6

Impulse and Linear Momentum

In the first five chapters of the text, we developed and applied the principles of Newtonian physics; the focus was on forces exerted on objects and on careful description of the details of the resulting motion. In this chapter, we introduce a new approach involving physical quantities that remain *constant* for a system of one or more objects when the external environment has no net effect on the system. When the same physical quantity *does* change due to the action of the environment we can account for that change, as we can always find a new system where this quantity remains constant. This means that the physical quantity is *conserved.*

Section 6.1 illustrates this approach by considering the mass of a system of objects. The remainder of the chapter focuses on the ideas of impulse and momentum.

The content-based learning goals for impulse and momentum are listed below.

Students should be able to:

1. Explain the concept of a "system" and systems thinking in physics.
2. Explain the difference between the terms "constant" and "conserved". Recognize that momentum is a conserved quantity, but not necessarily constant in a particular system.
3. Choose a system and initial and final states when analyzing a process involving impulse and momentum.
4. Represent processes involving impulse and momentum using bar charts. Be able to identify the system, initial and final states, and decide whether momentum is constant or not in the process. If momentum is not constant, they need to be able to account for the change through impulse.
5. Compare and contrast force and impulse, impulse and momentum, force and momentum.
6. Apply momentum and impulse to solve problems in one and two dimensions.

7. Explain the relationship between the generalized impulse-momentum principle and Newton's laws.
8. Analyze inelastic collisions.
9. Be able to use momentum and impulse ideas to explain how practical applications such as air bags work to save lives.

We have broken the chapter into three parts:

I. *Helping students develop the main ideas involving impulse and momentum*
II. *Applying the general impulse-momentum principle to physical processes, including the use of words, sketches, impulse-momentum bar charts (a new representation), and equations*
III. *Applying the impulse-momentum principle and knowledge from the first three chapters to interesting real-life physical processes*

For each part, we provide examples of activities that can be used in the classroom and brief discussions of anticipated student difficulties with the subject matter.

Chapter subject matter	**Related textbook and ALG sections**	**Textbook videos**	**ALG videos**
Helping students develop the main ideas involving impulse and momentum	6.1- 6.3	OET 6.1 (p149), TET 6.2 (p150)	6.2.3
Applying the general impulse-momentum principle to physical processes, including the use of words, sketches, impulse-momentum bar charts (a new representation), and equations	6.4, 6.5	6.1 (p158)	
Applying the impulse-momentum principle and knowledge to interesting real-life physical processes	6.6. 6.7	p164	
Nontraditional end-of-chapter questions and problems Choose answer and explanation (CAE): Q6.8, Q6.14 Evaluate (reasoning or solution…) (EVA): Q6.15, Q6.16, Q6.18, P6.74			

Linearization (LIN): P6.47 Multiple possibility and tell all (MPO): Q6.17, P6.5 Jeopardy (JEO): P6.25, P6.26, P6.34 Design an experiment (or pose a problem) (DEX): P6.78, P6.77 Problem based on real data (that students can collect by themselves) (RED): P6.72, P6.73, P6.74

Brief summary of student difficulties with impulse and momentum

Students need to be reminded that before they do any analysis, they need to carefully identify a system and keep track of the initial and final momenta of each object in the system. The system choice is often fairly obvious, but we learn later that choosing a one-object system or a two-object system allows students to answer different questions. Objects that are outside the system can change its momentum; this does not mean that the momentum if not *conserved*, it only means that in that particular system it is not *constant*. The difficulty understanding the difference between constancy and conservation is even bigger for instructors who are used to the word conserved being used in place of constant. Another difficulty stems from identifying initial and final states of the system, without this step no process can be analyzed. Finally, students need to remember that momentum and impulse are vector quantities. To have numerical values students need to choose the axis and work with components.

I. Students develop the main Ideas of impulse and momentum

Students first meet the idea of a conserved quantity (mass) in Section 6.1. The mass of an isolated system remains constant. If the system is not isolated, mass can enter or leave the system, but this is compensated for by the mass of the environment decreasing or increasing, respectively (or in some new system the mass is constant). This is what makes mass a conserved quantity. The section provides a concrete introduction to this important subject and shows students a useful representation for a conserved quantity: a bar chart. The goal of this section is to help students develop a conceptual understanding of, and representational abilities relating to, conserved quantities so that they can transfer these abilities to the more abstract quantity of momentum. We suggest that students do the ALG Activities 6.1.1-6.1.4 and then read the book (ALG Activity 6.1.5).

In Section 6.2, the goal is for students to invent a new physical quantity that is a product quantity. So far, they have used data to invent ratio quantities—the rate of change of position, the rate of change of velocity. This time they have to invent a physical quantity that is the product of two others, which (perhaps surprisingly) is more difficult than a quantity that is the ratio of two others. In addition, one of the quantities in this product is a vector and the other one is a scalar. Here students see again that when a vector quantity in physics is multiplied by a scalar the result is a new quantity (new units), not the old quantity of a different magnitude. And finally, for a system, they need to realize that these new quantities add together as vectors. All activities in Section 6.2 of the ALG help students invent this quantity and test whether it is constant in an isolated system.

