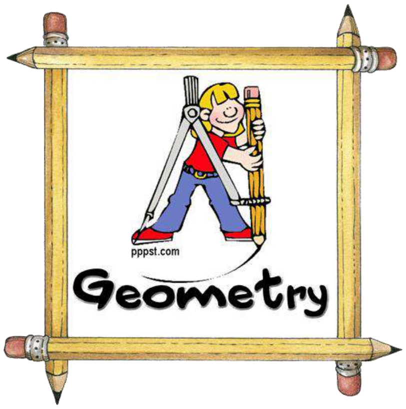
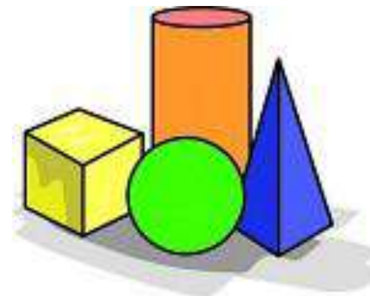
Integrated Algebra: Geometry



Topics of Study:

- Perimeter and Circumference
- Area
  - Shaded Area
  - Composite Area
- Volume
- Surface Area
- Relative Error

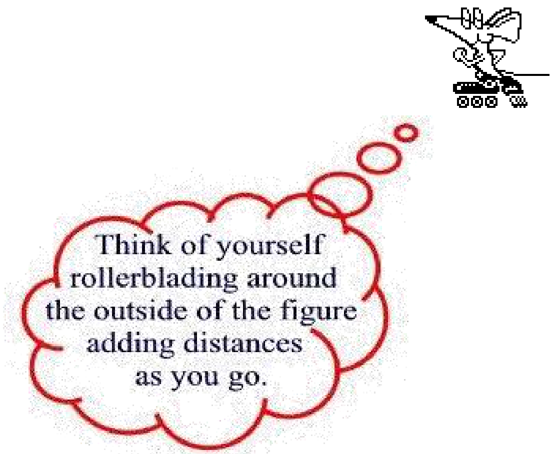


Links to Useful Websites & Videos:

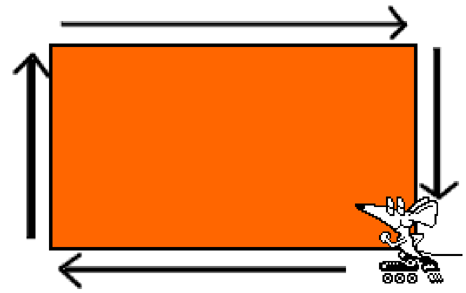
- **Perimeter and Area (Website)**
- **Perimeter and Area (Video)**
- **Perimeter and Area of Composite Figures (Video)**
- **Volume and Surface Area (Website)**
- **Volume and Surface Area of a Cylinder (Video)**
- **Relative Error (Video)**

Perimeter and Circumference:
Information/Formulas You Need to Know

Perimeter is the word used to describe the distance around the outside of a figure. It is measured in **units**.



To find the perimeter,
add together the lengths of all of the sides of the figure.



Triangle	3 sides
Quadrilateral	4 sides
Pentagon	5 sides
Hexagon	6 sides
Heptagon	7 sides
Octagon	8 sides
Nonagon	9 sides
Decagon	10 sides
Dodecagon	12 sides

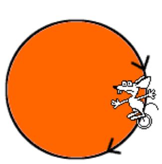
Refresh your polygon memories:

When working with perimeter, references may be made to the names of polygons. Listed at the left are some of the more common polygons whose names you should know.

Remember that "**regular polygons**" are polygons whose sides are all the same length and whose angles are all the same size. Not all polygons are "regular".

Circumference is the word used to describe the distance around a circle. It is also measured in **units.**

Like perimeter, the circumference is the distance around the outside of the figure. Unlike perimeter, in a circle there are no straight segments to measure, so a special formula is needed.

