

# Instructor's Guide for College Physics: Explore and Apply, 2nd Edition

Eugenia Etkina, Rutgers University

David Brookes, Cal State University Chico

Gorazd Planinsic, University of Ljubljana, Slovenia

Alan Van Heuvelen, Rutgers University

©2019 | Pearson

# 1

## Introduction

The purpose of this Instructor's Guide is to assist you in helping your students learn algebra-based physics using the textbook *College Physics: Explore and Apply*, the *Active Learning Guide* (ALG), Mastering Physics, and other support materials. This guide will walk you through the approach we take when we teach, as well as other approaches you might take in using this material in your course. Although both the textbook and the ALG grew out of the Investigative Science Learning Environment, an approach we developed at Rutgers University and California State University, Chico, the textbook itself can work with any curriculum and any instructional approach.

### An effective approach to teaching physics

Discussing an effective approach to teaching physics should start with a few foundational principles of learning for which we will not provide theoretical arguments but which will consider as similar to geometrical axioms that do not require proof. These foundational principles are as follows:

1. Learning is a physical process that involves changes in the brain and body of the learner. It happens when the learner establishes new neuronal connections to the existing ones. The corollary of this principle is that no one can learn by observing somebody else or listening to somebody else without a purposeful effort to connect these sensory experiences to what they already know and to actively test new ideas. This corollary is known as *active learning*.
2. Learning is a social process that involves people sharing, debating and testing their ideas in interactions with others. The corollary of this principle is that it is very difficult to learn something in solitude without being socially engaged with other people. This corollary is known as *group work*.

There are many curricular approaches that use these principles to design student activities and assessment instruments and to create written texts. In this regard, our approach is no different to others. It encourages students to actively construct their own knowledge by connecting it to what they already know and by collaborating with others. However, we go a step further. We help our students learn physics by actively constructing their knowledge in collaboration with others by engaging them in the activities that mirror activities of physicists when they create and apply knowledge. In other words, we engage students in doing physics while learning it. Why is this approach useful?

We believe that one of the primary goals of K-12 education and higher education in particular is to help students become independent thinkers. To contribute constructively to society, they need to be able to critically analyze information and to generate new knowledge based on the assumptions of the physical and social environment in which they operate. Thus, physics education at any level needs to be concerned with more than students learning the “final product of physics.” Our students need to know where this final product came from. They need to understand the rules of the intellectual “game” that is “played” by those who develop this product and, most importantly, be able to engage in this process of creating, evaluating, and applying new knowledge.

This is especially important now when K-12 education follows the Next Generation Standards which view the learning of science in a three-dimensional way: students learning science should not only master disciplinary core ideas, but be engaged in the practices of science, and be skilled in the ideas that permeate all topics (called cross-cutting concepts). A student, learning science in K-12 classrooms, will come to college knowing how to design experiments, argue from evidence, evaluate claims, and so forth. A student who understands the process of generating knowledge can use this understanding to become a lifelong learner and a critical evaluator of claims in any field.

The idea that students should learn physics by practicing it forms the basis for the Investigative Science Learning Environment (ISLE)<sup>1</sup>. This framework originated in the work of Eugenia Etkina in the early 1990s. She designed a logical progression of student learning of physics that mirrors the processes in which physicists engage while constructing and applying knowledge. This progression was enriched in the early 2000s when Alan Van Heuvelen added his multiple-representation approach. While the logical flow represents a path for thinking, the multiple representations are thinking tools. When Gorazd Planinšič joined the team in 2014, he added his unique experience with educational experiments, which made ISLE’s experimental base much more practical and yet much more exciting.

---

<sup>1</sup> Publications concerning the ISLE philosophy are listed at the end of this chapter, references numbered 1-8.

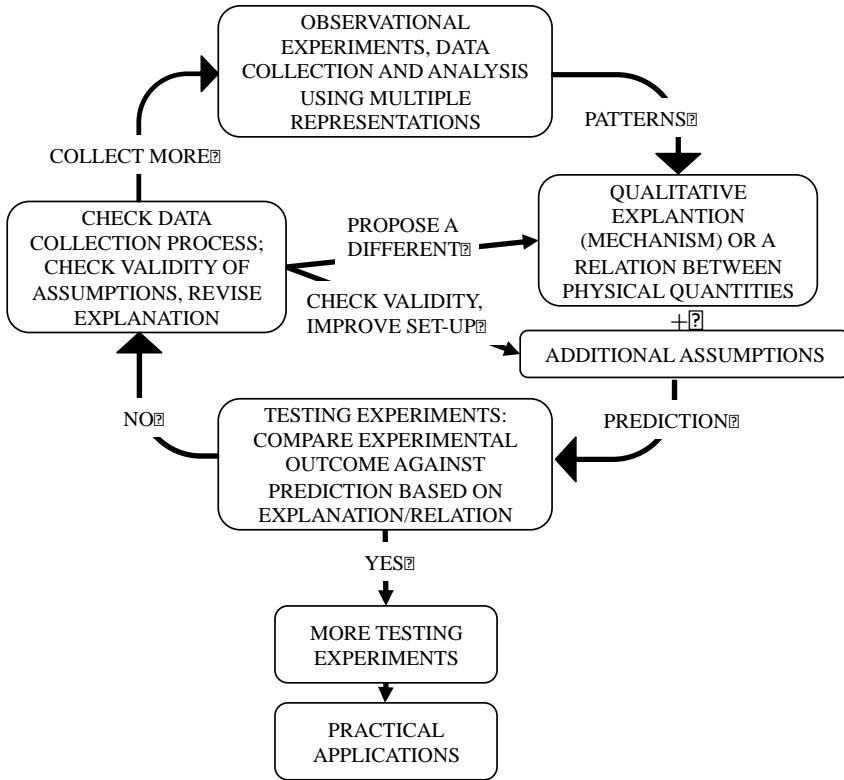
Students who learn physics through ISLE learn by engaging in the processes of *doing* physics—processes that mirror the way physicists practice physics. The social foundation of this process is collaboration: at all stages, students work in groups and as a whole class community, presenting and arguing their ideas and findings. First, students working in groups observe and analyze so-called *observational* experiments. Observational experiments can be qualitative or quantitative with rigorous data analysis. The goal is to identify useful patterns in the data. The key difference between this type of experiment and the other two (see below) is that students do not make any predictions before the outcomes of the observational experiments; they are expected to collect data with “open eyes,” without expectations, and describing what they see in simple non-technical words.

These patterns in the observations and analyses lead to the development of physics explanations (causal and/or mechanistic) and/or mathematical models. These can be collectively called *hypotheses*. Next, students use the hypotheses they have created to design *testing* experiments whose outcomes they can predict using the developed hypotheses. They then compare their predictions with the actual outcomes. If the models or hypotheses survive the testing, they become the basis for future real-world problems and applications (*application* experiments). If not, students might reject the models or hypotheses. Third, this experimentation and model building leads to the development of what we will term physical theories, such as Newton’s theory of universal gravitation or the kinetic molecular theory. These three roles of experimentation, from observation to testing to application, form the backbone of the ISLE learning cycle (see the figure below). They also form the structure upon which the textbook and ALG are built. Students who learn physics through ISLE engage and develop two types of reasoning: (1) *Inductive* reasoning includes both finding patterns in the data, and *analogical* reasoning when they invent casual or mechanistic explanations/hypotheses for the patterns. (2) *Hypothetico-deductive* reasoning is employed when students use the invented explanations/hypotheses to make predictions about the outcomes of the testing experiments. The hypothetico-deductive logical reasoning chain that we use in the book and ALG, and that we encourage you to use with your students, is as follows:

*If*            the explanation (statement of the pattern/explanation/hypothesis) I invented  
                 is correct  
*and*        I do such and such (description of the testing experiment),  
*then*       so and so should happen (prediction of the outcome of the testing  
                 experiment).

However, it did not happen, *therefore* I need to revise my explanations. Or: And it did happen, *therefore* my explanation is not disproved.

Note that the statement after *if* is NOT the description of the experiment (if I do such and such) but the description of the hypothesis under test. It is important that students practice this logical chain when they design experiments to test their ideas.



For students to take ownership of their own learning, they need to develop certain scientific abilities or “habits of mind.” We often complain that when encountering a physics problem, students proceed to search for a formula, whereas a physicist in a similar situation starts with a sketch and qualitative analysis of relevant information. This book helps students develop the ability to represent physical processes in multiple ways and thus approach problems in an expert way. We also help students develop the ability to evaluate the result using different methods. As you look through the ALG and the textbook, you will see that in addition to asking students to solve problems, we include many activities that specifically target the development of these scientific abilities. We also developed self-assessment and assessment rubrics that your students can use to improve their work and you can use for grading. All rubrics are available for free on the website listed in the footnote.<sup>2</sup>

---

<sup>2</sup> To learn more about scientific abilities, their development and assessment use References 3, 6, 9, 11, 13 and to download the scientific abilities rubrics, go to <https://sites.google.com/site/scientificabilities/>

Do we have evidence that the ISLE approach helps students learn physics and develop scientific abilities? Over the past 16 years, instructors using the ISLE philosophy, the first version of the ALG, and their own ISLE-inspired materials have found that their students not only learn physics concepts (as assessed by physics education research-based tests such as the Force Concept Inventory [FCI], the Mechanics Baseline test [MB], the Conceptual Survey of Electricity and Magnetism [CSEM], and others) but become expert problem solvers, can design and evaluate their own experiments, communicate, and most importantly see physics as a process based on evidence as opposed to a set of rules that come from the book.

The list of research papers describing student learning in ISLE-based classes is given at the end of this chapter.<sup>3</sup>

## Learning principles that serve as the foundation of the learning system

*College Physics: Explore and Apply* and the *Active Learning Guide* (ALG) adhere to several important principles.

### **Principle 1: Experiments are the foundation of knowledge creation in physics**

The main principle is that students practice developing physics concepts by following steps similar to those of physicists developing and applying knowledge. The first introduction to a concept or a relation happens when students observe simple experiments (called *observational* experiments). These experiments need to be simple enough that students can analyze them and find patterns (either qualitative or quantitative) in the data, and develop explanations (causal or mechanistic) for the patterns or quantitative relations (often multiple explanations are possible and students are encouraged to propose as many as they can). They then learn how to test the explanations and relations in new *testing* experiments. Sometimes the outcomes of the experiments might cause them to reject the explanations; often they help students keep them. Students see how scientific ideas develop from evidence and are tested by evidence, and how evidence sometimes causes us to reject the proposed explanations. Finally, students learn how tested explanations and relations are applied for practical purposes (*application* experiments) and to problem solving. This strategy helps students answer the question “how do I know what I know?”—a foundational question in science.

**Principle 2: Concept first, name second**—Wherever possible, we try to help students devise an image, and where possible, a kinesthetic “feeling” for the concept

---

<sup>3</sup> See References 9-15 on the list.

before giving it a name. This helps students connect new ideas to what they already know and makes new knowledge more accessible in their brains.

**Principle 3: Careful language**—We try to use very careful language. For example, we help students understand that force is a physical quantity characterizing an interaction between two objects and is not a property of one object and not a physical entity. To do this, we use the expression “the force that object A exerts on B,” not “the force of A” or “the force of A on B.” We also avoid using the names of the forces and grammatical constructions that might make students think that forces belong to objects—the weight *of* an object or the tension *in* the rope. Instead we talk about the force exerted by Earth on an object, or the force exerted by the rope on an object. We do not use the term centripetal force, so that students do not think that it is an additional force. When talking about energy, we always talk about the gravitational potential energy of a system (two or more objects), not of a single particle. When dealing with thermodynamics we use the word *heating* instead of heat to underline an energy transfer “process” as opposed to a “substance” that an object gains. The goal of this careful use of language is to eliminate unnecessary difficulties that traditional use of words adds to the learning of physics.<sup>4</sup>

**Principle 4: Building a bridge between words and mathematics**—We introduce physical representations that help students “translate” between the language of words and the language of mathematics. These concrete representations can be used to analyze physical situations and experiments and to solve problems. These representations continue to appear throughout the book. For example, force diagrams are used in Chapters 2, 3, 4, 5, 6, 7, 8, 9, 11, 14, 17, 27, and others. Bar charts (momentum and energy) are used in Chapters 5, 6, 8, 11, 14, 26, 27, and 28. These graphical representations form the backbone of students’ problem-solving strategy. The goal of this bridge is to help our students create concrete referents in their minds when they reason about abstract ideas—the same purpose behind placing a picture of an apple next to the word “apple” when teaching a child that word.

**Principle 5: Making sense of mathematics**—When students devise/derive a new mathematical expression or obtain a numerical result, we stop to ask whether the expression/result makes sense. To answer this question, we evaluate the units, apply limiting cases, or use common sense. We also emphasize the difference between mathematical expressions that define a quantity operationally (operational definitions) and expressions that show how this quantity depends on other quantities (cause-effect relationships). For example, one can define the quantity of

---

<sup>4</sup> Our research and recommendations concerning the use of language in physics are in References 28-30.

electric current as  $I = \frac{Dq}{Dt}$  and it will be an operational definition (as the current

does not depend on the charge or time interval independently), but  $I = \frac{DV}{R}$  is a

cause-effect relationship (as these two variables are independent of each other). The goal of making sense of mathematics is to help our students be comfortable with the mathematical language of physics and understand that this way of representing the world must agree with other representations.

**Principle 6: Moving away from a plug-and-chug problem-solving approach—**

To help students avoid searching for an equation in which they insert numbers to get an answer for a problem (so-called plug-and-chug problem solving), we introduce a myriad of nontraditional types of problems. These are listed in the table below. The problems develop specific reasoning skills, and, if you are teaching an AP Physics course, they are extremely helpful for your students. Research shows that these problems promote higher levels of cognition and improve conceptual understanding and problem-solving skills.

Type of problem	Keywords	Description
Ranking tasks (RAT)	Rank, compare	Students have to rank the values of a certain physical quantity for different situations, in descending or ascending order.
Choose answer and explanation (CAE)		Students have to choose the correct answer <i>and</i> the correct matching explanation (cause-effect or mechanistic) in order to get full credit.
Choose measuring procedure (MEP)	Procedure, method	Students have to choose (or propose) the correct (or the best) experimental procedure that will allow them to measure/determine a certain quantity.
Evaluate (reasoning, solution...) (EVA)	Evaluate, your friend says..., agree, reconcile, comment on, how will the answer change,	Students have to critically evaluate the reasoning of some (imaginary) people or evaluate the suggested solution to a problem (given either in words, graphs, diagrams, or as an equation). Students have to recognize productive ideas (even when they are embedded in incorrect answers) and differentiate them from



	discuss, how do we know, compare and contrast	unproductive ideas.
Make judgment (based on data) (MJU)	Decide, reject, do the data..., justify, (in)consistent, hypothesis	Students have to make a judgment about one or more hypotheses, based on data or other forms of evidence that are given in the problem, sometimes taking uncertainties into account.
Linearization (LIN)		First, students have to write an equation that describes the relevant situation. Then they have to rearrange the equation to obtain a linear function (note that the independent and the dependent variables in this function can be any function of data given in the problem). Students then draw the graph, plot the best-fit line, and determine the unknown quantities using the best-fit line. These problems help students combine knowledge of physics, the ability to “read and write” with graphs, the ability to manipulate equations, and the ability to recognize linear dependence in non-standard situations.
Multiple possibility (MPO)	Tell all, say everything you can, make a list, as many, give (three) examples, what can you infer, relevant	Students have to list as many quantities as they can that can be determined based on data given in the problem, or tell everything they can about the physical attributes of the objects that appear in the text, or the relations between them. Normally, students are required to determine the values for only few of the quantities that they identify. These problems allow all students to feel successful.
Jeopardy (JEO)	Jeopardy, invent a problem, pose a problem	Students have to convert a representation of a solution into a problem statement. If the solution is given in the form of an equation, they need to understand the meaning of the quantities and their units. Such problems emphasize the value of units.
Design an experiment or	Design, invent, write	Students have to design an experiment, an experimental procedure, or a device that will

pose a problem (DEX)	your own, pose, describe experiments, devise	<p>allow them to measure/determine certain physical quantities or that would meet specific requirements.</p> <p>Students have to pose a problem that involves certain objects with given characteristics. Often there is an additional requirement that solving the problem should involve the use of a particular physics topic, law, or principle. Students may also need to do an additional literature search.</p>
Problem based on real data (RED)		<p>Students have to solve problems that are based on real data, obtained in real-life situations, often using easily available equipment and/or equipment that is typically used in student labs. The types of problems may be traditional or any of the types presented above. Students need to deal with uncertainties, anomalous data and assumptions, and to propose meaningful models.</p>

## Organization of the textbook and the Active Learning Guide (ALG)

The textbook and the ALG follow a path from mechanics (force, momentum, and energy for linear motion, and force, momentum, and energy for rotational motion), to vibrations and mechanical waves, to fluids and thermal physics and then to electricity and magnetism. Then we proceed to the study of light, electromagnetic waves, special relativity, and modern physics. To help students learn, the textbook and the ALG addresses each physics idea or concept twice—once qualitatively to understand the phenomena and the second time quantitatively to learn to use the mathematical concept. As the chapter progresses, students synthesize the new concept or concepts learned by applying them to real-life situations. This logic allows students to use the same idea at least three times in each chapter.

## Textbook elements and how to use them

The format of this textbook is flexible and designed to accommodate any teaching style (you do not need to use the ISLE approach to use the book or the ALG). Students can construct the relevant ideas using several approaches: by performing

the activities described in the ALG in a lab or by observing and discussing experiment table activities in lecture and then reading the book at home and working through worked examples. Students can also read the book before they come to class and use the videos to view most of the experiments and work on worked examples in class without seeing the textbook solutions, and checking with the solutions later. Finally, if you use Mastering Physics, you can assign your students to complete the experiment activities before class.

Each textbook chapter follows the same format and contains the same elements:

1. **Teaser questions, “be sure you know how to” list, and chapter-opening story.** Each chapter starts with three interesting questions that are answered later in the chapter. The “be sure you know how to” three-item list encourages students to review the most important ideas and skills from the previous chapters that they need for the new chapter. The chapter-opening story is connected to the first interesting question but does not answer it: it expands on the question and provides a hook for the chapter material. Following the story, the first paragraph shows how the knowledge that students developed in the previous chapters is insufficient or incomplete but will be useful for building the ideas in the coming chapter.
2. **Observational experiment tables.** These tables describe simple experiments that students use to devise patterns and later explain them. Many of the experiments exist as videos that students can access via a special website or through links in the eText. We suggest that students perform the experiments in a lab, observe them in a class setting, or view the available videos, and then analyze the data provided in the experiment table.
3. **Testing experiment tables.** These tables describe experiments that can be used to test the patterns or hypotheses that student learned about while analyzing the observational experiments. Each table describes one or more experiments, predicts the outcome based on the hypotheses under test, describes the outcome, and concludes whether the hypothesis under test should be rejected or should be kept for the time being. If students are performing testing experiments in a lab, make sure they make predictions based on the hypotheses under test, and not on their intuition, and only then perform the experiment. If they are not performing or observing the actual experiment, but the video of the testing experiment is available, they can watch the video after they make the prediction.
4. **Physics Tool Boxes.** Physics Tool Boxes focus specifically on the steps students should take to master a specific physics representation, such as a motion diagram, force diagram, momentum or energy bar chart, ray diagram, and so forth. It is best if you follow the same steps in class when students first learn the representation.

5. **Tips.** The Tips are short notes focusing the attention of the students on common difficulties. You can emphasize their importance by asking a question to which a particular Tip is the answer.
6. **Worked examples.** These examples follow a four-step problem-solving strategy (sketch and translate, simplify and diagram, represent mathematically, and solve and evaluate) that is adapted to the content of every chapter. For example, in linear dynamics students use motion and force diagrams in the second step, and in geometrical optics they use ray diagrams. The purpose of following the same strategy repeatedly is to help students create effective problem-solving habits and increase their confidence. Almost every chapter has a special “Skills” section where each step of this strategy is described in detail and illustrated for a specific example. All other examples follow the steps but do not provide lengthy descriptions. We suggest that you model problem solving in class using a few worked examples so students see the progression of steps. Skipping steps and using shortcuts make students view every new problem as a completely new challenge and prevent transfer of problem-solving skills.
7. **Conceptual exercises.** These qualitative questions require students to apply the first two steps of the strategy: sketch and translate, and simplify and diagram. They do not require calculations, and they help students build multiple-representation abilities.
8. **Quantitative exercises.** These quantitative one- or two-step simple exercises focus on mathematical representations and evaluation of the result only. The quantitative exercises employ the last two steps of the strategy. The purpose of these exercises is to help students practice new equations and evaluate their solutions.
9. **End-of-section review questions.** The goal of the review questions is to help students learn how to read a science textbook effectively. Many of these questions ask the reader to explain why a particular statement is true. Such questions have been found to be the most beneficial for reading comprehension.
10. **Applications spread throughout each chapter.** Every chapter shows students how to apply the concepts that they learned in the chapter to solve problems based on the applications of those ideas. For example, in Chapter 2 (“Kinematics: Motion in One Dimension”) students learn why tailgating is dangerous, in Chapter 17 (“Electric Charge, Force, and Energy”) they learn how to explain the operation of a Van de Graaff generator, in Chapter 26 (“Special Relativity”) students learn how the clock ticks from GPS satellites enable vehicles on Earth to calculate their position with precision.
11. **End-of-chapter conceptual questions.** These questions are a mix of multiple-choice questions (perfect for using in lectures with a student response system) and open-ended conceptual questions that require deep

reasoning. You can discuss some of them in class, letting students work in groups or having a whole class discussion, or assign them for homework.

12. **End-of-chapter problems.** The problems are grouped by textbook sections and marked with asterisks that indicate level of difficulty. Special icons indicate problems with a biological foundation, and problems that ask students to estimate an answer. As discussed previously, several types of problems are nontraditional and do not have only one correct answer. Students should get credit for the reasoning process, articulation of assumptions, and considering multiple possibilities. It is necessary to solve those nontraditional problems in class at first to show students how to reason and then assign them for homework. The general problems at the end of the problem set can be more complex, often require estimations, and may use ideas from previous chapters.

## Active Learning Guide

The *Active Learning Guide* workbook by Eugenia Etkina, David Brookes, Gorazd Planinsic, and Alan Van Heuvelen consists of carefully-crafted in-class cycles of activities that provide an opportunity for the students to conduct observational experiments, find the patterns, develop explanations and conduct the testing experiments for those explanations described in the textbook before they read it.

Additional activities help them practice using Physics Tools (different representations and purposefully develop specific reasoning skills). These learning cycles are interspersed with “pivotal” activities that serve different purposes. These are:

- (a) to help students invent foundational concepts/physical quantities;
- (b) to introduce and familiarize students with a new representational technique;
- (c) to give students practice with a representational technique;
- (d) to directly address an idea that we *know* students struggle with (the idea is to encourage that struggle so that students reach a resolution either (1) through their own discussion or (2) by the instructor giving a “time for telling” lecture at the end of the activity); and
- (e) to provide scaffolding to students to work through an example or a passage in the textbook.

The ALG also contains multiple experiments that can be used in labs. Whether the activities are assigned or not, students can always use this workbook to reinforce the concepts they have read about in the text, to practice applying the concepts to real-world scenarios, or to work with sketches, diagrams, and graphs that help them visualize the physics.

Another type of activity is the Reading Exercise which asks students to read the textbook section and answer the Review Question or some other additional questions. They are present in every ALG section and, depending on the content,

are placed either at the end (if we think that students can invent relevant ideas through activities) or at the beginning of the chapter (if we think that it is more beneficial to read the chapter first and then proceed to the activities). We consider reading the textbook a crucial skill that students need to develop when learning physics. Research shows that most students have tremendous difficulties comprehending scientific texts and need to be taught specifically how to do this. The Review Questions are research-based and have the goal to help your students work with scientific text. This goal will be achieved only if reading the textbook becomes a habit. We also do *not* recommend that students read the book before coming to class on a regular basis. As our goal is that students do not believe that physics knowledge comes from authority but develops through the analysis of experimental evidence and reasoning, it is crucial that students have an opportunity to participate in this process firsthand and only then read the description of the same process in the textbook.

All activities in the ALG are marked either for class (a large enrollment course lecture, a small section for problem solving, a high school lesson) or for a lab (small section, up to 26 students with necessary equipment). If you are teaching a small enrollment course, you can use the activities marked Class for any course meetings (including labs). If you use a studio format, then all activities can be performed in the same room. Most experiments are performed by the students in groups, but some are suited for one set-up by the instructor. If you do not have the necessary equipment, you can use the videos or data tables that we provide. Each activity has a list of equipment. A common piece is a small whiteboard and dry-erase markers that students use to present the results of their discussions. Almost all ALG activities encourage students to work in groups, come to a consensus and share it with other groups. Additionally, you will see that some activities are marked “pivotal” while others are not. The activities marked “pivotal” are the activities that we think are so important in the knowledge construction process that they should not be skipped. Our concern was that you may be overwhelmed by the volume and variety of available activities in the ALG. If you feel that way, focus on the pivotal activities (these are the ones we always use in our own classes) and include more as you feel is appropriate or as you get more comfortable.

If you have questions on how to use the ALG and the textbook to implement the ISLE method, we invite you attend a workshop “Learning Physics by Practicing Science: Introduction to ISLE” offered at every summer AAPT meeting, or contact either Eugenia Etkina at [Eugenia.etkina@gmail.com](mailto:Eugenia.etkina@gmail.com) or David Brookes at [dtbrookes@gmail.com](mailto:dtbrookes@gmail.com).

## Mastering Physics

**Mastering<sup>TM</sup> Physics** is the leading online homework, tutorial, and

assessment platform designed to improve results by engaging students with powerful content. The Instructor Resources page of Mastering Physics includes links to the ALG chapters, the IG chapters, and other supporting materials such as Ready-to-Go modules that will provide you with additional help in planning your classes, the test bank (which has been heavily revised for the second edition), PowerPoint lecture outlines and clicker questions, and the solutions manuals for the textbook problems and ALG activities. The textbook and ALG videos are all available in the student-accessible Study Area. In the Item Library, you will find the textbook's end-of-chapter questions and problems, as well as tutorials (that provide hints and answer-specific feedback) that you can assign for homework, and interactive Observational and Testing Experiment Tables.

The interactive Observational and Testing Experiment Tables in Mastering are new for the second edition of this book. These are activities for students to do before coming to class, as an alternative to working through some of the ALG activities or prior to reading the textbook. These activities focus students' attention on observational experiments, helping them learn to identify patterns in the data, and on testing experiments, helping them learn how to make a prediction of an outcome of an experiment using an idea being tested, not personal intuition. Both skills are very important in science, but are very difficult to develop.

## Learning Objectives

We have developed learning objectives that we want students to achieve by the end of the course. The objectives are measurable. You can use ALG activities, textbook worked examples, and end-of-chapter problems and questions to help students achieve these objectives and to assess them (for the latter purpose you can also use problems from the test bank). The objectives are grouped around general learning objectives and learning objectives specific to a particular content area. The general learning objectives are listed below; content-specific learning objectives are listed at the beginning of every chapter in this Instructor's Guide.

### *General learning objectives*

#### **A. Scientific abilities objectives**

These abilities<sup>5</sup> are developed throughout the year and can be assessed using the

---

<sup>5</sup> An ability is not an innate skill with which a person is born but instead a developed "habit of mind", something that students do habitually but not automatically. Students engage in science practices to develop scientific abilities. To learn more about scientific abilities, use Reference 3 and the website <https://sites.google.com/site/scientificabilities/>.

rubric published in the appendix to the Instructor's Guide. We have done research on using the rubrics and reflective journals in different universities and high schools in the US and around the world, and we are finding very consistent results. It takes time for the students to master these scientific abilities – from 8 to 10 weeks, assuming they are actually using rubrics to self-assess and revise their work and the instructor gives them appropriate feedback. The use of rubrics for self-evaluation by the students is invaluable. The grading time by instructors using the rubrics instead of providing personal comments decreases dramatically. Note the letters A, B, C, etc. in the text below: they refer to the rubrics available at <https://sites.google.com/site/scientificabilities/>

1. ***Ability to represent phenomena and knowledge in multiple ways.*** Through the course students will develop the ability to represent phenomena and knowledge in multiple ways and use non-mathematical representations to create mathematical representations. This ability has two components: 1. “reading,” and 2. “writing” with a representation. “Reading” means reading out or extracting all possible information from a representation (for example, interpreting a position-versus-time graph) and “writing” means using a representation to describe a phenomenon or using one representation to create another representation (for example, constructing a Newton's second law equation consistently from a force diagram). This is assessed by rubric A.
2. ***Ability to design and conduct scientific investigations.*** Through the course students will develop the ability to design experiments to either (a) investigate a phenomenon to find a pattern (these are *observational* experiments assessed by rubric B) or (b) to test one or multiple hypotheses (these are testing experiments assessed by rubric C). The two types of experiments require different sub-abilities; those are clearly described in the rubrics. For observational experiments it is critically important that students are *not* asked to make any predictions. The epistemological goal of an observational experiment is to gather data and look for patterns in that data. For the testing experiments, it is especially important that students can differentiate between a hypothesis and a prediction and make predictions based on hypotheses (ideas, relations) being tested, not their intuition. Note that students should not perform testing experiments before they make predictions.
3. ***Ability to apply physics ideas to solve practical real-world problems through designing experiments and/or calculations.*** Once students have tested their ideas, eliminated unworkable hypotheses, and established multiple ways of representing the remaining ideas, they can use those ideas to solve a real-world practical problem. For example, design two independent experiments to measure the coefficient of static friction between your shoe and a floor tile. These activities form a “capstone” for each learning cycle and help students to



see how what they are learning is deeply connected to their everyday experiences. This ability is evaluated by rubric D.

4. ***Ability to collect, represent and evaluate data from a real experiment and from a video of an experiment.*** This ability involves methods of collecting and recording the data, representing data with graphs, and estimating and evaluating uncertainties. We suggest evaluating uncertainties conceptually, using the weakest link rule<sup>6</sup>. This ability is evaluated by rubric G.
5. ***Ability to evaluate the reasonableness of an idea or calculated/experimental result.*** This ability involves students learning (in a variety of contexts) to deploy evaluation techniques such as examining whether a result makes physical sense (comparing with everyday experience), analyzing the consistency of units, using limiting or special cases, analyzing the consistency of representations, coming up with an independent method to measure or calculate something, evaluating an idea with a testing experiment, or constructing an appropriate scientific argument to support or reject an idea. This ability is evaluated by rubric E.
6. ***Ability to effectively communicate ideas to others***  
Through the course students will learn to communicate their ideas, experimental designs, scientific conclusions, and justify their reasoning processes using words and/or diagrams. This ability is evaluated by rubric F.
7. ***Ability to reflect*** Each week, many of us who use ISLE ask our students to write a reflective journal (Reference 14) based on the following three prompts: 1. Write a paragraph about what you learned this week. Include in your paragraph a discussion about how, if your friend questioned the truth of what you learned, you would convince your friend that what you learned is true. 2. Is there anything that remains unclear from this week? 3. If you were the professor, what question would you ask to determine whether your students had learned this week's material? Developing ability of self-reflective learning is one of the key components of life-long learning.<sup>7</sup>
8. ***Ability to work with a scientific text (in any format)*** As one of the goals of our learning system is to help our students develop scientific habits of mind, we need to help our students learn to read, comprehend, evaluate and apply new information in a new context.

---

<sup>6</sup> The document describing the weakest link rule and teaching students to use it is at <https://drive.google.com/file/d/0By53x8SYAF1ILWhORU5OTnIHbHc/view>

<sup>7</sup> For the benefits of productive reflection for learning read Reference 15.

Special Review Questions at the end of each section in the textbook serve this purpose. We suggest that pose similar questions on the exams so that students know that you value such skills.

## **B. Learning about learning/Conceptions of learning**

In addition the scientific abilities learning objectives listed above, we believe it is essential to include a learning goal for our students that they will develop greater intellectual maturity, curiosity and perseverance in the face of challenge. It can be incredibly difficult to create a student-centered classroom where they are in charge of their investigations and at least partially directing their own learning. Students are used to being told what to know and now we're asking them to be the authors of their own knowledge. Many students are intimidated by this idea, and so we as instructors see that we need to have an additional learning goal in which students slowly shift from being consumers of knowledge to authors of knowledge. We try to explicitly convey this idea to our students at every opportunity. We let them know that physics is a knowledge "game" and part of the purpose of the course is for them to learn to play that game better. While we don't have a good method for evaluating this learning goal, it is our experience that student "buy-in" is a good indicator as to whether they have shifted in their conceptions of what learning is. We can often see this from the types of comments that students leave on their evaluations of the class.

## **C. General mathematical abilities**

The mathematical skills that the students work toward developing during the whole year are listed below.

1. Vector operations, including adding/subtracting vectors and finding vector and scalar components.
2. Proportional reasoning.
3. Explaining equations with words.
4. Performing algebraic manipulations of equations.
5. Understanding the difference between an operational definition and a cause-effect relationship.
6. Making and interpreting graphs based on collected data. Drawing graphs of functions and writing functions from graphs.
7. Converting units.
8. Using trigonometric functions, especially sine and cosine.
9. Using logarithmic functions.

## **D. Problem solving abilities**

The main objective here is that the students solve problems in an expert manner, not by searching for equations that contain variables mentioned in the text of the

problem. To achieve this objective, we have identified important steps you should encourage your students to use, and which are in the four-step problem-solving strategy followed consistently in the textbook:

- *Sketch and translate* – translate words to physics, draw a labeled sketch with knowns and unknowns;
- *Simplify and diagram* – state assumptions and draw physics representations;
- *Represent mathematically* – convert the physics representation into algebraic equations and work with the equations to derive the expression for the unknown quantity;
- *Solve and evaluate* – plug in the numbers and evaluate the result (units, common sense, limiting cases and consistency with the representations in step 2).

We recommend that when you are assessing student work on problems, the quality of these steps, not the final answer, is the basis for the grade. However, many non-traditional problems do not fit into this step-by-step procedure. You need to use your own judgment and worked examples in the textbook to make grading decisions. All problems can be evaluated using the following three criteria: clarity, consistency, and completeness. Are the steps that the student took solving the problem clear and do they require no effort to comprehend? Are the different representations consistent with each other? Is the solution complete, does it answer the question posed in the problem? Note that these criteria do not involve correctness.

## ***Learning objectives specific to particular content areas (see the beginning of each IG chapter)***

## **Structure and use of the Instructor's Guide**

Each chapter of this Instructor's Guide has the same structure. We first introduce the content-based learning goals of the chapter and summarize in a table the main themes of the chapter and the activities that address them, specifically focusing on the non-traditional problems and videos to help you find them easily in the textbook and ALG. We then provide a brief summary of student difficulties with the content of this chapter and where appropriate an overview of the innovations regarding the content. After the introductory part the chapter proceeds with the sections dedicated to the discussion of each theme. We suggest how to approach it in class, what activities to do and in what order, the important aspects of the activities and the pitfalls. We also recommend specific end-of-chapter questions and problems.

We recommend the following sequence of actions when you use all of the

materials that we provide for each chapter (assuming that you familiarize yourself with this chapter).

1. Read carefully the textbook chapter and go over the worked examples.
2. Go over the ALG activities and think about which ones your students might do and what equipment you might need to gather.
3. Read the IG chapter. Consult the available powerpoint slides and Ready-to-Go module for this chapter in the Instructor Resources page of Mastering Physics. Check out non-traditional questions and problems marked in the first table.
4. Return to the textbook and ALG to make the choice of the sequence (what will your students do in class in groups, what experiments you will show them, what EOC questions and problems you will assign, etc.) Check the Observational Experiment Tables and Testing Experiment Tables available on Mastering Physics for students to work on their own. You can assign them for initial exploration or for formative assessment if the students did these experiments in class.
5. Make a plan for the week, prepare and test needed equipment, and assign activities to your students.

In addition to the ALG activities in individual chapters, we have an ALG Appendix with the activities that use material from multiple chapters and can be used at the end of the course (or before the AP Exams) to help students decide on relevant concepts. These activities are very challenging and are likely to puzzle even your brightest students. Finally, in the list of references below, we provide papers that describe additional activities (Section III. Practical suggestions for teaching using ISLE framework) that are not included in the textbook or ALG.

As we said at the beginning of this chapter, ISLE is an approach to teaching and learning physics, i.e. a philosophy, not a set of curricular materials. Thus the ISLE process can be used in any course – not just an introductory physics. In addition, all of the materials that we provide can be used in teacher preparation (to learn physics by future elementary teachers and how to teach physics by future physics teachers). Eugenia Etkina has been running one of the largest and most successful physics teacher preparation program in the US using these materials and produced over 130 highly successful physics teachers. The papers describing the program and the foundations of physics teacher preparation are References 31–34 in Section V. of the reference list below.

## Acknowledgments

We would like to acknowledge that several activities in the ALG (in Chapters 25

and 30) were designed by Michael Gentile, and activities with spacetime diagrams in Chapter 26 were designed by Paul Bunson. We thank them deeply for their contributions. We also thank Suzanne Brahmia and Carolyn Sealfon for their contributions to the learning objectives in this IG.

## **A list of publications about the use of ISLE in the classroom**

We provide below the information about peer-refereed papers that describe ISLE philosophy, supporting materials, research on students learning and practical issues. The list of publications about the use of ISLE in the classroom is broken into five groups:

- I. Publications about ISLE and its specific aspects as a methodological framework for learning and teaching physics.
- II. Research concerning students engaged in ISLE and learning in ISLE classrooms.
- III. Practical suggestions for teaching using the ISLE framework.
- IV. Language we use.
- V. ISLE and physics teacher preparation.

### ***I. Publications about ISLE and its specific aspects as a methodological framework for learning and teaching physics***

1. Etkina, E. (2015). Millikan award lecture: Students of physics—Listeners, observers, or collaborative participants in physics scientific practices? *American Journal of Physics*, 83(8), 669-679. DOI: 10.1119/1.4923432.
2. Etkina, E., & Van Heuvelen, A. (2007). Investigative Science Learning Environment—A Science Process Approach to Learning Physics, in E. F. Redish and P. Cooney (Eds.), *Research Based Reform of University Physics (AAPT)*, Online at [http://per-central.org/per\\_reviews/media/volume1/ISLE-2007.pdf](http://per-central.org/per_reviews/media/volume1/ISLE-2007.pdf)
3. Etkina, E., Van Heuvelen, A., White-Brahmia, S., Brookes, D. T., Gentile, M., Murthy, S., Rosengrant, D., & Warren, A. (2006). Developing and assessing student scientific abilities. *Physical Review. Special Topics, Physics Education Research*. 2, 020103.
4. Etkina, E. & Planinsic, G. (2014). Thinking like a scientist. *Physics World, March*, 48-51.

5. Poklinek Čančula, M., Planinsic, G. & Etkina, E. (2015). Analyzing patterns in experts' approaches to solving experimental problems. *American Journal of Physics*, 83(4), 366-374.
6. Etkina, E., Murthy, S., & Zou, X. (2006). Using introductory labs to engage students in experimental design. *American Journal of Physics*. 74, 979–982.
7. Etkina, E., Van Heuvelen, A., Brookes, D. & Mills, D. (2002). Role of experiments in physics instruction—A process approach. *The Physics Teacher*, 40 (6), 351–355.
8. Brookes, D. T., & Etkina, E. (2010). Physical phenomena in real time. *Science*, 330, 605–606.

## ***II. Research concerning students engaged in ISLE and learning in ISLE classrooms***

9. Etkina, E., Karelina, A., Ruibal-Villasenor, M., Jordan, R., Rosengrant, D., & Hmelo-Silver, C. (2010). Design and reflection help students develop scientific abilities: Learning in introductory physics laboratories. *Journal of the Learning Sciences*, 19, 1, 54–98.
10. Gregorcic, B., Planinsic, G. & Etkina, E. (2017) Doing science by waving hands: Talk, symbiotic gesture, and interaction with digital content as resources in student inquiry, *Physical Review, Physics Education Research*, 13, 020104.
11. Etkina, E., Karelina, A., Murthy, S., & Ruibal-Villasenor, M. (2009). Using action research to improve learning and formative assessment to conduct research. *Physical Review. Special Topics, Physics Education Research*, 5, 010109.
12. Rosengrant, D., Van Heuvelen, A., & Etkina, E. (2009). Do students use and understand free body diagrams? *Physical Review, Special Topics, Physics Education Research*, 5, 010108.
13. Etkina, E., Karelina, A., & Ruibal-Villasenor, M. (2008). How long does it take? A study of student acquisition of scientific abilities. *Physical Review, Special Topics, Physics Education Research*, 4, 020108.
14. Harper, K., Etkina, E., & Lin, Y. (2003). Encouraging and analyzing student questions in a large physics course: Meaningful patterns for instructors. *Journal of Research in Science Teaching*, 40 (8), 776–791.

15. May, D., & Etkina, E. (2002). College physics students' epistemological self-reflection and its relationship to conceptual learning. *American Journal of Physics*, 70 (12), 1249–1258.

### **III. Practical suggestions for teaching using the ISLE framework**

16. Etkina, E. and Planinsic, G. (2015) Defining and developing “Critical Thinking” through devising and testing multiple explanations of the same phenomenon. *The Physics Teacher*, 53, 432-437. DOI: 10.1119/1.4931014.
17. Etkina, E., Planinsic, G., Vollmer, M. (2013). A simple optics experiment to engage students in scientific inquiry. *American Journal of Physics*, 81(11), 815-822.
18. Planinsic, G., & Etkina, E. (2014). Light emitting diodes: A hidden treasure. *The Physics Teacher*, 52(2), 94-99.
19. Etkina, E. & Planinsic, G. (2014). Light emitting diodes: Exploration of underlying physics. *The Physics Teacher*, 52(4), 212-218.
20. Planinsic, G. & Etkina, E. (2014). Light emitting diodes: Learning new physics. *The Physics Teacher*, 53(4) 212-218.
21. Planinsic, G., and Etkina, E. (2015). Light emitting diodes: Solving complex problems. *The Physics Teacher*, 53(5), 291-297.
22. Planinsic, G., Gregorcic, B., & Etkina, E. (2014). Learning and teaching with a computer scanner. *Physics Education*, 49(5), 586-595.
23. Planinsic, G. (2015) Using artistic drawings to create physics problems. *The Physics Teacher*, 53 (10), 443-444.
24. Richards, A.J., & Etkina, E. (2013). Kinaesthetic learning activities and learning about solar cells. *Physics Education*, 48(5), 578-586.
25. Planinsic, G. & Etkina, E. (2012). Bubbles that change the speed of sound. *The Physics Teacher*, 50(8), 458-462.
26. Etkina, E., & Andre, K. (2002). Weekly Reports: Student reflections on learning. *Journal of College Science Teaching*, 31 (7), 476–480.
27. Planinsic, G., and Etkina, E. (2015). Popping balloon with a spaghetti. *The Physics Teacher*, 53(5), 309-310. DOI: 10.1119/1.4917443.

#### **IV. Language we use**

28. Brookes, D. T. & Etkina, E. (2007). Using conceptual metaphor and functional grammar to explore how language used in physics affects student learning. *Physical Review, Special Topics, Physics Education Research*, 3, 010105, 16 pages.
29. Brookes, D. & Etkina, E. (2009). Force, ontology and language. *Physical Review, Special Topics, Physics Education Research*, 5, 010110, 16 pages.
30. Brookes, D. T., & Etkina, E. (2015). The importance of language in students' reasoning about heat in thermodynamic processes. *International Journal of Science Education*, 37(5-6), 659-779.

#### **V. ISLE and physics teacher preparation**

31. Etkina, E., Gregorcic, B., and Vokos, S. (2017) Organizing physics teacher professional education around productive habit development: A way to meet reform challenges. *Physical Review, Physics Education Research*, **13**, 010107.
32. Etkina, E. (2010). Pedagogical content knowledge and preparation of high school physics teachers. *Physical Review Special Topics Physics Education Research*, 6, 020110.
33. Etkina, E. (2015). Using early teaching experiences and a professional community to prepare pre-service teachers for every-day classroom challenges, to create habits of student-centered instruction and to prevent attrition. In C. Sandifer and E. Brewe (Eds.), *Recruiting and Educating Future Physics Teachers: Case Studies and Effective Practices* (249-266). College Park, MD: American Physical Society.
34. Etkina, E. (2011). Pedagogical content knowledge and preparation of physics teachers. In D. Meltzer, and P. Shaffer (Eds.), *Teacher Education in Physics* (103-128). College Park, MD: American Physical Society.



# 2

## Kinematics: Motion in One Dimension

In Chapter 2, students will learn to describe motion using sketches, motion diagrams, graphs, and algebraic equations, and look for consistency between different representations. The content-based learning objectives are listed below.

**Students should be able to:**

1. “Read and write” with motion diagrams (students can interpret a given diagram and draw a diagram for a given scenario).
2. “Read and write” with motion graphs; reading in this context means being able to interpret a graph, to write a mathematical function for the motion on the graph, to find the slope of the graph and the area between the graph and the time axis; writing in this context means being given a function for any kinematics quantity and using it to draw all three graphs (position-versus-time, velocity-versus-time, and acceleration-versus-time).
3. “Read and write” with vectors.
4. Find consistency or inconsistency between different representations of motion.
5. Operate with average quantities and differentiate between the quantity and its change ( $\bar{v}$  and  $\Delta \bar{v}$  for example).
6. Compare and contrast displacement, velocity, and acceleration, and solve problems involving these quantities.
7. Compare and contrast position, displacement, distance, and path length, and solve problems involving these quantities.

8. Explain where the equations for position as a function of time for motion at constant velocity and at constant acceleration come from.
9. Explain the meaning of the term “reference frame” and analyze the same motion using different reference frames.

The chapter subject matter is broken into four parts:

- I. *What is motion and how do we describe it qualitatively?*
- II. *Some of the quantities used to describe motion and a graphical description of motion*
- III. *Use of the above to describe constant velocity and constant acceleration motion*
- IV. *Developing and using the skills needed to analyze motion in real processes*

For each part, we provide examples of activities that can be used in the classroom, brief discussions of why we introduce the content in a particular order and use of these activities to support the learning, and common student difficulties. We suggest that before doing any planning of the instruction, you first read the textbook Chapter 2, then go through the ALG activities in Chapter 2, and only then continue reading this chapter.

In the table below we provide an overview of the chapter concerning subject matter goals, book sections, and ALG sections. We provide a list of non-traditional problems, which help students develop science practices and epistemic cognition (epistemic cognition is the highest level of cognition when a person can evaluate the situation and assumptions, make decisions and evaluate the results). We also include a list of videos so that you can watch them prior to planning your classes related to this chapter (the videos are available in the MasteringPhysics Study Area and via the following link: <https://goo.gl/s2MerO> )

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
What is motion and how do we describe it qualitatively?	2.1, 2.2	OET 2.1 (p15)	
Some of the quantities used to describe motion and a graphical description of motion	2.3, 2.4, 2.5		
Use of the above to describe constant velocity and constant acceleration motion	2.6, 2.7, 2.8		2.6.7 2.7.1
Developing and using the skills needed to analyze motion in real processes	2.9		2.9.14

**Non-traditional end-of-chapter questions and problems**

Choose answer and explanation (CAE): Q2.8

Choose measuring procedure (MEP): Q2.11

Evaluate (reasoning or solution...) (EVA): Q2.28

Make judgment (based on data) (MJU): P2.71

Multiple possibility and tell all (MPO): P2.18, P2.24, P2.25, P2.39, P2.59

Jeopardy (JEO): Q2.22, P2.17

Design an experiment (or pose a problem) (DEX): Q2.27

Problem based on real data (that students can collect by themselves) (RED):  
P2.71, P2.75, P2.76, RP2

*Brief summary of student difficulties with kinematics*

Differentiating between velocity and acceleration, between velocity and displacement, and between position and displacement, understanding the difference between a physical quantity and the change in that quantity, interpreting graphs (including slopes and areas), interpreting signs of physical quantities (thinking that negative acceleration means slowing down), thinking that when velocity is zero acceleration must be zero.

## I. What is motion, and how do we describe it qualitatively?

Because any description of motion depends on the observer, it is important that students begin kinematics by constructing the concept of relative motion and a reference frame. As with all Investigative Science Learning Environment (ISLE) cycles, students start with observational experiments and describe what they see (see Chapter 1 for the introduction to ISLE). Working through activities in Section 2.1 in the ALG, students should devise the idea that different observers see the same motion differently. Humans are naturally egocentric, thinking their own reference frame is the preferred one. Once students can see that there is some difficulty in defining motion egocentrically, they are ready to read Section 2.1 of the textbook about the definitions of motion and reference frame. The analysis of physical processes in many parts of the book depends on carefully defined reference frames. It is important to get off to a good start in this first section because of the mathematical description of motion in later chapters, especially when dealing with Newton's laws, the Doppler effect, and special relativity.

The next step in student learning is to master describing motion qualitatively. They do it by constructing and applying motion diagrams. This is the first of many useful representational tools that students will learn as they progress through the material. Motion diagrams are very important tools not only in kinematics but also in dynamics as they introduce students to the  $\Delta \vec{v}$  vector that will later help them determine the direction of the sum of the forces on a force diagram. Activities in the ALG Section 2.2 lead them through this process. Pay attention to the activities labeled as

PIVOTAL – in our experience they should not be skipped. Students learn to draw motion diagrams first by constructing dot diagrams that represent the position of an object every second (ALG 2.2.1 or the experiments described in the Observational Experiment Table 2.1 in the textbook). Activity 2.2.2 serves as a formative assessment of their understanding that the location of each dot simultaneously means where the object was and at what time due to the original agreement of how the locations are marked.

When you or your students conduct observational experiments like those described in the ALG activities, it is crucial that the students try to describe what they see in their own words without trying to throw out “physics-y” sounding words. Thus an appropriate response to “What is different in the motion of cars 1 and 2 in ALG Activity 2.2.2?” might be “Car 2 covers a greater distance every second as compared to car 1” or “Car 2 is moving faster than car 1” instead of “Car 2 has more power (momentum, energy, etc.)” Dots that get closer together (as in ALG Activity 2.2.3) mean that “The object is slowing down.” With regards to motion, students generally have excellent physical insight and understand what is going on. The difficult part is to build the bridge between their intuitive physical understanding and the more precise verbal *and mathematical* meanings that we reserve for the words “velocity” and “acceleration” in physics.

After this, students can proceed to the formal motion diagrams that use position dots, velocity arrows, and velocity change arrows—such as those in ALG Activity 2.2.5. ALG Activity 2.2.5 correlates to the motion diagram Physics Tool Box 2.1 at the end of Section 2.2 in the textbook. This Physics Tool Box summarizes the construction of the diagrams. Notice how we use the photos with blinking LEDs in this section. They provide a visual connection between real motion and motion diagrams. It is important that students analyze Figure 2.2 in the textbook to make this connection.

The biggest difficulty for students with motion diagrams is determining the direction of the velocity change arrows. This skill is crucial in dynamics. Conceptual Exercise 2.1 helps students with this. The ALG activities in this section and the corresponding end-of-chapter (EOC) problems (2.1; 2.6) help students practice motion diagrams. They need to learn how to read and write with them – basically, they need to be able to tell a story looking at a diagram, and to create a diagram based on a story. In general it is a good practice that students work in groups through worked examples in the text, without looking at the solution, and then compare their solution to the one presented in the book, and after that they proceed to the *Try it yourself* step.

## II. Introduce some of the quantities used to describe motion and a graphical description of motion

This part of learning about motion starts with helping students learn about vectors and operations with vectors (Section 2.3). If you do not have time in class for them to do the activities in the ALG, they can read the textbook and do Conceptual Exercise 2.2 before completing EOC problems for this section. Note that vector arithmetic is not easy for students and it is difficult to learn about vectors out of physical context. Thus we recommend that students use this section as a reference every time they need to carry out any operations with vectors. There is an important aspect of using vectors that will come up later in many chapters. Multiplying a vector by a scalar in mathematics means that we just lengthen or shorten the same vector (equivalent to an addition or subtraction of another vector). In physics, the meaning is different because we might multiply a vector quantity that has some units (for example, m/s) by a scalar quantity that has other units (such as kg, for example). The result is a *new* quantity that cannot be obtained by addition or subtraction from the original vector quantity. This important difference needs to be discussed when students create such quantities as velocity, acceleration, momentum,  $\vec{E}$  field, etc.

In order to describe motion qualitatively, students need to learn the language that physicists use in kinematics. Research on student learning shows that they tend to confuse the terms describing motion – distance and speed, displacement and velocity, velocity and acceleration, and so forth. The reason for this is that in everyday life people use these words interchangeably and often just say “motion” – this word covers everything. Thus it is really important that students construct images of these words before they learn the definitions.

We suggest that students do the activities in Section 2.4 of the ALG (especially 2.4.1 and 2.4.2) before reading Section 2.4, which defines in the textbook the quantities time, time interval, position, displacement, distance, and path length, as well as the scalar component of displacement along an axis for one-dimensional motion. The Tip about subscripts on page 21 emphasizes their importance. Make sure your students pay attention to this Tip. Be sure that students can distinguish between a displacement vector and the scalar component of the displacement along an axis. Students can practice these quantities through ALG Activities 2.4.3 and 2.4.4 and problems in the related EOC section. ALG Activities 2.4.5 and 2.4.6 introduce students to experimental uncertainties. After they work through them, they can finish reading the end of the section that introduces students to significant digits that are later practiced in the worked examples.

In Section 2.5, students learn to represent one-dimensional motion using data tables and kinematics graphs. Research shows that students often tend to think of the graphs as sketches of the motion and not as abstract representations of the motion. For example, if an object is on the negative side of the origin and moving toward the origin

with positive velocity, they might think that the object is moving in the negative direction because the graph line is in the negative region. This problem is addressed in part if students develop a correspondence between the graphs and the motion diagrams (see Figure 2.14). A useful strategy to help students more with this difficulty is to ask them to act out the graph. Another important feature of Section 2.5 is the description of the same motion using two different reference frames (see Conceptual Exercise 2.3). This clearly indicates the importance of having a well-defined reference frame for describing the motion of a particular object. After completing the section, you can now have students try EOC Question 24 and Problem 10. Note that students can learn this material either by reading the book first and then doing the ALG activities, or by first doing the ALG activities and then reading the book.

### III. Use the above to describe constant velocity and constant acceleration motion

We use the same approach to help students construct the concepts of velocity and acceleration: they analyze position-versus-time data and define the slope of the position-versus-time graph as velocity (ALG Activity 2.6.1). The same is repeated for the acceleration—only this time students analyze velocity-versus-time data and the corresponding graph and then define the slope of the latter graph as acceleration (ALG Activities 2.7.1–2.7.2: here, note that when students plot velocity-versus-time graphs, they often have difficulty with the idea that the clock reading at which the object is moving with that average velocity is in the *middle* of the time interval under consideration. Be prepared to address this point if students ask questions).

We suggest that students first work in groups with these and other activities in the ALG Sections 2.6 and 2.7 and only after completing relevant activities read the book Sections 2.6 and 2.7. Notice that the terms velocity and acceleration appear in the book only after students construct these ideas using the data (graphs). Worked Examples 2.4–2.6 help students to analyze different types of motion and to start using problem-solving strategies.

It is important to note that in ALG Section 2.6 students encounter the first testing experiments (a crucial element of the ISLE framework) where they need to make predictions before they conduct the experiments. In both Activities 2.6.2 and 2.6.7 the idea being tested is whether the equation for the displacement of an object traveling at constant velocity applies to the motion of the objects that they study. They need to use this equation to make predictions about the outcome of the experiment. In Activity 2.6.2 the prediction will match the outcome of the experiment and in 2.6.7 it will not. Thus students meet for the first time a situation when they need to reject the hypothesis under test. Make sure that they do not skip these two activities.

Students often have difficulties with the signs of kinematics quantities. The signs are reviewed in Section 2.6—see Example 2.4 and in the Tip following it (pages 27 and 28). Pay particular attention to this Tip and consider using a clicker question to

stimulate discussion in class. For example, in mathematics the upward direction is positive, but in physics often the downward direction chosen as positive simplifies the solution. Students should learn that in physics we choose the positive direction according to the situation. EOC Problems 24 and 25 are good problems to assign to students to practice all of the above.

Students tend to think that anything that is slowing down has a *negative* acceleration and anything that is speeding up has a *positive* acceleration. It can take a while for them to realize that when an object is speeding up, the velocity and acceleration have the same sign, but when an object is slowing down, velocity and acceleration have opposite signs. Corresponding ALG Activities are 2.6.3, 2.6.4, 2.6.5 and 2.7.3.

It is important that students derive the mathematical expression for the displacement of an object moving at constant acceleration using either the idea of average velocity or using the graphical method (both are presented in Section 2.8). Being able to explain where such a complicated equation came from empowers them. It is also useful to use the limiting case analysis for this equation – does it become a familiar equation describing constant velocity motion when acceleration is zero (ALG Activity 2.9.6)? At the end, ask them to make a summary list of the ideas and then use the list to analyze real processes in the last three sections. ALG Activity 2.8.3 achieves the same goal.

