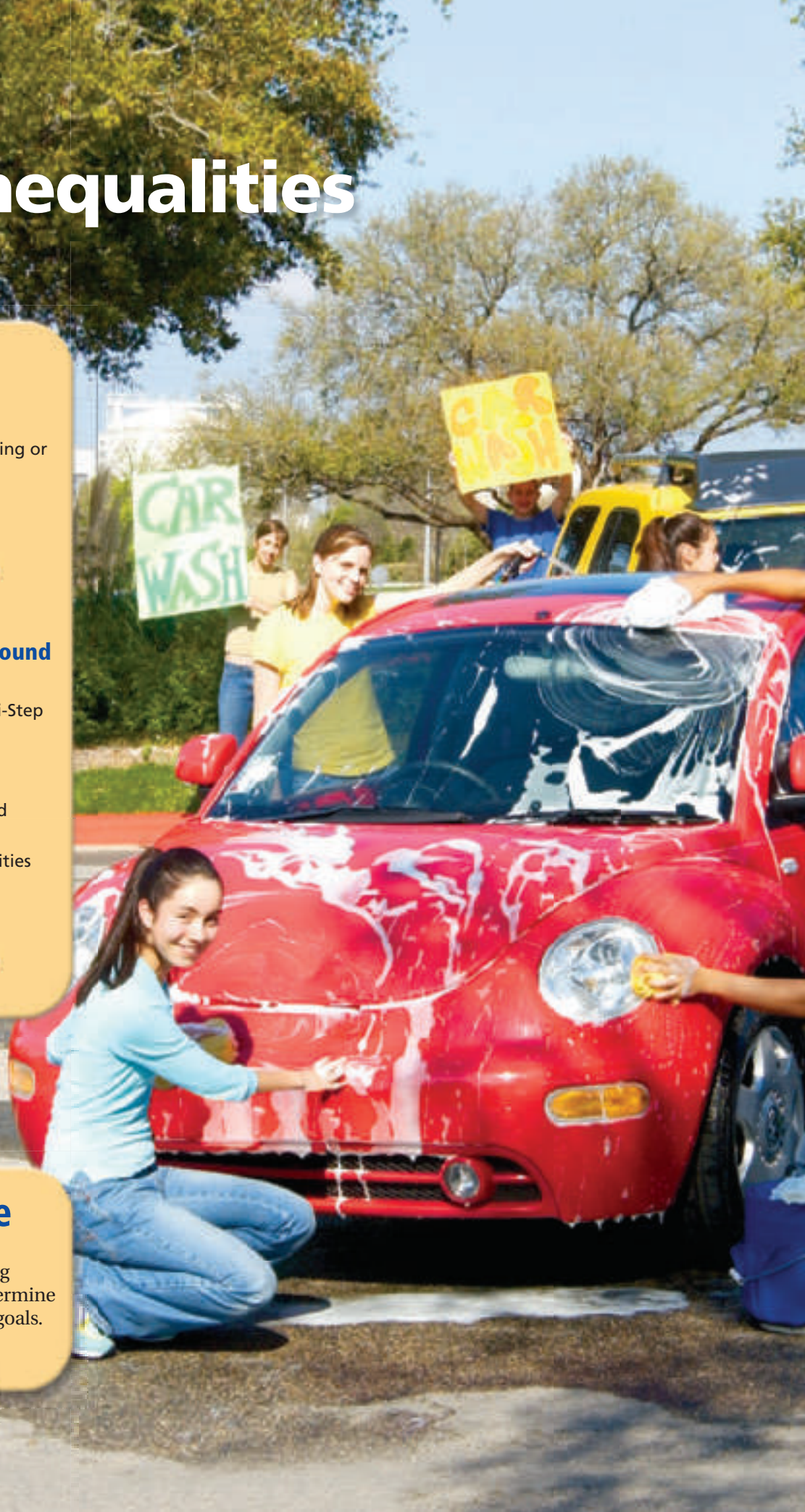# Inequalities

For a *Good* Cause

You can use the concepts in this chapter to plan for a fund-raising event. Inequalities help you determine how to reach your fund-raising goals.

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ARE YOU READY?

✓ Vocabulary

Match each term on the left with a definition on the right.

1. equation
2. evaluate
3. inverse operations
4. like terms
5. solution of an equation

A. mathematical phrase that contains operations, numbers, and/or variables

B. mathematical statement that two expressions are equivalent

C. value of a variable that makes a statement true

D. terms that contain the same variable raised to the same power

E. to find the value of an expression

F. operations that "undo" each other

✓ Evaluate Expressions

Evaluate each expression for $a = 2$ and $b = 6$.

6. $b - a$
7. ab
8. $b \div a$
9. $a + b$

✓ Compare and Order Real Numbers

Compare. Write $<$, $>$, or $=$.

10. $10 \;\rule{1em}{0.8em}\; 21$
11. $5.27 \;\rule{1em}{0.8em}\; 5.23$
12. $20\% \;\rule{1em}{0.8em}\; 0.2$
13. $\frac{1}{3} \;\rule{1em}{0.8em}\; \frac{2}{5}$

✓ Combine Like Terms

Simplify each expression by combining like terms.

14. $6x + x$
15. $-8a + 3a$
16. $9x^2 - 15x^2$
17. $2.1x + 4.3x$

✓ Distributive Property

Simplify each expression.

18. $2(x + 3)$
19. $(3 - d)5$
20. $4(r - 1)$
21. $3(4 + m)$

✓ Solve One-Step Equations

Solve.

22. $s - 3 = 8$
23. $-7x = 21$
24. $y + 11 = 2$
25. $\frac{h}{2} = 6$
26. $t + 2 = -2$
27. $6x = 42$
28. $r - 8 = -13$
29. $\frac{y}{3} = -12$

Study Guide: Preview

Where You've Been

Previously, you
- learned the properties of equality.
- solved equations by using inverse operations.
- solved equations with variables on both sides.

In This Chapter

You will study
- the properties of inequality.
- how to solve inequalities by using inverse operations.
- how to solve inequalities with variables on both sides.
- how to solve compound inequalities.

Where You're Going

You can use the skills in this chapter
- in all your future math classes, including Geometry.
- in other classes, such as Health, Chemistry, Physics, and Economics.
- in the real world to plan a budget, to find cost-efficient services, and to set financial goals.

Key Vocabulary/Vocabulario

compound inequality	desigualdad compuesta
inequality	desigualdad
intersection	intersección
solution of an inequality	solución de una desigualdad
union	unión

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The prefix *in-* means "not." An *equality* states that two things are equal. Use these meanings to write your own definition for the word **inequality**.

2. The word *compound* means "consisting of two or more parts." What do you think a **compound inequality** might be?

3. The **intersection** of two roads is the place where the two roads overlap. What do you think the *intersection* of two graphs would be?

4. The word **union** begins with the root *uni-*. List some other words that begin with *uni-*. What do all of these words have in common?

Reading and Writing Math

Study Strategy: Use Your Notes Effectively

Taking notes helps you arrange, organize, and process information from your textbook and class lectures. In addition to taking notes, you need to use your notes before and after class effectively.

Step 1: Before Class
- Review your notes from the last class.
- Then preview the next lesson and write down any questions you have.

Step 2: During Class
- Write down main ideas.
- If you miss something, leave a blank and keep taking notes. Fill in any holes later.
- Use diagrams and abbreviations. Make sure you will understand any abbreviations later.

Step 3: After Class
- Fill in the holes you left during class.
- Highlight or circle the most important ideas, such as vocabulary, formulas, or procedures.
- Use your notes to quiz yourself.
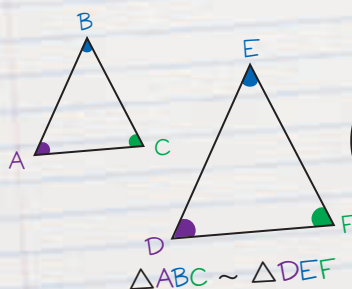
10/3   Lesson 2-7   Applications of Proportion

How do I know if figures are similar?

Similar figures—same shape, but maybe not same size

Corresponding sides and angles—same relative position.

Similar figures if corr. sides are proportional and corr. angles are same.

Use the symbol ~ to show figures are sim.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$m\angle A \cong m\angle D$$
$$m\angle B \cong m\angle E$$
$$m\angle C \cong m\angle F$$

$\triangle ABC \sim \triangle DEF$

Try This

1. Look at the next lesson in your textbook. Write down some questions you have about the material in that lesson. Leave space between each question so that you can write the answers during the next class.

2. Look at the notes you took during the last class. List three ways you can improve your note-taking skills.

Graphing and Writing Inequalities

Who uses this?
Members of a crew team can use inequalities to be sure they fall within a range of weights. (See Example 4.)

The athletes on a lightweight crew team must weigh 165 pounds or less. The acceptable weights for these athletes can be described using an *inequality*.

An **inequality** is a statement that two quantities are not equal. The quantities are compared by using one of the following signs:

$<$	$>$	$\leq$	$\geq$	$\neq$
$A < B$	$A > B$	$A \leq B$	$A \geq B$	$A \neq B$
A is less than *B*.	*A* is greater than *B*.	*A* is less than or equal to *B*.	*A* is greater than or equal to *B*.	*A* is not equal to *B*.

A **solution of an inequality** is any value that makes the inequality true.

EXAMPLE **1** **Identifying Solutions of Inequalities**

Describe the solutions of $3 + x < 9$ in words.

Test values of x that are positive, negative, and 0.

x	-2.75	0	5.99	6	6.01	6.1
$3 + x$	0.25	3	8.99	9	9.01	9.1
$3 + x \overset{?}{<} 9$	$0.25 \overset{?}{<} 9$	$3 \overset{?}{<} 9$	$8.99 \overset{?}{<} 9$	$9 \overset{?}{<} 9$	$9.01 \overset{?}{<} 9$	$9.1 \overset{?}{<} 9$
Solution?	Yes	Yes	Yes	No	No	No

When the value of x is a number less than 6, the value of 3 + x is less than 9. When the value of x is 6, the value of 3 + x is equal to 9. When the value of x is a number greater than 6, the value of 3 + x is greater than 9.

It appears that the solutions of $3 + x < 9$ are numbers less than 6.

Writing Math
The solutions of the inequality in Example 1 can be written in *set-builder notation* as $\{x \mid x < 6\}$, which is read as "the set of all real numbers x such that x is less than 6."

CHECK IT OUT! **1.** Describe the solutions of $2p > 8$ in words.

An inequality like $3 + x < 9$ has too many solutions to list. You can use a graph on a number line to show all the solutions.

The solutions are shaded and an arrow shows that the solutions continue past those shown on the graph. To show that an endpoint is a solution, draw a solid circle at the number. To show that an endpoint is not a solution, draw an empty circle.
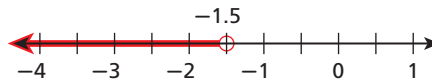
Know it! Note

Graphing Inequalities

WORDS	ALGEBRA	GRAPH
All real numbers less than 5	$x < 5$	
All real numbers greater than -1	$x > -1$	
All real numbers less than or equal to $\frac{1}{2}$	$x \le \frac{1}{2}$	
All real numbers greater than or equal to 0	$x \ge 0$	

EXAMPLE 2 **Graphing Inequalities**
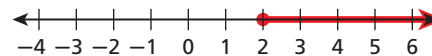
Graph each inequality.

A $b < -1.5$

Draw an empty circle at -1.5. Shade all the numbers less than -1.5 and draw an arrow pointing to the left.

B $r \ge 2$

Draw a solid circle at 2. Shade all the numbers greater than 2 and draw an arrow pointing to the right.

CHECK IT OUT! Graph each inequality.

2a. $c > 2.5$ **2b.** $2^2 - 4 \ge w$ **2c.** $m \le -3$

Student to Student

Graphing Inequalities

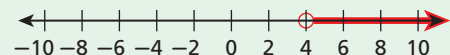*To know which direction to shade a graph, I write inequalities with the variable on the left side of the inequality symbol. I know that the symbol has to point to the same number after I rewrite the inequality.*
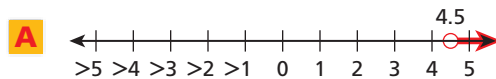
For example, I write $4 < y$ as $y > 4$.

Now the inequality symbol points in the direction that I should draw the shaded arrow on my graph.

Victor Solomos
Palmer High School

EXAMPLE 3 **Writing an Inequality from a Graph**

Write the inequality shown by each graph.

A

4.5

>5 >4 >3 >2 >1   0   1   2   3   4   5

Use any variable. The arrow points to the right, so use either > or ≥.
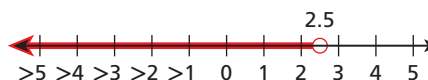*The empty circle at 4.5 means that 4.5 is not a solution, so use >.*

$h > 4.5$

B

>5 >4 >3 >2 >1   0   1   2   3   4   5
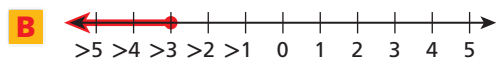
Use any variable. The arrow points to the left, so use either < or ≤.
The solid circle at −3 means that −3 is a solution, so use ≤.

$m \leq -3$

CHECK IT OUT!

3. Write the inequality shown by the graph.

2.5

>5 >4 >3 >2 >1   0   1   2   3   4   5

EXAMPLE 4 *Sports Application*

The members of a lightweight crew team can weigh no more than 165 pounds each. Define a variable and write an inequality for the acceptable weights of the team members. Graph the solutions.

Let *w* represent the weights that are allowed.

Athletes may weigh	no more than	165 pounds.
w	≤	165

Reading Math

"No more than" means "less than or equal to."

"At least" means "greater than or equal to."

$w \leq 165$

165

0   30   60   90   120   150   180
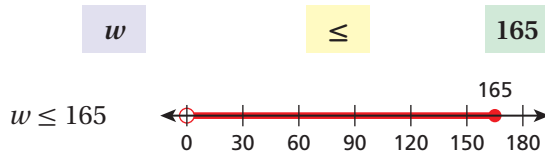
Stop the graph at 0 because a person's weight must be a positive number.

CHECK IT OUT!
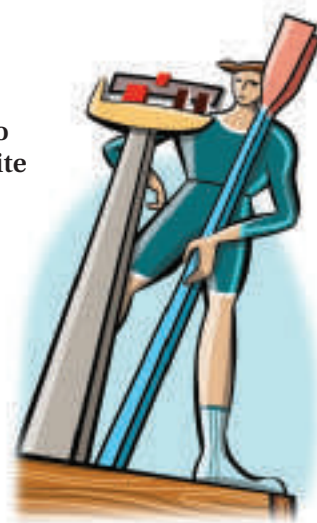
4. A store's employees earn at least $8.25 per hour. Define a variable and write an inequality for the amount the employees may earn per hour. Graph the solutions.

THINK AND DISCUSS

1. Compare the solutions of $x > 2$ and $x \geq 2$.

2. GET ORGANIZED Copy and complete the graphic organizer. Draw a graph in the first row and write the correct inequality in the second row.

Know it! Note

Inequality	Graph
x 1	
	>5 >4 >3 >2 >1 0 1

3-1 **Exercises**

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KEYWORD: MA7 Parent

GUIDED PRACTICE

1. Vocabulary How is a *solution of an inequality* like a solution of an equation?

SEE EXAMPLE **1**
p. 168

Describe the solutions of each inequality in words.

2. $g - 5 \geq 6$ **3.** $-2 < h + 1$ **4.** $20 > 5t$ **5.** $5 - x \leq 2$

SEE EXAMPLE **2**
p. 169

Graph each inequality.

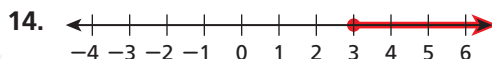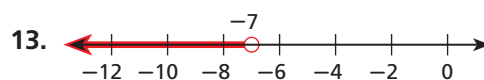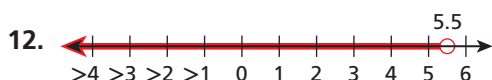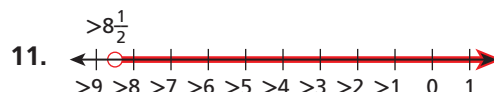**6.** $x < -5$ **7.** $c \geq 3\frac{1}{2}$ **8.** $(4 - 2)^3 > m$ **9.** $p \geq \sqrt{17 + 8}$

SEE EXAMPLE **3**
p. 170

Write the inequality shown by each graph.

10.

11.

12.

13.

14.

15.

SEE EXAMPLE **4**
p. 170

Define a variable and write an inequality for each situation. Graph the solutions.

16. There must be at least 20 club members present in order to hold a meeting.

17. A trainer advises an athlete to keep his heart rate under 140 beats per minute.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
18–21	1
22–25	2
26–31	3
32–33	4

Extra Practice
Skills Practice p. S8
Application Practice p. S30

Describe the solutions of each inequality in words.

18. $-2t > -8$ **19.** $0 > w - 2$ **20.** $3k > 9$ **21.** $\frac{1}{2}b \leq 6$

Graph each inequality.

22. $7 < x$ **23.** $t \leq -\frac{1}{2}$ **24.** $d > 4(5 - 8)$ **25.** $t \leq 3^2 - 2^2$

Write the inequality shown by each graph.

26.

27.

28.

29.

30.

31.
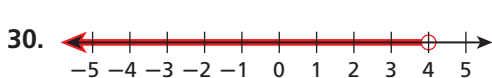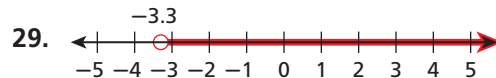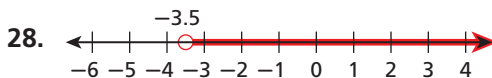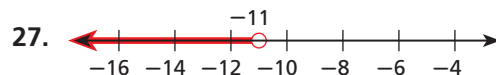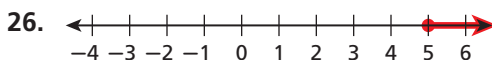
Define a variable and write an inequality for each situation. Graph the solutions.

32. The maximum speed allowed on Main Street is 25 miles per hour.

33. Applicants must have at least 5 years of experience.

Write each inequality in words.

34. $x > 7$ **35.** $h < -5$ **36.** $d \leq 23$ **37.** $r \geq -2$

Write each inequality with the variable on the left. Graph the solutions.

38. $19 < g$ **39.** $17 \geq p$ **40.** $10 < e$ **41.** $0 < f$

Define a variable and write an inequality for each situation. Graph the solutions.

42. The highest temperature ever recorded on Earth was 135.9°F at Al Aziziyah, Libya, on September 13, 1922.

43. Businesses with profits less than $10,000 per year will be shut down.

44. You must be at least 46 inches tall to ride the Indiana Jones Adventure ride at Disney's California Adventure Park.

45. Due to a medical condition, a hiker can hike only in areas with an elevation no more than 5000 feet above sea level.

Write a real-world situation that could be described by each inequality.

46. $x \geq 0$ **47.** $x < 10$ **48.** $x \leq 12$ **49.** $x > 8.5$

Match each inequality with its graph.