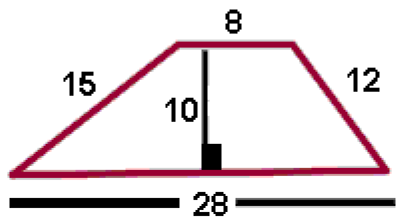
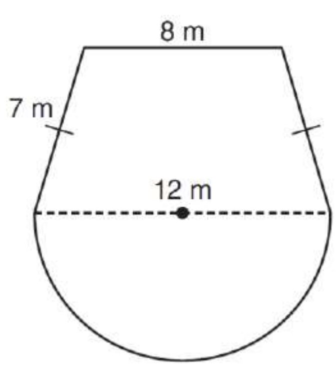


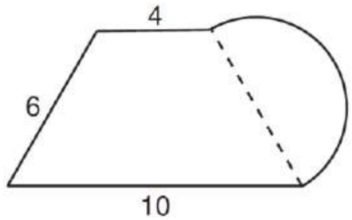
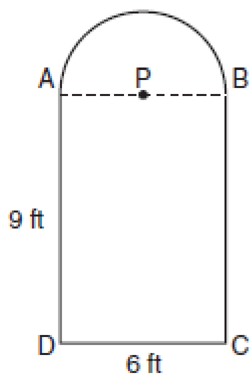
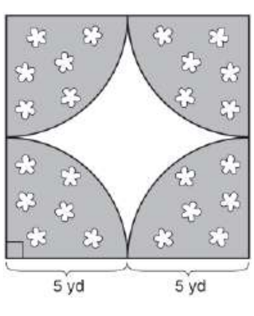
$$C = 2\pi r = \pi d$$

$C = 2\pi r$ Use when you know the radius.

$C = \pi d$ Use when you know the diameter.

Perimeter and Circumference:
Practice Problems

#	Question	Your Work	Final Answer
1	Find the perimeter of the trapezoid below, in units. 		
2	The circumference of a circle is 10π feet. What is the diameter of the circle?		
SAMPLE	A garden is in the shape of an isosceles trapezoid and a semicircle, as shown in the diagram below. A fence will be put around the perimeter of the entire garden. Which expression represents the length of fencing, in meters, that will be needed?  [A] $22+6\pi$ [B] $22+12\pi$ [C] $15+6\pi$ [D] $15+12\pi$	*Only find the perimeter around the outside of the figure.* ***Do not include the dashed line inside of the figure as part of the perimeter.*** *That dashed line is simply showing you the diameter.* *Remember to* ***only take half of the circumference of the circle****!* 7 + 8 + 7 + 6π 22 + 6π units	**Choice A**

3	What is the perimeter of the figure shown below, which consists of an isosceles trapezoid and a semicircle?  [A] $20+3\pi$ [B] $20+6\pi$ [C] $26+3\pi$ [D] $26+6\pi$		
4	Serena's garden is a rectangle joined with a semicircle, as shown in the diagram below. Line segment *AB* is the diameter of semicircle *P*. Serena wants to put a fence around her garden. Calculate the length of fence Serena needs to the *nearest tenth of a foot*.  		
5	A designer created a garden, as shown in the diagram below. The garden consists of four quarter-circles of equal size inside a square. The designer put a fence around *both* the inside and the outside of the garden. Which expression represents the amount of fencing, in yards, that the designer used for the fence?  [A] $40+10\pi$ [B] $40+25\pi$ [C] $100+10\pi$ [D] $100+25\pi$		

<u>AREA</u>:
<u>Information/Formulas You Need to Know</u>

<u>**AREA**</u> is the number of square units needed to <u>cover</u> a two-dimensional (flat) region. It is measured in <u>**square units**</u>.

Picture the square tiles (1 foot x 1 foot) that cover our classroom floor. How many tiles are there? If you count them up, that is the area of our classroom floor, in square feet. Formulas are a shortcut for physically counting square units.

IMPORTANT INFORMATION REGARDING THE AREA FORMULAS:

Height of a <u>Parallelogram</u>…

- A parallelogram has two sets of parallel sides. A rhombus is a *type* of parallelogram in which all sides are congruent.
- The <u>**height**</u> of a triangle, trapezoid, and parallelogram makes a right angle with (is <u>*perpendicular*</u> to) its base(s). The height does not have to be vertical because the shape can be turned.
- To find the area of a parallelogram, multiply the base (length) by its height.

Height/Bases of a <u>Trapezoid</u>…

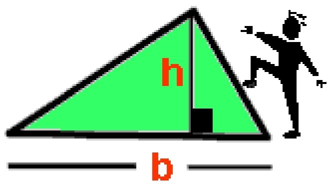
- Unlike the parallelogram, the trapezoid has exactly one set of parallel sides, called bases. The height is the perpendicular distance between the bases.
- The area formula for a trapezoid is very similar to that of a triangle except that the two bases must be added together first.

