*

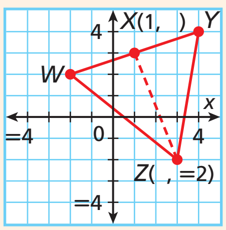
Try This

Find the length of the line segment that connects each pair of points.

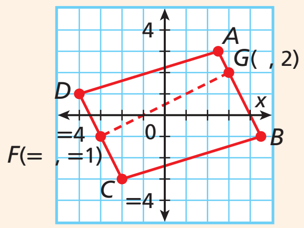
10. $F(-8, 3)$ and $G(-11, -4)$ **11.** $S(30, -15)$ and $T(-55, 40)$ **12.** $W(0.5, 1.2)$ and $X(0.6, 2.5)$

Find the length of each dashed line segment.

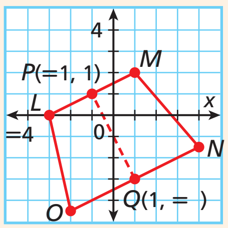
13. the altitude of $\triangle WYZ$



14. the height of parallelogram *ABCD*



15. the height of trapezoid *LMNO*

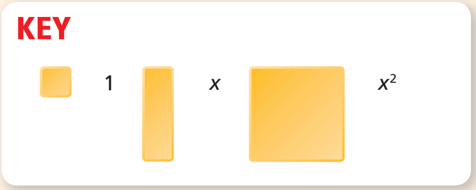


9-8 Algebra LAB

Model Completing the Square

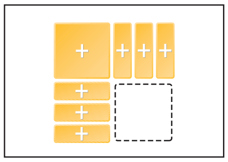
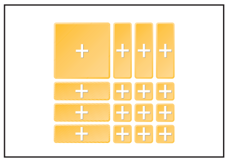
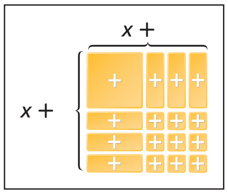
Use with Lesson 9-8

One way to solve a quadratic equation is by using a procedure called *completing the square*. In this procedure, you add something to a quadratic expression to make it a perfect-square trinomial. This procedure can be modeled with algebra tiles.



Activity

Use algebra tiles to model $x^2 + 6x$. Add unit tiles to complete a perfect-square trinomial. Then write the new expression in factored form.

MODEL		ALGEBRA
	*Arrange the tiles to form part of a large square.* *Part of the square is missing. How many one-tiles do you need to complete it?*	$x^2 + 6x$
	*Complete the square by placing 9 one-tiles on the mat.* $x^2 + 6x + 9$ *is a perfect-square trinomial.*	$x^2 + 6x + 9$
	*Use the length and the width of the square to rewrite the area expression in factored form.*	$(x + 3)^2$

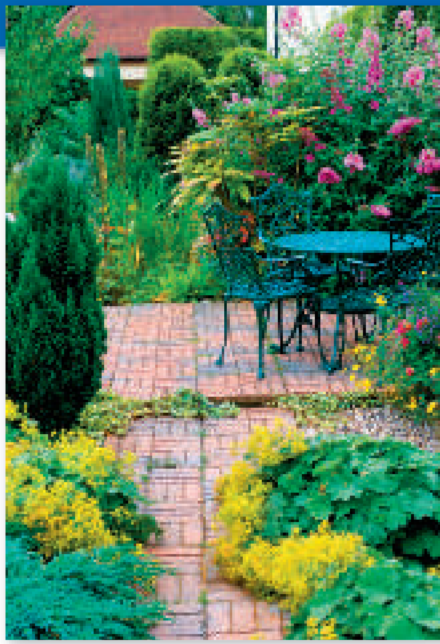
Try This

Use algebra tiles to model each expression. Add unit tiles to complete a perfect-square trinomial. Then write the new expression in factored form.

1. $x^2 + 4x$ **2.** $x^2 + 2x$ **3.** $x^2 + 10x$ **4.** $x^2 + 8x$

5. **Make a Conjecture** Examine the pattern in Problems 1–4. How many unit tiles would you have to add to make $x^2 + 12x$ a perfect-square trinomial?

9-8 Completing the Square



Objective
Solve quadratic equations by completing the square.

Vocabulary
completing the square

Who uses this?
Landscapers can solve quadratic equations to find dimensions of patios. (See Example 4.)

In the previous lesson, you solved quadratic equations by isolating x^2 and then using square roots. This method works if the quadratic equation, when written in standard form, is a perfect square.

When a trinomial is a perfect square, there is a relationship between the **coefficient of the x-term** and the **constant term**.

$$x^2 + 6x + 9 \qquad x^2 - 8x + 16$$

$$\left(\frac{6}{2}\right)^2 = 9 \qquad \left(\frac{-8}{2}\right)^2 = 16$$

Divide the coefficient of the x-term by 2, then square the result to get the constant term.

An expression in the form $x^2 + bx$ is not a perfect square. However, you can use the relationship shown above to add a term to $x^2 + bx$ to form a trinomial that is a perfect square. This is called **completing the square**.

Know it! Note — Completing the Square

WORDS	NUMBERS	ALGEBRA
To complete the square of $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$ to the expression. This will form a perfect square trinomial.	$x^2 + 6x + \square$ $x^2 + 6x + \left(\frac{6}{2}\right)^2$ $x^2 + 6x + 9$ $(x + 3)^2$	$x^2 + bx + \square$ $x^2 + bx + \left(\frac{b}{2}\right)^2$ $\left(x + \frac{b}{2}\right)^2$

EXAMPLE 1 Completing the Square

Complete the square to form a perfect square trinomial.

A $x^2 + 10x + \square$

$x^2 + 10x$ — *Identify b.*

$\left(\frac{10}{2}\right)^2 = 5^2 = 25$ — *Find $\left(\frac{b}{2}\right)^2$.*

$x^2 + 10x + 25$ — *Add $\left(\frac{b}{2}\right)^2$ to the expression.*

B $x^2 - 9x + \square$

$x^2 + -9x$

$\left(\frac{-9}{2}\right)^2 = \frac{81}{4}$

$x^2 - 9x + \frac{81}{4}$

CHECK IT OUT! Complete the square to form a perfect square trinomial.

1a. $x^2 + 12x + \square$ **1b.** $x^2 - 5x + \square$ **1c.** $8x + x^2 + \square$

To solve a quadratic equation in the form $x^2 + bx = c$, first complete the square of $x^2 + bx$. Then you can solve using square roots.

Know It! Note

Solving a Quadratic Equation by Completing the Square	
Step 1	Write the equation in the form $x^2 + bx = c$.
Step 2	Find $\left(\frac{b}{2}\right)^2$.
Step 3	Complete the square by adding $\left(\frac{b}{2}\right)^2$ to both sides of the equation.
Step 4	Factor the perfect-square trinomial.
Step 5	Take the square root of both sides.
Step 6	Write two equations, using both the positive and negative square root, and solve each equation.

EXAMPLE 2 Solving $x^2 + bx = c$ by Completing the Square

Solve by completing the square.

A $x^2 + 14x = 15$

Step 1	$x^2 + 14x = 15$	*The equation is in the form $x^2 + bx = c$.*
Step 2	$\left(\frac{14}{2}\right)^2 = 7^2 = 49$	*Find $\left(\frac{b}{2}\right)^2$.*
Step 3	$x^2 + 14x + 49 = 15 + 49$	*Complete the square.*
Step 4	$(x + 7)^2 = 64$	*Factor and simplify.*
Step 5	$x + 7 = \pm 8$	*Take the square root of both sides.*
Step 6	$x + 7 = 8$ or $x + 7 = -8$ $x = 1$ or $x = -15$	*Write and solve two equations.*

The solutions are 1 and −15.

Check

$x^2 + 14x$	$= 15$
$(1)^2 + 14(1)$	15
$1 + 14$	15
15	15 ✓

$x^2 + 14x$	$= 15$
$(-15)^2 + 14(-15)$	15
$225 - 210$	15
15	15 ✓

Helpful Hint

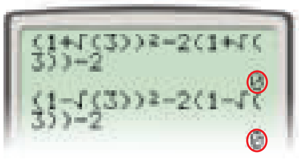
$(x + 7)(x + 7) = (x + 7)^2$. So the square root of $(x + 7)^2$ is $x + 7$.

B $x^2 - 2x - 2 = 0$

Step 1	$x^2 + (-2x) = 2$	*Write in the form $x^2 + bx = c$.*
Step 2	$\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$	*Find $\left(\frac{b}{2}\right)^2$.*
Step 3	$x^2 - 2x + 1 = 2 + 1$	*Complete the square.*
Step 4	$(x - 1)^2 = 3$	*Factor and simplify.*
Step 5	$x - 1 = \pm\sqrt{3}$	*Take the square root of both sides.*
Step 6	$x - 1 = \sqrt{3}$ or $x - 1 = -\sqrt{3}$ $x = 1 + \sqrt{3}$ or $x = 1 - \sqrt{3}$	*Write and solve two equations.*

The solutions are $1 + \sqrt{3}$ and $1 - \sqrt{3}$.

Check Use a graphing calculator to check your answer.



Writing Math

The expressions $1 + \sqrt{3}$ and $1 - \sqrt{3}$ can be written as one expression: $1 \pm \sqrt{3}$, which is read as "1 plus or minus the square root of 3."

CHECK IT OUT!

Solve by completing the square.

2a. $x^2 + 10x = -9$ **2b.** $t^2 - 8t - 5 = 0$

EXAMPLE 3 Solving $ax^2 + bx = c$ by Completing the Square

Solve by completing the square.