50. $x \geq 5$ **A.**

51. $x < 5$ **B.**

52. $x > 5$ **C.**

53. $x \leq 5$ **D.**
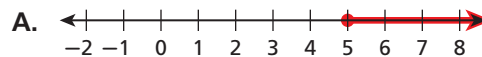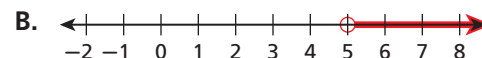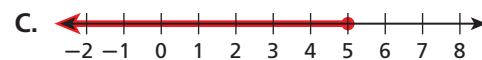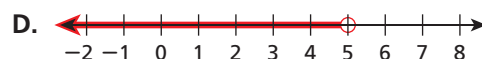
54. /// **ERROR ANALYSIS** /// Two students graphed the inequality $4 > b$. Which graph is incorrect? Explain the error.
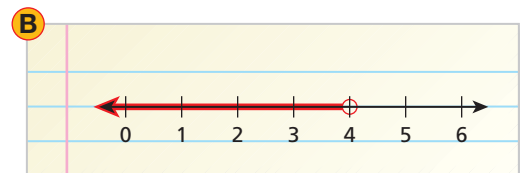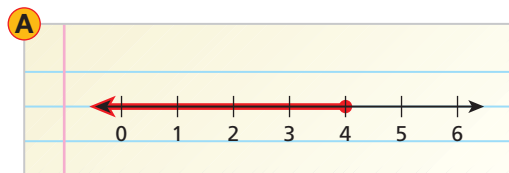
55. This problem will prepare you for the Multi-Step Test Prep on page 186.

     a. Mirna earned $125 baby-sitting during the spring break. She needs to save $90 for the German Club trip. She wants to spend the remainder of the money shopping. Write an inequality to show how much she can spend.

     b. Graph the inequality you wrote in part **a.**

     c. Mirna spends $15 on a bracelet. Write an inequality to show how much money she has left to spend.

56. Critical Thinking Graph all positive integer solutions of the inequality $x < 5$.

57. Write About It Explain how to write an inequality that is modeled by a graph. What characteristics do you look for in the graph?

58. Write About It You were told in the lesson that the phrase "no more than" means "less than or equal to" and the phrase "at least" means "greater than or equal to."

  a. What does the phrase "at most" mean?

  b. What does the phrase "no less than" mean?

59. Which is NOT a solution of the inequality $5 - 2x \geq -3$?

  (A) 0   (B) 2   (C) 4   (D) 5

60. Which is NOT a solution of the inequality $3 - x < 2$?

  (F) 1   (G) 2   (H) 3   (J) 4

61. Which graph represents the solutions of $-2 \leq 1 - t$?
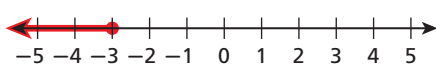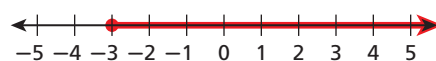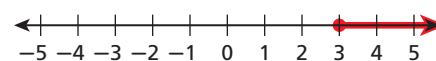
CHALLENGE AND EXTEND

Give a value for x and a value for y that make each inequality true.

62. $x + y \leq |x + y|$ **63.** $x^2 < xy$ **64.** $x - y \geq y - x$

Complete each statement. Write $<$ or $>$.

65. If $a > b$, then b ⬜ a.   **66.** If $x > y$ and $y > z$, then x ⬜ z.

67. Name a value of x that makes the statement $0.35 < x < 1.27$ true.

68. Is $\frac{5}{6}$ a solution of $x < 1$? How many solutions of $x < 1$ are between 0 and 1?

69. Write About It Explain how to graph all the solutions of $x \neq 5$.

SPIRAL REVIEW

Add or subtract. *(Lesson 1-2)*

70. $-7 + 5$ **71.** $6 - (-4)$ **72.** $8 - 13$ **73.** $12 + (-5)$

Simplify each expression. *(Lesson 1-7)*

74. $x + 3x$ **75.** $x + (x + 1) + (x + 2)$ **76.** $5 + (x + 3) + 5 + 2(x + 3)$

77. There are twice as many girls in Sally's class as boys. Write a rule for the number of girls in Sally's class. Find the number of girls if there are 8 boys. *(Lesson 1-8)*

78. A video club charges a \$12 membership fee plus \$2.00 for each movie rental. Write a rule for the cost of renting x videos. Find the cost of renting 3, 7, and 15 videos. *(Lesson 1-8)*

Solve each equation. Check your answer. *(Lesson 2-4)*

79. $2b - 6 = b + 3$ **80.** $-3(2 - x) = 5x + 2$ **81.** $2(y + 1) = 2y + 1$

3-2 Solving Inequalities by Adding or Subtracting

Objectives

Solve one-step inequalities by using addition.

Solve one-step inequalities by using subtraction.

Who uses this?

You can use inequalities to determine how many more photos you can take. (See Example 2.)

Tenea has a cell phone that also takes pictures. After taking some photos, Tenea can use a one-step inequality to determine how many more photos she can take.

Solving one-step inequalities is much like solving one-step equations. To solve an inequality, you need to isolate the variable using the properties of inequality and inverse operations.

Know it! Note

Properties of Inequality

Addition and Subtraction

WORDS	NUMBERS	ALGEBRA
Addition You can add the same number to both sides of an inequality, and the statement will still be true.	$3 < 8$ $3 + 2 < 8 + 2$ $5 < 10$	$a < b$ $a + c < b + c$
Subtraction You can subtract the same number from both sides of an inequality, and the statement will still be true.	$9 < 12$ $9 - 5 < 12 - 5$ $4 < 7$	$a < b$ $a - c < b - c$

These properties are also true for inequalities that use the symbols $>$, $\geq$, and $\leq$.

EXAMPLE 1 Using Addition and Subtraction to Solve Inequalities

Solve each inequality and graph the solutions.

Helpful Hint

Use an inverse operation to "undo" the operation in an inequality. If the inequality contains addition, use subtraction to undo the addition.

A $x + 9 < 15$

$x + 9 < 15$
$\underline{\quad -9 \quad -9}$
$x + 0 < \quad 6$
$\quad\quad x < 6$

Since 9 is added to x, subtract 9 from both sides to undo the addition.

B $d - 3 > -6$

$d - 3 > -6$
$\underline{\quad +3 \quad +3}$
$d + 0 > -3$
$\quad\quad d > -3$

Since 3 is subtracted from d, add 3 to both sides to undo the subtraction.

Solve each inequality and graph the solutions.

C $0.7 \geq n - 0.4$

$\quad\quad 0.7 \geq n - 0.4$ — *Since 0.4 is subtracted from n, add 0.4 to both sides to undo the subtraction.*

$\quad\quad \underline{+\,0.4 \quad\quad +\,0.4}$

$\quad\quad 1.1 \geq n - 0$

$\quad\quad n \leq 1.1$

$$\xleftarrow{\hspace{3cm}}\bullet\!\!\xrightarrow{\hspace{1cm}}$$
$-5\ -4\ -3\ -2\ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5$ (point at 1.1)
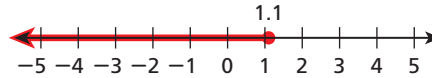
CHECK IT OUT! Solve each inequality and graph the solutions.

1a. $s + 1 \leq 10$ **1b.** $2\frac{1}{2} > -3 + t$ **1c.** $q - 3.5 < 7.5$

Since there can be an infinite number of solutions to an inequality, it is not possible to check all the solutions. You can check the endpoint and the direction of the inequality symbol.

The solutions of $x + 9 < 15$ are given by $x < 6$.

Step 1 Check the endpoint.

Substitute 6 for x in the related equation $x + 9 = 15$. The endpoint should be a solution of the equation.

$$x + 9 = 15$$
$$\frac{6 + 9 \quad | \quad 15}{15 \quad | \quad 15\ \checkmark}$$

Step 2 Check the inequality symbol.

Substitute a number less than 6 for x in the original inequality. The number you choose should be a solution of the inequality.

$$x + 9 < 15$$
$$\frac{4 + 9 \ | < | \ 15}{13 \ | < | \ 15\ \checkmark}$$

EXAMPLE 2 *Problem Solving Application*

PROBLEM SOLVING

The memory in Tenea's camera phone allows her to take up to 20 pictures. Tenea has already taken 16 pictures. Write, solve, and graph an inequality to show how many more pictures Tenea could take.

1 Understand the Problem

The **answer** will be an inequality and a graph that show all the possible numbers of pictures that Tenea can take.

List the important information:
- Tenea can take up to, or *at most*, 20 pictures.
- Tenea has taken 16 pictures already.

2 Make a Plan

Write an inequality.
Let p represent the remaining number of pictures Tenea can take.

Number taken	plus	number remaining	is at most	20 pictures.
16	+	p	$\leq$	20

3 Solve

$$16 + p \le 20$$

$\underline{-16 \qquad -16}$ *Since 16 is added to p, subtract 16 from both sides to undo the addition.*

$$p \le 4$$

It is not reasonable for Tenea to take a negative or fractional number of pictures, so graph the nonnegative integers less than or equal to 4.

Tenea could take 0, 1, 2, 3, or 4 more pictures.

4 Look Back

Check   Check the endpoint, 4.   Check a number less than 4.

$16 + p = 20$		$16 + p \le 20$	
$16 + 4$	20	$16 + 2$ $\le$	20
20	20 ✓	18 $\le$	20 ✓

Adding 0, 1, 2, 3, or 4 more pictures will not exceed 20.

CHECK IT OUT! 2. The Recommended Daily Allowance (RDA) of iron for a female in Sarah's age group (14–18 years) is 15 mg per day. Sarah has consumed 11 mg of iron today. Write and solve an inequality to show how many more milligrams of iron Sarah can consume without exceeding the RDA.

EXAMPLE 3   *Sports Application*

Josh can bench press 220 pounds. He wants to bench press at least 250 pounds. Write and solve an inequality to determine how many more pounds Josh must lift to reach his goal. Check your answer.

Let p represent the number of additional pounds Josh must lift.

220 pounds	plus	additional pounds	is at least	250 pounds.
220	+	p	$\ge$	250

$$220 + p \ge 250$$

$\underline{-220 \qquad -220}$ *Since 220 is added to p, subtract 220 from both sides to undo the addition.*

$$p \ge 30$$

Check   Check the endpoint, 30.   Check a number greater than 30.

| $220 + p = 250$		$220 + p \ge 250$	
$220 + 30$	250	$220 + 40$ $\ge$	250
250	250 ✓	260 $\ge$	250 ✓

Josh must lift at least 30 additional pounds to reach his goal.

CHECK IT OUT! 3. **What If...?** Josh wants to try to break the school record of 282 pounds. Write and solve an inequality to determine how many more pounds Josh needs to break the school record. Check your answer.

THINK AND DISCUSS

1. Show how to check your solution to Example 1B.

2. Explain how the Addition and Subtraction Properties of Inequality are like the Addition and Subtraction Properties of Equality.

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an inequality that you must use the specified property to solve. Then solve and graph the inequality.

Properties of Inequality

Addition   Subtraction
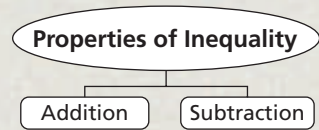
3-2 Exercises
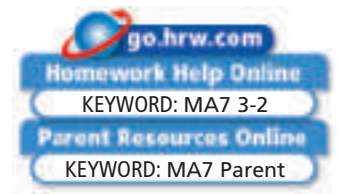
GUIDED PRACTICE

SEE EXAMPLE 1
p. 174

Solve each inequality and graph the solutions.

1. $12 < p + 6$ **2.** $w + 3 \geq 4$ **3.** $-5 + x \leq -20$ **4.** $z - 2 > -11$

SEE EXAMPLE 2
p. 175

5. **Health** For adults, the maximum safe water temperature in a spa is 104°F. The water temperature in Bill's spa is 102°F. The temperature is increased by t°F. Write, solve, and graph an inequality to show the values of t for which the water temperature is still safe.

SEE EXAMPLE 3
p. 176

6. **Consumer Economics** A local restaurant will deliver food to your house if the purchase amount of your order is at least $25.00. The total for part of your order is $17.95. Write and solve an inequality to determine how much more you must spend for the restaurant to deliver your order.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
7–10	1
11	2
12	3

Extra Practice
Skills Practice p. S8
Application Practice p. S30

Solve each inequality and graph the solutions.

7. $a - 3 \geq 2$ **8.** $2.5 > q - 0.8$ **9.** $-45 + x < -30$ **10.** $r + \dfrac{1}{4} \leq \dfrac{3}{4}$

11. **Engineering** The maximum load for a certain elevator is 2000 pounds. The total weight of the passengers on the elevator is 1400 pounds. A delivery man who weighs 243 pounds enters the elevator with a crate of weight w. Write, solve, and graph an inequality to show the values of w that will not exceed the weight limit of the elevator.

12. **Transportation** The gas tank in Mindy's car holds at most 15 gallons. She has already filled the tank with 7 gallons of gas. She will continue to fill the tank with g gallons more. Write and solve an inequality that shows all values of g that Mindy can add to the car's tank.

Write an inequality to represent each statement. Solve the inequality and graph the solutions.

13. Ten less than a number x is greater than 32.

14. A number n increased by 6 is less than or equal to 4.

15. A number r decreased by 13 is at most 15.

Solve each inequality and graph the solutions.

16. $x + 4 \le 2$ **17.** $-12 + q > 39$ **18.** $x + \dfrac{3}{5} < 7$ **19.** $4.8 \ge p + 4$

20. $-12 \le x - 12$ **21.** $4 < 206 + c$ **22.** $y - \dfrac{1}{3} > \dfrac{2}{3}$ **23.** $x + 1.4 \ge 1.4$

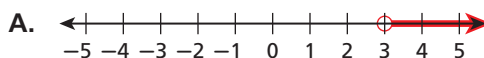**24.** Use the inequality $s + 12 \ge 20$ to fill in the missing numbers.

     a. $s \ge \blacksquare$ **b.** $s + \blacksquare \ge 30$ **c.** $s - 8 \ge \blacksquare$

25. **Health** A particular type of contact lens can be worn up to 30 days in a row. Alex has been wearing these contact lenses for 21 days. Write, solve, and graph an inequality to show how many more days Alex could wear his contact lenses.

Special-effects contact lenses are sometimes part of costumes for movies. All contact lenses should be worn under an eye doctor's supervision.

Solve each inequality and match the solution to the correct graph.

26. $1 \le x - 2$ **A.**

27. $8 > x - (-5)$ **B.**
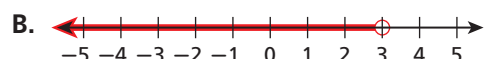
28. $x + 6 > 9$ **C.**
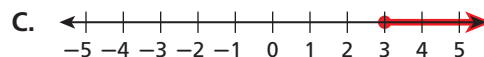
29. $-4 \ge x - 7$ **D.**
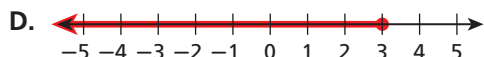
30. **Estimation** Is $x < 10$ a reasonable estimate for the solutions to the inequality $11.879 + x < 21.709$? Explain your answer.

31. **Sports** At the Seattle Mariners baseball team's home games, there are 45,611 seats in the four areas listed in the table. Suppose all the suite level and club level seats during a game are filled. Write and solve an inequality to determine how many people p could be sitting in the other types of seats.

Mariners Home Game Seating	
Type of Seat	**Number of Seats**
Main bowl	24,399
Upper bowl	16,022
Club level	4,254
Suite level	936

32. **Critical Thinking** Recall that in Chapter 2 a balance scale was used to model solving equations. Describe how a balance scale could model solving inequalities.

33. **Critical Thinking** Explain why $x + 4 \ge 6$ and $x - 4 \ge -2$ have the same solutions.

34. **Write About It** How do the solutions of $x + 2 \ge 3$ differ from the solutions of $x + 2 > 3$? How do the graphs of the solutions differ?

MULTI-STEP TEST PREP

35. This problem will prepare you for the Multi-Step Test Prep on page 186.

     a. Daryl finds that the distance from Columbus, Ohio, to Washington, D.C, is 411 miles. What is the round-trip distance?

     b. Daryl can afford to drive a total of 1000 miles. Write an inequality to show the number of miles m he can drive while in Washington, D.C.

     c. Solve the inequality and graph the solutions on a number line. Show that your answer is reasonable.

36. Which is a reasonable solution of $4.7367 + p < 20.1784$?

   Ⓐ 15   Ⓑ 16   Ⓒ 24   Ⓓ 25

37. Which statement can be modeled by $x + 3 \leq 12$?

   Ⓕ Sam has 3 bottles of water. Together, Sam and Dave have at most 12 bottles of water.

   Ⓖ Jennie sold 3 cookbooks. To earn a prize, Jennie must sell at least 12 cookbooks.

   Ⓗ Peter has 3 baseball hats. Peter and his brothers have fewer than 12 baseball hats.

   Ⓙ Kathy swam 3 laps in the pool this week. She must swim more than 12 laps.

38. Which graph represents the solutions of $p + 3 < 1$?
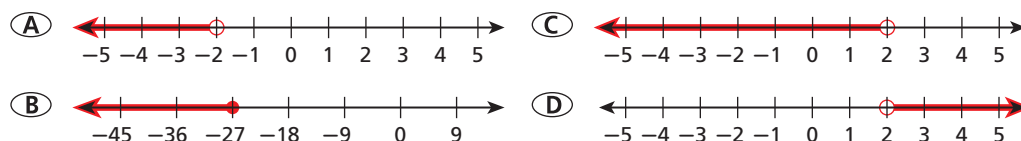
39. Which inequality does NOT have the same solutions as $n + 12 \leq 26$?

   Ⓕ $n \leq 14$ Ⓖ $n + 6 \leq 20$ Ⓗ $10 \geq n - 4$ Ⓙ $n - 12 \leq 14$

CHALLENGE AND EXTEND

Solve each inequality and graph the solutions.

40. $6\frac{9}{10} \geq 4\frac{4}{5} + x$ **41.** $r - 1\frac{2}{5} \leq 3\frac{7}{10}$ **42.** $6\frac{2}{3} + m > 7\frac{1}{6}$

Determine whether each statement is *sometimes, always,* or *never* true. Explain.

43. $a + b > a - b$

44. If $a > c$, then $a + b > c + b$.

45. If $a > b$ and $c > d$, then $a + c > b + d$.

46. If $x + b > c$ and $x > 0$ have the same solutions, what is the relationship between b and c?

SPIRAL REVIEW

Solve each equation for the indicated variable. *(Lesson 2-5)*

47. $2x + 3y = 9$ for y **48.** $P = 4s$ for s **49.** $2a + ab = c$ for a

50. $p + e = f$ for e **51.** $2s - k = 11$ for k **52.** $5m + n = 0$ for m

Find the value of x in each diagram of similar figures. *(Lesson 2-7)*

53.   **54.**
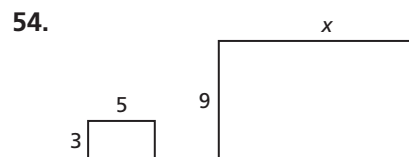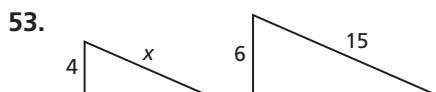
Write the inequality shown by each graph. *(Lesson 3-1)*
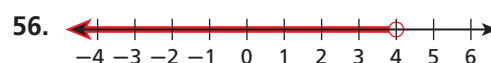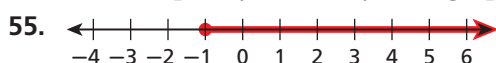
55.   **56.**

3-3 Solving Inequalities by Multiplying or Dividing

Who uses this?
You can solve an inequality to determine how much you can buy with a certain amount of money. (See Example 3.)

Remember, solving inequalities is similar to solving equations. To solve an inequality that contains multiplication or division, undo the operation by dividing or multiplying both sides of the inequality by the same number.

The rules below show the properties of inequality for multiplying or dividing by a positive number. The rules for multiplying or dividing by a negative number appear later in this lesson.

"This is all I have, so I'll take 3 pencils, 3 notebooks, a binder, and 0.9 calculators."