<u>Circles</u>...

- diameter = radius + radius **OR** diameter = 2 • radius
- diameter $\neq$ radius^2
- A sector of a circle is just another word for a “section” of a circle that includes its center. Think of a sector as a slice of pizza.
- **Use the π button on your calculator (2ND ^), not the decimal approximation! Do not round any numbers until you reach your final answer!**

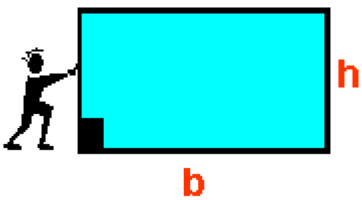
<u>Area Formulas</u>

***All of these formulas, except for the area of a trapezoid, must be memorized. The formula for the area of a trapezoid is given on the Integrated Algebra Regents Reference Sheet.**



Area of a Triangle

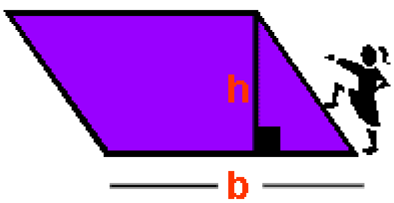
$$A = \frac{1}{2}bh$$



Area of a Rectangle

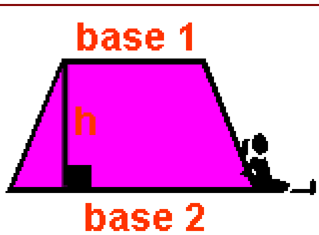
$$A = bh$$

$$A = lw$$



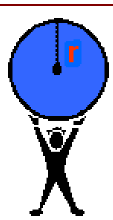
Area of a Parallelogram

$$A = bh$$



Area of a Trapezoid

$$A = \frac{1}{2}h(b_1 + b_2)$$



Area of a Circle

$$A = \pi r^2$$

Area of Sectors of a Circle

(Sectors are similar to "pizza pie slices" of a circle.)

Semi-Circle

(half of circle = half of area)

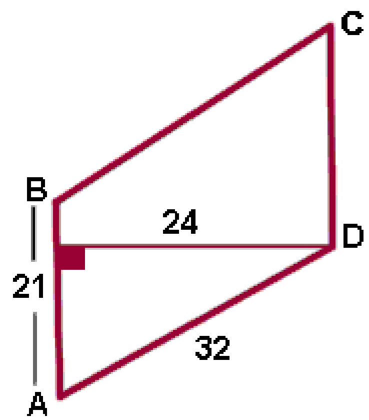
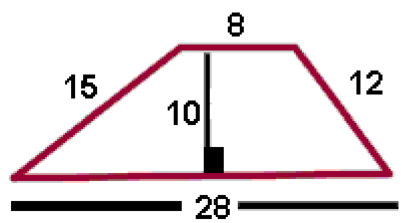
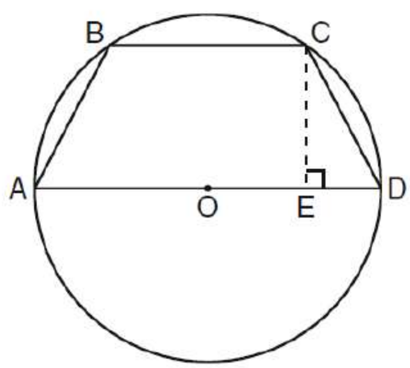
$$A = \frac{1}{2}\pi r^2$$

Quarter-Circle

(1/4 of circle = 1/4 of area)

$$A = \frac{1}{4}\pi r^2$$

Area:
Practice Problems

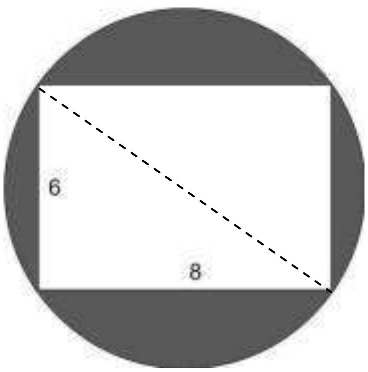
#	Question	Your Work	Final Answer
6	Find the area of parallelogram ABCD. 		
7	Find the area of the trapezoid below: 		
8	In the diagram below, the circumference of circle *O* is 16π inches. Then length of *BC* is three-quarters the length of diameter *AD* and *CE* = 4 inches. Calculate the area, in square inches, of trapezoid *ABCD*. 		