Specifically, ALG Activities 6.2.1 and 6.2.2 and the textbook's Observational Experiment Table 6.1 describe a series of experiments in which two carts collide. Students need to consider the system as the combination of two carts and analyze and compare the initial and final amounts of different quantities describing that system. They find that the sum of the mass times velocity of all the objects in an isolated system ($\Sigma m\vec{v}$) is the same for both the initial and final states of the system. The product $m\vec{v}$ is given the name *linear momentum* of an object and the sum is given the name *momentum of the system*.

If you are showing students similar experiments in class, it is important not to say the word momentum before students come up with the concept (this is true for any new quantity). You will notice that no new concept is defined in the book before students have a chance to "construct" it from concrete experience, either by doing and analyzing a real experiment or making sense of provided data. Helping students to create "an image" in the brain prior to providing an abstract definition makes the memories of the concept more accessible. It is also important to recognize that if you let students invent the quantity on their own, they may come up with other quantities that remain constant for an isolated system. For example, for *elastic* collisions the sum of the products mass times speed of all objects remains constant. Students might come up with this quantity and simply ignore the one type of (*inelastic*) collision where $\sum mv$ does not remain constant before and after the collision. It is helpful to remind them that they are trying to find a physical quantity that stays the same before and after the collision for *all* possible cases; they are not allowed to ignore one case. This quantity is the sum of the products of mass and velocity of all objects in the system.

The constancy rule for the momentum of a multi-object system is tested in ALG Activities 6.2.3 and 6.2.4 and in the textbook Testing Experiment Table 6.2. If you are not using the ALG, encourage students to make predictions before reading the outcomes of the testing experiments in the textbook. Insist that they explicitly use the rule under test when making the predictions. End-of-chapter Questions 3, 6, 7, 18 are appropriate formative assessment questions here and Problems 1-9 can be assigned for homework.

We suggest that you focus on the following important points:

1. Start by identifying a system and keep track of the initial and final momentum of each object in the system.
2. Decide whether the system can reasonably be considered isolated. Equal magnitude and oppositely directed forces can be ignored, since in combination they do not change the momentum of the system. However, friction forces can be considerable and each case needs to be examined separately.

3. Remember that momentum is a vector quantity analyzed using the momentum components along each coordinate axis. This requires a well-defined coordinate system and the use of correct signs for the momentum components.

In Section 6.3, students rearrange Newton's second law for a one-particle system to come up with the idea of the impulse that an external force exerts on the system object (ALG Activity 6.3.1). They also use Newton's second and third laws to understand momentum constancy for an isolated system. After students see how the momentum constancy idea follows from Newton's laws, it is helpful to emphasize that momentum is not really something new. It is something they have already learned, just reconceptualized in a different language, the language of a conserved physical quantity. Another important point is that intuitively student have a great feeling for momentum, as they know that a heavier faster moving object does more damage than a slower or a lighter one. They just need to give this feeling a name!

ALG Activities 6.3.2–6.3.4 can be done in class before students read the book (ALG Activity 6.3.5). We have a wealth of questions and problems for them to really understand the vector nature of impulse and its relationship to momentum. End of chapter Questions: 12, 13, 14, 26 and EOC Problems 10-26.

II. Develop skills to apply the generalized impulse-momentum principle in a way that leads to understanding

In Section 6.4, students learn the generalized impulse-momentum principle in vector and component forms. It can be used for any process with an isolated or non-isolated system. They use a new qualitative way to represent such processes—impulse-momentum bar charts. These charts serve a similar role to that played by force diagrams in Newtonian physics. They have bars on the left for components of the initial momenta of the objects in the system, then a bar in the middle to indicate the component of the impulse exerted and then bars on the right reserved for the components of momenta in the final state. Students draw the bar charts after they have identified the system and initial and final states and the positive direction(s). When drawing bar charts, the main difficulties are how to decide what objects are in the system and how to represent the motion of objects described in the problem statement as bars. Students often have trouble with the idea of indicating momentum in the negative direction using downward bars and correctly incorporating external impulses into the bar chart. We suggest that students start by working on the ALG Activity 6.4.1 because at the end of it they are asked to read the textbook and learn how to draw bar charts. However, this skill comes on the "need-to-know" basis not as an isolated assignment. The ALG activities that follow (Activities 6.4.2 – 6.4.9) will help students practice making and evaluating bar charts (reading and writing with the bar charts) and, finally, Activity

6.4.10 makes them question the difference in the meanings of the terms conserved and constant.