Jonny Hawkins/CartoonResource.com

Properties of Inequality

Know it!
Note

Multiplication and Division by Positive Numbers

WORDS	NUMBERS	ALGEBRA
Multiplication You can multiply both sides of an inequality by the same *positive* number, and the statement will still be true.	$7 < 12$ $7(3) < 12\,(3)$ $21 < 36$	If $a < b$ and $c > 0$, then $ac < bc$.
Division You can divide both sides of an inequality by the same *positive* number, and the statement will still be true.	$15 < 35$ $\dfrac{15}{5} < \dfrac{35}{5}$ $3 < 7$	If $a < b$ and $c > 0$, then $\dfrac{a}{c} < \dfrac{b}{c}$.

These properties are also true for inequalities that use the symbols $>$, $\geq$, and $\leq$.

E X A M P L E   1   **Multiplying or Dividing by a Positive Number**

Solve each inequality and graph the solutions.

A $3x > -27$

$3x > -27$

$\dfrac{3x}{3} > \dfrac{-27}{3}$ *Since x is multiplied by 3, divide both sides by 3 to undo the multiplication.*

$1x > -9$

$x > -9$

-9

−10 −8 −6 −4 −2 0 2 4 6 8 10

Solve each inequality and graph the solutions.

B $\frac{2}{3}r < 6$

$\frac{2}{3}r < 6$ *Since r is multiplied by $\frac{2}{3}$, multiply both sides by the reciprocal of $\frac{2}{3}$.*

$\frac{3}{2}\left(\frac{2}{3}r\right) < \frac{3}{2}(6)$

$r < 9$
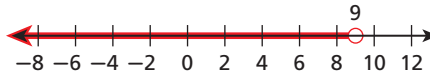
Solve each inequality and graph the solutions.

1a. $4k > 24$ **1b.** $-50 \geq 5q$ **1c.** $\frac{3}{4}g > 27$

If you multiply or divide both sides of an inequality by a negative number, the resulting inequality is not a true statement. You need to reverse the inequality symbol to make the statement true.

$5 > -3$ *5 is **greater than** -3.*

$5(-2) \quad -3(-2)$ *Multiply both sides by -2.*

$-10 \quad 6$ *You know that -10 is less than 6, so use the symbol for **less than.***

$-10 < 6$
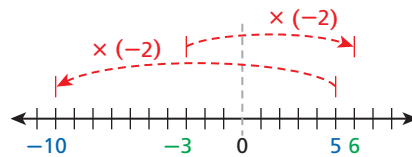
Multiplying both sides by a negative number changes the sign of both sides of the inequality.

This means there is another set of properties of inequality for multiplying or dividing by a negative number.

Properties of Inequality

Multiplication and Division by Negative Numbers

WORDS	NUMBERS	ALGEBRA
Multiplication		
If you multiply both sides of an inequality by the same *negative* number, you must reverse the inequality symbol for the statement to still be true.	$8 > 4$ $8(-2) < 4(-2)$ $-16 < -8$ $-16 < -8$	If $a > b$ and $c < 0$, then $ac < bc$.
Division		
If you divide both sides of an inequality by the same *negative* number, you must reverse the inequality symbol for the statement to still be true.	$12 > 4$ $\frac{12}{-4} < \frac{4}{-4}$ $-3 < -1$ $-3 < -1$	If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.
These properties are also true for inequalities that use the symbols $<$, $\geq$, and $\leq$.		

EXAMPLE 2 Multiplying or Dividing by a Negative Number

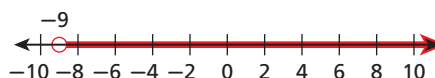Solve each inequality and graph the solutions.

A $-8x > 72$

$$\dfrac{-8x}{-8} < \dfrac{72}{-8}$$

Since x is multiplied by −8, divide both sides by −8. Change > to <.

$$x < -9$$
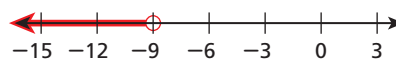
B $-3 \le \dfrac{x}{-5}$

$$-5(-3) \ge -5\left(\dfrac{x}{-5}\right)$$

Since x is divided by −5, multiply both sides by −5. Change ≤ to ≥.

$$15 \ge x \;(\text{or } x \le 15)$$

CHECK IT OUT! Solve each inequality and graph the solutions.

2a. $10 \ge -x$ **2b.** $4.25 > -0.25h$

EXAMPLE 3 *Consumer Application*

Ryan has a $16 gift card for a health store where a smoothie costs $2.50 with tax. What are the possible numbers of smoothies that Ryan can buy?

Let s represent the number of smoothies Ryan can buy.

$2.50	times	number of smoothies	is at most	$16.00.
2.50	•	s	≤	16.00

$$2.50s \le 16.00$$

$$\dfrac{2.50s}{2.50} \le \dfrac{16.00}{2.50}$$

Since s is multiplied by 2.50, divide both sides by 2.50. The symbol does not change.

$$s \le 6.4$$

Ryan can buy only a whole number of smoothies.

Ryan can buy 0, 1, 2, 3, 4, 5, or 6 smoothies.

CHECK IT OUT! **3.** A pitcher holds 128 ounces of juice. What are the possible numbers of 10-ounce servings that one pitcher can fill?

THINK AND DISCUSS

1. Compare the Multiplication and Division Properties of Inequality and the Multiplication and Division Properties of Equality.

2. GET ORGANIZED Copy and complete the graphic organizer. In each cell, write and solve an inequality.

Solving Inequalities by Using Multiplication and Division		
	By a Positive Number	**By a Negative Number**
Divide		
Multiply		

3-3 **Exercises**

go.hrw.com
Homework Help Online
KEYWORD: MA7 3-3
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1
p. 180

Solve each inequality and graph the solutions.

1. $3b > 27$ **2.** $-40 \geq 8b$ **3.** $\dfrac{d}{3} > 6$ **4.** $24d \leq 6$

5. $1.1m \leq 1.21$ **6.** $\dfrac{2}{3}k > 6$ **7.** $9s > -18$ **8.** $\dfrac{4}{5} \geq \dfrac{r}{2}$

SEE EXAMPLE 2
p. 182

9. $-2x < -10$ **10.** $\dfrac{b}{-2} \geq 8$ **11.** $-3.5n < 1.4$ **12.** $4 > -8g$

13. $\dfrac{d}{-6} < \dfrac{1}{2}$ **14.** $-10h \geq -6$ **15.** $12 > \dfrac{t}{-6}$ **16.** $-\dfrac{1}{2}m \geq -7$

SEE EXAMPLE 3
p. 182

17. Travel Tom saved $550 to go on a school trip. The cost for a hotel room, including tax, is $80 per night. Write an inequality to show the number of nights Tom can stay at the hotel.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
18–29	1
30–41	2
42	3

Extra Practice
Skills Practice p. S8
Application Practice p. S30

Solve each inequality and graph the solutions.

18. $10 < 2t$ **19.** $\dfrac{1}{3}j \leq 4$ **20.** $-80 < 8c$ **21.** $21 > 3d$

22. $\dfrac{w}{4} \geq -2$ **23.** $\dfrac{h}{4} \leq \dfrac{2}{7}$ **24.** $6y < 4.2$ **25.** $12c \leq -144$

26. $\dfrac{4}{5}x \geq \dfrac{2}{5}$ **27.** $6b \geq \dfrac{3}{5}$ **28.** $-25 > 10p$ **29.** $\dfrac{b}{8} \leq -2$

30. $-9a > 81$ **31.** $\dfrac{1}{2} < \dfrac{r}{-3}$ **32.** $-6p > 0.6$ **33.** $\dfrac{y}{-4} > -\dfrac{1}{2}$

34. $-\dfrac{1}{6}f < 5$ **35.** $-2.25t < -9$ **36.** $24 \leq -10w$ **37.** $-11z > 121$

38. $\dfrac{3}{5} < \dfrac{f}{-5}$ **39.** $-k \geq 7$ **40.** $-2.2b < -7.7$ **41.** $16 \geq -\dfrac{4}{3}p$

42. Camping The rope Roz brought with her camping gear is 54 inches long. Roz needs to cut shorter pieces of rope that are each 18 inches long. What are the possible number of pieces Roz can cut?

Solve each inequality and graph the solutions.

43. $-8x < 24$ **44.** $3t \leq 24$ **45.** $\dfrac{1}{4}x < 5$ **46.** $\dfrac{4}{5}p \geq -24$

47. $54 \leq -9p$ **48.** $3t > -\dfrac{1}{2}$ **49.** $-\dfrac{3}{4}b > -\dfrac{3}{2}$ **50.** $216 > 3.6r$

Write an inequality for each statement. Solve the inequality and graph the solutions.

51. The product of a number and 7 is not less than 21.

52. The quotient of h and -6 is at least 5.

53. The product of $-\dfrac{4}{5}$ and b is at most -16.

54. Ten is no more than the quotient of t and 4.

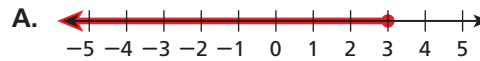**55. Write About It** Explain how you know whether to reverse the inequality symbol when solving an inequality.

56. Geometry The area of a rectangle is at most 21 square inches. The width of the rectangle is 3.5 inches. What are the possible measurements for the length of the rectangle?
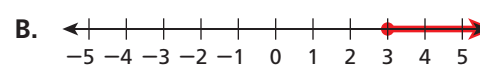
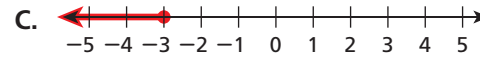**Solve each inequality and match the solution to the correct graph.**
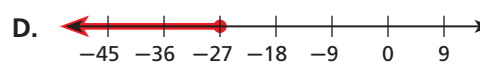
57. $-0.5t \geq 1.5$

A.

$$\longleftarrow \;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;\bullet\;\;|\;\;|\;\;|\longrightarrow$$
$$-5\;\;-4\;\;-3\;\;-2\;\;-1\;\;0\;\;1\;\;2\;\;3\;\;4\;\;5$$

58. $\dfrac{1}{9}t \leq -3$

B.

$$\longleftarrow\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;\bullet\;\;|\;\;|\longrightarrow$$
$$-5\;\;-4\;\;-3\;\;-2\;\;-1\;\;0\;\;1\;\;2\;\;3\;\;4\;\;5$$

59. $-13.5 \leq -4.5t$

C.

$$\longleftarrow\;\;|\;\;\bullet\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\longrightarrow$$
$$-5\;\;-4\;\;-3\;\;-2\;\;-1\;\;0\;\;1\;\;2\;\;3\;\;4\;\;5$$

60. $\dfrac{t}{-6} \leq -\dfrac{1}{2}$

D.

$$\longleftarrow\;\;|\;\;|\;\;\bullet\;\;|\;\;|\;\;|\;\;|\;\;|\longrightarrow$$
$$-45\;\;-36\;\;-27\;\;-18\;\;-9\;\;0\;\;9$$

Animals

Orangutans weigh about 3.5 pounds at birth. As adults, female orangutans can weigh as much as 110 pounds, and male orangutans can weigh up to 300 pounds.

61. **Animals** A wildlife shelter in North Carolina is home to native species of birds, mammals, and reptiles. If cat chow is sold in 20 lb bags, what is the least number of bags of cat chow needed for one year at this shelter?

Food Consumed at a Wildlife Shelter per Week	
Type of Food	**Amount of Food (lb)**
Grapes	4
Mixed seed	10
Peanuts	5
Cat chow	10
Kitten chow	5

62. **Education** In order to earn an A in a college math class, a student must score no less than 90% of all possible points. One semester, students with at least 567 points earned an A in the class. Write an inequality to show the numbers of points possible.

63. **Critical Thinking** Explain why you cannot solve an inequality by multiplying both sides by zero.

64. /// **ERROR ANALYSIS** /// Two students have different answers for a homework problem. Which answer is incorrect? Explain the error.

A

$$9m \geq -27$$
$$\frac{9m}{9} \geq \frac{-27}{9}$$
$$m \leq -3$$

B

$$9m \geq -27$$
$$\frac{9m}{9} \geq \frac{-27}{9}$$
$$m \geq -3$$

65. Jan has a budget of $800 for catering. The catering company charges $12.50 per guest. Write and solve an inequality to show the numbers of guests Jan can invite.

MULTI-STEP TEST PREP

66. This problem will prepare you for the Multi-Step Test Prep on page 186.

a. The Swimming Club can spend a total of $250 for hotel rooms for its spring trip. One hotel costs $75 per night. Write an inequality to find the number of rooms the club can reserve at this hotel. Let n be the number of rooms.

b. Solve the inequality you wrote in part **a.** Graph the solutions on a number line. Make sure your answer is reasonable.

c. Another hotel offers a rate of $65 per night. Does this allow the club to reserve more rooms? Explain your reasoning.

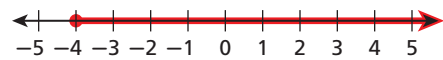**67.** Which inequality does NOT have the same solutions as $-\frac{2}{3}y > 4$?

Ⓐ $12 < -2y$ Ⓒ $-\frac{3}{4}y > \frac{9}{2}$

Ⓑ $\frac{y}{2} < -12$ Ⓓ $-3y > 18$

68. The solutions of which inequality are NOT represented by the following graph?

Ⓕ $\frac{x}{2} \geq -2$ Ⓗ $3x \geq -12$

Ⓖ $-5x \geq 20$ Ⓙ $-7x \geq 28$

69. Which inequality can be used to find the number of 39-cent stamps you can purchase for $4.00?

Ⓐ $0.39s \geq 4.00$ Ⓒ $\frac{s}{0.39} \leq 4.00$

Ⓑ $0.39s \leq 4.00$ Ⓓ $\frac{4.00}{0.39} \leq s$

70. Short Response Write three different inequalities that have the same solutions as $x > 4$. Show your work and explain each step.

CHALLENGE AND EXTEND

Solve each inequality.

71. $2\frac{1}{3} \leq -\frac{5}{6}g$ **72.** $\frac{2x}{3} < 8.25$ **73.** $2\frac{5}{8}m > \frac{7}{10}$ **74.** $3\frac{3}{5}f \geq 14\frac{2}{5}$

75. Estimation What is the greatest possible integer solution of the inequality $3.806x < 19.902$?

76. Critical Thinking The Transitive Property of Equality states that if $a = b$ and $b = c$, then $a = c$. Is there a Transitive Property of Inequality using the symbol $<$? Give an example to support your answer.

77. Critical Thinking The Symmetric Property of Equality states that if $a = b$, then $b = a$. Is there a Symmetric Property of Inequality? Give an example to support your answer.

SPIRAL REVIEW

Write the power represented by each geometric model. *(Lesson 1-4)*

78.   **79.**   **80.**
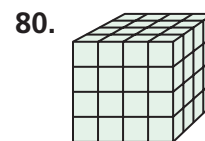
Find the unit rate. *(Lesson 2-6)*

81. Twelve gallons of gas cost $22.68.

82. A tree grows four feet in six years.

83. A student types 105 words in 3 minutes.

Solve each inequality and graph the solutions. *(Lesson 3-2)*

84. $x + 5 \geq 3$ **85.** $t - \frac{1}{4} < \frac{3}{4}$ **86.** $4 > x - 1$ **87.** $6 > b - 8$

MULTI-STEP TEST PREP

Simple Inequalities

Remember the Alamo! The Spanish Club is planning a trip for next summer. They plan to travel from Fort Worth, Texas, to San Antonio, Texas. They can spend only $550 for the entire trip.

1. The treasurer of the club budgets $60 for gasoline. The current gas price is $1.95/gallon. The school van gets an average of 20 miles per gallon of gasoline. Determine how many miles they can drive on this budget. Round your answer to the nearest mile.

2. The distance from Fort Worth to San Antonio is 266 miles. Write an inequality that can be used to solve for the number of miles m that they can drive while in San Antonio. Solve your inequality and graph the solutions.

3. The treasurer budgeted $200 for hotel rooms for one night. The club chose a hotel that charges $58 per night. Write an inequality that can be used to solve for the number of rooms they can reserve n. What is the maximum number of rooms that they can reserve in the hotel?

4. Use the maximum number of rooms you found in part **3.** How much will the club spend on hotel rooms?

5. The club members plan to spend $80 on food. They also want to see attractions in San Antonio, such as SeaWorld and the Alamo.

   Write an inequality that can be solved to find the amount of money available for seeing attractions. What is the maximum amount the club can spend seeing attractions?

6. Write a summary of the budget for the Spanish Club trip. Include the amount they plan to spend on gasoline, hotel rooms, food, and attractions.

READY TO GO ON?

Quiz for Lessons 3-1 Through 3-3

3-1 Graphing and Writing Inequalities

Describe the solutions of each inequality in words.

1. $-2 < r$ **2.** $t - 1 \le 7$ **3.** $2s \ge 6$ **4.** $4 > 5 - x$
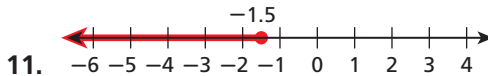
Graph each inequality.

5. $x > -2$ **6.** $m \le 1\frac{1}{2}$ **7.** $g < \sqrt{8 + 1}$ **8.** $h \ge 2^3$

Write the inequality shown by each graph.

9.

10.

11.
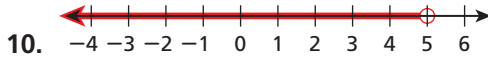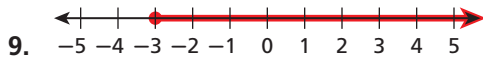
Write an inequality for each situation and graph the solutions.

12. You must purchase at least 5 tickets to receive a discount.

13. Children under 13 are not admitted to certain movies without an adult.

14. A cell phone plan allows up to 250 free minutes per month.

3-2 Solving One-Step Inequalities by Adding and Subtracting

Solve each inequality and graph the solutions.

15. $k + 5 \le 7$ **16.** $4 > p - 3$ **17.** $r - 8 \ge -12$ **18.** $-3 + p < -6$

19. Allie must sell at least 50 gift baskets for the band fund-raiser. She already sold 36 baskets. Write and solve an inequality to determine how many more baskets Allie must sell for the fund-raiser.

20. Dante has at most $12 to spend on entertainment each week. So far this week, he spent $7.50. Write and solve an inequality to determine how much money Dante can spend on entertainment the rest of the week.

3-3 Solving One-Step Inequalities by Multiplying and Dividing

Solve each inequality and graph the solutions.

21. $-4x < 8$ **22.** $\dfrac{d}{3} \ge -3$ **23.** $\dfrac{3}{4}t \le 12$ **24.** $8 > -16c$

25. A spool of ribbon is 80 inches long. Riley needs to cut strips of ribbon that are 14 inches long. What are the possible numbers of strips that Riley can cut?

3-4 Solving Two-Step and Multi-Step Inequalities

Objective
Solve inequalities that contain more than one operation.

Who uses this?
Contestants at a county fair can solve an inequality to find how many pounds a prize-winning pumpkin must weigh. (See Example 3.)

At the county fair, contestants can enter contests that judge animals, recipes, crops, art projects, and more. Sometimes an average score or average weight is used to determine the winner of the blue ribbon. A contestant can use a multi-step inequality to determine what score or weight is needed in order to win.

Inequalities that contain more than one operation require more than one step to solve. Use inverse operations to undo the operations in the inequality one at a time.

E X A M P L E 1 Solving Multi-Step Inequalities

Solve each inequality and graph the solutions.

A $160 + 4f \leq 500$

$$160 + 4f \leq 500$$
$$\underline{-160 \qquad -160}$$
$$4f \leq 340$$
$$\frac{4f}{4} \leq \frac{340}{4}$$
$$f \leq 85$$

Since 160 is added to 4f, subtract 160 from both sides to undo the addition.