<u>SHADED AREA</u>:
<u>Information/Formulas You Need to Know</u>

In order to find the area of a shaded region…

1. **Find the area of the *entire* figure that is shaded.**
2. **Find the area of the figure that is not shaded.**
3. **Subtract the areas.**

<u>SAMPLE PROBLEM:</u>
Find the area of the shaded region below. Round the area to the nearest hundredth.



In order to find the diameter and radius of this circle, use the Pythagorean Theorem.

$6^2 + 8^2 = c^2$

$36 + 64 = c^2$

$100 = c^2$

$10 = c$

Therefore, the diameter of the circle is 10 units long, and the radius is 5 units long.

Step 1: Find the area of the *entire* figure that is shaded.

Area of the Circle = πr^2

Area of the Circle = $\pi \bullet 5^2$

Area of the Circle = $\pi \bullet 25$ or 25π square units ← *Exact area in terms of pi.*

Step 2: Find the area of the figure that is not shaded.

Area of the Rectangle = Length • Width

Area of the Rectangle = 6 • 8

Area of the Rectangle = 48 square units

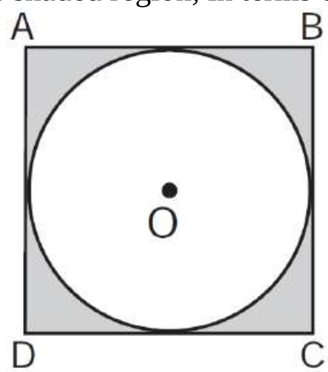
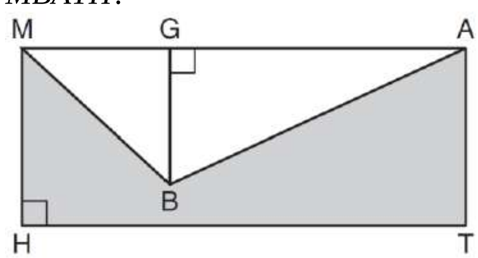
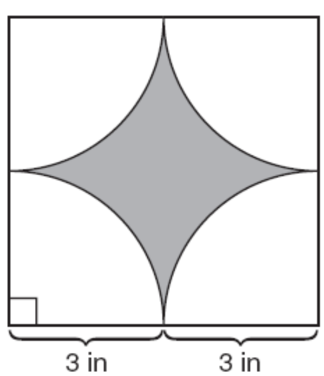
Step 3: Subtract the areas.

Shaded Area = Area of the Circle – Area of the Rectangle

Shaded Area = $25\pi - 48$ ← *Enter this into your calculator.*

Shaded Area = 30.54 square units ← *Final, rounded answer*

SHADED AREA:
Practice Problems

#	Question	Your Work	Final Answer
9	In the In the diagram below, circle *O* is inscribed in square *ABCD*. The square has an area of 36. What is the area of the shaded region, in terms of π? 		
10	In the diagram below, *MATH* is a rectangle, $GB = 4.6$, $MH = 6$, and $HT = 15$. What is the area of polygon *MBATH*?  1) 34.5 2) 55.5 3) 90.0 4) 124.5		
11	A designer created the logo shown below. The logo consists of a square and four quarter-circles of equal size. Express, in terms of π, the exact area, in square inches, of the shaded region. 		

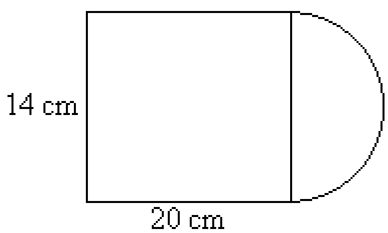
COMPOSITE AREA
Information You Need to Know

In order to find area of a composite figure (which is made of two or more figures)…

1. **Find the area of each individual figure. (Watch out for figures like semicircles, for which you have to divide the area by two.)**
2. **Add the individual areas together.**

SAMPLE PROBLEM

Find the composite area of the rectangle and semicircle. Round to the nearest hundredth.



Notice that in this figure, the width of the rectangle (14 cm) is also the diameter of the semi-circle.