A $-2x^2 + 12x - 20 = 0$

Step 1 $\dfrac{-2x^2}{-2} + \dfrac{12x}{-2} - \dfrac{20}{-2} = \dfrac{0}{-2}$ — *Divide by −2 to make a = 1.*

$x^2 - 6x + 10 = 0$ — *Write in the form $x^2 + bx = c$.*

$x^2 - 6x = -10$

$x^2 + (-6x) = -10$

Step 2 $\left(\dfrac{-6}{2}\right)^2 = (-3)^2 = 9$ — *Find $\left(\frac{b}{2}\right)^2$.*

Step 3 $x^2 - 6x + 9 = -10 + 9$ — *Complete the square.*

Step 4 $(x - 3)^2 = -1$ — *Factor and simplify.*

There is no real number whose square is negative, so there are no real solutions.

B $3x^2 - 10x = -3$

Step 1 $\dfrac{3x^2}{3} - \dfrac{10}{3}x = \dfrac{-3}{3}$ — *Divide by 3 to make a = 1.*

$x^2 - \dfrac{10}{3}x = -1$

$x^2 + \left(-\dfrac{10}{3}x\right) = -1$ — *Write in the form $x^2 + bx = c$.*

Step 2 $\left(-\dfrac{10}{3} \cdot \dfrac{1}{2}\right)^2 = \left(-\dfrac{10}{6}\right)^2 = \dfrac{100}{36} = \dfrac{25}{9}$ — *Find $\left(\frac{b}{2}\right)^2$.*

Step 3 $x^2 - \dfrac{10}{3}x + \dfrac{25}{9} = -1 + \dfrac{25}{9}$ — *Complete the square.*

$x^2 - \dfrac{10}{3}x + \dfrac{25}{9} = -\dfrac{9}{9} + \dfrac{25}{9}$ — *Rewrite using like denominators.*

Step 4 $\left(x - \dfrac{5}{3}\right)^2 = \dfrac{16}{9}$ — *Factor and simplify.*

Step 5 $x - \dfrac{5}{3} = \pm\dfrac{4}{3}$ — *Take the square root of both sides.*

Step 6 $x - \dfrac{5}{3} = \dfrac{4}{3}$ or $x - \dfrac{5}{3} = -\dfrac{4}{3}$ — *Write and solve two equations.*

$x = 3$ or $x = \dfrac{1}{3}$

The solutions are 3 and $\frac{1}{3}$.

Remember!
Dividing by 2 is the same as multiplying by $\frac{1}{2}$.

CHECK IT OUT! **Solve by completing the square.**

3a. $3x^2 - 5x - 2 = 0$

3b. $4t^2 - 4t + 9 = 0$

EXAMPLE 4 *Problem-Solving Application*

PROBLEM SOLVING

A landscaper is designing a rectangular brick patio. She has enough bricks to cover 144 square feet. She wants the length of the patio to be 10 feet greater than the width. What dimensions should she use for the patio? Round to the nearest hundredth of a foot.

1 Understand the Problem

The **answer** will be the length and width of the patio.

Geometry

List the important information:

- There are enough bricks to cover 144 square feet.
- One edge of the patio is to be 10 feet longer than the other edge.

2 Make a Plan

Set the formula for the area of a rectangle equal to 144, the area of the patio. Solve the equation.

3 Solve

Let x be the width.
Then $x + 10$ is the length.

Use the formula for area of a rectangle.

ℓ	$\cdot$	w	$=$	A
length	times	width	=	area of patio
$x + 10$	$\cdot$	x	$=$	144

Step 1 $x^2 + 10x = 144$ — *Simplify.*

Step 2 $\left(\frac{10}{2}\right)^2 = 5^2 = 25$ — *Find $\left(\frac{b}{2}\right)^2$.*

Step 3 $x^2 + 10x + 25 = 144 + 25$ — *Complete the square by adding 25 to both sides.*

Step 4 $(x + 5)^2 = 169$ — *Factor the perfect-square trinomial.*

Step 5 $x + 5 = \pm 13$ — *Take the square root of both sides.*

Step 6 $x + 5 = 13$ or $x + 5 = -13$ — *Write and solve two equations.*

$x = 8$ or $x = -18$

Negative numbers are not reasonable for length, so $x = 8$ is the only solution that makes sense.

The width is 8 feet, and the length is $8 + 10$, or 18, feet.

4 Look Back

The length of the patio is 10 feet greater than the width. Also, $8(18) = 144$.

CHECK IT OUT!

4. An architect designs a rectangular room with an area of 400 ft^2. The length is to be 8 ft longer than the width. Find the dimensions of the room. Round your answers to the nearest tenth of a foot.

THINK AND DISCUSS

1. Tell how to solve a quadratic equation in the form $x^2 + bx + c = 0$ by completing the square.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write and solve an example of the given type of quadratic equation.

Solving Quadratic Equations by Completing the Square

$x^2 + bx \quad c$	$ax^2 + bx \quad c$	$x^2 + bx + c \quad 0$

9-8 Exercises

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Homework Help Online
KEYWORD: MA7 9-8
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

1. Vocabulary Describe in your own words how to *complete the square* for the equation $1 = x^2 + 4x$.

SEE EXAMPLE 1
p. 645

Complete the square to form a perfect square trinomial.

2. $x^2 + 14x + \square$ **3.** $x^2 - 4x + \square$ **4.** $x^2 - 3x + \square$

SEE EXAMPLE 2
p. 647

Solve by completing the square.

5. $x^2 + 6x = -5$ **6.** $x^2 - 8x = 9$ **7.** $x^2 + x = 30$

8. $x^2 + 2x = 21$ **9.** $x^2 - 10x = -9$ **10.** $x^2 + 16x = 92$

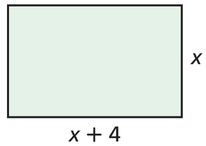
SEE EXAMPLE 3
p. 647

11. $-x^2 - 5x = -5$ **12.** $-x^2 - 3x + 2 = 0$ **13.** $-6x = 3x^2 + 9$

14. $2x^2 - 6x = -10$ **15.** $-x^2 + 8x - 6 = 0$ **16.** $4x^2 + 16 = -24x$

SEE EXAMPLE 4
p. 647

17. Multi-Step The length of a rectangle is 4 meters longer than the width. The area of the rectangle is 80 square meters. Find the length and width. Round your answers to the nearest tenth of a meter.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
18–20	1
21–26	2
27–32	3
33	4

Extra Practice
Skills Practice p. S21
Application Practice p. S36

Complete the square to form a perfect square trinomial.

18. $x^2 - 16x + \square$ **19.** $x^2 - 2x + \square$ **20.** $x^2 + 11x + \square$

Solve by completing the square.

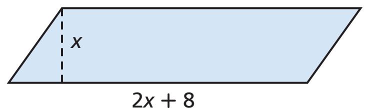
21. $x^2 - 10x = 24$ **22.** $x^2 - 6x = -9$ **23.** $x^2 + 15x = -26$

24. $x^2 + 6x = 16$ **25.** $x^2 - 2x = 48$ **26.** $x^2 + 12x = -36$

27. $-x^2 + x + 6 = 0$ **28.** $2x^2 = -7x - 29$ **29.** $-x^2 - x + 1 = 0$

30. $3x^2 - 6x - 9 = 0$ **31.** $-x^2 = 15x + 30$ **32.** $2x^2 + 20x - 10 = 0$

33. Geometry The base of a parallelogram is 8 inches longer than twice the height. The area of the parallelogram is 64 square inches. What is the height?



Solve each equation by completing the square.

34. $3x^2 + x = 10$ **35.** $x^2 = 2x + 6$ **36.** $2a^2 = 5a + 12$

37. $2x^2 + 5x = 3$ **38.** $4x = 7 - x^2$ **39.** $8x = -x^2 + 20$

40. Hobbies The height in feet h of a water bottle rocket launched from a rooftop is given by the equation $h = -16t^2 + 320t + 32$, where t is the time in seconds. After the rocket is fired, how long will it take to return to the ground? Solve by completing the square. Round your answer to the nearest tenth of a second.

Complete each trinomial so that it is a perfect square.

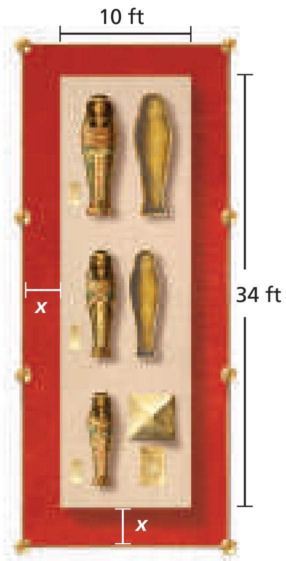
41. $x^2 + 18x + \square$ **42.** $x^2 - 100x + \square$ **43.** $x^2 - 7x + \square$

44. $x^2 + \square x + 4$ **45.** $x^2 - \square x + \frac{81}{4}$ **46.** $x^2 + \square x + \frac{1}{36}$

47. **Multi-Step** A roped-off area of width x is created around a 34-by-10-foot rectangular museum display of Egyptian artifacts, as shown. The combined area of the display and the roped-off area is 640 square feet.

a. Write an equation for the combined area.

b. Find the width of the roped-off area.



48. **Graphing Calculator** Compare solving a quadratic equation by completing the square with finding the solutions on a graphing calculator.

a. Complete the square to solve $2x^2 - 3x - 2 = 0$.

b. Use your graphing calculator to graph $y = 2x^2 - 3x - 2$.

c. Explain how to use this graph to find the solutions of $2x^2 - 3x - 2 = 0$.

d. Compare the two methods of solving the equation. What are the advantages and disadvantages of each?

49. **ERROR ANALYSIS** Explain the error below. What is the correct answer?

$x^2 + 4x \quad 77$

$x^2 + 4x + 4 \quad 77 + 4$

$(x + 2)^2 \quad 81$

$x + 2 \quad 9$

$x \quad 7$

Solve each equation by completing the square.