Since f is multiplied by 4, divide both sides by 4 to undo the multiplication.

Remember!
Subtracting a number is the same as adding its opposite.
$7 - 2t = 7 + (-2t)$

B $7 - 2t \leq 21$

$$7 - 2t \leq 21$$
$$\underline{-7 \qquad -7}$$
$$-2t \leq 14$$
$$\frac{-2t}{-2} \geq \frac{14}{-2}$$
$$t \geq -7$$

Since 7 is added to −2t, subtract 7 from both sides to undo the addition.

Since t is multiplied by −2, divide both sides by −2 to undo the multiplication.

Change ≤ to ≥.

CHECK IT OUT! Solve each inequality and graph the solutions.

1a. $-12 \geq 3x + 6$ **1b.** $\dfrac{x + 5}{-2} > 3$ **1c.** $\dfrac{1 - 2n}{3} \geq 7$

To solve more complicated inequalities, you may first need to simplify the expressions on one or both sides by using the order of operations, combining like terms, or using the Distributive Property.

EXAMPLE 2 | **Simplifying Before Solving Inequalities**

Solve each inequality and graph the solutions.

A $-4 + (-8) < -5c - 2$
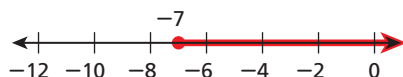
$-12 < -5c - 2$

$\underline{+2 \qquad\qquad +2}$

$-10 < -5c$

$\dfrac{-10}{-5} > \dfrac{-5c}{-5}$

$2 > c \text{ (or } c < 2)$

Combine like terms. Since 2 is subtracted from $-5c$, add 2 to both sides to undo the subtraction.
Since c is multiplied by -5, divide both sides by -5 to undo the multiplication.
Change $<$ to $>$.
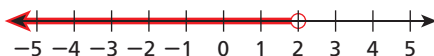
B $-3(3 - x) < 4^2$

$-3(3 - x) < 4^2$

$-3(3) - 3(-x) < 4^2$

$-9 + 3x < 4^2$

$-9 + 3x < 16$

$-9 + 3x < 16$

$\underline{+9 \qquad\quad +9}$

$3x < 25$

$\dfrac{3x}{3} < \dfrac{25}{3}$

$x < 8\dfrac{1}{3}$

Distribute -3 on the left side.
Simplify the right side.
Since -9 is added to $3x$, add 9 to both sides to undo the addition.
Since x is multiplied by 3, divide both sides by 3 to undo the multiplication.
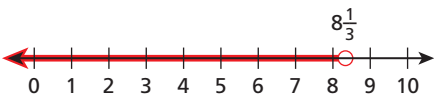
C $\dfrac{4}{5}x + \dfrac{1}{2} > \dfrac{3}{5}$

$10\left(\dfrac{4}{5}x + \dfrac{1}{2}\right) > 10\left(\dfrac{3}{5}\right)$

$10\left(\dfrac{4}{5}x\right) + 10\left(\dfrac{1}{2}\right) > 10\left(\dfrac{3}{5}\right)$

$8x + 5 > 6$

$\underline{-5 \quad -5}$

$8x \qquad > 1$

$\dfrac{8x}{8} > \dfrac{1}{8}$

$x > \dfrac{1}{8}$

Multiply both sides by 10, the LCD of the fractions.
Distribute 10 on the left side.
Since 5 is added to $8x$, subtract 5 from both sides to undo the addition.
Since x is multiplied by 8, divide both sides by 8 to undo the multiplication.
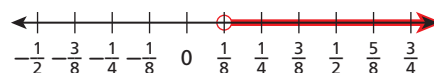
CHECK IT OUT! Solve each inequality and graph the solutions.

2a. $2m + 5 > 5^2$ **2b.** $3 + 2(x + 4) > 3$ **2c.** $\dfrac{5}{8} < \dfrac{3}{8}x - \dfrac{1}{4}$

EXAMPLE **3** *Gardening Application*

To win the blue ribbon for the Heaviest Pumpkin Crop at the county fair, the average weight of John's two pumpkins must be greater than 819 lb. One of his pumpkins weighs 887 lb. What is the least number of pounds the second pumpkin could weigh in order for John to win the blue ribbon?

Let p represent the weight of the second pumpkin. The average weight of the pumpkins is the sum of each weight divided by 2.

(887	plus	p)	divided by	2	must be greater than	819.
(887	+	p)	÷	2	>	819

$$\frac{887 + p}{2} > 819$$ *Since 887 + p is divided by 2, multiply both sides by 2 to undo the division.*

$$2\left(\frac{887 + p}{2}\right) > 2(819)$$

$$887 + p > 1638$$ *Since 887 is added to p, subtract 887 from both sides to undo the addition.*

$$\underline{-887 \qquad -887}$$
$$p > \quad 751$$

The second pumpkin must weigh more than 751 pounds.

Check   Check the endpoint, 751.   Check a number greater than 751.

$$\frac{887 + p}{2} = 819 \qquad\qquad \frac{887 + p}{2} > 819$$

$$\begin{array}{c|c} \frac{887 + 751}{2} & 819 \\ \hline \frac{1638}{2} & 819 \\ 819 & 819 \checkmark \end{array} \qquad\qquad \begin{array}{c|c} \frac{887 + 755}{2} & > & 819 \\ \frac{1642}{2} & > & 819 \\ 821 & > & 819 \checkmark \end{array}$$

CHECK IT OUT!

3. The average of Jim's two test scores must be at least 90 to make an A in the class. Jim got a 95 on his first test. What grades can Jim get on his second test to make an A in the class?

THINK AND DISCUSS

1. The inequality $v \geq 25$ states that 25 is the ___?___. (*value of v, minimum value of v,* or *maximum value of v*)

2. Describe two sets of steps for solving the inequality $\frac{x + 5}{3} > 7$.

3. GET ORGANIZED Copy and complete the graphic organizer.

Know it! Note

Solving Multi-Step Equations and Inequalities

How are they alike?   How are they different?

3-4 **Exercises**

go.hrw.com
Homework Help Online
KEYWORD: MA7 3-4
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

SEE EXAMPLE **1**
p. 188

Solve each inequality and graph the solutions.

1. $2m + 1 > 13$ **2.** $2d + 21 \leq 11$ **3.** $6 \leq -2x + 2$ **4.** $4c - 7 > 5$

5. $\dfrac{4 + x}{3} > -4$ **6.** $1 < 0.2x - 0.7$ **7.** $\dfrac{3 - 2x}{3} \leq 7$ **8.** $2x + 5 \geq 2$

SEE EXAMPLE **2**
p. 189

9. $4(x + 2) > 6$ **10.** $\dfrac{1}{4}x + \dfrac{2}{3} < \dfrac{3}{4}$ **11.** $4 - x + 6^2 \geq 21$

12. $4 - x > 3(4 - 2)$ **13.** $0.2(x - 10) > -1.8$ **14.** $3(j + 41) \leq 35$

SEE EXAMPLE **3**
p. 190

15. Business A sales representative is given a choice of two paycheck plans. One choice includes a monthly base pay of $300 plus 10% commission on his sales. The second choice is a monthly salary of $1200. For what amount of sales would the representative make more money with the first plan?

PRACTICE AND PROBLEM SOLVING

Solve each inequality and graph the solutions.

16. $4r - 9 > 7$ **17.** $3 \leq 5 - 2x$ **18.** $\dfrac{w + 3}{2} > 6$ **19.** $11w + 99 < 77$

20. $9 \geq \dfrac{1}{2}v + 3$ **21.** $-4x - 8 > 16$ **22.** $8 - \dfrac{2}{3}z \leq 2$ **23.** $f + 2\dfrac{1}{2} < -2$

24. $\dfrac{3n - 8}{5} \geq 2$ **25.** $-5 > -5 - 3w$ **26.** $10 > \dfrac{5 - 3p}{2}$ **27.** $2v + 1 > 2\dfrac{1}{3}$

28. $4(x + 3) > -24$ **29.** $4 > x - 3(x + 2)$ **30.** $-18 \geq 33 - 3h$

31. $-2 > 7x - 2(x - 4)$ **32.** $9 - (9)^2 > 10x - x$ **33.** $2a - (-3)^2 \geq 13$

34. $6 - \dfrac{x}{3} + 1 > \dfrac{2}{3}$ **35.** $12(x - 3) + 2x > 6$ **36.** $15 \geq 19 + 2(q - 18)$

37. Communications One cell phone company offers a plan that costs $29.99 and includes unlimited night and weekend minutes. Another company offers a plan that costs $19.99 and charges $0.35 per minute during nights and weekends. For what numbers of night and weekend minutes does the second company's plan cost more than the first company's plan?

Solve each inequality and graph the solutions.

38. $-12 > -4x - 8$ **39.** $5x + 4 \leq 14$ **40.** $\dfrac{2}{3}x - 5 > 7$

41. $x - 3x > 2 - 10$ **42.** $5 - x - 2 > 3$ **43.** $3 < 2x - 5(x + 3)$

44. $\dfrac{1}{6} - \dfrac{2}{3}m \geq \dfrac{1}{4}$ **45.** $4 - (r - 2) > 3 - 5$ **46.** $0.3 - 0.5n + 1 \geq 0.4$

47. $6^2 > 4(x + 2)$ **48.** $-4 - 2n + 4n > 7 - 2^2$ **49.** $\dfrac{1}{4}(p - 10) \geq 6 - 4$

50. Use the inequality $-4t - 8 \leq 12$ to fill in the missing numbers.

    a. $t \geq \blacksquare$ **b.** $t + 4 \geq \blacksquare$ **c.** $t - \blacksquare \geq 0$

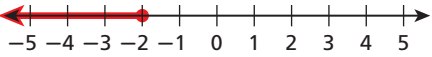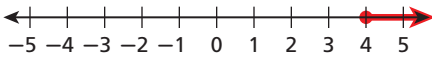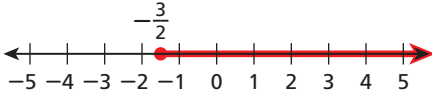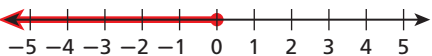   **d.** $t + 10 \geq \blacksquare$ **e.** $3t \geq \blacksquare$ **f.** $\dfrac{t}{\blacksquare} \geq -5$

Write an inequality for each statement. Solve the inequality and graph the solutions.

51. One-half of a number, increased by 9, is less than 33.

52. Six is less than or equal to the sum of 4 and $-2x$.

53. The product of 4 and the sum of a number and 12 is at most 16.

54. The sum of half a number and two-thirds of the number is less than 14.

Solve each inequality and match the solution to the correct graph.

55. $4x - 9 \geq 7$

A.
-5 -4 -3 -2 -1 0 1 2 3 4 5

56. $-6 \geq 3(x - 2)$

B.
-5 -4 -3 -2 -1 0 1 2 3 4 5

57. $-2x - 6 \geq -4 + 2$

C.
$-\frac{3}{2}$
-5 -4 -3 -2 -1 0 1 2 3 4 5

58. $\frac{1}{2} - \frac{1}{3}x \leq \left(\frac{2}{3} + \frac{1}{3}\right)^2$

D.
-5 -4 -3 -2 -1 0 1 2 3 4 5

59. Entertainment A digital video recorder (DVR) records television shows on an internal hard drive. To use a DVR, you need a subscription with a DVR service company. Two companies advertise their charges for a DVR machine and subscription service.

EASY ELECTRONICS
$225 for DVR machine
$400 for lifetime subscription

CABLE SOLUTIONS
$275 for DVR machine
$15 per month for service

For what numbers of months will a consumer pay less for the machine and subscription at Easy Electronics than at Cable Solutions?

60. Geometry The area of the triangle shown is less than 55 square inches.
   a. Write an inequality that can be used to find x.
   b. Solve the inequality you wrote in part **a.**
   c. What is the maximum height of the triangle?

$(2x + 3)$ in.

5 in.
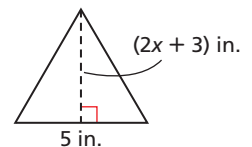
61. This problem will prepare you for the Multi-Step Test Prep on page 210.

   a. A band wants to create a CD of their last concert. They received a donation of $500 to cover the cost. The CDs cost $350 plus $3 per CD. Complete the table to find a relationship between the number of CDs and the total cost.

   b. Write an equation for the cost C of the CDs based on the number of CDs n.

   c. Write an inequality that can be used to determine how many CDs can be made with the $500 donation. Solve the inequality and determine how many CDs the band can have made from the $500 donation.

Number	Process	Cost
1	350 + 3	353
2		
3		
10		
n		

62. Critical Thinking What is the least whole number that is a solution of $4r - 4.9 > 14.95$?

63. Write About It Describe two sets of steps to solve $2(x + 3) > 10$.

64. What are the solutions of $3y > 2x + 4$ when $y = 6$?

  (A) $7 > x$ (B) $x > 7$ (C) $x > 11$ (D) $11 > x$

65. Cecilia has $30 to spend at a carnival. Admission costs $5.00, lunch will cost $6.00, and each ride ticket costs $1.25. Which inequality represents the number of ride tickets x that Cecilia can buy?

  (F) $30 - (5 - 6) + 1.25x \le 30$ (H) $30 - (5 + 6) \le 1.25x$

  (G) $5 + 6 + 1.25x \le 30$ (J) $30 + 1.25x \le 5 + 6$

66. Which statement is modeled by $2p + 5 < 11$?

  (A) The sum of 5 and 2 times p is at least 11.

  (B) Five added to the product of 2 and p is less than 11.

  (C) Two times p plus 5 is at most 11.

  (D) The product of 2 and p added to 5 is 11.

67. Gridded Response A basketball team scored 8 points more in its second game than in its first. In its third game, the team scored 42 points. The total number of points scored in the three games was more than 150. What is the least number of points the team might have scored in its *second* game?

CHALLENGE AND EXTEND

Solve each inequality and graph the solutions.

68. $3(x + 2) - 6x + 6 \le 0$ **69.** $-18 > -(2x + 9) - 4 + x$ **70.** $\dfrac{2 + x}{2} - (x - 1) > 1$

Write an inequality for each statement. Graph the solutions.

71. x is a positive number.   **72.** x is a negative number.

73. x is a nonnegative number.   **74.** x is not a positive number.

75. x times negative 3 is positive.   **76.** The opposite of x is greater than 2.

SPIRAL REVIEW

Find each square root. *(Lesson 1-5)*

77. $\sqrt{49}$ **78.** $-\sqrt{144}$ **79.** $\sqrt{\dfrac{4}{9}}$

80. $\sqrt{196}$ **81.** $-\sqrt{1}$ **82.** $\sqrt{10{,}000}$

83. Video rental store A charges a membership fee of $25 and $2 for each movie rental. Video rental store B charges a membership fee of $10 and $2.50 for each movie. Find the number of movie rentals for which both stores' charges are the same. *(Lesson 2-4)*

Solve each inequality and graph the solutions. *(Lesson 3-3)*

84. $2x < -8$ **85.** $\dfrac{a}{-2} \le -3$ **86.** $\dfrac{1}{4} < \dfrac{t}{12}$

Solving Inequalities with Variables on Both Sides

Objective
Solve inequalities that contain variable terms on both sides.

Who uses this?
Business owners can use inequalities to find the most cost-efficient services. (See Example 2.)

Some inequalities have variable terms on both sides of the inequality symbol. You can solve these inequalities like you solved equations with variables on both sides.

Use the properties of inequality to "collect" all the variable terms on one side and all the constant terms on the other side.

EXAMPLE 1 **Solving Inequalities with Variables on Both Sides**

Solve each inequality and graph the solutions.

A $x < 3x + 8$

$$x < 3x + 8$$

To collect the variable terms on one side, subtract x from both sides.

$$\underline{-x \quad\quad -x}$$
$$0 < 2x + 8$$

Since 8 is added to $2x$, subtract 8 from both sides to undo the addition.

$$\underline{-8 \quad\quad\quad -8}$$
$$-8 < 2x$$

$$\frac{-8}{2} < \frac{2x}{2}$$

Since x is multiplied by 2, divide both sides by 2 to undo the multiplication.

$$-4 < x \;(\text{or } x > -4)$$

Helpful Hint

Your first step can also be to subtract $3x$ from both sides to get $-2x < 8$. When you divide by a negative number, remember to reverse the inequality symbol.

B $6x - 1 \le 3.5x + 4$

$$6x - 1 \le 3.5x + 4$$

Subtract $6x$ from both sides.

$$\underline{-6x \quad\quad\quad -6x}$$
$$-1 \le -2.5x + 4$$

Since 4 is added to $-2.5x$, subtract 4 from both sides to undo the addition.

$$\underline{-4 \quad\quad\quad\quad -4}$$
$$-5 \le -2.5x$$

$$\frac{-5}{-2.5} \ge \frac{-2.5x}{-2.5}$$

Since x is multiplied by -2.5, divide both sides by -2.5 to undo the multiplication. Reverse the inequality symbol.

$$2 \ge x$$

CHECK IT OUT!

Solve each inequality and graph the solutions.

1a. $4x \ge 7x + 6$ **1b.** $5t + 1 < -2t - 6$

EXAMPLE **2** *Business Application*

The *Daily Info* charges a fee of $650 plus $80 per week to run an ad. The *People's Paper* charges $145 per week. For how many weeks will the total cost at *Daily Info* be less expensive than the cost at *People's Paper*?

Let w be the number of weeks the ad runs in the paper.

Daily Info fee	plus	$80 per week	times	number of weeks	is less expensive than	*People's Paper* charge per week	times	number of weeks.
$650	+	$80	•	w	<	$145	•	w

$$650 + 80w < 145w$$

$$\underline{\quad -80w \quad -80w \quad}$$ *Subtract 80w from both sides.*

$$650 \quad\quad < \quad 65w$$ *Since w is multiplied by 65, divide both sides by 65 to undo the multiplication.*

$$\frac{650}{65} < \frac{65w}{65}$$

$$10 < w$$

The total cost at *Daily Info* is less than the cost at *People's Paper* if the ad runs for more than 10 weeks.

CHECK IT OUT!

2. A-Plus Advertising charges a fee of $24 plus $0.10 per flyer to print and deliver flyers. Print and More charges $0.25 per flyer. For how many flyers is the cost at A-Plus Advertising less than the cost at Print and More?

You may need to simplify one or both sides of an inequality before solving it. Look for like terms to combine and places to use Distributive Property.

EXAMPLE **3** **Simplifying Each Side Before Solving**

Solve each inequality and graph the solutions.

A $6(1-x) < 3x$

$$6(1-x) < 3x$$ *Distribute 6 on the left side of the inequality.*

$$6(1) - 6(x) < 3x$$ *Add 6x to both sides so that the coefficient of x is positive.*

$$6 - 6x < 3x$$

$$\underline{\quad +6x \quad +6x \quad}$$

$$6 \quad < 9x$$

$$\frac{6}{9} < \frac{9x}{9}$$ *Since x is multiplied by 9, divide both sides by 9 to undo the multiplication.*
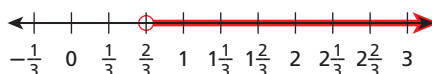
$$\frac{2}{3} < x$$

Number line from $-\frac{1}{3}$, 0, $\frac{1}{3}$, $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2, $2\frac{1}{3}$, $2\frac{2}{3}$, 3 with open circle at $\frac{2}{3}$.

Solve each inequality and graph the solutions.