Step 1: Find the area of the rectangle:

Area of the Rectangle = Length • Width

Area of the Rectangle = 14 • 20

Area of the Rectangle = 280 square centimeters

Step 2: Find the area of the semi-circle:

Area of the Semi-circle = $\frac{1}{2}\pi r^2$

Area of the Semi-circle = $\frac{1}{2} \cdot \pi \cdot 7^2$

Area of the Semi-circle = $\frac{1}{2} \cdot \pi \cdot 49$

Area of the Semi-circle = 24.5π square centimeters

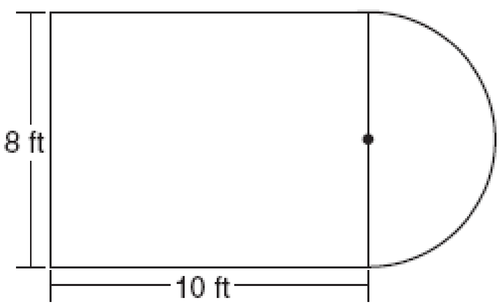
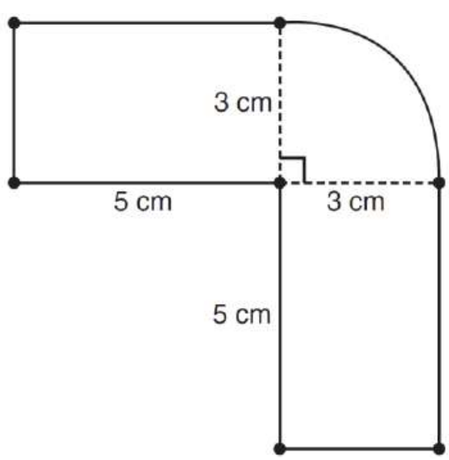
Step 3: Add the individual areas together:

Composite Area = Area of Rectangle + Area of Semi-Circle

Composite Area = $280 + 24.5\pi$

Composite Area ≈ 356.97 square centimeters

COMPOSITE AREA
Practice Problems

#	Question	Your Work	Final Answer
12	Luis is going to paint a basketball court on his driveway, as shown in the diagram below. This basketball court consists of a rectangle and a semicircle.  Which expression represents the area of this basketball court, in square feet? 1) 80 2) $80 + 8\pi$ 3) $80 + 16\pi$ 4) $80 + 64\pi$		
13	The figure shown below is composed of two rectangles and a quarter circle.  What is the area of this figure, to the *nearest square centimeter?* 1) 33 2) 37 3) 44 4) 58		

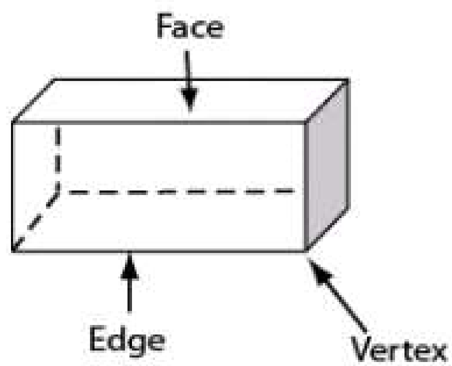
THREE-DIMENSIONAL FIGURES
Information You Need to Know

Three-dimensional figures, or **solids**, can be measured in 3 directions, such as length, width, and height. They enclose a part of space.

A **polyhedron** is a solid whose faces (sides) are polygons. In other words, all of the faces are flat surfaces. (The plural form of this noun is polyhedra.)

A **prism** is a polyhedron that has two congruent, parallel bases that are polygons.

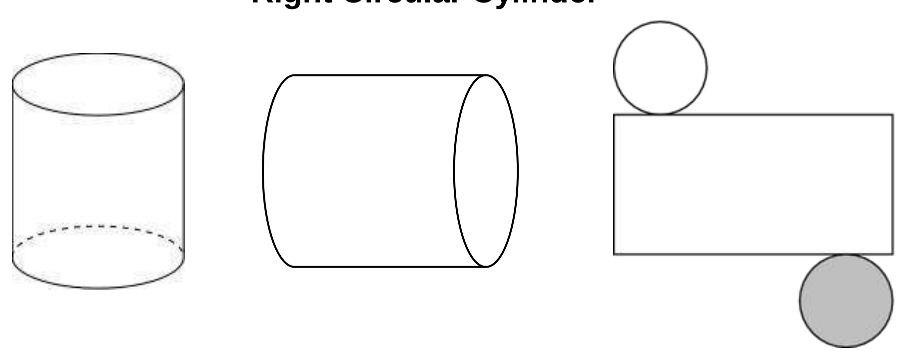
Right Rectangular Prism
Rectangular Solid



All faces are rectangles.