50. $5x^2 - 50x = 55$

51. $3x^2 + 36x = -27$

52. $28x - 2x^2 = 26$

53. $-36x = 3x^2 + 108$

54. $0 = 4x^2 + 32x + 44$

55. $16x + 40 = -2x^2$

56. $x^2 + 5x + 6 = 10x$

57. $x^2 + 3x + 18 = -3x$

58. $4x^2 + x + 1 = 3x^2$

59. **Write About It** Jamal prefers to solve $x^2 + 20x - 21 = 0$ by completing the square. Heather prefers to solve $x^2 + 11x + 18 = 0$ by factoring. Explain their reasoning.

60. **Critical Thinking** What should be done to the binomial $x^2 + y^2$ to make it a perfect-square trinomial? Explain.

MULTI-STEP TEST PREP

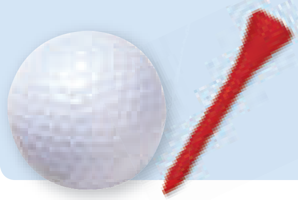
61. This problem will prepare you for the Multi-Step Test Prep on page 660.

The function $h(t) = -16t^2 + vt + c$ models the height in feet of a golf ball after t seconds when it is hit with initial velocity v from initial height c feet. A golfer stands on a tee box that is 32 feet above the fairway. He hits the golf ball from the tee at an initial velocity of 64 feet per second.

a. Write an equation that gives the time t when the golf ball lands on the fairway at height 0.

b. What number would be added to both sides of the equation in part **a** to complete the square while solving for t?

c. Solve the equation from part **a** by completing the square to find the time it takes the ball to reach the fairway. Round to the nearest tenth of a second.



62. **Write About It** Compare solving an equation of the form $x^2 + bx + c = 0$ by completing the square and solving an equation of the form $ax^2 + bx + c = 0$ by completing the square.

TEST PREP

63. What value of c will make $x^2 + 16x + c$ a perfect-square trinomial?

Ⓐ 32 Ⓑ 64 Ⓒ 128 Ⓓ 256

64. What value of b will make $x^2 + b + 25$ a perfect-square trinomial?

Ⓕ 5 Ⓖ $5x$ Ⓗ 10 Ⓙ $10x$

65. Which of the following is closest to a solution of $3x^2 + 2x - 4 = 0$?

Ⓐ 0 Ⓑ 1 Ⓒ 2 Ⓓ 3

66. **Short Response** Solve $x^2 - 8x - 20 = 0$ by completing the square. Explain each step in your solution.

CHALLENGE AND EXTEND

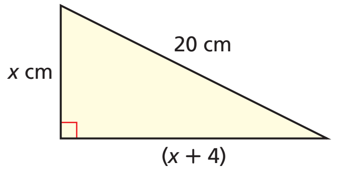
Solve each equation by completing the square.

67. $6x^2 + 5x = 6$ **68.** $7x + 3 = 6x^2$ **69.** $4x = 1 - 3x^2$

70. What should be done to the binomial $ax^2 + bx$ to obtain a perfect-square trinomial?

71. Solve $ax^2 + bx = 0$ for x.

72. **Geometry** The hypotenuse of a right triangle is 20 cm. One of the legs is 4 cm longer than the other leg. Find the area of the triangle. (*Hint:* Use the Pythagorean Theorem.)



SPIRAL REVIEW

Graph the line with the given slope and y-intercepts. *(Lesson 5-6)*

73. slope $= 4$, y-intercept $= -3$ **74.** slope $= -\frac{2}{3}$, y-intercept $= 4$

75. slope $= -2$, y-intercept $= -2$ **76.** slope $= -\frac{4}{3}$, y-intercept $= 0$

Multiply. *(Lesson 7-8)*

77. $(x - 4)^2$ **78.** $(x - 4)(x + 4)$ **79.** $(4 - t)^2$

80. $(2z + 3)^2$ **81.** $(8b^2 - 2)(8b^2 + 2)$ **82.** $(2x - 6)(2x + 6)$

Solve using square roots. *(Lesson 9-7)*

83. $5x^2 = 5$ **84.** $x^2 + 3 = 12$ **85.** $5x^2 = 80$

86. $9x^2 = 64$ **87.** $25 + x^2 = 250$ **88.** $64x^2 + 3 = 147$

Solve. Round to the nearest hundredth. *(Lesson 9-7)*

89. $12 = 5x^2$ **90.** $3x^2 - 4 = 15$ **91.** $x^2 - 7 = 19$

92. $6 + x^2 = 72$ **93.** $10x^2 - 10 = 12$ **94.** $2x^2 + 2 = 33$

9-9 The Quadratic Formula and the Discriminant

Objectives

Solve quadratic equations by using the Quadratic Formula.

Determine the number of solutions of a quadratic equation by using the discriminant.

Vocabulary

discriminant

Why learn this?

You can use the discriminant to determine whether the weight in a carnival strength test will reach a certain height. (See Exercise 4.)

In the previous lesson, you completed the square to solve quadratic equations. If you complete the square of $ax^2 + bx + c = 0$, you can derive the *Quadratic Formula.* The Quadratic Formula is the only method that can be used to solve *any* quadratic equation.

Numbers		Algebra
$2x^2 + 6x + 1 = 0$		$ax^2 + bx + c = 0,\ a \neq 0$
$\frac{2}{2}x^2 + \frac{6}{2}x + \frac{1}{2} = \frac{0}{2}$	*Divide both sides by a.*	$\frac{a}{a}x^2 + \frac{b}{a}x + \frac{c}{a} = \frac{0}{a}$
$x^2 + 3x + \frac{1}{2} = 0$		$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
$x^2 + 3x = -\frac{1}{2}$	*Subtract $\frac{c}{a}$ from both sides.*	$x^2 + \frac{b}{a}x = -\frac{c}{a}$
$x^2 + 3x + \left(\frac{3}{2}\right)^2 = -\frac{1}{2} + \left(\frac{3}{2}\right)^2$	*Complete the square.*	$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
$\left(x + \frac{3}{2}\right)^2 = \frac{9}{4} - \frac{1}{2}$	*Factor and simplify.*	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$
$\left(x + \frac{3}{2}\right)^2 = \frac{9}{4} - \frac{2}{4}$	*Use common denominators.*	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$
$\left(x + \frac{3}{2}\right)^2 = \frac{7}{4}$	*Simplify.*	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$
$x + \frac{3}{2} = \pm\frac{\sqrt{7}}{2}$	*Take square roots.*	$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$
$x = -\frac{3}{2} \pm \frac{\sqrt{7}}{2}$	*Subtract $\frac{b}{2a}$ from both sides.*	$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-3 \pm \sqrt{7}}{2}$	*Simplify.*	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Remember!

To add fractions, you need a common denominator.

$$\frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2}{4a^2} - \frac{c}{a}\left(\frac{4a}{4a}\right)$$
$$= \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$
$$= \frac{b^2 - 4ac}{4a^2}$$

Know it! Note

The Quadratic Formula

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

EXAMPLE 1 Using the Quadratic Formula

Solve using the Quadratic Formula.

A $2x^2 + 3x - 5 = 0$

$2x^2 + 3x + (-5) = 0$ — *Identify a, b, and c.*

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — *Use the Quadratic Formula.*

$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-5)}}{2(2)}$ — *Substitute 2 for a, 3 for b, and −5 for c.*

$x = \frac{-3 \pm \sqrt{9 - (-40)}}{4}$ — *Simplify.*

$x = \frac{-3 \pm \sqrt{49}}{4} = \frac{-3 \pm 7}{4}$ — *Simplify.*

$x = \frac{-3 + 7}{4}$ or $x = \frac{-3 - 7}{4}$ — *Write as two equations.*

$x = 1$ or $x = -\frac{5}{2}$ — *Solve each equation.*

B $2x = x^2 - 3$

$1x^2 + (-2x) + (-3) = 0$ — *Write in standard form. Identify a, b, and c.*

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}$ — *Substitute 1 for a, −2 for b, and −3 for c.*

$x = \frac{2 \pm \sqrt{4 - (-12)}}{2}$ — *Simplify.*

$x = \frac{2 \pm \sqrt{16}}{2} = \frac{2 \pm 4}{2}$ — *Simplify.*

$x = \frac{2 + 4}{2}$ or $x = \frac{2 - 4}{2}$ — *Write as two equations.*

$x = 3$ or $x = -1$ — *Solve each equation.*

Helpful Hint

You can graph the related quadratic function to see if your solutions are reasonable.

CHECK IT OUT! Solve using the Quadratic Formula.

1a. $-3x^2 + 5x + 2 = 0$ **1b.** $2 - 5x^2 = -9x$

Many quadratic equations can be solved by graphing, factoring, taking the square root, or completing the square. Some cannot be solved by any of these methods, but you can always use the Quadratic Formula to solve any quadratic equation.

EXAMPLE 2 Using the Quadratic Formula to Estimate Solutions

Solve $x^2 - 2x - 4 = 0$ using the Quadratic Formula.

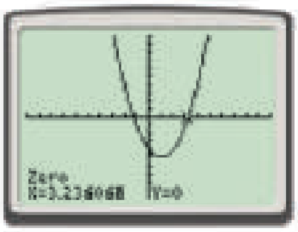
$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$

$x = \frac{2 \pm \sqrt{4 - (-16)}}{2} = \frac{2 \pm \sqrt{20}}{2}$

$x = \frac{2 + \sqrt{20}}{2}$ or $x = \frac{2 - \sqrt{20}}{2}$

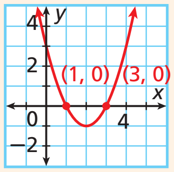
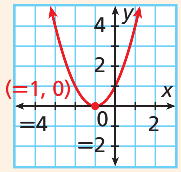
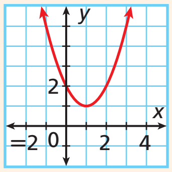
Use a calculator: $x \approx 3.24$ or $x \approx -1.24$.