B $-0.2x + 0.9 \geq 1.6x$

$\begin{array}{rl} -0.2x + 0.9 \geq & 1.6x \\ \underline{+\,0.2x \qquad\quad +\,0.2x} & \\ 0.9 \geq & 1.8x \end{array}$ *Since $-0.2x$ is added to 0.9, subtract -0.2 from both sides. Subtracting -0.2 is the same as adding 0.2.*

$\dfrac{0.9}{1.8} \geq \dfrac{1.8x}{1.8}$ *Since x is multiplied by 1.8, divide both sides by 1.8 to undo the multiplication.*

$\dfrac{1}{2} \geq x$
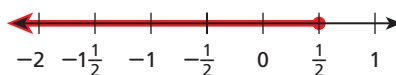
Solve each inequality and graph the solutions.

3a. $5(2 - r) \geq 3(r - 2)$ **3b.** $0.5x - 0.3 + 1.9x < 0.3x + 6$

There are special cases of inequalities called *identities* and *contradictions*.

Know it! Note

Identities and Contradictions

WORDS	ALGEBRA
Identity	
When solving an inequality, if you get a statement that is always true, the original inequality is an identity, and all real numbers are solutions.	$\begin{array}{r} 1 + x < 7 + x \\ \underline{-\,x \qquad\quad -\,x} \\ 1 < 7\;✓ \end{array}$
Contradiction	
When solving an inequality, if you get a false statement, the original inequality is a contradiction, and it has no solutions.	$\begin{array}{r} x + 7 < \;\; x \\ \underline{-\,x \quad\; -\,x} \\ 7 < 0\;✗ \end{array}$

These properties are also true for inequalities that use the symbols $>$, $\geq$, and $\leq$.

E X A M P L E   **4**   **Identities and Contradictions**

Solve each inequality.

A $x + 5 \geq x + 3$

$\begin{array}{rl} x + 5 \geq & x + 3 \\ \underline{-\,x \qquad} & \underline{-\,x} \\ 5 \geq & 3\;✓ \end{array}$ *Subtract x from both sides.*
        True statement

All values of x make the inequality true.
All real numbers are solutions.

B $2x + 6 < 5 + 2x$

$\begin{array}{rl} 2x + 6 < & 5 + 2x \\ \underline{-\,2x \qquad} & \underline{-\,2x} \\ 6 < & 5\;✗ \end{array}$ *Subtract $2x$ from both sides.*
        False statement

No values of x make the inequality true.
There are no solutions.

Solve each inequality.

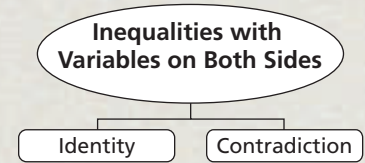**4a.** $4(y - 1) \geq 4y + 2$ **4b.** $x - 2 < x + 1$

THINK AND DISCUSS
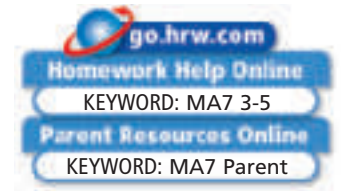
1. Explain how you would collect the variable terms to solve the inequality $5c - 4 > 8c + 2$.

2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, give an example of an inequality of the indicated type.

Inequalities with Variables on Both Sides

| Identity | Contradiction |

Exercises

GUIDED PRACTICE

SEE EXAMPLE 1
p. 194

Solve each inequality and graph the solutions.

1. $2x > 4x - 6$
2. $7y + 1 \leq y - 5$
3. $27x + 33 > 58x - 29$
4. $-3r < 10 - r$
5. $5c - 4 > 8c + 2$
6. $4.5x - 3.8 \geq 1.5x - 2.3$

SEE EXAMPLE 2
p. 195

7. **School** The school band will sell pizzas to raise money for new uniforms. The supplier charges $100 plus $4 per pizza. If the band members sell the pizzas for $7 each, how many pizzas will they have to sell to make a profit?

SEE EXAMPLE 3
p. 195

Solve each inequality and graph the solutions.

8. $5(4 + x) \leq 3(2 + x)$
9. $-4(3 - p) > 5(p + 1)$
10. $2(6 - x) < 4x$
11. $4x > 3(7 - x)$
12. $\frac{1}{2}f + \frac{3}{4} \geq \frac{1}{4}f$
13. $-36.72 + 5.65t < 0.25t$

SEE EXAMPLE 4
p. 196

Solve each inequality.

14. $2(x - 2) \leq -2(1 - x)$
15. $4(y + 1) < 4y + 2$
16. $4v + 1 < 4v - 7$
17. $b - 4 \geq b - 6$
18. $3(x - 5) > 3x$
19. $2k + 7 \geq 2(k + 14)$

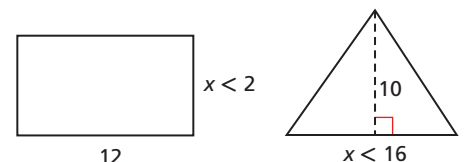## PRACTICE AND PROBLEM SOLVING

Solve each inequality and graph the solutions.

20. $3x \leq 5x + 8$
21. $9y + 3 > 4y - 7$
22. $1.5x - 1.2 < 3.1x - 2.8$
23. $7 + 4b \geq 3b$
24. $7 - 5t < 4t - 2$
25. $2.8m - 5.2 > 0.8m + 4.8$

26. **Geometry** Write and solve an inequality to find the values of x for which the area of the rectangle is greater than the area of the triangle.

$x < 2$

12

10

$x < 16$

Solve each inequality and graph the solutions.

27. $4(2 - x) \le 5(x - 2)$

28. $-3(n + 4) < 6(1 - n)$

29. $9(w + 2) \le 12w$

30. $4.5 + 1.3t > 3.8t - 3$

31. $\frac{1}{2}r + \frac{2}{3} \ge \frac{1}{3}r$

32. $2(4 - n) < 3n - 7$

Solve each inequality.

33. $3(2 - x) < -3(x - 1)$

34. $7 - y > 5 - y$

35. $3(10 + z) \le 3z + 36$

36. $-5(k - 1) \ge 5(2 - k)$

37. $4(x - 1) \le 4x$

38. $3(v - 9) \ge 15 + 3v$

Solve each inequality and graph the solutions.

39. $3t - 12 > 5t + 2$

40. $-5(y + 3) - 6 < y + 3$

41. $3x + 9 - 5x < x$

42. $18 + 9p > 12p - 31$

43. $2(x - 5) < -3x$

44. $-\frac{2}{5}x \le \frac{4}{5} - \frac{3}{5}x$

45. $-2(x - 7) - 4 - x < 8x + 32$

46. $-3(2r - 4) \ge 2(5 - 3r)$

47. $-7x - 10 + 5x \ge 3(x + 4) + 8$

48. $-\frac{1}{3}(n + 8) + \frac{1}{3}n \le 1 - n$

Recreation

49. Recreation A red kite is 100 feet off the ground and is rising at 8 feet per second. A blue kite is 180 feet off the ground and is rising at 5 feet per second. How long will it take for the red kite to be higher than the blue kite? Round your answer to the nearest second.

50. Education The table shows the enrollment in Howard High School and Phillips High School for three school years.

School Enrollment			
	Year 1	Year 2	Year 3
Howard High School	1192	1188	1184
Philipps High School	921	941	961

a. How much did the enrollment change each year at Howard?

b. Use the enrollment in year 1 and your answer from part **a** to write an expression for the enrollment at Howard in any year x.

c. How much did the enrollment change each year at Phillips?

d. Use the enrollment in year 1 and your answer from part **c** to write an expression for the enrollment at Phillips in any year x.

e. Assume that the pattern in the table continues. Use your expressions from parts **b** and **d** to write an inequality that can be solved to find the year in which the enrollment at Phillips High School will be greater than the enrollment at Howard High School. Solve your inequality and graph the solutions.

The American Kitefliers Association has over 4000 members in 35 countries. Kitefliers participate in festivals, competitions, and kite-making workshops.

MULTI-STEP TEST PREP

51. This problem will prepare you for the Multi-Step Test Prep on page 210.

a. The school orchestra is creating a CD of their last concert. The cost of creating the CDs is $400 + 4.50 per CD. Write an expression for the cost of creating the CDs based on the number of CDs n.

b. The orchestra plans to sell the CDs for $12. Write an expression for the amount the orchestra earns from the sale of n CDs.

c. In order for the orchestra to make a profit, the amount they make selling the CDs must be greater than the cost of creating the CDs. Write an inequality that can be solved to find the number of CDs the orchestra must sell in order to make a profit. Solve your inequality.

Write an inequality to represent each relationship. Solve your inequality.

52. Four more than twice a number is greater than two-thirds of the number.

53. Ten less than five times a number is less than six times the number decreased by eight.

54. The sum of a number and twenty is less than four times the number decreased by one.

55. Three-fourths of a number is greater than or equal to five less than the number.

56. Entertainment Use the table to determine how many movies you would have to rent for Video View to be less expensive than Movie Place.

	Membership Fee ($)	Cost per Rental ($)
Movie Place	None	2.99
Video View	19.99	1.99

57. Geometry In an acute triangle, all angles measure less than 90°. Also, the sum of the measures of any two angles is greater than the measure of the third angle. Can the measures of an acute triangle be x, $x - 1$, and $2x$? Explain.

58. Write About It Compare the steps you would follow to solve an inequality to the steps you would follow to solve an equation.

59. Critical Thinking How can you tell just by looking at the inequality $x > x + 1$ that it has no solutions?

60. ///ERROR ANALYSIS/// Two students solved the inequality $5x < 3 - 4x$. Which is incorrect? Explain the error.

A

$$5x \quad 3 - 4x$$
$$\underline{< 4x \quad < 4x}$$
$$9x \quad 3$$
$$x \quad \frac{1}{3}$$

B

$$5x \quad 3 - 4x$$
$$\underline{- 4x \quad - 4x}$$
$$x \quad 3$$

TEST PREP

61. If $a - b > a + b$, which statement is true?
- Ⓐ The value of a is positive.
- Ⓒ The value of a is negative.
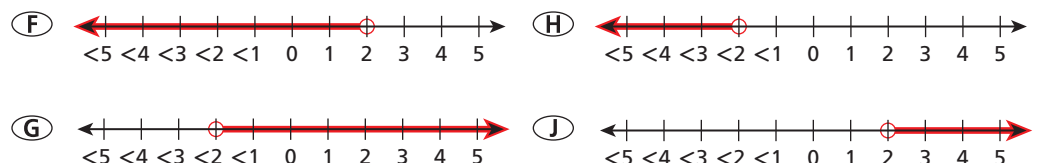- Ⓑ The value of b is positive.
- Ⓓ The value of b is negative.

62. If $-a < b$, which statement is true?
- Ⓕ $a < b$
- Ⓖ $a > b$
- Ⓗ $a < -b$
- Ⓙ $a > -b$

63. Which is a solution of the inequality $7(2 - x) > 4(x - 2)$?
- Ⓐ -2
- Ⓑ 2
- Ⓒ 4
- Ⓓ 7

64. Which is the graph of $-3x < -6$?

Ⓕ
$$\begin{array}{c} \text{<5 <4 <3 <2 <1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5} \end{array}$$

Ⓗ
$$\begin{array}{c} \text{<5 <4 <3 <2 <1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5} \end{array}$$

Ⓖ
$$\begin{array}{c} \text{<5 <4 <3 <2 <1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5} \end{array}$$

Ⓙ
$$\begin{array}{c} \text{<5 <4 <3 <2 <1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5} \end{array}$$

65. Short Response Write a real-world situation that could be modeled by the inequality $7x + 4 > 4x + 13$. Explain how the inequality relates to your situation.

CHALLENGE AND EXTEND

Solve each inequality.

66. $2\frac{1}{2} + 2x \geq 5\frac{1}{2} + 2\frac{1}{2}x$

67. $1.6x - 20.7 > 6.3x - (-2.2x)$

68. $1.3x - 7.5x < 8.5x - 29.4$

69. $-4w + \dfrac{-8 - 37}{9} \leq \dfrac{75 - 3}{9} + 3w$

70. Replace the square and circle with numbers so that the inequality is an identity.
$\square - 2x < \bigcirc - 2x$

71. Replace the square and circle with numbers so that the inequality is a contradiction.
$\square - 2x < \bigcirc - 2x$

72. Critical Thinking Explain whether there are any numbers that can replace the square and circle so that the inequality is an identity. $\square + 2x < \bigcirc + x$

SPIRAL REVIEW

73. The ratio of the width of a rectangle to the length is $2:5$. The length is 65 inches. Find the width. *(Lesson 2-6)*

74. Find the simple interest paid after 6 months on a loan of $5000 borrowed at a rate of 9%. *(Lesson 2-9)*

Define a variable and write an inequality for each situation. Graph the solutions. *(Lesson 3-1)*

75. Participants must be at least 14 years old.

76. The maximum speed on a certain highway is 60 miles per hour.

Algebra LAB

Use with Lesson 3-6

Truth Tables and Compound Statements

A compound statement is formed by combining two or more simple statements. A compound statement is either true or false depending on whether its simple statements are true or false.

Activity 1

• Let *P* be "Cindy is at least 17 years old."   • Let *Q* be "Cindy has a driver's license."

If…	then *P* is	and *Q* is	so *P* AND *Q* is
Cindy is 18 years old. Cindy has a driver's license.	True	True	True
Cindy is 17 years old. Cindy does not have a driver's license.	True	False	False
Cindy is 16 years old. Cindy has a driver's license.	False	True	False
Cindy is 15 years old. Cindy does not have a driver's license.	False	False	False

P AND *Q* is true when _____?_____.

Try This

For each pair of simple statements, tell whether *P* AND *Q* is true or false.

1. *P:* Many birds can fly; *Q:* A zebra is an animal.

Activity 2

• Let *P* be "Paul plays tennis."   • Let *Q* be "Paul has brown eyes."

If…	then *P* is	and *Q* is	so *P* OR *Q* is
Paul plays tennis. Paul has brown eyes.	True	True	True
Paul plays tennis. Paul has green eyes.	True	False	True
Paul does not play tennis. Paul has brown eyes.	False	True	True
Paul does not play tennis. Paul has green eyes.	False	False	False

P OR *Q* is true when _____?_____.

Try This

For each pair of simple statements, tell whether *P* OR *Q* is true or false.

2. *P:* The number 12 is even; *Q:* The number 12 is a composite number.

Solving Compound Inequalities

Objectives
Solve compound inequalities in one variable.

Graph solution sets of compound inequalities in one variable.

Vocabulary
compound inequality
intersection
union

Who uses this?
A lifeguard can use compound inequalities to describe the safe pH levels in a swimming pool. (See Example 1.)

The inequalities you have seen so far are simple inequalities. When two simple inequalities are combined into one statement by the words AND or OR, the result is called a **compound inequality**.

Compound Inequalities

WORDS	ALGEBRA	GRAPH
All real numbers greater than 2 AND less than 6	$x > 2$ AND $x < 6$ $2 < x < 6$	0 2 4 6 8
All real numbers greater than or equal to 2 AND less than or equal to 6	$x \geq 2$ AND $x \leq 6$ $2 \leq x \leq 6$	0 2 4 6 8
All real numbers less than 2 OR greater than 6	$x < 2$ OR $x > 6$	0 2 4 6 8
All real numbers less than or equal to 2 OR greater than or equal to 6	$x \leq 2$ OR $x \geq 6$	0 2 4 6 8

EXAMPLE 1 *Chemistry Application*

A water analyst recommends that the pH level of swimming pool water be between 7.2 and 7.6 inclusive. Write a compound inequality to show the pH levels that are within the recommended range. Graph the solutions.

Let p be the pH level of swimming pool water.

7.2	is less than or equal to	pH level	is less than or equal to	7.6
7.2	$\leq$	p	$\leq$	7.6

$7.2 \leq p \leq 7.6$

7.1 7.2 7.3 7.4 7.5 7.6 7.7

Helpful Hint

The phrase "between 7.2 and 7.6 *inclusive*" means that the numbers 7.2 and 7.6 are included in the solutions. Use a solid circle for endpoints that are solutions.

CHECK IT OUT!

**1.** The free chlorine level in a pool should be between 1.0 and 3.0 parts per million inclusive. Write a compound inequality to show the levels that are within this range. Graph the solutions.

In this diagram, oval *A* represents some integer solutions of $x < 10$, and oval *B* represents some integer solutions of $x > 0$. The overlapping region represents numbers that belong in both ovals. Those numbers are solutions of *both $x < 10$ and $x > 0$.*
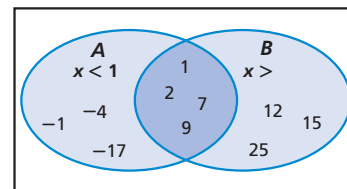
You can graph the solutions of a compound inequality involving AND by using the idea of an overlapping region. The overlapping region is called the **intersection** and shows the numbers that are solutions of both inequalities.
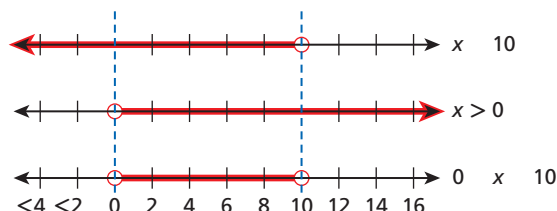
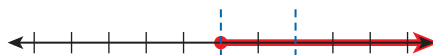**EXAMPLE 2** **Solving Compound Inequalities Involving AND**

Solve each compound inequality and graph the solutions.

A $4 \leq x + 2 \leq 8$

$$4 \leq x + 2 \quad \text{AND} \quad x + 2 \leq 8 \qquad \textit{Write the compound inequality using AND.}$$
$$\underline{-2 \qquad -2} \qquad \qquad \underline{-2 \quad -2} \qquad \textit{Solve each simple inequality.}$$
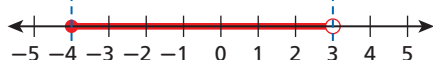$$2 \leq x \qquad \text{AND} \quad x \qquad \leq 6$$

Graph $2 \leq x$.

Graph $x \leq 6$.

Graph the intersection by finding where the two graphs overlap.

Remember!

The statement $-5 \leq 2x + 3 \leq 9$ consists of two inequalities connected by AND. Example 2B shows a "shorthand" method.
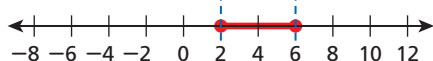
B $-5 \leq 2x + 3 < 9$

$$-5 \leq 2x + 3 < 9 \qquad \textit{Since 3 is added to 2x, subtract 3 from each part}$$
$$\underline{-3 \qquad -3 \quad -3} \qquad \textit{of the inequality.}$$
$$-8 \leq 2x \qquad < 6$$

$$\frac{-8}{2} \leq \frac{2x}{2} < \frac{6}{2} \qquad \textit{Since x is multiplied by 2, divide each part of the}$$
$$-4 \leq x < 3 \qquad \textit{inequality by 2.}$$

Graph $-4 \leq x$.

Graph $x < 3$.

Graph the intersection by finding where the two graphs overlap.
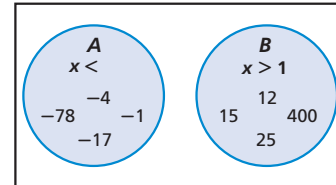
CHECK IT OUT! Solve each compound inequality and graph the solutions.