## IV. Develop and use the skills needed to analyze motion in real processes

In Section 2.8, students are introduced for the first time to a general problem-solving strategy, a recurring feature that is adapted to different content throughout the book. The strategy involves representing processes in multiple ways—words, sketches, diagrams, graphs, and equations (some or all of these)—and then evaluating the result using one or more of three methods: limiting-case analysis, unit analysis, and reasonability of the answer analysis. In addition, encourage students to always check whether the representations they used to analyze the problem *are consistent with each other*. The multiple representation strategy is emphasized in Equation Jeopardy Examples 2.10 and 2.11, where students are given an equation that describes an unknown process. Students are asked to interpret the equation and use it to construct a sketch of a process that is *consistent* with the equation, then a motion diagram that is *consistent* with the sketch and the equation, and finally a word problem that is *consistent* with all of these representations. Students really need to understand the symbols used in the equations to complete these tasks. Finally, a general kinematics multiple representation problem-solving strategy is outlined in Example 2.12. We illustrate every step of the strategy for the problem described in that example. We repeat the same process in almost every chapter, using the same four steps but adapting them for the content in that chapter. Note the content of Example 2.12. The

problem concerns tailgating and allows the students to see why keeping the distance is crucial in driving and how human reaction time affects safety on the road. It allows students to connect the abstract representations of the idealized motions that they have studied so far to a real-life situation—the analysis of which is important for saving lives. You can use this problem as a “hook” for the whole chapter.

It is really important that you practice the problem solving strategy with your students. One effective way to do this is to present the problem and ask each student to work on it alone. After working alone, they can consult with one or more neighboring students to see if they agree about their work. If not, they can try to reconcile their differences. Then you can work through the solution with student input, but making sure that their suggestions are marked with the relevant steps of the problem-solving strategy (for example, if they say: label the knowns and unknowns, you might tell them that we call this procedure “Translate” as they are translating the English language into the language of physical quantities), so the solution seems to reflect their thinking. And finally you can reflect on the crucial general steps that helped them make the solution and decide whether the final answer makes sense – these will be the steps of the problem solving strategy that we want them to practice for traditional problems: *Sketch and translate* (imagine the problem situation, draw a picture, and translate English words into physical quantities); *Simplify and diagram* (make necessary simplifying assumptions and draw a specific representation, such as a motion diagram or a graph); *Represent mathematically* (convert the latter representation into a mathematical representation) and *Solve and evaluate* (find the numerical answer and evaluate it). The last step requires evaluation of consistency of all representations used to solve the problem.

Note that this strategy works well for traditional problems but does not work very well for non-traditional problems, which we have plenty of in the book and in the ALG. Do not force students into the steps if they need to design an experiment, evaluate somebody else’s solution and so forth. Consult the solutions available for instructors to see what we expect from the students when they are solving a particular problem.

There are a few remaining important points to make. At the beginning of this Instructor’s Guide chapter, we gave the numbers for the non-traditional problems included in this chapter. Make sure that students attempt them all. In the ALG we have several problems that use real-time data and encourage students to make estimations and even seek information on the Internet. Examples are ALG Activities 2.6.8, 2.9.10, 2.9.11, and 2.9.12. We also have several ALG activities that can be used as performance-based assessment of student learning. These are ALG Activities 2.9.13, 2.9.15, and 2.9.16.

Finally, do not skip the Reading Exercises in the ALG. They “force” students to answer the Review Questions at the end of each section. You might want to assign these questions as homework after students have worked on the ALG activities and other problems in class.

Use the list of content-based learning objectives at the beginning of this chapter and General Learning Objectives from Chapter 1, end-of-chapter problems and



questions, ALG activities and selected problems from the test bank to create your exam.

# 3

## Newtonian Mechanics

In Chapter 2, students learned how to describe motion using sketches, motion diagrams, graphs, and algebraic equations. In this chapter, students learn how to explain *why* objects move the way they do and add a new representation to their toolbox—a force diagram. They also learn how to test their ideas, a process that is different in science than in everyday life. This chapter focuses on Newton’s laws and their applications to one-dimensional motion. The content-based learning goals of the chapter are listed below.

### **Students should be able to:**

1. Identify a system for analysis and objects interacting with the system.
2. “Read and write” with force diagrams, labeling forces with two subscripts.
3. Find force components along chosen axes (in one dimension).
4. Find consistency between a motion diagram and a force diagram for a system (recognize the relationship between  $\Sigma \vec{F}$  and  $\Delta \vec{v}$ ). Explain how we know that objects do not move in the direction of the sum of the forces exerted on them (by referring to the experiments).
5. Explain the role of inertial reference frames for using Newton’s laws to analyze motion and thus the role of Newton’s first law in the set of laws.
6. Describe the experiments from which they developed Newton’s laws and use the laws to predict the outcomes of simple one –dimensional processes.
7. Write Newton’s second law in component form for a system using a force diagram.
8. Compare and contrast Newton’s second and third laws.

9. Explain the difference between an operational definition of acceleration

$$\left( \vec{a} = \frac{\Delta \vec{v}}{\Delta t} \right) \text{ and a cause-effect relationship } \left( \vec{a} = \frac{\Sigma \vec{F}}{m} \right)$$

10. Apply Newton's laws to solve problems.

11. Explain why objects have the same free-fall acceleration on Earth.

In what follows, we have broken the chapter into five parts, with the early focus on qualitative reasoning and the latter on quantitative problem solving. For each part, we provide examples of activities that can be used in the classroom and brief discussions of the motivations for using these activities.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
System, interactions, force diagrams, adding and measuring forces	3.1, 3.2		3.1.2
Developing a conceptual relationship between force and motion	3.3	OET 3.1 (p57)	3.3.2, 3.3.6
Inertial reference frames and Newton's first law	3.4	OET 3.3 (p60), 3.1 (p60)	
Mass, Newton's second law, the component form of Newton's second law, the gravitational force, and applying Newton's second law	3.5; 3.6, 3.7	OET 3.4 (p61)	3.5.1; 3.7.2
Newton's third law	3.8	OET 3.6 (p71), TET 3.7 (p72)	
<b>Nontraditional end-of-chapter questions and problems</b> Choose answer and explanation (CAE): Q3.8 Evaluate (reasoning or solution...) (EVA): P3.8, P3.32 Multiple possibility and tell all (MPO): P3.13 Jeopardy (JEO): P3.21, P3.22, P3.23, P3.24 Design an experiment (or pose a problem) (DEX): P3.8 Problem based on real data (that students can collect by themselves) (RED): P3.12, P3.32			

*Brief summary of student difficulties with one-dimensional dynamics*

The most difficult is the meaning of the word “force” as a quantity that characterizes an interaction between two objects as opposed to the motion of an object. The reason for this difficulty is the language we use in everyday life. The difficulty that stems from our teaching is thinking that  $m\vec{a}$  is a force and using  $m\vec{a}$  to calculate any force. Other common difficulties include thinking that objects move in the direction of forces, and that any two forces that are the same in magnitude and opposite in direction are Newton’s third law forces. When drawing force diagrams for an object of interest, students mistakenly put forces exerted *by* the object of interest on some other object.

## I. System, interactions, force diagrams, adding and measuring forces

For students to understand how changes in motion of an object relate to the sum of the forces exerted on that object, they need more than words and equations. In Active Learning Guide (ALG) Activity 3.1.1, students physically experience that they need to push up to hold a heavy ball as opposed to a light ball (a bowling ball or a medicine ball will work for a heavy ball, and it is best if every student in the class has this physical experience). This activity leads them to the force diagram as a representation of forces.

To draw a force diagram, students first need to choose a system and identify touching objects and nontouching objects (Earth) that interact with and exert forces on the system. Two things are important here. First, that the students circle the system and only consider the forces exerted *on* the system, not the forces that the system exerts on other objects. Second, students might say that gravity pulls the ball down; however, gravity is the name of a phenomenon, not the name of the *object* that is exerting the gravitational force. And they might say that air pushes down on the objects (make sure you let them test this hypothesis, using ALG Activity 3.1.2). This process is described in Section 2.1, which you can use as a guide for your first lecture. However, it is best if the students do the ALG activity in the lab or recitation and then have a discussion in the lecture.

In Section 3.1 of the textbook, Physics Tool Box 3.1 helps students learn to construct force diagrams (see ALG Activity 3.1.3). Follow the steps from the Tool Box with your students and emphasize the importance of each step. Then ask students to repeat the process using one of the end-of-chapter questions or problems (for example, Problem 1 or 3). Remember that they are not yet completing a problem solution—just learning to represent processes using sketches and force diagrams. Somewhere along the way, you can briefly review how to graphically add forces

(Section 3.2) so that students can determine the direction of the sum of the forces exerted on an object.

Language use is very important in this chapter. We say: “Object A exerts a force on object B” instead of “Force X acts on object B.” The “A on B” language communicates the nature of force as an interaction between two objects as opposed to an entity belonging to the system—for example, the “weight of an object” or “tension in the rope.” We always label each force with two subscripts with no shortcuts. When the force label is  $\vec{F}_{\text{Earth on system}}$ , we read it as “the force that Earth exerts on the system” and not “the force of Earth on the system” to underscore that Earth does not have a force. We also do not use the term “gravity” as in the force of gravity. Gravity is not an object and cannot be used as a subscript on a force diagram. It also becomes confusing when students try later to apply the third law: if gravity is what exerts a force on an object, then according to the third law, the object should exert an oppositely directed force on gravity (the same reasoning applies to the friction and normal forces). We also do not use the terms weight and tension in the rope. In this chapter, we do not introduce the term normal force. Instead, we talk about the force that the surface exerts on an object because in all situations in this chapter the contact forces are exerted only in the vertical direction (i.e. no situations in this chapter involve friction). Activities that help students understand the nature of the normal force (without using this term) are in Section 3.2 of the ALG.

Always labeling forces with two subscripts might seem cumbersome and unnecessary, but a consistent use of two subscripts prevents students from putting on their force diagrams labels such as “the force of acceleration”, “the force of motion” and so forth, as well as thinking that the normal force is the Newtonian third law pair force to the weight of the object. In fact, this force notation improves students’ performance using Newton’s third law tremendously.

## **II. Developing a conceptual relationship between motion quantities and the sum of the forces, and using the relationship to reason qualitatively about processes**

At the beginning of their study, many students believe that an object’s velocity is in the same direction as the sum of the forces exerted on it because most of them think of force as a property of motion, not the cause of motion. Section 3.3 addresses this difficulty. We suggest that students first do ALG Activities 3.3.1–3.3.3 in labs or recitations, or in the lecture, and then read the textbook. If students start putting a “force of motion” label on their force diagrams, ask them to include two subscripts to identify the two interacting objects. The key that students need to learn is that if they

can't identify the object exerting the force, that force shouldn't be included on the diagram. Without doing qualitative activities such as those described in Sections 3.3 and 3.4, students may have the same ideas at the end of their study as they had at the beginning.

You can also discuss the experiments in Observational Experiment Table 3.1 in the textbook and ask students to identify a pattern based on the outcomes of these experiments: How are the sum of the forces and motion quantities related? Most will find that the only consistent response that works for all three experiments is that the  $\Sigma \vec{F}$  from their force diagrams and  $\Delta \vec{v}$  from their motion diagrams are in the same direction. Some observational experiments definitely contradict the idea that the velocity is in the same direction as  $\Sigma \vec{F}$ . In our experience, these observational experiments are not sufficient for students to confidently establish the pattern relating  $\Delta \vec{v}$  and  $\Sigma \vec{F}$ . Thus, the observational experiments are immediately followed by a testing experiment in which students are asked to design an experiment to test two alternate hypotheses: (1)  $\vec{v}$  of the object always points in the direction of  $\Sigma \vec{F}$  and (2)  $\Delta \vec{v}$  of the object always points in the direction of  $\Sigma \vec{F}$ . The goal is for students to find a counterexample that shows that either (1) or (2) is not *always* true (ALG Activity 3.3.3).

At this point, a special note about testing experiments is in order. If you are trying to follow the logical sequence of observations-patterns-explanations-testing closely by having students think up experiments to test the two preceding hypotheses, then you will discover that students have great difficulty with the idea that they should be trying to *disprove* each hypothesis rather than prove it. A significant portion of your students will suggest that pushing a stationary object as a testing experiment. Students need to learn why this is not a good testing experiment. If you ask students to draw the directions of  $\vec{v}$ ,  $\Delta \vec{v}$ , and  $\Sigma \vec{F}$ , they should quickly discover that all three vectors point in the same direction. Thus the proposed testing experiment is unable to distinguish hypothesis (1) from hypothesis (2). It helps to encourage students to think about the reasoning process of hypothesis testing as a crime or medical mystery. Many TV shows actually depict the scientific reasoning processes of the protagonists very accurately. Students who are familiar with these TV shows will quickly realize that it is much more productive to disprove an idea than to keep confirming an idea that may indeed be incorrect. And finally, doing testing experiments is a good opportunity for students to practice *if-then* reasoning, which is often used incorrectly: *If* I do such and such, *then* such and such will happen. The correct *if-then* logical chain actually consists of three words: *if-and-then*: *If* my hypothesis is correct *and* I do such and such, *then* such and such will happen. It is very useful to practice this line of reasoning with your students.

After students have devised the pattern connecting the force and change in motion, they can now use motion diagrams, force diagrams, and this new pattern to reason qualitatively about physical processes, as in the ALG Activities 3.3.4–3.3.6 and EOC Questions 1, 16, and Problems 7, 8, 11, 12, 13. Help students understand that if enough information is given to use a motion diagram to determine the direction

of the acceleration, they can then construct a force diagram to answer a question about an unknown force. On the other hand, if they know enough to construct a force diagram with the known relative magnitudes of the forces, they can use this information to answer a question about the direction of the acceleration.

### **III. Inertial reference frames and Newton's first law**

In Section 3.3 students learned that objects change their motion when the sum of the forces exerted on them is not zero. However, in the real world, they observe objects that change their motion without any visible external influences. To explain this seeming contradiction, students need to understand the role of the observer in these cases. That is why we tie Newton's first law to the existence of inertial reference frame observers.

Students can do ALG Activity 3.4.1 and read Section 3.4 in the textbook (make sure they watch the videos!), or you can use the material of this section to have a discussion in the lecture. The goal is to apply the rule relating the motion and force diagrams to explain the motion of an object from the point of view of two different observers. For one observer the rule works; for the other it does not. Note that we formulate Newton's first law slightly differently than other textbooks do, emphasizing *the importance of identifying the observer*. Consider a system that does not interact with the environment or whose interactions with the environment are balanced (the sum of the forces exerted on the system is zero). Only an observer in an inertial reference frame sees this system moving at constant velocity. In this approach, Newton's first law serves as a definition of reference frames for observers for whom the subsequent second and third law would be true. In summary, Newton's second law (an object's acceleration is proportional to the sum of the forces and inversely proportional to its mass) holds true in inertial reference frames. EOC Questions 2, 4 and Problems 14 and 15 are appropriate here.

### **IV. Newton's second law, the component form of Newton's second law, the gravitational force, and applying Newton's second law**

In Section 3.5 in the textbook and ALG, students construct a quantitative version of Newton's second law, including the introduction of mass. We suggest that they work on the activities in the ALG before they read the textbook. This can be done either in a lab or in a lecture/recitation (class) setting. We pay particular attention to making sure students distinguish the dependent variable (acceleration) from the independent

variables (usually  $\Sigma \vec{F}$  and  $m$ ). In other words, the acceleration of an object depends on the sum of the forces exerted on it and on the object's mass. Notice that the way we formulate Newton's second law is different from traditional approaches because we focus on the cause-effect relationship between the sum of the forces and acceleration. We also discuss the difference between an operational definition of acceleration as

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \text{ and the relation that explains what affects acceleration: } \vec{a}_{\text{system}} = \frac{\Sigma \vec{F}_{\text{on system}}}{m_{\text{system}}}.$$

We want students to understand that while the operational definition of a quantity provides a method of determining its value, it does not *explain* what the value depends on. The cause-effect relation does. We continue to emphasize this difference throughout the book.

Section 3.6 outlines the logic that helps students come up with the expression for the gravitational force that Earth exerts on an object. You can follow a similar progression in class and then assign this section for students to read and analyze, or alternatively students can do ALG Activities 3.6.1 and 3.6.2, and then read the book and answer the Review Questions.

In all of the problems, we use a multiple representation strategy (Section 3.7) involving a word description of the problem, which is used to help construct a sketch representing the problem process. After choosing the system, students should use the system in the sketch to help construct a motion diagram and a force diagram (if both are possible). Students should develop the habit of checking to see that the force diagram and the motion diagram are consistent.

At this point, students learn to use a force diagram to help apply Newton's second law in component form to the problem process. If necessary, they can apply one of the kinematics equations to the process. They use the equations to get a quantitative answer for the problem. Finally, they need to evaluate the answer for units, consistency with the diagrams and sketch, order of magnitude, and its consistency with limiting-case analysis. You can do one example for the students and then ask them to work on ALG Activity 3.7.1 on their own. This is the worked example in the textbook, and once they finish, they can quickly compare their solution to the textbook solution. Note an important mathematical element in the solution for Example 3.7 in the textbook. We always add force components and use a minus sign to indicate the direction of the force, not the operation of subtraction. In physics, the minus sign can communicate many different ideas: direction, absence, subtraction, change, etc. Thus every time the sign is used, you need to discuss its meaning with your students. In the case of forces and Newton's second law, the forces are added and so we keep the plus sign signifying that operation for each scalar force component (while the components themselves can be negative numbers).

The ALG and the book include special problems that support this multiple representation strategy—for example, Jeopardy problems (EOC problems 22–24). In an Equation Jeopardy problem, students are given equations that are the applications of Newton's second law and perhaps a kinematics equation to an unknown process.



They use the Newton's second law component equations to construct a force diagram, and the diagram and kinematics equation to construct a sketch of a process that is consistent with the equations. It is impossible for students to solve Equation Jeopardy problems unless they understand the meaning of the symbols in the equations and the nature of the equations. With algebra-based physics students, such problems are appropriate after students have gotten used to applying the multiple-representation problem-solving strategy in the usual forward direction. Note EOC Problems 26, 31, and 32. These are non-traditional problems and they combine student applications of Newton's second law and the development of physics reasoning skills. Do not skip EOC Questions 7–14 and 17–19.

## V. Newton's third law

Students have lots of ideas related to Newton's third law (Section 3.8). They understand that when heavy/light and fast/slow moving objects collide, the effects on the two objects are different. But they often confuse the damage or accelerations of the objects with the forces one object exerts on the other. Students need to reconcile their intuition with the physics approach to the world. The sequence of ALG Activities 3.8.1–3.8.2 forms a coherent lab, which we have found to be very successful in helping students start developing their understanding of the third law.

After they do this lab, they can proceed with ALG Activities 3.8.3 and 3.8.4, or you might pose the following scenario: A semi-trailer truck runs straight into the back of a stopped car. Which object exerts a bigger force on the other? To answer this question, it is important to consider each vehicle as a separate system. First ask your students to draw force diagrams for the truck and car during the collision, indicate the direction of  $\Delta\vec{v}$  for each object (comparing before to just after the collision), and then ask them to compare the directions and magnitudes of the  $\Delta\vec{v}$ s. After students have talked to each other, remind them of Newton's second law and ask which object had the larger acceleration. Often at this point, many students go back to the lab experience and realize that the force that the car and the truck each exert on the other are the same in magnitude, but the truck's acceleration is tiny compared to the car's acceleration because the truck is so much heavier.

It is important that students realize that although Newton's third law may seem counterintuitive, it is consistent with their everyday experience. They simply need to make a distinction between the forces exerted by each object on the other and the acceleration of each object. It is generally a very helpful strategy to ask students "What is your system?" when they analyze a situation involving Newton's third law. Note that we do not formulate Newton's third law in terms of action-reaction but instead focus on the two forces that two objects exert on each other. The double-subscript notation that we have been using for forces helps students tremendously here because they learn in the section on Newton's second law that they can only add forces that are exerted on the same object to find the sum in the second law. EOC

Questions 6, 8, 26 and 27 and Problems 36–42 will help students with Newton’s third law.

Do not skip activities in the ALG Section 3.9: 3.9.3 is a linearization problem that is very useful for the development of students’ mathematical skills and the lab presented in 3.9.4. This lab is highly motivational and connects to student lives. Make sure that they understand that all force and motion diagrams are drawn by the observer on the ground (the issue goes back to the inertial and non-inertial reference frames).

Finally, the EOC General Problems section in the textbook contains a series of estimations problems. If you wish your students to master the skill of estimation, do not skip them. We especially recommend Problems 47–50.

# 4

## Applying Newton's Laws

In Chapter 3, students learned to apply Newton's laws in situations in which other objects exerted forces on the object of interest only along the line of motion. Most everyday life processes involve forces that are *not* all directed only along the axis of motion. In this chapter, students learn to apply Newton's laws in component form to these more complex processes. The content-based goals of the chapter are listed below.

### **Students should be able to:**

1. Explain the difference between a vector, vector component, and scalar components.
2. Find the scalar components of vectors in two dimensions.
3. Resolve the force that a surface exerts on an object into two vector components: parallel (the friction force) and perpendicular (the normal force) to the surface.
4. Apply Newton's second law to situations with multiple connected objects, objects on inclined planes, and objects on rough surfaces (problems involving friction).
5. Test Newton's second law experimentally.
6. Apply the independence of horizontal and vertical motions to analyze projectile motion situations.
7. Determine the coefficient of friction (static or kinetic) experimentally.
8. Apply Newton's laws to explain complex real-life situations.
9. Give examples of situations that cannot be explained using Newton's laws.

We have broken the chapter into three parts. For each part, we provide examples of activities that can be used in the classroom and a brief discussion of the motivation for using these activities.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Force components and Newton's second law in component form	4.1, 4.2	OET 4.1 (p89)	
Friction, and skills for analyzing processes involving forces in two dimensions	4.3, 4.4	OET 4.2 (p91), TET 4.3 (p92)	4.4.16
Projectile motion and applications	4.5 and 4.6	TET 4.6 (p103)	4.5.1, 4.5.3, 4.5.9
<b>Nontraditional end-of-chapter questions and problems</b> Ranking tasks (RAT): P4.55, P4.56 Choose answer and explanation (CAE): Q4.4 Choose measuring procedure (MEP): P4.25 Evaluate (reasoning or solution...) (EVA): P4.39, P4.74 Make judgment (based on data) (MJU): P4.63 Linearization (LIN): P4.86 Multiple possibility and tell all (MPO): P4.18, P4.48, P4.93 Jeopardy (JEO): P4.14, P4.38, P4.59 Design an experiment (or pose a problem) (DEX): P4.92 Problem based on real data (that students can collect by themselves) (RED): P4.19, P4.75			

*Brief summary of student difficulties with two-dimensional dynamics*

It traditional instruction students often assume that the normal component of the force that the surface exerts on the object is always equal to  $m\vec{g}$  and that friction force is always equal to  $\mu m\vec{g}$ . Students have difficulty identifying force components for different choices of coordinate axes, they might continue calculating any force as a product of mass and acceleration. Trigonometry poses a big hurdle, as well as vector arithmetic.

## I. Force components and Newton's second law in component form

One way to introduce students to components is to assign Active Learning Guide (ALG) Activities 4.1.1–4.1.3, which correlate to textbook Section 4.1. Notice the grid in Activity 4.1.1 – it helps students see the exact magnitude of the component. The next step is to help students understand that a force vector can be replaced by two perpendicular vector component forces, which when added graphically equal the original force, that is,  $\vec{F} = \vec{F}_x + \vec{F}_y$ . Force vectors are difficult to manipulate mathematically. Thus, instead of using vector components, we use scalar components ( $F_x, F_y$ ). The signs of these scalar components indicate the orientation of the vector components relative to an  $x$ - $y$  coordinate system. The process for determining these scalar components is summarized in Physics Tool Box 4.1 and illustrated in a concrete way in Example 4.1, which involves three strings pulling on a knot (the system). The three forces that the strings exert on the knot are sketched on a background grid, which makes the components apparent by visual inspection. The components can also be calculated to check for consistency of the mathematical and diagrammatic representations.

After introducing this component method, you might ask students to work on their own on Problems 6 and 7 and then check with their neighbors to see if they agree. You can then go quickly through the solution to these problems. Having the vectors on a grid helps students develop intuition for calculating the force components mathematically. Tell students that in the future, they will not have the grids and will use only the mathematical method for determining the scalar components, which we call simply force components.

As usual, students label forces with two subscripts indicating object A that exerts a force on object B. Now we add an additional subscript indicating the  $x$  or  $y$  component of the force—for example,  $F_{A \text{ on } B, x}$  and  $F_{A \text{ on } B, y}$ . This may seem cumbersome, but students accept it quickly. As noted in Chapter 3, the A on B language communicates the nature of force as an interaction between two objects as opposed to an entity belonging to the system—for example the “weight of an object” or “tension in the rope.”

Example 4.1 serves as a motivation to write Newton's second law in component form as opposed to vector form (textbook Section 4.2). We suggest that students first work on Activities 4.2.1 and 4.2.2 in the ALG and then read the textbook (Physics Tool Box 4.2). Appropriate EOC Problems are 1 through 8.

## II. Friction, and skills for analyzing processes involving forces in two dimensions

Notice that the section discussing friction is now earlier in the textbook and the ALG compared to the first edition. The reason for the change is that we truly want students to understand that the normal force and the friction force are two perpendicular vector components of the *same* force that the surface exerts on the object placed on it. To achieve this goal we took a new approach. It is implemented in ALG Activity 4.3.1 and in the Observational Experiment Table 4.1. Students are asked to draw a force diagram for a block that is being pulled across a table with a spring scale, but the block does not start moving. Given that the block interacts with three objects (Earth, the spring scale and the table) there must be three forces exerted on it that add to zero. Students have to draw the force exerted by the surface at an angle to it to balance both the downward force exerted by Earth and the horizontal force exerted by the spring. Then they learn how to decompose this force into two perpendicular components and how to name them “normal force” and “friction force” (in this case, static friction force). Make sure you review OET 4.1 in the textbook before the students carry out the ALG activities to make sure that they are drawing the force diagrams using three not four forces. The expression for the maximum static friction force is based on the patterns that students find working with ALG Activities 4.3.2–4.3.4 or in the textbook’s Observational Experiment Table 4.2. Because students often think that it is the mass/weight of the object and not the normal force that determines the magnitude of the maximum static friction force, they test this idea in Testing Experiment Table 4.3. Table 4.4 in the textbook completes the experimental foundation for the maximum static friction force law. The application of static friction in walking presents a great opportunity for students to see how the friction model they have developed in the class can be applied to real-world scenarios.

Student learning of kinetic friction is based on the steps they used for static friction and thus takes little time. It is important to discuss possible mechanisms for friction, from very rough to very smooth interacting surfaces.

Now students are ready to tackle some interesting problems. We suggest that they first work through the Conceptual Exercise 4.2 in the textbook and then move to EOC Problems 8–29. EOC Questions 1–3, 15, and 20–22 are helpful here.

However, if you feel that your students are not ready to solve complex problems, they will benefit from reviewing the problem-solving strategy and all worked examples in Section 4.4. In all of the problems, we use the multiple-representation strategy that was already used in Chapters 2 and 3. Students use the word description of the problem to construct a sketch representing the problem process and to translate the text into the language of physical quantities. After choosing the system, students draw a motion diagram for the system to help determine the

direction of the  $\Delta \vec{v}$  vector and then draw the force diagram. The motion diagram helps draw the force diagram because it shows explicitly the direction of the acceleration. Knowing the direction of the acceleration, students can figure out the relative lengths of the force vectors on their force diagram. Students are often uncertain with which forces they should align their coordinate axes. It is important to keep reminding them throughout this chapter and the next that the best rule of thumb is to align one of their coordinate axes with the *acceleration* found in the motion diagram. This makes it easier to apply Newton's second law in component form. Drawing force lengths on graph paper might be a helpful process here. Students could rotate their graph paper so that they see the "alignment" with a now-shifted  $x$ - and  $y$ -axis.

We want students to develop the ability to check for consistency between the force diagram and the motion diagram. In other words, does the sum of the forces in the force diagram align with the direction of the  $\Delta \vec{v}$  vector found in the motion diagram? Then they use the force diagram to help apply Newton's second law in component form to the problem process. It is important that students understand that components in the same direction whose values can be positive or negative are always added to find acceleration. If needed, kinematics equations can be applied. Students have to produce the quantitative solution before they plug in the numbers. After students use the equations to get a quantitative answer for the problem, they should evaluate it for units, consistency with the diagrams and the sketch, order of magnitude, and its consistency with limiting-case analysis. The unit and limiting-case analyses can be done, when convenient, before the numerical answer.

Students find the idea of a limiting case rather hard to grasp. They will need multiple exposures to it before they become comfortable with it. You can do one example together with your students (for example, worked Example 4.7) and then ask them to do the following *Try It Yourself* exercise before they work on another problem on their own.

We chose Example 4.7 for another reason. It involves a process in which two objects are connected together (for example, a modified Atwood machine). Given that we want students to develop general problem solving approaches, we suggest that they analyze such problems in a general way rather than using problem-specific methods. It helps if students choose separate systems (one for each object) and construct motion and force diagrams for each object, including coordinate axes for each diagram. They then apply Newton's second law in component form for each object; indicate any constraints that are appropriate (for example,  $a_{1,x} = a_{2,y}$ ,  $T_{\text{String on 1}} = T_{\text{String on 2}} \dots$ ); simplify the equations and solve simultaneous equations to answer the question. As we mentioned before here, limiting-case analysis is particularly useful to evaluate the final equation used to calculate the answer. For example, what if the mass of object 2 is zero—does the equation then provide an expected outcome? Note that it is helpful to view a pulley as a simple string redirector as opposed to a rotating object.

ALG activities in Section 4.4 (4.4.1–4.4.14) can be used in class. This way students can work in groups and help each other. At the end, we suggest that you quickly go through the solution with their input for key parts. ALG Activity 4.4.15 is a lab that might take 2.5–3 hours for your students to complete. ALG Activity 4.4.16 is a good candidate for formative assessment at the end of this section. You can assign it as quiz or put it on the exam. EOC problems in the respective section are of increasing difficulty. We recommend Problems 38–40, 43, 45, 47 and EOC Questions 10–17 and 27–30.

Finally, a note on problems with friction (ALG Activities 4.4.12 and 4.4.14). These types of problems address two common student difficulties: (1) when there is no friction, the force that the surface exerts on the system is always perpendicular to the surface but not necessarily vertical; (2) students often believe that the magnitude of the normal force equals the magnitude of the gravitational force that Earth exerts on the object  $mg$ .

### III. Projectile motion and applications

In Section 4.5, students apply the kinematics equations and Newton’s second law to analyze projectile motion. The process starts with simple qualitative observational experiments when a person moving at constant speed horizontally throws a ball straight up (the activities in the ALG and described experiments in the textbook). To explain why the ball always returns to the hands of the thrower, the students need to hypothesize that the horizontal and vertical motions of the ball are independent. ALG Activity 4.5.3 and Testing Experiment Table 4.6 in the textbook encourage them to test this idea. The related video can be used in class to supplement real experiments as the video allows students to see the process in slow motion.

There are a variety of qualitative questions to emphasize the idea that projectile motion is a combination of constant velocity in the horizontal direction and acceleration in the vertical direction, due to the downward force that Earth exerts on the projectile (ALG Activity 4.54). The quantitative analysis of projectile motion is pretty traditional. However, we have a wealth of activities for students to apply their knowledge. ALG Activity 4.5.6 is an extremely successful lab that our students have been enjoying for years, and Activities 4.5.7, 4.5.8, and 4.5.9 will engage your students in exciting data analysis of some really cool experiments. They will analyze photo and video data, and work with provided data. These are all the skills that they will need in the future. The videotaped experiments allow students to test their solutions instantly. Helpful EOC Questions are 6, 7, and 24–26 and Problems 55–63.

The goal of the last section (Section 4.6) is to apply what students have learned to explain the motion of a car. If you ask students what makes a car move, a common answer is “the engine.” However, if the engine is a part of the car, then it cannot exert an external force on the car and thus cannot affect its motion. A detailed analysis of the process presented in the section helps resolve this issue. In addition, students find



it particularly difficult to understand how it is possible that the road can exert a static friction force on the driving wheels of the car that points *forward* in the direction of the acceleration of the car when it is speeding up. It is instructive to bring a bicycle to class and ask students to discuss with their neighbors, or in groups, the direction of the friction force of the road on the back wheel (the driving wheel) and on the front wheel (the nondriving wheel) when the bicycle is speeding up.

Likewise, students struggle to understand how it is possible that the static friction force exerted by the pavement on a shoe can point in the same direction that the person is walking for the “pushing off” phase of the motion. It is always helpful when students are stuck to ask them how the bicycle wheel or shoe would slip relative to the surface if the friction force were not present. Once students have figured all this out, you can challenge them to think about the direction of the friction force on the wheels of a bicycle or car when the vehicle is slowing down by applying the brakes.

Once students feel comfortable with the idea that static friction can take on different values and different directions on the wheels of the car/bicycle, the class can end with a discussion about how electronic anti-lock braking systems (ABS) in cars work. Essentially they work to keep static friction at its maximum value by monitoring when the wheel starts to slip relative to the road surface and modulating the force exerted by the brakes on the inside of the wheel. By doing this, the force exerted by the road on the car wheels is maximized, bringing the car to a stop in the shortest possible distance (Section 4.6). EOC Problem 71 will be good practice for students.

Notice the linearization activity in the ALG, Activity 4.6.4. Not only will it strengthen your students understanding of friction forces, but it will also improve their mathematical reasoning. A similar problem in the textbook is EOC Problem 86. If you are teaching an AP course, these problems are extremely useful.

Finally, the EOC General Problems section offers a variety of different types of problems that will challenge your students. We recommend 74, 75, 78, 83, 86, 89, and 91–93.

# 5

## Circular Motion

Chapter 5 presents the greatest challenge students have faced so far: constant speed circular motion. To master the concepts of constant speed circular motion we suggest that students start with qualitative dynamics, then proceed to kinematics and after that combine qualitative dynamics and expression for radial acceleration to make Newton's second law for circular motion. The content-based learning goals for constant speed circular motion are listed below.

### **Students should be able to:**

1. Describe experiments that allow us to infer that when an object moves at constant speed in a circular path the sum of the forces exerted on it must point toward the center.
2. Use the velocity method to determine the direction of acceleration of an object moving at constant speed in a circular path.
3. Carry out a graphical derivation of  $a_r = \frac{v^2}{r}$ , explain why it makes sense and how to test it experimentally.
4. Be able to determine the sum of the forces exerted on an object moving at constant speed in a circle experimentally.
5. Apply Newton's law(s) to processes involving circular motion at constant speed with multiple forces in two dimensions.
6. Be able to demonstrate how to get to Newton's law of universal gravitation from the Moon's data and Newton's second and third laws.
7. Explain why objects are not "weightless" in orbit.

We have broken the chapter into three parts. For each part, we provide examples of activities that can be used in the classroom, common student difficulties, and a brief discussion of the motivation for using these activities.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Qualitative analysis of constant speed circular motion	5.1, 5.2	OET 5.1 (p119), TET 5.2 (p120)	5.1.4
Quantitative analysis of constant speed circular motion	5.3, 5.4		5.3.5
Newton's universal law of gravitation	5.5		
<p align="center"><b>Nontraditional end-of-chapter questions and problems</b></p> <p>Choose answer and explanation (CAE): Q5.3, Q5.19  Choose measuring procedure (MEP): P5.15  Evaluate (reasoning or solution...) (EVA): Q5.3, Q5.16, Q5.17, P5.37  Linearization (LIN): P5.71  Multiple possibility and tell all (MPO): P5.10  Jeopardy (JEO): P5.32, P5.33  Design an experiment (or pose a problem) (DEX): P5.34, P5.36, P5.54, P5.62, P5.63, P5.65  Problem based on real data (that students can collect by themselves) (RED): Q5.15, P5.11, P5.12, P5.70</p>			

*Brief summary of student difficulties with constant speed circular motion*

The biggest difficulty comes from reconciling the everyday life experience of feeling you are being thrown outward when a car is making a turn and the concept that the sum of the forces exerted on an object moving at constant speed in a circle points toward the center of the circle. The second difficulty stems from a seeming conflict between the motion being accelerated and, at the same time, being constant-speed. Finally, common language referring to “microgravity” or “weightlessness” of astronauts in orbit make the students think that objects in orbit are weightless.

## I. Qualitative analysis of constant speed circular motion

Students begin by performing and analyzing three experiments in which objects move in horizontal constant speed circular motion (ALG Activity 5.1.1). They analyze the situations with force diagrams and build a pattern in which, in each case, the sum of the forces exerted on the object of interest points toward the center of the circle (ALG Activity 5.1.2). They develop an explanation or hypothesis that the sum of the forces exerted on an object moving in a circle at constant speed *always* points toward the center of the circle (ALG Activity 5.1.3). They then proceed to test this hypothesis in ALG Activity 5.1.4 by predicting what will happen to a ball rolling around inside a metal ring when a piece of the ring is removed. We suggest that students finish working on this section by reading the textbook section or working on ALG Activity 5.1.6.

At this point, students know that the sum of the forces exerted on an object moving at a constant speed in a circle points toward the center. Now the question is: why is such a net force needed? To answer this question, students need to examine the motion more carefully and figure out that, despite moving at a constant speed, the object is accelerating because the direction of its velocity is changing. How to determine the direction of this acceleration? To answer this question and to make the logical connections described above, students need to work with ALG Activity 5.2.1 which at the end sends them to the textbook where they learn the velocity method to estimate the direction of an object's velocity change and therefore its acceleration during 2D motion (this method was developed by Fred Reif and colleagues).

The goal of this method is to build on what students have already learned: how to *add* vectors head to tail. Thus it is conceptually easier for them to think of adding a  $\Delta\vec{v}$  vector to the  $\vec{v}_i$  vector to get a *resultant*  $\vec{v}_f$  rather than subtracting  $\vec{v}_i$  from  $\vec{v}_f$  to get a resultant  $\Delta\vec{v}$ . However, more advanced students quickly become comfortable with the idea that subtracting a vector is simply equivalent to adding a  $-\vec{v}_i$ . Our experience is that it pays for students to see both ways and choose whatever method they feel comfortable with. It is important that the students draw velocity vectors for the object equally distant from the point at which they want to determine the direction of acceleration of the object, just before and just after that point. They also need to label the velocity vectors properly. If they use the technique in the book, they then determine what vector needs to be added to the initial velocity vector to produce the final velocity vector. This is the velocity change vector, which also indicates the direction of the acceleration of the object at the point of interest, which is positioned exactly between the points at which they determined initial and final velocity vectors.

After students have gotten acquainted with the method, they should complete ALG Activities 5.2.2 (or Example 5.1 in the textbook). This activity can readily be turned into multiple-choice in-class clicker questions. For example, you can apply the method in Activity 5.2.2 to one point and then ask one-third of the students to apply it to point A, a second third to point B, and the final third to point C. The primary goal

of these activities is for students to devise a key pattern: When an object is moving in a circle at a constant speed, its acceleration always points toward the center of the circle. From these activities, students have learned two important lessons. First, even though the speed is constant, there is acceleration because the *direction of the velocity* is changing. Second, for constant speed circular motion, the acceleration points toward the center of the circle. It is worth mentioning at this point that when an object moves in a circular path but not at constant speed, its acceleration will not point toward the center of the circle (building on ALG Activity 5.2.4).

Now students are ready to connect dynamics and kinematics. Both the acceleration and the sum of the forces point toward the center of the circle, just as it should be if Newton's second law were valid for circular motion! This connection is made in the ALG Activity 5.2.3.

Students can either proceed to the application of their new ideas by doing ALG Activity 5.2.7 or if you wish them to actually test these ideas, they can do Activities 5.2.5 and 5.2.6. In these activities, the situation is a little more complicated because the speed of the pendulum is changing, however, if they assume that the speeds right before the pendulum passes the lowest point and right after are the same, they can make a fascinating prediction concerning the reading of the scale – it should be greater! When students see that their prediction matches the outcome, they clap! Note that according to PER research, only a few physics graduate students can reason to the prediction that matches the outcome of this particular experiment. Your students will be very proud of themselves if they can do it!

Now students are ready to read the textbook – ALG Activity 5.2.8. The tip before Conceptual Exercise 5.1 is very important. Students have a tendency to draw very small diagrams, which makes it very difficult for them to determine the velocity change arrow. Ask them to make the diagrams larger, and it will help them a great deal.

Students can then use the graphical velocity technique, force diagrams, and Newton's second law to reason qualitatively about circular motion questions (for example, EOC Questions 1–3, 5, 6, or 9). Do not skip ALG Activity 5.2.9 as many students have this question!

Students often add extra forces in the direction of motion to their diagrams—for example, in a force diagram for a pendulum ball while passing the bottom of its swing. They also have a tendency to add an outward-pointing force because of the sensation of being pushed outward when they themselves are moving in a circular path (such as being in a car taking a turn at high speed). To deal with these issues, insist that students be able to identify the external object exerting each force. They will not be able to give satisfying answers in either case. We also do not ever use the term centripetal force in the book. It is simply a synonym for the radial component of net force, but when it has a special name it becomes confusing for students. Students tend to interpret centripetal force as an additional force exerted on the object and include it on their force diagrams.

## II. Quantitative analysis of constant speed circular motion

In Section 5.3, students examine the kinematics of circular motion quantitatively to develop an expression for the magnitude of the radial acceleration of an object moving at constant speed  $v$  in a circle of radius  $R$ :  $a_r = v^2/R$ .

We suggest that they start with the ALG Activities 5.3.1 and 5.3.2 and then read the textbook answers to the questions posed in these activities. They analyze experiments (Observational Experiment Tables 5.3 and 5.4). The key to ALG Activity 5.3.1 or Experiments 2 and 3 in the textbook table is understanding that the acceleration of an object is the ratio of the velocity change over the time interval during which this change occurred. Students tend to focus on the velocity change and overlook the time interval. Make sure students read the analysis (the second column) of the experiments in both tables. Also, notice the *Try It Yourself* question after Quantitative Exercise 5.3 that helps students see the importance of choosing correctly the radius of the circle. To test Newton's second law for circular motion, students can do ALG Activities 5.3.3 and 5.3.4. Activity 5.3.3 is a full lab that we have been running for years. It works very well with the students. Do not skip ALG Activity 5.3.4 – it can be a homework exercise. EOC Questions 8, 10, and 24 and Problems 4 – 8 are useful here.

Section 5.4 applies the component form of Newton's second law, kinematics, and our qualitative reasoning abilities to circular motion problems. Students must learn to use a coordinate system with a radial axis (toward the center of the circle) and for some problems, a vertical y-axis. Before doing traditional problems, it is helpful for students to practice drawing force diagrams and writing Newton's second law without solving for anything. ALG Activity 5.4.1 serves this purpose. Again, it is a good idea to use graph paper that students can rotate to assist them in the choice of the coordinate system when they are drawing force diagrams. A problem-solving box outlines the multiple representation strategy for circular motion problems (ALG Activity 5.4.2). Finally all of the ALG activities in the section are suitable for class work. Activity 5.4.9 is another lab that will help your students apply Newton's second law to circular motion. We suggest that you choose between this lab and the lab in Activity 5.3.3. Activity 5.4.10 is an excellent opportunity for the students to practice their evaluation skills. The wrong solution has mistakes that students often make.

Note the wealth of non-traditional EOC problems in this chapter as shown in the table at the beginning here: make sure you assign most of them so that students develop a variety of reasoning skills and science practices.

At the end of Section 5.4 there is a subsection entitled "Conceptual Difficulties with Circular Motion" that helps students reconcile their everyday knowledge with what they just learned (if the students did ALG Activity 5.2.9 they already discussed

this issue). Students are invited to analyze the situation of a passenger in a turning car from the point of view of the observer on the ground or from the point of view of the observer in the car. The latter cannot explain why she/he starts sliding toward the door when the car turns. This means that the car is not an inertial reference frame. This is an excellent opportunity to revisit the circumstances under which Newton's second law holds.

You may want to refer to this section much earlier in the development of circular motion (that is why this activity comes earlier in the ALG). From early on, more curious students will recognize that the feeling of being “thrown to the left” when you turn right (or vice versa) in a car does not seem to be consistent with the idea that the sum of the forces on the person in the car point toward the center of the circle. This is a great opportunity to turn a lecture into an interactive discussion. You could invite students to turn to their neighbors and discuss how they could reconcile the feeling of being thrown away from the center of the circle with the idea that the net force points toward the center of the circle. It is an opportunity for students to integrate their knowledge of Newton's first law with the idea that even though the speed of an object moving in a circle may not be changing, the *direction* of its motion is changing.

Excellent EOC Questions to ponder: 15, 19, and 24. EOC problems to assign for class and homework: 9, 11, 12, 15, 23, 28, 29 32, and 33. Note that in Problem 11 students encounter the explanation of rolling as a combination of circular and translational motion.

## II. Newton's law of universal gravitation and large-scale circular motion

One of the goals of the textbook is to help students see the origins of the concepts and relations that comprise the body of introductory physics. The approach we took to help students learn Newton's law of universal gravitation is consistent with this goal. Following Newton's own reasoning, students analyze the data for the circular motion of the Moon and compare the Moon's acceleration to the free-fall acceleration of the objects on Earth. Then they apply Newton's second law to explain why objects on Earth fall with the same acceleration (independent of their mass) and thus reason that the force that Earth exerts on an object is proportional to the object's mass. Finally, they use Newton's third law to come up with the idea that the force that Earth exerts on an object should be proportional to the mass of Earth as well. Students at this level can follow this reasoning leading to the force being proportional to  $1/r^2$  and the product of the two masses involved  $Mm$ . The proportionality constant  $G$  was determined after Newton's time. As a testing experiment, students then can use Newton's laws plus the law of universal gravitation to derive Kepler's third law, well known empirically at that time. This logical progression is reflected in ALG Activities 5.5.1–5.5.4. You may want students to work on those activities in groups before the whole-class discussion.

An important issue here is the one of weightlessness of astronauts in orbit (ALG Activity 5.5.8). Activity 5.5.6 came from a conversation with a student –it addresses another common difficulty often not recognized by teachers. The rest of the activities in this chapter of the ALG are very engaging – make sure your students have a chance to debate the doomsday scenario in Activity 5.5.12! EOC Questions 12–16 and Problems 37–45 are useful here.

Finally, there is a myriad of general problems that will help your students strengthen their conceptual understanding and develop reasoning skills. We recommend; 54, 55, 57–60, 66, 70 and 71 (this is a linearization problem).



# 6

## Impulse and Linear Momentum

In the first five chapters of the text, we developed and applied the principles of Newtonian physics; the focus was on forces exerted on objects and on careful description of the details of the resulting motion. In this chapter, we introduce a new approach involving physical quantities that remain *constant* for a system of one or more objects when the external environment has no net effect on the system. When the same physical quantity *does* change due to the action of the environment we can account for that change, as we can always find a new system where this quantity remains constant. This means that the physical quantity is *conserved*.

Section 6.1 illustrates this approach by considering the mass of a system of objects. The remainder of the chapter focuses on the ideas of impulse and momentum.

The content-based learning goals for impulse and momentum are listed below.

### Students should be able to:

1. Explain the concept of a “system” and systems thinking in physics.
2. Explain the difference between the terms “constant” and “conserved”. Recognize that momentum is a conserved quantity, but not necessarily constant in a particular system.
3. Choose a system and initial and final states when analyzing a process involving impulse and momentum.
4. Represent processes involving impulse and momentum using bar charts. Be able to identify the system, initial and final states, and decide whether momentum is constant or not in the process. If momentum is not constant, they need to be able to account for the change through impulse.
5. Compare and contrast force and impulse, impulse and momentum, force and momentum.
6. Apply momentum and impulse to solve problems in one and two dimensions.

7. Explain the relationship between the generalized impulse-momentum principle and Newton's laws.
8. Analyze inelastic collisions.
9. Be able to use momentum and impulse ideas to explain how practical applications such as air bags work to save lives.

We have broken the chapter into three parts:

- I. *Helping students develop the main ideas involving impulse and momentum*
- II. *Applying the general impulse-momentum principle to physical processes, including the use of words, sketches, impulse-momentum bar charts (a new representation), and equations*
- III. *Applying the impulse-momentum principle and knowledge from the first three chapters to interesting real-life physical processes*

For each part, we provide examples of activities that can be used in the classroom and brief discussions of anticipated student difficulties with the subject matter.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Helping students develop the main ideas involving impulse and momentum	6.1- 6.3	OET 6.1 (p149), TET 6.2 (p150)	6.2.3
Applying the general impulse-momentum principle to physical processes, including the use of words, sketches, impulse-momentum bar charts (a new representation), and equations	6.4, 6.5	6.1 (p158)	
Applying the impulse-momentum principle and knowledge to interesting real-life physical processes	6.6. 6.7	p164	
<p align="center"><b>Nontraditional end-of-chapter questions and problems</b></p> <p>Choose answer and explanation (CAE): Q6.8, Q6.14            Evaluate (reasoning or solution...) (EVA): Q6.15, Q6.16, Q6.18, P6.74            Linearization (LIN): P6.47</p>			

Multiple possibility and tell all (MPO): Q6.17, P6.5

Jeopardy (JEO): P6.25, P6.26, P6.34

Design an experiment (or pose a problem) (DEX): P6.78, P6.77

Problem based on real data (that students can collect by themselves) (RED): P6.72, P6.73, P6.74

### *Brief summary of student difficulties with impulse and momentum*

Students need to be reminded that before they do any analysis, they need to carefully identify a system and keep track of the initial and final momenta of each object in the system. The system choice is often fairly obvious, but we learn later that choosing a one-object system or a two-object system allows students to answer different questions. Objects that are outside the system can change its momentum; this does not mean that the momentum is not *conserved*, it only means that in that particular system it is not *constant*. The difficulty understanding the difference between constancy and conservation is even bigger for instructors who are used to the word conserved being used in place of constant. Another difficulty stems from identifying initial and final states of the system, without this step no process can be analyzed. Finally, students need to remember that momentum and impulse are vector quantities. To have numerical values students need to choose the axis and work with components.

## **I. Students develop the main Ideas of impulse and momentum**

Students first meet the idea of a conserved quantity (mass) in Section 6.1. The mass of an isolated system remains constant. If the system is not isolated, mass can enter or leave the system, but this is compensated for by the mass of the environment decreasing or increasing, respectively (or in some new system the mass is constant). This is what makes mass a conserved quantity. The section provides a concrete introduction to this important subject and shows students a useful representation for a conserved quantity: a bar chart. The goal of this section is to help students develop a conceptual understanding of, and representational abilities relating to, conserved quantities so that they can transfer these abilities to the more abstract quantity of momentum. We suggest that students do the ALG Activities 6.1.1-6.1.4 and then read the book (ALG Activity 6.1.5).

In Section 6.2, the goal is for students to invent a new physical quantity that is a product quantity. So far, they have used data to invent ratio quantities—the rate of change of position, the rate of change of velocity. This time they have to invent a physical quantity that is the product of two others, which (perhaps surprisingly) is more difficult than a quantity that is the ratio of two others. In addition, one of the quantities in this product is a vector and the other one is a scalar. Here students see

again that when a vector quantity in physics is multiplied by a scalar the result is a new quantity (new units), not the old quantity of a different magnitude. And finally, for a system, they need to realize that these new quantities add together as vectors. All activities in Section 6.2 of the ALG help students invent this quantity and test whether it is constant in an isolated system.

Specifically, ALG Activities 6.2.1 and 6.2.2 and the textbook's Observational Experiment Table 6.1 describe a series of experiments in which two carts collide. Students need to consider the system as the combination of two carts and analyze and compare the initial and final amounts of different quantities describing that system. They find that the sum of the mass times velocity of all the objects in an isolated system ( $\sum m\vec{v}$ ) is the same for both the initial and final states of the system. The product  $m\vec{v}$  is given the name *linear momentum* of an object and the sum is given the name *momentum of the system*.

If you are showing students similar experiments in class, it is important not to say the word momentum before students come up with the concept (this is true for any new quantity). You will notice that no new concept is defined in the book before students have a chance to “construct” it from concrete experience, either by doing and analyzing a real experiment or making sense of provided data. Helping students to create “an image” in the brain prior to providing an abstract definition makes the memories of the concept more accessible. It is also important to recognize that if you let students invent the quantity on their own, they may come up with other quantities that remain constant for an isolated system. For example, for *elastic* collisions the sum of the products mass times speed of all objects remains constant. Students might come up with this quantity and simply ignore the one type of (*inelastic*) collision where  $\sum mv$  does not remain constant before and after the collision. It is helpful to remind them that they are trying to find a physical quantity that stays the same before and after the collision for *all* possible cases; they are not allowed to ignore one case. This quantity is the sum of the products of mass and velocity of all objects in the system.

The constancy rule for the momentum of a multi-object system is tested in ALG Activities 6.2.3 and 6.2.4 and in the textbook Testing Experiment Table 6.2. If you are not using the ALG, encourage students to make predictions before reading the outcomes of the testing experiments in the textbook. Insist that they explicitly use the rule under test when making the predictions. End-of-chapter Questions 3, 6, 7, 18 are appropriate formative assessment questions here and Problems 1-9 can be assigned for homework.

We suggest that you focus on the following important points:

1. Start by identifying a system and keep track of the initial and final momentum of each object in the system.
2. Decide whether the system can reasonably be considered isolated. Equal magnitude and oppositely directed forces can be ignored, since in combination

they do not change the momentum of the system. However, friction forces can be considerable and each case needs to be examined separately.

3. Remember that momentum is a vector quantity analyzed using the momentum components along each coordinate axis. This requires a well-defined coordinate system and the use of correct signs for the momentum components.

In Section 6.3, students rearrange Newton's second law for a one-particle system to come up with the idea of the impulse that an external force exerts on the system object (ALG Activity 6.3.1). They also use Newton's second and third laws to understand momentum constancy for an isolated system. After students see how the momentum constancy idea follows from Newton's laws, it is helpful to emphasize that momentum is not really something new. It is something they have already learned, just reconceptualized in a different language, the language of a conserved physical quantity. Another important point is that intuitively students have a great feeling for momentum, as they know that a heavier faster moving object does more damage than a slower or a lighter one. They just need to give this feeling a name!

ALG Activities 6.3.2–6.3.4 can be done in class before students read the book (ALG Activity 6.3.5). We have a wealth of questions and problems for them to really understand the vector nature of impulse and its relationship to momentum. End of chapter Questions: 12, 13, 14, 26 and EOC Problems 10-26.

## **II. Develop skills to apply the generalized impulse-momentum principle in a way that leads to understanding**

In Section 6.4, students learn the generalized impulse-momentum principle in vector and component forms. It can be used for any process with an isolated or non-isolated system. They use a new qualitative way to represent such processes—impulse-momentum bar charts. These charts serve a similar role to that played by force diagrams in Newtonian physics. They have bars on the left for components of the initial momenta of the objects in the system, then a bar in the middle to indicate the component of the impulse exerted and then bars on the right reserved for the components of momenta in the final state. Students draw the bar charts after they have identified the system and initial and final states and the positive direction(s). When drawing bar charts, the main difficulties are how to decide what objects are in the system and how to represent the motion of objects described in the problem statement as bars. Students often have trouble with the idea of indicating momentum in the negative direction using downward bars and correctly incorporating external impulses into the bar chart. We suggest that students start by working on the ALG Activity 6.4.1 because at the end of it they are asked to read the textbook and learn how to draw bar charts. However, this skill comes on the “need-to-know” basis not as an isolated assignment. The ALG activities that follow (Activities 6.4.2 – 6.4.9) will

help students practice making and evaluating bar charts (reading and writing with the bar charts) and, finally, Activity 6.4.10 makes them question the difference in the meanings of the terms conserved and constant.

Notice ALG Activity 6.4.4 and matching Example 6.3 in the text (the happy and sad balls). They represent a very productive exercise that allows students to wrestle with the difficulties described. First, demonstrate in class the experiment where the sad ball fails to knock over the wooden block, but the happy ball does. In a small studio-class environment you can then ask each group of students to draw a bar chart of the process on a whiteboard with the goal of explaining why the happy and sad balls had different effects on the block (they should ignore the impulse exerted by Earth during the collision). Students will naturally ask whether or not to include the block in the system. You can encourage some groups to treat the ball as the system and other groups to treat the ball and block together as a system during the collision. Either approach will work, and great discussions can follow when they present their work to the rest of the class.

In a large lecture, you can break the exercise into a couple of steps: (1) Ask students to consider the ball as an isolated system and then vote on the correct impulse-momentum bar chart. They should then discuss their votes with each other and vote again; (2) Ask them to do the same analysis considering both the board and the ball as a system. Ask them to vote on the bar chart, then discuss with neighbors, then vote again. In this case it is important to recognize that the board receives some momentum from the sad ball (it rocks back and forth) even though it does not fall over. What students take away from this exercise is not only that the direction of motion matters, but that a bar chart is a tremendously powerful way to analyze a physical process.

Often students want to know why there isn't an "initial" and "final" impulse. Students are learning that impulse does not describe a state of the system in the way that momentum does. Momentum "resides" in the system, whereas impulse is the mechanism through which the momentum of the system can be changed—it does not reside in the system. It is ontologically distinct from momentum, and this point must be emphasized. Impulse quantifies a continuous process that adds or takes momentum away from the system from the initial state all the way through to the final state. The shading of the area where students have to place the impulse bar indicates that impulse is a quantity fundamentally different from momentum. On a bar chart, the total height of the bars in the initial and impulse regions must equal the total height of the bars in the final region. To help students with this, we overlay a grid onto each bar chart.