A **cylinder** has two parallel, congruent bases that are circles. It is not a polyhedron, but you can relate it to a prism.

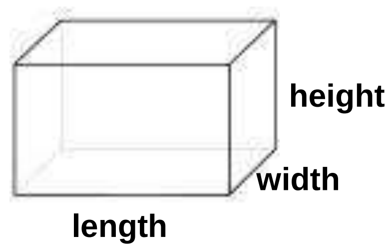
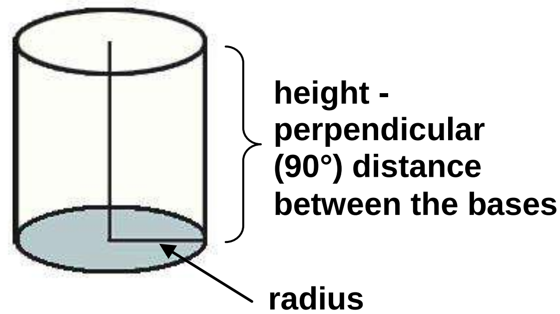
Right Circular Cylinder



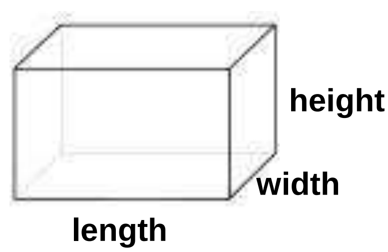
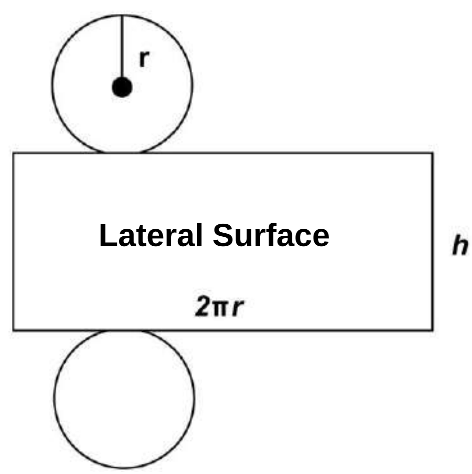
Interestingly, when a right circular cylinder is "opened up," its net consists of a rectangle and two circles.

VOLUME & SURFACE AREA
OF PRISMS AND CYLINDERS
Information/Formulas You Need to Know

Volume is the number of **cubic units** that are needed to completely **FILL** the inside of a three-dimensional space.

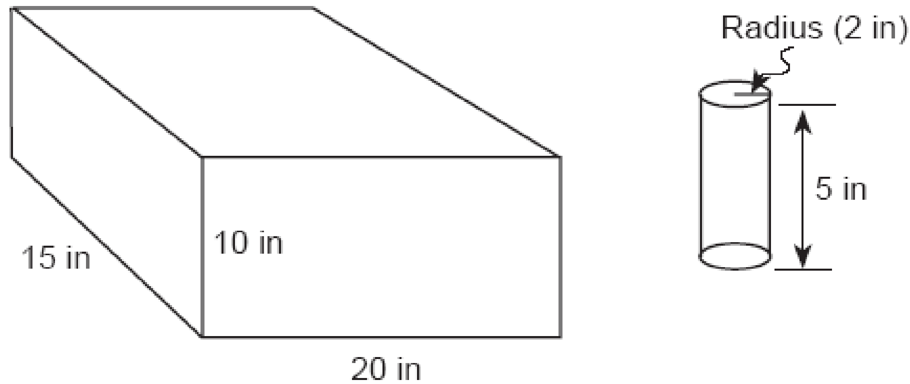
Right Rectangular Prism	**Right Circular Cylinder**
	
$\mathbf{V} = \boldsymbol{l} \bullet \boldsymbol{w} \bullet \boldsymbol{h}$ *Memorize this!	$V = \pi r^2 h$ *This formula is provided for you on the Regents Reference Sheet. ☺

Surface Area: number of **square units** that are needed to completely **COVER** the surfaces of a three-dimensional space.