**2a.** $-9 < x - 10 < -5$ **2b.** $-4 \leq 3n + 5 < 11$

In this diagram, circle *A* represents some integer solutions of $x < 0$, and circle *B* represents some integer solutions of $x > 10$. The combined shaded regions represent numbers that are solutions of *either x < 0 or x > 10.*
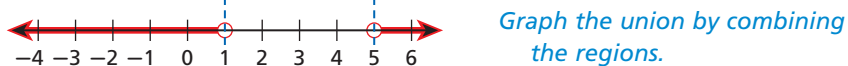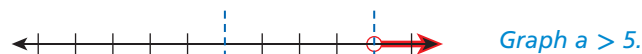

You can graph the solutions of a compound inequality involving OR by using the idea of combining regions. The combined regions are called the **union** and show the numbers that are solutions of either inequality.
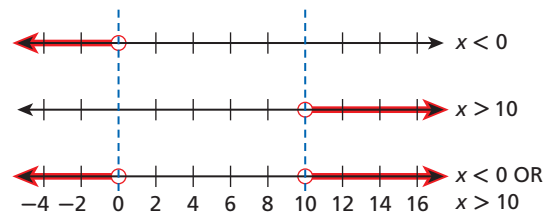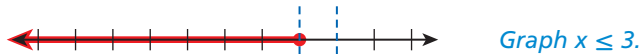

E X A M P L E **3**   **Solving Compound Inequalities Involving OR**

Solve each compound inequality and graph the solutions.

A $-4 + a > 1$ OR $-4 + a < -3$

$$-4 + a > \quad 1 \text{ OR} -4 + a < -3$$

$$\underline{+4 \qquad +4 \quad +4 \qquad +4} \qquad \textit{Solve each simple inequality.}$$

$$a > \quad 5 \text{ OR} \qquad a < \quad 1$$

  Graph a > 5.

Graph a < 1.

Graph the union by combining the regions.

B $2x \le 6$ OR $3x > 12$

$$2x \le 6 \text{ OR } 3x > 12$$

$$\frac{2x}{2} \le \frac{6}{2} \qquad \frac{3x}{3} > \frac{12}{3} \qquad \textit{Solve each simple inequality.}$$

$$x \le 3 \text{ OR} \quad x > \quad 4$$

  Graph x ≤ 3.

Graph x > 4.

Graph the union by combining the regions.

Solve each compound inequality and graph the solutions.

3a. $2 + r < 12$ OR $r + 5 > 19$

3b. $7x \ge 21$ OR $2x < -2$

Every solution of a compound inequality involving AND must be a solution of both parts of the compound inequality. If no numbers are solutions of *both* simple inequalities, then the compound inequality has no solutions.

The solutions of a compound inequality involving OR are not always two separate sets of numbers. There may be numbers that are solutions of both parts of the compound inequality.
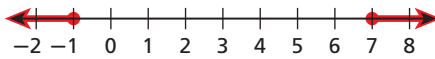
EXAMPLE 4

Writing a Compound Inequality from a Graph

Write the compound inequality shown by each graph.

A

The shaded portion of the graph is not between two values, so the compound inequality involves OR.

On the left, the graph shows an arrow pointing left, so use either < or ≤.
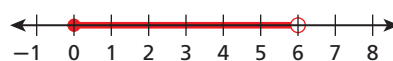*The solid circle at −1 means −1 is a solution, so use ≤.*

$x \leq -1$

On the right, the graph shows an arrow pointing right, so use either > or ≥.
The solid circle at 7 means 7 is a solution, so use ≥.

$x \geq 7$

The compound inequality is $x \leq -1$ OR $x \geq 7$.

B

The shaded portion of the graph is between the values 0 and 6, so the compound inequality involves AND.

The shaded values are to the right of 0, so use > or ≥.
The solid circle at 0 means 0 is a solution, so use ≥.

$x \geq 0$

The shaded values are to the left of 6, so use < or ≤.
The empty circle at 6 means 6 is not a solution, so use <.

$x < 6$

The compound inequality is $x \geq 0$ AND $x < 6$.

Helpful Hint

The compound inequality in Example 4B can also be written with the variable between the two endpoints.
$0 \leq x < 6$

CHECK IT OUT! **Write the compound inequality shown by the graph.**

4a.

4b.
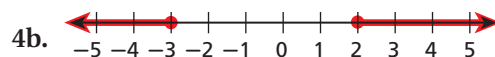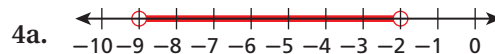
THINK AND DISCUSS

1. Describe how to write the compound inequality $y > 4$ AND $y \leq 12$ without using the joining word AND.

2. GET ORGANIZED Copy and complete the graphic organizers. Write three solutions in each of the three sections of the diagram. Then write each of your nine solutions in the appropriate column or columns of the table.

x > 5 AND x < 1	x > 5 OR x < 1

3-6

Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA7 3-6
Parent Resources Online
KEYWORD: MA7 Parent
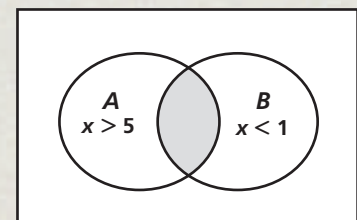
GUIDED PRACTICE

1. **Vocabulary** The graph of a(n) ___?___ shows all values that are solutions to both simple inequalities that make a compound inequality. (*union* or *intersection*)

SEE EXAMPLE 1
p. 202

2. **Biology** An iguana needs to live in a warm environment. The temperature in a pet iguana's cage should be between 70° F and 95°F inclusive. Write a compound inequality to show the temperatures that are within the recommended range. Graph the solutions.

SEE EXAMPLE 2
p. 203

Solve each compound inequality and graph the solutions.

3. $-3 < x + 2 < 7$

4. $5 \leq 4x + 1 \leq 13$

5. $2 < x + 2 < 5$

6. $11 < 2x + 3 < 21$
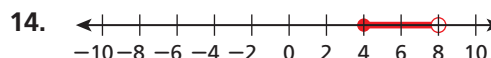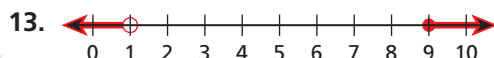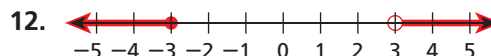
SEE EXAMPLE 3
p. 204

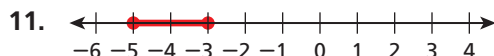7. $x + 2 < -6$ OR $x + 2 > 6$

8. $r - 1 < 0$ OR $r - 1 > 4$

9. $n + 2 < 3$ OR $n + 3 > 7$

10. $x - 1 < -1$ OR $x - 5 > -1$

SEE EXAMPLE 4
p. 205

Write the compound inequality shown by each graph.

11.

```
←+——●——+——●——+——+——+——+——+——+——+——→
  −6 −5 −4 −3 −2 −1  0  1  2  3  4
```

12.

```
←——————●——+——+——+——+——○——————→
  −5 −4 −3 −2 −1  0  1  2  3  4  5
```

13.

```
←——————○——+——+——+——+——+——+——+——●——→
   0  1  2  3  4  5  6  7  8  9  10
```

14.

```
←+——+——+——+——+——+——●———————○——+——→
 −10 −8 −6 −4 −2  0  2  4  6  8  10
```

PRACTICE AND PROBLEM SOLVING

15. **Meteorology** Earth's atmosphere is made of several layers. A layer called the stratosphere extends from about 16 km above Earth's surface to about 50 km above Earth's surface. Write a compound inequality to show the altitudes that are within the range of the stratosphere. Graph the solutions.

Solve each compound inequality and graph the solutions.

16. $-1 < x + 1 < 1$

17. $1 \leq 2n - 5 \leq 7$

18. $-2 < x - 2 < 2$

19. $5 < 3x - 1 < 17$

20. $x - 4 < -7$ OR $x + 3 > 4$

21. $2x + 1 < 1$ OR $x + 5 > 8$

22. $x + 1 < 2$ OR $x + 5 > 8$

23. $x + 3 < 0$ OR $x - 2 > 0$

Write the compound inequality shown by each graph.

24.

```
←——————+——○——+——+——+——○——+——+——→
  −3 −2 −1  0  1  2  3  4  5  6  7
```

25.

```
←——————+——○——+——●——+——+——+——+——→
  −3 −2 −1  0  1  2  3  4  5  6  7
```

26.

```
                          5
←+——+——○——+——+——+——+——●——+——+——→
 −10 −8 −6 −4 −2  0  2  4  6  8  10
```

27.

```
←+——+——+——○——+——○——+——+——+——→
 −5 −4 −3 −2 −1  0  1  2  3  4  5
```
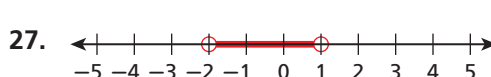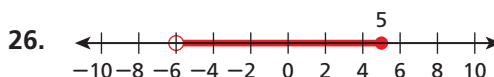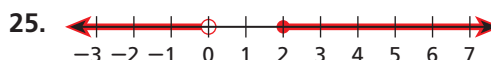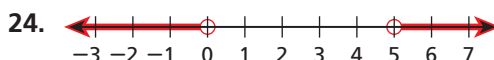
28. **Music** A typical acoustic guitar has a range of three octaves. When the guitar is tuned to "concert pitch," the range of frequencies for those three octaves is between 82.4 Hz and 659.2 Hz inclusive. Write a compound inequality to show the frequencies that are within the range of a typical acoustic guitar. Graph the solutions.

29. This problem will prepare you for the Multi-Step Test Prep on page 210. Jenna's band is going to record a CD at a recording studio. They will pay $225 to use the studio for one day and $80 per hour for sound technicians. Jenna has $200 and hopes to raise an additional $350 by taking pre-orders for the CDs.

   a. Explain how the inequality $200 \leq 225 + 80n \leq 550$ can be used to find the number of hours Jenna and her band can afford to use the studio and sound technicians.

   b. Solve the inequality. Are there any numbers in the solution set that are not reasonable in this situation?

   c. How much more money does Jenna need to raise if she wants to use the studio and sound technicians for 6 hours?

Write and graph a compound inequality for the numbers described.

30. all real numbers between -6 and 6

31. all real numbers less than or equal to 2 and greater than or equal to 1

32. all real numbers greater than 0 and less than 15

33. all real numbers between -10 and 10 inclusive

34. Transportation The cruise-control function on Georgina's car should keep the speed of the car within 3 mi/h of the set speed. Write a compound inequality to show the acceptable speeds s if the set speed is 55 mi/h. Graph the solutions.

35. Chemistry Water is not a liquid if its temperature is above 100°C or below 0°C. Write a compound inequality for the temperatures t when water is not a liquid.

Solve each compound inequality and graph the solutions.

36. $5 \leq 4b - 3 \leq 9$ **37.** $-3 < x - 1 < 4$

38. $r + 2 < -2$ OR $r - 2 > 2$ **39.** $2a - 5 < -5$ OR $3a - 2 > 1$

40. $x - 4 \geq 5$ AND $x - 4 \leq 5$ **41.** $n - 4 < -2$ OR $n + 1 > 6$

42. Sports The ball used in a soccer game may not weigh more than 16 ounces or less than 14 ounces at the start of the match. After $1\frac{1}{2}$ ounces of air was added to a ball, the ball was approved for use in a game. Write and solve a compound inequality to show how much the ball might have weighed before the air was added.

43. Meteorology Tornado damage is rated using the Fujita scale shown in the table. A tornado has a wind speed of 200 miles per hour. Write and solve a compound inequality to show how many miles per hour the wind speed would need to increase for the tornado to be rated "devastating" but not "incredible."

Fujita Tornado Scale		
Category	**Type**	**Wind Speed (mi/h)**
F0	Weak	40 to 72
F1	Moderate	73 to 112
F2	Significant	113 to 157
F3	Severe	158 to 206
F4	Devastating	207 to 260
F5	Incredible	261 to 318

44. Give a real world situation that can be described by a compound inequality. Write the inequality that describes your situation.

45. Write About It How are the graphs of the compound inequality $x < 3$ AND $x < 7$ and the compound inequality $x < 3$ OR $x < 7$ different? How are the graphs alike? Explain.

Chemistry

The element gallium is in a solid state at room temperature but becomes a liquid at about 30°C. Gallium stays in a liquid state until it reaches a temperature of about 2204°C.

46. Critical Thinking If there is no solution to a compound inequality, does the compound inequality involve OR or AND? Explain.

47. Which of the following describes the solutions of $-x + 1 > 2$ OR $x - 1 > 2$?

   Ⓐ all real numbers greater than 1 or less than 3

   Ⓑ all real numbers greater than 3 or less than 1

   Ⓒ all real numbers greater than -1 or less than 3

   Ⓓ all real numbers greater than 3 or less than -1

48. Which of the following is a graph of the solutions of $x - 3 < 2$ AND $x + 3 > 2$?
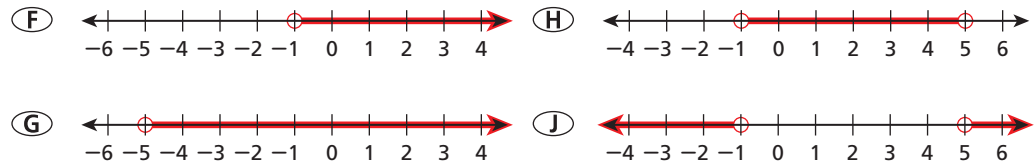
49. Which compound inequality is shown by the graph?

   Ⓐ $x \le 2$ OR $x > 5$ Ⓒ $x \le 2$ OR $x \ge 5$

   Ⓑ $x < 2$ OR $x \ge 5$ Ⓓ $x \ge 2$ OR $x > 5$

50. Which of the following is a solution of $x + 1 \ge 3$ AND $x + 1 \le 3$?

   Ⓕ 0   Ⓖ 1   Ⓗ 2   Ⓙ 3

CHALLENGE AND EXTEND

Solve and graph each compound inequality.

51. $2c - 10 < 5 - 3c < 7c$ **52.** $5p - 10 < p + 6 < 3p$

53. $2s \le 18 - s$ OR $5s \ge s + 36$ **54.** $9 - x \ge 5x$ OR $20 - 3x \le 17$

55. Write a compound inequality that represents all values of x that are NOT solutions to $x < -1$ OR $x > 3$.

56. For the compound inequality $x + 2 \ge a$ AND $x - 7 \le b$, find values of a and b for which the only solution is $x = 1$.

SPIRAL REVIEW

Simplify each expression. Justify each step. *(Lesson 1-7)*

57. $4(x - 3) + 7$ **58.** $5x - 4y - x + 3y$ **59.** $6a - 3(a - 1)$

Generate ordered pairs for each function for $x = -2, -1, 0, 1,$ and 2. Graph the ordered pairs and describe the pattern. *(Lesson 1-8)*

60. $y = -2x + 2$ **61.** $y = x^2 - 1$ **62.** $y = x^2 + (-2)$

Solve each inequality and graph the solutions. *(Lesson 3-4)*

63. $3m - 5 < 1$ **64.** $2(x + 4) > 6$ **65.** $11 \le 7 - 2x$

Triangle Inequality
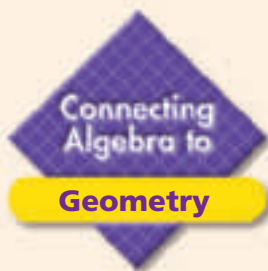
Connecting Algebra to Geometry

For any triangle, the sum of the lengths of any two sides is greater than the length of the third side.

See Skills Bank page S60

The sides of this triangle are labeled a, b, and c. You can use the Triangle Inequality to write three statements about the triangle.

$$a + b > c \qquad a + c > b \qquad b + c > a$$

Unless all three of the inequalities are true, the lengths a, b, and c cannot form a triangle.
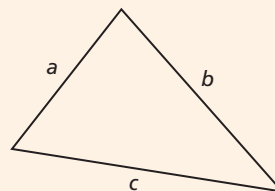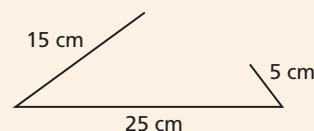

Example 1

Can three side lengths of 25 cm, 15 cm, and 5 cm form a triangle?

a. $25 + 15 > 5$ **b.** $25 + 5 > 15$ **c.** $15 + 5 > 25$
    $40 > 5$ *True* $30 > 15$ *True* $20 > 25$ *False*

One of the inequalities is false, so the three lengths will not make a triangle. The situation is shown in the figure to the right.

Example 2

Two sides of a triangle measure 8 ft and 10 ft. What is the range of lengths of the third side?

Start by writing three statements about the triangle. Use x for the unknown side length.

a. $8 + 10 > x$ **b.** $8 + x > 10$ **c.** $x + 10 > 8$
   $18 > x$ $8 + x - 8 > 10 - 8$ $x + 10 - 10 > 8 - 10$
                    $x > 2$ $x > -2$

The third side must be shorter than 18 ft.   *The third side must be longer than 2 ft.*   *This provides no new useful information.*

From part **a,** the third side must be shorter than 18 ft. And from part **b,** it must be longer than 2 ft. An inequality showing this is $2 < x < 18$.

Try This

Decide whether the three lengths given can form a triangle. If not, explain.

1. 14 ft, 30 ft, 10 ft   **2.** 11 cm, 8 cm, 17 cm   **3.** $6\frac{1}{2}$ yd, 3 yd, $2\frac{3}{4}$ yd

Write a compound inequality for the range of lengths of the third side of each triangle.

4.   **5.**   **6.**
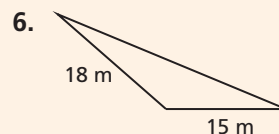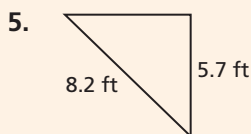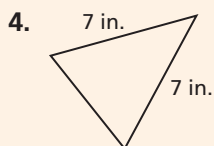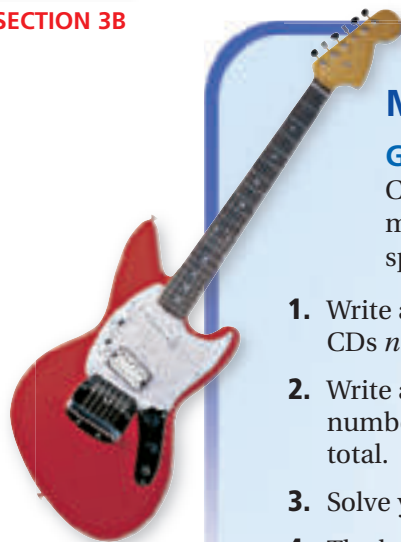
Multi-Step and Compound Inequalities

Guitar Picks Cullen and his band are interested in recording a CD of their music. The recording studio charges $450 to record the music and then charges $5 for each CD. The band is required to spend at least $1000 for the total of the recording and CD charges.

1. Write an equation for the cost C of the CDs based on the number of CDs n.

2. Write an inequality that can be used to determine the minimum number of CDs that must be burned at this studio to meet the $1000 total.

3. Solve your inequality from Problem 2.

4. The band orders the minimum number of CDs found in Problem 3. They want to sell the CDs and make at least as much money as they spent for the recording studio and making the CDs. Write an inequality that can be solved to determine the minimum amount the band should charge for their CDs.

5. Solve your inequality from Problem 4.

6. If the band has 30 more CDs made than the minimum number found in Problem 4 and charges the minimum price found in Problem 5, will they make a profit? If so, how much profit will the band make?

Quiz for Lessons 3-4 Through 3-6

3-4 Solving Two-Step and Multi-Step Inequalities

Solve each inequality and graph the solutions.

1. $2x + 3 < 9$ **2.** $3t - 2 > 10$ **3.** $7 \geq 1 - 6r$

Solve each inequality.

4. $2(x - 3) > -1$ **5.** $\frac{1}{3}a + \frac{1}{2} > \frac{2}{3}$

6. $2^2 - x > 4(3 - 5)$ **7.** $24b + 5 - 6b \leq 41$

8. $-7(7 + x) \geq -50$ **9.** $11d - (-2) < -15d$

10. $15 < 5(m - 7)$ **11.** $2 + (-6) > 8p$

12. The average of Mindy's two test scores must be at least 92 to make an A in the class. Mindy got an 88 on her first test. What scores can she get on her second test to make an A in the class?