After students learn how to draw bar charts and use them to apply the generalized impulse-momentum equation to a particular process, they can apply this method at home to solve EOC Problems 27–34. We also have a novel EOC Questions that are will be challenging and helpful for the students. These are 9, 11–14.

In Section 6.5, students learn general problem-solving skills for using momentum to analyze physical processes. ALG Activity 6.5.1 is the same as Example 6.4 in the textbook. It describes a process that students need to analyze. They then represent the

process with a sketch. Then they use the sketch to represent the process with a bar chart. Finally, they use the bar chart to apply the generalized impulse-momentum equation to the process.

To determine the impulse caused by the force that an external object exerts on a system object, students will need the time interval that the force is exerted. In the case of an object coming to rest (Example 6.5), often it is the stopping distance that is known, not the time interval the object took to stop. An important method for converting a stopping distance to a stopping time interval is summarized and used in this section. It provides a good opportunity for the students to review kinematics.

The ALG activities in Section 6.5 provide multiple opportunities to practice the problem solving strategy. Activity 6.5.6 can be used as a lab. At home students can tackle EOC problems in Section 6.5. We especially recommend Problems 45, 46, 47, 49, 50 and 69, 74 and 76.

### **III. The application of the generalized impulse-momentum principle to analyze some interesting real-world applications**

Jet propulsion is the subject of Section 6.6. It does *not* include the continuous ejection of fuel from a rocket and its continual mass change—a subject that requires the use of calculus. However, it addresses conceptually the common belief that the fuel has to push against something for the rocket to accelerate. We suggest that students work on the ALG Activity 6.6.1 first and then read the textbook section (ALG Activity 6.6.2)

Section 6.7 is dedicated to two-dimensional collisions. The major issue here is to help students recognize that motions in  $x$ - and  $y$ -directions are independent and need to be treated separately, thus they need two bar charts for each collision – one for each direction. ALG Activity 6.7.3 is rather helpful here.

After this difficulty is overcome, the next one is whether the system can be isolated. Students may be confused about why the force of friction exerted by the surface on the cars is ignored when the cars collide (textbook Example 6.7) or why we don't worry about the gravitational force exerted by Earth on objects when they break apart (Example 6.8). Students should recognize that in collision events the initial and final states of the system are very close together in time. When this is the case, external forces are multiplied by a small  $\Delta t$ , resulting in a small or negligible impulse contribution to the bar chart. This only works if the interaction time is short and/or the external forces are small relative to the interaction forces between the objects that are included in the system.

End of chapter problems in Section 6.7 can be assigned in class or for homework, we especially recommend 60 and 62. Problem 60 changes the context of collisions as often students think that what applies to cars does not apply to celestial objects. Problem 62 uses non-traditional representations for the studies of momentum to help

students find consistency. Notice the problems without numerical values in this chapter. Research indicates that students have tremendous difficulties with these compared to similar problems that have numbers. Do not skip these (Problems 21 and 76). Finally, EOC Question 26 will challenge even your most advanced students.



# 7

## Work and Energy

In the last chapter, students learned about two conserved quantities—mass and momentum. Here they encounter the idea of the total energy of a system and the means to change it by doing work on the system. Just as the quantity *momentum* is either constant for a system of one or more objects if the system is isolated or can change when an external impulse is exerted on the system, the total *energy* of the system is constant if the system is isolated and changes if an external force does work on it. This approach allows us to incorporate the changes in the internal energy of the system into the generalized work-energy equation and removes the need to discuss conservative and nonconservative forces. As you read through the chapter, you will learn the details of this approach. Here we summarize briefly:

1. To determine the energy of a system, one must first choose a system. Any choice is allowable, but given the objective, often certain choices of system are better than others.
2. Total energy is a property of a system. Different forms of energy describe the interactions between objects in the system and their motion. The various forms of energy can be converted from one to another within the system.
3. External objects (parts of the environment) can do work on the system (positive or negative) and thus change the system's total energy. Objects within the system cannot do work on the system.
4. The energy conversions within the system and the changes of the total energy due to work done by external forces can be represented on a bar chart.
- 5 In cases that involve friction, we include both surfaces of interacting objects in the system. Thus when one object slides across the surface of the other object, there is no force of friction doing work on the system (this force is an internal force) but mechanical energy is converted into internal energy of the system.

Many physics education research studies have found that students have difficulties differentiating between work and energy and often double count them. For example, in a system including Earth and another object (and therefore possessing gravitational potential energy) students will still reason that Earth does work on the system. The approach used in the book not only allows us to address this problem but also helps with the first law of thermodynamics and all other areas of physics in which energy is the primary focus—electrostatic interactions, photoelectric effect, atomic spectra, fission, and so on. The approach establishes a link not only between different areas of physics but also between physics and chemistry. The idea of energy arises first in mechanics and develops throughout the text. The content-based learning objectives in the chapter are as listed below.

**Students should be able to:**

1. Connect energy to everyday experiences.
2. Recognize the role of a system in energy analysis, be able to analyze the same situation using different systems and explain the benefits and drawback of different choices.
3. Calculate the work done by constant and variable forces (such as an elastic force). Understand that the sign of work does not mean direction but addition and subtraction.
4. Differentiate between energy and related ideas (work, power, momentum, force).
5. Recognize that energy is a conserved quantity but not necessarily constant in a particular system.
6. Represent processes using work-energy bar charts and convert the bar chart into a mathematical statement of a generalized work-energy principle, and use the bar chart to evaluate numerical solutions.
7. Account for conversions of different forms of energy into other forms within the system, including conversions into internal energy.
8. Explain where the mathematical expressions for different form of energy came from, explain how to choose the zero level for gravitational potential energy and why the gravitational potential energy is negative when zero is chosen at infinity.
9. Design experiments to test energy conservation.
10. Analyze collisions using momentum and energy.

We have broken the chapter into six parts:

- I. *Students develop the main ideas of work and energy* Sections 7.1 and 7.2 help students devise work as a physical quantity, explain the different forms of energy, and introduce a new bar chart to represent work-energy processes qualitatively.
- II. *Develop quantitative expressions for the different types of energy*

- III. *Apply the generalized work-energy principle to interesting real-world processes in a way that leads to understanding.* Section 7.6 teaches how to use the generalized work-energy principle in a multiple-representation strategy to analyze interesting real-life physical processes.

The last three sections of the chapter focus on special work-energy processes:

- IV. *Apply momentum and energy conservation to different types of collisions*  
 V. *Develop the idea of power as the time rate of system energy conversion from one form to another or the time rate at which work is done on a system*  
 VI. *Construct a new expression for the gravitational potential energy of a system*

For each part, we provide examples of activities that can be used in the classroom and brief discussions of anticipated student difficulties with the subject matter.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Develop the main ideas of work and energy	7.1, 7.2	OET 7.1 (p177), OET 7.2 (p179), TET 7.3 (p182)	
Develop quantitative expressions for the different types of energy	7.3–7.5		
Apply the generalized work-energy principle situations that leads to understanding	7.6		7.6.11
Apply momentum and energy conservation to different types of collisions	7.7	OET 7.7 (p200)	7.7.1, 7.7.2, 7.7.4
Develop the idea of power	7.8		
Construct a new expression for the gravitational potential energy of a system	7.9		
<b>Nontraditional end-of-chapter questions and problems</b>  Ranking tasks (RAT): P7.72 Choose answer and explanation (CAE): Q7.10, Q7.14, Q7.22 Evaluate (reasoning or solution...) (EVA): Q7.13, Q7.16, Q7.21, P7.29, P7.43, P7.44, P7.75			

Make judgment (based on data) (MJU): P7.52, P7.53  
Multiple possibility and tell all (MPO): P7.16, P7.28  
Jeopardy (JEO): P7.39, P7.40, P7.41, P7.42  
Design an experiment (or pose a problem) (DEX): Q7.17, P7.74, P7.80, P7.82

### *Brief summary of student difficulties with work and energy*

Students might think that work is a property of a system; this mistake often carries over to thermodynamics. Given work can be positive or negative, students might think that work is a vector quantity and has direction. Students still might have difficulties identifying a system and initial and final states, and identifying the zero level of the gravitational potential energy. As we said above, the failure to identify a system and external objects often leads to double counting. Changes in internal energy of the system are imperceptible: this creates another difficulty – students might think that energy disappears. Often students think that the change in gravitational potential energy of the system is exactly equal to the change in its kinetic energy and vice versa (compensatory reasoning). Students might confuse work with energy, energy with force, and so forth. Thus they might think that both the elastic force and elastic potential energy are directly proportional to the first power of displacement.

## **I. Students develop the main ideas of work and energy**

Active Learning Guide (ALG) Activity 7.1.1 and Observational Experiment Table 7.1 lead students through four simple experiments in which a carefully defined system (Earth is part of the system) either gains the ability to break a piece of chalk or becomes warmer. Observational Experiment Table 7.1 matches the ALG activity but includes the answers. These experiments are easily reproduced in a lecture setting. (When we teach the material, we have students use the ALG table first and then read the answers in the textbook at home.) Students analyze the experiments by indicating the direction of an external force (represented by an arrow) exerted on one of the objects in the system as it moves from an initial state to a final state as well as the direction of the object's displacement (also represented by an arrow). They see a pattern: when the chalk-crushing ability of the system increases, the force and the displacement are in the same direction. Positive work is said to be done on the system, causing something in the system to increase so it could break the chalk or become warmer. This “something” that changes in the system when work is done on it is called *energy*.

The book *does not* define the total energy of a system as the ability to do work but instead as the sum of the different types of energy that change when work is done on the system. Energy comes in different forms, such as gravitational potential energy, kinetic energy, elastic potential energy, and internal energy (the warming and structural changes of objects). Other forms of energy (electric potential energy, rest energy, and so on) will be encountered later in the book and seamlessly fit into the framework developed in this chapter.

ALG Activities 7.1.2 and 7.1.3 and Observational Experiment Table 7.2 help students extend their understanding of work to the cases of negative and zero work. They perform and analyze (or read) three experiments in which the system either loses some of its ability to break the chalk or its ability does not change. The students' analysis of the force arrow and the system displacement arrow indicates that the arrows are in the opposite directions or perpendicular to each other, respectively. In the first two of these three experiments, negative work is said to be done, whereas in the third experiment (perpendicular force and displacement) zero work is said to be done. The patterns identified in ALG Activities 7.1.1-7.1.3 and textbook Tables 7.1 and 7.2 lead to a formal definition of work:  $W = Fd \cos \theta$ . It is helpful to point out to students how using the cosine function captures all the features of positive, zero, and negative work that they found in the observational experiments.

Students experience two difficulties with the concept of work. The first is linguistic. Many times, students will ask, "how is it possible that I can hold a heavy object in my hand and yet I am doing no work according to the definition?" It helps to remind students that physicists have simply reappropriated an everyday word into physics, and that in physics, "work" has a much more specialized meaning than in everyday language. Also, in the above example, physics does not consider the "microscopic" work done by the contracting muscles supporting the object. This example is discussed in detail in the textbook, on page 180, using two different systems.

The second difficulty is the abstractness of the mathematical expression for work, especially the meaning of the negative sign. It does not mean direction, but means subtraction. A whole class discussion of the meanings of the negative sign in physics can be very useful at this point. So far students have encountered the meaning of the minus sign related to direction. Now the negative sign has nothing to do with direction, but indicates an operation – taking something away. They might bring another meaning of the negative sign familiar to them, negative temperatures. This negative sign does not mean a special direction, but means direction along a scale of temperatures and represents the relative value with respect to a chosen zero. Research shows tremendous difficulties that students have with the negative sign, thus do not skip this discussion; initiate it! Try posing EOC Problems 1-3 as clicker questions in a large class or for group discussion in a studio classroom.

ALG Activity 7.1.5 helps students construct the concept of internal energy change due to external forces doing work on the system. Note that in the case of friction, we include both surfaces of the interacting objects in the system and talk about the work

we do on the system by pulling the crate being converted into the internal energy change. You might need to help your students here: use the material in the Observational Experiment Table 7.1 (experiment 4) and its discussion on page 178 for guidance.

Note that we suggest that students perform all ALG activities in Section 7.1 before they read the textbook (Activity 7.1.9 is the Reading exercise) but you could assign them to read the chapter first and then do Activities 7.1.4-7.1.8 in class. EOC Questions 1, 2, 5, 6, and 9 and Problems 1–9 are useful here.

We defined the energy of a system as something that changes when external objects do work on the system. The logical consequence of such a definition is the hypothesis that the energy of an isolated system is constant. Students test this hypothesis by doing the experiment in ALG Activity 7.2.1 or by analyzing the experiments described in Testing Experiment Table 7.3 in Section 7.2 (the experiments in Table 7.3 can be done in a whole-class setting). They then learn to analyze processes in which work is done on the system using qualitative work-energy bar charts. A bar chart has initial and final bars for each type of energy of the system. These are separated by a central shaded region for one or more bars representing the work done on the system by external objects. The bar chart can then be translated into an equation:

$$E_i + W = E_f$$

$$(U_{gi} + K_i + U_{si}) + W = (U_{gf} + K_f + U_{sf} + \Delta U_{int})$$

Table 7.4 provides examples of processes in which energy is converted from one form to another, represents these conversions with bar charts, and introduces proper language. We suggest that you use the situations in the table to analyze with the whole class through a discussion and then let students read the book. Corresponding activities in the ALG are Activities 7.2.2-7.2.5; all of them are marked as pivotal.

Physics Tool Box 7.1 on page 184 describes how to construct the bar charts and helps students apply bar charts to more complex problems. Although in the textbook it is placed after Table 7.4, you should encourage your students read it before working on the experiments in Table 7.4. Students need to explicitly go through all of the reasoning about a physical situation by constructing a bar chart for the process (choosing the system, defining initial and final states, and considering what energies the system possesses in each state). They can later convert the information in the chart to the mathematical form of the work-energy principle. A couple of bar chart activities in class provide a great opportunity to generate discussion among students as they try to make sense of what they are doing. The advantage of using bar charts is that students can analyze any process qualitatively without overburdening their minds with mathematical symbols in an equation—symbols that may have little meaning to them at this early stage. Research shows that this qualitative reasoning leads to better intuitive understanding of work and the changes of different types of energy, and

ultimately leads to superior reasoning and problem-solving ability. Later they will use the bar chart to write the generalized work-energy principle.

There are several important issues to consider: (1) Students have difficulties choosing the initial and final states of the process. Help them by explicitly asking every time they start a new problem what initial and final stages of the process they wish to consider; (2) They need to choose a system. For work-energy processes it is often advantageous to choose a fairly large system so that all the energy changes are internal to the system rather than expressed as work done by external objects. Table 7.4 provides simple examples of such system choices. You can ask students to construct bar charts for these or other examples. Use simple ones at the beginning; (3) It is usually easiest to include the gravitational interaction between Earth and an object as gravitational potential energy. To do this, Earth must be included in the system. However, if Earth is not included in the system, *the object by itself does not have any gravitational potential energy*, and the effect of gravitational interaction is expressed as work done by Earth.

Students struggle with the idea of choosing a zero point for gravitational potential energy of an object-Earth system. It helps to go through explicit examples in class. For example, holding a marker in your hand, you can ask students to discuss how much gravitational potential energy the system (marker and Earth) has relative to your desk as opposed to the floor. In trying to resolve this issue, students will come to recognize that they always need to choose an explicit zero point when discussing gravitational potential energy. In addition, thinking about gravitational potential energy of an object-Earth system, students reevaluate their experience of the physical world. An object on its own does not possess gravitational potential energy. An increase in gravitational potential energy arises from the fact that the instructor (an external agent) moved the marker and Earth farther apart from each other. When the marker and Earth fall back together, they are able to crush chalk between them. Most students probably have not thought about the idea that Earth “falls” toward the falling marker. However, although Earth “falls” we do not consider its motion in the analysis of work-energy processes because the distance it moves is negligible. Please also note that in this initial encounter with energy, we ignore processes involving friction until Section 7.5.

The generalized work-energy principle (Equation 7.3) in the textbook deals with the work of external forces exerted on the system of interacting objects, not the work of the net force exerted on one object. The work of external forces changes the total energy of the system including the internal energy. We do not specifically emphasize the kinetic energy theorem that equates the work done on point-like object by the net force with the change of this object’s kinetic energy, because this theorem does not take into account the changes in the internal energy of the object.

Note Conceptual Exercise 7.2 in this section of the textbook (page 185) that analyzes a pole-vaulting jump (do not skip it!). It teaches students how to analyze energy-related situations where internal energy change plays a significant effect. Here the internal energy (chemical energy) of the pole-vaulter decreases to enable her to do

the jump. Note that this example connects to the chapter opening photo and the story. We will return to pole-vaulting quantitatively later in the chapter. EOC Problems 39 and 40 are useful here.

## II. Develop quantitative expressions for the different types of energy

ALG Activity 7.3.1 and textbook Section 7.3 show students how to derive quantitative expressions for the gravitational potential energy of an object-Earth system and for kinetic energy. Textbook Section 7.4 leads students to Hooke's law. We use the same approach in the derivation of each type of energy in the textbook: Choose a system in which only the needed type of energy changes when external forces do work, and derive an expression for this work that depends on quantities describing the initial and the final state of the system. The sections describe the systems and external forces we chose step by step.

ALG Activity 7.4.1 in which the students invent the expression for the Hooke's law is framed as a testing experiment (a similar experiment in the textbook is framed as observational). Note the difference in the mental steps that the students need to take when the experiments are framed differently. In an observational experiment, the data that students collect without prior expectations lead to the creation of a hypothesis. In the testing experiment the students need to design an experiment, make a prediction based on the hypothesis that they are testing and only then run the experiment. They should have a very clear expectation of the outcome if their hypothesis is correct. After the students invent Hooke's law they proceed to the development for the expression for the elastic potential energy using the same approach as the used for gravitational potential energy and kinetic energy.

The approach for including friction is different from some other textbooks (ALG Activity 7.5.1 and textbook Section 7.5). Typically, a surface that exerts a friction force on an object in the system is excluded from the system, and we consider the work done on the system by this surface (an external object). This approach would be fine if the system's object could be considered a point-like object that does not have internal energy. But in real processes (for example, a car or skier skidding to a stop), we cannot consider the skidding object as a point-like object. The friction force exerted on the object causes the contacting surface to warm and possibly have structural changes—like scraping some rubber off a car tire. There are internal energy changes in the system object and similar changes in the surface exerting the friction force. If the work done by friction is equal to the friction force times the displacement of the object, it poses an unresolvable difficulty of the internal energy coming from nowhere. The fact is that the distance over which the friction force is exerted is less than the distance that the object moves. We can avoid these difficulties if we include both contacting surfaces in the system and we never talk about the work done by friction. Instead, we consider how much work is done by an external agent on the



object-surface system in order to keep the object moving at a constant velocity over some distance  $s$ . By making this choice, we can consider friction as an internal mechanism that increases the system's internal energy:

$$\Delta U_{\text{int}} = +f_k s$$

rather than considering it as a force exerted by an external object. Here  $s$  is the distance the center of mass of the object travels. Note that  $f_k s$  is not equal to work done by the friction force, because displacements of points where two objects touch each other are in general smaller than  $s$  due to deformations at the microscopic level. Most students accept this easily because they are not familiar with any other method.

Students can immediately apply the outcomes of these derivations to EOC Problems 10-29. We especially recommend 16, 28 and 29. They can do some of these in class in groups and at home. You can use multiple-choice Questions 10-13 for formative assessment in class. The real fun, however, is in the next part of the chapter, where students apply the generalized work-energy principle to interesting problems. The work-energy approach makes it much easier to solve the same problems that students solved earlier in the course using Newton's second law and kinematics. Additionally all of the videos of the experiments that we used in the Newton's law chapters can be re-analyzed here with the work-energy approach.

### III. Apply the generalized work-energy principle to interesting real-world processes

Section 7.6 introduces students to the problem-solving strategy for situations where the generalized work-energy principle is applicable (Example 7.6, which is turned into ALG Activity 7.6.7). Students can work with the ALG activities in Section 7.6 in class and lab. (Note that ALG Activities 7.6.3 and 7.6.6 can be combined in a very dense 3-hour lab for stronger students, but otherwise just Activity 7.6.6 may take them almost 3 hours).

Note Example 7.8 in the textbook: we are revisiting pole-vaulting quantitatively. Here students calculate the decrease in the internal energy of the pole vaulter and have an opportunity to analyze the process again. You might be wondering why we emphasize pole-vaulting so much, as most of our students will never experience it. However, the analysis of the pole-vaulting process prepares students to understand the mundane but very conceptually difficult process of walking up and down stairs (EOC Problem 46). When do we spend more internal energy? We encourage you to turn the material in the textbook on pages 197-198 into an interesting discussion. Conceptual Exercise 7.9 can serve as formative assessment of the discussion – students can do it in groups in class and then read the textbook at home.

It is very difficult to decide what EOC problems to assign for homework – there are so many excellent problems that not only put energy in context but also help students develop various science reasoning skills. We especially recommend Problems 33, 35, and 39-44, and, as already mentioned, 46.

## IV. Apply momentum and energy constancy to different types of collisions

Energy and momentum are used to analyze three different types of collisions in Observation Experiment Table 7.7 in Section 7.7. The collisions involve different types of pendulum balls hitting carts of different rigidity. In each collision, the system is the ball and cart; the system can be considered isolated. Students find that momentum is constant in all of the collisions, but kinetic energy is constant only in the collision in which the pendulum ball and cart results in no structural changes in either of the objects in the system. From these observations, three types of collisions emerge and are summarized in Table 7.8. It is good to go through these different types of collisions with students and to emphasize that they can use momentum constancy during all collisions.

Deciding when to use energy and when to use momentum in a problem like Example 7.10 is one of the most difficult challenges for students. The best way to help them figure it out is in a lab – see ALG Activity 7.7.2. It is a full lab and students have an opportunity to revise their assumptions multiple times. Simply telling students that they have to use momentum in an inelastic collision because they cannot account for all the energy will not have much effect on their understanding. The best way for students to build understanding is for them to explicitly draw energy *and* momentum bar charts for the collision part of the problem and then discuss their bar charts with each other. By exposing and resolving the conceptual issues associated with these multistep problems themselves, students will slowly become comfortable in tackling these problems on their own. Students can practice analyzing collisions solving EOC Problems 47-51. Problems 52 and 53 will present a real challenge to all of your students.

Finally ALG Activity 7.7.4 presents a real challenge. Here students encounter a system that does not consist of objects that can be considered rigid: there is motion inside the objects. The activity presents data for the collision of two carts, one of which has a box filled with sugar and ball bearings. These move during the collision. Not only is the kinetic energy converted into internal energy, but the momentum of the system exhibits a very strange behavior. Students will need both energy and momentum bar charts to analyze the process.

## **V. Develop the idea of power as the time rate of system energy conversion from one form to another or the time rate at which work is done on a system**

Notice that we define power as the rate of conversion of the system's energy from one form to another. That is why the language here is "the power of a process." It is important to emphasize that power is also a rate (ratio-type) quantity, similar to velocity and acceleration—the rate of energy conversion. The concept of power in Section 7.8 is primarily used later in the course—especially in Chapter 19—for electric circuits.

We suggest that students first work on the ALG Activities 7.8.1 and 7.8.2 before they read the textbook. Example 7.11 is a unique example not only addressing power but teaching students how to linearize data to find a relation. Do not skip it! EOC problems in this section vary from rather traditional to very innovative. Make sure you assign a few estimation problems and do not skip Problem 63.

## **VI. Construct a new expression for the gravitational potential energy of a system**

Section 7.9 introduces students to a new expression for the gravitational potential energy of a system to be used when  $g$  cannot be considered constant. Here the system approach to work-energy problems is especially important. If we consider the gravitational potential energy of two objects separated by an infinite distance to be zero, then an external force has to do positive work in order to slowly separate two initially closely-positioned objects so that the distance between them is infinite (this positive work brings the final energy of the system to zero). Representing this process with a bar chart yields the conclusion that the initial gravitational potential energy of two objects separated by a distance less than infinity is negative.

This is a challenging idea for students. It is helpful to begin Section 7.9 with a clicker or discussion question in class. The question presents students with two configurations of a pair of planets. In configuration A, the planets are close to each other; in configuration B, the same two planets are farther apart from each other. The question is, "Which configuration has the greater gravitational potential energy?" Students find this question surprisingly difficult, and it will engender a lively and productive discussion. Many students will incorrectly say that configuration A has greater gravitational potential energy because the force between the planets is greater. Given the opportunity and time, most students can resolve this confusion by discussing it with their peers. (This is ALG Activity 7.9.1). The next step is to reason

that the gravitational potential energy of a system is negative if the zero is chosen at infinity. It is addressed in the ALG Activity 7.9.2.

The derivation of the expression for gravitational potential energy when the objects are far apart involves the use of calculus; thus the book outlines the procedure but does not go into mathematical detail. To help students reconcile the final expression for gravitational potential energy of two objects

$$U_g = -G \frac{m_E m_O}{r_{EO}}$$

(with zero potential energy when the objects are infinitely far apart) with the familiar  $U_g = m_O g y$ , they could compare the work needed to lift 100-kg of supplies to the International Space Station using both methods and find the results to be very close. The idea that the energy zero point is when the objects are infinitely far apart requires radical conceptual restructuring for students who have become familiar with  $U_g = m_O g y$ . ALG Activities 7.9.3 and 7.9.4 address this issue. Do not skip them!

Practicing drawing bar charts for different scenarios will greatly help students make the transition. Later, students will apply the new expression to derive an expression for escape speed and learn about black holes. Perhaps the best reason for introducing this subject is its mathematical similarity to the expression for electrical potential energy that students will encounter in Chapter 17. ALG Activities 7.9.5–7.9.7 serve as motivation for the students – they can solve real-life (and nature) problems with the physics that they have learned so far.

Finally, note that students often have considerable difficulty understanding situations where the gravitational potential energy of a system is negative. This is yet another context in which they encounter the negative sign. We really want them to understand that we consider the energy of two objects attracted to each other to be negative because we need to do positive work to bring those objects infinitely away from each other. At this distance their energy of interaction is zero. The work is positive because the force we exert and the displacement of the object are in the same direction. Thus we added a positive number to some other number and we got zero – this means that this other number has to be negative.

Practicing determining escape speed as in EOC Problems 66 and 67 helps students understand the negative nature of the gravitational potential energy. It is crucial that they draw the energy bar chart representing what happens when we send an object away from Earth or the solar system, and only then convert it into an equation.

General Problems 71-74 represent an excellent conclusion to the chapter – they combine everything students learned about energy and momentum and encourage students to practice their higher reasoning skills. Problems 76 and 77 present real assessment of what students learned, given that without real understanding of systems they cannot solve them. Notice Problem 75: it combines a traditional problem with a novel exercise – the evaluation of someone else’s reasoning process. In addition, it requires students to use knowledge from Chapter 5

# 8

## Extended Bodies at Rest

Chapters 6 and 7 developed and applied the work-energy and impulse-momentum principles. In the work-energy chapter, we started moving away from modeling objects in a system as point-like objects because we wanted to incorporate the internal energy changes of these objects. In this chapter, we start a new approach involving rigid, extended bodies (not point-like) that are in static equilibrium. The content-based learning objectives are listed below.

### **Students should be able to:**

1. Find the center of mass of an object experimentally and mathematically. Give examples showing that mass is not necessarily symmetrically distributed around the center of mass.
2. Calculate the torques of given forces.
3. Design experiments to determine unknown forces using knowledge of torques and equilibrium.
4. Write the conditions of equilibrium and apply these conditions to analyze real-life situations.
5. Explain why certain equilibrium states are stable and certain unstable, give real-life examples, and analyze real-life situations, including biological systems.

We have broken the chapter into four parts:

- I. *A qualitative introduction to rigid bodies and to the idea of center of mass*
- II. *The development of the idea of torque and the conditions (translational and rotational) for the static equilibrium of rigid extended bodies*
- III. *A quantitative method for determining an object's center of mass*

IV. *The techniques and skills needed to apply equilibrium conditions to interesting examples, including the biomechanics of the human body. This part also includes a section on the stability of equilibrium.*

For each part, we provide examples of activities that can be used in the classroom and brief discussions of anticipated student difficulties with the subject matter.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Develop a qualitative introduction to rigid bodies and to center of mass	8.1	OET 8.1 (p219)	
Develop the idea of torque and the conditions for the static translational and rotational equilibrium of rigid bodies	8.2, 8.3	OET 8.3 (p227)	
Develop a quantitative method for determining an object's center of mass	8.4		
Develop the techniques and skills needed to apply equilibrium conditions to interesting examples including the biomechanics of the human body	8.5, 8.6	TET 8.4 (p238)	8.6.3
<b>Nontraditional end-of-chapter questions and problems</b>  Ranking tasks (RAT): P8.67 Evaluate (reasoning or solution...) (EVA): Q8.11, Q8.14, P8.56, P8.57, P8.58, Multiple possibility and tell all (MPO): Q8.16, P8.3, P8.17, P8.18 Design an experiment (or pose a problem) (DEX): Q8.22, P8.58, P8.59 Problem based on real data (that students can collect by themselves) (RED): P8.17, P8.56			

*Brief summary of student difficulties with torque and center of mass*

Students might confuse torque with force, and determining the sign of torque might pose a difficulty as well. Many students think that a force exerted on an object farther away from axis of rotation exerts a larger torque than a force exerted closer to the axis. The greatest of all difficulties is the confusion created by the term “center of mass” because mass is not symmetrically distributed around the center of mass. Most students would initially think that the masses of the parts of an object on two sides of the center of mass are equal (because the point is called the center of mass). This is an excellent testable idea!

## **I. Develop a qualitative introduction to rigid bodies and to center of mass**

The chapter starts by introducing students to a new model of an object, a rigid body, early in Section 8.1. One of the main properties of a rigid body is its center of mass. To come up with this concept, students can do Active Learning Guide (ALG) Activities 8.1.1–8.1.2 in a lab or lecture and then read through a simple observational experiment described in the textbook’s Table 8.1. The experiments involve pushing a flat board on a smooth surface with a pencil eraser (see Table 8.1) first when the board is by itself and next when a heavy object is placed at one of its corners. Students find that if they push along certain lines through the board, the board slides without turning. The intersection of these points is called the *center of mass* of the board. ALG Activity 8.1.3 tests these ideas and simultaneously makes students realize that the effect of the gravitational forces exerted on the extended object can be replaced by a single force exerted at the center of mass. This is a very important idea used later when applying the conditions of static equilibrium to extended bodies. Students can practice these ideas by doing ALG Activity 8.1.4 in recitations and labs, and after that reading Section 8.1 in the textbook. Alternatively you can assign to them to read the textbook section before class and then do activities in ALG Section 8.1, but turn Activities 8.1.1 and 8.1.2 into testing experiments. Use multiple choice questions 1 and 2 for formative assessment.

## **II. Develop the idea of torque and the conditions for the static translational and rotational equilibrium of rigid bodies**

ALG Activities 8.2.1–8.2.5 and textbook Section 8.2 help students develop the idea of the torque produced by a force exerted on an extended body as they observe simple experiments involving objects and several spring scales exerting forces on a meter stick at various locations perpendicular to the stick. Students observe patterns in the strength and locations of the pulling forces that cause the stick to remain at rest. If

you start torque in a lab or recitation, then students can do the ALG activities and collect their own data. If you start in a lecture, they can use the data in Figure 8.5 in the textbook. The next step is to include the angle between the forces and the stick—students observe and analyze the effect of the angle by doing ALG Activities 8.2.7 and 8.2.8 (they need to collect their own data there) or by analyzing the data provided in the textbook’s Table 8.2. They devise an expression for the turning effect of a force exerted on the stick (the *torque* that the force exerts on the object):

$$\tau = \pm Fl \sin \theta$$

Note that in this book we incorporate the vector nature of the torque through the sign rather than using the cross product. We also do not use the term lever arm, as this is additional (somewhat obscurely named) terminology that often confuses students. Because students “invent” the definition of the torque, slowly adjusting it to fit multiple experiments, the process helps them differentiate between force and torque. Finally, the torque represents another product quantity, similar to momentum, but the difference is that we treat it as a scalar. Several aspects of using the above equation are very important:

(1) Students need to construct an extended-body force diagram that incorporates the points at which forces are exerted on the object;

(2) Students need to carefully define an axis of rotation even if the object is not rotating. This is needed to determine the distance  $l$  that appears in the expression for the torque produced by each force exerted on the extended body. A subsection early in Section 8.2 focuses on the importance of defining an axis of rotation;

(3) Students often have difficulty deciding the sign of a torque. Figure 8.11 in the textbook just before Example 8.2, provides a concrete method to determine the sign. It is worth going over this method in the tip as a short interlude in a lecture. Recitation instructors should also be taught the method and encourage students to use it.

The experiments in Section 8.2 both in the textbook and the ALG used to develop the idea of torque also lead to a first introduction to the condition for rotational condition of equilibrium of an extended body. However, in the EOC problems we separate the skills of determining the torque from the analysis of the equilibrium. EOC Question 6 and Problems 1-4 help students practice how to determine the torques of different forces.

In Section 8.3, students develop the first and second conditions of static equilibrium further (see Activities 8.3.1 and 8.3.2 in the ALG and Table 8.3 in Section 8.3 in the textbook). Here, it is important to discuss again with students the difference between force and torque, as they often confuse these two physical quantities. Students can test these conditions by working through the experiment in Example 8.3, either as a whole class observational experiment in lecture or by performing the experiment in lab. In lecture, students can first work individually and use the equilibrium conditions to predict the readings on two scales supporting a beam at its ends while a massive block rests off-center on the beam. After students make their initial predictions, they can compare their results with those of



neighboring students. After the groups agree on their predictions, the instructor performs the experiment so that students can compare their predictions to the experiment's outcome. It is an exciting example of how physics principles can be used to make predictions that match the outcomes of the experiments.

In the text, student read about predictions made using two different axes of rotation, an important reminder to carefully choose the axis of rotation. Although the result will be the same regardless of choice, usually certain choices of axes will make the problem easier to solve. Alternatively, students can test the conditions of equilibrium in a lab using ALG Activity 8.3.2. EOC Problems 5-16 focus on just translational condition of equilibrium (sum of the forces is zero) and Problems 17-25 on both types of equilibrium. Questions 7 and 12-19 will be extremely useful.

### **III. Develop a quantitative method for determining an object's center of mass**

Section 8.4 introduces students to a method to quantitatively determine the center of mass of a multiple-point object and, in principle, of a continuous rigid body. The latter generally requires the use of calculus and is not done in the book. The key concept that students learn in this section is that the term “center of mass” is often misleading, because the mass of the object is not necessarily symmetrically distributed around the center of mass. Instead it should be called a “center of torque.” Given that many students take the term center of mass literally, it is important that they work through ALG Activities 8.4.1 and 8.4.2 and the subsection “Mass distribution and the center of mass,” including Figure 8.17 and ALG Activity 8.4.3.

In a lab, you can take a meter stick, attach a small but heavy object to one of its ends, and then ask students to calculate the center of mass of the combined system. After students do this, you can ask them whether the mass of the system to the left of the center of mass is equal to the mass of the system to the right. If they answer that the masses are equal, you can ask them how they can check this answer. Students have no trouble coming up with the way to do it, and they become very surprised to find those masses unequal. Problem 36 addresses this issue. ALG Activity 8.4.2 is a good laboratory activity to help students see that the mass of an object is not symmetrically distributed around the center of mass. Finally, Question 11 in the EOC is an excellent opportunity to consider the location of a center of mass of empty and full vessels. It is a unique question that stumbles experts but your students should be able to figure it out!

Also note that observant students will likely (given the opportunity) ask some variant of the following question: “If I add a new object to a rigid body (e.g., a seesaw) and the question asks, ‘where must I move the pivot so that the system balances again?’ should I treat this new object as exerting a torque or should I think of it as shifting the center of mass to a new location?” Allowing students to work through and resolve this question can be a great learning experience. At the end, students should

realize that the two views are equivalent (assuming you're on the surface of Earth), but, as they have been learning throughout previous chapters, it is all about how we choose the system. If we take the rigid body as our system and think of the new object as external to that system, then we say it exerts a torque on the rigid body. If we choose the new object as part of the rigid body system, then we can say that the center of mass of the rigid body has moved.

## **IV. Develop the techniques and skills needed to apply equilibrium conditions to interesting examples, including the biomechanics of the human body**

Textbook Section 8.5 as well as ALG Section 8.5 adapt the standard multiple representation problem-solving strategy for solving problems involving static equilibrium. We place considerable emphasis on biomechanics: the biceps muscle lifting the arm, the Achilles tendon lifting the heel of the foot off the ground, and the back muscle lifting a load while in a bent position.

When solving these and other problems in these sections, it is important to emphasize to students that the same three aspects of the problem solving, described previously in Section II, are important: (1) To construct an extended-body force diagram. Note that students have great difficulty in including the force exerted by the hinge/joint on the object/bone of interest. It is also not always obvious in what direction that force is pointing. Students need considerable practice with this; (2) To carefully define an axis of rotation, even if the object is not rotating; and (3) To choose the correct signs of torques.

The issue of the directionality of torque is probably the trickiest conceptual hurdle that students struggle with in this chapter. Although you or recitation instructors may have used Figure 8.11 on page 225 and introduced the idea of clockwise and counterclockwise, students will naturally use what is familiar to them. Thus students may want to assign a + sign to torques due to upward-pointing forces and a – sign to torques due to downward-pointing forces. Some students may alternatively want to suggest that torques to the left of the pivot balance torques to the right of the pivot, irrespective of the direction of the force.

A good way to help students feel more comfortable with the new concept of clockwise and counterclockwise directions is to present them with the following clicker question in lecture: Give students an extended-body force diagram for a uniform beam pivoted about its center. The diagram should show two forces to the left of the pivot, one pointing up and one pointing down, and two forces to the right of the pivot, one pointing up and one pointing down. Then ask students to find the signs of the four torques due to these four forces. Give students a chance to revise

their responses by talking to their neighbors. A lot of productive discussion and learning will result.

A fourth aspect not discussed yet is the importance of the distinction between torque and force, which students often confuse. In the case of static equilibrium, it is important that the students examine whether the forces add to zero along the  $x$ -axis and independently along the  $y$ -axis (the signs of the force components depend on the chosen coordinate system). Each force can also produce a torque on the extended body, the sum of which students need to be examined independently. The signs of the torques are given by hints (see p. 225)—in which way the force tends to rotate the extended body about the axis of rotation (+ for counterclockwise and – for clockwise).

Students can work though the following questions and problems: all activities marked as pivotal in the ALG Section 8.5; EOC Question 16, Problem 4, problems in Section 8.5 and general Problems 46-57.

Section 8.6 concerns the stability of equilibrium. Few books discuss this issue but it is very important in everyday life. If your students first work through the ALG activities and then read the book, we recommend all of the activities in the ALG Section 8.6. If they read the book first, then they can do ALG Activities 8.6.2 and 8.6.3. The main idea is that an extended body does not tip if it rests on a surface, and a vertical line passing through the object's center of mass passes between the bottom supports of the object. This rule explains why a standing person might fall when a train leaves a station. Another rule that accounts for stable equilibrium is whether or not an object's center of mass is below a fixed axis of rotation. We have several excellent questions and problems that will help your students practice these ideas: EOC Questions 8 and 10 and Problems 41-45.

Note ALG Activities 8.6.3 and 8.6.4. Both can be turned to relatively short labs that require easy available equipment. Activity 8.6.4 requires applying/combining the knowledge of forces/torques and the knowledge of friction. Students like it because of its practical value: it represents a simple method for measuring coefficient of static friction.

# 9

## Rotational Motion

In Chapter 8, students learned to analyze situations in which rigid, extended bodies were in equilibrium (constant rotational velocity, often zero) despite many forces being exerted on them and producing torques. In this chapter, we analyze situations in which the rotational velocity of rigid bodies changes. The content-based learning objectives are listed below.

**Students should be able to:**

1. Represent rotational motion using kinematics quantities.
2. Compare and contrast physical quantities characterizing linear and rotational motion.
3. Explain how rotational inertia is different from mass, and determine the rotational inertia of selected objects around a specified axis.
4. Apply Newton's second law for rotational motion to solve problems.
5. Represent rotational momentum using bar charts and use them to write mathematical representations.

The chapter is broken into four parts:

- I. *A kinematics description of rotational motion*
- II. *A dynamics explanation of rotational motion: a relationship between the net torque exerted on an extended body, the rotational acceleration of the body, and its rotational inertia.* This is all put together into a rotational version of Newton's second law.
- III. *A development of the ideas of rotational momentum and rotational kinetic energy*
- IV. *The techniques and skills needed to apply rotational motion concepts to interesting everyday examples.*

For each of these parts, we provide examples of activities that can be used in the classroom and brief discussions of anticipated student difficulties with the subject matter.

<b>Chapter subject matter</b>	<b>Related textbook and ALG sections</b>	<b>Textbook videos</b>	<b>ALG videos</b>
Kinematics description of rotational motion	9.1		
Develop the rotational analog of Newton's second law	9.2, 9.3	OET 9.2 (p257), TET 9.3 (p258)	9.3.5, 9.3.6, 9.3.10
Develop the ideas of rotational momentum and rotational kinetic energy	9.4, 9.5	OET 9.6 (p267), 9.1 (p272)	9.4.3, 9.5.3
Techniques and skills for applying rotational motion concepts to everyday examples	9.6		
<b>Nontraditional end-of-chapter questions and problems</b>  Ranting tasks (RAT): Q9.6 Choose answer and explanation (CAE): Q9.11 Choose measuring procedure (MEP): P9.68 Evaluate (reasoning or solution) (EVA): P9.25, P9.40, P9.68 Multiple possibility and tell all (MPO): P9.10, P9.11 Jeopardy (JEO): P9.26, P9.27, P9.49, P9.55 Problem based on real data (that students can collect by themselves) (RED): P9.69,			

### *Brief summary of student difficulties with rotational motion*

Students have troubles distinguishing between rotational velocity and rotational acceleration, and determining the sign of rotational acceleration (negative rotational acceleration does not mean slowing down). The radian measure of angles is also very difficult for students and needs special attention. Combining knowledge of translational motion with the knowledge of rotational motion is a real challenge.

## I. Kinematics description of rotational motion

Section 9.1 introduces students to the kinematics quantities rotational position, rotational velocity, and rotational acceleration, as well as to the relationships of the quantities with each other. ALG Activities 9.1.1 and 9.1.2 are excellent for helping students see the difference between rotational and translations quantities. In Activity 9.1.1, one student holds the end of a stick that is two meters long. Other students hold it at different distances from the end. Now the student holding the end slowly turns in place. Everyone can see the outer students running while the inner students are walking slowly.

Activity 9.1.3 helps connect different representations of rotational motion: it is truly unique, do not skip it! ALG Activity 9.1.4 helps students connect linear acceleration to rotational acceleration and practice linearizing data. Finally, Activity 9.1.5 serves the purpose of synthesizing the information that students have about these two motions. After they complete the activities, they can read the textbook section or proceed in the opposite way – first read the book and then do the activities as formative assessment of their understanding of the book text. Make sure that they pay attention to Table 9.1 in the textbook; it helps students solidify an analogy that connects new ideas about rotational motion to their existing understanding of linear (translational) motion.

As in Chapter 8, we assume that a torque that tends to cause counterclockwise rotation is positive and a torque that tends to cause clockwise rotation is negative. The same sign convention applies to the resulting rotational acceleration. Students often have problems determining the sign of rotational acceleration, just as they have trouble determining translational acceleration for translational motion. It is often easier for them to understand that when rotational velocity in the positive counterclockwise direction is increasing in magnitude, the rotational acceleration is also positive (and the object's rotation is speeding up). However, it is important that they understand that when the rotational velocity is negative and increasing, the rotational acceleration is negative but the object's rotation is not slowing down, it is speeding up.

Note that students are not accustomed to radians as a measure of angles; you will need to provide them with additional help as they work through the subsection on the units of rotational position in Section 9.1. It is useful to ask students to draw a one-radian angle on graph paper so they can clearly see that the arc length for one radian is equal to the radius of the circle.

For homework all problems in EOC Section 9.1 are useful, but we especially recommend Problems 2–4, 6, 10, 11, 14, and 15.

## II. Develop the rotational analog of Newton's second law

It is difficult (but not impossible) to derive a quantitative form of Newton's second law for rotational motion from observational experiments. Likewise, a full mathematical understanding of rotational dynamics is beyond the scope of an introductory algebra-based physics course. Therefore, we have decided to develop the rotational analog of Newton's second law as follows. First, in Section 9.2 students start with *qualitative* observational and testing experiments that help them build a conceptual understanding of the factors that affect the angular acceleration of an object (namely, the net torque exerted on the object, and the mass and mass distribution of the object). Then in textbook Section 9.3 and the ALG Activity 9.3.4, students can derive an equation for the rotational acceleration of a small object attached to a light stick that can move on a smooth surface in a circular path, having a force  $\vec{F}$  exerted on it tangentially to the circle.

ALG Activities 9.2.1 and 9.2.2 and textbook Observational Experiment Table 9.2 in Section 9.2 describe the experiments that help students devise qualitative patterns relating the torques produced by external forces exerted on a rigid body and the body's resulting rotational acceleration. It is very important here that they can clearly see the rotating tire and analyze the *changes* in its rotation. Given that students often confuse force and torque, Observational Experiment Table 9.2 helps them see the difference in the effects of these, and see for themselves that the rotational acceleration depends on the net torque, not just on the forces. The table raises the question of the role of mass in the process, which is answered in Testing Experiment Table 9.3. Parallel activities in the ALG are Activities 9.2.2 and 9.2.3. Because the activities in the textbook and in the ALG for this section are rather different, students can either do the ALG activities first and then read the book or read the book first and then do the ALG activities in labs and in problem-solving recitations to help students become comfortable with finding net torque and understanding its relationship to rotational acceleration. We also recommend EOC question 6, a simple ranking task.

Sections 9.3 in both the ALG and textbook help students construct the quantitative form of Newton's second law for rotational motion using a small object moving in a circle. We heavily rely on the analogies between translational and rotational motion that students continuously utilize. Similar to how the sum of the forces exerted on a point-like object and its mass determine the object's resulting translational acceleration, the net torque produced by exerted forces along with the mass of the rigid body should determine the rotational acceleration of the rigid body.

ALG Activity 9.3.4 and textbook Section 9.3 help students reason to the rotational form of Newton's second law for a simple case of a point-like object moving in a circle at the end of a massless rigid rod:

$$\alpha = \tau / mr^2$$

In this case, only one external object exerts a force on the circling object, and  $\tau$  is the torque produced by that force. The above can be generalized for the case of multiple torques exerted on the object and for extended bodies with rotational inertia  $I$ :

$$\alpha = \Sigma \tau / I$$

This generalization cannot be done rigorously without calculus. The second half of Section 9.3 in the textbook presents a heuristic approach by adding up contributions of several point-like objects to the overall moment of inertia of a rigid body. After students come up with the rotational form of Newton's second law, it is useful to emphasize the similarity between the mathematical descriptions of translational and rotational dynamics again (Table 9.4 in the textbook). If you wish, you can ask students to fill out the table before they see it in the book and then consult with the book. To help students develop facility with the law, use the rest of the activities in ALG Section 9.3 and EOC Questions 3 and 11 and Problems 18, 19, 21-28, and 39. Note that EOC Problem 25 involves work done in a rotation-based situation and contrasts work and force.

Students encounter several difficulties with this material:

1. *The magnitude of the force that a string wrapped around a rotating disc exerts on an object of mass  $m$  hanging at the other end.* Often students think that the magnitude of the force that the string exerts on the disc is  $mg$ . Because the hanging object almost certainly accelerates, the magnitude of the force that the string exerts on the disc should be less than the magnitude of the gravitational force that Earth exerts on the hanging object.
2. *Multi-object systems.* If a problem involves a rotating disk and one or more objects hanging from strings that pass around the disk, it is more productive if students choose these objects as separate systems for analysis and draw separate force diagrams for each object, including coordinate axes. It is easiest if the axes are oriented so that a positive (or negative) rotational acceleration for the disk produces a positive (or negative) linear acceleration for each of the hanging objects. This technique is used in the Atwood machine example (Example 9.4).
3. *Any problem (such as Examples 9.4 and 9.5) that involves one or more objects hanging from or otherwise interacting with a rotating object.* Such a problem requires students to make the connection between the linear acceleration of the hanging object(s) and the rotational acceleration of the arm. This is a very challenging conceptual hurdle for students and requires special attention from instructors.

Problems involving a rotating extended body attached to one or more hanging point-like objects take considerable time to solve and are probably not good problems for tests—especially multiple-choice tests. You could instead show a process in a sketch and ask students to choose the best force diagram for one or more of the objects shown in the sketch. For example, if a point-like object hangs from a string



that wraps around a disk and the point-like object is moving down at increasing speed, you could ask students to choose the best of three force diagrams with different tension forces relative to the gravitational force Earth exerts on the hanging object. For hand-graded problems, you could ask students to draw a force diagram for the hanging object, then make sure the forces have the correct relative magnitudes.

### III. Develop the ideas of rotational momentum and rotational kinetic energy

Section 9.4 helps students construct a new physical quantity, rotational momentum. Although the observational experiments described in Table 9.6 are qualitative, they help establish a rule relating the changes in rotational velocity and rotational inertia of a spinning object:

For an isolated extended body, when its rotational inertia  $I$  decreases, its rotational speed  $\omega$  increases, and vice versa.

It is important to note that because the rotational inertia depends on the distribution of mass around the axis, the rotational inertia of a body can change while the mass of the system does not. This property of inertia allows for much richer/surprising experiments in rotational dynamics compared to linear/translational dynamics.

After this we introduce the quantity of rotational momentum again using the analogy with translational motion. Students learn how to apply the rule in Example 9.6, jumping on a merry-go-round. The reasoning is based on the rule formulated on the basis of Table 9.6: the rotational inertia decreases and the magnitude of its rotational velocity increases. These experiments lead to the construction of the physical quantity rotational inertia using analogical reasoning:  $I$  is the rotational analog of the mass  $m$  of a point-like object and  $\omega$  is the rotational analog of the object's translational velocity  $\vec{v}$ . Thus it is reasonable to propose a new quantity—rotational momentum  $L$  of a turning object (analogous to linear momentum  $\vec{p} = m\vec{v}$ )—that should be equal to  $L = I\omega$ . The analogy between the impulse-momentum equation and corresponding rotational quantities leads to a general rotational impulse-rotational momentum principle. Note that while we treat rotational momentum as a scalar with signs, we discuss the vector nature of rotational momentum on page 270 in the textbook.

We suggest that students arrive at these ideas first by doing Activities 9.4.1-9.4.3 and then read the textbook (Activity 9.4.4). Notice that we use rotational momentum bar charts similar to linear momentum bar charts in Chapter 6. Students apply the principle quantitatively to the spin-up of a pulsar as its size decreases. Students can apply and practice the concept by working on EOC Questions 8, 10, 14, 16, 19, and 12 and Problems 41, 42, 45, 47, and 49.

Section 9.5 helps students construct the expression for rotational kinetic energy. We suggest that the students first do the ALG activities in Section 9.5 and then read the textbook. Note the testing experiment on page 272 in the textbook – you can

easily perform it in class, pending that before the students watch the experiment (or watch the video) they make the prediction of the outcome of a race down an incline between a solid disk and a hollow hoop, both of the same mass and radius, using their knowledge of rotational kinetic energy. They later read about how to use the expression for rotational kinetic energy to analyze rotating disks (flywheels) used for energy storage. EOC Problems 50, 52, 55, and 57 help students practice rotational kinetic energy ideas.

The first reading passage at the end of the EOC problems describes a classic experiment with two bottles of the same mass, one filled with water and another filled with snow, rolling down an inclined plane. This passage problem helps students understand the difference between rolling and sliding. (Note that the water does not rotate with the rotating bottle that it is in; thus it only undergoes translational motion.) The experiment is easy to perform in class. In any case, you can treat this experiment as a testing experiment—have students use their understanding of rotational motion to predict the outcome, then observe the experiment and have them revise their reasoning, if necessary. This is a good activity with which to use the think-pair-share technique. After you ask the questions, let students think quietly for 1 to 2 minutes (think), then turn to their neighbors to discuss their answers and come to a consensus (pair), and finally report the results of that consensus to the rest of the class (share).

## **IV. The techniques and skills needed to apply rotational motion concepts to interesting everyday examples**

Section 9.6 uses ideas developed in the chapter to analyze the effect of Earth's tides on the rotational speed of Earth, and therefore on the length of its day. The example is important not only because it applies to what students have just learned (changing the rotational velocity of Earth) but also because it gives students an opportunity to learn about tides and to go back to Chapter 5, where they learned the law of universal gravitation. You can either let students read the textbook first and then do ALG Activity 9.6.1 or, even better, let them do the activity first and then read the book. Notice that the second reading passage is dedicated to tidal energy. General problems will be an excellent opportunity for students to synthesize and apply everything they learned in this chapter to biological problems such as Problems 65-67. Problems 68-71 are novel; they make students not only apply new knowledge but develop important reasoning skills such as reasoning from evidence, reasoning with multiple representations, and evaluation.

# Vibrational Motion

The main goal of this chapter is to help students learn a new kind of motion—vibrational motion—and a new model of motion—simple harmonic motion. Specifically we want them to learn how to analyze vibrational motion using motion, forces, and energy approaches using all the tools that they have developed before – motion diagrams, force diagrams, and energy bar charts. We also want them to learn the mathematical details of simple harmonic motion as a model of real motion.

Content-based learning objectives for the chapter are listed below.

Students should be able to:

1. Compare and contrast vibrational motion with linear and circular motion.
2. Represent vibrational motion using motion diagrams, kinematics graphs, force diagrams, and energy bar charts.
3. Explain vibrational motion using forces and an energy approach.
4. Differentiate between vibrational motion and SHM as a model.
5. Relate trigonometric functions to SHM model.
6. Explain how to derive the expression for the period of an object on a spring.
7. Explain why the period of a simple pendulum does not depend on the mass.
8. Design an observational experiment to find out what quantities affect the period of an object on a spring and simple pendulum and a two independent application experiments to determine the spring constant of a spring.
9. Design an experiment to measure someone's mass using only a spring.
10. Apply knowledge of forces and energy to explain damping and resonance qualitatively.

The chapter is broken into five parts, starting with qualitative analysis and ending with more quantitative applications. For each part, we describe instructional sequences, ideas and activities that can be used in labs, and in class. We also provide brief discussions of the motivation for using these activities.

- I. *Qualitative description of vibrational motion*
- II. *Describing vibration using kinematics, forces, and energy*

- III. *Applying the same techniques to a simple pendulum*
- IV. *Developing the skills to analyze vibrational motion*
- V. *Including friction and driving forces in vibrational motion*

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Qualitative description of vibrational motion	10.1	OET 10.1 (p285), OET 10.2 (p286)	10.1.3, 10.1.4
Describing vibration using kinematics, forces, and energy	10.2-10.4	10.1 (p294)	10.2.5 10.4.4
Applying the same techniques to a simple pendulum	10.5		
Developing the skills to analyze vibrational motion	10.6		
Including friction and driving force in vibrational motion	10.7, 10.8	OET 10.6 (p306)	10.7.1, 10.8.2
<b>Nontraditional end-of-chapter questions and problems</b>  Multiple possibility and tell all (MPO): Q10.10, P10.19, P10.45, P10.71 Evaluate (reasoning or solution...) (EVA): P10.68 Make judgment (based on data) (MJU): P10.70 Jeopardy (JEO): P10.40 Design and experiment (or pose a problem) (DEX): Q10.12, Q10.13, P10.35			

*Brief summary of student difficulties with vibrational motion*

If your students have mastered Newton's laws and conservation laws and are fluent with motion diagrams, force diagrams, and energy bar charts, there will not be many difficulties in this chapter assuming that they are familiar with trigonometric functions and their graphs. Mathematics represents the biggest hurdle in this chapter and we suggest that you do not assume that students are fluent with sine and cosine functions.

The meaning of each symbol in the position-, velocity-, and acceleration-versus-time functions need to be discussed every time they write the equations.

## I. Qualitative description of vibrational motion

We suggest that students first observe different examples of vibrational motion to identify common features of this phenomenon. They can use objects attached to springs, pendulums, and any other vibrating systems. The experiments are described in ALG Activity 10.1.1. If you are starting in a large lecture, you can let them observe the experiments and then answer the ALG questions. Help your students focus on important features of this new type of motion: the equilibrium position where the vibrating object is at rest when undisturbed and the movement of the object when disturbed past equilibrium in two directions. Students should also find that some other object (like a spring), if moved away from the equilibrium position, exerts a force on the vibrating object that tends to return (restore) it to the equilibrium position. ALG Activity 10.1.2 asks students to explain these observations.

The next step is to qualitatively analyze vibrational motion in more detail (ALG Activities 10.1.3–10.1.5). We start with the example of a cart attached to a horizontal spring, as this is the easiest case in which restoring force can be clearly identified. It can be difficult to find a spring that stretches and compresses that will allow you to set up this apparatus for an actual demonstration. The work-around solution is to attach a partially stretched spring to each end of a dynamics cart, attaching those springs to the opposite ends of the dynamics track. However, this setup can be more confusing than enlightening to students. Likewise, an object attached to a vertical spring represents a more complex situation, and a pendulum is even more complex. Thus we made a video of a cart attached to a horizontal spring that we encourage your students to analyze. This is the simplest and the easiest experiment for kinematics, dynamics, and energy analysis.