Check reasonableness



CHECK IT OUT! **2.** Solve $2x^2 - 8x + 1 = 0$ using the Quadratic Formula.

If the quadratic equation is in standard form, the **discriminant** of a quadratic equation is $b^2 - 4ac$, the part of the equation under the radical sign. Recall that quadratic equations can have two, one, or no real solutions. You can determine the number of solutions of a quadratic equation by evaluating its discriminant.

Equation	$x^2 - 4x + 3 = 0$	$x^2 + 2x + 1 = 0$	$x^2 - 2x + 2 = 0$
Discriminant	$a = 1, b = -4, c = 3$ $b^2 - 4ac$ $(-4)^2 - 4(1)(3)$ $16 - 12$ 4 The discriminant is **positive**.	$a = 1, b = 2, c = 1$ $b^2 - 4ac$ $2^2 - 4(1)(1)$ $4 - 4$ 0 The discriminant is **zero**.	$a = 1, b = -2, c = 2$ $b^2 - 4ac$ $(-2)^2 - 4(1)(2)$ $4 - 8$ -4 The discriminant is **negative**.
Graph of Related Function	Notice that the related function has **two *x*-intercepts**. 	Notice that the related function has **one *x*-intercept**. 	Notice that the related function has **no *x*-intercepts**. 
Number of Solutions	**two real solutions**	**one real solution**	**no real solutions**

Know It! Note

The Discriminant of Quadratic Equation $ax^2 + bx + c = 0$

If $b^2 - 4ac > 0$, the equation has **two** real solutions.

If $b^2 - 4ac = 0$, the equation has **one** real solution.

If $b^2 - 4ac < 0$, the equation has **no** real solutions.

EXAMPLE 3 Using the Discriminant

Find the number of solutions of each equation using the discriminant.

A $3x^2 + 10x + 2 = 0$

$a = 3, b = 10, c = 2$

$b^2 - 4ac$

$10^2 - 4(3)(2)$

$100 - 24$

76

$b^2 - 4ac$ is positive.

There are two real solutions.

B $9x^2 - 6x + 1 = 0$

$a = 9, b = -6, c = 1$

$b^2 - 4ac$

$(-6)^2 - 4(9)(1)$

$36 - 36$

0

$b^2 - 4ac$ is zero.

There is one real solution.

C $x^2 + x + 1 = 0$

$a = 1, b = 1, c = 1$

$b^2 - 4ac$

$1^2 - 4(1)(1)$

$1 - 4$

-3

$b^2 - 4ac$ is negative.

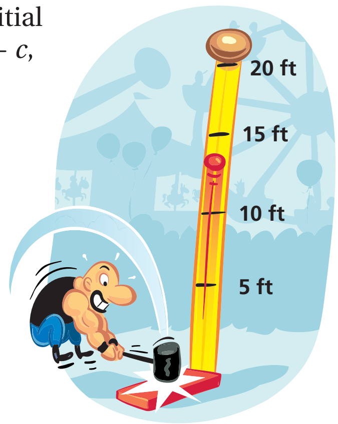
There are no real solutions.

CHECK IT OUT!

Find the number of solutions of each equation using the discriminant.

3a. $2x^2 - 2x + 3 = 0$ **3b.** $x^2 + 4x + 4 = 0$ **3c.** $x^2 - 9x + 4 = 0$

The height h in feet of an object shot straight up with initial velocity v in feet per second is given by $h = -16t^2 + vt + c$, where c is the beginning height of the object above the ground.



EXAMPLE 4 Physics Application

Helpful Hint

If the object is shot straight up from the ground, the initial height of the object above the ground equals 0.

A weight 1 foot above the ground on a carnival strength test is shot straight up with an initial velocity of 35 feet per second. Will it ring the bell at the top of the pole? Use the discriminant to explain your answer.

$h = -16t^2 + vt + c$	
$20 = -16t^2 + 35t + 1$	*Substitute 20 for h, 35 for v, and 1 for c.*
$0 = -16t^2 + 35t + (-19)$	*Subtract 20 from both sides.*
$b^2 - 4ac$	*Evaluate the discriminant.*
$35^2 - 4(-16)(-19) = 9$	*Substitute −16 for a, 35 for b, and −19 for c.*

The discriminant is positive, so the equation has two solutions. The weight will reach a height of 20 feet so it will ring the bell.

CHECK IT OUT!

4. **What if...?** Suppose the weight is shot straight up with an initial velocity of 20 feet per second. Will it ring the bell? Use the discriminant to explain your answer.

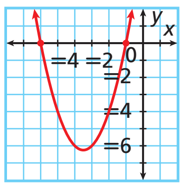
There is no one correct way to solve a quadratic equation. Many quadratic equations can be solved using several different methods.

EXAMPLE 5 Solving Using Different Methods

Solve $x^2 + 7x + 6 = 0$. Show your work.

Method 1 Solve by graphing.

$y = x^2 + 7x + 6$ *Write the related quadratic function and graph it.*



The solutions are the x-intercepts, -6 and -1.

Method 2 Solve by factoring.

$x^2 + 7x + 6 = 0$	
$(x + 6)(x + 1) = 0$	*Factor.*
$x + 6 = 0$ or $x - 1 = 0$	*Use the Zero Product Property.*
$x = -6$ or $x = -1$	*Solve each equation.*

Method 3 Solve by completing the square.

$x^2 + 7x + 6 = 0$	
$x^2 + 7x = -6$	
$x^2 + 7x + \frac{49}{4} = -6 + \frac{49}{4}$	*Add $\left(\frac{b}{2}\right)^2$ to both sides.*
$\left(x + \frac{7}{2}\right)^2 = \frac{25}{4}$	*Factor and simplify.*
$x + \frac{7}{2} = \pm\frac{5}{2}$	*Take the square root of both sides.*
$x + \frac{7}{2} = \frac{5}{2}$ or $x + \frac{7}{2} = -\frac{5}{2}$	*Solve each equation.*
$x = -1$ or $x = -6$	

Method 4 Solve using the Quadratic Formula.

$1x^2 + 7x + 6 = 0$ *Identify a, b, and c.*

$x = \dfrac{-7 \pm \sqrt{7^2 - 4(1)(6)}}{2(1)}$ *Substitute 1 for a, 7 for b, and 6 for c.*

$x = \dfrac{-7 \pm \sqrt{49 - 24}}{2} = \dfrac{-7 \pm \sqrt{25}}{2} = \dfrac{-7 \pm 5}{2}$ *Simplify.*

$x = \dfrac{-7 + 5}{2}$ or $x = \dfrac{-7 - 5}{2}$ *Write as two equations.*

$x = -1$ or $x = -6$ *Solve each equation.*

CHECK IT OUT! **Solve. Show your work.**

5a. $x^2 + 7x + 10 = 0$ **5b.** $-14 + x^2 = 5x$ **5c.** $2x^2 + 4x - 21 = 0$

Notice that all of the methods in Example 5 produce the same solutions, **−1** and **−6**. The only method you cannot use to solve $x^2 + 7x + 6 = 0$ is using square roots. Sometimes one method is better for solving certain types of equations. The table below gives some advantages and disadvantages of the different methods.

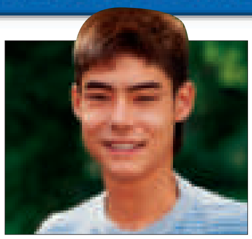
Know it! Note

Methods of Solving Quadratic Equations

METHOD	ADVANTAGES	DISADVANTAGES
Graphing	• Always works to give approximate solutions • Can quickly see the number of solutions	• Cannot always get an exact solution
Factoring	• Good method to try first • Straightforward if the equation is factorable	• Complicated if the equation is not easily factorable • Not all quadratic equations are factorable.
Using square roots	• Quick when the equation has no *x*-term	• Cannot easily use when there is an *x*-term
Completing the square	• Always works	• Sometimes involves difficult calculations
Using the Quadratic Formula	• Always works • Can always find exact solutions	• Other methods may be easier or less time consuming.

Student to Student

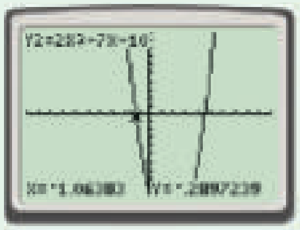
Solving Quadratic Equations



Binh Pham
Johnson High School

No matter what method I use, I like to check my answers for reasonableness by graphing.

I used the Quadratic Formula to solve $2x^2 - 7x - 10 = 0$. I found that $x \approx -1.09$ and $x \approx 4.59$. Then I graphed $y = 2x^2 - 7x - 10$. The x-intercepts appeared to be close to −1 and 4.5, so I knew my solutions were reasonable.



THINK AND DISCUSS

1. Describe how to use the discriminant to find the number of solutions to a quadratic equation.

2. Choose a method to solve $x^2 + 5x + 4 = 0$ and explain why you chose that method.

3. Describe how the discriminant can be used to determine if an object will reach a given height.

4. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write the number of real solutions.

The number of real solutions of $ax^2 + bx + c =$ when0

$b^2 = 4ac > 0$ is ▢.	$b^2 = 4ac < 0$ is ▢.	$b^2 = 4ac$ 0 is ▢.

9-9 Exercises

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GUIDED PRACTICE

1. Vocabulary If the *discriminant* is negative, the quadratic equation has ___?___ solution(s). (*no*, *one*, or *two*)

SEE EXAMPLE 1 p. 653

Solve using the Quadratic Formula.