Right Rectangular Prism	**Right Circular Cylinder**
	
$SA = 2lw + 2hw + 2lh$	$SA = 2\pi r^2 + 2\pi rh$
Both of these formulas are provided for you on the Regents Reference Sheet. ☺	

VOLUME SAMPLE PROBLEM

In the accompanying diagram, a rectangular container with the dimensions 10 inches by 15 inches by 20 inches is to be filled with water, using a cylindrical cup whose radius is 2 inches and whose height is 5 inches. What is the maximum number of full cups of water that can be placed into the container *without the water overflowing the container*?



Step 1: Volume of Rectangular Container

Volume of Rectangular Container = $l \cdot w \cdot h$

Volume of Rectangular Container = 15 • 20 • 10

Volume of Rectangular Container = 300 cubic inches

Step 2: Volume of Cylindrical Cup

Volume of Cylindrical Cup = $\pi \cdot r^2 \cdot h$

Volume of Cylindrical Cup = $\pi \cdot 2^2 \cdot 5$

Volume of Cylindrical Cup = 20π cubic inches

Step 3: Divide

$$\frac{300 \text{ cubic inches}}{20\pi \text{ cubic inches}} \approx 47.7 \text{ cups}$$

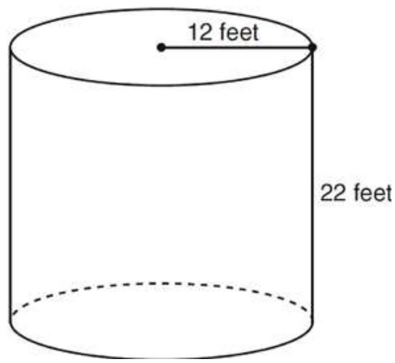
The question asks how many *full* cups of water can be placed into the container without the water overflowing, so do not round up to 48. *Round down* to **<u>47 cups</u>**.

VOLUME
Practice Problems

#	Question	Your Work	Final Answer
14	A fish tank with a rectangular base has a volume of 3,360 cubic inches. The length and width of the tank are 14 inches and 12 inches, respectively. Find the height, in inches, of the tank.		
15	A cylindrical oil storage tank has a height of 18 meters and a diameter of 14 meters. If the tank is 50% full, how many kiloliters of oil, to the nearest tenth, does it contain? (1 cubic meter = 1 kiloliter)		
16	A rectangular container with the dimensions 4 inches by 8 inches by 3 inches is to be filled with water, using a cylindrical cup whose radius is 1.5 inches and whose height is 4 inches. What is the maximum number of full cups of water that can be placed into the container without the water overflowing the container?		

SURFACE AREA SAMPLE PROBLEM

The cylindrical tank shown in the diagram below is to be painted. The tank is open at the top. Only the outside, not the inside of the can, needs to be painted. Each can of paint covers 600 square feet. How many cans of paint must be purchased to complete the job?



Step 1: Find the surface area of the cylinder, except for the circular top.

The surface area formula for a cylinder is SA = $2\pi r^2 + 2\pi r$h. However, we are only painting one of the two circles, so we need to adjust the formula: SA = $\pi r^2 + 2\pi r$h.

Surface Area = $\pi r^2 + 2\pi r$h

Surface Area = $\pi \bullet 12^2 + 2 \bullet \pi \bullet 12 \bullet 22$

Surface Area = $144\pi + 528\pi$

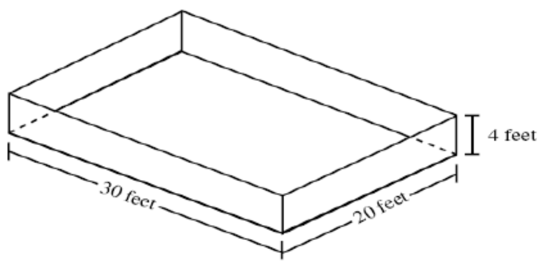
Surface Area = 672π square feet

Step 2: Divide to determine the number of cans of paint needed.

$\dfrac{672\pi \text{ square feet}}{600 \text{ square feet}}$ **≈ 3.51858… cans of paint**

Since we cannot buy a fractional part of a can of paint, we must round our answer to a whole number. The answer must be *rounded up* to ensure that we have enough paint to cover the entire cylinder. Therefore, the correct answer is **4 cans of paint**.