When students represent the motion of the carts with motion diagrams, force diagrams, and bar charts, encourage them to place the representations for the same instants under each other so the pattern is easier to see.

ALG Activities 10.1.3–10.1.5 are probably the two most important activities for students to build their understanding of vibrational motion. We recommend allowing students to spend a good chunk of time on them. If you can't afford lecture time, they could be done in recitation. After students work through these representations on their own, they can read textbook Observational Experiment Tables 10.1 and 10.2 for answers to the ALG activities. Note that after this analysis, you can define the physical quantities of the amplitude and period for the students, for by then they should have a

pretty good understanding of the meanings. Reading textbook Section 10.1 (ALG Activity 10.1.6) and working on EOC Questions 1-4 and Problems 1-4 will solidify your students' conceptual understanding of vibrational motion.

## II. Describing vibration using kinematics, forces, and energy

Sections 10.2–10.4 both in the ALG and the textbook help develop the strategies needed to describe vibrational motion quantitatively. Students start with the data from the observational experiment of the motion of an object attached to a horizontal spring used in the ALG Activity 10.1.3 (ALG Activity 10.2.1 – do not skip it!). However, this time there is a motion detector at the end of the track that records position-, velocity- and acceleration-versus-time graphs for the cart. If you have similar equipment it would be great for the students to do the experiment and see the graphs, however, being able to analyze and make sense out of data collected by somebody else is also a very valuable skill. ALG Activity 10.2.1 asks students to focus on whether the three graphs are consistent with each other and whether they are consistent with motion and force diagrams constructed earlier.

It is very important that students work through these consistency checks. If they rush or are rushed through these elementary steps, they will fail to see the connections between all their representations (pictorial, graphical, and mathematical). The end result is a weak understanding that manifests itself as rote equation matching instead of deep and thoughtful analysis when students have to solve difficult problems.

Next, students need to establish the mathematical equations that describe the three graphs they have observed. Notice that we do not use the quantity  $\omega$  in the equations, just the period. We made this choice so that we do not confuse students: they can visualize the period of vibrations easily, but angular velocity is a much more complicated concept not really needed when they first encounter vibrational motion. ALG Activities 10.2.2–10.2.4 guide students through using the circular motion analogies (carefully crafted) to come up with their own time dependencies for the position, velocity, and acceleration. If they are not fluent with sines and cosines, it is often helpful to review the unit circle with them to develop and image and conceptual understanding of the sin and cos. ALG Activity 10.2.5 involves a real-life application of their knowledge – the analysis of the video of a flying humming bird. Do not miss this!

After working on these ALG activities, students can read the textbook section and meet the simple harmonic motion model of vibrational motion. Note here the difference between vibrational motion as a physical phenomenon and simple harmonic motion (SHM) as a *model* of such motion. Working through Examples 10.2 and 10.3 (an Equation Jeopardy problem that provides an exercise for interpreting a kinematics vibrational motion equation) in this section will be extremely helpful for students, as well as EOC Questions 5-8 and Problems 5-12.

The next step is to find out what variables affect the period of vibrational motion of an object attached to a spring, and how. The problem with simply deriving the relationship  $T = 2\pi \sqrt{m/k}$  from Newton's second law is that it doesn't allow students to test the idea that amplitude might affect the period. The most effective approach is to ask students to briefly brainstorm what variables they think affect the period and whether each variable will make the period longer or shorter. They should keep the list available and proceed to the ALG Activities 10.3.1-10.3.3. Ideally these three activities can occupy the whole lab period, but at the end they will be able to say whether and how the quantities that they put on their list actually affect the period. In this way, students *disprove* for themselves that amplitude matters. This helps convince them more than pointing to the equation for the period where there is no amplitude. Textbook Section 10.3 shows students a path to the mathematical relation between  $k$ ,  $m$ , and  $T$  using Newton's second law and the mathematical expressions for  $x(t)$  and  $a(t)$ . EOC Problems in the same section (13-20) are of increasing difficulty, if you need to challenge your students, we especially recommend 17, 19, 20, as well as General Problems 58, 59, 61, 63, and 71.

In textbook Section 10.4, students apply the ideas of work and energy in the analysis of vibrational motion. Table 10.4 has a summary of the potential energy, the kinetic energy, and the total energy of a cart-spring system at different times during one cycle of vibration. Students learn how to develop an expression for the energy of this system at different times. This expression leads to a useful equation relating the cart's maximum speed to the amplitude of vibration. All of the work-energy ideas are used in Example 10.5 to answer many questions about the cart-spring system. ALG activities in the corresponding Section 10.4 can be done before students read the textbook or after. If you wish to challenge your students we recommend ALG Activity 10.4.4. To explain the strange behavior of the toy, students need to combine the concepts of elastic potential energy, gravitational potential energy, and internal energy. EOC Problems 21-29, and 67 can be used with students for homework.

### III. Applying the same techniques to a simple pendulum

The same approach, from observations and qualitative analysis to the explanations and quantitative analysis, can be applied to a pendulum. We mostly focus on the model of a real pendulum, the simple pendulum, in which the bob is considered a point-like object, the string does not stretch, and the amplitude of vibrations is small.

The motion and force analyses of the simple pendulum are more complicated than those of an object attached to a spring because the sum of the forces exerted on the pendulum bob (the forces exerted by the string and by Earth) is **not** pointed toward the equilibrium position. To analyze the motion of the pendulum, it is beneficial to set up a new coordinate system, with a radial axis and a tangential axis. The restoring force in the case of the pendulum is the tangential component of the net force, but it is

just one component of the net force. This component always points toward the equilibrium position and is responsible for the change in the magnitude of the velocity vector. The radial component of the net force makes the bob move in an arc and is responsible for the change in the direction of the velocity vector. Students conduct the qualitative analysis of the pendulum's motion in ALG Activity 10.5.1. They will require considerable help with this activity. If they get stuck, they can check their work with beginning of the textbook section 10.5 (Figure 10.10). ALG Activities 10.5.2-10.5.4 can occupy the whole lab period, where students come up with and test the expression for the period of the pendulum; Activity 10.5.5 can serve as a part of homework. It is crucial that students read the textbook after they do some of the experiments as they have a very strong intuition for the period of a pendulum, but this intuition is valid for real pendulums, not the simple one. Thus they need physical experience first and then the reading exercise.

In the textbook (Section 10.5), they first read the text and examine Figure 10.10, which shows how to represent the vibrational motion of a simple pendulum in multiple ways: a motion diagram, force diagrams, and gravitational potential and kinetic energies. Table 10.5 provides data related to the effect of a pendulum string length, the amplitude of vibration, and the pendulum bob mass on the period and frequency of the pendulum. Analysis of the data should help students see that the period depends only on the length of the string. A derivation of the period and frequency of a pendulum is consistent with this outcome. The textbook applies the pendulum analysis to the swinging frequency of a leg and the dependence of leg length on the number of steps per unit time. Students can try EOC Problems 35, 37, 38, 40, 41, and 49.

## IV. Developing the skills to analyze vibrational motion

We use a vertical spring in Example 10.7 in Section 10.6 to illustrate how to adapt the book's general problem-solving methods to vibrational motion problems and show how to approach multiple-possibility problems, problems that have multiple answers depending on the assumptions. It is best if your students first attempt this problem on their own (as in the ALG Activity 10.6.1) and then carefully study our solution in the textbook. Example 10.8 in the textbook involves a skier who runs into and attaches to a padded cart with a spring on the other side. The skier and cart undergo vibrational motion at the end of the ski run. Both examples represent typical problems for simple harmonic motion. The third example in this section, Example 10.9, discusses the vibration of  $\text{CO}_2$  molecules in the atmosphere and the way the absorption of infrared radiation from Earth contributes to greenhouse warming.

Notice that in Examples 10.7 and 10.8, both motion diagrams and energy bar charts are used to analyze problems situations. After working thorough these examples students should proceed to the ALG activities in Section 10.6. Most of them represent practice problems posed in different ways. Two of the Activities, 10.6.7 and 10.6.8,



can be used as a complete lab. EOC problems in Section 10.6 and General Problems 66-68 and 70 and 71 can be assigned for homework. Note that Problem 71 is a multiple-possibility problem that allows the students to practice making and following up on the assumptions that they learned by working through Example 10.7

## **V. Including friction and driving forces in vibrational motion**

Section 10.7 is short section dedicated to the damping of vibrating systems (underdamped, critically damped, and overdamped oscillators). We have a nice video that helps students visualize small and large damping in the ALG (Activity 10.7.1). We suggest that students work with this activity first and then read the book. EOC problems in Section 10.7 can be assigned for homework.

Section 10.8 introduces students to forced vibrations qualitatively. We suggest that students do the experiments described in the ALG Activities 10.8.1 and 10.8.2 – these can be done in a lab or in class – and then proceed to reading the book. By analyzing the observational experiments in the ALG and experiments in the Observational Experiment Table 10.6, students should come to the following conclusions: the amplitude of forced vibrations increases when the frequency of the application of the external force is the same as the natural frequency of the vibrating system. The explanation for this pattern comes from the energy analysis. When these two frequencies are the same, the external force exerted on the vibrating system does positive work on it, and the energy of the system increases. The idea is then tested in the textbook with a simple experiment with several pendulums of different lengths hanging from a sagging horizontal cord. However, if you start with the ALG activities, this experiment is used as an observational experiment. Therefore the students can learn that the same experiment can serve either purpose depending on how much we know beforehand. When it is used as an observational experiment, we use the data to make explanations or hypotheses; when it is used as a testing experiment, we use those hypotheses to predict its outcome. However, we cannot use the same experiment for both purposes. This particular experimental set-up is easy to make, and we strongly recommend that you do it. It allows students to analyze forced vibrations deeply and develop confidence in their ideas. Real-world examples of forced vibrations include a marching band that caused a bridge to collapse, breaking a wine glass with the correct frequency sound, and vibrations of the Tacoma Narrows Bridge. We strongly recommend that you work with your students though EOC Problem 56 and actually do the experiment described in Problem 54. Will they get a similar result?

Finally, General Problems 62, 64, 65, 68 and 68 are excellent problems to challenge your students at the end of this chapter. Problem 62 is especially useful because students can easily do the experiment themselves as a testing or an application experiment.

# 11

## Mechanical Waves

Chapter 10, on vibrational motion, laid a foundation for the study of mechanical waves in this chapter. In Chapter 10, students learned to describe and explain the vibrational of one point-like object. In this chapter students will learn how vibrations originated at one location spread through the medium. Mathematically speaking, instead of describing the motion of an object using a sinusoidal function of one variable—time—they will learn how to describe motion using a sinusoidal function of two variables, time and the position of the vibrating object. The content-based learning objectives are listed below:

**Students should be able to:**

1. Explain the role of a source and a medium for wave propagation.
2. Explain why it is better to write the famous wave relation as  $\lambda = Tv$  rather than  $v = f\lambda$ .
3. Explain the difference between an operational definition of wave speed on a string and a cause-effect relationship.
4. Write the wave equation  $y = A\cos\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$  and explain how it relates to the physical quantities describing traveling waves.
5. Explain and justify which physical quantities in the wave equation are a consequence of the wave source and which are a consequence of the medium in which the wave travels (or both).
6. “Read and write” with wave motion graphs  $y(t)$  and  $y(x)$ , and with wave fronts representations.
7. Describe the differences between traveling and standing waves (in terms of phases of different points and their amplitudes).
8. Calculate resonant frequencies in strings and open and closed pipes.
9. Explain the role of impedance in wave propagation.
10. Use Huygens principle to explain reflection and refraction of waves.

11. Use the superposition principle to explain beats and constructive and destructive interference and standing waves.

The chapter is broken into six parts:

- I. *A qualitative analysis and kinematics description of waves*
- II. *A dynamics and work-energy analysis of waves*
- III. *Wave interference*
- IV. *The application of these ideas to sound waves*
- V. *Standing waves on string instruments and in pipes*
- VI. *The Doppler effect for sound*

For each part, we suggest activities that can be used in a lab, lecture, and recitations to help students develop and test these ideas. We also provide brief discussions of the motivations for using these activities.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
A qualitative analysis and kinematics description of waves	11.1, 11.2	OET11.1 (p316), 11.1 (p318)	11.1.3, 11.1.4, 11.1.6
A dynamics and work-energy analysis of waves	11.3, 11.4		
Wave reflection and interference	11.5, 11.6	TET 11.3 (p331)	11.5.1, 11.6.2, 11.6.4, 11.6.5,
Application of these ideas to sound waves	11.7		
Standing waves on string instruments and in pipes	11.8, 11.9	OET 11.5 (p336)	11.8.6
Constructing and applying the Doppler effect for sound	11.10		
<b>Nontraditional end-of-chapter questions and problems</b>			

Ranking tasks (RAT): P11.13, P11.59  
Evaluate (reasoning or solution...) (EVA): P11.51  
Make judgment (based on data) (MJU): P11.43, P11.65  
Multiple possibility and tell all (MPO): P11.3, P11.7, P11.9, P11.50, P11.63  
Designing an experiment (or pose a problem) (DEX): Q11.16, P11.18, P11.25, P11.37, P11.75  
Problem based on real data (that students can collect by themselves) (RED): P11.58, P11.79

### *Brief summary of student difficulties with mechanical waves*

There are several conceptual and several mathematical difficulties. The biggest conceptual difficulty is the fact that the propagation of wave is not the propagation of parts of medium but rather the propagation of disturbance (deformation) of the medium. Another difficulty is the understanding that there are several speeds involved in wave motion – the instantaneous speed of each particle in the medium and the speed of propagation of energy in the medium. When we talk about wave speed we commonly mean the latter but this is not evident for students. The mathematical difficulty comes from the wave equation being a function of two variables, a situation students have not encountered before. Therefore they often confuse period of a wave with its wavelength and try to find the wavelength on a  $y(t)$  graph. Finally, students have conceptual difficulty understanding superposition (that waves move through each other and do not collide).

## **I. A Qualitative analysis and kinematics description of waves**

Students start to learn about wave motion by studying the motion of individual pulses. ALG Activities 11.1.1 and 11.1.2 help students conduct initial observations of wave motion. These observations help students to see the difference between transverse and longitudinal pulses, and their propagation in the medium. It is best if students not only perform the experiments described in the ALG activities and observe the motion of the individual Slinky coils and the generated pulses, but also video the experiments (using their phones if they have high speed cameras). If it is not possible for your students to conduct the experiments themselves, you can demonstrate the ALG experiments or experiments described in textbook Section 11.1, Observational Experiment Table 11.1.

When students observe pulses traveling on a Slinky, suggest that they put a ribbon on one of the Slinky coils and focus their attention on the fact that the disturbance moves along the Slinky, but the ribbon moves perpendicular to the Slinky (or along it) as the pulse passes, but does not travel along the Slinky. Students tend to think that

the material through which a wave disturbance travels actually moves in the same direction—perhaps because ocean waves push us toward the shore. Give students time to explore the idea that the harder you shake the Slinky, the faster the wave moves. In our experience, most students haven’t distinguished between frequency and amplitude yet. They suspect that the more “oomph” you put into shaking the Slinky, the faster the pulse will move. A bigger “oomph” is some combination of a quicker hand movement and larger amplitude. Students can explore this idea and refute it for themselves through careful experimentation with the Slinky. Note that the Slinky is a unique piece of equipment as it allows the students to observe both transverse and longitudinal pulses (waves).

ALG Activity 11.1.3 moves students from a pulse to a wave. If you have a body of water nearby, it is worth taking students outside to perform this activity; if not, the video will do. The main point of the experiment is to question whether the water moves outward as the wave propagates. Once students come up with the explanation they can test it in the next activity, Activity 11.1.4. We suggest that you first ask them to design an experiment to test two competing hypothesis: (1) the water moves outward or (2) the disturbance moves but the water parts only oscillate in place. In our experience students design an experiment very similar to the one videoed. They can make predictions based on both hypotheses and then watch the video. ALG Activities 11.1.5 and 11.1.6 help students construct the idea that the interactions between the parts of the medium are necessary for any mechanical wave to propagate, the video in Activity 11.1.6 of a wave generated by hitting a person’s arm provokes a lot of interest. Finally, Activity 11.1.7 helps them observe and explain reflection of the pulses.

Textbook Section 11.1 introduces students to wave fronts and different types of mechanical waves—water waves and sound waves.

After students explore pulse/wave motion qualitatively and transition to wave motion at the end of Section 11.1, they can proceed to working with Section 11.2, which helps them define the physical quantities that describe wave motion: period, frequency, amplitude, and propagation speed. We focus on the main idea here that the frequency of a wave is determined only by the wave source, the speed is determined only by the medium (as we only deal with cases where there is no dispersion), and the wavelength is the combination of the two. The quantity of the wavelength appears only after students see how to construct a mathematical description of a sinusoidal wave. We suggest that you examine carefully the steps that lead to the wave function  $y(x,t) = A \cos[(2\pi/T)(t - x/v)]$  and the subsequent introduction of the wavelength in the textbook, and then guide the students through Activities 11.2.1-11.2.6 in the ALG. We use PhET simulations here because we found them to be especially effective for this purpose. Here is a brief summary of the logical progression.

We first construct a function for the position as a function of time for one point in the medium (we choose this point to be the vibrating source). Then we write the time

function describing the vibration of a point that is  $x$  meters away from the source. This function is similar, but we need to take into account that vibrations arrive at this point with a time delay equal to  $\frac{x}{v}$ . Thus we can write the function  $y(t)$  for every point of the medium as

$$A \cos \left[ \frac{2\pi}{T} \left( t - \frac{x}{v} \right) \right] = A \cos \left[ \frac{2\pi}{T} t - \frac{2\pi}{T} \frac{x}{v} \right],$$

which means that we have the function of two variables—time and position,  $y(x, t)$ . The new wave equation allows us to find the points in the medium that always have the same displacement at the same clock reading, the points that vibrate in phase. To find them, we set  $[(2\pi)/T](x/v) = 2\pi n$  where  $n$  is any integer. Therefore the distance between those points is  $x = nTv$ . The closest of those points ( $n = 1$ ) are said to be separated by a distance called the wavelength. Thus we can define the cause-effect relation for the wavelength  $\lambda = Tv = \frac{v}{f}$ . This approach helps students understand that the wavelength depends on the motion of the source (period of the wave) and the properties of the medium (wave speed).

A common way of writing the same equation as  $v = f\lambda$  does not allow students to see this cause-effect relationship, and unnecessary confusion results. To understand wavelength operationally, it helps students to think of it as the distance the wave travels in the medium in the time of one period of the source oscillation. It is useful to have students summarize and organize the large number of new physical quantities that they have discovered and the relationships among them. They need to realize that frequency and period depend on the behavior of the source. Wave speed depends only on properties of the medium in which it travels. Wavelength is determined both by the source *and* by the medium in which the wave travels ( $\lambda = \frac{v}{f} = vT$ ).

Having students organize their ideas at this point serves as an excellent foundation for the more difficult wave topics that are still to come (such as standing waves).

Students can work on textbook Example 11.1, an Equation Jeopardy problem in which students learn to tell everything they can about a wave whose wave equation is known. After this example they can work on the ALG Activity 11.2.7

The following Questions are useful here 1-5, 8, 10, 11-15. We also recommend Problems 1- 13. Problem 13 is especially helpful for understanding wave equation.

## II. A dynamics and work-energy analysis of waves

After students have learned to construct the function that describes the motion of each point in a wave, the next step is to investigate what physical quantities affect wave speed. A conceptual activity that is useful to start with is ALG Activity 11.3.1, and then students can proceed to ALG Activities 11.3.2 and 11.3.3. The latter two

activities provide data concerning the speed of wave propagation along several strings. The data analysis allows students to construct the cause-effect relationship that connects the speed of the wave to the properties of the medium. They do not need equipment to do these activities, so you can use them in a lecture setting or a problem-solving session. The following activities in this section of the ALG allow the students to practice the invented expression for the speed and finally, Activity 11.3.6 can be used in a lab as it takes a significant amount of time. In this activity students will design an experiment that allows them to determine the speed of a pulse on the rope and compare the result with the value of the speed obtained from the expression for the speed on the string (cause-effect relationship) that they discovered earlier. They can practice assessing assumptions and uncertainties in addition to the mathematical relations for wave speed.

A similar setup that students use in the ALG Activity 11.3.6 is described in the textbook. Table 11.2 in Section 11.3 provides the data and the analysis showing how  $A$ ,  $f$ , and the force you exert pulling on the end of the string affect the speed  $v$  of a wave on the string. Force is the only one of these three quantities that affects the wave speed. Other experiments indicate that the mass per unit length of the string also affects the wave speed. These ideas lead to an expression for the wave speed. Since the wave speed depends only on properties of the medium and the wave frequency depends only on the source vibration frequency, the wavelength depends on the wave speed and the frequency. Students can work on EOC Problems 16–19.

In Section 11.4 students analyze single wave pulses produced in two- and three-dimensional media. The energy of the pulse spreads over greater surface areas for each case. Thus, the amplitude of the wave decreases as it spreads from the source of the wave. This leads to the introduction of a new quantity, the wave intensity, which is used later in this chapter and in subsequent chapters. They can first do ALG Activity 11.4.1 and then work through EOC Problems 20–23 here.

### III. Wave interference

Textbook Section 11.5 and ALG Activity 11.5.1 describe experiments with wave pulses that are partially reflected and partially transmitted if passing from one medium to another medium with different density (a mass-like quantity) or a different coupling of the particles in the medium (a force-like quantity). Students can observe these experiments in lecture to start thinking about the impedance of a medium and impedance matching to reduce the reflection of waves at the boundary of different media. We provide biological examples of this important idea. Use end-of-chapter Problems 26, 28, 29, and 30.

To construct the principle of superposition, it is best if students start with ALG Activity 11.6.1 as an observational experiment and Activity 11.6.2 as a testing experiment. If you ask students to explain what they observed in Activity 11.6.1, some will suggest that the two pulses passed through each other, but others will suggest that the pulses bounced off of each other and were reflected back. You can use the second

experiment (one upright, one inverted pulse) to test both of these ideas. Ask students to predict what they will see based on each idea (passing through and bouncing back) before you run the second experiment. Once it is established that the two pulses pass through each other, students need to observe how the pulses add together when they're on top of each other. Once these activities are done (could be in a lab or in lecture) the students can read the textbook, which describes similar experiments in Section 11.6.

We then apply this idea of pulses adding or subtracting to waves created in swimming pool water by two synchronously vibrating beach balls. Looking along a line that is equidistant from the two vibrating beach balls, one observes a resultant wave that has twice the amplitude of a single wave. Students should be able to construct the superposition principle analyzing these simple observations. This analysis is very important for helping students understand Young's double slit interference experiment, which they will encounter later in wave optics. ALG Activities 11.6.3, 11.6.4, and 11.6.5 help students construct and apply Huygens' principle. We suggest that you do not skip these activities to prepare students for wave optics. In the textbook, Huygens' principle comes after students work on Example 11.4 (see below). It starts with an analysis of an imaginary observational experiment that leads to the Huygens' "idea." The idea is then tested in Testing Experiment Table 11.3.

In the same section, Section 11.6, you will see how the textbook's general problem-solving strategy is adapted to problems involving wave motion (ALG Activity 11.6.7 and Example 11.4 show the general steps (on the left side of a problem-solving table) and their application to the problem (on the right side)). Students can work on EOC Problems 32-36.

## **IV. The application of these ideas to sound waves**

Since students encountered sound in Section 11.1, Section 11.7 focuses primarily on the physiological impression related to the loudness of sound. The intensity of sound is a physical measure of the variation of pressure of sound waves relative to atmospheric pressure. However, our impression of sound loudness is measured by a quantity called intensity level—defined in this section and applied in Quantitative Example 11.6 to the sound in a busy classroom. ALG Section 11.7 is full of mathematically-oriented activities doing which students can practice graphical analysis, wave equation and so forth using the data from sound waves.

What follows is the frequency of sound, the subjective impression of the pitch of a sound, and complex sounds, which are in part related to the quality of a sound. A complex sound, which consists of multiple frequencies of single-frequency sound waves heard simultaneously, is analyzed in terms of a waveform and a frequency spectrum. The waveforms from musical instruments and the corresponding frequency



spectra are first introduced here. Students can work on EOC Problems 39, 40, 41, 43 and 44 in lectures, recitations or homework.

## **V. Standing waves on string instruments and in pipes**

ALG Activity 11.8.1 is crucial for student understanding of how standing waves form on strings. If you do not have lab time, make sure you invite two students in front of the class in lecture and they carry out this experiment. The same experiment is described in Observational Experimental Table 11.5 in textbook Section 11.8. The key elements to helping students understand how standing waves are formed are that the wave is being reflected at each boundary, that the waves produced by the source adds (superimpose) to the waves that were produced earlier and reflected from the boundary, and that the harmonic motion of the source needs to be synchronized with the time it takes for the wave to move back and forth on the string. Once students understand these elements, they can work out the mathematical relationship for standing waves (ALG Activity 11.8.2). ALG Activity 11.8.3 is a perfect testing experiment for the mathematical expression that they derive. Finally, ALG Activity 11.8.4 is a full-scale lab that solidifies their understanding (another, more sophisticated testing experiment). We have been using this lab for years with great success. Students can then work on ALG Activity 11.8.5, EOC Questions 10 and 11, and EOC Problems 47-52. Problem 51 is excellent for formative assessment. Finally, general Problem 79 is a perfect problem to conclude this section!

The process is repeated in Section 11.9 for the first standing wave in an open-open pipe. ALG Activity 11.9.1 is an excellent introduction. After this we suggest that the students read Section 11.9 in the textbook and then return to ALG Activities 11.9.2–11.9.6. The case of standing wave frequencies of an open-closed pipe is more complicated because a wave has to complete two round-trips, so it is reinforced constructively to produce the fundamental frequency. The standing wave frequencies of various musical instruments are related to the open-open and open-closed pipe frequencies. EOC Problems that address this topic are 63–65.

## **VI. Constructing and applying the Doppler effect for sound**

Earlier in the chapter, the frequency of observed waves equaled the frequency of the source. To start students' exploration of the Doppler effect, if you have experiments described in Observational Experiment Table 11.6 in Section 11.10, you can use them instead and proceed with the students using the guidance of this table.

Students observe that the frequency of an observed wave is greater than the frequency of the source of the wave if the relative motion of the source and observer

is toward each other; the observed frequency is less if the source and observer are moving apart. Then they can work on ALG Activities 11.9.1-11.9.3 to construct a conceptual understanding of the Doppler effect (they can be done in a lab or in a class setting) and then read the textbook chapter (ALG Activity 11.10.4).

You can repeat the textbook derivation for the frequency change with the source moving relative to the medium through which the wave travels and with the observer moving relative to the medium. Combining these two expressions leads to the Doppler effect equation for general case. If you follow the derivation, the students will build this concept using their own observations.

Students can work on ALG Activity 11.10.5, which encourages them to make sense of the equation. The Doppler equation with  $\pm$  on the top and  $\mp$  on the bottom can be confusing to use. We provide the rules, but a good check is to recall the outcomes of the Table 11.6 experiments and choose the plus or minus sign that matches the situation being addressed. We use the Doppler equation to show one method to measure blood speed (students can work on ALG Activity 11.10.6 and read the text) and the changing frequency of a buzzer moving in a circle (Example 11.9 in the textbook). Students can then work on EOC Question 9 and Problems 66-71, and 78. In class, it is sometimes fun to take a multipart problem like Problem 66 and have one-fourth of the class work on Part (a), one-fourth work on Part (b), and so forth.

# 12

## Gases

This is the first chapter that focuses on the properties of matter—in this case, primarily on gases. The next two chapters concern static and dynamic fluid phenomena (primarily liquids but a little on gases as well). The content-based learning objectives of this chapter are listed below.

**Students should be able to:**

1. Describe observational and testing experiments for the hypothesis that materials are made of microscopic particles that move randomly and have empty spaces between them.
2. Describe the model of an ideal gas and show that this model applies to air in the room.
3. Explain why gas exerts pressure on a container wall using microscopic models.
4. Apply Newton's laws and momentum conservation to derive the equation  $p = \frac{1}{3} m_0 n \bar{v}^2$  and explain qualitatively why the pressure depends on  $v^2$ .
5. Explain what absolute temperature is and how we know that it is a measure of the average kinetic energy of molecules.
6. Describe how to carry out isoprocesses experimentally and explain why they can serve as empirical testing of an ideal gas law that was derived microscopically.
7. Explain isoprocesses using the ideal gas model (qualitatively and quantitatively) microscopically.
8. "Read and write" with graphical representations of gas processes.
9. Apply gas laws to analyze real-life situations.
10. Explain diffusion and apply this explanation to real-life processes and especially biological phenomena.

We have broken the chapter into four parts:

- I. *A qualitative analysis of the structure of matter*
- II. *Development of the physical quantities pressure, density, and mass of a gas particle, and the development of the kinetic theory ideal gas model*
- III. *Temperature and the development and testing of the ideal gas law*
- IV. *Skills needed to analyze gas processes, including some real-world examples*

For each part, we provide examples of activities that can be used in the classroom and brief discussions of anticipated student difficulties with the subject matter.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
A qualitative analysis of the structure of matter	12.1	TET 12.1 (p353)	
Development of the physical quantities pressure, density, the mass of a gas particle, and the development of the kinetic theory of gases	12.2, 12.3	TET 12.2 (p357)	
Temperature and the development and testing of the ideal gas law	12.4-12.6		
Skills needed to analyze gas processes including some real-world examples	12.7, 12.8	12.1 (p383)	
<b>Nontraditional end-of-chapter questions and problems</b>  Choose measuring procedure (MEP): P12.35 Evaluate (reasoning or solution...) (EVA): P12.17, P12.33, P12.41, P12.42 Make judgment (based on data) (MJU): P12.59 Linearization (LIN): P12.72 Multiple possibility and tell all (MPO): Q12.27, P12.63, P12.65 Jeopardy (JEO): P12.47, P12.48, P12.56 Design an experiment (or pose a problem) (DEX): Q12.22, Q12.26, P12.28, P12.33, P12.34 Problem based on real data (that students can collect by themselves) (RED): P12.57, P12.59			

*Brief summary of student difficulties with gases*

The main difficulties relate to the consistency of macroscopic descriptions and microscopic explanations of gas processes and to the confusion between the terms “heat” and temperature, due to a vague meaning of the term “heat” in everyday life. In addition students tend to confuse temperature with kinetic energy of individual particles. It is difficult for them to visualize an isothermal process and an isobaric process.

## I. A qualitative analysis of the structure of matter

We follow a “from microscopic to macroscopic” approach to the kinetic molecular theory (KMT) of gases. We start from an evidence-based microscopic analysis of the behavior of gases. We devise the ideal gas model based on a few assumptions and Newton’s laws and use it to derive the expression  $PV = \frac{1}{3}Nm_{\text{particle}}v^2$ . We then use experimental data to connect the average kinetic energy of particles to the temperature of the gas  $\overline{K} = \frac{3}{2}kT$  and use this result to arrive at the canonical ideal gas law in the form  $PV = nRT$ . We finally test this equation by using it to predict the outcomes of real gas processes. In this approach, Boyle’s and Charles’s laws become the consequences of the microscopic gas analysis.

Students have learned about atoms and molecules throughout their earlier schooling. However, they are often unable to provide any evidence for the existence of atoms and molecules. Likewise, they have heard the word evaporation but have no idea what this word really means at the level of KMT. A set of activities in Section 12.1 of the ALG, Activities 12.1.1–12.1.4, and textbook Section 12.1 address these difficulties by engaging students in the analysis of an experiment in which they observe a streak of water or alcohol left on a paper by wiping a moist cotton ball across the paper. The streak slowly disappears from the edges, eventually vanishing entirely. If you decide to do these activities interactively in class, be careful with students’ language. When you wipe alcohol on the board and ask the students what they see, they will invariably say “the alcohol is evaporating,” without understanding what this really means. One strategy is to plead ignorance and ask them to describe what they see in nontechnical terms. Students should be able to come up with something like “the alcohol is disappearing gradually.”

The goal of the ALG activities and the textbook section is to help students come up with an explanation for the gradual aspect of the disappearance and then suggest mechanisms that explain the disappearance itself (these are two different aspects of the observational experiment). They relatively easy come up with a hypothesis to explain the former – the gradual aspect by hypothesizing that alcohol is made of

smaller parts. But they need to be explicitly pushed to come up with mechanistic explanations of disappearance – how did those parts disappear? Here, again, if you ask students to suggest mechanisms, they may say “evaporation.” However, evaporation is not a mechanism; it is simply a technical term that gives a name to the phenomenon that they are studying. They then devise testing experiments to rule out one or more of these possible mechanisms (see the textbook Table 12.1). To do this, they need to think of experiments whose outcomes they can predict using each of these mechanisms, perform the experiments, and then compare the outcomes to the predictions. All of the experiments are easy to perform in class, so it is best if students go through this process by doing the ALG activities before they read Section 12.1. You can have these experiments prepared and ready to execute in class time.

Students might come up with mechanisms that are not mentioned in the section. In this case, you will need to help them think of new testing experiments. By the end of this process, usually there is only one possible mechanism that has not been disproved—that the liquid is made of small pieces (particles) that are in continual random motion. The section provides other experiments that can be explained using the same mechanism, such as observing Brownian motion of pollen in water under microscope. With all of this supporting evidence for an atomic model of matter (including explanations of macroscopic properties of different states of matter), students can then be introduced to the qualitative ideal gas model. The students can then use the textbook EOC Questions 1 and 2. While the qualitative analysis of the structure of matter may seem rather trivial for university students to engage in, it is a rare opportunity for students to practice the full cycle of scientific reasoning. (The cycle starts from observations, moves to developing multiple explanations, and then testing those explanations with the goal of eliminating some of them.) In our experience, students find the exercise highly enjoyable and rewarding.

## **II. Development of the physical quantities pressure, density, the mass of a gas particle, and the development of the kinetic theory of gases**

The goal of ALG Activities 12.2.1–12.2.3 and textbook Sections 12.2 and 12.3 is to help students develop an understanding of physical quantities that are necessary to describe gases macroscopically and microscopically, as well as to devise a qualitative and quantitative microscopic explanation of pressure as the collisions of gas particles with the walls of the container.

A great way to begin this development is to start by blowing up a balloon in front of the class. Now that you have established that atoms and molecules are in a constant state of motion, students are quite comfortable with the idea that air molecules *inside* the balloon are bouncing off the walls of the balloon, pushing outward, and the more

molecules you add, the more outward push there is (hence, the balloon inflates). Yet at the same time, many students think of air as empty space (we ourselves often speak of the air this way when we say, “Throw an object into the air.”). Because of the way we talk about air, students often do not conceptualize air outside of the container as something that consists of moving particles that exert pressure on an object immersed in it. Thus, if you ask students what is stopping the balloon from expanding further (ALG Activity 12.2.1), many will suggest that it is the rubber skin preventing further expansion (to add to the analysis, you might want to ask your students to choose an element of the balloon’s skin and draw a force diagram for it after the balloon stops expanding). At this point, either a particularly sharp student or you, the instructor, can introduce the alternative explanation that it is the air molecules outside of the balloon that are bombarding the walls of the balloon from the *outside*, preventing it from expanding further (again, a force diagram here is very helpful for analysis). Students can then test these two competing ideas using in the ALG Activity 12.2.2 (Testing Experiment Table 12.2).

As an in-class activity you can ask students to make two predictions about the behavior of the balloon inside the vacuum jar, one for each explanation. Remember to remind students: A prediction is *not* a guess, it is a statement about what will happen if a particular explanation is true. It is important to differentiate between the physical quantity of pressure and the physical phenomenon of gas pressure that is explained by collisions of the particles with the walls of the container. Quantitative Exercise 12.1 emphasizes the concept that atmospheric air exerts pressure and uses the definition of pressure as a physical quantity to estimate the force exerted by the air against the front of a person’s body.

Do not skip ALG Activities 12.2.4 and 12.2.5 that solidify student understanding of the concept of gas pressure.

The physical quantity of density follows; students practice by estimating the density of a person’s body in Quantitative Exercise 12.2 (material related to density is revisited again in Chapter 13 in relationship to floating and sinking). Finally, Avogadro’s number, atomic mass, and the mole are introduced toward the end of Section 12.2 and used in Example 12.3 to estimate the average distance between air particles (about 30 times the diameter of an air particle). This exercise is extremely important because it suggests the ideal gas model could be used to describe atmospheric air and also because it continues the theme of helping students learn how to do estimations. The rest of the activities in the ALG Section 12.2 supplement Section 12.2 of the textbook. EOC Questions 3-6 and Problems 1-8 can be used for practice.

ALG Activities 12.3.1–12.3.3 and textbook Section 12.3 help students develop the quantitative aspect of the ideal gas model. It is important that students connect the derivation of the  $PV = \frac{1}{3} Nm_{\text{particle}} \overline{v^2}$  to their previous knowledge of momentum. Encourage them to use impulse-momentum bar charts when they derive the expression for the force that the wall exerts on a particle hitting it (ALG Activity

12.3.2). It is also important to examine the equation after students derive it (ALG Activity 12.3.3). For example, doubling the number of the particles inside the container doubles the gas pressure, but doubling the root-mean-squared (rms) speed leads to the increase of the pressure by 4 times. Rewriting the above equation as

$$PV = \frac{2}{3} N \left( \frac{1}{2} \overline{mv^2} \right) = \frac{2}{3} N \overline{K}$$

helps students establish the connection between pressure and the average kinetic energy of particles:

$$PV = \frac{2}{3} N \left( \frac{1}{2} \overline{mv^2} \right) = \frac{2}{3} N \overline{K}$$

Students can now use this equation to estimate the average root-mean-square speed of air particles (textbook Example 12.4). When they calculate the result, it is important to stop and ask whether this result seems to make sense. If the speed of the particles is so great, why does the smell of perfume from an open bottle take minutes to spread from one corner of a room to the other? In general, it is useful to establish a rule that students must evaluate every new equation that is derived in the book by looking at limiting cases and seeing that it is consistent with the rest of their physics knowledge.

EOC Problems 9-15 help students develop student facility in using these ideas. We hope that by the end of these sections, students have developed a robust physical intuition for what a gas is really like at a microscopic level: the speed of the gas molecules, the amount of space between them, and the fact that a gas exerts a large force on the walls of its container due to the collective action of a vast number ( $10^{23}$ ) of individual molecules.

### III. Temperature and the development and testing of the ideal gas law

In Section 12.3, students derived the relation between the pressure exerted by the gas particles on the wall of a container and their average kinetic energy. From experience, students know there is a connection between the temperature of a gas and its pressure. How then does temperature enter into the above-mentioned relation? After students review Celsius and Fahrenheit scales, they learn how to establish the connection between pressure and temperature through constructing a new temperature scale. This process is based on the observation that  $PV/N$  is a constant for any two gases in thermal equilibrium with each other, and that  $PV/N$  increases with the temperature (in Celsius) of the gas. Using proportional reasoning and some provided data, students arrive at the relation between the quantities used to describe gases:



$$\frac{PV}{N} = k_B T$$

where  $T$  is the temperature measured on the kelvin scale. Given students know from Section 12.3 that  $PV = 2/3 N \bar{K}$ , they can establish the connection between the new temperature and the average kinetic energy of particles  $\bar{K} = 3/2 k_B T$ . After students calculate the value of the proportionality constant, they can develop another form of the ideal gas law by rewriting  $N$  in terms of the number of moles  $n$  of gas:

$$PV = nRT$$

In class, you can either follow ALG Activities 12.4.1—12.4.5 or the discussion described in textbook Section 12.4 to connect  $PV = 2/3 N \bar{K}$  to the temperature of the gas.

Textbook Section 12.5 outlines the steps that students might take to test the gas law ( $PV = nRT$ ) by predicting what will happen to a constant-mass sample of gas if one of the macroscopic physical quantities is held constant while a second one changes. They do it for three different pairs of variables in Table 12.5 (encourage students to work through the steps outlined in the table before actually reading the text), eventually arriving at the mathematical descriptions and experimental tests of various isoprocesses. Table 12.6 summarizes the processes. Students then work through the problems that apply those ideas to breathing and to what happens to a bottle carried by a passenger on an airplane flight. Make sure that students read the subsection “Reflection on the process of construction of knowledge” in this textbook section. It outlines the logical steps involved and puts the knowledge that students developed so far in the historical perspective. Both Examples 12.6 and 12.7 that follow the reflection as well as the explanation of breathing will be very useful for the students.

Note ALG Activities 12.5.3 and 12.5.4 that allow the students to analyze real-life processes with real data using gas laws. ALG Activities 12.5.5–12.5.7 help students represent gas processes in multiple ways and analyze them qualitatively and quantitatively.

The speed distribution of gas particles is the subject of Section 12.6. Although we do not derive Maxwell’s distribution in the textbook and do not describe testing experiments, a historical account for the Stern’s testing experiment is turned into an ALG Activity 12.6.3. The novel aspect of this activity is that the students have to read a relatively long passage and find purposefully inserted mistakes.

Students can apply the concepts in Sections 12.3–12.6 by answering EOC Questions 11–15 and solving Problems 28–34.

## IV. Skills needed to analyze gas processes, including some real-world examples

Section 12.7 of the ALG (Activities 12.7.1–12.7.3) helps students master graphical representations of gas processes that allow them to connect macroscopic descriptions to microscopic explanations. The key to these activities is positioning the graphs with respect to each other in a way that the students can keep track of the changes of the same variable – they need to be of the same size on different graphs. Thus you can put the graphs ***P-vs-V*** and ***P-vs-T*** next to each other to keep the pressure changes the same and the graph ***V-vs-T*** under the ***P-vs-T*** graph to keep the temperature changes the same. Use these activities before the students try textbook Example 12.8, which provides the techniques that students can use to adapt the general problem-solving procedure to gas law problems (the example involves the size of a scuba diver’s lungs as they rise to the surface). After that, make sure that students work through Example 12.9 that describes an exiting real-world application of gas laws carried out by the authors during a flight and afterward. These examples can be followed by ALG Activities 12.7.4–12.7.6

The next section, 12.8, discusses two exciting examples – the lifetime of the Sun and diffusion of gases. The latter has huge biological applications, thus it is important that students work through both parts. We suggest that the students first do ALG Activities 12.8.1–12.8.3 and then proceed to 12.8.4 – reading the textbook chapter. ALG Activity 12.8.2 is a possible lab experiment. Students are now ready to work on the more complex Problems 35, 27, 44–51, and 53–56. The General Problems 57–59, 65, 66, and 72 are not only excellent in helping your students apply the concepts, but also continue developing scientific abilities. Students will return to apply the concepts of temperature, thermal energy, and energy transfer in Chapters 15 and 16 when they study thermodynamics.

# 13

## Static Fluids

In Chapter 12, we constructed the ideal gas model and used it to explain the behavior of gases. The next two chapters concern static and dynamic fluids phenomena (primarily liquids but a little on gases as well). Content-related learning objectives are as listed below.

### Students should be able to:

1. Explain why the pressure in fluids changes with depth.
2. Use force diagrams to determine pressure of fluids as a function of depth.
3. Explain how we know that pressure in fluids is  $p = p_{atm} + \rho gh$ .
4. Apply the expression in objective 3 to resolve Pascal's paradox and other static fluids-related phenomena.
5. Describe how to measure pressure using a column of liquid.
6. Explain where Archimedes' Principle (or the expression for the buoyant force) comes from and apply it to solve practical problems.
7. Analyze static fluids problems using energy bar charts.
8. Design two independent experiments to determine the density of an object.
9. Explain floating and sinking, including the stability of boats.
10. Describe how knowledge of fluid pressure and the buoyant force applies to biological phenomena and real-life situations.

We have broken this static fluids chapter into four parts:

- I. *Density*. In Section 13.1, students are reintroduced to the mathematical concept of density and also summarize a pattern that they probably already know about the relationship between density and sinking or floating. Instead of trying to explain this pattern straight away, we put it aside and return to explain this pattern at the beginning of Section 13.7, only *after* students have developed a complete understanding of buoyant force.

- II. *Development of the concepts of pressure variation in a fluid and the dependence of pressure on the depth in the fluid* (Sections 13.2–13.4).
- III. *Derivation of an expression for the buoyant force that a fluid exerts on an object in the fluid and application of this and other fluid concepts to problem solving.* In Section 13.5, students start with an observational experiment that allows them to observe patterns related to the buoyant force that a fluid exerts on an object floating or immersed in it. They then proceed directly to develop a mathematical explanation for the buoyant force using Pascal's second law. Finally, they interpret the mathematical model physically as Archimedes' principle. Section 13.6 develops students' understanding of buoyant force through simple applications.
- IV. *Interesting examples of fluid statics in the real world.* In Section 13.7, students have a chance to put it all together: First we connect back to Section 13.1 and formalize the relationship between densities and floating and sinking in a fluid. Then students can learn about ship design and stability, ballooning, altitude sickness, scuba diving, and decompression sickness.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Density	13.1		
Development of the concepts of pressure variation in a fluid and the dependence of pressure on the depth in the fluid	13.2-13.4	OET 13.2 (p392)	13.3.5
Derivation of an expression for the buoyant force that a fluid exerts on an object in the fluid and application of this and other fluid concepts to problem solving	13.5, 13.6	OET 13.4 (p398)	
Interesting real-world examples of fluid statics	13.7		
<b>Nontraditional end-of-chapter questions and problems</b>			
Ranking tasks (RAT): Q13.1, Q13.39, P13.7, P13.23, P13.41, P13.52, P13.53, P13.59			
Choose measuring procedure (MEP): Q13.17			

Evaluate (reasoning or solution...) (EVA): P13.14, P13.28, P13.38, P13.46, P13.56, P13.88  
Make judgment (based on data) (MJU): P13.30, P13.89  
Multiple possibility and tell all (MPO): P13.22, P13.25, P13.83  
Design an experiment (or pose a problem) (DEX): Q13.19, Q13.20, Q13.32, Q13.35, P13.58, P13.71  
Problem based on real data (that students can collect by themselves) (RED): Q13.24, P13.73, P13.89

*Brief summary of student difficulties with hydrostatic pressure and buoyant force*

Students might think that the pressure is determined by the mass of the fluid above a certain level, not the height of the liquid column. For the buoyant force, the main difficulty is reconciling that although hydrostatic pressure increases with depth, the buoyant force does not change after the object is totally submerged. Students also have difficulty applying Newton's third law in static fluid cases – they forget that the submerged object also exerts a force on a fluid. Other difficulties include consistently applying the concept that the buoyant force does not depend on the density of the object.

## I. Density

Students learned about density in Chapter 12. However, the concept of density as the slope of the mass versus volume graph is difficult for the students, and thus we recommend ALG Activities 13.3.1 and 13.3.2 (these two activities can be done in a lab at any time as no previous knowledge is required). In this chapter, students connect an object's density with its ability to float or sink in a fluid. While most students have probably already learned this idea in a middle school science class, we offer a more novel example of a helium balloon immersed in air. Students may not have considered this example in the same category as wooden and metal blocks, respectively, floating and sinking in a beaker of water. It is probably a good idea to draw their attention to this. An interesting example involves the lower density of ice relative to water and its effect on the ability of fish and plants to survive winter freezes in ponds and lakes. You can follow up with EOC Questions 3, 5, 18, 19 20 and Problems 1-6, 9, 12, 11, 83 and 84 and with the Reading Passage "Lakes freeze from top down."

## II. Development of the concepts of pressure variation in a fluid and the dependence of pressure on the depth in the fluid

The goal of Part II is for students to develop, test, and apply two related ideas about the behavior of pressure in fluids: Pascal's first and second laws. The key ideas that students have to develop for Pascal's first law are (1) that a fluid exerts pressure in all directions (ALG Activity 13.2.1) and (2) that a change in pressure is redistributed equally throughout the fluid (textbook Section 13.2, particularly Figures 13.4a and b and ALG Activity 13.2.2). For Pascal's second law, students need to realize that pressure increases with depth and to develop a "layer" model. In this model, we break the fluid up into layers and choose one layer of a fluid as a system. Because each layer of the fluid is in a state of static equilibrium, the forces exerted on it from all directions must cancel each other. In the downward direction, two objects exert forces on the layer—all of the layers pushing from above and Earth. In the upward direction, it is only the fluid that exerts a force. Therefore, if we compare two layers inside the fluid, one closer to the top and the other one closer to the bottom, the upward force exerted on the layer that is deeper in the fluid must be larger than the upward force exerted on the layer that is closer to the top (ALG Activities 13.3.1–13.3.6, and textbook Section 13.3).

Notice that while the ALG uses data tables in Activities 13.3.3 and 13.3.4 to help students construct the expression for variation of pressure with depth, and a real experiment in Activity 13.3.5 to achieve the same goal, the textbook uses a mathematical derivation to develop Pascal's second law:  $P_1 = P_2 + \rho_{\text{fluid}}(y_2 - y_1)g$ . One difficulty that students face here is to recognize that the pressure of a fluid depends on its height above the level of interest, not the total mass of the fluid. ALG Activity 13.3.6 helps students test both ideas. Notice Conceptual Exercise 13.3 in the textbook. It involves some non-traditional representations of pressure with depth and application of Pascal's laws. We suggest that you assign it to students as an exercise to do in groups in class and only then show them the solution in the textbook.

We would like to discuss now an issue that was not brought up neither in the textbook, nor in the ALG, but poses a difficulty for advanced students who are seeking coherence in their physics knowledge. From the previous chapter on gases, students learned that the microscopic mechanism of pressure is the transfer of momentum to the walls of the container by moving particles and consequently the force that they exert on the walls. If a liquid is incompressible and the temperature inside the liquid does not change, how can the pressure increase with depth? The answer to this question lies in the difference between gases and liquids. Because the density of a liquid is about 1000 times larger than the density of gases, one would expect that pressure in liquids due to moving particles is also larger by the same factor with respect to gas at the same temperature. However, in liquids the pressure that moving particles exert on the walls due to their random motion is greatly reduced due to the intermolecular attraction (we can think of this attraction as creating "negative" pressure which when added to the

pressure due to random motion, decreases the net pressure on the walls). The intermolecular attraction (among other factors) depends on the density of the liquid. Even minuscule changes in the density lead to considerable changes. Because the density of liquid does indeed change with depth, even though very slightly, this increase is enough to decrease the “negative” contribution to pressure due to molecular interaction and explain the change of pressure with depth.

Textbook Section 13.4 offers students applications of Pascal’s second law – measurement of atmospheric pressure, and some others. Torricelli hypothesized that it is atmospheric air pressure exerted on the surface of water that pushes the water 10 meters up an evacuated tube. A test of this idea, described in Table 13.3, was performed with an evacuated tube in mercury. Torricelli predicted the mercury would rise to a much lower height – about 760 mm, based on the difference in densities, and this is exactly what happened. This experiment led to a method to measure pressure (a mercury barometer). Students learn how to apply this idea to a diving bell lowered into water in Example 13.5. End-of-chapter questions and problems to help students develop understanding and facility in using these ideas in problem solving include: for Section 13.2, Questions 1, 2 and 23 and Problems 18 - 21, 23, 24, 25; for Section 13.3, Questions 4 and 38 and Problems 23, 26, 29, 30, 31, 32, 37, 38 and 41; and for Section 13.4, Questions 9, 21, 22, 24, and 39, and Problems 44-48 and 86-88.

### **III. Derivation of an expression for the buoyant force that a fluid exerts on an object in the fluid and application of this and other fluid concepts to problem solving**

Many students hold an idea that air pushes down on objects. ALG Activities 13.5.1 and 13.5.2 help students develop and test the concept that the net force exerted by a fluid on a submerged object points up.

There are two ways that you could approach the development of the buoyant force. The textbook starts with a qualitative development and proceeds to a derivation, while the ALG uses an empirical method.

Qualitative development for the quantity of the buoyant force is described in the textbook Table 13.4 on pages 398-399, where a solid block is lowered slowly into water by a string that is suspended at the other end to a spring scale. The scale reading decreases as the block becomes more submerged and is then constant after the block is completely submerged but lowered even deeper in the fluid. ALG Activity 13.5.3 addresses the same goals only it takes students further forward by actually allowing them to construct a relation between the force exerted on the submerged object and its submerged volume. Given Activity 13.5.4 repeats the experiment for a different liquid, students have an opportunity to deduce the expression for buoyant force from those two activities.

Students need to carefully examine the data and find the patterns to decide how the net force that the liquid exerts on the submerged object depends on various physical quantities. Notice that in ALG Activities 13.5.3 and 13.5.4 there are two objects of different density (students have to deduce this), which allows students to devise the patterns and to see that the force that fluid exerts on the submerged object does not depend on the mass of the object but on the submerged volume. The data from the observational experiments is sufficient to develop the relationship empirically.

In the textbook, we present an alternate method where the buoyant force relationship is derived from Pascal's second law and then given physical meaning as Archimedes' principle. Tips for using the principle to determine the magnitude of the upward buoyant force that a fluid exerts on a partially or totally submerged object appear at the end of Section 13.5. It is important to remind students that the density in the expression for the buoyant force is the density of the fluid and not of the object in the fluid. The biggest difficulty here is to recognize that the buoyant force does not change after the object is totally submerged (notice the last readings for both blocks in ALG Activity 13.3.5) and it does not depend on the density of the object once it is totally submerged. For a floating object, the density determines how deeply it is submerged and thus indirectly affects the buoyant force. ALG Activities 13.5.5–13.5.9 help students test and solidify their ideas. This is the subtle part of the pattern that students have to observe, and you may need to draw their attention to it. We suggest that students first work with the ALG activities, then read the textbook (ALG Activity 13.5.10) and then proceed to EOC questions and problems: Questions 6, 7, 10–14, 15, 16, 25, 27, 36 and Problems 49–51 and 52–59.

Numerous activities in Section 13.6 of the ALG will allow your students to apply ideas of hydrostatic pressure and buoyant force to practical problems, experiments, and develop various science practices – reasoning with representations and evaluation. Activities 13.6.8 and 13.6.9 form a short lab. If you do not have lab time, Activity 13.6.11 perfectly substitutes for a lab (if you do have a lab we suggest that your students do this activity in class or at home). The key here is that students use their knowledge of dynamics. ALG Activity 13.6.10 is an excellent activity to help students apply Newton's third law to problems involving buoyant forces.

EOC Problems 70–76 will challenge your students to apply their knowledge in many different situations. Several of these problems are non-traditional and require applications of force diagrams and energy bar-charts.

## **IV. Interesting examples of fluid statics in the real world**

Textbook Section 13.7 begins by revisiting and explaining the well-known empirical rule that if an object is less dense than the fluid it is immersed in, it will float, whereas if it is more dense than the fluid, it will sink. Section 13.7 also has many interesting real-world problems that involve buoyancy and other fluid statics phenomena:



- The conditions needed for ships to be stable
- The number of people a life raft can hold
- Ballooning
- Scuba diving and breathing methods as the diver descends, so external and internal pressures balance
- Oxygen overload during scuba diving
- Decompression sickness when ascend during a dive

Note that we do not have a parallel section of the ALG here as we think that students have enough opportunities to practice using EOC problems. Specifically, they could solve all problems in EOC Section 13.7 and General Problems 89-93.

# Fluids in Motion

In Chapter 13, we developed ideas about the pressure in static fluids and how pressure variation leads to an upward buoyant force that the fluid exerts on an object that is partially or totally submerged in the fluid. In this chapter, we consider two new effects: the effect of moving fluid on the pressure in the fluid, and the forces exerted by the fluid on an object with respect to which the fluid is moving. The content-based learning objectives are as listed below.

**Students should be able to:**

1. Describe simple observational experiments that led to Bernoulli's effect.
2. Describe fluid flow rate with relevant physical quantities.
3. Explain Bernoulli's effect and state its limitations.
4. Use bar charts to derive and apply Bernoulli's equation to real-life situations.
5. Explain the meaning and the difference between: types of drag forces (laminar and turbulent) and the two Reynolds numbers.
6. Explain and apply Poiseuille's equation.
7. Describe real-life and especially biology-related phenomena relevant to Bernoulli's principle and Poiseuille's equation.

We have broken this chapter into three parts:

- I. *A qualitative introduction to Bernoulli's effect, and ideas concerning flow rate and types of fluid flow*
- II. *A quantitative development of Bernoulli's equation and the skills and applications for applying it quantitatively*
- III. *Viscous fluid flow and drag forces*

For each part, we provide examples of activities that can be used in the classroom and brief discussions of motivation for those activities and anticipated student difficulties with the subject matter.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
A qualitative introduction to Bernoulli's effect, and ideas concerning flow rate and types of fluid flow	14.1-14.3	OET 14.1 (p416),  TET 14.2 (p417)	
A quantitative development of Bernoulli's equation and the skills and applications for applying it quantitatively	14.4, 14.5		
Viscous fluid flow and drag forces	14.6, 14.7		
<b>Nontraditional end-of-chapter questions and problems</b>  Ranking tasks (RAT): P14.45 Choose answer and explanation (CAE): Q14.9 Evaluate (reasoning or solution...) (EVA): Q14.10, Q14.11, Q14.15, P14.6, P14.48 Jeopardy (JEO): P14.9, P14.10, P14.11, P14.12, P14.13, P14.14, P14.15, Design an experiment (or pose a problem) (DEX): P14.36, P14.46			

### *Brief summary of student difficulties with fluids in motion*

The biggest difficulty is understanding that pressure in a moving fluid decreases when the speed of the fluid increases, because it is rather counterintuitive. Another difficulty is reconciling the continuity equation to Bernoulli's effect – the liquid in a narrow tube exerts less pressure than the liquid in a wider tube. Sometimes students do not differentiate between viscosity and density (oil is less dense than water but more viscous). Finally, they tend to use Bernoulli's principle in cases where it is not applicable. Conditions of applicability are difficult to define because seemingly small changes can make these phenomena much more complex than presented in an introductory physics courses. Other difficulties relate to a large number of new equations describing the drag force.