2. $x^2 - 5x + 4 = 0$ **3.** $2x^2 = 7x - 3$ **4.** $x^2 - 6x - 7 = 0$

5. $x^2 = -14x - 40$ **6.** $3x^2 - 2x = 8$ **7.** $4x^2 - 4x - 3 = 0$

SEE EXAMPLE 2 p. 653

8. $2x^2 - 6 = 0$ **9.** $x^2 + 6x + 3 = 0$ **10.** $x^2 - 7x + 2 = 0$

11. $3x^2 = -x + 5$ **12.** $x^2 - 4x - 7 = 0$ **13.** $2x^2 + x - 5 = 0$

SEE EXAMPLE 3 p. 654

Find the number of solutions of each equation using the discriminant.

14. $2x^2 + 4x + 3 = 0$ **15.** $x^2 + 4x + 4 = 0$ **16.** $2x^2 - 11x + 6 = 0$

17. $x^2 + x + 1 = 0$ **18.** $3x^2 = 5x - 1$ **19.** $-2x + 3 = 2x^2$

20. $2x^2 + 12x = -18$ **21.** $5x^2 + 3x = -4$ **22.** $8x = 1 - x^2$

SEE EXAMPLE 4 p. 655

23. Hobbies The height above the ground in meters of a model rocket on a particular launch can be modeled by the equation $h = -4.9t^2 + 102t + 100$, where t is the time in seconds after its engine burns out 100 m above the ground. Will the rocket reach a height of 600 m? Use the discriminant to explain your answer.

SEE EXAMPLE 5 p. 655

Solve. Show your work.

24. $x^2 + x - 12 = 0$ **25.** $x^2 + 6x + 9 = 0$ **26.** $2x^2 - x - 1 = 0$

27. $4x^2 + 4x + 1 = 0$ **28.** $2x^2 + x - 7 = 0$ **29.** $9 = 2x^2 + 3x$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
30–32	1
33–35	2
36–38	3
39	4
40–42	5

Extra Practice

Skills Practice p. S21
Application Practice p. S36

Solve using the Quadratic Formula.

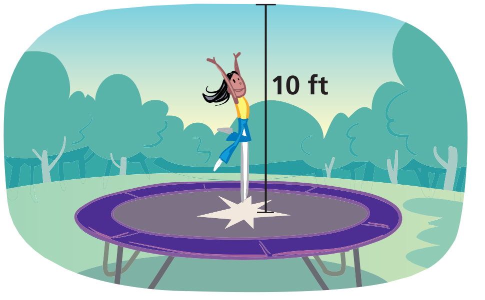
30. $3x^2 = 13x - 4$ **31.** $x^2 - 10x + 9 = 0$ **32.** $1 = 3x^2 + 2x$

33. $x^2 - 4x + 1 = 0$ **34.** $3x^2 - 5 = 0$ **35.** $2x^2 + 7x = -4$

Find the number of solutions of each equation using the discriminant.

36. $3x^2 - 6x + 3 = 0$ **37.** $x^2 - 3x - 8 = 0$ **38.** $7x^2 + 6x + 2 = 0$

39. **Multi-Step** A gymnast who can stretch her arms up to reach 6 feet jumps straight up on a trampoline. The height of her feet above the trampoline can be modeled by the equation $h = -16x^2 + 12x$, where x is the time in seconds after her jump. Do the gymnast's hands reach a height of 10 feet above the trampoline? Use the discriminant to explain. (*Hint:* Let $h = 10 - 6$, or 4.)



Solve. Show your work.

40. $x^2 + 4x + 3 = 0$ **41.** $x^2 + 2x = 15$ **42.** $x^2 - 12 = -x$

Write each equation in standard form. Use the discriminant to determine the number of solutions. Then find any real solutions using the Quadratic Formula.

43. $2x = 3 + 2x^2$ **44.** $x^2 = 2x + 9$ **45.** $2 = 7x + 4x^2$

46. $-7 = x^2$ **47.** $-12x = -9x^2 - 4$ **48.** $x^2 - 14 = 0$

Multi-Step **Use the discriminant to determine the number of *x*-intercepts. Then use the Quadratic Formula to find them.**

49. $y = 2x^2 - x - 21$ **50.** $y = 5x^2 + 12x + 8$ **51.** $y = x^2 - 10x + 25$

52. Copy and complete the table.

Quadratic Equation	Discriminant	Number of Solutions
$x^2 + 12x - 20 = 0$	■	■
$8x + x^2 = -16$	■	■
$0.5x^2 + x - 3 = 0$	■	■
$-3x^2 - 2x = 1$	■	■

53. **Sports** A diver begins on a platform 10 meters above the surface of the water. The diver's height is given by the equation $h(t) = -4.9t^2 + 3.5t + 10$, where t is the time in seconds after the diver jumps.

a. How long does it take the diver to reach a point 1 meter above the water?

b. How many solutions does your equation from part **a** have?

c. Do all of the solutions to the equation make sense in the situation? Explain.

54. **Critical Thinking** How many solutions does the equation $x^2 = k$ have when $k > 0$, when $k < 0$, and when $k = 0$? Use the discriminant to explain.

55. **Write About It** How can you use the discriminant to save time?

MULTI-STEP TEST PREP

56. This problem will prepare you for the Multi-Step Test Prep on page 660.

The equation $0 = -16t^2 + 80t + 20$ gives the time t in seconds when a golf ball is at height 0 feet.

a. Will the height of the ball reach 130 feet? Explain.

b. Will the golf ball reach a height of 116 feet? If so, when?

c. Solve the given quadratic equation using the Quadratic Formula.

TEST PREP

57. How many solutions does $4x^2 - 3x + 1 = 0$ have?

Ⓐ 0 Ⓑ 1 Ⓒ 2 Ⓓ 4

58. For which of the following conditions does $ax^2 + bx + c = 0$ have two solutions?

I. $b^2 = 4ac$

II. $b^2 > 4ac$

III. $a = b, c = b$

Ⓕ I only Ⓖ II only Ⓗ III only Ⓙ II and III

59. **Extended Response** Use the equation $0 = x^2 + 2x + 1$ to answer the following.

a. How many solutions does the equation have?

b. Solve the equation by graphing.

c. Solve the equation by factoring.

d. Solve the equation by using the Quadratic Formula.

e. Explain which method was easiest for you. Why?

CHALLENGE AND EXTEND

60. **Agriculture** A rancher has 80 yards of fencing to build a rectangular pen. Let w be the width of the pen. Write an equation giving the area of the pen. Find the dimensions of the pen when the area is 400 square yards.

61. **Agriculture** A farmer wants to fence a four-sided field using an existing fence along the south side of the field. He has 1000 feet of fencing. He makes the northern boundary perpendicular to and twice as long as the western boundary. The eastern and western boundaries have to be parallel, but the northern and southern ones do not.

a. Can the farmer enclose an area of 125,000 square feet? Explain why or why not. (*Hint:* Use the formula for the area of a trapezoid, $A = \frac{1}{2}h(b_1 + b_2)$.)

b. What geometric shape will the field be?

SPIRAL REVIEW

Solve each equation by completing the square. *(Lesson 9-8)*

62. $x^2 - 2x - 24 = 0$ **63.** $x^2 + 6x = 40$ **64.** $-3x^2 + 12x = 15$

Factor each polynomial by grouping. *(Lesson 8-2)*

65. $s^2r^3 + 5r^3 + 5t + s^2t$ **66.** $b^3 - 4b^2 + 2b - 8$ **67.** $n^5 - 6n^4 - 2n + 12$

Order the functions from narrowest graph to widest. *(Lesson 9-4)*

68. $f(x) = 0.2x^2,\ g(x) = 1.5x^2 + 4,\ h(x) = x^2 - 8$ **69.** $f(x) = -\frac{1}{5}x^2 + 5,\ g(x) = \frac{1}{6}x^2$

CHAPTER 9

SECTION 9B

MULTI-STEP TEST PREP

Solving Quadratic Equations

Seeing Green A golf player hits a golf ball from a tee with an initial velocity of 80 feet per second. The height of the golf ball t seconds after it is hit is given by $h = -16t^2 + 80t$.

1. How long is the golf ball in the air?
2. What is the maximum height of the golf ball?
3. How long after the golf ball is hit does it reach its maximum height?
4. What is the height of the golf ball after 3.5 seconds?
5. At what times is the golf ball 64 feet in the air? Explain.

READY TO GO ON?

Quiz for Lessons 9-5 Through 9-9

9-5 Solving Quadratic Equations by Graphing

Solve each equation by graphing the related function.

1. $x^2 - 9 = 0$
2. $x^2 + 3x - 4 = 0$
3. $4x^2 + 8x = 32$
4. The height of a fireworks rocket launched from a platform 35 feet above the ground can be approximated by $h = -5t^2 + 30t + 35$, where h is the height in meters and t is the time in seconds. Find the time it takes the rocket to reach the ground after it is launched.

9-6 Solving Quadratic Equations by Factoring

Use the Zero Product Property to solve each equation.

5. $(x + 1)(x + 3) = 0$
6. $(x - 6)(x - 3) = 0$
7. $x(x + 3) = 18$
8. $(x + 2)(x - 5) = 60$

Solve each quadratic equation by factoring.

9. $x^2 - 4x - 32 = 0$
10. $x^2 - 8x + 15 = 0$
11. $x^2 + x = 6$
12. $-8x - 33 = -x^2$
13. The height of a soccer ball kicked from the ground can be approximated by the function $h = -16t^2 + 64t$, where h is the height in feet and t is the time in seconds. Find the time it takes for the ball to return to the ground.

9-7 Solving Quadratic Equations by Using Square Roots

Solve using square roots.