3-5 Solving Inequalities with Variables on Both Sides

Solve each inequality and graph the solutions.

13. $5x < 3x + 8$ **14.** $6p - 3 > 9p$ **15.** $r - 8 \geq 3r - 12$

Solve each inequality.

16. $3(y + 6) > 2(y + 4)$ **17.** $4(5 - g) \geq g$

18. $5(t + 3) < 5t - 3$ **19.** $-2(6 + h) < 3(1 + h)$

20. $4x < 4(x - 1)$ **21.** $3(1 - x) \geq -3(x + 2)$

22. $9d > 3(1 - d)$ **23.** $16(s - 2) \leq 4(4s - 5)$

24. $5q \leq 2(q + 3)$ **25.** $3n > 5(n - 2)$

26. Phillip has $100 in the bank and deposits $18 per month. Gil has $145 in the bank and deposits $15 per month. For how many months will Gil have a larger bank balance than Phillip?

3-6 Solving Compound Inequalities

Solve each compound inequality and graph the solutions.

27. $-2 \leq x + 3 < 9$ **28.** $m + 2 < -1$ OR $m - 2 > 6$

29. $-3 \geq x - 1$ AND $x - 5 > 2$ **30.** $-2 > r + 2$ OR $r + 4 < 5$

31. $-2x > -8$ AND $x + 7 \geq 6$ **32.** $5 > y + 9$ OR $y - 4 > 2$

33. It is recommended that a certain medicine be stored in temperatures above 32° F and below 70° F. Write a compound inequality to show the acceptable storage temperatures for this medicine.

Solving Absolute-Value Inequalities

Objective
Solve inequalities in one variable involving absolute-value expressions.

When an inequality contains an absolute-value expression, it can be written as a compound inequality. The inequality $|x| < 5$ describes all real numbers whose distance from 0 is less than 5 units. The solutions are all numbers between -5 and 5, so $|x| < 5$ can be written as $-5 < x < 5$, which is the compound inequality $x > -5$ AND $x < 5$.

Know it! Note

Absolute-Value Inequalities Involving <

WORDS	NUMBERS
The inequality $|x| < a$ (when $a > 0$) asks, "What values of x have an absolute value less than a?" The solutions are numbers between $-a$ and a.	$|x| < 5$ $-5 < x < 5$ $x > -5$ AND $x < 5$

GRAPH	ALGEBRA
|← a units →|← a units →| $-a$ 0 a	$|x| < a$ (when $a > 0$) $-a < x < a$ $x > -a$ AND $x < a$

The same properties are true for inequalities that use the symbol $\leq$.

EXAMPLE 1   Solving Absolute-Value Inequalities Involving <

Solve each inequality and graph the solutions. Then write the solutions as a compound inequality.

A $|x| + 3 < 12$
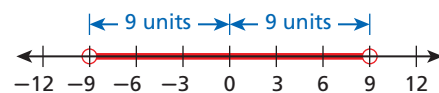
$$|x| + 3 < 12$$
$$\underline{-3 \phantom{<} -3}$$
$$|x| < 9$$

Since 3 is added to $|x|$, subtract 3 from both sides to undo the addition.

|← 9 units →|← 9 units →|

$-12 \;\; -9 \;\; -6 \;\; -3 \;\; 0 \;\; 3 \;\; 6 \;\; 9 \;\; 12$

Think, "The distance from x to 0 is less than 9 units."

$$x > -9 \text{ AND } x < 9$$
$$-9 < x < 9$$

Write as a compound inequality.

B $|x| - 7 < -3$
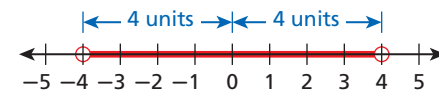
$$|x| - 7 < -3$$
$$\underline{+7 \phantom{<} +7}$$
$$|x| < 4$$

Since 7 is subtracted from $|x|$, add 7 to both sides to undo the subtraction.

|← 4 units →|← 4 units →|

$-5 \;\; -4 \;\; -3 \;\; -2 \;\; -1 \;\; 0 \;\; 1 \;\; 2 \;\; 3 \;\; 4 \;\; 5$

Think, "The distance from x to 0 is less than 4 units."

$$x > -4 \text{ AND } x < 4$$
$$-4 < x < 4$$

Write as a compound inequality.

Helpful Hint

Just like solving absolute-value equations, when you solve absolute-value inequalities, isolate the absolute-value expression.

Solve each inequality and graph the solutions. Then write the solutions as a compound inequality.

C $2|x| \le 6$

$\dfrac{2|x|}{2} \le \dfrac{6}{2}$ *Since $|x|$ is multiplied by 2, divide both sides by 2.*

$|x| \le 3$
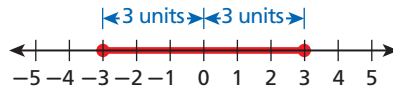
Think, "The distance from x to 0 is less than or equal to 3 units."

$x \ge -3$ AND $x \le 3$ *Write as a compound inequality.*

$-3 \le x \le 3$

D $|x + 3| - 4.5 \le 7.5$

$|x + 3| - 4.5 \le \quad 7.5$ *Since 4.5 is subtracted from $|x+3|$, add 4.5 to both sides.*

$\underline{\quad +4.5 \qquad +4.5\quad}$

$|x + 3| \qquad \le \quad 12$

Think, "The distance from x to −3 is less than or equal to 12 units."

$x + 3 \ge -12$ AND $x + 3 \le \quad 12$ *x + 3 is between −12 and 12, inclusive.*

$\underline{\quad -3 \qquad -3 \qquad\quad -3 \quad -3\quad}$
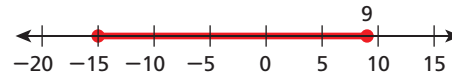
$x \qquad \ge -15$ AND $x \qquad \le \quad 9$

$x \ge -15$ AND $x \le 9$ *Write as a compound inequality.*

$-15 \le x \le 9$

CHECK IT OUT! Solve each inequality and graph the solutions. Then write the solutions as a compound inequality.

1a. $|x| + 12 < 15$ **1b.** $|x| - 6 < -5$

The inequality $|x| > 5$ describes all real numbers whose distance from 0 is greater than 5 units. The solutions are all numbers less than −5 or greater than 5. The inequality $|x| > 5$ can be written as the compound inequality $x < -5$ OR $x > 5$.

Know it! Note

Absolute-Value Inequalities Involving >

WORDS	NUMBERS
The inequality $|x| > a$ (when $a > 0$) asks, "What values of x have an absolute value greater than a?" The solutions are numbers less than $-a$ or greater than a.	$|x| > 5$ $x < -5$ OR $x > 5$
GRAPH	**ALGEBRA**
	$|x| > a$ (when $a > 0$) $x < -a$ OR $x > a$

The same properties are true for inequalities that use the symbol $\ge$.

EXAMPLE 2 **Solving Absolute-Value Inequalities Involving >**
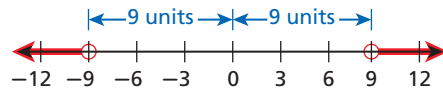
Solve each inequality and graph the solutions. Then write the solutions as a compound inequality.

A $|x| + 5 > 14$

$|x| + 5 > 14$ *Since 5 is added to $|x|$, subtract 5 from*
$\underline{\quad -5 \quad -5}$ *both sides to undo the addition.*
$|x| \quad\quad > 9$

|←—9 units —→|←—9 units —→|

Think, "The distance from x to 0 is greater than 9 units."

−12 −9 −6 −3   0   3   6   9   12

$x < -9 \text{ OR } x > 9$ *Write as a compound inequality.*

B $|x| - 20 > -13$

$|x| - 20 > -13$ *Since 20 is subtracted from $|x|$, add 20*
$\underline{\quad +20 \quad +20}$ *to both sides to undo the subtraction.*
$|x| \quad\quad > 7$
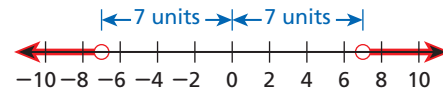
|←—7 units —→|←—7 units —→|

Think, "The distance from x to 0 is greater than 7 units."

−10 −8 −6 −4 −2   0   2   4   6   8   10

$x < -7 \text{ OR } x > 7$ *Write as a compound inequality.*

C $|x - 8| + 5 \geq 11$

$|x - 8| + 5 \geq \quad 11$ *Since 5 is added to $|x - 8|$, subtract 5*
$\underline{\quad\quad -5 \quad\quad -5}$ *from both sides to undo the addition.*
$|x - 8| \quad\quad \geq \quad 6$

Think, "The distance from x to 8 is greater than or equal to 6 units."

$x - 8 \leq -6 \text{ OR } x - 8 \geq 6$

$x - 8 \leq -6 \text{ OR } x - 8 \geq \quad 6$ *Solve the two inequalities.*
$\underline{+8 \quad +8 \quad\quad\quad +8 \quad +8}$
$x \quad\quad \leq \quad 2 \text{ OR } x \quad\quad \geq \quad 14$

−2   0   2   4   6   8   10   12   14   16   18
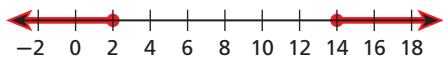
$x \leq 2 \text{ OR } x \geq 14$ *Write as a compound inequality.*

CHECK IT OUT! Solve each inequality and graph the solutions. Then write the solutions as a compound inequality.

2a. $|x| + 10 \geq 12$

2b. $|x| - 7 > -1$

2c. $\left| x + 2\frac{1}{2} \right| + \frac{1}{2} \geq 4$

Exercises

Solve each absolute-value inequality and graph the solutions.

1. $|x| - 5 \leq -2$ **2.** $|x| - 6 > 16$ **3.** $|x + 1| - 7.8 < 6.2$

4. $|x + 5| - 4\frac{1}{2} \geq 7\frac{1}{2}$ **5.** $|3x| + 2 < 8$ **6.** $|x| + 2.9 > 8.6$

Write and solve an absolute-value inequality for each expression. Graph the solutions on a number line.

7. all numbers whose absolute value is less than or equal to 15

8. all numbers that have an absolute value greater than 7

9. all numbers less than 3 units from 2 on the number line

10. all numbers at least 2 units from 8 on the number line
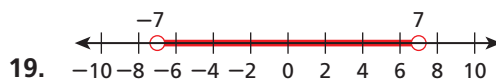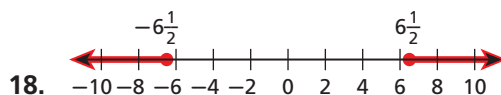
11. Find all values of x that make $|x - 5|$

  a. less than 11.   **b.** at least 4.   **c.** less than or equal to 8.

Tell whether the given value of x is a solution of the inequality.

12. $|x| > 3; x = -5$ **13.** $|2x| \leq 8; x = 6$

14. $|x - 1.2| < 5.4; x = 6.6$ **15.** $\left|x + 3\frac{1}{2}\right| - 2\frac{1}{4} \geq 5\frac{1}{4}; x = -11$

Write an absolute-value inequality for each graph.

16.

17.

18.

19.
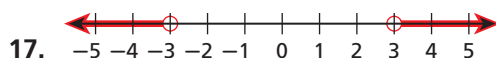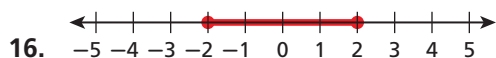
20. /// **ERROR ANALYSIS** /// A student solved the absolute-value inequality below. Find and explain the error(s) in the student's work. What is the correct answer?

$$x - 6 \ + 3 \geq 8$$
$$x - 6 \ + 3 \geq \ 8$$
$$\underline{\qquad\ -3 \quad -3\qquad}$$
$$x - 6 \ \geq \ 5$$
$$-5 \geq \ x - 6 \geq \quad 5$$
$$\underline{+6 \qquad\ +6 \qquad +6}$$
$$|\ \geq \ x \qquad \geq \quad ||$$

21. **Write About It** Describe how to use an absolute-value inequality to find all the values on a number line that are within 5 units of -6.

Study Guide: Review

Vocabulary

compound inequality 202   **intersection** 203   **union** 204

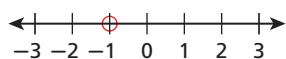**inequality** 168   **solution of an inequality** 168

**Complete the sentences below with vocabulary words from the list above.**

1. A(n) ___?___ is a mathematical statement that two quantities are not equal.

2. The numbers that are solutions to either inequality of a compound inequality is the ___?___.

3. A statement formed by combining two simple inequalities with the words AND or OR is a(n) ___?___.

4. The numbers that are solutions to both inequalities of a compound inequality is the ___?___.

5. Any value that makes the inequality true is a(n) ___?___.

3-1 Graphing and Writing Inequalities *(pp. 168–173)*

EXAMPLES

■ **Graph the inequality** $y > -1$.

Draw an empty circle at −1.
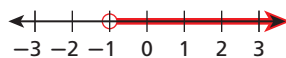
Shade all the numbers greater than −1.

■ **Write the inequality shown by the graph.**
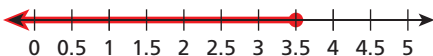
$n \le 3.5$ *Use the variable n. The arrow points left, so use either < or ≤. The closed circle means 3.5 is a solution, so use ≤.*

■ **Write an inequality for the situation and graph the solutions.**

Applicants for a driver's permit must be at least 16 years old.

age	must be at least	16 years
a	$\ge$	16
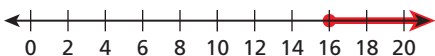
EXERCISES

Graph each inequality.

6. $x > -3$

7. $p \le 4$

8. $-1 > t$

9. $r \ge 9.5$

10. $2(3 - 5) < k$

11. $w < 3$

Write the inequality shown by each graph.

12.

13.

14.
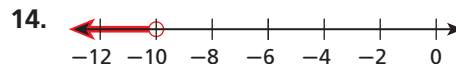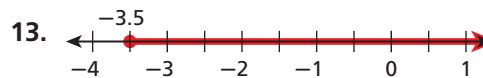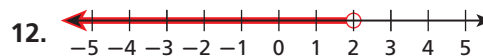
Define a variable and write an inequality for each situation. Graph the solutions.

15. The temperature must be at least 72°F.

16. No more than 12 students were present.

17. It takes less than 30 minutes to complete the lab activity.

3-2 Solving Inequalities by Adding or Subtracting (pp. 174–179)

EXAMPLES

Solve each inequality and graph the solutions.

■ $x + 6 > 2$

$$x + 6 > 2$$
$$\underline{-6 \quad -6}$$ *Since 6 is added to x, subtract 6 from both sides.*
$$x > -4$$

−5 −4 −3 −2 −1 0 1 2 3 4 5

■ $n - 1.3 < 3.2$

$$n - 1.3 < 3.2$$ *Since 1.3 is subtracted from x,*
$$\underline{+1.3 \quad +1.3}$$ *add 1.3 to both sides.*
$$n < 4.5$$

0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
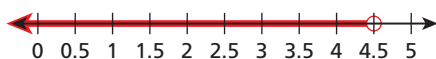
EXERCISES

Solve each inequality and graph the solutions.

18. $t + 3 < 10$

19. $k - 7 \le -5$

20. $-1 < m + 4$

21. $x + 2.3 \ge 6.8$

22. $w - 3 < 6.5$

23. $4 > a - 1$

24. $h - \dfrac{1}{4} < \dfrac{3}{4}$

25. $5 > 7 + v$

26. Tammy wants to run at least 10 miles per week. So far this week, she ran 4.5 miles. Write and solve an inequality to determine how many more miles Tammy must run this week to reach her goal.

27. Rob has a gift card for $50. So far, he has selected a shirt that costs $32. Write and solve an inequality to determine the amount Rob could spend without exceeding the gift card limit.

3-3 Solving Inequalities by Multiplying or Dividing (pp. 180–185)

EXAMPLES

■ Solve $\dfrac{p}{-3} \le 6$ and graph the solutions.

$$\frac{p}{-3} \le 6$$ *Since p is divided by −3,*
multiply both sides by −3.
$$-3 \cdot \frac{p}{-3} \ge -3 \cdot 6$$
$$p \ge -18$$ *Change ≤ to ≥.*

−21 −18 −15 −12 −9 −6 −3
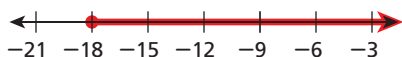
■ **What possible numbers of pizzas that cost $5.50 each can be purchased with $30?**
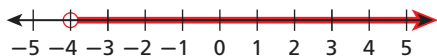
Let n represent the number of pizzas that can be purchased.

$5.50	times	number of pizzas	is at most	$30.
5.50	•	n	≤	30

$$5.50n \le 30$$ *Since n is multiplied by 5.50,*
$$\frac{5.50n}{5.50} \le \frac{30}{5.50}$$ *divide both sides by 5.50.*
$$n \le 5\frac{5}{11}$$

Only a whole number of pizzas can be purchased, so 0, 1, 2, 3, 4, or 5 pizzas can be purchased.

EXERCISES

Solve each inequality and graph the solutions.

28. $3a \le 15$

29. $-18 < 6t$

30. $\dfrac{p}{4} > 2$

31. $\dfrac{2}{5}x \le -10$

32. $-3n < -18$

33. $\dfrac{g}{-2} > 6$

34. $-2k < 14$

35. $-3 > \dfrac{1}{3}r$

36. $27 < -9h$

37. $-0.4g > -1$

38. What are the possible numbers of notebooks costing $1.39 that can be purchased with $10?

39. The senior class is selling lanyards as a fundraiser. The profit for each lanyard is $0.75. Write and solve an inequality to determine the number of lanyards the class must sell to make a profit of at least $250.

3-4 Solving Multi-Step Inequalities (pp. 188–193)

EXAMPLES

Solve each inequality and graph the solution.

■ $18 + 3t > -12$

$$18 + 3t > -12$$
$$\underline{-18 \qquad -18}$$
$$3t > -30$$

Since 18 is added to 3t, subtract 18 from both sides.

$$\frac{3t}{3} > \frac{-30}{3}$$

Since t is multiplied by 3, divide both sides by 3.

$$t > -10$$

■ $3^2 - 5 \le 2(1 + x)$

$$3^2 - 5 \le 2(1 + x)$$

Simplify the left side using order of operations.

$$9 - 5 \le 2(1 + x)$$
$$4 \le 2(1 + x)$$

Distribute 2 on the right side.

$$4 \le 2(1) + 2(x)$$
$$4 \le 2 + 2x$$

Since 2 is added to 2x, subtract 2 from both sides

$$\underline{-2 \qquad -2}$$
$$2 \le 2x$$

$$\frac{2}{2} \le \frac{2x}{2}$$

Since x is multiplied by 2, divide both sides by 2.
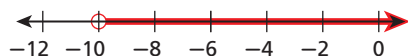
$$1 \le x$$
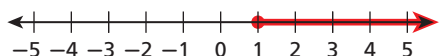
■ **Car rental company A charges $45 per day to rent a certain car. Car rental company B charges $30 per day plus $0.20 per mile to rent a similar car. For how many miles is the cost at car rental company B more than the cost at car rental company A?**

Let m represent the number of miles traveled per day.

$30	plus	$0.20	times	m	is more than	$45.
30	+	0.20	•	m	>	45

$$30 + 0.20m > \quad 45$$
$$\underline{-30 \qquad\qquad -30}$$
$$0.20m > \quad 15$$

Since 30 is added to 0.20m, subtract 30 from both sides.

$$\frac{0.20m}{0.20} > \frac{15}{0.20}$$

Since m is multiplied by 0.20, divide both sides by 0.20.

$$m > 75$$

The cost at company B is higher for more than 75 miles.