## I. A Qualitative introduction to Bernoulli's effect, and ideas concerning flow rate and types of fluid flow

There are many simple and fun experiments that students can use to find a pattern in the relationship between the speed of a moving fluid and the pressure that it exerts on a surface that it moves across. Remember, let students first observe the phenomena without predicting! Their natural surprise is the opportunity for them to try to explain why the phenomena are happening.

If you start in a lab, students can perform Activities 14.1.1–14.1.4 in the ALG to come up with a qualitative version of Bernoulli's principle. Then students can do ALG Activity 14.1.5 – the textbook reading exercise.

If you start this chapter in lecture, follow the sequence from Section 14.1 in the textbook. Table 14.1 in Section 14.1 describes an observational experiment and the analysis. Make sure that you ask your students to work in pairs to draw the force diagram for the piece of paper and then to devise a causal explanation for it. Once they come up with an explanation, you could ask them to use this explanation to predict the outcome of the experiment described in Testing Experiment Table 14.2. It is easy to conduct it in lecture if you have a camera to project the image on the screen. Alternatively, you can project the video provided.

This is a good opportunity to remind students that their predictions should be based on the explanation being tested. The tested explanation is then accepted as Bernoulli's effect, which students then apply to explain how people snore. This progression is a self-contained ISLE cycle: conducting observational experiments, inferring patterns, constructing an explanation, designing a testing experiment, making predictions of their outcomes using the constructed explanation, conducting a testing experiment, comparing outcomes to the prediction, and making judgments about the explanations. You can use ALG Activities 14.1.3 and 14.1.4 and EOC Questions 1 and 3.

The next step is the concept of flow rate. The textbook provides an operational definition for the flow rate (Equation 14.1) and then proceeds to the derivation of what affects the flow rate—average speed  $v$  through a tube or vessel and the cross-sectional area  $A$  (Equation 14.2). Matching ALG activities are in Section 14.2.

We suggest that students try them first and then proceed to working with the text. We recommend that you use an activity like ALG Activity 14.2.3 either for an interactive class discussion with clicker responses or for discussion in recitation. Students can be separately comfortable with Bernoulli's effect and the continuity equation for incompressible fluids. However, when asked to put both ideas together to explain what is going on in an activity like ALG Activity 14.2.3, they are likely to be confused. Students struggle with the intuitive idea that if a space is “constricted” (the narrowing pipe), the pressure should be higher. Letting students talk to each other while trying to explain ALG Activity 14.2.3 (with an accompanying live demo, if you have it) can help students to work through their difficulties with this rather

counterintuitive result. You can help students think about this problem by encouraging them to compare the speed of the fluid at two different points. Asking them to think about what causes the fluid to speed up should lead them to think about the need for a difference in pressure between two points. For the fluid to accelerate, the pressure would need to be higher in the region where the fluid is moving slower as compared to the region where it is moving faster. Having students think about the pressure difference naturally focuses their thinking for the next section, where they have to derive the complete quantitative form of Bernoulli's equation. EOC Questions 2, 6, 13, and Problems 1-6 will be helpful for the students.

Section 14.3 introduces students to laminar-streamline and turbulent flow and helps contrast the two. We suggest that the students do ALG Activity 14.3.1 first and then read the book (ALG Activity 14.3.2).

## II. A quantitative development of Bernoulli's equation and the skills for applying it quantitatively

Textbook Section 14.4 and ALG Activity 14.4.1 (done after 14.3.1) lead students through the derivation of Bernoulli's equation using the work-energy principle. Following the derivation, it would be good to show how the math is consistent with the outcomes of the experiments in Tables 14.1 and 14.2 in the textbook. If you feel that your students are not motivated by derivations, you can show them the concepts behind the derivation using Bernoulli bar charts—the fluid analog of a work-energy bar chart. Bernoulli bar charts have the same underlying idea as work-energy bar charts, except they use energy density instead of energy, and pressure difference instead of work. They provide an insightful way to analyze a fluid-dynamic process and help students make a conceptual bridge from the physical system to the mathematical representation. Consider the following form of Bernoulli's equation that represents the energy conservation for a unit volume of fluid:

$$\frac{1}{2}\rho v_1^2 + \rho g y_1 + (P_1 - P_2) = 1 \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Here is how this equation can be reconceptualized using a bar chart: A system, consisting of a unit volume of fluid and Earth, has kinetic and potential energy densities that change when the system is subjected to a pressure difference. We can represent the process using bars for kinetic energy density  $1/2\rho v^2$ , gravitational potential energy density  $\rho g y$  for the two states, and pressure difference  $P_1 - P_2$ . Once students construct a bar chart for a process, they can use it to help them apply Bernoulli's equation to the process.

ALG Activities 14.4.2 and especially 14.4.3 are excellent for testing and applying Bernoulli's equation. EOC Questions 9, 14 and 15 are very useful for helping your students practice Bernoulli's equation, connect it to their previous knowledge and practice using bar charts to analyze fluid flow.

The next section in the textbook and ALG is dedicated to the development of problem solving skills. We suggest that the students first work on the ALG Activities 14.5.1–14.5.5 and only then start solving traditional problems. These ALG Activities and EOC Problems 7–15 help students practice moving from the sketches of the physical processes to bar charts to equations and from equations to bar charts and sketches of the processes.

ALG Activity 14.5.6 and worked Example 14.2 in the textbook show students how to analyze fluid dynamics processes quantitatively. There are several key strategies to emphasize. First, Bernoulli's equation is applied to two different positions or points in a moving fluid. It is important to have a sketch of the process and to have the points clearly identified. Second, one point is chosen in order to determine some unknown quantity needed to solve the problem (for example, the pressure at the output of a pump). The second point is chosen at a place where everything about the moving fluid is known. Third, to specify the gravitational potential energy density, the student must specify a vertical  $y$ -axis and choose an origin or zero point, just as they did for gravitational potential energy. Fourth, when constructing a qualitative Bernoulli bar chart, it is often easiest to make the gravitational energy density bars first, then the kinetic energy bars, and finally the pressure bars, so that the sum of the three terms at position 1 equals the sum at position 2. Remember that the bar heights are qualitative and often unknown exactly. The pressure bars are adjusted to make the three terms on the left side of the chart equal to the three on the right side. Also, note that the pressure bars are due to external agents and are placed in a shaded region (like the work bars when doing work-energy processes). ALG Activities 14.6.7–14.6.9 are excellent follow up activities, as well as EOC Problems 19, 21, 24, 25, 27, 28 and 46–48.

Section 14.6 in the textbook not only introduces students to the application of the general problem-solving strategy to the fluid dynamics problems but also discusses three important applications of Bernoulli's equation: a high-speed wind lifting the roof up off a house; the rapid flow of blood past a narrow, plaque-constricted region in a blood vessel, thus dislodging and lifting the plaque off the vessel wall; and explanations of how airplanes fly. Do not miss the latter – it involves a calculation showing that Bernoulli's equation alone *cannot* explain the lift.

### III. Viscous fluid flow and drag forces

We believe that it is critically important for student motivation and interest that they see the physics they study as connected to their experience of everyday life. Thus we have devoted the final part to studying fluids in a more realistic setting when the effects of viscosity and of conversion of kinetic energy of fluid into internal energy

are considered. Section 14.6 starts with a qualitative discussion of friction, leading to the idea of friction flow in fluids (viscous fluid flow). Four factors are identified that affect the flow rate through a tube or vessel with a viscous fluid flowing through it: (1) the pressure decrease from the input to the output of the vessel, (2) the radius of the vessel, (3) the length of the vessel, and (4) the viscosity of the fluid. Table 14.3 provides data for analyzing how each quantity affects the flow rate. After such an analysis, students will have an easier time understanding where Poiseuille's law, which determines the pressure decrease across a vessel needed for a particular flow rate, comes from. Notice the unit analysis that we use to determine the units for the viscosity. Quantitative Exercise 14.5 applies this idea to the analysis of the effect of flow rate and pressure drop across a narrow part of an artery with plaque.

You may consider skipping this entire section, but there is a useful connection that could be made between Poiseuille's law and Ohm's law and the resistivity equation in DC circuits. Although the functional relationship is not the same, the qualitative relationship is similar (larger  $r$ , larger flow rate; larger  $l$ , lower flow rate, assuming constant pressure difference). If you intend to use water-pipe analogies when covering DC circuits next semester, it could be extremely useful to have students work through the real-life subtleties of fluid flow now. They can then draw on that understanding when working with Ohm's law and the resistivity equation.

We recommend that the students read the textbook section first (ALG Activity 14.6.1) and analyze worked examples and then work through ALG activity 14.6.2. We recommend EOC Questions 7 and 17 and Problems 29, 30, 33, 37, 50, 51 and 52.

The drag force that a fluid exerts on an object moving relative to the fluid is examined in Section 14.7 for the cases of laminar flow and turbulent flow (both equations are given without derivation or backup data—a rare case in this textbook). It is important to emphasize that in one equation the drag force is proportional to speed and in the second one, the speed squared. Also emphasize that in everyday life (driving cars), the square law applies (i.e. driving a little bit faster may result in using a lot more gas) but also that the shape of cars can significantly decrease mileage per gallon of gas. Because both drag force equations (Equations 14.9 and 14.11) contain several new symbols, it is important to ask your students to explain what each term means and to describe what they imagine when they see each equation. It is easier to do it first in the context of a particular problem, and then ask students to describe a situation for which a particular equation might be relevant. The latter idea is applied to the terminal speed of a skydiver (Example 14.6) and former to the motion of a ball in a liquid (Example 14.7). Example 14.7 is an example of a multiple possibility problem that develops epistemic cognition – an ability to evaluate the effects of assumptions on the solution. This is an ability that is extremely important in the workplace and yet not developed in the graduates of universities: do not skip it!

Note ALG Activity 14.7.2 – it combines the understanding of the drag force with the ability to analyze and interpret graphical data.

EOC Questions 10 and 11 and Problems 39, 40, 41, 45, 54-58 serve as good applications for the concepts in Section 14.7.

# First Law of Thermodynamics

In Chapters 13 and 14, we developed ideas about static and dynamic fluids. The focus was on the physical quantities force and pressure and the microscopic mechanisms that explain processes in the fluids. In this chapter, we bring the energy approach into the analysis, and we add a new mechanism through which the energy of a system can change—heating. Until now, students have only known about work as a mechanism of energy transfer. We also help them connect the microscopic understanding of the gas processes to the work-heating-energy concepts. The content-based learning objectives for this chapter are listed below.

## Students should be able to:

1. Understand and be able to explain the cause-effect relationship between internal energy changes, work, and heating.
2. Distinguish state variables from processes that cause the system state to change.
3. Represent thermodynamics processes using energy bar charts and use the bar charts to write mathematical descriptions of the processes (the first law of thermodynamics)
4. Explain isoprocesses using the first law of thermodynamics
5. Apply the first law to analyze problems involving all three quantities—internal energy change, work, and heating.
6. Use  $P$ -vs- $V$ ,  $V$ -vs- $T$  and  $P$ -vs- $T$  graphs to analyze thermodynamics processes.
7. Apply the law of energy constancy to calorimetry problems.
8. Design two independent experiments to determine specific heat of an unknown objects.



9. Apply knowledge of mechanisms of energy transfer (conduction, convection, radiation and evaporation) to explain real-life processes and biological examples.
10. Design an experiment to test the first law of thermodynamics.

We have broken this first chapter on thermodynamics into three parts:

- I. *Define the work done on a gas, the internal energy of a gas, heating, and the first law of thermodynamics; apply the first law of thermodynamics to gas processes*
- II. *Develop the ideas of temperature change and state changes*
- III. *Apply the knowledge of heating mechanisms and the first law of thermodynamics to real-life phenomena*

For each part, we provide examples of activities that can be used in the classroom and brief discussions of anticipated student difficulties with the subject matter.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Define the work done on a gas, energy provided through heating to the gas, the internal energy of a gas, and the first law of thermodynamics  Apply the first law of thermodynamics to gas processes (so-called iso-processes)	15.1-15.3  15.4	TET 15.2 (p447)  15.1 (p453)	15.1.2, 15.2.2, 15.3.1
Develop the ideas of temperature change and state changes	15.5, 15.6		15.5.7,
Apply the knowledge of heating mechanisms and the first law of thermodynamics to real-life phenomena	15.7	15.2(p468)	15.7.2, 15.7.3, 15.7.4, 15.7.5
<b>Nontraditional end-of-chapter questions and problems</b>			
Evaluate (reasoning or solution...) (EVA): Q15.11, P15.7, P15.35, P15.53, P15.31, P15.69			

Make judgment (based on data) (MJU): P15.69  
Multiple possibility and tell all (MPO): Q15.17, Q15.20, Q15.27, P15.20, P15.21, P15.23, P15.27, P15.69,  
Jeopardy (JEO): P15.8, P15.9  
Design an experiment (or pose a problem) (DEX): Q15.20  
Problem based on real data (that students can collect by themselves) (RED): P15.69

There are a few ideas that make our approach different from the traditional approach:

1. We do not use the term *heat* for the mechanism of energy transfer because students often think that heat is something that belongs to the system. We use the word *heating* instead.
2. We continue to use the system approach—the system has energy, and this energy can be changed through two mechanisms: work and heating. The mechanism of work explains the energy transfer when there is mechanical motion of some parts of the system or volume change due to the interaction with the environment. The mechanism of heating explains the energy transfer when the system and the environment are at different temperatures and no work is done.
3. We add space for the heating bar on the energy bar chart next to the space for work so that it becomes the work-heating-energy bar chart, showing that all energy is transferred to the system from the environment through work and heating (the energy can be positive or negative); we also rearrange the energy bar chart to represent total energy change instead of different types of energies before and after.

We only have one statement for the first law of thermodynamics:

$$W_{\text{Environment on System}} + Q_{\text{Environment to System}} = \Delta E_{\text{System}}$$

*Brief summary of student difficulties with the first law of thermodynamics*

The main difficulty comes from the language used when we talk about heat as a substance that is inside an object. This leads to students thinking that heat is a state function (the same applies to work). That is why we do not use the term heat in this book, but use heating instead. Another two difficulties are the understanding that (a) the system might receive some energy through heating and still cool down (in a way, heating might indirectly lead to cooling) and (b) the system might receive (or lose) energy through heating, but the temperature of the system does not change—when phase changes occur. Yet another difficulty is the confusion between temperature, internal energy, and heating as the mechanism of energy transfer (everyday language is once more to blame). Other difficulties include consistency in identifying the system and environment and remembering that the system does not do work on the environment, but the environment does work on the system (the same applies to heating).

# I. Define the work done on a gas, energy provided through heating to the gas, the internal energy of a gas, and the first law of thermodynamics

In Section 15.1, students learn how to apply their knowledge of ideal gases and the definition of mechanical work done by an external force on a point-like object as the object moves to derive the expression for the internal thermal energy of the ideal gas  $U_{\text{th}} = 3/2NkT$  and work done by a piston on the gas in a cylinder (the gas is the system) as the gas volume changes:

$$W_{\text{Piston on Gas}} = -F_{\text{P on G}} \Delta x = -(P \cdot A) \left( \frac{\Delta V}{A} \right) = -P \Delta V$$

Notice the minus sign here. We only consider the work done by the environment on the system. When the volume of the gas decreases, the environment does positive work on the system, and when the volume of the gas increases, the environment does negative work. It is important to emphasize here that work is not a state variable but instead depends on the particular way that the gas takes to move from the initial state to the final state of the gas (at lower or higher temperature, for example); the work done by the environment on a gas  $W_{\text{Environment on gas}}$  is the negative of the area under the  $P$ -versus- $V$  graph for that process. In Section 15.4, this property of work is contrasted with the thermal energy of a gas. Thermal energy is a **state function**, which depends only on the temperature of the gas:

$$U_{\text{th}} = N \left( \frac{3}{2} k_{\text{B}} T \right) = \frac{3}{2} nRT$$

when in a particular state. ALG Activities 15.1.1-15.1.3 help students derive both expressions. Students can either work on these activities in groups and then you have a summarizing discussion in class or you can lead the discussion by using Section 15.1, which shows both derivations. EOC Questions 1 and 2 and Problems 1-5 will be helpful here.

Section 15.2 is dedicated to the discussion of two ways to change the energy of a system; before there was only one (work). We suggest that students do ALG Activities 15.2.1-15.2.3 and then work with the textbook section (ALG Activity 15.2.4). ALG Activity 15.2.1 requires you to do the experiment, thus it is probably better if you start with this section. Students can work in groups answering the question, but only the instructor should perform the experiment. Make sure you wear safety goggles too. Activity 15.2.3 involves a video and thus does not require lab time, however students are more successful if they work in groups on this activity. Both activities help students devise and apply the new quantity that describes the process of transfer of energy to the system. Students realize that there are several

cases where the work-energy principle they studied in Chapter 7 cannot explain the transfer of energy. They may feel that the principle is incorrect or that it needs modification—adding a new mechanism of energy transfer, which we define as **heating**. Heating  $Q$  is a physical quantity that characterizes the process during which an amount of energy is transferred from the environment to the system when they are at different temperatures. Students should understand that the word heating means a process through which the energy of a system changes, not some substance that resides in the system (and eventually, that the heating does not necessarily leads to warming up of the system). In addition, they should understand that in some instances there can be energy transferred to the system through the process of heating, but the system does not “heat up”—the temperature does not change (or it might even go down).

The textbook Observational Experiment Table 15.1 in Section 15.2 serves the same purpose as the above ALG activities. It helps the students solidify the understanding that work alone cannot explain the changes in the internal thermal energy of a gas in many situations. Testing Experiment Table 15.2 represents a simplified version of a historical experiment performed by James Joule to investigate the equivalence of work and heating as two mechanisms of changing the internal energy of a system.

Section 15.3 leads students through the development of the first law of thermodynamics (we only have one statement of the law as in our approach the system does not do work on the environment). Notice that ALG Section 15.3 starts with the Activity 15.3.1 that resembles Joule’s experiment. It can serve as a formative assessment of student work with the previous section of the textbook. ALG Activities 15.3.2–15.3.3 lead students in the application of the first law of thermodynamics to real-world processes, and finally, ALG Activities 15.3.4 and 15.3.5 are metacognitive.

It is important that students take away from this section that this new principle (the first law of thermodynamics) succeeds in explaining many phenomena that the old work-energy principle could not—such as the change of the temperature of water (the system) placed on a hot electric stove (the environment), or the change in the temperature of a hot, hard-boiled egg (the system) placed in a bowl of cold water (the environment). The first law of thermodynamics is the extension of the work-energy principle to thermodynamic processes. It is important to emphasize again and again that heating and work do not belong to the system but describe the processes through which the energy of the system changes.

We put the discussion about the differences between temperature, thermal energy, and heating here given that students tend to confuse them (they are confused by our everyday language and speech). The two Tips next to the statement of the first law in the textbook (page 449) are also extremely important; make sure that the students do not miss them.

EOC Question 8 is crucial for helping students distinguish the properties of the three quantities involved in the first law: do not skip it! Another useful Question is 9,

and Problems 6 and 7. They will help students apply the first law of thermodynamics qualitatively. In all these examples, work is done *on* the system and the energy is provided *to* the system through heating (both can be positive or negative). The system does not do work.

The next step is applying the first law of thermodynamics to different processes. Textbook Section 15.4 and ALG Activities 15.4.1-15.4.4 help students combine the microscopic approach to the gas processes with the work-heating-energy approach. This builds their knowledge and helps them to recognize the fundamental differences between the concept of energy as a state function and the concepts of work and heating that are energy transfer *processes*. To help students learn these ideas, we encourage them to represent the processes using words, graphs, bar charts, and equations, and to explain what happens using a microscopic approach (molecules and their motion) and a first law of thermodynamics approach (positive or negative energy transfer to the system and the changes in the system's energy).

The beginning of textbook Section 15.4 dedicates a large amount of space to the discussion of the mechanism of energy transfer to and through a wall of a container. It is a rather unique discussion and we strongly recommend that you conduct it in class. Students can have the discussion with you before they do the ALG activities or after, as long as they question how energy is transferred to and through solid objects. The key here is to understand that the old model of elastic collisions of the molecules with the walls of the container cannot explain transfer of energy. If we use an elastic collisions model, we cannot explain how kinetic energy of fast (hot) gas molecules transfers energy to the molecules of the cold wall. Because the wall molecules vibrate, the incoming gas molecule colliding elastically with the wall molecules can either speed up or slow down the vibrating molecule of the wall, depending on during which part of the vibration the incoming gas molecule collides with the wall molecule.

To explain how the molecules of the hot gas in a flame transfer energy to the container walls, we need to model the collisions differently. In this new model, when a molecule hits the surface of the wall, instead of rebounding immediately, it sticks to the wall for a very short time and then leaves the surface because it is “knocked” by the vibrating wall atoms (see the magnified part of Figure 15.5 in the textbook). Hot gas molecules have a large velocity (and thus kinetic energy). When they stick to the surface, their kinetic energy is transferred to the wall atoms. As a result, the wall atoms start to vibrate with larger amplitudes. When the gas molecules detach from the surface and fly away, their velocity (and therefore kinetic energy) is smaller than it was before they stuck to the surface. This fundamental difference between the model of collisions that we used when we derived the ideal gas law and when we try to explain gas processes using energy transfer shows to students that in physics no model can explain the richness of the world; we modify models as we go deeper into the understanding of natural phenomena. The “stickiness” model is used later to explain the transfer of energy during isoprocesses microscopically. We strongly

recommend that you discuss with your students the material on pages 451-453 and then assign the reading of these pages for homework again.

The issue of work and heating not being state functions is addressed in ALG Activity 15.4.1. This activity is based on research work that shows that even after instruction on the first law, students still think that when a system undergoes a process and returns to the original state, all of the quantities of work, heating, and internal energy acquire initial values, i.e. all changes are equal to zero. It is useful to have students work through ALG Activity 15.4.1 in groups and resolve any discrepancies that they encounter through discussion with each other. The idea that the internal energy of the gas is a state function but heating and work are not is one of the most challenging ideas for students to figure out. Example 15.3 in the textbook outlines the problem-solving strategy for using the first law of thermodynamics to analyze gas processes.

Different representations that we use for the analysis solidify their understanding and further develop their ability to reason with multiple representations. Research shows that students have tremendous difficulties connecting macroscopic descriptions of the gas processes to the microscopic explanations. The goal of this section is to help with this difficulty. Thus, we suggest that students first discuss the section material in class with you, then read the textbook section, and only after that carry out the activities in the ALG Section 15.4. EOC Problems to use are 8-13.

## II. Develop the ideas of temperature change and state change

Activities in textbook Sections 15.5 and 15.6 and in the same sections of the ALG help students construct the physical quantities of specific heat and latent heat. The sequence in ALG Activities 15.5.1–15.5.4 can be done in class to help students construct the quantity of specific heat before they do the lab (ALG Activities 15.5.6 – 15.5.8, choose one or more of those for the lab).

Note that we define the **specific heat** of a substance as the physical quantity equal to the amount of energy (and not heating) that needs to be added to 1 kilogram of the substance to increase its temperature by 1°C. Often this energy is added through heating, but the temperature of a substance can change by doing work or by both processes. Thus we have  $\Delta U = cm\Delta T$  instead of the traditional  $Q = cm\Delta T$ , an expression that promotes an unnecessary difficulty. Only when there is no work involved in the process can we say that  $Q = cm\Delta T$ .

While students work with solids and liquids and there is no change of state, the temperature changes when the system's energy is changed through heating. However, if a gas is involved or the material changes state, the heating might not lead to the temperature change. Thus it is important to discuss this nuance in language with students. Just as with the word “work,” physics has recruited the everyday word “heating” and endowed it with a much more specialized meaning. In everyday life,

the verb “heating” is equated with warming, but in physics this is not the case. In our experience, students need to be reminded of this fact more than once.

For example, when energy is provided through the mechanism of heating, it can lead to changes in state (Section 15.6)—melting and freezing or boiling and condensing. The energy in joules needed to melt a mass  $m$  of a solid at its melting temperature or the energy released when a mass  $m$  of the liquid freezes at that same temperature is:

$$\Delta U = \pm mL_f \quad (15.6)$$

$L_f$  is the heat of fusion of the substance (see Table 15.5). The plus sign is used when the substance melts and the minus sign when it freezes. Note that the phrase “heat of fusion” is confusing because in our common language heat is often associated with warming (i.e., temperature change). In the case of melting or freezing, there is no temperature change. However, there is still a change in internal energy, the potential energy of interactions of the particles in the system. The same arguments apply to the heat of vaporization: the energy in joules  $\Delta U$  needed to transform a mass  $m$  of a liquid at its boiling temperature into the gaseous state at the same temperature is

$$\Delta U = \pm mL_v \quad (15.7)$$

where  $L_v$  is the heat of vaporization of the substance. The plus sign is the energy needed to boil the liquid, and the minus sign is the energy released when the gas condenses.

You might be surprised by the choice of axes on the graphs showing the independent variable on the y-axis and the dependent variable on the x-axis (Figure 15.1 in the textbook). We made this choice to help students connect the graphs to the mathematical expressions that follow.

The ALG activities in Section 15.6 show a sequence you can use in class. Notice ALG Activities 15.6.8 and 15.6.9. They are excellent for helping your students synthesize all of the ideas in this chapter. EOC Questions 6–9 and Problems 29–37 can follow.

### III. Heating mechanisms and important real-world applications of the first law thermodynamics

Section 15.7 is dedicated to the discussion of specific mechanisms through which heating occurs (conduction, convection, radiation, and evaporation) and applications of these ideas to the greenhouse effect and climate control and for body temperature control. We suggest that student first read the textbook and then carry out the ALG activities in this section. Activities 15.7.1–15.7.5 are new and will be very challenging for the students. The labs do not require any additional equipment because we

provide videos. Here you can use EOC Questions 10, 11, 14, 17, 19-27 and Problems 41-47, 52-55, and 58-60. General Problems 64, 68, and 69 will challenge your students and peak their curiosity.



# Second Law of Thermodynamics

In Chapter 15, students learned about the new method of energy transfer—heating—and how to include this method in the work-energy equation: the first law of thermodynamics. In this chapter, students find that many processes that are allowed by the first law of thermodynamics actually do not occur. This leads to the development of the second law of thermodynamics and a way to decide whether an energy conversion process is possible. The chapter then applies the second law to analyze the efficiency of various practical devices like motors and refrigerators. Content-based learning objectives of the chapter are listed below.

**Students should be able to:**

1. Explain the difference between reversible and irreversible processes, and give examples.
2. Explain the meaning of the macroscopic and microscopic statements of the second law of thermodynamics.
3. Explain how thermodynamic engines work.
4. Calculate the efficiency of some thermodynamic engines.
5. Explain the relationship between the second law of thermodynamics and the evolution of complex life systems.

This chapter on the second law thermodynamics is broken into three parts:

- I. *Examine reversible and irreversible processes*
- II. *Develop a statistical reason for irreversibility and connect this statistical approach to a thermodynamic approach*
- III. *Apply the second law of thermodynamics and efficiency of processes to the analysis of thermodynamics engines and thermodynamic pumps*

For each part, we provide examples of activities that can be used in the classroom and brief discussions of anticipated student difficulties with the subject matter.

*Brief summary of student difficulties with the second law of thermodynamics*

It is difficult for the students to make connections between the macroscopic and microscopic statements of the second law, and conceptualize entropy as both a microscopic and a macroscopic quantity. Keeping track of what the system and the environment are and what does work on what in thermodynamic engines and pumps is a challenge. Another difficulty is finding the correct system for analysis and clearly specifying its initial and final states. Finally, while in the previous chapter students worked with the terms work and heating that are familiar from everyday life, now they meet the term *entropy* that is completely new and for which they have no mental image. The lack of this image and the very abstractness of the notion of entropy (possibly the most abstract idea that students meet in a physics course) make this concept very difficult for students.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Examine the types of energy changes that make some processes irreversible	16.1	16.1 (p477)	16.1.2a, 16.1.2b, 16.1.2c, 16.1.2d
Develop a statistical reason for irreversibility and connect this statistical approach to a thermodynamic approach	16.2, 16.3	16.2 (p479)	16.3.4
Apply the second law of thermodynamics and efficiency of processes to the analysis of thermodynamics engines and thermodynamic pumps	16.4, 16.5		16.4.11
<b>Nontraditional end-of-chapter questions and problems</b>  Ranking tasks (RAT): P16.32 Evaluate (reasoning or solution) (EVA): Q16.10, P16.14 Multiple possibility and tell all (MPO): Q16.14, Q16.18 Jeopardy (JEO): P16.36			

## **I. Examine the types of energy changes that make some processes irreversible**

In the first part of textbook Section 16.1 and ALG Activity 16.1.1, students analyze processes that are never observed to move in the reverse direction (in time), although the reversed processes are consistent with the first law of thermodynamics (this can be done in class with students working in groups and sharing their ideas). They find that when the processes progress in the direction that is “observed” in nature, organized energy is (eventually) converted to disorganized (non-organized) energy. In the reverse process, disorganized energy should be spontaneously converted to more organized energy, but those processes are not observed in nature. Students encounter three more examples when they work through Conceptual Exercise 15.1. ALG Activities 16.1.2-16.1.4 continue the investigation concerning reversible and irreversible processes and connect them to the types of energy involved – more or less organized. Specifically, Activity 16.1.2 includes “flipped” videos of four different processes. Students need to analyze them from the energy point of view and decide which processes could occur naturally. Activity 16.1.5 asks them to test the newly invented ideas by predicting the outcomes of the experiments that they are already familiar with solidifying their understanding of the connection between reversible and irreversible processes and more or less organized forms of energy.

The direction of the process is examined in Observational Experiment Table 16.1, this time with respect to spontaneous energy transfer in an isolated system when its different parts are at different temperatures. This discussion is needed to introduce a simple version of a cyclic thermodynamic engine and to compare it to a mechanical engine. To summarize, the ALG activities in Section 16.1 and textbook Section 16.1 help students construct three patterns: (1) We do not observe complete conversions of the internal thermal energy of a system (kinetic energy of random motion of particles) to organized macroscopic kinetic energy or to gravitational potential energy in isolated systems; (2) Observations show that in isolated systems, thermal energy spontaneously always transfers from hot to cold substances; (3) Thermodynamic engines convert less organized form of energy into more organized ones, but cannot convert all of the available energy, unlike mechanical engines. These are all qualitative statements of the second law of thermodynamics. You can use EOC Questions 1, 3, 4, and 18 and Problems 1-5 in addition to the ALG activities. Make sure that your students read and discuss the textbook section (ALG Activity 16.1.7) because the textbook has more information than the ALG activities taken together in this section.

## **II. Develop a statistical reason for irreversibility and connect this statistical approach to a thermodynamic approach**

Textbook Section 16.2 and the ALG activities in Section 16.2 provide students with questions and exercises to learn a statistical method to count all the different ways in which the atoms in a gas can occupy two halves of a box (or how 4 coins can be arranged). The least probable distribution is with all atoms bunched together on the same half, and the most probable distribution is with equal numbers of atoms on each half. The latter is a less-organized distribution. For small numbers of particles, there is a reasonable chance for them to all be in the same half. But as the number of atoms in the box increases, it is overwhelmingly more probable for the atoms to be equally distributed between the two halves of the box. We introduce the “count” of microstates as the number of ways to have a certain number of atoms in one half and the rest in the other half (a particular macrostate). The textbook then defines the entropy of a particular macrostate as a physical quantity that is proportional to the natural logarithm ( $\ln$ ) of the count. Example 16.3 helps students find that the entropy increases as a fixed amount of gas expands. A denser gas has more potential to do work on some other system and is more “useful.” The expanded gas has less potential to do work. Thus, this increasing entropy of an expanding gas is consistent with the idea that entropy increases as a system loses its potential to do useful work on some other system. The statistical version of the second law of thermodynamics is that isolated systems tend to proceed in the direction of less organized structure, or in other words, in direction of increasing entropy. Problems 6–8 and 14–15 help students understand this statistical approach in addition to the ALG activities.

The statistical approach becomes cumbersome if applied to large numbers of particles. In Section 16.3, we use this approach to help create a thermodynamic expression for entropy change, which involves the physical quantity of the energy transferred through heating from environment to a system and the average value of the absolute temperature of the system. This allows us to formulate the second law of thermodynamics, which is that the sum of the entropy changes of a system and its environment must always increase during any allowed process. At the end of Section 16.3, there is a short discussion about how humans manage to grow and increase in complexity, what seems like a contradiction to the second law. You can decide if this is a brief discussion that you want to hold with your own class. Students can use the thermodynamic approach to entropy change by working on the ALG Activities in Section 16.3 and EOC Questions 6-9, 13, 14, 16 and Problems 16-21. Note ALG Activity 16.3.4; it is one of the few cases for which students are actually able to calculate the change of the entropy (it also involves estimations).

### **III. Apply the second law of thermodynamics and efficiency of processes to the analysis of thermodynamic engines and pumps**

In Section 16.4, the first and second laws of thermodynamics along with the definitions of efficiency (and Carnot's maximum efficiency) are used to analyze thermodynamic engines, refrigerators, and thermodynamic pumps. Notice that we do not use the terms "heat engines" and "heat pumps" in order to prevent students from thinking that heat is a fluid that can be transferred from one system to another. When discussing thermodynamic pumps, it is helpful to mention that the pumps gather energy from Earth and its atmosphere. These types of renewable energy concepts are very important these days, and students have a genuine interest in them.

The second law of thermodynamics is a moderately difficult topic. It is important here to be careful about work. In previous chapters we agreed that the system did not do work; only the environment did work on the system—positive or negative. Here the processes are more complicated and the system and the environment interchange. When students use bar charts to analyze processes, the work represented on the bar chart is always the work done on a chosen system, but in other instances one system can do work on another system. Caution students to be very careful when they talk about work. It needs to be clear what does work on what, and what system is used for the analysis of energy conservation. Notice Conceptual Exercise 16.5 in the textbook: it uses the same air-filled bottle as a thermodynamic engine but this time analyzes the process with graphs and bar charts. ALG Activities 16.4.1–16.4.10 provide students with multiple opportunities to work with engines and pumps. We especially recommend Activity 16.4.11 that introduces students to a historical Sterling engine. ALG Activity 16.4.10 provides an example of a hypothetical experiment that students can design to determine the average efficiency of a human body.

EOC Problems recommended here are 23-27, 32-34. General Problems 35-38 will help your students practice engines and efficiencies even more.

# 17

## Electric Charge, Force, and Energy

We have completed our investigations of mechanics, gases and liquids, and thermodynamics. We now start five chapters concerning electricity and magnetism. The conceptual arrangement of Chapters 17 and 18 is slightly different than most other textbooks. Chapter 17 combines the concepts of electric force and electrical potential energy, whereas Chapter 18 is dedicated to the analysis of electric field as the medium of interaction, using the quantities of  $\vec{E}$  field and  $V$  field (electric potential). Both force and potential energy involve an interaction between two or more objects. How these objects interact is an action-at-a-distance problem. Chapter 18 serves as the counterpoint to this, introducing a vector field (the  $\vec{E}$  field) and a scalar field (the  $V$  field) to address the action-at-a-distance problem. We feel that students need to understand why physicists need the field representation. Content-based learning objectives are listed below.

### **Students should be able to:**

1. Design an experiment to test a hypothesis: electric and magnetic interactions are the same.
2. Explain how we know that there are only two types of electric charge; how to charge and discharge objects.
3. Explain macroscopic and microscopic differences between conductors and dielectrics.

4. Explain interactions of charged objects, and charged and neutral objects, using microscopic pictures of charge distribution.
5. Describe the experiment from which Coulomb's law can be inferred, describe how Coulomb found proportionality of the force to the magnitude of the charges, and apply Coulomb's law to situations using force diagrams and Newton's laws.
6. Construct and evaluate energy bar charts for situations involving static electricity.
7. Explain why the electric potential energy of a system of two oppositely-charged objects is negative and of two like-charged objects is positive.
8. Apply knowledge of forces, momentum, and energy to solve complex problems combining mechanics and electrostatics.
9. Describe how knowledge of electric charges helps explain real-life phenomena and biological applications.

Chapter 17 is broken into three parts:

- I. *Qualitative analysis of electrostatic interactions of charged objects, and of charged and neutral objects, including interactions of charged particles with conductors and dielectrics*
- II. *Quantitative analysis of electrostatic interactions: Coulomb's law and the electrical potential energy of systems of charged point particles*
- III. *Applications of electrostatics knowledge to problems and real-life phenomena*

For each part, we provide examples of activities that can be used in the classroom and brief discussions of anticipated student difficulties with the subject matter.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Qualitative electrostatic interactions, including interactions of charged objects with conductors and dielectrics	17.1-17.3	OET 17.1 (p501), OET 17.2 (p503), TET 17.3 (p505)	
Quantitative analysis of electrostatic interactions: Coulomb's force law and the electrical potential energy of systems of	17.4, 17.5		

charged point particles			
Applications of electrostatics knowledge to problems and real-life phenomena	17.6, 17.7		
<p align="center"><b>Nontraditional end-of-chapter questions and problems</b></p> <p>Ranking tasks (RAT): P17.31          Evaluate (reasoning or solution...) (EVA): Q17.27, Q17.29, P17.29, P17.36          Make judgment (based on data) (MJU): Q17.19, Q17.23, P17.2          Multiple possibility and tell all (MPO): P17.26, P17.30, P17.41, P17.42, P17.50          Jeopardy (JEO): P17.32, P17.33, P17.34, P17.35          Design an experiment (or pose a problem) (DEX): Q17.15, Q17.16, Q17.19, Q17.21, Q17.26,</p>			

*Brief summary of student difficulties with electric force and energy*

It is important to know that students confuse electric interactions with magnetic one, given that magnetic ones are more familiar. They often call magnetic poles charges and vice versa. This language confusion actually reflects the belief of many students that magnetism explains electric interactions. However, the biggest difficulty for students is going back to mechanics and using force diagrams and work-energy bar charts to analyze processes, especially applying Newton's third law to the interactions of charged objects. Another difficulty is dealing with positive and negative potential energies of the systems of charged objects. But most importantly, a hidden difficulty is a similarly simple question of charging an object positively. Students tend to think that we can add both positively and negatively charged particles to objects. Thus the concept of charging something positively by taking away negatively charged particles is really difficult.

## **I. Qualitative electrostatic interactions, including interactions of charged objects with conductors and dielectrics**

Electric charge exploration starts with students learning the new types of interaction. Physics instructors commonly start this chapter by letting students observe the interaction of charged objects with neutral objects: comb and hair, a balloon rubbed with wool and a wall. Notice that in both the textbook and the ALG, the exploration starts with objects rubbed with some other object, and only after students come up with



the concept of two different types of electric charges do they proceed to the investigations of charged and neutral objects.

To begin, students can observe and debate the patterns in the experiments described in Table 17.1 or perform ALG Activity 17.1.1 in a lab. Students observe repulsion of the same objects rubbed with the same material as well as the attraction of objects rubbed with different materials and the object and the material with which it is rubbed (this part might be tricky due to the leakage of charge though your hands and sweat on the rubbing materials, thus we suggest using rubber gloves and/or standing on a Styrofoam board). Thus, rubbing makes objects acquire a new property—to be able to attract or repel other objects after rubbing. The textbook provides a brief history of the exploration of this phenomenon and shows the term that physicists invented for this new property, *electric charge*. It is important that the term electric charge comes after students have had experience with the phenomenon. Students should realize that we never see electric charge—it is an invented concept to help explain the observed attractive and repulsive forces.

A summary of six observations and patterns explained by the introduction of electric charge and the electrostatic force between charged objects appears in the middle of Section 17.1. The experiments in the ALG Activities 17.1.3-17.1.4 and Observational Experiment Table 17.2 help students expand their knowledge of charged objects as they observe that both positively and negatively charged objects attract neutral objects. EOC Questions 1 and 2 can be used here.

Below we list a few suggestions for ALG Activities 17.1.3 and 17.1.4. Note that the term *charge diagram* refers to a sketch on which the students show with pluses and minuses the electric charges inside or on the surface of objects. The number of pluses or minuses reflects the magnitude of the charge.

Activity 17.1.3: When we do this activity, we expect students to come up with two models of the internal structure of the material: 1. A model in which the electrons are free to move around the material so that they all congregate on one side, leaving the other side positive. We don't worry too much at this stage about whether they say the electrons move or whether the + and – charges move. This is not the time to get into the “correct” model – just the idea that some charge in the material can move around. 2. A model in which the electrons in the atoms are not free to move from the atom. Here the electrostatic force exerted by the charged rod causes the atoms to “distort” and become dipoles. In our experience with college science majors, they've had enough science in high school and that at least 1 group in a section of 6 is able to come up with the “dipole” model (the free charge model is very common, they almost always come up with that). And if no group is coming up with the dipole model, we will “seed” the model with a group who we know has members in it who've taken university level chemistry already. Many times we will observe a “good” group is already struggling to create this model, so it is more of a matter of just guiding them to formalize it by getting them to draw multiple atoms and asking them how each atom is going to interact with the other, creating a full set of dipoles through the material, just leaving one side with a + surface charge and the other end with a –

surface charge and effectively everything inbetween is canceling out. The purpose is to get them to think about the details of the microscopic model: the electrons are free to move uninhibited versus the electrons are bound to the atom and can't move.

Activity 17.1.4: After students work through part b, we suggest that the instructor scaffolds or leads a summary discussion at the end of these experiments: Both models are correct. The first model describes what we call “conductors” the other describes what we call “insulators” or “dielectrics.”

The next step is for the students to come up with a mechanism explaining the interaction of charged objects (ALG Activity 17.2.1). So far, students are familiar with the attraction of objects due to the gravitational force. However, many have experienced the attraction and repulsion of magnets prior to taking the physics class. If you ask students to explain why rubbed objects repel and attract each other, the most common answer will be “they are magnets.” This is a very testable hypothesis. ALG Activity 17.2.2 guides students through the reasoning process necessary to design an experiment to test this hypothesis. Alternately, you could have students discuss and propose experiments during the lecture. As long as you have the equipment available and a camera (or even an overhead projector) to project the experiment on the screen, all of them will see that both poles of the magnet attract the objects rubbed with different materials. Later in the chapter, students need to return to this observation and realize that the reason for such attraction is the conductive nature of the magnet; it behaves as any conductor in the presence of an electrically charged object. Textbook Testing Experiment Table 17.3 in Section 17.2 summarizes the reasoning process and outcomes of a possible testing experiment.

Another model that students often come up with to explain the interaction is the particle model—when two objects are rubbed, one loses some particles and the other gains them, so the one that has lost “wants” them back. Although it is not a very scientific mechanism, it is basically similar to the one physicists have (except the “wanting” part), so this is a productive aspect of student reasoning on which you can build. The textbook exposes students to the fluid model of electric charge and the oil drop experiment, which indicated that electric charge came in tiny quantized units. The textbook then uses a contemporary model to explain experiments discussed in the first two sections. Students can practice these ideas by answering EOC Questions 3, 4, and 14-16.

Often, it is difficult for students to understand how one can charge an object positively if positively charged particles cannot be added to the object. You can help here by using a number example. A neutral object has a total zero electric charge. When one takes away a few negatively charged particles, the procedure is similar to subtracting a negative number from zero; for example,  $0 - (-5)$ ; the result is a positive number:  $0 - (-5) = 0 + 5 = +5$ .

Students can now perform ALG Activities 17.3.1–17.3.3 to deepen and apply an understanding of the internal structure of conductors and dielectrics and get familiar with how electroscopes work. If you did ALG Activities 17.1.3 and 17.1.4, students should have already constructed the main idea that both conductors and dielectrics have

positively and negatively charged particles inside them before they come in contact with other objects. In the presence of external charged objects, negatively charged particles in the metals move to a different location, and in dielectrics they just shift within a molecule, creating polarized molecules, or reorient if the molecule was already polarized. In both cases, a surface charge is created, but in one case it is free (conductors) and in the other case it is bound (dielectrics). Students can then use each of the two models to make predictions about the possible outcomes of testing the experiment in ALG Activity 17.3.1.

Another good testing experiment involves bringing a charged rod near one side of two touching metal soda cans without touching them and then separating them while the charged rod is still there and then removing the rod. (The experiment must be done very carefully so the cans do not get discharged during the separation.) Students need to predict what happens to the charges of the cans. Because of the movement of negative charge from one can to the other, the cans are oppositely charged. However, when the experiment is repeated with two touching plastic bottles, they do not become charged.

After students have established that the free electron model applies to metals and the polarization model applies to Styrofoam and plastics, students can read Section 17.3 and apply their understanding of conductors to the behavior of electroscopes (ALG Activities 17.3.2 and 17.3.3). These activities help students apply their understanding of conductors to explain how an electroscope works and how to charge objects without rubbing – by induction. Note that for an electroscope to work properly it should be positioned on a horizontal surface, any tilt will distort the deflection. If the weather is humid you can improve the results using a hair dryer. We do not see the electroscope Activities 17.3.2 and 17.3.3 as “pivotal,” but they are great for an exploration if you have extra time to spend on electrostatics.

In general, when students make predictions or reason about electrostatics phenomena, make sure they draw the distribution of charged particles inside the objects, i.e. charge diagrams, and reason qualitatively with them. The section ends with several applications: determination that the human body is a conductor, the idea of grounding, and a review of the properties of electric charge.

Now students are ready to answer EOC Questions 4, 5, 18, 22, and solve EOC Problems 1 and 4.

## **II. Coulomb’s force law and the electrical potential energy of systems of charged point particles**

So far, students have observed that electric forces that charged objects exert on other charged objects decrease with the increase of the distance between them. ALG Activity 17.4.1 presents students with fabricated data to use in constructing

Coulomb's law. The results of the analysis are given in the textbook in the discussion following Table 17.4 in Section 17.4. If you want students to first attempt the task on their own, it is better to start with the ALG here (alternatively you can use Direct measurement videos available through Mastering Physics). If you use the ALG activity, the students will analyze pretend data to develop an expression for the magnitude of the force that electric charge 1 exerts on electric charge 2—Coulomb's law. It is important that students realize that the law (as we have formulated it) is used only to find the magnitude of the force. The direction of the forces exerted on a particular charged object by other charged objects can be determined from a force diagram for the object of interest and by whether the forces exerted on the system object are attractive or repulsive. The signs of the charges in Coulomb's law correctly determine the direction of the force *only* in a properly defined radial coordinate system. We feel that including this radial axis is too much unnecessary mathematical overhead for students. We have found that students are more comfortable with the approach when Coulomb's law defines the magnitude of the force and the direction is inferred separately. Asking students to construct force diagrams for each of multiple charges is a very useful exercise (see later ALG Activities 17.6.1-17.6.3 and end-of-chapter Problem 11). Encourage students to compare and contrast Coulomb's law with the law of universal gravitation – what are the similarities? What are the differences?

Students should also understand that two charged objects exert equal magnitude and opposite direction forces on each other, even if one object has a much greater charge (see ALG Activity 17.4.4 and textbook Conceptual Exercise 17.3). Finally, for problems involving charges in a plane and two-dimensional forces, it is important to use a complete problem-solving approach, which includes a sketch, force diagram, and Newton's second law in component form. Students could have forgotten this process from the first semester, and if you do such problems for electrostatics, the process needs careful review. A series of two ALG activities done in a lab (Activities 17.4.5 and 17.4.6) provides a unique opportunity for your students to review mechanics and simultaneously learn a way to measure electrostatic forces with an electronic platform scale. These can be a part of the lab for electrostatics at any time.

You can use the following activities: ALG Activities 17.4.2 and 17.4.3, Examples 17.4 and 17.5 in the textbook, EOC Questions 6 - 9, and Problems 12, 13, 15, 16-19.

Finally, it is important that you discuss the limitations of Coulomb's law. Why does it work for only point-like charged objects? What happens when two metal charged spheres are brought next to each other and are located at a distance comparable to their radii? (EOC Question 28.)

The electric potential energy of a system of two or more point-like charged objects is the subject of Section 17.5. Electric potential energy is glossed over in many textbooks, and yet it forms the fundamental grounding and understanding for the more abstract quantity of electric potential. In our experience, the time devoted to understanding electric potential energy pays off later when students have to wrestle with electric potential. Both the ALG and the textbook follow the same approach.

Student learning of electric potential energy is based on the system approach, which initially involves using bar charts to analyze situations involving electrically charged objects qualitatively. From this analysis, students invent a new type of energy—electric potential energy. Only then do they proceed to the construction of the mathematical description of this energy of two like and two unlike charged objects. We recommend starting off the section on electric potential energy in class with the ALG Activities 17.5.1-17.5.3, making sure that students analyze the situations using the bar charts (textbook Figures 17.15 and 17.16) and follow up with a clicker question, in which you present students with pairs of configurations of two charged objects and ask them which configuration has more electric potential energy. For example, you could ask students to compare the electric potential energy of a system consisting of a positively charged object and a negatively charged object when they are two different distances apart. Then you could ask students to compare the electric potential energy of a system of two negatively charged objects (or two positively charged objects) when they are placed at a different distance apart. In our experience, this is a challenging activity for students because there is more than one variable involved (charge and distance), but it helps them to keep track of how electric potential energy should increase or decrease based on the signs of the charged objects and the distance apart.

It is very important to establish where the electric potential energy of a system is zero. For simplicity, we always assume that the electric potential energy of a system of two electrically charged objects is zero when the distance between the objects is infinite. The signs of the potential energies can cause difficulties. To help students with the signs, use textbook Example 17.6 to analyze energy of the system of two opposite charged objects and then Figure 17.18, in which students examine the graph of the potential energy versus separation distance.

Finally, a general discussion concerning the fact that a system of objects that are attracted to each other has negative electric potential energy (assuming that the same objects being separated by an infinitely large distance do not interact and thus the system does not have any electric potential energy) is extremely important. Discussing an analogy between a gravitational potential energy of system object-Earth and electric potential energy of two oppositely charged objects might help students understand that if you have to do positive work to separate two objects in the system, the most reasonable choice is to assign a negative sign for the energy of their interaction. Comparing and contrasting positive and negative electric potential energies of different system is also very important. Make sure that you assign the reading exercise (ALG Activity 17.5.5) here. It is different from our common reading exercises and it poses specific questions in addition to the Review Question.

At the end of the section, students meet the concept of the electric potential energy of a system of multiple charged objects. EOC Problems 27 and 28 provide good practice here.

Students can further work on EOC Questions 10–12 and 29. These can be followed up with Problems 20-23 and 25-29. Other end-of-chapter problems for

Sections 17.6 and 17.7 can also be integrated into the instruction with Sections 17.4 and 17.5 (see the problem list at the end of Part III).

### **III. Applications of electrostatics knowledge to problems and real-life phenomena**

ALG Activity 17.6.1 and Example 17.7 in Section 17.6 guide students on how to use the strategies used to solve electrostatics problems and illustrates these strategies for the example problem. The example involves electrostatic force. Example 17.8 uses the same strategies for an energy problem—radon decay in the lungs. We recommend that students, after working through ALG Activity 17.6.1, compare their solution to the textbook and then proceed to working in groups through the ALG Activities 17.6.2-17.6.12 and 17.6.14, which is a linearization problem. Activity 17.6.13 can be used as a lab experiment. All EOC problems in this section are very useful but we especially recommend 31-36, 44-46.

Section 17.7 has more examples: the operation of a Van de Graaff generator, a free electron acceleration near a Van de Graaff that causes atom ionization and a 10-cm long spark, the operation of a Wimshurst generator, and a photocopy machine. All of the end-of-chapter problems in this section are actually appropriate for use in Sections 17.4 and 17.5. At the end of the chapter we recommend that students work on EOC problems including Problems 53, 54 and 55. Both reading passages might spark students' curiosity and interest.

# 18

## The Electric Field

In Chapter 17, students developed ideas concerning electric charge, force, and energy using a point-like charge action-at-a-distance interaction model. In this chapter, they learn a field approach for these same interactions—especially important for the electric circuit analysis in Chapter 19. The content-based learning objectives for the chapter are listed below.

### Students should be able to:

1. Explain the difference between the concept of a field as a medium for interactions and physical quantities characterizing it:  $\vec{E}$  and  $V$  fields.
2. Compare and contrast the physical quantities of  $\vec{E}$  and  $V$  fields.
3. Compare and contrast operational definitions of  $\vec{E}$  and  $V$  fields and cause-effect relationships for the same quantities.
4. “Read and write” with different representations of electric field such as  $\vec{E}$  field vectors,  $\vec{E}$  field lines, and equipotential surfaces.
5. Apply the superposition principle to calculate  $\vec{E}$  and  $V$  fields for situations involving multiple charged objects, including infinitely large metal plates.
6. Explain grounding and shielding qualitatively and grounding quantitatively through electric potential.
7. Give examples of applications of our knowledge of electric field to real-life and especially biological examples.
8. Apply knowledge of electric fields and work/energy to explain how a capacitor works.
9. Compare and contrast an operational definition of capacitance of a capacitor and a cause-effect relationship for capacitance of a parallel-plate capacitor.
10. Explain how to derive an expression for electric potential energy of a capacitor and how it depends on a dielectric inside it (macroscopically and microscopically).

This chapter is broken into five parts:

- I. *Qualitative and quantitative development of the electric field concept and the quantity of the  $\vec{E}$  field*
- II. *Development of the physical quantity of electric potential  $V$  ( $V$  field), using it to analyze processes, and relating the electric field and the  $V$  field*
- III. *Conductors and dielectrics in electric fields*
- IV. *Capacitors*
- V. *Applying these ideas to understand electrocardiography*

For each part, we provide examples of activities that can be used in the classroom and brief discussions of anticipated student difficulties with the subject matter.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Qualitative and quantitative development of the electric field concept and the quantity of the $\vec{E}$ field	18.1, 18.2	OET 18.1 (p539)	
Development of the electric potential $V$ ( $V$ field), using it to analyze processes, and relating the $\vec{E}$ field and the $V$ field	18.3, 18.4		18.4.4
Conductors and dielectrics in electric fields	18.5, 18.6		
Capacitors	18.7		
Applying these ideas to understand electrocardiography	18.8		
<b>Nontraditional end-of-chapter questions and problems</b> Ranking tasks (RAT): P18.32, P18.33, P18.57 Evaluate (reasoning or solution...) (EVA): Q18.7, Q18.16, Q18.17, P18.61 Make judgment (based on data) (MJU): P18.58 Linearization (LIN): P18.59 Multiple possibility and tell all (MPO): P18.18, P18.42, P18.52 Jeopardy (JEO): P18.19, P18.20, P18.30, P18.31			



**Design an experiment (or pose a problem) (DEX): Q18.10, P18.34**

Before we proceed to the analysis of individual sections and activities, we should discuss the language used in this chapter. Because students have tremendous difficulties with the concept of field as “altered space” and the concept of electric potential, we have moved away from the traditional terminology. We wish to help students conceptually distinguish between the ideas that (1) there is an altered region of space (altered by the presence of one or more electrically charged objects) with specific properties and (2) each point in this region of space can be described by several physical quantities. We will use the term *field* (electric or some other field) to describe an altered region of space. We call the physical quantity which is traditionally called electric field,  $\vec{E}$  field, and we call the quantity traditionally called electric potential,  $V$  field (and electric potential). The former is the force-like vector quantity, and the latter is the energy-like scalar quantity. We represent  $\vec{E}$  field with  $\vec{E}$  field vectors and  $\vec{E}$  field lines, and  $V$  field with equipotential surfaces. In summary, when charged objects are present, each point in space can be described by a vector field ( $\vec{E}$  field) and a scalar field ( $V$  field).

*Brief summary of student difficulties with electric fields*

While students have difficulty with the concept of the field itself and the physical quantity of  $\vec{E}$  field and especially the superposition principle, the concept of electric potential ( $V$  field) is even more challenging. One possible reason is that in general forces are more tangible to students than energies. The understanding that both quantities do not describe charged objects but describe points in space (or two different points in space) is probably the main challenge here.

## **I. Qualitative development of the concept of electric field and quantitative development of the physical quantity of $\vec{E}$ field**

To help students create the concept of the field as the medium for interactions, both the textbook and the ALG start by establishing the analogy between gravitational and electrostatic interactions (Section 18.1 and ALG Activities 18.1.1–18.1.4) and the activities that help students devise a field-based model for both. In this model, an electrically charged object creates a disturbance in space. When another object is placed in this disturbance, its motion is affected. Thus, two charged objects can interact with each other without direct contact (the same logic applies to the objects with mass, although we do not distinguish between gravitational and inertial mass). Both the beginning of the textbook Section 18.1 and the ALG Activity 18.1.1 introduce a “blanket” or “elastic sheet” analogy to help students visualize this

invisible agent that allows two objects at a distance to interact. Please read the description of the analogy and its limitations carefully before you decide if you want to use it – it has its pluses and minuses.