14. $3x^2 = 48$
15. $36x^2 - 49 = 0$
16. $-12 = x^2 - 21$
17. Solve $3x^2 + 5 = 21$. Round to the nearest hundredth.

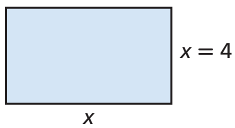
9-8 Completing the Square

Complete the square for each expression.

18. $x^2 - 12x + \square$
19. $x^2 + 4x + \square$
20. $x^2 + 9x + \square$

Solve by completing the square.

21. $x^2 + 2x = 3$
22. $x^2 - 5 = 2x$
23. $x^2 + 7x = 8$
24. The length of a rectangle is 4 feet shorter than its width. The area of the rectangle is 42 square feet. Find the length and width. Round your answer to the nearest tenth of a foot.



9-9 Using the Quadratic Formula and the Discriminant

Solve using the Quadratic Formula. Round your answer to the nearest hundredth.

25. $x^2 + 5x + 1 = 0$
26. $3x^2 + 1 = 2x$
27. $5x + 8 = 3x^2$

Find the number of solutions of each equation using the discriminant.

28. $2x^2 - 3x + 4 = 0$
29. $x^2 + 1 + 2x = 0$
30. $x^2 - 5 + 4x = 0$

CHAPTER 9 Study Guide: Review

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. The ___?___ is the highest or lowest point on a parabola.

2. A quadratic function has a ___?___ if its graph opens upward and a ___?___ if its graph opens downward.

3. A ___?___ can also be called an x-intercept of the function.

4. Finding the ___?___ can tell you how many real-number solutions a quadratic equation has.

5. ___?___ is a process that results in a perfect-square trinomial.

9-1 Identifying Quadratic Functions *(pp. 590–597)*

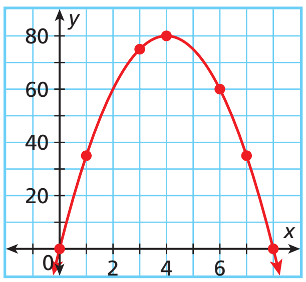
EXAMPLE

■ **Use a table of values to graph $y = -5x^2 + 40x$.**

Step 1 Make a table of values.
Choose values of x and use them to find values of y.

x	0	1	3	4	6	7	8
y	0	35	75	80	60	35	0

Step 2 Plot the points and connect them with a smooth curve.



EXERCISES

Tell whether each function is quadratic. Explain.

6. $y = 2x^2 + 9x - 5$

7. $y = -4x + 3$

8. $y = -\frac{1}{2}x^2$

9. $y = 5x^3 + 8$

Use a table of values to graph each quadratic function.

10. $y = 6x^2$

11. $y = -4x^2$

12. $y = \frac{1}{4}x^2$

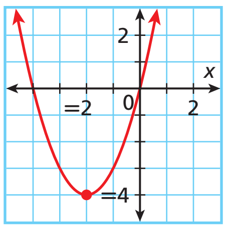
13. $y = -3x^2$

Tell whether the graph of each function opens upward or downward. Explain.

14. $y = 5x^2 - 12$

15. $y = -x^2 + 3x - 7$

16. Identify the vertex of the parabola. Then give the minimum or maximum value of the function.

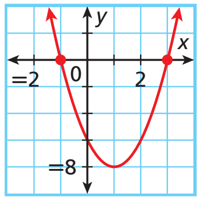


9-2 Characteristics of Quadratic Functions *(pp. 599–605)*

EXAMPLE

■ **Find the zeros of $y = 2x^2 - 4x - 6$ from its graph. Then find the axis of symmetry and the vertex.**

y 2x² = 4x = 6



Step 1 Use the graph to find the zeros.
The zeros are -1 and 3.

Step 2 Find the axis of symmetry.

$x = \frac{-1+3}{2} = \frac{2}{2} = 1$ *Find the average of the zeros.*

The axis of symmetry is the vertical line $x = 1$.

Step 3 Find the vertex.

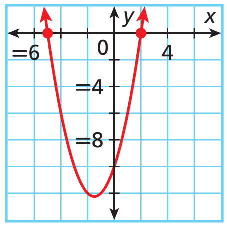
$y = 2x^2 - 4x - 6$
$y = 2(1)^2 - 4(1) - 6$ *Substitute 1 into the function to find the y-value of the vertex.*
$y = -8$

The vertex is $(1, -8)$.

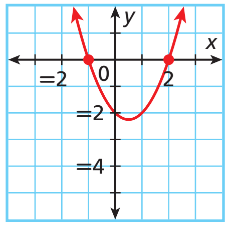
EXERCISES

Find the zeros of each quadratic function from its graph. Check your answer.

17. $y = x^2 + 3x - 10$

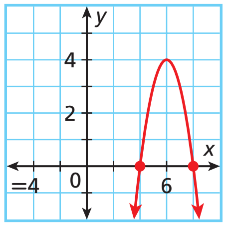


18. $y = x^2 - x - 2$

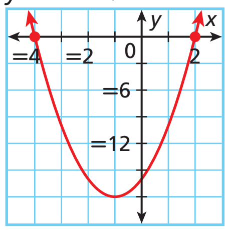


Find the axis of symmetry and vertex of each parabola.

19. y =x² + 12x = 2



20. y 2x² + 4x = 16



9-3 Graphing Quadratic Functions *(pp. 606–611)*

EXAMPLE

■ **Graph $y = 2x^2 - 8x - 10$.**

Step 1 Find the axis of symmetry.

$x = \frac{-b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$

The axis of symmetry is $x = 2$.

Step 2 Find the vertex.

$y = 2x^2 - 8x - 10$
$y = 2(2)^2 - 8(2) - 10$
$y = -18$

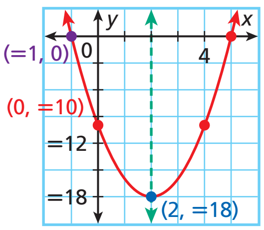
The vertex is $(2, -18)$.

Step 3 Find the y-intercept.
$c = -10$

Step 4 Find one more point on the graph.

$y = 2(-1)^2 - 8(-1) - 10 = 0$
Use $(-1, 0)$.

Step 5 Graph the axis of symmetry and the points. Reflect the points and connect with a smooth curve.



EXERCISES

Graph each quadratic function.

21. $y = x^2 + 6x + 6$

22. $y = x^2 - 4x - 12$

23. $y = x^2 - 8x + 7$

24. $y = 2x^2 - 6x - 8$

25. $3x^2 + 6x = y - 3$

26. $2 - 4x^2 + y = 8x - 10$

27. Water that is sprayed upward from a sprinkler with an initial velocity of 20 m/s can be approximated by the function $y = -5x^2 + 20x$, where y is the height of a drop of water x seconds after it is released. Graph this function. Find the time it takes a drop of water to reach its maximum height, the water's maximum height, and the time it takes the water to reach the ground.

9-4 Transforming Quadratic Functions *(pp. 613–619)*

EXAMPLE

- **Compare the graph of $g(x) = 3x^2 - 4$ with the graph of $f(x) = x^2$. Use the functions.**
  - Both graphs open upward because $a > 0$.
  - The axis of symmetry is the same, $x = 0$, because $b = 0$ in both functions.
  - The graph of $g(x)$ is narrower than the graph of $f(x)$ because $|3| > |1|$.
  - The vertex of $f(x)$ is $(0, 0)$. The vertex of $g(x)$ is translated 4 units down to $(0, -4)$.
  - $f(x)$ has one zero at the origin. $g(x)$ has two zeros because the vertex is below the origin and the parabola opens upward.

EXERCISES

Compare the widths of the graphs of the given quadratic functions. Order functions with different widths from narrowest graph to widest.

28. $f(x) = 2x^2$, $g(x) = 4x^2$

29. $f(x) = 6x^2$, $g(x) = -6x^2$

30. $f(x) = x^2$, $g(x) = \frac{1}{3}x^2$, $h(x) = 3x^2$

Compare the graph of each function with the graph of $f(x) = x^2$.

31. $g(x) = x^2 + 5$

32. $g(x) = 3x^2 - 1$

33. $g(x) = 2x^2 + 3$

9-5 Solving Quadratic Equations by Graphing *(pp. 622–627)*

EXAMPLE

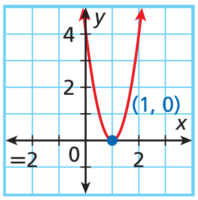
- **Solve $-4 = 4x^2 - 8x$ by graphing the related function.**

Step 1 Write the equation in standard form:

$$0 = 4x^2 - 8x + 4.$$

Step 2 Graph the related function:

$$y = 4x^2 - 8x + 4.$$



Step 3 Find the zeros.
The only zero is 1. The solution is $x = 1$.

EXERCISES

Solve each equation by graphing the related function.

34. $0 = x^2 + 4x + 3$

35. $0 = x^2 + 6x + 9$

36. $-4x^2 = 3$

37. $x^2 + 5 = 6x$

38. $-4x^2 = 64 - 32x$

39. $9 = 9x^2$

40. $-3x^2 + 2x = 5$

9-6 Solving Quadratic Equations by Factoring *(pp. 630–635)*

EXAMPLE

- **Solve $3x^2 - 6x = 24$ by factoring.**

$3x^2 - 6x = 24$	*Write the equation in standard form.*
$3x^2 - 6x - 24 = 0$	
$3(x^2 - 2x - 8) = 0$	*Factor out 3.*
$3(x + 2)(x - 4) = 0$	*Factor the trinomial.*
$3 \neq 0$, $x + 2 = 0$ or $x - 4 = 0$	*Use the Zero Product Property.*
$x = -2$ or $x = 4$	*Solve each equation.*

EXERCISES

Solve each quadratic equation by factoring.