EXERCISES

Solve each inequality and graph the solutions.

40. $3x + 4 < 19$

41. $7 \le 2t - 5$

42. $\dfrac{m + 3}{2} > -4$

43. $9 - 3r > -9$

44. $\dfrac{5 - 4p}{7} < 3$

45. $5 < 2g + 19$

46. $2(x + 5) < 8$

47. $-4(2 - 5) > (-3)^2 - h$

48. $\dfrac{1}{5}x + \dfrac{1}{2} > \dfrac{4}{5}$

49. $0.5(b - 2) \le 4$

50. $\dfrac{1}{3}y - \dfrac{1}{2} > \dfrac{2}{3}$

51. $6 - 0.2n < 9$

52. Carl's Cable Company charges $55 for monthly service plus $4 for each pay-per-view movie. Teleview Cable Company charges $110 per month with no fee for movies. For what number of movies is the cost of Carl's Cable Company less than the cost of Teleview?

Use the table for Exercise 53.

Month	Account Balance ($)
1	245
2	275
3	305
4	335

53. If the pattern continues, for how many months will the account balance be less than $1000?

54. Company A has a sales position with a yearly salary of $42,000. Company B has a similar sales position with a salary of $39,000 plus 1% commission on yearly sales. For what amount of yearly sales is the salary at company A greater than the salary and commission at company B?
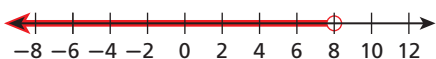
3-5 Solving Inequalities with Variables on Both Sides (pp. 194–200)

EXAMPLES

■ Solve $b + 16 < 3b$ and graph the solutions.

$$b + 16 > 3b$$

Subtract b from both sides so that the coefficient of b is positive.

$$\underline{-b \qquad -b}$$
$$16 > 2b$$

$$\frac{16}{2} > \frac{2b}{2}$$

Since b is multiplied by 2, divide both sides by 2.

$$8 > b$$

−8 −6 −4 −2 0 2 4 6 8 10 12

Solve each inequality.

■ $3(1 - k) > 4 - 3k$

$3(1) - 3(k) > 4 - 3k$ *Distribute the 3.*
$3 - 3k > 4 - 3k$
$\underline{+3k \qquad +3k}$ *Add 3k to both sides.*
$3 > 4$ *False statement*

There are no solutions.

■ $12x - 1 > \quad 12x - 19$ *Subtract 12x from*
$\underline{-12x \qquad -12x}$ *both sides.*
$-1 > \qquad -19$ *True statement*

All real numbers are solutions.

EXERCISES

Solve the inequality and graph the solutions.

55. $5 + 2m < -3m$
56. $y \le 6 + 4y$
57. $4c - 7 > 9c + 8$
58. $-3(2 - q) \ge 6(q + 1)$
59. $2(5 - x) < 3x$
60. $3.5t - 1.8 < 1.6t + 3.9$

Solve each inequality.

61. $d - 2 < d - 4$
62. $2(1 - x) > -2(1 + x)$
63. $4(1 - p) < 4(2 + p)$
64. $3w + 1 > 3(w - 1)$
65. $5(4 - k) < 5k$
66. $3(c + 1) > 3c + 5$

67. Hanna has a savings account with a balance of $210 and deposits $16 per month. Faith has a savings account with a balance of $175 and deposits $20 per month. Write and solve an inequality to determine the number of months Hanna's account balance will be greater than Faith's account balance.

3-6 Solving Compound Inequalities (pp. 202–208)

EXAMPLES

Solve each compound inequality and graph the solutions.

■ $-3 < c + 5 \le 11$ *Since 5 is added to c, subtract*
$\underline{-5 \qquad -5 \ -5}$ *5 from each part of the*
$-8 < c \qquad \le 6$ *inequality.*

Graph c > −8 and c ≤ 6.

Graph the intersection.

−10 −8 −6 −4 −2 0 2 4 6 8 10

■ $-2 + t \ge \quad 2$ OR $t + 3 < \quad 1$
$\underline{+2 \qquad +2 \quad -3 \quad -3}$ *Solve the simple*
$t \ge \quad 4$ OR $\quad t < -2$ *inequalities.*

Graph t ≥ 4 and t < −2.
Graph the union.

−5 −4 −3 −2 −1 0 1 2 3 4 5
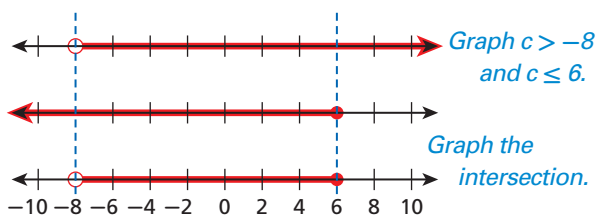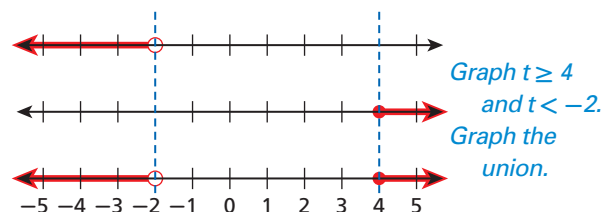
EXERCISES

Solve each compound inequality and graph the solutions.

68. $-4 < t + 6 < 10$
69. $-8 < k - 2 \le 5$
70. $-3 + r > 4$ OR $r + 1 < -1$
71. $n - 4 > 5$ AND $2 > n + 3$
72. $p + 8 > 6$ AND $12 \ge p + 7$
73. $3 < s + 9$ OR $1 > s - 4$

74. One day, the high temperature was 84°F and the low temperature was 68°F. Write a compound inequality to represent the day's temperatures.

75. The table shows formulas for the recommended heart rates during exercise for a person who is a years old. Write and solve a compound inequality to determine the heart rate range for a 16-year-old person.

Recommended Heart Rate Range	
Lower Limit	$0.5 \times (220 - a)$
Upper Limit	$0.9 \times (220 - a)$

CHAPTER TEST

Describe the solutions of each inequality in words.

1. $-6 \leq m$ **2.** $3t > 12$ **3.** $-x \geq 2$ **4.** $2 + b \leq 10$

Graph each inequality.

5. $b > -3$ **6.** $2.5 < c$ **7.** $y \leq -\sqrt{25}$ **8.** $3 - (4 + 7) \geq h$

Write the inequality shown by each graph.

9.

10.
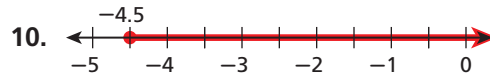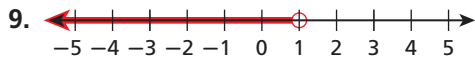
Write an inequality for the situation and graph the solutions.

11. Madison must run a mile in no more than 9 minutes to qualify for the race.

Solve each inequality and graph the solutions.

12. $d - 5 > -7$ **13.** $f + 4 < -3$ **14.** $4.5 \geq s + 3.2$ **15.** $g + (-2) \leq 9$

16. Students need at least 75 hours of volunteer service to meet their graduation requirement. Samir has already completed 48 hours. Write and solve an inequality to determine how many more hours he needs to complete.

Solve each inequality and graph the solutions.

17. $-2c \leq 2$ **18.** $3 > \dfrac{k}{2}$ **19.** $\dfrac{4}{5}x \leq -8$ **20.** $\dfrac{b}{3} > -7$

21. Marco needs to buy premium gasoline for his car. He has \$20 in his wallet. Write and solve an inequality to determine how many gallons of gas Marco can buy.

Gasoline Prices (\$)		
Regular	Plus	Premium
2.05	2.12	2.25

Solve each inequality and graph the solutions.

22. $3x - 8 < 4$ **23.** $-2(c - 3) > 4$ **24.** $5 \leq \dfrac{3}{4}n - 2^4$ **25.** $3 - 2a \leq -15 + (-9)$

Solve each inequality.

26. $2k - 6 > 3k + 2$ **27.** $2(5 - f) \leq f + 12$ **28.** $\dfrac{3}{2}d \leq -\dfrac{1}{2}d + 6$

29. Dion needs to rent a moving van for the day. Company A charges \$75 plus \$0.25 for every mile driven. Company B charges \$50 plus \$0.75 for each mile. For how many miles is company B less expensive than company A?

Solve each compound inequality and graph the solutions.

30. $-1 \leq x - 3 < 3$ **31.** $t + 7 < 3 \text{ OR } t - 1 > 4$ **32.** $d - 2 < 5 \text{ AND } d + 1 \geq 7$

33. The driving school instructor has asked Lina to stay within 2 miles of the posted speed limits. The current road has a speed limit of 45 mi/h. Write a compound inequality to show Lina's acceptable speeds s.

FOCUS ON SAT STUDENT-PRODUCED RESPONSES

Ten questions on the SAT require you to enter your answer in a special grid like the one shown. You do not have to write your answer in the boxes at the top of the grid, but doing this may help you avoid errors when filling in the grid. The circles must be filled in correctly for you to receive credit.

You cannot enter a zero in the first column of the grid. This is to encourage you to give a more accurate answer when you need to round. For example, $\frac{1}{16}$ written as a decimal is 0.0625. This should be entered in the grid as .063 instead of 0.06.

You may want to time yourself as you take this practice test. It should take you about 9 minutes to complete.

---

1. Mailing a standard-sized letter in 2005 by first-class mail cost $0.37 for a letter weighing 1 ounce or less and $0.23 for each additional ounce. How much did it cost, in dollars, to send a standard-sized letter that weighed 3 ounces?

2. If $p = q - 2$ and $\frac{q}{3} = 9$, what is the value of p?

3. Give the maximum value of x if
   $12 - 3(x + 1) \geq \frac{1}{2}(3 - 5)$.

4. Give the minimum value of x if $2x + y \leq 7x - 9$ and $y = -3$.

5. For what integer value of x is $2x - 9 < 5$ and $x - 1 > 4$?

6. What the minimum value of x that satisfies the inequality $z - 7.3 \geq 4.1$?

7. To be eligible for financial aid, Alisa must work at least 15 hours per week in a work-study program. She wants to spend at least 5 more hours studying than working each week. What is the minimum number of hours per day (Monday through Friday) that she must study to meet this goal and be eligible for financial aid?

8. For all real numbers a and b, define the operation # as follows:
   $$a \# b = 2a - b$$
   Given $a = 3$ and $a \# b = 1$, what is the value of b?

TEST TACKLER

Standardized Test Strategies

Short Response: Understand Short Response Scores

To answer a short-response question completely, you must show how you solved the problem and explain your answer. Short response questions are scored using a 2-point scoring rubric. A sample scoring rubric is provided below.

EXAMPLE 1

Short Response An online company offers free shipping if the cost of the order is at least $35. Your order currently totals $26.50. Write an inequality to show how much more you need to spend to qualify for free shipping. Solve the inequality and explain what your answer means.

2-point response:

Let c be the amount I must add to my order.
c plus the amount I already ordered must be at least $35.

$c + 26.50 \geq 35$

$c + 26.50 \geq 35$
$c + 26.50 - 26.50 \geq 35 - 26.50$
$c \geq 8.50$

Check:
$8.50 + 26.50 \geq 35$ ✓

To get free shipping on the order, I must spend at least $8.50 more since $8.50 + $26.50 is at least $35.

The student wrote and solved an inequality correctly. The student defined the variable used in the inequality, answered the question in a complete sentence, and showed an explanation for the work done.

1-point response:

$c + 26.50 > 35$
$c > 8.50$
$8.50

The student did not define the variable. The student gave a correct answer, but the inequality symbol shown in the student's work is incorrect. No explanation was given.

0-point response:

$9.25

The student gave an answer that satisfies the problem, but the student did not show any work or give explanation.

Scoring Rubric:

2 points: The student writes and correctly solves an inequality, showing all work. Student defines the variable, answers the question in a complete sentence, and provides an explanation.

1 point: The student writes and correctly solves an inequality but does not show all work, does not define the variable, or does not provide an explanation.

1 point: The student writes and solves an inequality but gives an incorrect answer. The student shows all work and provides an explanation for the answer.

0 points: The student gives no response or provides a solution without showing any work or explanation.

Read each sample and answer the questions that follow by using the scoring rubric below.

Scoring Rubric:

2 points: The student demonstrates a thorough understanding of the concept, correctly answers the question, and provides a complete explanation.

1 point: The student correctly answers the question but does not show all work or does not provide an explanation.

1 point: The student makes minor errors resulting in an incorrect solution but shows and explains understanding of the concept.

0 points: The student gives a response but shows no work or explanation, or the student gives no response.

Sample A
Short Response Write a real-world situation that can be modeled by the inequality $25s - 75 \geq 250$. Solve for s and explain how the value of s relates to your situation.

Student's Answer

A painter rents a booth at the county fair for $75. The artist sells his paintings for $25 each. If he makes at least $250 in profit, he can buy a new easel.

The artist has to sell at least 13 paintings.

1. What score should the student's answer receive? Explain your reasoning.

2. What additional information, if any, should the student's answer include in order to receive full credit?

Sample B
Short Response How do the solutions of $3s - 10 < 15 - 2s$ and $-34 + 9s \leq 4s - 9$ differ? How are the solutions alike? Include a graph in your explanation.

Student's Answer

3. What score should the student's answer receive? Explain your reasoning.

4. What additional information, if any, should the student's answer include in order to receive full credit?

Sample C
Short Response Explain the difference between the solution of the equation $x - 6 = 2x + 9$ and the solutions of the inequality $x - 6 < 2x + 9$.

Student's Answer

The equation has a solution of x = –15, and the inequality has a solution of x > –15. The equation is true only when x equals –15. The inequality is true for all values greater than –15.

5. What score should the student's answer receive? Explain your reasoning.

6. What additional information, if any, should the student's answer include in order to receive full credit?

CUMULATIVE ASSESSMENT, CHAPTERS 1–3

Multiple Choice

1. Which algebraic expression means "5 less than y"?

- (A) $5 - y$
- (B) $y - 5$
- (C) $5 < y$
- (D) $5 \div y$

2. Which is a simplified expression for $5 + 2(x - 5)$?

- (F) $2x$
- (G) $2x + 5$
- (H) $2x - 5$
- (J) $7x - 35$

3. If $t + 8 = 2$, find the value of $2t$.

- (A) -12
- (B) -6
- (C) 12
- (D) 20

4. The length of the rectangle is $2(x + 1)$ meters and the perimeter is 60 meters. Find the length of the rectangle.

$2(x + 1)$

4

- (F) 12 meters
- (G) 26 meters
- (H) 28 meters
- (J) 56 meters

5. Samantha deposited some money in her account in June. She deposited twice that amount in August. Samantha has less than \$600 in her account. If she made no other withdrawals or deposits, which inequality could be used to determine the maximum amount Samantha could have deposited in June?

- (A) $2x < 600$
- (B) $2x > 600$
- (C) $3x < 600$
- (D) $3x > 600$

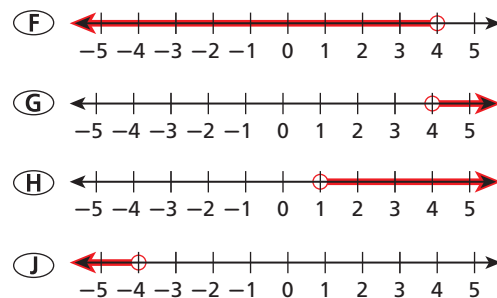**6.** Which proportion could be used to determine the ratio of the areas of these similar rectangles?

4 cm

10 cm

6 cm

15 cm
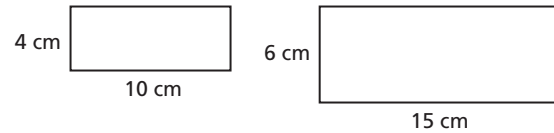
- (F) $\dfrac{2}{3}$
- (G) $\dfrac{2}{5}$
- (H) $\dfrac{4}{9}$
- (J) $\dfrac{4}{25}$

7. For which inequality is -2 a solution?

- (A) $2x < -4$
- (B) $-2x < 4$
- (C) $-2x > -4$
- (D) $-2x < -4$

8. Which graph shows the solutions of $-2(1 - x) < 3(x - 2)$?

- (F) ![number line from -5 to 5, shaded left with open circle at 4]
- (G) ![number line from -5 to 5, shaded right with open circle at 4]
- (H) ![number line from -5 to 5, shaded right with open circle at 1]
- (J) ![number line from -5 to 5, shaded left with open circle at -4]

9. Which compound inequality has no solution?

- (A) $x > 1$ OR $x < -2$
- (B) $x < 1$ AND $x > -2$
- (C) $x < 1$ OR $x < -2$
- (D) $x > 1$ AND $x < -2$

10. Which inequality has the same solutions as $p < -2$?

 (F) $p + 1 < -2$

 (G) $p + 4 < 2$

 (H) $2p + 1 < -4$
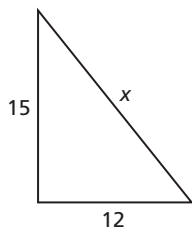
 (J) $3p < -12$

11. What is the greatest integer solution of $5 - 3m > 11$?

 (A) 0

 (B) -1

 (C) -2

 (D) -3

Gridded Response

12. The sum of the measures of any two sides of a triangle must be greater than the measure of the third side. What is the greatest possible integer value for x?

13. After 2 years, the simple interest paid on an investment of $2500 was $175. What percent was the interest rate?

14. Amy's bowling score in her third game was 10 points less than her score in the first game and 5 points more than her score in the second game. The total points for all three games was no more than 275. What is the highest number of points Amy could have scored in her first game?

15. Trevor needs a 93 on his second quiz to have a quiz average of 90. What score did Trevor receive on his first quiz?

16. The radius of a circle can be determined by the formula $r = \sqrt{\frac{A}{\pi}}$. What is the length in meters of the radius of a circle that has area of 314 square meters? (Use 3.14 for π.)

Short Response

17. Write 2 different inequalities that have the same solution as $n > 3$ such that

 a. the first inequality uses the symbol $>$ and requires addition or subtraction to solve.

 b. the second inequality uses the symbol $<$ and requires multiplication or division to solve.

18. Alison has twice as many videogames as Kyle. Maurice has 5 more videogames than Alison. The total number of videogames is less than 40.

 a. Write an inequality to represent this situation.

 b. Solve the inequality to determine the greatest number of videogames Maurice could have.

19. Donna's Deli delivers lunches for $7 per person plus a $35 delivery fee. Larry's Lunches delivers lunches for $11 per person.

 a. Write an expression to represent the cost of x lunches from Donna's Deli. Write an expression to represents the cost of ordering x lunches from Larry's Lunches.

 b. Write an inequality to determine the number of lunches for which the cost of Larry's Lunches is less than the cost of Donna's Deli.

 c. Solve the inequality and explain what the answer means. Which restaurant charges less for an order of 10 lunches?

Extended Response

20. Aleya has two employment opportunities. Company A offered her a yearly salary of $31,000. Company B offered her a similar position with a yearly salary of $27,000 plus 2.5% commission on her total sales for the year.

 a. Let x represent Aleya's total sales for the year at company B. Write an expression to represent the total income after one year at company B.

 b. Use your expression from part **a** to write an inequality that could be solved to determine the amount of sales for which the yearly income at company A would be greater than that at company B.

 c. Solve the inequality from part **b** and explain the meaning of the solution in relation to Aleya's decision to work for company A or company B.

 d. How much more than the salary at company A would Aleya make after one year at company B if her total sales for the year was $200,000?