ALG Activity 18.1.4 asks students to use the field model of electrostatic interaction to explain the described experiments and to design new experiments in which metal objects act as shields. After students have constructed the concept of the field as the medium for interactions, they learn two physical quantities that characterize the field—the force-type quantity of  $\vec{E}$  field and the energy-type quantity of  $V$  field. We use the same approach to help students invent both. For the quantity of  $\vec{E}$  field we consider one electrically charged object (called the source) and analyze the forces exerted on another object (the test object) that is placed in the field. We rely on the analogy with the idea of a gravitational field:

$$\vec{g} = \frac{\vec{F}_{\text{Field on Object}}}{m_{\text{Object}}} \quad \text{and} \quad \vec{E}_{\text{due to } Q} = \frac{\vec{F}_{Q\text{field on } q_{\text{test}}}}{q_{\text{test}}}$$

Notice that both equations represent operational definitions for the  $\vec{g}$  field and  $\vec{E}$  field at a particular location. It is important to label the test object with a special subscript to avoid confusion (we continue to stress the difference between the source and test objects throughout the chapter). To devise the cause-effect relationships that explain the value of the  $\vec{g}$  field and  $\vec{E}$  field, and show that those values do not depend on the test objects, students use the law of universal gravitation and Coulomb's law.

To help students develop the physical quantity of  $\vec{E}$  field at a particular location that students should conceptualize as the force that *would be* exerted on a unit charge placed at that location, we first use the case of the gravitational field created by Earth and analyzing its effects on test objects placed near by. We come up with the quantity of force divided by the mass of the test object in ALG Activities 18.1.6 and 18.1.7. The purpose of these activities is to get students to start thinking of field as an entity created by the source object and to understand that the definition of  $\vec{E}$  field (net force)/charge or a  $\vec{g}$  field (net force/mass) are human-created operational definitions. If students are struggling to see the point of this, we suggest to hold a whole-class discussion at the end of Activity 18.1.7, explaining that the only way to create a mass-independent quantity is to divide by mass (same for charge). Using the gravitational field first is a nice familiar context for this discussion.

ALG Activity 18.1.10 and textbook Observational Experiment Table 18.1 help students devise the superposition principle for  $\vec{E}$  fields. Following it, Physics Tool Box 18.1 shows how to estimate graphically the net  $\vec{E}$  field due to multiple charges, including an example of the field due to an electric dipole on the heart at one instant during the heartbeat cycle (students can do ALG Activities 18.1.11 and 18.1.12 here (notice the complex nature of diagrams in Activity 18.1.12); textbook Conceptual Exercise 18.1 is a worked example for ALG Activity 18.1.11).

So far, students represent electric field quantitatively and graphically with  $\vec{E}$  field vectors. The next step is to learn to represent it with  $\vec{E}$  field lines (the introduction is in the textbook subsection; students can then do ALG Activity 18.1.13). We use this representation because it allows students to form concrete images of the abstract concept and helps avoid overlapping vectors (as in Activity 18.1.12). To draw a line, students need to first draw  $\vec{E}$  field vectors at several locations and then draw the line to which those vectors are tangent. Conceptual Exercise 18.2 helps students draw the  $\vec{E}$  field lines for a large, uniformly charged glass plate. ALG Activity 18.1.14 can follow this example. The students need to remember three important things about the  $\vec{E}$  field lines representation. The first is that the direction tangent to the line at a specific location indicates the direction of a force that would be exerted on a positive test charge if it were placed in this location. The force direction does not mean the direction of motion; it only means the direction of acceleration. The second is that the relative number of lines originating from or ending on the charged objects indicate the magnitude of the charge of the object around which the  $\vec{E}$  field is being represented. The third is that density of  $\vec{E}$  field lines (how close they are) in selected region of space indicates the magnitude of the  $\vec{E}$  field in that region of space.

EOC Questions appropriate here are 1, 6, 7, 9–12 and Problems 1–6.

Section 18.2 illustrates methods for solving two common types of problems involving  $\vec{E}$  fields:

(1) In some problems, a source charge distribution is given and students are asked to determine the  $\vec{E}$  field at a particular location. Students could have forgotten how to add vector components; thus it is important to go slowly through such a problem-solving process. The process is illustrated in the text in Example 18.3. You can start with ALG Activities 18.2.1–18.2.2 and then have students review at home the worked example and follow up with the *Try It Yourself* activity for that example, which is also done in steps, with answers for each step.

(2) The second type of problem involves analyzing a process involving various forces, including forces caused by a given  $\vec{E}$  field. A problem-solving strategy is outlined on the left side of the table in the textbook Example 18.4 and illustrated for a problem on the right side of the table. The strategy is used again in Example 18.5 to determine the deflection of a tiny charged ink ball in an ink-jet printer. Students in class can try EOC Question 1 and work on ALG Activities 18.2.4–18.2.8. Homework can be Problems 7–20, especially valuable are 10, 11, 15, 16, 19, and 20.

## II. Developing the physical quantity of $V$ field (electric potential), using it to analyze processes, and relating the $\vec{E}$ field and the $V$ field

We follow the same logic when helping students develop the concept of another quantity characterizing electric field at each location. While the  $\vec{E}$  field at a particular location is indicative of a force that the field *would* exert on a unit charge (or force divided by the test charge, assuming that the test charge is positive) placed at that location, the quantity of  $V$  field at a particular location is indicative of the electric potential energy that the field-unit-charge system *would* have if a unit test charge were placed at that location. What is important to understand here is that for neither the  $\vec{E}$  or  $V$  field is a test charge needed; they exist at a particular location without any test charge placed there.

We suggest that the students start constructing the idea of electric potential by working through ALG Activities 18.3.1–18.3.3 and then read the textbook.

In Section 18.3, students find that the ratio of the electric potential energy of the system of a source charge  $Q$  and a test charge  $q$  separated by distance  $r$  and the test charge ( $(kQq/r)/q = kQ/r$ ) is independent of the test charge. Thus, it can represent an energy field (we call it  $V$  field or electric potential) at a distance  $r$  from the source charge. Note again the difference in the operational definition of the electric potential and the cause-effect relations for the potential of a specific point. If you look at Equation 18.6 in the textbook, it says  $V = U_{Q\text{test}}/q_{\text{test}} = k_C Q/r$ . The left side of the equation  $U_{Q\text{test}}/q_{\text{test}}$  represents the operational definition of electric potential and the right side  $k_C Q/r$  represents a cause-effect relationship for a specific case of electric field created by a single point like charged object with a charge  $Q$  at a distance  $r$  from that object.

After the students read the textbook Section 18.3 (ALG Activity 18.3.4) they can proceed with the ALG Activities 18.3.5–18.3.9. ALG Activity 18.3.9 and textbook Quantitative Exercise 18.6 help students develop the concept of superposition of the  $V$  field due to multiple charges by determining the  $V$  field (electric potential) in body tissue caused by the dipole charge on the heart. Students will be happy that the  $V$  field is a scalar quantity, and they can simply add it for multiple source charges. However, the sign of the  $V$  field at some point due to a source charge depends on the sign of the source charge. Do not skip pivotal ALG Activity 18.3.10 that helps students practice superposition principle for electric potential and simultaneously strengthen their evaluation skills.

The potential difference between two points in space is especially important in problem solving. Positively charged particles accelerate toward regions with lower potential, and negative particles accelerate toward regions with higher electric potential. This idea is used in Example 18.7 in the analysis of an X-ray machine. Section 18.3 ends with an introduction to equipotential surfaces and their behavior relative to electric field lines (ALG Activity 18.3.7). There are two major things that students need to learn: the closeness of the lines representing the equipotential surfaces separated by the same amount of electric potential (for example, 10 V) is related to how fast the electric potential changes in that region. This in turn relates to the magnitude of the  $\vec{E}$  field in the region (for example closer to a positive point-like charge the equipotential surfaces are closer together, this means that the electric potential changes more rapidly). The second aspect of the equipotential surfaces is that their values decrease in the direction of the  $\vec{E}$  field lines (both aspects can be clearly illustrated in a discussion using Figures 18.9 and 18.10 in the textbook). Figure 18.11 and the elastic sheet analogy help the students connect the  $\vec{E}$  field vectors and the equipotential surfaces.

In class you can try EOC Questions 4, 8, 19, 29 and 30 and Problems 21–27 and 30.

In Section 18.4 students derive a quantitative relation  $\vec{E}$  field and  $V$  field ( $E_x = -\Delta V/\Delta x$ ). We suggest that they work with the ALG Activities 18.4.1–18.4.4 and then work with the textbook Section 18.4 (ALG Activity 18.4.5). Make sure that you assign Quantitative Exercise 18.8 for them to work through – it really solidifies their understanding of electric potential and its relations to the energy of a system. Appropriate EOC Questions 26 and 27 are and Problems 28, and 31–33 and 57–59 (these last ones are especially helpful).

### III. Conductors and dielectrics in electric field

The logical progression of Section 18.5 and matching activities in the ALG is electric field of a charged conductor (ALG Activity 18.5.1), grounding (ALG Activity 18.5.2), and a conductor in an external electric field–shielding (ALG Activities 18.5.3 and 18.5.4).

First students learn that charged conductors produce electric fields with  $\vec{E}$  field vectors perpendicular to their surfaces. Then they reason that the  $\vec{E}$  field inside a charged conductor must be zero and the  $V$  field must be constant. They then apply these ideas to explain why grounding some potentially charged electric device causes almost all of the charge to be transferred to Earth. Students encountered the idea of grounding in Chapter 17, but that chapter did not explain it quantitatively. The concept of all points of a charged conductor having the same value of the  $V$  field helps students explain grounding quantitatively by analyzing the redistribution of

charges on the surfaces of two unequal size spherical conductors. This is a unique exercise, and we suggest that you start it with ALG Activity 18.5.2 and then let students work through the textbook material in the subsection “Electric field of a charged conductor” and work on the EOC Problem 37. In this section, students also learn that the ratio of charge/surface area is greater on a small surface than on a larger surface to which it is connected—an idea used later to understand how lightning rods work. To develop the concept of shielding, students can start with ALG Activities 18.5.3 and 18.5.4 and then read the textbook material. EOC Problems in Section 18.5 34–38 and EOC Questions 3, 20, 21 and 32 will help students understand conductors in electric field better.

Section 18.6 is dedicated to the effect of electric fields on dielectric materials. We discuss the polarization of the electric charge in atoms with the reorientation of permanent molecular dipoles leading to internal fields in the dielectrics with  $\vec{E}$  field vectors pointing opposite the  $\vec{E}$  field vectors of the external fields causing the polarization. This leads to the introduction of the dielectric constant and a reduction in the force between charged objects when in a dielectric material. This is important in biology because salts, which do not dissolve when in air, dissolve into sodium and chlorine ions when in blood. We suggest that students work with the textbook material before they attempt the ALG activities in this section. You can use EOC Questions 4, 22, 24, 25, and Problems 39–42 here.

## IV. Capacitors

Section 18.7 is dedicated to capacitors: their general nature, the electric field between the plates separated by a distance  $d$ , the operational definition of the capacitance as  $C = q/|\Delta V|$ , and the cause-effect relation  $C_{\text{parallel plate capacitor}} = \frac{\kappa A}{4\pi k d}$  that explains how the capacitance depends on the physical properties of the capacitor (the area of the plates, the distance separating them, and the dielectric constant of the material between them). We suggest that students carry out ALG activities in Section 18.7 (18.7.1–18.7.3) first, then work with the textbook before the section “Energy of a capacitor” (ALG Activity 18.7.4) and then return to the ALG Activity 18.7.5 – the derivation of the energy of the capacitor. We suggest that you help the students work through this activity by leading the whole class discussion stimulated by the questions in the activity.

We make a connection to biology in Example 18.10 that explores the capacitance of body cells estimated along with the total electric charge separated from the inside to the outside of the cell walls. After students derive the expression for the energy stored in the capacitor by charge separation of capacitor plates, they apply it to estimate the energy stored by the charge separation across body cells (Example 18.11). Finally, an expression for the energy density stored in an electric field is developed. Problems 43–48 and 49 (especially!) can be used with students in lectures and recitations. We return

to capacitors in Chapter 19, DC circuits, and recommend that you combine the material in this chapter and in Chapter 19 to study capacitors together.

One of the issues that come up with capacitors is whether capacitors store electric charge. As the total charge of a capacitor is zero, they do not store any charge, but they store electric potential energy due to charge separation. Somebody had to do work separating those charges to “charge” the capacitor, thus the capacitor possesses the energy.

## **V. Applying these ideas to understand electrocardiography**

Section 18.8 applies the chapter ideas to how electrocardiographs, with pads placed on the skin, are able to monitor the operation of the heart deep inside the body. Notice that the material on lightning is removed from the textbook and placed in the ALG with the supporting questions (ALG Activity 18.8.2). You can ask students to work on EOC Problems 52–54 and General Problems 55 and 56. For life-sciences students we strongly recommend EOC Problems 60–62 as well as the Reading Passages.

# DC Circuits

In Chapter 18, students learned two physical quantities that describe the electric field quantitatively using the quantities of  $\vec{E}$  field and  $V$  field—electric potential. They also learned that an electrically charged particle will accelerate if placed in the region with an electric field. In this chapter, students extend this idea to the motion of electric charge in electric circuits. Content-based learning objectives of the chapter are listed below.

**Students should be able to:**

1. Explain why only a closed circuit loop will light a bulb. Be able to trouble shoot wrongly connected circuits. Build simple circuits involving series and parallel connections and measure current through and potential difference across circuit elements.
2. Compare and contrast the physical quantities of electric current, potential difference, resistance, and electric power. Make predictions concerning potential difference, current, and resistance in DC circuits.
3. Reason qualitatively about series and parallel circuits using the concept of potential difference and resistance. Apply this reasoning to home wiring.
4. Explain why the slope of the current-versus-voltage curve is *not* equal to the inverse of the resistance of an element (except when the graph is a straight line that goes through the origin). Compare and contrast operational definition of resistance  $\left(R = \frac{\Delta V}{I}\right)$  with the cause-effect relationship  $\left(R = \rho \frac{l}{A}\right)$ .
5. Design an experiment to test whether a particular circuit element is ohmic or non-ohmic.
6. Compare and contrast resistors, incandescent light bulbs, LEDs, and capacitors in DC circuits.



7. Use Kirchhoff's rules to reason about circuits quantitatively.
8. Find the equivalent resistance of complex circuits.
9. Design an experiment to test which model for a regular battery is better: a source of constant current or source of constant voltage.
10. Design an experiment to estimate the internal resistance of a battery and opening voltage of an LED.

We have broken the chapter into four parts, with the early focus on qualitative reasoning and the latter on quantitative problem solving:

- I. *Electric current, batteries, simple circuits, Ohm's law, and qualitative analysis of circuits*
- II. *Joule's law, Kirchhoff's rules, series and parallel circuits with resistors, capacitors in DC circuits, and skills for quantitatively solving circuit problems*
- III. *Properties of resistors, superconductivity, and semiconductors*

For each part, we provide examples of activities that can be used in the classroom and brief discussions of the motivations for using these activities. The most important advice that we could give to the instructors is that helping their students reason about DC circuits primarily using the concept of potential difference is much more beneficial than reasoning primarily through current.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Electric current, batteries, simple circuits, Ohm's law, and qualitative analysis of circuits	19.1-19.5	OET 19.1 (p573), OET 19.2 (p579)  TET 19.4 (p585) OET 19.5 (p586) OET 19.6 (p587)	
Joule's law, Kirchhoff's rules, series and parallel circuits with resistors, capacitors in DC circuits, and skills for quantitatively solving circuit problems	19.6-19.9	19.1 (p599)	
Properties of resistors, superconductivity, and semiconductors	19.10		

**Nontraditional end-of-chapter questions and problems**

Ranking tasks (RAT): P19.16, P19.17, P19.51

Choose measuring procedure (MEP): P19.78

Evaluate (reasoning or solution...) (EVA): Q19.25 Q19.28, P19.24, P19.71

Make judgment (based on data) (MJU): Q19.12

Multiple possibility and tell all (MPO): P19.13, P19.25, P19.53, P19.54

Jeopardy (JEO): P19.39

Design an experiment (or pose a problem) (DEX): Q19.19, P19.9, Q19.26, P19.60, P19.70, P19.74

Problems based on real data (RED): P19.52

*Brief summary of student difficulties with DC circuits*

The biggest difficulty that students encounter is the concept of a complete circuit. Because we often make an analogy between electric current and water flow, students often think that electric charges making up current can flow in any downward directed wire. Another common difficulty is thinking that when they can make local changes to the circuit (such as removing or adding a resistor) it only affects the branch where the change was made without affecting the rest of the circuit. The next difficulties are thinking that a conventional battery is a source of constant current and the current is being used up as the charge move through a series circuit. Another one is thinking that adding resistors always increases the total resistance of the circuit.

Finally, thinking that Ohm's law  $\left( I = \frac{\Delta V}{R} \right)$  applies to only ohmic resistors.

A difficulty "induced" by instruction is the incorrect idea that the resistance of a circuit element is determined by the slope of the line on the  $I$ -vs- $\Delta V$  graph.

## **I. Electric current, batteries, simple circuits, Ohm's law, and qualitative analysis of circuits**

It is important to start the development of concepts related to electric circuits by building qualitative understanding. Research shows that students who can solve complex problems involving multiple current loops following mathematical procedures based on Kirchhoff's rules, often do not understand circuits conceptually. Thus textbook Sections 19.1–19.2 and 19.5 and ALG Sections 19.1, 19.2 and 19.5 focus on this qualitative understanding.

The first step is to understand that while the potential difference between two locations is necessary for electrically charged particles to relocate, the motion of charges due to an electric field leads to the equalizing of potentials, which stops this movement. ALG Activities 19.1.1 and 19.1.2 and textbook Observational Experiment

Table 19.1 in Section 19.1 help students develop this understanding. These experiments can be done in labs or in class and serve as a bridge between electrostatics and DC circuits. By analyzing the patterns in the experiments, students infer the charge transfer by analyzing the motion of a light metal-coated ball or the light from a neon bulb.

Students' analysis of the experiments should lead them to the conclusion that if there is a potential difference between two points in some region and there are charged particles that can move in this region, this motion of charges leads to a spark, a glow of a neon bulb, or some mechanical motion of a conducting object. However, all of these effects are short-lived as the motion of the charged particles in the electric field leads to the equalizing of potentials.

To help students visualize this process, the textbook makes an analogy of the water flow between two containers, in which water is at different heights, emphasizing that it is not the difference in the amount of liquid in the containers but the pressure difference (the height of the water in the containers) that makes the water flow, due to the gravitational pull of Earth. After the levels of the water in the containers equalize, no more water will flow. If we want the flow to continue, some external agent must lift the water up. This analogy helps students understand the role of a battery in a circuit. Note that this analogy works best if students have thoroughly learned and understood the chapters on fluid statics and dynamics. This is true for any analogy. It is best if the analogies come from students not from you. In this case they really understand the base they are using to understand the new concept.

At this point the textbook and the ALG diverge a little. Although the textbook does not discuss in detail how to light a bulb with a battery and wires until Section 19.3 (Observational Experiment Table 19.2) we recommend that in labs (or in a classroom if you have enough equipment for groups) students learn to light a bulb as soon as possible (ALG Activities 19.1.3 and 19.1.4). They are needed to help students develop a concept of a complete circuit as soon as possible and then slowly refine it as they progress through the chapter. Lightning a bulb with a battery and one wire now became a standard activity, but it is important that students can also do it with two wires. This is the first time that you have an opportunity to discuss the concept of a complete circuit with them. However, in order to be able to do this, students need to have an image of what is inside the lightbulb; how it is constructed. Since breaking the bulb can be dangerous (broken glass), you might need to explain at some point the internal construction of the lightbulb. Ask them not only which circuit configurations worked and what was common to all of them but also what configurations did not work and what was common to all of them. Once the students have a firm understanding of what is needed for a complete circuit, everything else will much easier, the important of this concept is analogous to the importance of force as a measure of interaction in dynamics.

ALG Activity 19.1.4 is extremely important as it encourages students to develop analogies for the elements of electric circuits and their role in the process of creating continuous charge flow through the circuit. Students often don't understand what an

analogy is but they can grasp it quickly with a little bit of scaffolding or guidance. The key advice to help them make progress is to let them know that the *interrelationships* between the elements of their source domain (what they are familiar with) have to match the interrelationships between the elements in the target domain (the electric circuit). For example: what is the relationship between the battery and the electrons? The battery in some way (not yet fully understood) makes the electrons move in an organized way. Connecting wires create a conduit within which the electrons move. The same should apply to your source domain: A pipe can be a conduit for water that is moved by means of a pump.

The second thing to do is to try to steer students *away* from the water pipe analogy. Let only a few groups (usually the stronger ones) work on the water pipe analogy because it is a difficult analogy where they don't have a complete understanding of the source domain. For other groups try to seed ideas or encourage them to think about other life experiences that they have a better understanding of: Here are a few of the creative examples that our students have come up with over the years. These should give you a sense of what is possible: A stream of people (electrons) passing by an ATM where they draw money (the battery). They then walk along a sidewalk to some store where they spend their money to watch some sort of entertainment (the lightbulb) before looping back along another sidewalk to the ATM. Traffic on the highway and a construction project that narrows the road is another popular one. The gas station is the battery in this example. Another good one for biology majors/pre-meds is the cardiovascular system. Here, remember that the blood is the electrons but the battery is a two-stage process of heart and lungs adding oxygen to the blood. The blood then delivers the oxygen to the muscles returning de-oxygenated. A frequent analogy is one in which runners run around a track that has an obstacle in it. A drink stand provides energy to the runners so that they can keep running around and climbing over/or running through the obstacle.

To help students better understand the role of a battery in a circuit you can use experiments and discussions in ALG Activities 19.2.1–19.2.3, which can be done in a lab or in class. In Activity 19.2.2 it is vital that students use their analogy to explain their observations here. This process allows them to modify their analogy to explain batteries in series or parallel and discard their analogy if they find they are unable to explain the connection. Note: An analogy, if it is a good one, doesn't always need to be discarded if it can't explain something. You can also let them know that their analogy is a useful one, but is simply not able to account for multiple batteries and that is okay. They can keep their analogy as long as they understand its limitations. For example, a drink stand as a battery only attracts runners but does not repel them. Note for ALG Activity 19.2.2: It is important to test your equipment before giving it to the students: choose a lightbulb that glows approximately equally brightly when connected to a single battery than when connected to two parallel batteries (the resistance of the glowing bulb should be much larger than the internal resistance of the batteries).

The ideas concerning the role of the battery in the circuit are developed in the textbook Section 19.2. The section introduces the idea of emf as the work per unit charge that a battery does to maintain potential difference between its terminals. Note Conceptual Exercise 19.2 in this section—it introduces students to graphing of potential along the circuit. Students can then work on EOC Problems 1 and 2, and 10.

While, in the textbook Section 19.3, students look for a pattern in the way a wire and battery can be connected to get a flashlight bulb to light and develop the idea of a complete circuit, ideally students should already be familiar with this idea if they did the ALG activities in Section 19.1 discussed above. If they did, they should proceed with investigating the role of the battery more deeply by doing ALG Activities 19.3.1–19.3.4. The main purpose of these ALG activities is to help them deepen their analogical models of electric circuit using two analogies – running water and running people. They need to map circuit elements and their function to the elements and their function in these two analogical models for electric circuit. Following this step, students learn the symbols for various circuit elements and how to use these to measure current and potential difference. In our experience operating electronic multimeters is not an easy task, thus before students move to the activities in the next section they need to be comfortable using these to measure current and potential difference. Useful EOC Question is 15 and Problems are 4–7.

So far, students have learned a lot about circuits, but all of the explorations and reasoning were qualitative. Sections 19.4 in both the textbook and ALG engage them in their first quantitative explorations of circuits. If schedule and equipment allow, we suggest that students start with the ALG Activity 19.4.1 in a lab (before they read the book or discuss the material in class) and find that the ratio of potential difference over current is constant and independent of the current for commercial resistors (when they are used according to their specifications). You can help them through the discussion to invent the idea that the ratio  $I/\Delta V$  is called the physical quantity of conductance and the inverse ratio ( $R = \Delta V/I$ ) is the physical quantity of resistance ( $R = \Delta V/I$  is an operational definition of resistance). Note that later we will write Ohm's law as  $I = (1/R)\Delta V = \Delta V/R$  to underscore the cause-effect relationship: A current  $I$  is the result of a potential difference  $\Delta V$ . It is really important to note here that Ohm's law does not define the resistance in terms of the *slope* of the line (or curve), it defines it as a *ratio* between the potential difference and the current. Although for ohmic resistors the inverse of the slope of the graph line and the ratio of the potential difference and current for any value of the potential difference are the same, they are not the same for non-ohmic resistors. Here a word concerning language is important. Make sure that the grammar you use concerning current and potential difference match their meanings, i.e. we talk about *current through* a circuit element and *potential difference across* this element. We try to avoid the term voltage because it does not communicate the idea of difference and we do not use the term voltage drop because in our definition of voltage it is already a difference between two potentials.

In ALG Activity 19.4.2, students can test whether the lightbulb is an ohmic resistor. Although students only start building microscopic models of resistance in Section 19.10, the variable resistance of the lightbulb is a great opportunity to get students thinking about building those microscopic models. Note that students most likely will not make a connection between the brightness of the lightbulb and the temperature of the filament. They need some explicit prompting to think about it while they are trying to explain why the resistance increases as the bulb gets brighter.

After students become familiar with a commercial resistor and an incandescent lightbulb they will investigate a new and exciting element – an LED. The authors of the textbook E. Etkina and G. Planinsic created a library of activities to use with LEDs (these are published in four articles in *The Physics Teacher*<sup>1</sup>), almost all of those activities are used in the textbook and the ALG. They connect the study of electricity to the most widely used source of light nowadays, as opposed to an incandescent lightbulb whose days are over.

ALG Activity 19.4.4 asks students to light an LED and compare the conditions at which it is glowing bright to a lightbulb. Once students find the differences (the LED glows when the potential difference across it exceeds a certain value and its long leg should be connected to the positive terminal of the battery), they need to come up with causal explanations. Usually students devise two explanations – an LED allows the current pass in both direction but only glows when the current is in particular direction. The second explanation is that the LED works like a one-way switch that allows the current to pass in one direction, which also makes the LED glow, but does not allow the current in opposite direction. Both are testable explanations and Activity 19.4.5 encourages students to test them. One of the testing experiments that students frequently suggest is to connect an LED and a lightbulb in series to the battery with the intention to use the lightbulb as an indicator of current. When they observe that the LED glows but the lightbulb does not, it is the right moment to discuss with them the importance of assumptions (they assumed that any current will make the lightbulb glow). As a result, students will ask for more batteries to increase the current through the LED and the lightbulb, hoping to see the lightbulb glow. You should check before class what the maximum number of batteries in combination with your lightbulb is, that you can use without exceeding the maximum allowed current through your LED and whether this combination makes your lightbulb glow. We succeeded to make a regular 0.3 A lightbulb barely glow using a green LEDs that allows current of up to 70 mA and four 1.5-V batteries.

The next step is to figure out how the current through an LED depends on the potential difference across it (Activity 19.4.6). It is through this activity that the students discover the opening voltage and non-linear  $I$ -vs- $\Delta V$  dependence of the LED. If you use a variable power supply to vary voltage across the LED, it is easy to get all data points including the voltages across the LED that are smaller than the

---

<sup>1</sup> References 18–21 on the list of references in Chapter 1 of the Instructor Guide.

opening voltage. Using a battery and resistors, the measurement becomes more challenging but also offers additional opportunities for learning. Because the effective resistance of the LED increases with decreasing current, measuring points below the opening voltage cannot be done simply by adding resistors in series with the LED. The only approach that works is to build a potential divider. In any case, warn your students that the current through an LED should not exceed the recommended maximum value (about 20-30 mA for most LEDs although some LEDs can stand current of up to 70 mA). If you are using a 3-V source (two 1.5-V batteries) as the power supply, you only need to be careful with the red LEDs (use a 50- $\Omega$  resistor in series with the red LED when connecting it to a 3-V). Most other LEDs (green, blue and white) can be safely connected directly to 3 V.

ALG Activity 19.4.7 tests whether Ohm's law applies to an open switch. If you ask students what the potential difference across an open switch in a circuit is, the most common answer is zero. Students often think that if there is no current in a circuit, the potential difference across any element is zero. They arrive to this answer by applying Ohm's law only to the current and voltage, forgetting the importance of resistance. Do not skip this activity!

Textbook Section 19.4 addresses all of the issues that students tackle in the ALG activities, thus we recommend that the students read it after they perform the experiments and reason about them. Make sure that you discuss the tips in this section with the students; they capture the most difficult issues. Students can then work on EOC Questions 10, 11, 19–25 and 28 and Problems 12–20.

ALG Activities 19.5.1–19.5.6 and textbook Section 19.5 introduce students to simple circuits in which bulbs are connected in series and in parallel. Here the brightness of bulbs is used as an indicator of the relative electric current through the bulbs. There are a few important ideas that students need to understand here: specifically, that the battery is not a source of constant current but is a source of almost constant potential difference, and that current is not used up in series circuits but is the same through each element. Note that students tend to reason about electric circuits using current as the fundamental concept, although it is more productive to base the analysis on potential difference. Analysis of the circuits leads to a list of qualitative rules based on the experiments (see the list on page 588 in the textbook). In Conceptual Exercise 19.4 students are introduced to a new concept – the storage capacity of charge in the battery. Do not skip it as it relates lots of previous activities to students' interests. Note that the term is somehow misleading since no charge is stored in the battery, the quantity tells the total charge that the battery can move before it becomes "dead". You can use textbook EOC Questions 1–9 to help students further develop qualitative understanding of electric circuits. Alternatively, students can conduct lab experiments similar to the ALG activities.

## II. Joule's law, Kirchhoff's rules, series and parallel circuits with resistors, capacitors in DC circuits, and skills for quantitatively solving circuit problems

Sections 19.6–19.9 in both the ALG and the textbook help students construct a quantitative understanding of electric circuits. In Section 19.6, they learn how to derive an expression for the rate of electric energy conversion (the electric power) of a circuit element. To start thinking about power, we suggest that the students observe or perform the experiment described in the ALG Activity 19.6.1 and at the beginning of textbook Section 19.6, in which two *different* lightbulbs are connected in series. Given that so far students have only had identical lightbulbs in all experiments, they know that identical bulbs in series have the same brightness, which they explained as being the result of the same current through the bulbs. Now students observe that the bulbs in series have different brightnesses despite the fact that the current through them is the same. This finding suggests the question of what really affects the brightness of the bulb. ALG Activity 19.6.2 helps students derive the expression for power, and ALG Activity 19.6.3 is the testing experiment (it in a way repeats the experiment in Activity 19.6.1, so if you feel that students are fluent with the power relation, skip it). After students construct the concept of power, the most interesting question arises: why did LEDs replace incandescent lightbulbs in our homes? Depending on the equipment, students can design experiments in ALG Activity 19.6.4 of different levels of rigor to answer this question, but the bottom line is that LEDs are about 10 times more efficient! As this experiment is described in detail with the relevant data collected in the textbook, we suggest that the students first do the lab and then read the textbook.

Two more notes about the equipment used in the Activity 19.6.4 are important:

(1) Most commercially available LEDs have an epoxy drop lens on the top. Such LEDs emit light in a cone and not as a point source that is crucial for Activity 19.6.4. You can either order flat-top white LEDs or carefully cut off the spherical lens from a regular white LED using a hacksaw. After you cut the lens, remove any scratches in the sawed surface by brushing the surface with fine sandpaper (grade 600 or higher) and finally polish the surface with white toothpaste until the surface looks perfectly transparent.

(2) Use only fresh batteries in this activity. Note that even a smallest variation in emf of the battery can result in a huge variation in brightness of an LED (remember the  $I$ -vs- $\Delta V$  dependence of LEDs). If you are using rechargeable batteries, make sure you obtain those with 1.5-V emf (if you use two 1.2-V rechargeable batteries your white LEDs may barely glow). If you decide to use a variable power source instead of the batteries, the same warnings and suggestions mentioned earlier in connection with Activity 19.4.6 apply.



Notice ALG Activity 19.6.5 – it is a lab that connects their knowledge of DC circuits to the knowledge of thermodynamics – a really nice capstone activity! Students can then work on EOC Problems 21–26 and 72.

Sections 19.7 in both the ALG and the textbook are dedicated to Kirchhoff's loop and junction rules. This is also the section where students first encounter the concept of the internal resistance of a battery (page 924). Students devise the rules by analyzing simple circuits (ALG Activities 19.7.1–19.7.4) and apply those in Activities 19.7.5–19.7.9. Some textbooks introduce Kirchhoff's rules *after* students learn how to add resistors in series and parallel. We feel that it makes more sense for students to explore Kirchhoff's rules *before* adding resistors in series and parallel because deriving the mathematical expressions requires Kirchhoff's rules. Note the summary of the sign conventions for using Kirchhoff's rules on page 595, before the introduction of the junction rule.

It is important when using Kirchhoff's rules to indicate with arrows the anticipated directions of the current in each branch of a circuit. The signs of potential changes across resistive elements when using the loop rule depend on the way the loop is being traversed relative to the anticipated direction of the current. If the current direction is chosen incorrectly but the potential changes across resistive elements is included correctly in the loop rule, an error in current direction choice will just yield a negative sign when Kirchhoff's rules are solved for the current.

The textbook uses Kirchhoff's rules to derive the expressions for the equivalent resistance of series and parallel resistive parts of circuits (Section 19.8). Note that Example 19.7 returns to the power output of the resistors in series and parallel circuits but is done quantitatively. ALG Activities 19.8.1 and 19.8.2 lead students through the derivations of resistors in series and parallel and the follow up Activities 19.8.3–19.8.11 let them apply and practice the rules. Note EOC Questions 13 and 14 and Problems 27–39. Be sure that students add current direction arrows to their circuit diagrams and use the sign conventions correctly.

The next part of the respective sections is dedicated to capacitors in DC circuits. We suggest (if possible) that students start in a lab or class with Activities 19.8.12 and 19.8.13 to get first hand experience with charging and discharging a capacitor in a DC circuit and then read the relevant part of the textbook section. EOC Question 30 and Problems 40 and 41 will be very useful here. Notice that we do not derive expressions for the equivalent capacitance of capacitors in series or parallel.

A set of activities in the ALG Section 19.9 let students practice problem-solving skills and how to adapt the general problem-solving strategy to electric circuit problems. All problems in EOC Section 19.9 are appropriate in addition to the ALG activities, but we especially recommend Problems 49–52.

Note that one of most common difficulties that students have with electric circuits is approaching the problems locally. They look only at the change that occurs when a resistor is added to the circuit at the location of that resistor. Understanding that everything in the circuit changes, including the currents, is very important. Therefore, it is very useful to discuss with your students the local and global changes that occur

when an element is added or removed from the circuit. This discussion can occur when students work on any problem involving a complex circuit. Notice the “chair” ALG Activity 19.9.8 that helps students learn how to simplify complex circuits by removing the branches across which the electric potential does not change (or how to simplify circuits that contain nodes with equal potential). ALG Activity 19.9.8 will challenge your most advanced students. It requires that they combine several conceptual areas (thermodynamics and electricity) and several problem-solving strategies (linearization and interpretation of non-standard graphs).

### **III. Properties of resistors, superconductivity, and semiconductors**

Section 19.10 concerns resistive elements, including the effect of the shape and material on the resistance of a normal resistor, the microscopic nature of resistance, the history of superconductivity, and a discussion of electric current in semiconductors and its temperature dependence. We recommend starting with the ALG Activities 19.10.1–19.10.4, then reading the textbook sections and then returning to the ALG activities (especially Activity 19.10.12). EOC Question 12 and Problems 57, 62 and 63 will be very useful here, as well as General Problems 73 and 81.

Note the new problems and questions in the textbook related to the LEDs (Question 13 and Problems 18–20, 74 and 77). We also have non-traditional challenging problems in this section that will allow your students to practice putting together all of the ideas they learned in this chapter. The General Problems that we recommend are: 64, 66, 69, and 74–78. The reading passages and related problems will help your students connect what they learned in this chapter to electric processes in a human body.

# Magnetism

In Chapter 17, students learned about the action-at-a-distance electrostatic interaction between charged objects considered as point-like particles. In Chapter 18, they learned how to use a field approach to describe this interaction. In this chapter, students learn how moving charged particles produce magnetic fields and how those fields exert forces on other moving charged particles. Content-based learning objectives of this chapter are listed below.

**Students should be able to:**

1. Describe the sources of magnetic fields. Explain how magnetic fields are created. “Read and write” with  $\vec{B}$  field line representations.
2. Explain how to use a compass to determine the direction and relative magnitude of the  $\vec{B}$  field at a particular location.
3. Determine the directions of  $\vec{B}$  field vectors when the magnetic field is created by a bar magnet, horseshoe magnet, and by a current-carrying wire, loop and a solenoid.
4. Apply the right-hand rule for the fields and the right-hand rule for forces to analyze situations involving magnetic fields when magnetic fields are created by current-carrying wires.
5. Compare and contrast electric fields and magnetic fields.
6. Determine the magnitude of a magnetic force exerted on a current-carrying wire or a moving charged particle in uniform magnetic field.
7. Apply knowledge of magnetic forces, electric forces and Newton’s laws to solve complex problems. Use force diagrams to analyze situations.
8. Explain how an electric motor works using knowledge of torques.
9. Describe how knowledge of magnetic fields applies to real-life and biological phenomena. Describe and explain quantitatively the differences between dia-, para- and ferro- magnetic materials.
10. Design an experiment to determine the magnitude and direction of the  $\vec{B}$  field produced by a current-carrying wire, current-carrying solenoid and by an unmarked magnet.

The chapter is broken into five parts:

- I. *A qualitative description of magnetic interactions and magnetic fields*
- II. *The magnetic force exerted by a field on a current-carrying wire and on a moving charged particle*
- III. *A quantitative rule for magnetic fields created by electric currents*
- IV. *Skills for analyzing magnetic processes, including important applications involving magnetic and electric fields*
- V. *Properties of magnetic materials*

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
A qualitative description of magnetic interactions and magnetic fields	20.1, 20.2	OET 20.1 (p619)	
The magnetic force exerted by a field on a current-carrying wire and on a moving charged particle	20.3, 20.4	OET 20.2 (p621), TET 20.3 (p623), 20.1 (p628)	20.4.1
A quantitative rule for magnetic fields created by electric currents	20.5		
Skills for analyzing magnetic processes, including important applications involving magnetic and electric fields	20.6		
Properties of magnetic materials	20.7	20.2 (p639)	20.7.2
<p align="center"><b>Nontraditional end-of-chapter questions and problems</b></p> <p>Evaluate (reasoning or solution...) (EVA): P20.3, P20.6,  Multiple possibility and tell all (MPO): Q20.13, Q20.15, Q20.16, Q20.20, P20.1, P20.13, P20.27, P20.31, P20.36  Jeopardy (JEO): P20.12, P20.33, P20.34, P20.35  Design an experiment (or pose a problem) (DEX): Q20.14, Q20.12, Q20.15, Q20.16, P20.5, P20.25, P20.44  Problems based on real data (RED): P20.18, P20.32</p>			

For each part, we describe the logical sequence of the creating of new ideas and provide examples of activities that you can use in the labs, lectures, and problem-solving recitations. We also discuss student difficulties and motivation for both the Active Learning Guide and textbook activities.

### *Brief summary of student difficulties with magnetic fields*

The biggest difficulty comes from the three-dimensional nature of magnetic phenomena—students need to use their hands to determine the direction of the  $\vec{B}$  field and of the magnetic force. Students often forget that a magnetic field exerts a force only on moving charged particles but not on the stationary particles, and only if the particles are moving in a particular direction. Students get confused when to apply each of the right-hand rules. Some students think that a solenoid produces a magnetic field even when there is no current through the windings. Students often think that the force exerted by one pole of a bar magnet on a diamagnetic or paramagnetic material will change the direction if the poles are swapped (the magnet is turned around).

## **I. A qualitative description of magnetic interactions and magnetic fields**

Textbook Section 20.1 and ALG Activities 20.1.1–20.1.3 introduce students to the phenomenon of magnetism. ALG Activity 20.1.1 helps students characterize the interaction of a compass needle with permanent bar magnets. It builds the first step to the representation of a magnetic field in space with the arrow pointing from S to N inside the compass (without naming the magnetic field). If you want to devote laboratory or recitation time to introducing magnetism, students should start with the ALG activities while using the textbook as summary reading.

Textbook Section 20.2 introduces students to the concept of magnetic field, the physical quantity of the  $\vec{B}$  field, and the compass needle as an indicator of the magnetic field in a region. Unlike an electric field, whose vector characteristic preceded the concept of electric field lines,  $\vec{B}$  field lines come up before any quantities. We use multiple compass needles to help students explore the direction of magnetic field (using the orientation of the needle) and the magnitude (using the period of oscillation of the needle placed in a particular location). Use ALG Activities 20.2.2 for the former and 20.2.3 for the latter. In Activity 20.2.3 students learn that the farther from the magnet, the weaker the magnetic field.

A fun and useful activity that can be done in recitation or a studio classroom is to have groups of students use compasses to map  $\vec{B}$  field lines of different configurations of permanent magnets (such as dipole, quadrupole, and horseshoe magnets) on whiteboards and present their results to the rest of the class. Make sure the magnets are strong because students can be confused by the additional interaction of Earth's magnetic field. (Watch for conduit pipes under tables, too!). If you want to

encourage students to explore magnetism, we recommend cow magnets as the most economical, safe, and student-proof magnets we have encountered.

ALG Activity 20.2.4 addresses a common student difficulty when they confuse magnetic poles with oppositely charged objects. It is a variation of a similar experiment that they did in Chapter 17. They again find that charged objects of both signs attract both poles of the magnet due to the fact that a magnet (it does not matter whether it is a metal magnet or a ceramic magnet) has charged particles inside it that lead to the electric polarization of the magnet so that the side closer to the charged object becomes charged oppositely and thus attracts. This interaction is different from the interaction of two magnets where a pole of a magnet attracts one pole of another magnet but repels the other.

The next step is to establish that current-carrying wires produce a magnetic effect on a compass similar to that of a permanent magnet. ALG Activity 20.2.4 mirrors the historical experiment done by Oersted and simultaneously helps student construct the first right-hand rule for the direction of magnetic field created by a known source (a right-hand rule for determining the direction of the  $\vec{B}$  field lines produced by an electric current in a wire). Students observe the effect that the current-carrying wire exerts on the orientation of the compass needle and find patterns in that orientation. If you are starting this topic in a lab and have enough equipment, it is best if students perform this experiment in groups. If this is not possible and you are doing this experiment in a lecture setting, students can watch you perform the experiment while they look for patterns. Orient the wire without current along the natural orientation of the compass (aligned with Earth's magnetic field), so that when you turn on the current, students can observe the biggest deflection. After observing and discussing the experiment, students can read about the experiments in Observational Experiment Table 20.1. ALG Activity 20.2.5 helps students visualize the “circles” for the field lines using a wire and iron filings. If you want to map the field around a wire with compasses, it is best to conduct a carefully prepared lecture demonstration. A current of at least 15–20 A is needed to swamp the effect of Earth's magnetic field. The simplest way to produce currents of these magnitudes is to use a car battery. Make sure you do not keep the current on for more than about 10 or 15 s.

ALG Activity 20.2.7 helps students practice the newly established right-hand rule for the fields by applying it to determine the direction of the  $\vec{B}$  field created by currents through a wire, a wire loop or a solenoid. Textbook Conceptual Exercise 20.1 only uses a solenoid but shows students how to draw the  $\vec{B}$  field lines for the current-carrying solenoid. We suggest that students first attempt the ALG activity and then work through the textbook exercise. The  $\vec{B}$  field lines produced by a bar magnet are found to be very similar to the field lines produced by the current in a coil of wire—a useful idea needed later in the chapter. You can use EOC Questions 1, 4, 5, 12, 13, 14 and Problems 1–6 for practice and formative assessment.

## II. The magnetic force exerted by a field on a current-carrying wire and on a moving charged particle

After students learn that current-carrying wires create magnetic fields around them with field lines similar to those for permanent magnets, the next step is to investigate how magnetic fields affect the current-carrying wires and individual moving electrically charged particles. This investigation happens in two parts. Students develop a qualitative right-hand rule for the force and then develop quantitative expressions for the force exerted by the magnetic field on a current-carrying wire ( $F_{\vec{B} \text{ on } W} \propto IL \sin \theta$ ).

The textbook's Observational Experiment Table 20.2 and ALG Activity 20.3.1 provide data and observational experiments that students can use to construct a rule relating the direction of the force to the direction of the current/moving charges and the direction of the  $\vec{B}$  field. Alternately, you can bring an old CRT oscilloscope to class and have students observe how the beam is deflected by the presence of a magnetic field created by a permanent magnet.

It is important to remember that students may develop a very different hand rule than the one used in the textbook. Offer them the textbook right-hand rule for the magnetic force at the end of class discussion with the justification that communication will be easier if just one hand rule is used, and then stick to it.

Note the testing experiment for the direction of the magnetic force exerted on a current-carrying wire in the ALG Activity 20.3.2. It is not only encouraging students to use the newly constructed rule for the force to make predictions about the outcome of the experiment, but also makes them review Newton's laws and what a scale measures. If you do not have this apparatus you can conduct the testing experiment using a traditional "jumping wire" set up for which you need a long wire, a DC power supply (several A current), and a horseshoe magnet. Give students enough time to struggle through their first attempt to apply the right-hand rule and make predictions before they actually perform the experiment. Testing Experiment Table 20.3 is very effective if you turn it into a large, spectacular lecture demonstration. Give students enough time to make their predictions before the experiment is performed.

ALG Activity 20.3.9 helps students practice the right-hand rule for the force, and Activity 20.3.10 makes them combine both rules to answer a question concerning how one current-carrying wire exerts a force on the neighboring current-carrying wire. Students struggle to distinguish between the source of the field and the test object that is interacting with the field. This is likely because the distinction is a choice of perspective. Whatever the reason, some students become fixated on the idea that either one magnetic field exerts a force on another magnetic field or one wire exerts a force on the other, completely ignoring the idea that the magnetic field is the intermediary. Activity 20.3.10 targets all these difficulties. Students find this activity surprisingly challenging, and it is worth devoting considerable time to having them work through it. You will probably need to keep reminding them to draw the field lines produced by the source and *then* focus on the direction of the field line *at the specific point* where the (test) wire is placed.

ALG Activities 20.3.5–20.3.7 serve as the starting point for later discussions about the torque exerted by a magnetic field on a loop of wire with a current through it. This is the second right-hand rule that students develop in this chapter, and often they confuse the two. One way to help is that every time they use a rule, explicitly ask whether they are interested in the field created by a known source (right-hand rule for the field) or in the force that the field whose source is unknown exerts on the test object – a moving charge or a current-carrying wire (right-hand rule for the force). Finally, ALG Activity 20.3.13 is an excellent activity for formative assessment. You can also use here textbook EOC Questions 6 and 17.

The second part of Section 20.3 helps develop a quantitative expression for the force that a magnetic field exerts on a current-carrying wire. It is important to note here that we have not established an operational definition for the magnitude of the  $\vec{B}$  field vector; yet this procedure will appear as the result of the analysis described below. Textbook Table 20.4 provides data for an imaginary experiment in which a current-carrying wire is placed in a uniform magnetic field produced by an electromagnet that does not change during the experiment. Students analyze the data to arrive at the conclusion that the magnitude of the force exerted on the current-carrying wire is directly proportional to the magnitude of the current, the length of wire, and the sine of the angle between the direction of the current and the direction of the  $\vec{B}$  field vector  $F_{\vec{B} \text{ on } W} \propto IL \sin \theta$ . In other words, students find that

$$\frac{F_{\vec{B} \text{ on } W}}{IL \sin \theta} = \text{constant}$$

This coefficient of proportionality is independent of the properties of the wire but depends on the strength of the magnet is used to define the magnitude of the  $\vec{B}$  field vector.

Note that the approach in the ALG is different. You can either use Activity 20.3.3 for the students to explore what affects the magnetic force and then have a discussion how the expression for the force can serve as an operational definition for the  $\vec{B}$  field or use Activity 20.3.4 that provides students with the data to construct the relationship



$F_{\vec{B} \text{ on } W} = IL \sin \theta$  directly, without an intermediate step as the textbook provides. You can choose either approach here.

If you start with ALG Activity 20.3.3 a few notes are important: If you get students to do the experiment in a lab, the best force-measuring device is a digital scale that measures to 0.01 g. Using the scale to determine the force that the magnet exerts on the wire is a great opportunity for students to go back to drawing force diagrams and applying Newton's third and second laws. Students might make the wrong prediction the first time around if they have not fully realized that the right-hand rule gives the direction of the force exerted by the magnet on the wire, not the force exerted by the wire on the magnet. This is a mistake that many can figure out on their own if you give them enough time. If you take this route and have students quantitatively investigate the force exerted by the field on the wire in lab, this apparatus does not really allow them to investigate the role of angle. When they are done with this experiment and present their results, we suggest having a short lecture, drawing the various aspects together into one equation and introduce the idea that angle matters: There is a "component" of  $I$  parallel to the field and another "component" perpendicular to the field.

After establishing the expression for the force, the textbook shows how to apply the expression to solve one standard problem and then uses it to develop an expression for the torque that a magnetic field exerts on a current loop. Students can start with ALG Activities 20.3.5 and 20.3.6 (if you have an old analogue ammeter, they can reverse-engineer it before doing the activity). Then you can apply the same ideas to explain how a simple motor works (textbook text on pages 626–627) and then hold a whole class discussion similar to the text on page 627 concerning the mathematical expression for the torque that magnetic field exerts on a current carrying loop. Students can practice the force ideas by working on textbook EOC Problems 7–12, 14, 15, 17, and 18 (which is also a student project or a lab experiment). If you have time, a great application experiment lab is to give students a neodymium magnet, a kebab skewer (for the shaft), a shoe box (for mounting the motor), tape, paper clips (for brushes), and solenoid wire and have them construct a working DC motor on their own using what they've learned. Note that this section in the textbook ends with magnetic dipole moment for which we do not have parallel ALG activities. It is important to introduce this quantity to the students, as it will be needed later, especially in atomic physics.

The next step is to develop the relationship for the force exerted on a moving charged particle. If you have the necessary equipment, students can start by performing ALG Activity 20.4.1 to see that the right-hand rule for the magnetic force needs to be adjusted when the moving particles have a negative sign. If you do not have the equipment, the link to the video is embedded in the activity. Following ALG Activities 20.4.2–20.4.4 let students practice newly developed rule in multiple contexts before proceeding to the discovery of the mathematical relation for the force exerted on a single charged particle. ALG Activity 20.4.5 to deduce the quantitative relation for the force using pretend data. Section 20.4 describes

experiments similar to those in ALG Activity 20.4.1 (with the same video) and provides a theoretical derivation for the magnitude of the force that the magnetic field exerts on a single moving charged particle starting with the force that the field exerts on moving charges in a wire. This leads to an expression for the magnitude and direction of the magnetic force that the field exerts on a moving charged object, considered as a point-like object.

The book then proceeds to study the circular motion of charged particles in a magnetic field and specifically to the deflection of cosmic rays by Earth's magnetic field (Example 20.4 and ALG Activity 20.4.8) and to the understanding of aurorae. Both the Example and the ALG activity are very difficult for students. They don't have enough knowledge to understand the full implications of the spiraling effect. If they can get to the point where they understand that the charged object is directed to the poles, we suggest finishing up with a brief lecture explaining the fact that the extra density of atmospheric air and concentration of charged particles results in visible light.

However, before the students even attempt ALG Activity 20.4.8 we have two preparatory activities that start students off with a simpler situation of a uniform magnetic field and ask them what sort of trajectory a charged particle will take if it approaches that field in a direction other than 90 degrees to the field. Students might have difficulty putting together circular motion in the  $x$ - and  $y$ -directions, with uniform linear motion in the  $z$ -direction to get a spiral or helix (ALG Activities 20.4.6 and 20.4.7). Once they can grasp this, ALG 20.4.8 and textbook Example 20.4 will be much easier for them to understand. You can use textbook EOC Questions 8, 10, 18, 21 and 27, and Problems 20, and 23–27.

It is worth noting here that unlike an electric field that has only one test object (a detector), namely a positively charged object, the magnetic field, can have four different test objects—a compass (a qualitative detector), a current-carrying straight wire, a current-carrying loop or an individual moving charged particle. Unlike the single operational definition for  $\vec{E}$  field as  $\vec{E} = \vec{F}_{\text{source on test}} / q_{\text{test}}$ , which defines both the direction and the magnitude, the direction of the  $\vec{B}$  field in this book is determined through either the right-hand rule for the fields (directly) or indirectly using the right-hand rule for the force if the force is known (the book does not use the vector product). The magnitude can be determined through several operational definitions:

$$B = \frac{F_{\vec{B} \text{ on } W \text{ max}}}{IL} \quad \text{or} \quad B = \frac{|\tau_{\vec{B} \text{ on coil max}}|}{NAI} \quad \text{or} \quad B = \frac{F_{\vec{B} \text{ on } q \text{ max}}}{qv} .$$

### III. A quantitative rule for magnetic fields created by electric currents

Observational Experiment Table 20.5 in Section 20.5 helps students develop an expression for the magnitude of the  $\vec{B}$  field created by a long, straight wire. The proportionality constant in that equation involves the magnetic permeability of the space (vacuum) surrounding the location of the magnetic field. The textbook then adapts the expression for the magnitude of the  $\vec{B}$  field caused by a long wire to other current situations and in particular to estimate the magnitude of the  $\vec{B}$  field due to the circular electron current in a hydrogen atom (13 T!). ALG Activity 20.5.2 can be used in a lab as a testing experiment for Equation 20.6. Students can then work on EOC Problems 29–32.

### IV. Skills for analyzing magnetic processes including important applications involving magnetic and electric fields

There are two main types of problems in this chapter: problems that involve finding a force exerted on a current carrying wire or on a moving charged particle by magnetic field the source of which is unknown (type 1) and problems involving figuring out the magnetic field created by known objects. In other words, the first type of problem deals with the magnetic force detectors (test objects) and the second type deals with the sources of magnetic field (type 2).

ALG Activity 20.6.1 asks students to apply the problem-solving strategy to solve a typical magnetic force problem (type 1). This is the problem solved in Example 20.6 in Section 20.6 in the textbook. Follow up Activities of increasing difficulty are 20.6.2, 20.6.3, 20.6.6, 20.6.7, 20.6.8. EOC Problems are 33, 35, 43.

Textbook Example 20.7 is a problem that involves using the strategy to determine an unknown magnetic field (Type 2), as well as ALG Activities 20.6.4 and 20.4.11. Students can then work on EOC Problems 34, 36, 37–39.

We also use the problem-solving strategy to explain important applications such as the mass spectrometer and intensity modulated radiation therapy (IMRT). Many of these involve a charged particle (ion) that moves through a perpendicular magnetic field. The field deflects positive ions to one side of the vessel through which the charged particles are moving and negative ions to the other side. These opposite charges on opposite walls cause an electric field that causes a balancing electric force—the ions now move straight ahead (Quantitative Exercise 20.9). Use EOC Problems 41–42, and all General problems.

## V. Properties of magnetic materials

Section 20.7 is dedicated to the properties of different types of magnetic materials. In the experiments described on textbook page 639, Figures 20.30–20.32 are the observational experiments that lead students to three different types of materials that behave differently in an external magnetic field—so-called diamagnetic materials, paramagnetic materials, and ferromagnetic materials. Each type of material is described at a microscopic level. Please read the text in this section carefully before assigning it to the students as it discusses a common misunderstanding of why a bar magnet exerts a force on a diamagnetic or paramagnetic materials. Students should perform ALG Activities 20.7.2 and 20.7.3 after class discussion of magnetic properties or after they read the textbook (ALG Activity 20.7.1). Then students can proceed to EOC Questions 9 and 26.

# 21

## Electromagnetic Induction

In Chapter 20, students learned that an electric current produces a magnetic field. This discovery in the 1800s led physicists to think that a magnetic field should be able to produce an electric current. In this chapter, students reproduce historical experiments and test ideas that led to the discovery that magnetic fields can indeed produce electric currents. They also learn about AC circuits. Content-based learning goals of the chapter are listed below.

### **Students should be able to:**

1. Design an experiment to create current in a coil that is not connected to a battery using two different methods.
2. Apply the concept of magnetic flux to solve problems.
3. Give examples of the situations where Lenz's law applies. Apply Lenz's law to solve problems.
4. Apply Faraday's law to explain real-life situations and solve problems.
5. Describe experiments that could have led to Faraday's law.
6. Explain how an electric generator works.
7. Describe applications of electromagnetic induction to real-life and biological phenomena.
8. Compare and contrast the behavior of resistors, capacitors, and inductors in AC circuits.
9. Describe the resistances of different circuit elements in AC circuits quantitatively.
10. Explain what rms current and voltage are.