41. $x^2 + 6x + 5 = 0$

42. $x^2 + 9x + 14 = 0$

43. $x^2 - 2x - 15 = 0$

44. $2x^2 - 2x - 4 = 0$

45. $x^2 + 10x + 25 = 0$

46. $4x^2 - 36x = -81$

47. A rectangle is 2 feet longer than it is wide. The area of the rectangle is 48 square feet. Write and solve an equation that can be used to find the width of the rectangle.

9-7 Solving Quadratic Equations by Using Square Roots *(pp. 636–641)*

EXAMPLE

- **Solve $2x^2 = 98$ using square roots.**

$\frac{2x^2}{2} = \frac{98}{2}$ *Divide both sides of the equation by 2 to isolate x^2.*

$x^2 = 49$

$x = \pm\sqrt{49}$ *Take the square root of both sides.*

$x = \pm 7$ *Use $\pm$ to show both roots.*

The solutions are −7 and 7.

EXERCISES

Solve using square roots.

48. $5x^2 = 320$

49. $-x^2 + 144 = 0$

50. $x^2 = -16$

51. $x^2 + 7 = 7$

52. $2x^2 = 50$

53. $4x^2 = 25$

54. A rectangle is twice as long as it is wide. The area of the rectangle is 32 square feet. Find the rectangle's width.

9-8 Completing the Square *(pp. 645–651)*

EXAMPLE

- **Solve $x^2 - 6x = -5$ by completing the square.**

$\left(\frac{-6}{2}\right)^2 = 9.$ *Find $\left(\frac{b}{2}\right)^2$.*

$x^2 - 6x + 9 = -5 + 9$ *Complete the square by adding $\left(\frac{b}{2}\right)^2$ to both sides.*

$x^2 - 6x + 9 = 4$

$(x-3)^2 = 4$ *Factor the trinomial.*

$\sqrt{(x-3)^2} = \sqrt{4}$ *Take the square root of both sides.*

$x - 3 = \pm 2$ *Use the $\pm$ symbol.*

$x - 3 = 2$ or $x - 3 = -2$ *Solve each equation.*

$x = 5$ or $x = 1$

The solutions are 5 and 1.

EXERCISES

Solve by completing the square.

55. $x^2 + 2x = 48$

56. $x^2 + 4x = 21$

57. $2x^2 - 12x + 10 = 0$

58. $x^2 - 10x = -20$

59. A homeowner is planning an addition to her house. She wants the new family room to have an area of 192 square feet. The contractor says that the length needs to be 4 more feet than the width. What will the dimensions of the new room be? Round your answer to the nearest hundredth of a foot.

9-9 Using the Quadratic Formula and the Discriminant *(pp. 652–659)*

EXAMPLE

- **Solve $x^2 + 4x + 4 = 0$ using the Quadratic Formula.**

The equation $x^2 + 4x + 4 = 0$ is in standard form with $a = 1$, $b = 4$, and $c = 4$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *Write the Quadratic Formula.*

$= \frac{-4 \pm \sqrt{4^2 - 4(1)(4)}}{2(1)}$ *Substitute for a, b, and c.*

$= \frac{-4 \pm \sqrt{16 - 16}}{2}$ *Simplify.*

$= \frac{-4 \pm \sqrt{0}}{2} = \frac{-4}{2} = -2$

The solution is $x = -2$.

EXERCISES

Solve using the Quadratic Formula.

60. $x^2 - 5x - 6 = 0$

61. $2x^2 - 9x - 5 = 0$

62. $4x^2 - 8x + 4 = 0$

63. $x^2 - 6x = -7$

Find the number of solutions of each equation using the discriminant.

64. $x^2 - 12x + 36 = 0$

65. $3x^2 + 5 = 0$

66. $2x^2 - 13x = -20$

67. $6x^2 - 20 = 15x + 1$

CHAPTER 9

Chapter Test

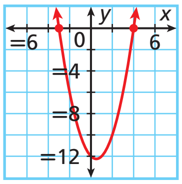
Tell whether each function is quadratic. Explain.

1. (10, 50), (11, 71), (12, 94), (13, 119), (14, 146)

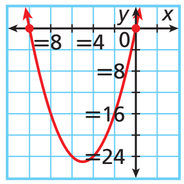
2. $3x^2 + y = 4 + 3x^2$

3. Tell whether the graph of $y = -2x^2 + 7x - 5$ opens upward or downward and whether the parabola has a maximum or a minimum.

4. Estimate the zeros of the quadratic function.



5. Find the axis of symmetry of the parabola.



6. Find the vertex of the graph of $y = x^2 + 6x + 8$.

7. Graph the quadratic function $y = x^2 - 4x + 2$.

Compare the graph of each function with the graph of $f(x) = x^2$.

8. $g(x) = -x^2 - 2$

9. $h(x) = \frac{1}{3}x^2 + 1$

10. $g(x) = 3x^2 - 4$

11. A hammer is dropped from a 40-foot scaffold. Another one is dropped from a 60-foot scaffold.

a. Write the two height functions and compare their graphs. Use $h(t) = -16t^2 + c$, where c is the height of the scaffold.

b. Use the graphs to estimate when each hammer will reach the ground.

12. A rocket is launched with an initial velocity of 110 m/s. The height of the rocket in meters is approximated by the quadratic equation $h = -5t^2 + 110t$ where t is the time after launch in seconds. About how long does it take for the rocket to return to the ground?

Solve by factoring.

13. $x^2 + 6x + 5 = 0$

14. $x^2 - 12x = -36$

15. $x^2 - 81 = 0$

Solve by using square roots.

16. $-2x^2 = -72$

17. $9x^2 - 49 = 0$

18. $3x^2 + 12 = 0$

Solve by completing the square.

19. $x^2 + 10x = -21$

20. $x^2 - 6x + 4 = 0$

21. $2x^2 + 16x = 0$

22. A landscaper has enough cement to make a patio with an area of 150 square feet. The homeowner wants the length to be 6 feet longer than the width. What dimensions should be used for the patio? Round to the nearest tenth of a foot.

Solve using the Quadratic Formula. Round to the nearest hundredth if necessary.

23. $x^2 + 3x - 40 = 0$

24. $2x^2 + 7x = -5$

25. $8x^2 + 3x - 1 = 0$

Find the number of solutions of each equation using the discriminant.

26. $4x^2 - 4x + 1 = 0$

27. $2x^2 + 5x - 25 = 0$

28. $\frac{1}{2}x^2 + 8 = 0$

COLLEGE ENTRANCE EXAM PRACTICE

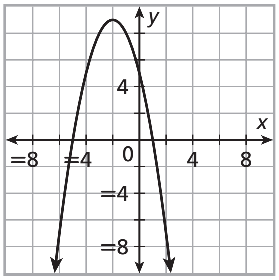
FOCUS ON SAT SUBJECT TESTS

In addition to the SAT, some colleges require the SAT Subject Tests for admission. Colleges that don't require the SAT Subject Tests may still use the scores to learn about your academic background and to place you in the appropriate college math class.

HOT TIP! Take the SAT Subject Test in mathematics while the material is still fresh in your mind. You are not expected to be familiar with all of the test content, but you should have completed at least three years of college-prep math.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

1. The graph below corresponds to which of the following quadratic functions?



(A) $f(x) = x^2 + 4x - 5$

(B) $f(x) = -x^2 - 4x + 3$

(C) $f(x) = -x^2 + 5x - 4$

(D) $f(x) = -x^2 - 4x + 5$

(E) $f(x) = -x^2 - 3x + 5$

2. What is the sum of the solutions to the equation $9x^2 - 6x = 8$?

(A) $\frac{4}{3}$

(B) $\frac{2}{3}$

(C) $\frac{1}{3}$

(D) $-\frac{2}{3}$

(E) $-\frac{8}{3}$

3. If $h(x) = ax^2 + bx + c$, where $b^2 - 4ac < 0$ and $a < 0$, which of the following statements must be true?

I. The graph of $h(x)$ has no points in the first or second quadrants.

II. The graph of $h(x)$ has no points in the third or fourth quadrants.

III. The graph of $h(x)$ has points in all quadrants.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) None of the statements are true.

4. What is the axis of symmetry for the graph of a quadratic function whose zeros are -2 and 4?

(A) $x = -2$

(B) $x = 0$

(C) $x = 1$

(D) $x = 2$

(E) $x = 6$

5. How many real-number solutions does $0 = x^2 - 7x + 1$ have?

(A) None

(B) One

(C) Two

(D) All real numbers

(E) It is impossible to determine.

CHAPTER 9

TEST TACKLER

Standardized Test Strategies

Extended Response: Explain Your Reasoning

Extended response test items often include multipart questions that evaluate your understanding of a math concept. To receive full credit, you must answer the problem correctly, show all of your work, and explain your reasoning. Use complete sentences and show your problem-solving method clearly.

EXAMPLE 1

Extended Response **Given $\frac{1}{2}x^2 + y = 4x - 3$ and $y = 2x - 12x$, identify which is a quadratic function. Provide an explanation for your decision. For the quadratic function, tell whether the graph of the function opens upward or downward and whether the parabola has a maximum or a minimum. Explain your reasoning.**

Read the solutions provided by two different students.

Student A

The quadratic function is $\frac{1}{2}x^2 + y = 4x - 3$ because it can be written in standard form, $y = -\frac{1}{2}x^2 + 4x - 3$, where a, b, and c are real numbers and $a \neq 0$. The other function, $y = 2x - 12x$, is not quadratic because there is no x^2-term.

The graph of this function will open downward because a, which is equal to $-\frac{1}{2}$, is less than 0. Because the parabola opens downward, the graph will have a maximum.