SURFACE AREA
Practice Problems

#	Question	Your Work	Final Answer
17	Andrew works for a company that built a goldfish pond for a local university. He has to plaster the interior sides of the pond, which is shaped like a rectangular prism with the dimensions shown in the picture below.  a. What is the total surface area of the interior sides and bottom of the pond? Show or explain your work. b. If the company charges $1.50 per square foot to plaster the pond, what will it cost to plaster the 4 interior sides and the bottom of the pond?		
18	Michael decides to give his tennis coach a gift at the end of the season. He buys him a can of tennis balls, and the can is shaped like a cylinder. The diameter of the can is 8 centimeters, and the height is 20 centimeters. a. How many square centimeters of wrapping paper will Michael need to completely cover the can? b. How many square centimeters of wrapping paper will Michael need if he decides not to cover the top of the can?		
19	A cylinder has a height of 11 feet and a volume of 275π cubic feet. What are the radius and surface area of this cylinder?		

RELATIVE ERROR
Information/Formulas You Need to Know

What is Error?

- Error in measurement is not the same as a mistake. It is actually the difference between the measured value and the true value of what you are measuring.
- No measurement is 100% accurate; it can always be more precise when a smaller unit is used.
- Area, surface area, and volume calculations depend on linear measurements, such as the length of a side/edge. Therefore, the error present in the initial linear measurement will build/compound when it is used towards calculations in square and cubic units. The error increases because the initial measurement is being multiplied over and over again.

Formula for Relative Error:

- Relative error is the ratio between the error in a measurement and the true value of the measurement.
- **Relative Error =** $\dfrac{|\text{measured value} - \text{true value}|}{\text{true value}}$
- Relative error is expressed as a decimal.
- The units cancel out, so there are no units.
- To convert "relative error" to "percent of error," simply multiply by 100.

SAMPLE PROBLEM

A carpenter measures the length of a piece of wood to be 60.7 inches. The exact measurement is 64 inches. Find the relative error in the carpenter's measurement to the nearest hundredth.

$$\frac{|60.7 - 64|}{64} = \frac{|-3.3|}{64} = \frac{3.3}{64} = .05$$

.05 is the relative error of the measurement.

RELATIVE ERROR
Practice Problems

#	Question	Your Work	Final Answer
20	The groundskeeper is replacing the turf on a football field. His measurements of the field are 130 yards by 60 yards. The actual measurements are 120 yards by 54 yards. Which expression represents the relative error in the measurement? 1) $\frac{(130)(60)-(120)(54)}{(120)(54)}$ 2) $\frac{(120)(54)}{(130)(60)-(120)(54)}$ 3) $\frac{(130)(60)-(120)(54)}{(130)(60)}$ 4) $\frac{(130)(60)}{(130)(60)-(120)(54)}$		
21	To calculate the volume of a small wooden cube, Ezra measured an edge of the cube as 2 cm. The actual length of the edge of Ezra's cube is 2.1 cm. What is the relative error in his **volume calculation** to the *nearest hundredth*? 1) 0.13 2) 0.14 3) 0.15 4) 0.16		
22	Sarah measures her rectangular bedroom window for a new shade. Her measurements are 36 inches by 42 inches. The actual measurements of the window are 36.5 inches and 42.5 inches. a. Using the measurements that Sarah took, determine the number of square inches in the area of the window. b. Determine the number of square inches in the actual area of the window. c. Determine the relative error in calculating the area. Express your answer as a decimal to the *nearest thousandth*.		

ANSWER KEY

#	Answer	Notes
1	63 units	Add up outside measurements only.
2	10 feet	
3	Choice (A)	Take half of the circumference of the circle.
4	33.4 feet	Take half of the circumference of the circle.
5	Choice (A)	The four quarter-circles are equivalent to one full circle.
6	504 square units	Use the two perpendicular sides.
7	180 square units	
8	56 square inches	
9	$(36 - 9\pi)$ square units	
10	55.5 square units	
11	$(36 - 9\pi)$ square inches	The “exact” answer is the unrounded one. Leave in terms of π.
12	Choice (2)	Take half of the area of the circle.
13	Choice (2)	
14	20 inches	
15	1385.4 kL	
16	5 cups	Round down so that the water does not overflow.
17	a. 1000 square feet b. $1500	
18	a. 604 square cm b. 553 square cm	Round up to completely cover the can with paint. Use your unrounded answer from part (a) to solve part (b).
19	5 feet	
20	Choice (1)	
21	Choice (2)	Calculate the measured and actual volumes first.
22	a. 1512 square inches b. 1551.25 square inches c. .025	