The chapter is broken into four parts, starting with qualitative analysis and ending with more quantitative applications:

- I. *Observe patterns in experiments to qualitatively develop the idea of electromagnetic induction and relate induction to changing magnetic flux*
- II. *Develop Lenz's law and Faraday's law*
- III. *Adapt the general problem-solving strategy to problems involving electromagnetic induction and apply the rules developed to analyze some important applications of electromagnetic induction*
- IV. *Investigate AC circuits and transformers*
- V. *Develop a deeper explanation for electromagnetic induction and see how it fits into the electricity and magnetism ideas developed in Chapters 17–20*

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Observe patterns in experiments to qualitatively develop the idea of electromagnetic induction and relate induction to changing magnetic flux	21.1, 21.2	OET 21.1 (p650)  TET 21.2 (p652)	21.1.1 21.1.2
Develop Lenz's law and Faraday's law	21.3, 21.4	OET 21.3 (p659)	
Adapt the general problem-solving strategy to problems involving electromagnetic induction and apply the rules developed to analyze some important applications of electromagnetic induction	21.5		21.5.14
Investigate AC circuits and transformers	21.6, 21.7	21.1 (p670) 21.2 (p672)	
Develop a deeper explanation for electromagnetic induction and see how it fits into the electricity and magnetism ideas developed in Chapters 14–17	21.8		
<b>Nontraditional end-of-chapter questions and problems</b>			
Ranking tasks (RAT): Q21.10			
Evaluate (reasoning or solution...) (EVA): Q21.4, Q21.18, P21.37, P21.38			
Make judgment (MJU): P21.15			

Multiple possibility and tell all (MPO): Q21.17, Q21.22, P21.25

Jeopardy (JEO): P21.31, P21.32, P21.33

Design an experiment (or pose a problem) (DEX): Q21.21, P21.1, P21.6, P21.29, P21.57, P21.59, P21.60, P21.62

*Brief summary of student difficulties with electromagnetic induction and AC circuits*

Students sometimes think that induced current is the result of attraction/repulsion between the magnet and electrons in a wire that makes the coil. Students have huge difficulties using the concept of magnetic flux. They confuse the flux with the rate of change (due to the term flux). When the field is non-homogenous (as for bar magnets) they forget that the field inside the chosen area can have field lines in both directions, thus canceling the flux. When they see that changing the magnetic field in one coil does not produce an induced current in a perpendicular coil, they explain this as the subtraction (cancellation) of the magnetic fields of both coils.

## **I. Observe patterns in experiments to qualitatively develop the idea of electromagnetic induction and relate the phenomenon of electromagnetic induction to changing magnetic flux**

The opening passage of Chapter 21 briefly describes transcranial magnetic stimulation (TMS), a method that helps treating certain diseases by inducing an electric current in a person's brain using electric current in a small coil placed on the scalp—completely noninvasive. How does this method work? To answer this question, students might start in class by observing and discussing experiments such as the textbook's Observational Experiment Table 21.1 (ALG Activity 21.1.3) or by doing ALG Activity 21.1.1 in a lab. The goal of these activities is to devise a rule describing when a current can be induced in a circuit that does not have a battery. However, ALG Activity 21.1.1 is much more open-ended as it is turned into a design lab. Your students receive a magnet and a coil connected to a galvanometer and need to figure out how to make the current in the coil without any battery, just using a magnet. Do not give them coils with lots of turns or very strong magnets so that it takes them a while to figure out what to do. Eventually they will find out that moving the magnet perpendicularly, in and out of the coil creates a current in the coil. You cannot overestimate the joy that students experience when this happens!

Alternatively, if you think that your students are not ready for such open-ended exercises, you can proceed either with the ALG Activity 21.1.3 or textbook activities in which students look for patterns in a variety of experiments involving a bar

magnet, a coil, and a galvanometer. What seems to be necessary in order to create a current in the coil? Most students immediately come up with this rule: when the magnet moves with respect to the coil, there is a current induced in the coil. They can test this rule in ALG Activities 21.1.2 and then 21.1.4.

It is important that when students make predictions, they actually use the rule they are testing. Sometimes students come up with another rule: the changing number of magnetic field lines through the coil leads to the current. Testing Experiment Table 21.2 provides experiments to test both rules. We suggest that you let your students make predictions by working together before observing the testing experiments and then reason about both rules. They can go back to the transcranial stimulation and realize that the first rule is inconsistent with the TMS method. As a result of multiple experiments, the second rule should be the one to survive. Students can use it to answer Conceptual Exercise 21.1. The textbook shows how to use this rule to explain how a dynamic microphone and a seismometer work. Students can work EOC Questions 1, 2, 12 and 18, and problems 1–4 and 7.

Notice ALG Activity 21.1.5. It focuses on the microscopic explanation of electromagnetic induction that is addressed in the textbook discussion on page 651. We suggest the activity as optional for those students who are thinking more deeply about induction and quickly arrive at the microscopic explanation that there is a force exerted on free charges in a wire moving relative to a magnetic field. If that comes up naturally in discussions, we suggest that you make time to assign this activity to them so that they can see that their idea (a) has a lot of merit and that the idea (b) does not explain all the phenomena of induction, in particular the cases where the magnetic field is turned on and off, because there is no relative motion between the coil and the magnet.

After students devise a qualitative rule, the next step is to find what determines the magnitude of the induced current. You can start with ALG Activity 21.2.1. For the experiment in this activity to work successfully, you need to have at least a couple of coils whose diameter is less than the diameter of your bar magnet. One recommendation is: have 2 coils with 100 and 200 turns, smaller than the diameter of the bar magnet and another pair of coils (100 and 200 turns) whose diameter exactly matches the diameter of the magnet, and another pair whose diameter is somewhat larger than the diameter of the magnet.

An alternative to Activity 21.2.1 that involves equipment is ALG Activity 21.2.2 or textbook Section 21.2, which describes the experiments indicating that three physical quantities that change are important for the number of  $\vec{B}$  field lines, whose change is responsible for the induced current: the magnitude of the  $\vec{B}$  field, the coil area  $A$ , and the orientation  $\theta$  of the coil relative to the  $\vec{B}$  field. Those quantities lead to a definition of magnetic flux  $\Phi$ .

Students find magnetic flux a very difficult concept. ALG Activity 21.2.3 provides a bridging analogy for flux. We are helping them to make an analogy to a more familiar example such as collecting rain in a rectangular box. In this example,



the relative orientation between the box and the rain affects how much rain is collected. Likewise, students can think of the loop as “collecting” field lines. To correctly account for how many  $\vec{B}$  field lines pass through the loop, we need to consider both the area of the loop *and* its orientation relative to those field lines. Once they understand the idea of flux at this simplified level, the rest of the chapter will be much easier: do not skip this activity! ALG Activities 21.2.4–21.2.6 help students practice the definition.

It is important to help students understand that flux itself does not cause an induced electric current—a *change* in flux does. The more quickly the flux changes, the greater the induced current. The magnitude of the induced current also depends on the number of turns in a coil (alternatively each turn can be seen as an additional surface area). Note that the term flux is confusing to the students—in everyday life, the word flux means change. Therefore a discussion of the “physics” meaning of the word flux is crucial here. EOC Question 3 and problems 9–11 can be used here.

## II. Develop Lenz’s law and Faraday’s law

The next step in students’ investigation of electromagnetic induction is to develop a rule for the direction of the induced current. They can do so directly by experimenting with the coil connected to a galvanometer and a magnet, but we find that the experiment is too complicated for the students to keep track of all variables. They can work with the material in textbook Section 21.3, Figure 21.8. Alternatively, ALG Activity 21.3.1 is clean enough for the students to see the pattern.

Students find that the direction of  $\vec{B}_{\text{ind}}$  is such that it always opposes the changes in the external flux—Lenz’s law. This is a difficult concept for students to develop, especially for cases where the magnitude of the  $\vec{B}_{\text{ext}}$  external field is decreasing, thus leading to  $\vec{B}_{\text{ind}}$  in the same direction. Our best advice is to give them a lot of practice. The textbook Physics Tool Box 21.1, Determining the Direction of an Induced Current, should receive extra attention if you want students to be able to apply Lenz’s law. Students can then work with ALG Activity 21.3.2 or with Conceptual Exercise 21.3 and then ALG Activity 21.3.3. Note eddy currents are introduced at the end of Section 21.3; this topic is a very useful application of Lenz’s law. We use it to explain the eddy current waste separator and braking systems for vehicles. You can use EOC Questions 5–8, and 19 and Problems 12–14 to help students practice applying Lenz’s law.

After students have investigated the phenomenon of electromagnetic induction qualitatively, it is time for the quantitative aspect. Observational Experiment Table 21.3 in Section 21.4 provides students with the data that allow them to connect that the magnitude of the induced current depends on the time interval that the external magnetic flux changes (the shorter the time interval, the greater the induced current).

Quantitative data provided in Figure 21.12 and the supporting discussion analyze the phenomenon quantitatively using measurable physical quantities. ALG Activity 21.4.1 achieves the same purpose and helps solidify this connection. However, those observations are not enough to write Faraday's law. Moreover, Faraday's law does not have current; it is formulated in terms of emf. It is important to discuss this issue. Students often talk about potential difference or voltage when discussing Faraday's law. This vernacular is incorrect. Changing flux induces a different electric field (with the lines that are closed loops) that causes an induced current in the coil (students encounter this at the end of the chapter). This field can do nonzero work moving electric charges inside the coil along a closed loop (we know that this is true because the loop gets warm). This means the field is unlike the electric field in electrostatics, which did not do any work if the charge moved along a closed loop. Practically, this means that it is ambiguous to talk about the potential difference between two points in this field. Contrast this case with a case when the same piece of wire is connected to a battery. Battery: When a test charge is placed inside a battery, the force exerted on it by the battery points in the direction opposite to the force exerted on it in the circuit. Induced emf: When a test charge is placed at any point inside a loop, the force that is exerted on it by the induced electric field is in the same direction through the whole loop (clockwise for example).

To quantitatively characterize the work done by this field, use the quantity of emf, which has a value that depends only on how quickly the external flux is changing. The potential difference between points of the loop should not be confused with the emf generated by the changing magnetic field. To help students construct Faraday's law

$$\mathcal{E}_{in} = N \left| \frac{\Delta\Phi}{\Delta t} \right| T$$

we suggest that students first work with ALG Activities 21.4.1 and 21.4.2 and then read Section 21.4 in the textbook, especially the text related to Figure 21.12. Note that the emf in these activities is not measured but is inferred from the measurements of the induced current (possible with the ammeter) and resistance of the coil in which the current is induced. Students can proceed to ALG Activity 21.4.3. EOC Questions 9 10, 15 and 16 can be used for formative assessment and Problems 16–18 provide more practice.

### III. Adapt the general problem-solving strategy to problems involving electromagnetic induction and apply the rules developed to analyze some important applications of electromagnetic induction

Example 21.5 in Section 21.5 shows how to adapt the textbook's general problem-solving procedure to electromagnetic induction problems (the parallel ALG activity is 21.5.1). We suggest that students first attempt this problem on their own following the problem-solving strategy and then check the solution in the textbook.

Notice also that textbook Section 21.5 has worked examples that each address the change of one physical quantity in the law of electromagnetic induction; make sure that you discuss all of the examples in class after students work on them on their own. Note that changes in  $\vec{B}_{\text{ex}}$ ,  $A$ , and coil orientation  $\theta$  relative to the  $\vec{B}$  field can each cause an induced emf.

Before starting these applications, it is good to review the math idea that non-changing quantities are like constants and can be moved outside the change symbols. For example, if only  $B$  is changing, Faraday's law becomes:

$$\mathcal{E}_{\text{in}} = N \left| \frac{\Delta[BA \cos \theta]}{\Delta t} \right| = N \left| A \cos \theta \frac{\Delta B}{\Delta t} \right|$$

Textbook Example 21.6 illustrates the analysis of how a changing magnetic field causes an induced emf in the case of the transcranial magnetic stimulation procedure. Later, students can work on EOC Problems 21, 23, 24, 28–31.

The analysis of a changing area causing an induced emf is illustrated in Example 21.7. This example involves a metal axle with wheels on the end rolling on two tracks. The tracks are connected together on the opposite ends by a lightbulb. A  $\vec{B}$  field points down between the tracks. Faraday's law can be used to determine the induced emf between the ends of the tracks due to the increasing area between the light bulb, rails, and rolling axle. The *same emf* can be determined by considering charge separation that occurs in the axle due to the magnetic force exerted on the free electrons in the axle moving perpendicular to the  $\vec{B}$  field.

All of the activities in the ALG Section 21.5 are useful for practice but we also want to make a few comments before you assign them to your students. ALG Activities 21.5.4 and 21.5.5 are designed to help students construct the concept of motional emf, so you might want to use these before students read the textbook section. However, these activities carry a huge amount of overhead in terms of time and intellectual effort for the students. These two activities really promote a deeper understanding of what is going on, but if you feel that you need to get past Chapter 21 quickly to other things, these two activities take up too much time. Later, students can try EOC Problems 25, 26 and 41 for motional emf and 22, 27, 41 and 42 for changing area.

The analysis in the textbook subsection “EMF of a generator” of the changing orientation of a coil relative to a  $\vec{B}$  field leads to the derivation of the induced emf of a generator and the discussion of electric power plants. Students work on EOC Problems 32, 34–38. Although the book does not go into the details of wind turbines, you can assign students to investigate how wind turbines work and how they are similar to or different from the regular generators.

There are many ALG activities in this section that address different aspects of the concept of electromagnetic induction and we suggest that you use as many as possible with your students. Notice Activity 21.5.14. It connects electromagnetic induction to the magnetic field of Earth, a concept that students might have forgotten by now. All General Problems will be very useful here, but we especially recommend 58, 59, and 61–63.

## IV. Investigate AC circuits and transformers

We preserved the integrity of the ISLE approach investigating AC circuits despite the mathematical complexity of the topic in Section 21.6. The textbook first describes experiments with AC circuits containing different elements and then shows how to analyze them mathematically. As our approach is very different from traditional approaches, we suggest that you first acquaint yourself with the steps using the textbook and then analyze ALG activities. After this you will be able to decide whether it is better to work with the students in class on the experiments and derivations presented in the textbook and then assign them work on the ALG Activities 21.6.2–21.6.7 or assign them the reading of the book first (ALG Activity 21.6.1) and then let them work through the rest of the ALG activities. Notice that we continuously ask students whether every new equation makes sense. This is a necessary step if you want them to connect the experiments that they observed (or read about) to mathematics.

There are three subsections in Section 21.6 in the textbook: resistors in AC circuits, capacitors in AC circuits, and inductors in AC circuits. For each of resistive elements we first describe simple experiments which reveal the new behavior of the element in an AC circuit compared to the DC circuit and then analyze this behavior mathematically by first considering the relationship between current and voltage and then focusing on the resistance/reactance. This systematic approach allows students to see the same elements of reasoning repeating for different circuit elements. We do not put all three elements in a circuit to consider the total resistance. This step involves physics and mathematics that is beyond the scope of the book.

If you plan to repeat the experiments described in the textbook and shown in the videos, you will need a function generator (a source of sinusoidal voltage with adjustable frequency and amplitude), an amplifier (to achieve suitable currents) and lightbulbs (we used car lightbulbs) in addition to capacitors, inductors, and resistors. You will also need a parallel-plate capacitor, a dielectric plate (you can use glass) and a ferromagnetic core (we used a core from a transformer).

Activities in the ALG do not require equipment, they can be done in class or in the lab, but they rely on student understanding of the material in the textbook. If you do not have much time, your best choice here is ALG Activity 21.6.2. EOC Questions to work on are 13, 14, 22, and 23 and Problems 43–48.

Section 21.7 uses Faraday's law to derive the effect of a relative number of input and output coils of a transformer on the input and output emfs. We apply the results of this derivation to see how to produce a 20,000-V spark in an automobile with only a 12-V battery. ALG Activity 261.7.2 is an excellent way for your students to really understand this material after they learn the basics of transformers either from class discussion or from reading the textbook. Students can work on EOC Questions 11 and 21, and EOC Problems 49 and 50.

Many of modern technologies that exploit the phenomenon of electromagnetic induction are not discussed in detail in the textbook. If you have more advanced students who need additional assignments or a capstone project, they can explore induction phenomena in everyday life. Some examples are how the magnetic strip on a credit card works, how electric guitar pickups work, magnetic levitation, and eddy currents.

## **V. Develop a deeper explanation for electromagnetic induction and see how it fits into the electricity and magnetism ideas developed in Chapters 17–20**

So far, students have learned that if the magnetic flux through a loop changes, there is an induced current in the loop. But they do not know the mechanism behind this process. Section 21.8 provides a deeper examination of why the phenomenon of electromagnetic induction exists and also summarizes the study of electricity and magnetism in Chapters 17–21.

The mechanism is that a changing  $\vec{B}$  field produces an  $\vec{E}$  field whose lines are loops that do not have a beginning or an end. We suggest that you first start with the ALG Activity 21.8.1 to help students invent the concept of this new type of  $\vec{E}$  field and then you can have a summarizing class discussion or they can read the textbook (ALG Activity 21.8.2). They will need this material when they study electromagnetic waves in Chapter 25. EOC Problem 53 will be useful here.

# Reflection and Refraction

In this chapter, we start the first of four sequential chapters involving light. The focus of this chapter is on light emission and light propagation in different media. (We do not address image formation in this chapter.) Students learn about two models explaining the behavior of light. In the first model, light is analogous to a stream of fast-moving bullets, and in the second, light is analogous to a wave. Content-based learning objectives are listed below.

**Students should be able to:**

1. Explain what is needed for us to see things.
2. Draw ray diagrams to represent how an extended source emits light.
3. Design an observational experiment to determine patterns in the behavior of a narrow beam of light (laser beam) incident on a mirror.
4. Design an observational experiment to determine patterns in the behavior of a laser beam incident on a tank of water.
5. Apply the law of reflection to solve problems.
6. Apply Snell's law to solve problems.
7. Design two independent experiments to determine the refractive index of a transparent material (one should involve total internal reflection).
8. Explain how a pinhole camera works.
9. Draw ray diagrams for light rays in complex situations (including prisms)

The chapter is broken into four parts:

- I. *Qualitative analysis of light propagation and emission*
- II. *Reflection, refraction, and total reflection of light*
- III. *Skills for analyzing light reflection and refraction and applications*
- IV. *Models of the nature of light*

For each part, we describe activities that you can use in labs, class meetings and homework.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Qualitative analysis of light propagation and emission	22.1		
Reflection, refraction, and total reflection of light	22.2-22.4		
Skills for analyzing light reflection and refraction and applications	22.5, 22.6		
Two models of light	22.7		
<b>Nontraditional end-of-chapter questions and problems</b>  Ranking tasks (RAT): P22.36 Evaluate (reasoning or solution...) (EVA): P22.3, Q22.35 Multiple possibility and tell all (MPO): P22.20 Jeopardy (JEO): P22.39, P22.40, P22.41 Design an experiment (or pose a problem) (DEX): Q22.15, P22.7, P22.10, P22.31, P22.51 Problem based on real data (that students can collect by themselves) (RED): Q22.12, Q22.35			

### *Brief summary of student difficulties with reflection and refraction*

In addition to thinking that light comes from our eyes and a source of light is not necessary to see objects, students have difficulties drawing ray diagrams for extended sources of light. Students often think that if a light ray bends during refraction, it should bend even if light is traveling perpendicular to the surface; when dealing with total internal reflection, they think of light only going from a more optically dense medium to a less dense medium without considering cases when light travels from a less to a more dense medium at an almost  $90^\circ$  angle. Technical difficulties include measuring angles using a protractor, consistently drawing normal lines when the media borders are curved or not horizontal or vertical, and considering the correct incident and reflective/refractive angles. These difficulties often come from not using a ruler when drawing incident and reflected/refracted rays especially when there is more than one reflective or

refractive surface.

## **I. A qualitative analysis of light propagation and emission**

In our experience, and based on research on students' understanding of optics, many difficulties that students have with optics may be traced back to inadequately developed models of the two most basic light phenomena: how light is emitted from a source or reflected off of an object, and the role of the human eye in perceiving light phenomena. We believe that instructors should pay extra attention to these foundational ideas. The question "How do we see things?" is a nontrivial one for many students. First, we need a source of light, then this light has to reach the object we want to see, and then light bounces off it and reaches our eyes. However, in ancient times people thought that their eyes emitted light rays that reached the objects, wrapped around them, and returned to their eyes, bringing them information about the object. Many people still have this idea in some form, and thus they often believe that if we sit in a dark room for a long time we will eventually be able to see. ALG Activity 22.1.1 addresses this issue. The ideal situation is if you have a completely dark room, where you can let your students sit for a while and then ask them if they see anything. This experiment works only if the room is completely dark, but it is extremely powerful when it works. Ask students to explain why they cannot see anything. To explain the outcome of this experiment, students need to come up with the idea that a source of light is needed to be able to see objects.

The next step is to help students construct two ideas: (1) light travels in straight lines in the same medium, and (2) one cannot see light from a source unless this light comes directly into our eyes or is reflected off some object and reaches the eyes of the observer. Students can complete ALG Activity 22.1.2 (the same activity is described in textbook Observational Experiment Table 22.1). Students observe and analyze the experiments involving a laser beam. The experiments can be easily and quickly reproduced in large or small classrooms. Note here that a laser is an artificially-made source of light that sends a very narrow parallel beam of light. When there are no obstacles, such as chalk dust, the beam itself is not seen but only the spot on the wall where the beam hits the wall. These simple experiments help students construct the idea that for us to see objects that do not emit light, light has to bounce off it and reach the observer's eyes.

The most important idea that students need to construct is a model of how light is emitted by a source. For this, students need the concept of a ray of light – a model of a phenomenon. We introduce it as a model that is an arrow that represents the direction that light travels (Section 22.1).

The key question that we pose to students is, does each point of an extended light source emit one ray or multiple rays? To answer this question, students are asked to test two models, one model with one ray from each point (rather like a child's depiction of the Sun) and one with multiple rays from each point. If you have a lab



period for this, then students can work on ALG Activity 22.1.3. If you do this material in a class setting (not in the lab), then the same activity can be done with students working in groups of two coming up with testing experiments and with you performing them. Make sure you have a piece of thick aluminum foil handy to cover the bulb, as one of the most common experiments requires it. Some possible experiments are described in the textbook's Testing Experiment Table 22.2. Testing experiments reject the one-ray model, and the multiple-ray model remains.

Give students lots of time to draw ray diagrams in order to make predictions of the outcomes of the different experiments using the two different models. This is likely the first time they are drawing ray diagrams, so they may need the extra time. The next ALG activities (22.1.4–22.4.6) help students solidify their understanding of this concept. Students can do these in a lab or talk about them in a class discussion after observing the experiment.

If you are doing ALG Activity 22.1.6 in a lecture setting (Conceptual Exercise 22.1 in the textbook), you will need to use a camera to capture the inverted flame of the candle on the screen. Students can make their own simple pinhole cameras using empty boxes of oatmeal (they have a cylindrical shape) that have a transparent matte plastic cover (that serves as a screen). Students poke a hole in the bottom of the can and see the inversed image of bright objects on the cover. Note that the photo in Figure 22.4 was made using a big cardboard box with a small hole and a GoPro camera positioned right next to the hole of the camera, capturing the image on the screen inside the box. If you are in a room with windows that you covered for the first ALG Activity 22.1.1, you can ask your students to predict what will happen if you make a small hole in the shade. If they say that they will see street objects upside down on the opposite wall, they really understand the concept of multiple rays. To help students practice drawing ray diagrams and explain shadows, use ALG Activities 22.1.7–22.1.9 and textbook EOC Questions 1–6, 18–20, 22–24 and Problems 1–6 and 8.

It is critically important that students build a robust model of how light is emitted from an extended source because the concept that each point of a light-emitting object sends multiple rays is crucial later for drawing ray diagrams for mirrors and lenses. The time your students spend on these basic ideas will pay dividends later.

## II. Reflection, refraction, and total internal reflection of light

If you are starting in a lecture setting, you can provide your students with the data presented in Table 22.3 in Section 22.2 so that they can find the pattern on their own. If you start this section in a lab, students can do ALG Activities 22.2.1 and 22.2.2 to construct and test the law of specular reflection. It is important to emphasize for students that the angles used are angles relative to the normal line and not relative to the surface. ALG Activity 22.2.3 provides good practice right away.

The book proceeds to a discussion of the contrast between specular and diffuse reflections. To help students learn this concept, you can first ask them to work on ALG Activity 22.2.4. This activity is an excellent opportunity for group discussion. It is critically important because students need to reconcile the ideas they developed earlier regarding diffuse reflection with the new phenomenon of specular reflection that they are currently studying. ALG Activities 22.2.5 and 22.2.6 and EOC Questions 7, 14 Problems 9–11, 14 and 15 provide good practice.

The next topic is refraction. We suggest that student get first-hand experience with the phenomenon: ALG Activity 22.3.1 and textbook Observational Experiment Table 22.4 in Section 22.3. Before students read this table in the textbook, it is best if they observe and analyze the direction of travel of a narrow light beam partially reflected from an interface between two different media and the direction of that same beam when partially transmitted (refracted) into the second medium. These activities allow students to construct the concept of refraction qualitatively. You may notice that we try to emphasize that there is always reflection *and* refraction at the boundary between different optical media. Too often the reflected rays are left out, causing students to develop unnecessary misunderstandings, such as thinking that there is no reflection when there is refraction. ALG Activities 22.3.2 and 22.3.3 provide additional practice with both refraction and reflection. It is really important that students draw normal lines habitually, carefully using a ruler and a pencil.

To devise Snell's law, students can work through ALG Activities 22.3.4 and 22.3.5 (parallel textbook Tables 22.5 and 22.6 provide angles of refraction for different angles of incidence). Analysis of the provided data leads to the introduction of the refractive indices of the media and to Snell's law. Example 22.3 uses Snell's law to determine the index of refraction of blood. You can assign textbook EOC Questions 8–10 and Problems 19–23, and 25–26.

The next step is to help students come up with the concept of an angle of the total internal reflection. As students have noted in the previous section, light bends away from the normal line when passing from an optically denser higher index of refraction material into an optically less dense lower index of refraction material. In ALG Activity 22.4.1, students use their knowledge of refraction to predict what will happen to light traveling from glass to air. After students make predictions and then observe the experiment, they can read the explanation presented in textbook Section 22.4. Note biological examples of the application of this phenomenon at the end of this section. EOC Questions 11 and 12 and Problems 27–33, 36, and 37 are good for practice.

### **III. Skills for analyzing light reflection and refraction and applications**

In Section 22.5 students practice using ray diagrams to help their reasoning and quantitative problem-solving for processes involving reflection, refraction, and total

internal reflection. The general problem-solving strategy of the textbook is adapted to such processes in Example 22.7. An Equation Jeopardy worked example follows: students have to make sense of an equation that is the answer to an unknown problem. Students can then work on ALG Activities 22.5.1–22.5.7 and textbook EOC Question 35 and Problems 38–45, and 50–53. Notice Problem 45: it will intrigue your students!

One of the common difficulties that students experience here is drawing light rays coming from people's eyes as opposed to coming from an object. If you notice your students doing this, ask them where the object is that emits the light.

Textbook Section 22.6 includes the following four interesting and important applications of this chapter. Fiber optics has widespread use in medicine, communications, and other applications. The book first treats it quantitatively (Example 22.9) and then proceeds to the qualitative discussion. We next discuss the operation of prisms along with some of its applications. When students work with prisms, it is important that they draw normal lines carefully. Here students learn also about dispersion (dependence of refractive index on color of light) – although we do not use this name here – the same idea comes back in the reading passage where students learn how rainbows are formed

Mirages are an interesting application of the refraction of light in air, whose index of refraction varies with the temperature. Finally, there is a brief analysis of why the sky is blue when looking in most directions away from the Sun. ALG activities in this section can serve as introduction to the above applications or done after you have a whole class discussion about them. Notice novel Activities 22.5.1 and 22.6.3. We hope that these activities will motivate your students as they directly connect the material of this chapter to their everyday life experiences. Students can work on EOC Questions 29–31 and Problems 46–49.

## IV. Models of the nature of light

Section 22.7 is dedicated to the explanations of the observed behavior of light. To explain the observed phenomena, scientists built analogical models using something familiar as a base for the analogy. Historically, there were two models (the particle fast-moving bullets model and the wave model) that successfully explained light propagation, reflection, and refraction. However, the two models predict different changes in light speed when light passes from one medium to another. The wave model predicts a smaller speed, and the particle model predicts a larger speed when light transitions from a less dense to a more dense optical medium. At the time of the development of ideas of refraction, light speed could not be measured in different media, and consequently the refraction phenomenon did not eliminate either the wave or the particle model. We follow the same approach, leaving the question unresolved, and will return to the subject in later chapters. This section is where you can introduce the students to Huygens' principle and let them work through the details of using the wave model to explain refraction. We suggest that you start with ALG Activities 22.7.1–22.7.4 (or similar activities) and then let students read the textbook (ALG

22.7.5). They can work on textbook EOC Questions 33 and 34. Note that it is not necessary to introduce Huygens' principle at the end of the chapter. It works equally well if you weave this section in earlier when students are first discovering the phenomenon of refraction.

# Mirrors and Lenses

Chapter 21 involved the study of reflection and refraction. In this chapter, we extend these ideas by studying the way in which mirrors and lenses form images of light from objects that reflect from the mirrors or pass through the lenses. Students will analyze experiments and test ideas involving image formation. Content-based learning objectives are listed below.

**Students should be able to:**

1. “Read and write” with ray diagrams for plane mirrors, curved mirrors, and lenses. Use ray diagrams to represent the problem situation and to evaluate the solution. Explain the role of three rays and the role of a focal plane. Provide examples.
2. Explain the difference between a real and a virtual image.
3. Use ray diagrams, the curved mirror equation, and the magnification equation to solve quantitative mirror problems. Translate between ray diagrams and equations.
4. Use ray diagrams, the thin lens equation, and the magnification equation to solve quantitative lens problems. Translate between ray diagrams and equations.
5. Solve moderate-difficulty multi-lens problems.
6. Explain how to choose a prescription for glasses.
7. Compare and contrast the human eye and a camera.
8. Design an experiment to determine the focal distance of a concave mirror.
9. Design an experiment to determine the focal distance of a convex lens.
10. Design an experiment to determine the focal distance of a concave lens.
11. Explain in detail how a magnifying glass works and how to find images produced by curved mirrors and lenses for different positions of an object.

The chapter is broken into five parts:

- I. *Analysis of image formation by plane mirrors*
- II. *Qualitative and quantitative analysis of image formation by curved mirrors*
- III. *Qualitative and quantitative analysis of image formation by lenses*
- IV. *The skills needed to solve mirror and lens problems, including the optics of the eye*
- V. *Angular magnification, magnifying glasses, telescopes, and microscopes*

For each part, we describe activities to help students acquire these ideas. Brief discussions of the motivations for using these activities are provided.

We begin this chapter by emphasizing the importance of students drawing ray diagrams to analyze the image production by different optical systems. Ray diagrams not only serve the same purpose as other representations we used before – motion diagrams, force diagrams, energy and momentum bar charts and so forth, but they also help students reconcile mathematics with real phenomena. Unlike a force diagram that is a completely mental construction, a ray diagram, similar to a circuit diagram represents real objects (sometimes these objects are virtual but this does not change the nature of this message). Therefore drawing ray diagrams will help students reconcile mathematics with real situations. But this is not enough. It is extremely important that students have plenty of opportunities to actually make the optical systems and find the images of the objects in the locations predicted by ray diagrams. We find again and again that even physics majors have not experienced finding an image of an object using a curved mirror or a lens.

<b>Chapter subject matter</b>	<b>Related textbook and ALG sections</b>	<b>Textbook videos</b>	<b>ALG videos</b>
Analysis of image formation by plane mirrors	23.1	OET 23.1 (p713), TET 23.2 (p714)	
Qualitative and quantitative analysis of image formation by curved mirrors	23.2, 23.3	OET 23.3 (p716), OET 23.4 (p719), OET 23.4 (p719)	
Qualitative and quantitative analysis of image formation by lenses	23.4, 23.5	OET 23.5 (p725)	
Skills needed to solve more complex mirror and lens problems, including the optics of the eye,	23.6, 23.7		

telescopes, and microscopes			
Angular magnification, magnifying glasses, telescopes, and microscopes	23.8, 23.9		
<p align="center"><b>Nontraditional end-of-chapter questions and problems</b></p> <p>Evaluate (reasoning or solution...) (EVA): P23.15, P23.31, P23.40, P23.57  Multiple possibility and tell all (MPO): P23.20, P23.21  Jeopardy (JEO): P23.75, P23.76, P23.77, P23.78  Design an experiment (or pose a problem) (DEX): Q23.26, P23.6, P23.7, P23.9, P23.10, P23.24, P23.35, P23.31, P23.69, P23.74, P23.81, P23.82  Problem based on real data (that students can collect by themselves) (RED): P23.23, P23.39, P23.40</p>			

### *Brief summary of student difficulties with mirrors and lenses*

One of the biggest student difficulties is reconciling the ray diagram with the real-life situation with real mirrors and lenses. Another difficulty is understanding the nature of virtual images. Remembering that each point of an extended source of light sends an infinite number of rays and we choose the convenient rays to draw diagrams poses a difficulty too. Finally, drawing diagrams without a ruler will lead to confusing results, thus requiring a pencil and a ruler to draw diagrams is very important.

## **I. Analysis of image formation by plane mirrors**

Students can figure out how to find an image of an object in a plane mirror by participating in the ALG Activities 23.1.1 and 23.1.2 and then analyze the ray diagrams for the plane mirror experiments in Observational Experiment Table 23.1 in class to develop a rule for the image location of an object that is placed in front of the mirror.

The rule that the image is the same distance behind the mirror as the object is in front is tested by experiments in the ALG Activity 23.1.3 and Testing Experiment Table 23.2. The tested rule is used to locate images of multiple point-like objects in the ALG Activity 23.1.4 and of an extended object in a tilted lamp in Conceptual Exercise 23.1. In Conceptual Exercise 23.2, students have to apply their understanding of ray diagrams and reflection to figure out the size of the smallest mirror that can be used in order to see their entire image in a mirror (a challenging activity). It is better to have students work though parallel ALG Activities 23.1.5 and 23.1.6 on their own in recitations or lab. If you want students to figure this out, they should first start with ALG Activity 23.1.5, which asks students what happens to the

size of an image in a plane mirror when the object is moved closer or farther away. ALG Activity 23.1.6 challenges students to use ray diagrams to find the smallest mirror necessary to see the entire image of one's body. EOC Questions 1–3 and Problems 1–5 can be used for more practice and formative assessment. EOC Question 17 is really challenging; assign it to your most advanced students. Note that students sometimes draw rays coming from the eyes of the observer as opposed to the object. Remind them to think about where the light rays start.

Note that students struggle with the concept of a virtual image, both in understanding its nature and in locating its perceived position. Work with students to help them understand that perceiving the position of an image is intimately tied into human binocular vision. The key idea is that we perceive an image from wherever the light rays appear to originate. Being able to locate the position of that image requires at least two rays coming from the same point of the object to reach the eye of the observer. ALG Activity 23.1.5 is an excellent supporting activity here.

## II. Qualitative and quantitative analysis of image formation by curved mirrors

Textbook Section 23.2 involves a qualitative analysis of image formation by curved mirrors in terms of three selected rays (we will call them principal rays here), and Section 23.3 involves a quantitative analysis (the derivation of the mirror equation). In many experiments in the textbook and ALG, for both curved mirrors and lenses, students use small mirrors and lenses that can be placed vertically on a piece of paper (they come as parts of optics kits). If you have optics kits with the sorts of mirrors/lenses that have light sources sending parallel beams, it will be even better (then you do not need laser pointers).

It is important that students continue developing the ray model of light in this chapter. Rays represent the direction of light travel from before to after reflection from a mirror. Rays are not real entities, just representations for the direction of light travel. We use the word beam for a narrow amount of light traveling from one place to another.

To analyze concave mirrors, students can work through ALG Activities 23.2.1–23.2.6 and then read Observational Experiment Table 23.3 in Section 23.2, where they see find confirmation to what they observed and analyzed already: that parallel rays reflect from a concave mirror and converge through a single focal point. Through observation and testing, these activities help them construct the principal rays. Figure 23.6 provides a geometric proof that the distance from the focal point to the mirror is one-half the radius of the curvature of the mirror.

Students need to solidify their ideas and use them to understand how to find the location of the image formed by a concave mirror. Textbook Physics Tool Box 23.1 and ALG Activities 23.2.6 and 23.2.7 are intended for this purpose. The Tool Box provides explicit instructions for determining the image distance from the mirror. The



nature of images (real or virtual, upright or inverted, and enlarged or reduced in size compared to the object) is discussed in detail and illustrated using ray diagrams. ALG Activity 23.2.8 is best used as whole class activity. In this activity, students draw ray diagrams to find the location of the image formed by a concave mirror for the three standard cases:  $s > 2f$ ,  $s < f < 2f$ , and  $s < f$ . This activity is key for building their understanding of the behavior of concave mirrors. Once students have completed this activity, they can work through textbook Conceptual Exercise 23.3, which illustrates the different object-image types. EOC Question 4, and parts of Problems 7 and 8 related to concave mirrors are useful here.

You can follow a similar sequence with convex mirrors starting with ALG Activities 23.2.10 and 23.2.11. Students can then read Observational Experiment Table 23.4 in Section 23.2. Here they see that parallel rays diverge after reflection from a convex mirror and seem to come from a single focal point on the other side of the mirror. Students can proceed to ALG Activity 23.2.11. Physics Tool Box 23.2 (parallel ALG Activity 23.2.12) provides explicit instruction for estimating the image distance for different objects at different distances from convex mirrors. Textbook Conceptual Exercise 23.3 illustrates the nature of images: real or virtual, upright or inverted, and enlarged or reduced in size compared to the object. Make sure that your students do not skip ALG Activity 23.2.13 that makes them explore the type, location, and the size of image produced by a convex mirror (similar to ALG Activity 23.2.8 above). EOC Questions 5, and 6 and Problems 7, 8 and 9, 11 and 13 can be used effectively for practice and formative assessment. In general, make it a priority that your students learn to draw ray diagrams and they draw them habitually for every problem they are asked to solve. Once they are fluent with ray diagrams, they can use them not only to set up problems but also to evaluate their answers (when appropriate).

An important issue needs addressing at this point (or you could address it earlier, after concave mirrors, if you prefer). Students can become so fixated on the principal rays that they have likely forgotten that each point on a light source emits rays in *all* directions. As a result, if you ask students what happens to an image if you cover half of a concave mirror, they will likely think that, since some of the principal rays get cut off, the image will get cut off or chopped in half. Make sure you have a discussion about this issue. Where should the observer be to still see the image and what will the image look like? Make sure you have the actual experiment on hand to demonstrate, once the students have resolved their confusion through discussion. Students should think of this as a testing experiment.

Another important issue is the focal plane. Rays parallel to each other but not parallel to the principal axis after reflection pass through the same point (Experiment 3 in the Observational Experiment Table 23.3). All of those points are located in a plane that is perpendicular to the principal axis and passes through the focal points; this plane is called the focal plane (all this is true only for the small angle approximation). ALG Activity 23.2.14 helps students apply this idea.

The image location process continues in Section 23.3, only now quantitatively. The object distance  $s$ , the image distance  $s'$ , and the mirror focal length  $f$  are

introduced in Figure 23.10 in the textbook. A step-by-step derivation of the mirror equation is provided relative to the symbols in that figure. A parallel ALG activity is 23.3.1. Note that the activity provides the derivation but then asks students questions that will help them make sense of the equation. Sign conventions for each of these three quantities are provided following Example 23.4 (it is very important that students understand these sign conventions). Next, students need to test the mirror equation. ALG Activity 23.3.2 is a lab that students can do after they learned the mirror equation. While this activity is relatively simple, we have found that after students have learned some difficult mathematics (here, the mirror equation) it is a very productive experience for them to have to instantiate that equation on real equipment. Students naturally have to confront important questions such as “which distances are  $s$  and  $s'$ ?” In figuring out these relatively simple procedural questions, students gain a solid understanding of the equation and the meaning of the symbols involved. Textbook Example 23.5 provides another test of the mirror equation in the textbook. Students can practice applying the equation in class using ALG Activities 23.3.3 and 23.3.4. EOC Question 14 and Problems 12–15, and 22 can be used for practice.

Linear magnification is then introduced and related to the image-object distances in Figure 23.11, leading to the linear magnification equation, which is applied in Example 23.6. A Tip indicates that the image size and location are independent of the size of the mirrors. A common student mistake is forgetting to invert the  $1/s'$  near the end of an example. EOC Problems 17–21 can be used for practice.

### III. Qualitative and quantitative analysis of image formation by lenses

You have probably noticed the pattern of development of ideas for mirrors: (1) students conduct observational experiments to find the principal rays, (2) students learn to use ray diagrams to find and describe images that are formed by the mirror, (3) an equation is developed, and (4) students test and apply the equation to real-world situations. The pattern of development for lenses is the same. If students have worked through mirrors already, the development of ideas in this section can go much more quickly, and we recommend developing concave and convex lenses simultaneously. In ALG Activities 23.4.1 and 23.4.2 and Observational Experiment Table 23.5 in Section 23.4, and by reading the text accompanying Figure 23.25, students learn about the focal points for convex and concave lenses. As always, we recommend that students first have physical experience with the lenses and only then read the book

Note that parallel rays that are not parallel to the axis of the lens focus at another point the same distance from the lens as the focal distance, but not on the axis—in the focal plane. With mirrors, there was an activity explicitly addressing this point (ALG Activity 23.2.14); for lenses an activity addressing this issue will come much later

(Activity 23.6.3); therefore, you need to make sure you focus students' attention on Experiment 3 in the Observational Experiment Table 23.3 to discuss the importance of the concept of the focal plane in class.

Light passing through a concave lens diverges on the other side as if it comes from a focal point on the same side as the light source. This point from which parallel light seems to diverge is called a virtual focal point. Students should understand that it is a virtual focal point because light does not actually pass through that point before reaching the lens (except the ray on the optical axis). ALG Activity 23.4.3 is the key activity in which students can practice drawing ray diagrams to locate the position and identify the nature of the image formed by concave and convex lenses. After making their own diagrams, student can read Physics Tool Box 23.3 and Table 23.6 to evaluate how they did. After finishing their diagrams, student can try to invent a practical application for the lens situation. Notice the discussion about locating the image of a point-like object that is on the principal axis in Figure 23.16 and the accompanying text. This is where students need the concept of the focal plane! Afterwards students should try a lens Jeopardy problem (Example 23.7, later addressed in the ALG Activity 23.6.3)—they are challenging for students and a lot of fun! Conceptual Exercise 23.8 involves the effect of covering up part of a lens on the location and size of the image formed by the lens. Make sure to pose this question in a whole class discussion. EOC Questions 7, 8, 10 and 11 and Problems 23–31 and 74, 75 and 78 can be used for practice (78 is a non-traditional problem that will challenge your most advanced students).

In Section 23.5, students can help develop the thin-lens equation and then test it in a lab (ALG Activity 23.5.1). ALG Activities 23.5.2 and 23.5.3 make an excellent pair of testing experiments that students can conduct in a lab. In the textbook, the equation is derived using the sketch in Figure 23.18. The thin-lens equation is found to survive extreme case evaluation. It is important for students to understand what is meant by a sharp image. The fact that we tend to call a sharp image a focused image in everyday language just adds to the confusion. They must also understand sign conventions used with the thin-lens equation presented in the thin-lens equation definition box. A computer projector example is provided in Example 23.9. Linear magnification is introduced and used in Example 23.10. The Tip at the end of the section about not forgetting to invert  $1/s'$  to find  $s'$  should be emphasized for the students. Students can try EOC Problems 32–35.

## **IV. Skills needed to solve more complex mirror and lens problems, including the optics of the eye, telescopes, and microscopes**

In Example 23.12, the general problem-solving strategy of the book is adapted to problems involving mirrors and lenses (ALG Activity 23.6.1). ALG activities that follow (23.6.2–23.6.5) represent non-traditional problems. If you wish your students to practice traditional word problems, EOC Problems 36–40, will be very useful. Notice EOC Problem 47: it is excellent for formative assessment.

Section 23.7, Single lens optical systems, involves a variety of practical single-lens applications, including photography and cameras, the human eye, optics of the eye, near-sighted vision, the power of eyeglass lenses, and far-sighted vision. All activities in the corresponding ALG section are excellent for practice (they can also be used in the Section 23.6) as well as EOC Problems 41–46 and 50–56.

## **V. Angular magnification, magnifying glasses, telescopes, and microscopes**

In Section 23.8 we first discuss the deficiency of using only linear magnification and then follow with the qualitative and quantitative development of the new quantity angular magnification. Angular magnification is applied to a magnifying glass. ALG Activity 23.8.1 is crucial for helping students understand that the image of a small object that someone is looking at with a magnifying glass is actually behind the object. If the object is placed on a desk, for example, the image is under the desk! Students can further work on EOC Problems 57–60, 76, 81, and 82. Both reading passages will be extremely useful too, not only for biologists!

In Section 23.9, we introduce two-lens applications, including the angular magnification of an object viewed through such a two-lens system. Section 23.9 starts with a step-by-step method to locate the image of a two-lens optical system, including the analysis in Figure 23.30. This is very useful for students if you wish to include telescopes and microscopes in your course. The two-lens method is applied to telescopes in Example 23.15 and Figure 23.31 and to microscopes in Example 23.16 and Figure 23.32. Students can follow up with EOC Problems 61–64, 67–69, 71–74, and 77 and 79.

# Wave Optics

In Chapter 22, the first chapter on optics, students learned that observable light phenomena can be explained using the ray or particle-bullet models of light and assuming that light behaves in a similar way to a wave. However, we did not resolve the issue and continued to study mirrors and lenses using a particle model of light (represented by light rays). In this chapter, students will learn about experimental evidence that cannot be explained by the particle-bullet model and start building the wave model of light.

We focus on interference and diffraction in this chapter and do not touch on the polarization of light, which is covered in the following chapter (Chapter 25, Electromagnetic Waves). Notice there is a slight change of language here—we do not use the term *diffraction grating* but instead call the gratings *interference gratings*, or sometimes just *gratings*. The reason for this is that in the book we are treating each slit in a grating as infinitely narrow. This simplifies the analysis to the interference (superposition) of simple circular waves, avoiding complications that result from taking into account the diffraction on finite-width slits. To emphasize this, we do not use the term diffraction grating but instead call the gratings interference gratings. We use the term diffraction to describe the pattern obtained when light is incident on a single slit. When we study diffraction, the slits are not considered infinitely narrow. We discuss the interference of light produced by different parts of the same slit. Because this model is not applicable to the grating (where light that passes through different slits interferes on the screen), we do not call it a diffraction grating.

Content-based learning goals of the chapter are listed below.

## Students should be able to:

1. “Read and write” with wave fronts and rays.
2. Use Huygens’ principle to explain interference and diffraction phenomena.
3. Apply the superposition principle to explain interference effects.

4. Analyze qualitatively and quantitatively situations involving laser light passing through two slits, one slit, multiple slits, gratings, and thin films. Identify path length difference and phase difference.
5. Determine resolving power of optical instruments, including eyes.
6. Design an experiment to determine the slit separation in a double-slit experiment.
7. Design an experiment to determine the slit width in a single-slit experiment.
8. Design an experiment to determine the wavelength of different colors of light using a grating.
9. Design an experiment to determine the number of slits per mm in a grating.
10. Apply knowledge of wave optics to explain and analyze technological and biological applications.

This chapter is broken into four parts:

- I. *Young's double-slit experiment and multiple-slit gratings. The relation between index of refraction, light speed, and wave coherence*
- II. *Thin-film interference*
- III. *Diffraction of light and resolving power*
- IV. *Skills for analyzing processes involving light waves*

For each part, we describe activities can be used in lecture and recitations to help students acquire these ideas and the motivation for using these activities.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Young's double-slit experiment, the relation between index of refraction, light speed, and wave coherence and gratings	24.1-24.3	TET 24.2 (p755),	
Thin-film interference	24.4	24.1 (p767)	
Diffraction of light and resolving power	24.5, 24.6		
Skills for analyzing processes involving light waves	24.7		
<b>Nontraditional end-of-chapter questions and problems</b>			
Evaluate (reasoning or solution...) (EVA): Q24.17, P24.27, P24.28, P24.62			

Make judgment (MJU): Q24.12, Q24.13, P24.19

Multiple possibility and tell all (MPO): Q24.24, P24.2, P24.4, P24.18, P24.38, P24.39, P24.46, P24.47, P24.50

Design an experiment (or pose a problem) (DEX): Q24.10, P24.2, P24.14, P24.49

Problem based on real data (that students can collect by themselves) (RED): P24.17, P24.45, P24.62, P24.67

*Brief summary of student difficulties with wave optics*

The biggest conceptual difficulty that comes from instruction is when students think that light is a wave after working through the material in this chapter. They need to distinguish between the phenomenon and the model. A wave is just one model of light phenomenon. One of the difficulties is moving between representations such as rays and wave fronts, another one is reconciling the mathematical description of double slit interference with the qualitative analysis and not connecting the waves language with the phenomenon (how does the interference maximum or minimum look in the real world?). A question that shows how good this reconciliation is, is when students have to decide how a certain change (slit separation, wavelength) will affect the total number of maxima that appear behind the grating. A major difficulty is considering the effect of a slit on a passing wave – what will happen to the interference pattern if one of the slits is covered? If a part of both slits is covered? Remembering that the frequency of the wave is determined by the source and the speed is determined by the medium and not by wavelength is also a challenge. Visually distinguishing the effects of diffraction on a slit (bending of light around it) and interference due to multiple slits is difficult unless students have seen the interference/diffraction pattern on a screen and analyzed their observations.

## I. Young's double-slit experiment and multiple-slit gratings

The investigation of the wave model of light assumes that students review the two model of light from Chapter 22. Now they begin by testing the two models of light by using them to predict the outcome of Young's double-slit experiment. Textbook Testing Experiment Table 24.1 uses the particle-bullet model to make the prediction and the ALG Activity 24.1.1 uses the wave model to make the prediction. Students make a prediction based on the particle-bullet model of light and observe that the pattern on a screen does not match the prediction (Testing Experiment Table 24.1). However, the wave model combined with Huygens' principle matches the outcome of the experiment (ALG Activities 24.1.2 and 24.1.3).

If you start this chapter in class (lecture) it is still extremely important that students observe what happens when laser light goes through two narrow slits and separate the effects of two slits from the effects of the width of each slit. They will not know which is which, thus you might need to point their attention to the important parts of the pattern on the screen.

Following the qualitative introduction to double-slit interference, students can analyze double-slit interference quantitatively by using ALG Activities 24.1.4–24.1.6 (these are best if students are in a lab and actually perform the testing experiment). You can lead them through the derivation in a class discussion using textbook Figure 24.2 for the 0th and 1st order maxima equal from the two slits to the screen and Figure 24.3 for the general expression for the  $m$ th order bright band. The relationship is then tested and summarized for *bright* bands. Notice the Tip following the summary box on page 756. It adapts the interference expression for the dark bands of the interference pattern on the screen.

Note that both the ALG and the textbook lead students first through the qualitative analysis of the interference process, and only after students understand what happens conceptually is the quantitative expression introduced/derived. The textbook also addresses a possible question that students might have at this point: if light is analogous to a wave, then what is oscillating in that wave? We do not provide an answer to this question yet, but only start the discussion; the answer will appear in Chapter 25 on electromagnetic waves. Help students work through Quantitative Exercise 24.1 to determine the wavelength of green light. Make sure that you assign ALG Activity 24.1.7, the Reading Exercise, they will need time to put together the qualitative and quantitative ideas from this section and reading the textbook after doing all the activities will help. Reading it before having physical experiences leads to the difficulties described above. EOC Questions 1, and 10–16 and Problems 1–7, and 46 and 57 can be used for practice and formative assessment.

Section 24.2 concerns important ideas that are necessary for light interference—monochromatic and coherent light sources. The section starts with a derivation of a relationship between the index of refraction  $n$  of a medium and the speed of light  $v$  in



that medium  $n = c/v$ . A Tip early in the section reminds students that the frequency of a light wave depends on the wave source and does not change as the wave passes from one medium to another. Thus, if the speed  $v$  changes from one medium to another, the wavelength  $\lambda$  also changes but not the frequency. A discussion of chromatic aberration of camera lenses is provided. Finally, the section focuses on monochromatic and coherent sources of light. We provide motivation for this discussion by asking a question: “Why don’t two lightbulbs produce an observable interference pattern on a screen?” The discussion is extremely important because an understanding of coherent waves is necessary for the comprehension of the thin-film interference that follows. We suggest that you have a whole class discussion concerning these issues using the textbook as a guide and only then assign to the students ALG Activity 24.1.2, the Reading Exercise. After that, use EOC Questions 2, 3, 5, 8, 17–19, and Problems 8 and 9.

ALG Activities 24.3.1–24.3.4 and textbook Section 24.3 are dedicated to the interference grating, a system of multiple, narrow slits. (Remind students that the distance  $d$  between adjacent slits is  $d = (\# \text{ of slits/m})^{-1}$ ). If you start this section in the lab and use the ALG activities, make sure that you have a whole class (or whole lab section) discussion after each activity. Students will need a lot of help if they are working on these activities without prior knowledge.

The textbook takes a slightly different path. We take the reader through a qualitative analysis of the interference process that occurs when there are more than two slits (in which we slowly add slits one by one and observe that the brightness of the bands increases and the width of the bright bands decreases, see Figure 24.11 showing the photos of this process). The book proceeds to the derivation of the quantitative expression in Quantitative Exercise 24.3. Now students can apply grating-related concepts to the interference of white light, a reflection grating from a CD and a DVD, and a spectrometer used to analyze spectral composition of light from a particular light source (see Example 24.4 for analysis of an H-atom spectrum). Note when introducing the term “interference grating” we focus only on the brightest spots – not on the fine-grain pattern between the brightest spots.

ALG Activity 24.3.4 addresses a concept that students sometimes have difficulty understanding—that it is not the total number of slits that determines how the interference pattern looks but the distance between the slits ( $\#$  of slits per mm). Students are invited to test this idea in a lab. Other ALG activities conceived as lab experiments are 24.3.5, and 24.3.6. Students can use EOC Questions 6, 9 and Problems 11–18 for practice and formative assessment. Question 9 and Problem 18 address the issue of the total number of maxima that appear behind the grating. This number does not directly “fall” from the equation. Students need to understand under which conditions new maxima appear.

## II. Thin-film interference

Section 24.4 is dedicated to thin-film interference. Let students observe the colors of a soap bubble before you proceed to the explanation. You can either set up a few containers with soap solution and wire frames with a side of 7–9 cm so students can see the film clearly. An even better way to make a soap bubble is described in the ALG Activity 24.4.1 (make sure that if you start in a lab, students do this activity). If you have not used this method before, make sure you practice blowing big bubbles. Notice that we suggest preparing the soap solution a day before class. Do not add glycerin to the solution unless it is absolutely necessary; it makes the bubbles last too long. The textbook provides an amazing video of a thinning soap bubble, make sure your students watch it, but only after they have observed the bubbles they made themselves. A video of the bubble is good, but nothing can substitute for a careful observation of the phenomenon. Everyone has seen this before, but they have not concentrated on the details. After students have had an opportunity to marvel at the colors of the bubble and observe the black color of the film on the top of the bubble right before the film breaks, you can proceed to helping students develop the explanation of the phenomenon in the simpler case of monochromatic light.

Several factors are important for this explanation: (a) light reflected from the front surface of the film and the back inside surface of the film interfere with each other; (b) a reflection leads to a phase change if the medium off which light reflects is optically denser than the first medium; (c) additional phase difference occurs due to the path length difference of the light traveling through the film and back out. This latter path length phase difference depends on the wavelength of the light while in the film (and therefore on the index of refraction of that film). All these factors contribute to the resulting interference if the film is thin enough for all of the waves to remain coherent. Students can test the explanation by predicting what they would see just before a soap bubble pops (when its thickness becomes very small due to sagging and evaporation) – read the text in the sub-section Soap bubble on pages 765–766 in the textbook. It is important to remember that colorful patterns observed on thin films are the result of interference of two reflected light beams that both have much smaller intensity than the light that goes through the film (EOC Problems 27 and 28 test these ideas).

We apply these same two phase-change ideas for other examples in this section, such as thin-film interference on a glass surface. Table 24.3 provides a summary of thin-film interference examples for monochromatic light. The section ends with more-detailed discussions of four interesting applications: (1) the continuous change in a soap bubble color as its thickness changes due to evaporation; (2) the complimentary (as opposed to rainbow) colors of the soap films irradiated by white light; (3) use of thin films on the lenses of optical instruments to reduce reflection; and (4) the color of light reflected from some bird feathers and from butterfly wings. Students can work on EOC Questions 21 and 22 and Problems 19–23 and 27 and 28 after they work through Section 24.4 (ALG Activity 24.4.2).

### III. Diffraction of light and resolving power

The next big theme is diffraction. In this chapter we do not use term diffraction to describe the phenomenon of diffraction (how and when do waves propagate in the region of geometrical shadow behind the obstacle). The term “diffraction” in this chapter means diffraction of waves on a single slit (and the resulting interference of diffracted waves). If your students are starting diffraction in a lab, they can do ALG Activity 24.5.1, or you can use it as a demonstration, keeping the spirit of the observational experiment, and then proceed to the quantitative explanation using ALG Activity 24.4.1. Students can work in pairs, answering the questions in this activity, followed by a whole-class discussion of the derivation, which can be followed by the testing experiment described in ALG Activity 24.5.2. This logical progression is repeated in the textbook Section 24.5, which starts with photos in Figures 24.21 and 24.22 and supporting text describing what happens when light passes through single slits of different widths. Students learn that the diffraction pattern of the transmitted light broadens as the slit thickness decreases. This is followed by a quantitative analysis of single-slit diffraction (see Figure 24.23).