Excellent Explanation

The response includes the correct answers along with a detailed explanation for each part of the problem. The explanation is written using complete sentences and is presented in an order that is easy to follow and to understand. It is obvious that this student knows how to determine and interpret a quadratic function.

Student B

$\frac{1}{2}x^2 + y = 4x - 3$ There is an x^2.

When I graphed the function on my calculator, I saw a parabola that opened downward.

It had a maximum.

Poor Explanation

The response includes the correct answers, but the explanation does not include details. The reason for defining the function as quadratic does not show knowledge of the concept. The student shows a lack of understanding of how to write and interpret a quadratic function in standard form.

HOT TIP! Include as many details as possible to support your reasoning. This increases the chance of getting full credit for your response.

Read each test item and answer the questions that follow.

Item A

The height in feet of a tennis ball x seconds after it is ejected from a serving machine is given by the ordered pairs $\{(0, 10), (0.5, 9), (1, 7), (1.5, 4), (2, 0)\}$. Determine whether the function is quadratic. Find its domain and range. Explain your answers.

1. What should a student include in the explanation to receive full credit?
2. Read the two explanations below. Which explanation is better? Why?

Student A

Range: $0 \le y \le 10$ Domain: $0 \le x \le 2$
Second differences are -1 : quadratic

Student B

The function is quadratic because the second differences are constant: -1. The domain and range are determined by the points $(0, 10)$ and $(2, 0)$. The range is $0 \le y \le 10$, and the domain is $0 \le x \le 2$.

Item B

The height of a golf ball can be approximated by the function $y = -5x^2 + 20x + 8$, where y is the height in meters above the ground and x is the time in seconds after the ball is hit. What is the maximum height of the ball? How long does it take for the ball to reach its maximum height? Explain.

3. A student correctly found the following answers. Use this information to write a clear and concise explanation.

Axis of symmetry is the vertical line at $x = 2$.
Vertex is at $(2, 28)$.
28 meters; 2 seconds
2 seconds versus 4 seconds

Item C

A science teacher set off a bottle rocket as part of a lab experiment. The function $h = -16t^2 + 96t$ represents the height in feet of a rocket that is shot out of a bottle with a vertical velocity of 96 feet per second. Find the time that the rocket is in the air. Explain how you found your answer.

4. Read the two responses below.
   a. Which student provided the better explanation? Why?
   b. What advice would you give the other student to improve his or her explanation?

Student C

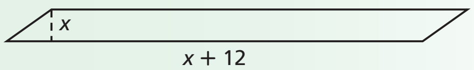
Graph the function $h = -16t^2 + 96t$, and then find the zeros. The first zero is when $t = 0$, when the rocket is launched. The second zero is when the rocket hits the ground: $t = 6$. The difference between 6 and 0 is the time that the rocket is in the air: 6 seconds.

Student D

6 seconds.
Graph the function to find how long the rocket is in the air, and find the values where it crosses the x-axis.

Item D

The base of a parallelogram is 12 centimeters more than its height. The area of the parallelogram is 13 square centimeters. Explain how to determine the height and base of the figure. What is the height? What is the base?



5. Read the following response. Identify any areas that need improvement. Rewrite the response so that it will receive full credit.

$x^2 + 12x + 36 = 49$ Complete the square.
$(x+6)^2 = 49$
$x + 6 = \sqrt{49}$
$x + 6 = \pm 7$; $x = 1$ or -13

base = 13, height = 1

CHAPTER 9

STANDARDIZED TEST PREP

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CUMULATIVE ASSESSMENT, CHAPTERS 1–9

Multiple Choice

1. Which expression is NOT equal to the other three?

Ⓐ 0^1

Ⓑ 1^1

Ⓒ 1^0

Ⓓ $(-1)^0$

2. Which function's graph is a translation of the graph of $f(x) = 3x^2 + 4$ seven units down?

Ⓕ $f(x) = -4x^2 + 4$

Ⓖ $f(x) = 10x^2 + 4$

Ⓗ $f(x) = 3x^2 - 3$

Ⓙ $f(x) = 3x^2 + 11$

3. The area of a circle is $\pi(9x^2 + 42x + 49)$. What is the circumference of the circle?

Ⓐ $\pi(3x + 7)$

Ⓑ $2\pi(3x + 7)$

Ⓒ $2\pi(3x + 7)^2$

Ⓓ $6x + 14$

4. CyberCafe charges a computer station rental fee of \$5, plus \$0.20 for each quarter-hour spent surfing. Which expression represents the total amount Carl will pay to use a computer station for three and a half hours?

Ⓕ $5 + 0.20(3.5)$

Ⓖ $5 + 0.20(3.5)(4)$

Ⓗ $5 + \frac{0.20}{3.5 \div 4}$

Ⓙ $5 + \frac{1}{4} \cdot \frac{0.20}{3.5}$

5. What is the numerical solution to the equation *five less than three times a number equals four more than eight times the number?*

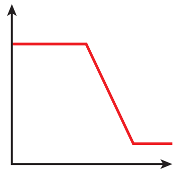
Ⓐ $-\frac{9}{5}$

Ⓑ $\frac{1}{11}$

Ⓒ $-\frac{1}{5}$

Ⓓ $\frac{1}{5}$

6. Which is a possible situation for the graph?



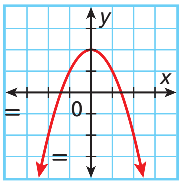
Ⓕ A car drives at a steady speed, slows down in a school zone, and then resumes its previous speed.

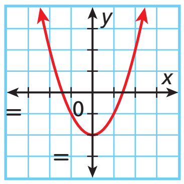
Ⓖ A child climbs the ladder of a slide and then slides down.

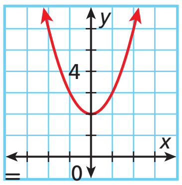
Ⓗ A person flies in an airplane for a while, parachutes out, and gets stuck in a tree.

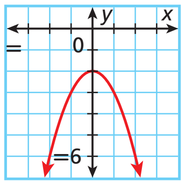
Ⓙ The number of visitors increases in the summer, declines in the fall, and levels off in the winter.

7. Which of the following is the graph of $f(x) = -x^2 + 2$?

Ⓐ 

Ⓒ 

Ⓑ 

Ⓓ 

8. The value of y varies directly with x, and $y = 40$ when $x = -5$. Find y when $x = 8$.

Ⓕ 25

Ⓖ -1

Ⓗ -8

Ⓙ -64

9. What is the slope of the line that passes through the points $(4, 7)$ and $(5, 3)$?

Ⓐ 4

Ⓑ $\frac{1}{4}$

Ⓒ -4

Ⓓ $-\frac{1}{4}$

HOT TIP! The problems on many standardized tests are ordered from least to most difficult, but all items are usually worth the same amount of points. If you are stuck on a question near the end of the test, your time may be better spent rechecking your answers to earlier questions.

10. Putting Green Mini Golf charges a \$4 golf club rental fee plus \$1.25 per game. Good Times Golf charges a \$1.25 golf club rental fee plus \$3.75 per game. Which system of equations could be solved to determine for how many games the cost is the same at both places?

(F) $\begin{cases} y = 4x + 1.25 \\ y = 3.75 + 1.25x \end{cases}$

(G) $\begin{cases} y = 4 - 1.25x \\ y = -3.75 + 1.25x \end{cases}$

(H) $\begin{cases} y = 1.25x + 4 \\ y = 3.75x + 1.25 \end{cases}$

(J) $\begin{cases} y = 1.25x - 4 \\ y = 1.25x + 3.75 \end{cases}$

11. Which graph has an axis of symmetry of $x = -2$?

(A) $y = 2x^2 - x + 3$

(B) $y = 4x^2 + 2x + 3$

(C) $y = x^2 - 2x + 3$

(D) $y = x^2 + 4x + 3$

12. Which polynomial is the product of $x - 4$ and $x^2 - 4x + 1$?

(F) $-4x^2 + 17x - 4$

(G) $x^3 - 8x^2 + 17x - 4$

(H) $x^3 + 17x - 4$

(J) $x^3 - 15x + 4$

Gridded Response

13. The length of a rectangle is 2 units greater than the width. The area of the rectangle is 24 square units. What is its width?

14. Find the value of the discriminant of the equation $y = -2x^2 + 3x + 4$.

15. Use the Quadratic Formula to find the positive solution of $4x^2 = 10x + 2$. Round your answer to the nearest hundredth.

Short Response

16. The data in the table shows ordered pair solutions to a linear function. Find the missing y-value. Show your work.

x	y
−2	−7
−1	−3
0	
1	5
2	9

17. Answer the following questions using the function $f(x) = 2x^2 + 4x - 1$.

a. Make a table of values and give five points on the graph.

b. Find the axis of symmetry and vertex. Show all calculations.

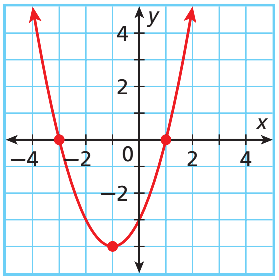
18. a. Show how to solve $x^2 - 2x - 8 = 0$ by graphing the related function. Show all your work.

b. Show another way to solve the equation in part **a**. Show all your work.

19. What can you say about the value of a if the equation $y = ax^2 - 8$ has no solutions? Explain.

Extended Response

20. The graph shows the quadratic function $f(x) = ax^2 + bx + c$.



a. What are the solutions of the equation $0 = ax^2 + bx + c$? Explain how you know.

b. If the point $(-5, 12)$ lies on the graph of $f(x)$, the point $(a, 12)$ also lies on the graph. Find the value of a.

c. What do you know about the relationship between the values of a and b? Use the coordinates of the vertex in your explanation.

d. Use what you know about solving quadratic equations by factoring to make a conjecture about the values of a, b, and c in the function $f(x) = ax^2 + bx + c$.