Notice ALG Activity 6.4.4 and matching Example 6.3 in the text (the happy and sad balls). They represent a very productive exercise that allows students to wrestle with the difficulties described. First, demonstrate in class the experiment where the sad ball fails to knock over the wooden block, but the happy ball does. In a small studio-class environment you can then ask each group of students to draw a bar chart of the process on a whiteboard with the goal of explaining why the happy and sad balls had different effects on the block (they should ignore the impulse exerted by Earth during the collision). Students will naturally ask whether or not to include the block in the system. You can encourage some groups to treat the ball as the system and other groups to treat the ball and block together as a system during the collision. Either approach will work, and great discussions can follow when they present their work to the rest of the class.

In a large lecture, you can break the exercise into a couple of steps: (1) Ask students to consider the ball as an isolated system and then vote on the correct impulse-momentum bar chart. They should then discuss their votes with each other and vote again; (2) Ask them to do the same analysis considering both the board and the ball as a system. Ask them to vote on the bar chart, then discuss with neighbors, then vote again. In this case it is important to recognize that the board receives some momentum from the sad ball (it rocks back and forth) even though it does not fall over. What students take away from this exercise is not only that the direction of motion matters, but that a bar chart is a tremendously powerful way to analyze a physical process.

Often students want to know why there isn't an "initial" and "final" impulse. Students are learning that impulse does not describe a state of the system in the way that momentum does. Momentum "resides" in the system, whereas impulse is the mechanism through which the momentum of the system can be changed—it does not reside in the system. It is ontologically distinct from momentum, and this point must be emphasized. Impulse quantifies a continuous process that adds or takes momentum away from the system from the initial state all the way through to the final state. The shading of the area where students have to place the impulse bar indicates that impulse is a quantity fundamentally different from momentum. On a bar chart, the total height of the bars in the initial and impulse regions must equal the total height of the bars in the final region. To help students with this, we overlay a grid onto each bar chart.

After students learn how to draw bar charts and use them to apply the generalized impulse-momentum equation to a particular process, they can apply this method at home to solve EOC Problems 27–34. We also have a novel EOC Questions that are will be challenging and helpful for the students. These are 9, 11–14.

In Section 6.5, students learn general problem-solving skills for using momentum to analyze physical processes. ALG Activity 6.5.1 is the same as Example 6.4 in the textbook. It describes a process that students need to analyze. They then represent the process with a sketch. Then they use the sketch to represent the process with a bar chart. Finally, they use the bar chart to apply the generalized impulse-momentum equation to the process.

To determine the impulse caused by the force that an external object exerts on a system object, students will need the time interval that the force is exerted. In the case of an object coming to rest (Example 6.5), often it is the stopping distance that is known, not the time interval the object took to stop. An important method for converting a stopping distance to a stopping time interval is summarized and used in this section. It provides a good opportunity for the students to review kinematics.

The ALG activities in Section 6.5 provide multiple opportunities to practice the problem solving strategy. Activity 6.5.6 can be used as a lab. At home students can tackle EOC problems in Section 6.5. We especially recommend Problems 45, 46, 47, 49, 50 and 69, 74 and 76.

III. The application of the generalized impulse-momentum principle to analyze some interesting real-world applications

Jet propulsion is the subject of Section 6.6. It does *not* include the continuous ejection of fuel from a rocket and its continual mass change—a subject that requires the use of calculus. However, it addresses conceptually the common belief that the fuel has to push against something for the rocket to accelerate. We suggest that students work on the ALG Activity 6.6.1 first and then read the textbook section (ALG Activity 6.6.2)

Section 6.7 is dedicated to two-dimensional collisions. The major issue here is to help students recognize that motions in *x*- and *y*-directions are independent and need to be treated separately, thus they need two bar charts for each collision – one for each direction. ALG Activity 6.7.3 is rather helpful here.

After this difficulty is overcome, the next one is whether the system can be isolated. Students may be confused about why the force of friction exerted by the surface on the cars is ignored when the cars collide (textbook Example 6.7) or why we don't worry about the gravitational force exerted by Earth on objects when they break apart (Example 6.8). Students should recognize that in collision events the initial and final states of the system are very close together in time. When this is the case, external forces are multiplied by a small Δt, resulting in a small or negligible impulse contribution to the bar chart. This only works if the interaction time is short and/or the external forces are small relative to the interaction forces between the objects that are included in the system.

End of chapter problems in Section 6.7 can be assigned in class or for homework, we especially recommend 60 and 62. Problem 60 changes the context of collisions as often students think that what applies to cars does not apply to celestial objects. Problem 62 uses non-traditional representations for the studies of momentum to help students find consistency. Notice the problems without numerical values in this chapter. Research indicates that students have tremendous difficulties with these compared to similar problems that have numbers. Do not skip these (Problems 21 and 76). Finally, EOC Question 26 will challenge even your most advanced students.