It is important to remind students that the equation describing single-slit diffraction, although mathematically identical to the equation for bright bands for multiple slits, applies to the angular deflection of the *dark bands* and not the bright bands, which are used for the bright bands in double-slit interference and grating interference.

Notice the discussion about the Poisson spot experiment which provides a very interesting example of the process of science. Poisson did not support the wave model of light. He considered it false because the prediction that followed from the wave model for the situation when light passes around a small ball seemed ridiculous, and he expected that it would not be observed: there should be a bright spot at the center of the shadow of a small ball irradiated by light. To his surprise, the bright spot was there! It provided support for the wave model of light. Section 24.5 ends with everyday examples of diffraction, including why we hear sound in all parts of a room with a tiny opening in the door, but see only a bright band of light coming from outside into the room. Students can work on EOC Problems 28–31 and 60.

Section 24.6 concerns the resolving ability of devices with circular apertures, such as circular lenses, including light entering the pupil of the eye. Why, for example, is it better to have a large-diameter telescope lens than a small one in order to detect small details on a distant object? We suggest that students start with ALG Activity 24.6.1 and then have a whole class discussion that follows the textbook Section 24.6. They can then try EOC Question 7 and Problems 28–31 and 33–35 for diffraction and 36, 37 and 41–43.

## IV. Skills for analyzing processes involving light interference

Example 24.7 in Section 24.7 adapts the general problem-solving strategy of the textbook to problems involving light interference (with parallel ALG Activity 24.7.1). In Example 24.8, the strategy is applied to studying diffraction patterns produced by different color LEDs. After these examples, students can proceed to the remaining ALG activities in Section 24.7, specifically, ALG Activities 24.7.5 and 24.7.6 can be used as experiments in the labs, and Activity 24.7.5 is a good lead into the observational experiments for black body radiation in Chapter 27. You can assign them all problems in Section 24.7 and General Problems here. We recommend the following non-traditional problems: 44, 45, 48, 49, 62, 65, 67, 69.

# Electromagnetic Waves

In Chapter 24, students learned the wave model of light and used it to explain what happens when light passes through narrow openings or reflects off thin films. Although the wave model of light is successful in explaining and predicting a great variety of light-related phenomena, it does not contain the mechanism of the wave motion. Specifically, it does not explain what physical quantities change inside the light wave. In this chapter, students learn the electromagnetic nature of the light waves through the analysis of simple experiments and subsequent testing of the newly constructed explanations. Content-based learning objectives in this chapter are listed below.

**Students should be able to:**

1. Conduct experiments to observe the polarization of light and use that data to explain the nature of electromagnetic waves.
2. Describe in their own words what is happening in an electromagnetic wave and what experiments demonstrate their basic properties.
3. Design an experiment to observe Brewster angle.
4. Explain how electromagnetic waves are emitted and detected.
5. Calculate the longest and shortest range of a radar.
6. Explain what light phenomena can be explained using a particle-bullet model, wave model, and electromagnetic wave model.
7. Do simple calculations of the energy density and intensity of electromagnetic waves.
8. Describe the scientific steps of the investigations/reasoning that led to the discovery of electromagnetic waves.

The chapter is broken into five parts:

- I. *Polarization of waves*
- II. *The discovery of electromagnetic waves*
- III. *Some important applications of electromagnetic waves*
- IV. *The electromagnetic spectrum and a quantitative description of waves*
- V. *Polarization and light reflection. Applications.*

For each part, we describe the sequence of activities that can be used in lecture and recitations to help students develop these ideas. We also provide brief discussions of motivations for using these activities.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Polarization of waves	25.1		
Discovery of electromagnetic waves	25.2		
Some important applications of electromagnetic waves	25.3		
The electromagnetic spectrum and a quantitative description of waves	25.4, 25.5		
Polarization and light reflection. Applications	25.6		
<b>Nontraditional end-of-chapter questions and problems</b>  Choose answer and explanation (CAE): Q25.10 Evaluate (reasoning or solution...) (EVA): Q25.25, P25.39, P25.44, P25.45 Make judgment (based on data) (MJU): Q25.18, P25.14, Multiple possibility and tell all (MPO): Q25.26, P25.20, P25.30, P25.31, P25.32, P25.39 Jeopardy (JEO): P25.30, P25.31, P25.32 Design an experiment (or pose a problem) (DEX): Q25.13, Q25.18, Q25.21, Q25.27, P25.27, P25.39			

*Brief summary of student difficulties with electromagnetic waves*

Difficulties include visualizing three-dimensional waves and reconciling between static electric and magnetic fields and dynamic fields in the wave. From previous

chapters students know that  $\vec{B}$  and  $\vec{E}$  cannot be in phase because one is the cause of the other, yet they are in phase in an electromagnetic wave. Although waves are transverse, there is nothing spatial that is moving perpendicular to the direction of propagation (representing  $\vec{E}$  and  $\vec{B}$  with vectors might suggest that there is something that extends perpendicular to this direction). This issue becomes important when students need to reconcile the transverse wave model for EM waves with the passing of EM waves through a narrow slit(s)-diffraction. Another difficulty involves visualizing angles of polarization and keeping track of the axes of polarizers and directions of polarizations of light.

## I. Polarization of waves

Although students learned about several types of experiments that can be explained with the wave model, Chapter 24 did not touch on the polarization of waves. Polarization is the subject of this section.

It is important that students get a firm conceptual grounding of polarization in the context of mechanical waves. Thus, this section invites students to explore polarization in the context of transverse and longitudinal waves on a Slinky. In Observational Experiment Table 25.1, students observe that transverse mechanical waves can be polarized but longitudinal mechanical waves cannot. This is an excellent activity to do as a lecture demonstration, or have students perform experiments on their own; the matching Activity in the ALG is 25.1.1. Students devise an explanation for these observations. They then observe that light can be polarized in a similar way to mechanical waves, leading to the hypothesis that light is a transverse wave. This is followed by a quantitative analysis of the effect of polarizers on light (ALG Activity 25.1.2 or textbook Observational Experiment Table 25.2).

The intensity of light passing through two polarizers decreases in proportion to the square of the cosine of the angle between the polarizers (ALG Activity 25.1.3). Here it is important to discuss the concept of an ideal polarizer that only absorbs light due to polarization.

Polarization experiments lead students to the conclusion that light is a transverse wave; however, what is vibrating in a transverse manner in the light wave is still a mystery. The section ends with two hypotheses concerning the mechanism for the transverse light wave: light travels in a transparent massless elastic medium called the ether, or light is a new type of wave that doesn't require a medium to travel in. Students can use EOC Questions 1, 2, 11 and 12 and Problems 1–3 for more practice.

## II. The discovery of electromagnetic waves

Because the mechanism that explains how a wave propagating in a vacuum can be transverse involves electric and magnetic fields, students need to review these and

the relationships among them. Textbook Section 25.2 starts with a qualitative summary of Maxwell's equations. We then discuss one of the consequences of the equations, that a changing magnetic field can produce a changing electric field, which in turn can produce a changing magnetic field, and on and on in a sort of feedback loop. Figure 25.4 in the textbook illustrates this process. Thus one can conceptualize a light wave as a wave of changing electric and magnetic fields whose  $\vec{E}$  and  $\vec{B}$  vectors are perpendicular to the direction of wave propagation; hence the mechanism explaining how a transverse wave can propagate in a vacuum. The equations even predict the speed  $c$  of the waves in a vacuum.

The development of an understanding of electromagnetic waves is an excellent example of how physics develops as encapsulated in the Investigative Science Learning Environment cycle. This idea that electromagnetic waves could propagate without a medium was first tested in experiments by Hertz (Testing Experiment Table 25.3). If you are running a process-focused physics course, you can have students read the textbook section about Hertz's experiments and how they tested various aspects of Maxwell's hypothesis about electromagnetic waves. This development quickly leads to applications such as transmitting information via radio waves. The end of Section 25.2 is dedicated to the emission of electromagnetic waves by antennas and to a conceptual explanation of the phase difference between the  $\vec{E}$  and the  $\vec{B}$  field vectors in a wave (in phase) compared to the phase difference in the antenna ( $\pi/2$ ) (see Figure 25.7).

It is a real accomplishment if your students can build a conceptual understanding of how a simple half-wave dipole antenna can produce an electromagnetic wave when driven by an oscillating emf. We recommend that students start this section with ALG Activity 25.2.3. This activity helps them build up a detailed step-by-step picture as the antenna passes through one complete charging cycle. Once they have this understanding, they can work through the end of textbook Section 25.2 with you in lecture or on their own. Here they learn that these antenna fields are called near fields (with the phase shift), which later, after leaving the antenna, progress by self-propagation to produce so-called far field waves (in phase). Students can then work on the EOC Questions 3–7, 13–15 and Problems 3–10.



### III. Some important applications of electromagnetic waves

Three important applications of electromagnetic waves are the subject of Section 25.3: (1) the operation of radar for keeping track of air vehicles, specifically, the longest and shortest distances to objects that can be detected with a particular radar; (2) the way in which the global positioning system works, including triangulation; and (3) the interesting history and the details of the operation of microwave cooking (ALG Activities 25.3.3–25.3.4). Given that you need a microwave for these activities, we suggest that you perform them and your students observe, discuss and calculate, working in groups. You can use EOC Problems 11–16. Note that in the first application (radar), we do not get into the details of pulsed Doppler radar but stick with the most elementary radar, in which location and speed of a moving object are determined by the time interval between received and transmitted pulses.

### IV. The electromagnetic spectrum and a quantitative description of waves

Section 25.4 is dedicated to the physical quantities of frequency and wavelength describing electromagnetic waves and the electromagnetic spectrum (see Table 25.4). It is important to note that the word spectrum stands in this case for the range of electromagnetic waves in terms of their frequency, not the energy distribution versus frequency, which is another meaning of this word. Section 25.5 follows with the mathematical description of electromagnetic waves and their energy. We use unit analysis to show that  $E_{\max} = cB_{\max}$  and use this relation as well as the arguments for in-phase vibrations developed earlier to create mathematical functions describing the electromagnetic waves (matching ALG Activities are 25.5.1–25.5.4). Notice textbook Figure 25.14 that shows how to use the right-hand rule to relate the  $\vec{E}$  and the  $\vec{B}$  field vectors and the travel direction of the wave. The section also shows how to develop the expressions for the energy density and the intensity of the waves. We apply the knowledge of intensity to determine  $\vec{E}$  field amplitude from the solar constant in Example 25.3. Students can now try ALG Activities 25.5.5 and 25.5.6 and EOC Questions 10, 19, 25 and 26 and Problems 17, 20–22, 26–29, 30–32, 34–37, and 47 and 48.

## V. Polarization and light reflection. Applications

Students learned at the beginning of the chapter that light could be polarized. Section 25.6 returns to the subject at a new level. First, students learn the mechanism behind the production of unpolarized light and then study a simplified model of a polarizer and its interaction with light. The section gets into the mechanism of light polarization, whereas the first section merely discussed the existence of the phenomenon. The next step is learning about polarization by reflection and the Brewster angle. Students can perform the experiments described in the textbook in the labs, or you can perform experiments in a lecture setting. Ask your students to analyze the observed phenomena following the guidance provided in the table or ALG Activity 25.6.1, and then have them work through ALG Activity 25.6.2 to lead them to the final result of the Brewster angle (Equation 25.8). Do not skip the discussion in the textbook on page 803 and the experiment in Figure 25.21. Example 25.4 involves reducing the glare of reflected light. The polarization by scattering includes an analysis of why the sky is blue and sunsets are red (Figures 25.23 and 25.24). The polarization of light from LCDs is analyzed in Figure 25.26. We suggest that your students do ALG Activity 25.6.5 after they read this subsection of the textbook; it is an excellent capstone activity not only for LCD screens but also for light polarization and color mixing in general without requiring any mathematics. The role of polarization in 3D movies is the subject of the last part of the section (Figure 25.27). Students can now try EOC Questions 9, and 21–23 and Problems 33–40, and 49.

# Special Relativity

In the previous four chapters, students learned the nature of light and its applications. We now start five chapters concerning contemporary physics involving quantum concepts and wave/particle duality of light and matter. It is impossible to understand these concepts without understanding Special Relativity – the topic of this chapter. Content-based learning objectives of this chapter are listed below.

**Students should be able to:**

1. Use Galilean transformations of velocities to analyze motion problems.
2. State Einstein's postulates and the reasons for proposing them.
3. Analyze situations using Einstein's postulates.
4. Apply the concepts of time dilation and length contraction to explain relevant phenomena.
5. Construct and evaluate spacetime diagrams from different frames of reference.
6. Apply Einstein's energy and momentum equations to solve problems.
7. Describe applications of special and general relativity to everyday life and astronomy.

This chapter is broken into seven parts. For each part, we describe the sequences of activities that can be used in class to help students acquire the ideas on relativity and brief discussions of the motivation for using these activities.

- I. *Ether, postulates of special relativity, and simultaneity*
- II. *Time dilation and length contraction*
- III. *Spacetime diagrams*
- IV. *Velocity transformations*
- V. *Relativistic momentum and energy*
- VI. *The Doppler effect for electromagnetic waves*
- VII. *General relativity and Global Positioning Systems*

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Ether, postulates of special relativity, and simultaneity	26.1-26.3		
Time dilation and length contraction	26.4, 26.5		
Spacetime diagrams	26.6		
Velocity transformations	26.7		
Relativistic momentum and energy	26.8, 26.9		
The Doppler effect for electromagnetic waves	26.10		
General relativity and GPS	26.11, 26.12		
<b>Nontraditional end-of-chapter questions and problems</b>			
Evaluate (reasoning or solution...) (EVA): Q26.24, Q26.27, P26.4, P26.21, P26.29, P26.55, P26.68			
Make judgment (based on data) (MJU): P26.68			
Multiple possibility and tell all (MPO): Q26.14, P26.9, P26.24			

It is important to note that this is the first chapter where students cannot perform actual experiments. Most of the experiments are thought experiments or historical experiments. Therefore, careful planning of the activities and engaging students in discussions is very important here. You might use the textbook sections and ALG activities as a basis for class discussions and then let students read the textbook and go through worked examples on their own. Conceptually, what unifies all the sections in this chapter is the use of the “need to know” approach. Although many of the equations (especially in the second part of the chapter) cannot be derived in this course, we can definitely provide a motivation for the students for why new, relativistic expressions for familiar quantities are needed, thus creating “the need to know” these new expressions.

In our many years of teaching we encountered, broadly speaking, two types of students when teaching special relativity. The first group are “theoretical enthusiasts” who have been waiting for this topic during the whole course and are excited to talk about things that never happen in their lives. The second group are “practical skeptics” who have little interest in learning something that is mostly contained in

equations and does not lead to any tangible experiments. Both groups can be successful learning special relativity but you need to motivate them in different ways.

### *Brief summary of student difficulties with special relativity*

If you have ever taught special relativity, you probably know that all concepts in it is very difficult for students because they are very counterintuitive and students have no real-life experience to connect to when learning these new ideas. Given that the following ideas are crucial to understanding everything else, we suggest that you give them as much attention as possible: relativity of simultaneity, Einstein's second postulate, and the concept of the proper reference frame. If your students master these ideas, the rest will come much more easily.

## **I. Ether, postulates of special relativity, and simultaneity**

Although students have learned about electromagnetic waves that do not need any medium to travel through, historically physicists thought that light waves needed a medium to travel (polarization of light created the need for a medium where sheer deformations could occur). Thus, they invented *ether* as the medium through which light propagates. How could one test the existence of ether? To understand the essence of the Michelson-Morley experiment that is an example of a testing experiment for the model of ether, students can use the boat-in-a-river analogy (textbook Section 26.1). Notice that we have not done traditional up-down-across stream boat problems in the kinematics chapter intentionally because we planned to return to the concept of relative motion here.

Specifically, there is a boat race in a stream. One boat travels up a stream and back down. The second boat travels the same distance relative to the shore but across the stream and back. Using classical analysis, we find that the upstream-downstream trip should take longer than the across-and-back stream trip. This type of thinking led to a test for the ether performed by Michelson and Morley (see Testing Experiment Table 26.1). No ether was found. Therefore, the concept of ether can be discarded. We suggest that you lead students through this logical progression and assign end-of-chapter Questions 1 and 2. We do not have any parallel activities for this section in the ALG, thus we suggest that you either have a discussion with students in class and then they can read the book, alternatively, you could assign them to read this section first and then have a discussion in class followed by EOC Questions 1-3 and Problems 1-4.

Independently of the Michelson-Morley experiment, Einstein (based on thought experiments) proposed two postulates that formed the basis for special relativity (see Section 26.2). We recommend that before you start this section, students start by answering EOC Questions 13 and 14 and then solve Problems 5 and 6 to make sure

that they students understand the difference between inertial and non-inertial reference frames. After that they can start working with the postulates.

The most difficult thing for students is to understand that light travels at the same speed in all inertial reference frames, independent of the velocities of these reference frames relative to each other. Students can do ALG Activities 26.2.1 and 26.2.2 to build a foundation for this postulate and read Section 26.2. Students can proceed to EOC Question 7.

Section 26.3 involves a discussion of events and the possibility that events occurred simultaneously in one inertial reference frame but not in another. We, unfortunately, do not have an ALG activity to help students invent this idea, so a discussion and textbook reading might be the best options, then students can work through ALG Activity 26.3.2. It is important that students work thorough this textbook section because they meet Alice and Bob – the actors in many situations in the chapter. Your “theoretical enthusiasts” will enjoy this section very much.

## II. Time dilation and length contraction

Section 26.4 involves a derivation of time dilation and the introduction of the proper time between two events, which occur at the same place in a particular inertial reference frame. It is crucially important for students to understand which reference frame is considered the proper reference frame. EOC Questions 4 and 5 address this issue head-on.

We suggest that you have a whole class discussion that leads to the Equation 26.1 (including Testing Experiment Table 26.2) and then assign them to work on the ALG Activities 26.4.2 and 26.4.3. The experiment discussed in Testing Experiment Table 26.2 can peak student interest as it considers muon production in the atmosphere (remember those “practical skeptics”?)— something that can be seen in an experiment. Muons would be unable to travel to Earth’s surface if it were not for time dilation. Now your students are ready for EOC Questions 4, 5, and 9 and Problems 7–13 and 59–61.

Section 26.5 involves a derivation of an expression for length contraction. Here we recommend that students start with working on ALG Activity 26.5.1, then you have a whole class discussion with them concerning the final result (or they read the textbook section – ALG Activity 26.5.2) and then they go back to working on ALG Activities 26.5.3 and 26.5.4. Activity 26.5.3 helps students apply the contraction idea to explain the interaction of two current-carrying wires. Quantitative Exercise 26.1 takes students back to the familiar muons to explain how they can reach Earth. Use EOC Questions 6, 21, 22 and Problems 15–21 for more practice.

### III. Spacetime diagrams

Section 26.6 is a new section in the textbook that did not exist in the previous edition. The textbook material was written in collaboration with Paul Bunson and Paul also devised the ALG activities in this section. If you are not familiar with spacetime diagrams we suggest that you first familiarize yourself with them by working through the section and then through the ALG activities. After that, you might be able to decide if your students are able to start by working through the ALG Activities 26.6.1 and 26.6.2 before any whole class discussion, or reading the book, or if they will need a discussion before they attempt the first activity. The ALG section suggests that they do Activity 26.6.1 and 26.6.2 first, then you can have a class discussion followed by textbook work, and then they can return to the ALG Activities 26.6.4 and 26.6.5. EOC Question 12 and Problems 22, 23 and 64 and 65 will be very useful after that.

### IV. Velocity transformations

ALG Section 26.7 starts with novel activities with a scanner (Activities 26.7.1 and 26.7.2) that you can use with the students to prepare them for the derivation of the Galilean velocity transformation in the textbook. If you have a computer scanner (for these activities you need a scanner that allows the top cover to be removed or completely opened), you can perform the experiments in front of the class and have students discuss the questions. If you do not have a scanner, they can use the photos in Activity 26.7.1. This way the outcome of the derivation in the textbook section will be related to those experiences.

To foster students' "need to know" the relativistic velocity transformation, we show that the classical way of adding velocities does not work for high speeds (ALG Activity 26.7.3). We do not derive the expression for a relativistic velocity transformation; we provide it without the derivation and ask the students to evaluate its application to the known limiting cases (ALG Activity 26.7.4). You might follow the same strategy in class. ALG Activity 26.7.6 and EOC Problems 24–29 can be used for more practice and homework.

### V. Relativistic momentum and energy

Students learn that momentum, when calculated using the classical definition, stops being a conserved quantity (here students need to take this conclusion on faith, as we do not provide data to support this idea), and this finding motivates the relativistic expression for momentum. We derive a relativistic expression for momentum by replacing the  $\Delta t$  in the classical expression by the proper time from the time dilation equation. Students can use this new relativistic expression for the momentum expression when analyzing a cosmic ray hitting a nitrogen nucleus in Example 26.4. Students can work on ALG Activities 26.3.3 and 26.3.4 and EOC Problems 30–32.

To motivate the new expression for energy, students can conduct a classical energy analysis of the electron that passes through a potential difference of 300,000 V. They will find that the electron would be traveling at a speed greater than the speed of light (Section 26.8). This finding means that energy expressions must also be revised for relativistic situations. We provide new relativistic expressions for particle rest energy, total energy, and kinetic energy without proof, and students can apply the new expressions in Quantitative Exercise 26.5 that determines the mass energy equivalent needed to warm a home in winter and cool it in summer for one year—about  $2 \times 10^{-7}$  kg! Example 26.6 analyzes an electron particle accelerator. Students can follow up with ALG Activities 26.9.2–26.9.5 (the astronomy connections will appeal to both groups of your students) and EOC Question 24 and Problems 40–44, 46, 47, 4–51, and 67.

## VI. The Doppler effect for electromagnetic waves

Section 26.10 starts with building the “need” for a new expression for the Doppler effect for electromagnetic waves compared to sound waves as the result of the postulates of special relativity. We treat the Doppler effect traditionally by providing the expression for electromagnetic radiation for high and low speeds. Example 26.8 discusses the use of Doppler radar for determining an automobile speed and the speed of baseballs and tennis balls.

The most motivating part of this section for both types of students is a discussion of evidence for the expanding universe as observed and measured by Edwin Hubble and Milton Humason, leading to an estimate of the age of the universe. Notice Figure 26.16 that uses spacetime diagrams to explain how all galaxies can move away from each other no matter where the observer is located. We suggest the whole class discussion followed by textbook work and then EOC Question 25 and Problems 52–56.

## VII. General relativity and Global Positioning Systems

Section 26.11 introduces students to the principle of equivalence, along with the idea of space curvature. We describe testing this idea in the historical experiments of starlight bent as it passes the Sun and also by the speed of precession of Mercury about the Sun. We touch upon gravitational time dilation and red shift along with gravitational waves and black holes. Students can try end-of-chapter Questions 9 and 26.

Section 26.12 discusses the way in which global positioning systems (GPS) work, along with the effect of special relativity and general relativity on GPS location. Students in lectures or recitations can try end-of-chapter Question 10. We suggest that you encourage students to work through second Reading Passage on “Quasars”.



# Quantum Optics

In this chapter, we build the photon model of light. Students have been developing their understanding of light for quite some time now. In Chapter 22 they started with a simple ray model and then learned that the ray-like behavior can be explained by assuming that light behaved like a stream of low-mass fast bullets (the particle-bullet model) or with the wave model. In Chapter 24 they encountered experimental evidence that could not be explained with the particle-bullet model at all and developed more confidence in the wave model of light. Later (in Chapter 25) they learned what was “waving” in the light wave and revised their model of light again to arrive at the electromagnetic wave model.

In this chapter, they revisit the behavior of light one more time and revise the model of light. This complex process reflects the complex nature of light: it cannot be adequately compared to any macroscopic object, and that is why the final model of light that the students will learn in this book—the photon model—is full of apparent contradictions. Neither the particle-bullet nor the wave model on its own can explain and predict light behavior completely. Only a model that combines those two contradictory ideas into one entity—a photon, which is not a particle and is not a wave but possesses the properties of both—can explain light as a real phenomenon. Because the photon model of light introduces students to something with which they have no direct experience, they often oversimplify it, thinking that it is just a particle model with a different name. The goal of this chapter is to help students avoid this confusion. Content-based learning objectives of this chapter are listed below.

**Students should be able to:**

1. Describe black body radiation using Stefan-Boltzmann’s and Wien’s laws, and solve problems using these laws.
2. Use Planck’s relation for quantum energy to estimate the energy of light quanta.
3. Describe the experimental features of the photoelectric effect (PEE) that could be explained using the electromagnetic wave model of light.

4. Describe the experimental features of the PEE that could not be explained using the electromagnetic wave model of light.
5. Apply Einstein's model of the PEE to explain all observable features qualitatively and quantitatively. Use energy bar charts to explain the PEE.
6. Use the photon model to solve simple quantitative problems concerning energy and momentum.
7. Explain how X-rays are produced and when they might be or might not be dangerous.
8. Estimate the number of photons emitted by the Sun every second and the number arriving to Earth.

We've broken the discussion of this chapter into three parts:

- I. *Quantum (photon) model of EM radiation*
- II. *X-rays*
- III. *Photocells, solar cells and LEDs*

For each part, we provide examples of activities that can be used in the classroom and brief discussions of anticipated student difficulties with the subject matter. Because there are few simple experiments that students can perform in the chapter, the ALG activities are limited, and the bulk of the problem solving should be based on the material in the end-of-chapter problems.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Quantum (photon) model of EM radiation	27.1–27.4	OET 27.2 (p854)	
X-rays	27.5		
Photocells , solar cells and LEDs	27.6	27.1 (p874)	
<b>Nontraditional end-of-chapter questions and problems</b>  Choose answer and explanation (CAE): Q27.11 Evaluate (reasoning or solution...) (EVA): P27.27, P27.40, P27.41, P27.42, P27.43 Make judgment (based on data) (MJU): P27.43 Jeopardy (JEO): P27.15 Design an experiment (or pose a problem) (DEX): P27.43			

*Brief summary of student difficulties with quantum optics*

The biggest difficulty with the photoelectric effect is that students need to combine together knowledge of energy, the internal structure of metals, electricity, electromagnetic waves, and optics to understand experiments that led to the development of the explanation of the photoelectric effect. Students have difficulties interpreting graphs, which adds to all of the above. Another difficulty is remembering that intensity of light is function of two variables: it is equal to the number of photons emitted per second multiplied by the energy of each photon. If a light source emits light of constant intensity and the frequency of light changes, the number of photons should change too.

## I. Quantum (photon) model of EM radiation

The goal of Section 27.1 is to introduce students to the quantum hypothesis that was born as an attempt to explain how objects radiate light (black body radiation). Students learn about black bodies and observational evidence related to their emission of light, including Stefan's law and Wien's law. These are useful in general for applications concerning radiation from the Sun (see Quantitative Exercise 27.1), the stars, and in general for ideas related to body temperature and global temperature control. However, the main emphasis of this section is the failed efforts in the late 1800s to explain the intensity-versus-frequency spectrum of black body radiation using the wave model of light and Planck's quantum hypothesis, which did provide an explanation for this radiation and initiated the idea of quanta of light. The mathematical details of Planck's light quanta idea for black body radiation are beyond the grasp of the students in this course, but the concept of light emission as quanta is very important for the upcoming investigation of the photoelectric effect. We suggest that students first work with the ALG Activity 27.1.1 and then you have a whole class discussion concerning black body radiation and Planck's hypothesis, or students work with the textbook (ALG Activity 27.1.2). This work can be followed by EOC Questions 1–3 and Problems 1–7, and 51. Note that Activity 27.1.1 will encourage your students to review almost everything that they learned in preceding 15 chapters.

Textbook Section 27.2 is dedicated to the photoelectric effect but starts with a short review of the structure of metals. Students are already familiar with the classical models of metal structure from Chapter 19. What is important here is the emphasis on the electric potential energy of the electron-ion lattice system, which is negative (if we consider the system to have zero energy when the electron is not interacting with the lattice). The minimum positive energy needed to remove a negative electron from the metal is called the work function ( $\phi$ , Greek phi). Thus, the energy of interaction of the electron with the lattice is  $-\phi$ .

The investigation of the photoelectric effect starts with experiments in which ultraviolet light discharges a charged electroscope. You can use ALG Activities 27.2.1 and 27.2.2 in class or lab (if the students watch the video). The experiment in 27.2.1 can be performed as a class demonstration and students can answer the question posed in the activity or, if you do not have the required materials, students can work in groups watching the video. They can also read Observational Experiment Table 27.2, which summarizes experiments in which ultraviolet light discharges a negatively charged electroscope (whereas visible light does not). Neither ultraviolet nor visible light discharge a positively charged electroscope.

After students perform the ALG activities (described in Table 27.2), it is useful to discuss possible mechanisms that can explain why light would have an effect on a charged electroscope and why it only discharges a negatively charged electroscope and not a positively charged one (see ALG Activity 27.2.2 and two proposed explanations in the textbook after Table 27.2, as well as EOC Problem 43 that suggests explanations which students need to evaluate).

After students devise a qualitative mechanism for the discharge of the electroscope by electromagnetic waves, they can proceed to more detailed investigations of the phenomenon. ALG Activities 27.2.3<sup>1</sup>–27.2.6 and textbook Observational Experiment Table 27.3 describe historical experiments performed by P. Lenard, including important patterns he observed (note that in the textbook we emphasize that for some metals even visible light can produce the same effect as the ultraviolet).

Engage students in the discussion to explain the patterns (ALG Activities 27.2.3–27.2.5). ALG activity 27.2.3 is an activity that may be worth spending a little extra time on. It requires students to coordinate their understanding of circuits and electric potential to comprehend what is happening in the experiment. This is very challenging for them and takes time. However, this activity can be used to reintroduce earlier ideas and reinforce the usefulness of their understanding of electrical phenomena for understanding real-world experiments. Finally, students should progress to ALG Activity 27.2.6, which mirrors textbook Table 27.3 and summarizes quantitative patterns of the observations of photoelectric effect.

Make sure that you use energy bar charts to help students represent the process (the textbook guides students with this revised representation). Students will be able to explain some of them using the electromagnetic wave model of light (with your help and guidance) but not the cutoff frequency, stopping potential, and the immediate nature of the phenomenon. The key point is that students need to realize that there are aspects of this experiment that the wave model of light cannot explain. Make sure that you discuss with the students every pattern in the observations and how electromagnetic wave model of light can or cannot explain each pattern. This

---

<sup>1</sup> Note there is an excellent PhET simulation. “Photoelectric effect.” that can be used here to help students visualize what is going on and even collect quantitative data. Note that the PhET does not take into account the effect of the anode material.

discussion is presented in the textbook pages 857–858. The result of this discussion should be that a new model of light is needed to explain all of the observations. Remind students about Planck’s hypothesis explaining the emission of light using the quantum idea and ask if a similar idea could be used to explain the patterns for the photoelectric effect. In rare cases, students construct the “photon” model during such discussions. At home, students can try Question 4.

Section 27.3 introduces students to Einstein’s photon model and shows how it is consistent with all of the experiments in Table 27.3. They can start with the ALG Activities 27.3.1–27.3.2 to see how the photon model explains all of the observed patterns qualitatively and then read the book (ALG Activity 27.3.3). Note that we write the photoelectric effect equation  $-\phi + hf = K_e$  so that we can represent the process using a bar chart. Here the work function with the negative sign in front of it represents the initial potential energy of the electron-lattice system,  $hf$  is the energy added to the system, and  $K_e$  is the final energy of the electron. Note that we carefully mention that in all experiments with photoelectric effect, the cathode and the anode are made of the same material. If they are made of different materials, due to the way we record ejected photoelectrons (through photocurrent), the work function in the equation should be the work function of the anode. Given that the explanation of why this is the case is beyond the level of this textbook, we simplified the situation by always having the cathode and the anode made of the same material.

Textbook Testing Experiment Table 27.4 describes a simple testing experiment for the photon explanation of the photoelectric effect. We provide the summary of how Einstein’s theory addressed the observations of the photoelectric experiment near the end of the section, along with a definition of a photon. Finally, make sure that students work through Example 27.6, which shows how to combine the energy bar chart analysis with the mathematics to analyze photoelectric effect problems. Students can work on EOC Questions 4, 5, 9, 11 and 12 (the latter two are especially useful), and Problems 10–15.

The goal of Section 27.4 is to deepen students’ understanding of the complex nature of light. Table 27.6 in Section 27.4 describes the evolution of the light models that followed the increased experimental sophistication of those physicists who worked with light. It is important that students slow down here and really think of the properties of photons that allow them to explain all of the existing experimental evidence. A photon is not just a light particle; it is an object that has simultaneously particle-like and wave-like properties. An exciting experiment with low-intensity light provides more evidence for these unique properties of photons that do not have any analogs in our macroscopic world.

We suggest that you first let students work through ALG Activities 27.4.1–27.4.4, and then proceed to the Activity 27.4.5, which involves Vavilov-Brumberg’s experiment (described in Testing Experiment Table 27.6). After they struggle with the questions, they can open the book and read the description of the outcome of the experiment.

ALG Activity 27.4.6 is very useful here to help students consolidate the understanding of the photon's peculiar properties and develop argumentation skills. A relatively simple derivation of photon momentum occurs at the end of the textbook section (parallel to ALG Activity 27.4.7). Note that the ALG Section 27.4 has a long list of various activities that can be done to help students understand the photon model and photon-based explanation of photoelectric effect deeper. Use your own judgment to choose the activities. From the EOC section we especially recommend Questions 6–8, 13–15 and on Problems 21–28.

## II. X-rays

The goal of textbook Section 27.5 is to introduce students to X-rays as photons of energy that is much higher than that of visible light. We use the historical path here, moving from cathode ray tubes and the discovery of the electrons to the discovery of X-rays by Roentgen, who was working with cathode ray tubes. There are two important points here. First, careful analysis of cathode rays allows the students to review mechanics and magnetic fields material. Second, Roentgen's accidental discovery of X-rays provides a particularly interesting example of the practice of science and the importance of careful observation and testing. The section describes the investigations that showed that properties of X-rays had all the properties of EM radiation, only with much shorter wavelength than other forms of this radiation known at the time. However, the fact that X-rays ionize gases provides the need for the photon model. Note Example 27.8 that combines knowledge of electricity, energy and photons. Do not skip it!

X-rays provide a good connection to medicine. Quantitative Exercises 27.9 and 27.10 provide estimates of the number of X-ray photons absorbed during a chest X-ray and the number absorbed from background radiation from our environment. EOC problems that students can work on to help develop qualitative and quantitative understanding include Problems 31, and 33–36.

## III. Photocells, solar cells and LEDs

We use photocells, solar cells and LEDs as examples of many interesting applications of photoelectric effect in Section 27.6. We suggest that students work on ALG Activities 27.6.1–27.6.5 that provide a unique introduction to semiconductor physics using LEDs that are already familiar to the students. Through these unique activities students learn more about the internal structure of LEDs and solar cells and simultaneously learn to test multiple hypotheses. None of the ALG activities requires prior knowledge of the internal structure of semiconductors but they prepare the students by creating “the need to know” of what is inside LEDs and solar cells.

After they work through these activities, they can read the textbook and learn the details of the microscopic structure of pure and doped semiconductors and p-n junction and its role in the production of a current in the presence of light in solar cells and of light when an LED is connected to a battery. Activities that are used in the ALG and many others that your students enjoy are described in the series of papers dedicated to LEDs published in *The Physics Teacher* [references 19 and 20 on the reference list in Chapter 1 of the Instructor Guide]. In the textbook we provide unique photographs and the video of the inside structure of LEDs that allow the students to see the p-n junction of a glowing LED. In addition, we recommend another paper (published in *Physics Education* [reference 24 on the reference list]) describing kinesthetic activities to help students understand the microscopic nature of pure and doped semiconductors and p-n junction. You can assign EOC Problems 37–41.

When the students finish working through the whole chapter the following General Problems will be challenging and motivating: Problems 42–46 and 54.

# Atomic Physics

In Chapter 26, students learned about the dual wave-particle nature of light. In this chapter, these ideas are used to help build atomic models to account for observations in early atomic physics and to develop the concept of the dual wave-particle nature of elementary particles. We continue to use the representations that students are already familiar with, such as force diagrams and energy bar charts, but in addition we introduce a new representation—the energy states diagram. We also are careful about language usage. Because an electron by itself can only have kinetic energy, we do not talk about the electron energy levels in the atom but instead talk about the energy states of the atom. This wording assumes that the kinetic energy of the nucleus is zero. Content-based learning objectives in this chapter are listed below.

**Students should be able to:**

1. Describe experimental evidence that led to the creation of a planetary model of the atom and experimental evidence that rejected this model.
2. Explain why the energy of a nucleus-electron system in an atom has to be negative.
3. Derive the ground state energy value of hydrogen atom using Bohr postulates.
4. Apply the uncertainty principle to reason about quantum phenomena.
5. Read and interpret symbolic representations of electrons in atoms (shells).
6. Describe the historical experiments and explanations that led to quantum mechanics
7. Use Bohr's model to predict wavelengths of visible spectral lines in the spectrum of hydrogen and use a spectroscope to compare the experimental outcome to the prediction. Interpret emission and absorption spectra.
8. Use Pauli's exclusion principle and atomic quantum numbers to reason about the behavior of electrons in atoms.
9. Describe the contemporary model of an atom.
10. Apply ideas of wave mechanics to simple situations.



This chapter is broken into five parts:

- I. *Early atomic models*
- II. *Bohr's model of the atom*
- III. *Spectral analysis and lasers*
- IV. *Quantum nature of atoms, the exclusion principle, and many-electron atoms*
- V. *The uncertainty principle*

For each part, we provide examples of activities that can be used in the classroom and brief discussions of anticipated student difficulties with the subject matter.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Early atomic models			
Bohr's model of the atom			
Spectral analysis and lasers			
Quantum nature of atoms, the exclusion principle, and many-electron atoms			
The uncertainty principle			
<b>Nontraditional end-of-chapter questions and problems</b>			
Evaluate (reasoning or solution...) (EVA): Q28.13, Q28.14, Q28.15, Q28.17, P28.27			
Make judgement (based on data) (MJU): Q28.18, Q28.22, Q28.28, P28.16, P28.58			
Design an experiment (or pose a problem) (DEX): P28.35, P28.56			

### *Brief summary of student difficulties with atomic physics*

The biggest difficulties that students experience with the material of this chapter are due to the fact that they need to imagine something that you never see and accept ideas that contradict physics that they have learned so far (Bohr's postulate that "the electron does not radiate although it accelerates", the inherent uncertainty of physics quantities, the probabilistic nature of events, and so forth. Other difficulties are understanding that the total energy of the atom is negative, why we need several atomic numbers and what they mean, and that "energy levels" in the Bohr's model are not spatial levels. Another major difficulty comes from an overwhelming amount of information. We suggest that you make decisions concerning what to keep and what to cut in this chapter.

## I. Early atomic models

As we said in the introduction to this chapter, students do not have concrete images of the models that they work with in this chapter. One of the ways to help them become comfortable with these abstract ideas is to build on their episodic memory, the memory that is connected to some stories and events. That is why there are more stories in this (and in the previous and the following two chapters) than in the earlier chapters.

A discussion of experiments leading to the “plum pudding” model and the Rutherford orbital models of the atom (Section 28.1) provides an excellent chance to help students base model building on experimental observations. ALG Activities 28.1.1–28.1.2 and 28.1.3 provide such observational experiments for the students and start their reasoning process concerning the structure of the particles that produce line spectra (Activity 28.1.1 is easy to reproduce in a lab or project on the screen in a lecture setting).

Because students are familiar with the plum pudding model of the atoms, you can focus their attention on how this model can account for the observational evidence. For example, you could ask students to think briefly about what type of scattering you might expect from positively charged elementary particles shot at a thin sheet of plum pudding atoms. Then ask them to reconcile their thinking with that of neighboring students. Proceed to ALG Activity 28.1.4, which describes the Geiger and Marsden experiments of alpha particle scattering. ALG Activity 28.1.5 helps students understand the nature of the experiment and the reasoning process through which Rutherford might have arrived at the nuclear model of the atom. ALG Activities 28.1.6 and 28.1.7 help them understand the model in depth and question its limitations—the difficulties of the planetary model and the need to revise it. Note that this section presents a historical development of the atomic model and illustrates a nonlinear path of science. Students can then work on EOC Questions 1, 2, 13, 14 and 18 and Problems 2–4.

## II. Bohr’s model of the atom

The next step in the investigation of the atomic structure is Bohr’s model. Although its applicability is very narrow and it does not represent an ideological shift from classical physics, we have included the model with all of its mathematical complexity in this book. The main reason is that it allows students to create a concrete image of the atom and thus provides the link between the planetary model and the quantum mechanics model of the atom that students learn later.

We suggest that students start with ALG Activity 28.2.1 or a discussion of a similar nature. ALG Activities 28.2.2–28.2.5 help students get a qualitative feel for the usefulness of the model. The derivation of Bohr radii and one-electron atom energy states takes considerable time in lectures (ALG Activity 28.2.6). Ask students to write down the fundamental equations used in the derivations (Coulomb’s force

law, Newton's second law for circular motion, the electrical and kinetic energies of the atom, and the postulate about quantized angular momentum). It is really important for the students to understand that the total energy of the atom is negative, and this is why it can be a bound system. Notice that we do not talk about energy levels of an electron in the atom, but instead we talk about the energy states of the atom, as without the nucleus the electron would only have kinetic energy. It is the atom as a system that has negative electric potential energy and total negative energy. We also do not talk about the electrons in excited states but atoms in excited states.

ALG Activities 28.2.7–28.3.10 help students become more comfortable with the model. Ask the students to confirm that the Balmer spectral lines are consistent with atomic transitions from higher states to the  $n = 2$  state—this is, in a way, a testing experiment.

Questions and problems that you can use to help students develop qualitative and quantitative understanding include ALG Activities 28.2.11 and 28.2.12, EOC Questions 3–5, 19, 20 and Problems 5–8, 10, 11, 20, 64, and 66.

### III. Spectral analysis and lasers

We spend considerable time on spectral analysis because it is one of the most important applications of the concepts in this chapter. Start students working on ALG Activities 28.3.1–28.3.6 or follow the textbook exploration in Section 28.3. There we discuss different ways that atoms are excited such as discharge tubes (electric field excitation), and hot atoms (as on the Sun) colliding (thermal excitation). Students investigate stellar spectra in Observational Experiment Table 28.2. Students often think that the dark lines in the stellar spectra are due to missing elements that do not emit light of particular wavelengths; thus, it is important that they think of how to test these ideas. Notice an important discussion on plants and chlorophyll at the end of the section, important for life science students.

Section 28.4 is dedicated to stimulated emission and methods to produce population inversion and laser light. We suggest that students work through ALG Activities 28.4.1–28.4.4, and, for both sections, on EOC Questions 6–9, 31–34 and Problems 18–22, and 25–28.

### IV. Quantum nature of atoms, the exclusion principle, and many-electron atoms

Textbook Section 28.5 describes the experiments and reasoning processes that led to the invention of the quantum numbers  $l$ ,  $m_l$ , and  $m_s$  for atomic states in multielectron atoms. The end of the section is dedicated to the summary of the quantum numbers along with the Pauli exclusion principle. Notice how this section

describes historical struggles along this path and is a perfect example of how models that would later be proven to be wrong stimulate the scientific progress. Although this section contains rather complex material, we hope that the story it is telling will be compelling for the students. EOC Questions 12, 25, 26 and EOC Problems 29 and 41–45 will be useful here.

Textbook Section 28.6 introduces students to a new concept of the wave nature of particles. The ALG sequence that parallels the textbook story is in Activities 28.6.1–28.6.5. It is important that you also assign Reading Exercise (ALG Activity 28.6.7) as it posed an important question concerning the nature of science. The textbook uses a similar experimental and reasoning progression as that outlined in the ALG activities and also describes testing experiments for the concept of an electron as a wave (Table 28.4) and the story of Davisson and Germer. Although the textbook introduces the students to wave functions, we do it only qualitatively and suggest that you do not put this material on tests.

Textbook Section 28.7 combines earlier results in the chapter to describe atomic states in multielectron atoms and to analyze the periodic table. Students learn about the concept of subshells, shells, and ground state configurations. The EOC Questions relevant to sections 28.6 and 28.7 are 10, 27 and 28 and EOC Problems 30–33, 35, 36 and 41.

## V. The uncertainty principle

Textbook Section 28.8 develops the position-momentum version of the uncertainty principle (through the analysis of particle diffraction through a single slit) and then proceeds to the energy-time version. We use this latter idea to explain electron tunneling and the possibility of mutations occurring in DNA replication. This happens if a proton in one of the strands that forms a bond between the electron pair on the opposite sides of the DNA spiral happens to be in the wrong well when the replication occurs. Students can practice the uncertainty principle by solving EOC Problems 55–58.

All problems in the General Problems section are useful to assign at the end of the section. The Reading Passages about electron microscope and electron transport chains in photosynthesis and metabolism are rather technical but might be interesting for bio-inclined students.

# Nuclear Physics

Chapter 28 involved atomic models and atomic applications, including spectroscopy. In this chapter, students learn the composition of a nucleus. We follow the historical path here and continue to use the tools that students have developed, such as energy bar charts. The history is especially important in this chapter, not only because it provides an excellent example of how physicists construct knowledge but also because in this chapter students meet female physicists for the first time. Until now, all the historical figures noted have been males. The content-based learning objectives in this chapter are listed below.

**Students should be able to:**

1. Describe experimental evidence concerning types of radioactive decay.
2. Explain three types of radioactive decay using microscopic descriptions.
3. Explain how we know that there are no electrons in the nucleus.
4. Apply the ideas of binding energy to explain fission and fusion.
5. Estimate at what temperature two protons could fuse together.
6. Estimate how long the Sun can shine if it is powered by fusion in 10% of its mass (surrounding the center).
7. Represent fission and fusion with energy bar charts.
8. Interpret graphs for half-life.
9. Apply the knowledge of nuclear physics to real-life and medical applications.
10. Describe the contemporary nuclear model and be able to discuss what the nucleus looks like.

The chapter is broken into four parts:

- I. *Early nuclear models*
- II. *Nuclear force, binding energy, and nuclear reactions*
- III. *Radioactive decay, half-lives, and radioactive dating*
- IV. *Ionizing radiation and its effects*

For each part, we provide examples of activities that can be used in the classroom and brief discussions of anticipated student difficulties with the subject matter.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Early nuclear models	29.1, 29.2		
Nuclear force, binding energy, and nuclear reactions	29.3, 29.4, 29.5		
Radioactive decay, half-lives, and radioactive dating	29.6, 29.7, 29.8		
Ionizing radiation and its effects	29.9		
<b>Nontraditional end-of-chapter questions and problems</b>  Evaluate (reasoning or solution...) (EVA): P29.25, P29.26, P29.61 Jeopardy (JEO): P29.24 Design an experiment (or pose a problem) (DEX): Q29.12, P29.35, P29.51			

## I. Early nuclear models

Section 29.1 tells a story of the early development of understanding of the constituents of nuclei (the studies of radioactivity), focusing on the experiments that led to the concept of a changing nucleus. It starts with Becquerel's accidental discovery that uranium salt placed on top of a covered photographic plate exposed the plate. Marie and Pierre Curie studied the nature of the radiation sources exposing the photographic plates (the textbook discussion on page 922). Rutherford and his group deflected the radiation in a magnetic field (Testing Experiment Table 29.1) and found that the radiation consisted of alpha particles, electrons, and gamma rays. This discovery ultimately led to a nuclear model whose constituents were alpha particles and electrons.

ALG Activities 29.1.2–29.1.6 provide a sequence of exercises for the students to do before or in parallel with reading the textbook. You can work through the ALG activities in class with them and then assign the textbook section for homework or let them read the textbook on their own and then do ALG activities. Note a short discussion of the process that led to the discovery of the proton by Rutherford and colleagues at the end of textbook Section 29.1.

The experiments described in Section 29.1 led physicists to believe that there were three types of radiation emitted by atomic nuclei, one of which behaved as an electron and one as an alpha-particle. Therefore, it was reasonable to hypothesize that electrons and alpha particles were inside the nuclei. Textbook Section 29.2 explores the possibility that the electron was a nuclear constituent. We show with calculations,

based on the uncertainty principle, that if that were the case, the positive kinetic energy of the electron would be too large compared with the negative electric potential energy of the interaction of the electron with the nucleus, therefore making it impossible for the electron to be bound inside the nucleus. The calculation is rather complicated, so you will need to decide whether your students are ready. If you do decide it is too much, you can discuss in qualitatively.

To prompt students to start thinking about the issue, they could start with ALG Activity 29.2.1 and then lead a whole-class discussion on the subject. Here it is productive to draw a bar chart for the nucleus as a system. Because the electrons cannot be in the nucleus (they were needed to explain the charge and the mass of the nuclei and especially alpha particles), then what gives the nucleus (and an alpha particle) its mass? A neutral particle was proposed as a nuclear constituent and was discovered at Cambridge University in experiments by Chadwick (ALG Activity 29.2.2). The new nuclear model now consisted of protons and neutrons. Neutrons explained the observations of isotopes for some nuclei (Observational Table 29.2). Notice the gradual building of the nuclear model, showing the process through which scientists continuously invent and refine models for the observed phenomena. It is another opportunity for students to get a clear picture of the process by which scientists build their knowledge.

We introduce the  ${}^A_Z\text{X}$  terminology for nuclei right after the neutron discussion. Students are fairly successful in acquiring the ideas in these two sections. To help develop facility with these ideas, they could work through ALG Activities 29.2.4 and 29.2.5 and Conceptual Exercises 29.1 and 29.2 in the textbook and then answer EOC Questions 1 -6, 10, and 11-15, and work on EOC Problems 1, 3, and 5.

## II. Nuclear force, binding energy, and nuclear reactions

Section 29.3 in the textbook provides a logical sequence of arguments that lead to the need of an attractive force that nucleons exert on each other inside the nucleus (ALG Activity 29.3.1). Consequently, because of this attraction, we can talk about the negative energy of interaction of nucleons.

We do not define this energy as binding energy—if we did, it would be negative and look unusual on the graphs. Therefore, we choose to define the binding energy of the nucleus as the magnitude of the energy that must be added to the nucleus to separate its constituents. For students, we start with the binding energy of the atom, and based on similar reasoning we predict the mass defect of the nucleus. We suggest that you have a whole-class discussion following the logic of the section, building up to Equation 29.1, with students working together in class repeating the calculations to make sure they understand what mass defect means. We use energy bar charts to keep track of all energies involved, thus make sure you help students interpret the bar charts

in Figure 29.7 in the textbook. Why is final nuclear energy negative? Students can work on ALG Activity 29.3.2, which connect the mass defect and binding energy to the idea of fusion. EOC problems to work through are 6, 7 and 10.

The goal of textbook Section 29.4 is to lead students to the rules governing nuclear reactions (Observational Experiment Table 29.3 or ALG Activities 29.4.1–29.4.3 present reactions that have been observed and the reactions that have not been observed). Students analyze the data and deduce two rules that apply to all reactions: the electric charge is the same before and after the reaction (charge conservation ( $Z$ )), and the number of nucleons in the nucleus or mass number are the same before and after the reaction (conservation of ( $A$ )).

The discussion that follows the invention of the two rules leads students to the idea that the energy should also be a conserved quantity in these reactions. Textbook Testing Experiment Table 29.4 provides students with the testing experiments for this rule. We do not describe the actual experiments in which the data could have been collected, just the outcomes, so this particular table is in a way a check-mark for the process through which the knowledge is constructed, rather than the description of a real experiment.

There is a very important consequence of the rules that students devise. If the products of the reaction have less mass than the reactants, the energy equivalent of the mass difference is provided to the reactants as kinetic energy of the random motion of the particles. Thus, nuclear reactions can produce kinetic energy, such as the thermal energy on the Sun due to fusion reactions and in nuclear power plants due to fission reactions – the subject of Sections 29.5 in the textbook and ALG. For nuclear reactions students can work on Problems 13, 14, 19–21, 27 and 28.

Notice the discussion in Section 29.5 concerning fusion and the bar charts in Figure 29.9 in the textbook. It is crucial that students connect the qualitative analysis of the fusion process with the bar charts before they start working on the worked examples – Quantitative Exercise 19.4 and Example 29.5. In Chapter 15 students did a calculation showing that the thermal energy of the Sun is not sufficient to maintain its luminosity for billions of year. Now they are exploring whether nuclear energy is a possible source. ALG Activities 29.5.1–29.5.3 achieve the same goals but we still recommend using textbook Figure 29.9 in the discussions.

Fission receives a great deal of attention too. We first present an example of how breaking of large nuclei can produce energy (the example of the actual historical experiment) and then describe the history of the discovery of fission, demonstrating once again the path that science takes from an unexpected result of an experiment to the creation of a new model and its subsequent testing.

The history of fission is important not only because it shows how scientists develop new models but also because one of the leaders in this process was Lise Meitner. Until now, students have met only two women that contributed to the development of physics—Marie Curie and her daughter Irène Joliot Curie. Lise Meitner is the third. We suggest that you spend time on the fission story to emphasize the importance of diversity in science and the unfortunate fate of many women who



did excellent work but were not recognized for their contributions. Lise Meitner is one of them, but another important name is Emmy Noether (1882–1935). Despite the fact that her work explained the reason for the existence of laws of conservation, most physics students complete their physics majors without knowing her name. ALG activities that you can use to help students figure out the fission process on their own before they read the textbook are Activities 29.5.4–29.5.6. Afterwards they can proceed to EOC Questions 19–22 and Problems 18, 19, 22–24 and 64–67.

### III. Radioactive decay, half-lives, and radioactive dating

Students learned about the three types of radioactive decay (alpha, beta, and gamma) in the first section of this chapter, but they did not have enough knowledge to explain the mechanism of those processes. Now that they are equipped with the knowledge of the constituents of the nucleus, they can dig into the mechanisms.

ALG Activity 29.4.2 that students have already done represents the work of Frederick Soddy. We suggest that they go back to this activity but this time have a discussion of the mechanism behind the reactions. You can either have a whole class discussion using the material in Section 29.6 of the textbook or let students read the book and complete ALG Activity 29.6.1. Notice the logic that leads to the neutrino hypothesis and the weak interaction idea (another demonstration of the path of science)—when evidence cannot be explained using existing models, physicists invent new models and subsequently test them experimentally. We suggest that you lead students through the worked examples in this section (Examples 29.7–29.9) so that they get a feel for the numbers involved in the processes, especially the energy that is released. Especially important is Example 29.9, which analyzes the energy absorbed in our bodies due to potassium-40 decay—an interesting biological example that provides a real-life connection for the students. EOC Questions that are useful here are 8, 9 and Problems are 26–29 and 33 and 34.

Students can develop the idea of the half-life of radioactive decay of a radioactive sample by working with ALG Activity 29.7.1 and reading Observational Experiment Table 29.5. They are capable of developing a relatively simple rule  $\frac{N}{N_0} = \frac{1}{2^n}$  for the ratio of radioactive nuclei remaining after  $n$  half-lives. Everything is fairly straightforward to this point, and many quantitative calculations can be done by students using just these ideas. The math becomes more difficult when the decay rate ( $\frac{\Delta N}{\Delta t} = \lambda N$ ) and exponential function are introduced. The decay rate equation needs some careful attention (why the minus sign in front of  $\frac{\Delta N}{\Delta t}$ , the meaning of  $\lambda$  and its relationship to half-life, and why the decay rate is proportional to  $N$ ). Students can work on ALG Activity 29.7.2 after this discussion. EOC Questions are 18 and Problems 35–41.

In Section 29.8, we derive an expression for the age of a radioactive sample with its application to carbon dating. As there are a reasonable number of equations based on the exponential function and used in applications in Sections 29.7 and 29.8, we suggest that you make an effort, after all of the equations have been derived, to organize them and their applications into a neat table so the material does not become an equation searching plug-and-chug effort by students. There are several interesting biological applications in these sections, including use of radioactive carbon nuclei to determine the source of carbon in plants, the volume of blood in the human body (Example 29.10), and determining the age of an old bone. All problems in ALG Section 29.8, and Problems 46–50 provide good practice for students.

## IV. Ionizing radiation and its effects

All of Section 29.9 is devoted to the definitions of quantities used to describe the absorption of ionizing radiation by the human body: absorbed dose (rad), biological effectiveness, and dose equivalent (rem). Conceptual Exercise 29.12 estimates the number of ions produced by a chest X-ray. We proceed to the discussion of the natural and human-made sources of ionizing radiation, along with the effects on the body of ionizing radiation. Students can apply the ideas in this section by working through ALG Activities 29.9.1 and 29.9.2 and EOC Problems 52–56.

The first Reading Passage will be interesting for biologists and the second one, for architects and art lovers.

# Particle Physics

Chapters 26, Relativity, and Chapter 29, Nuclear Physics, form the basis for some of the material in this chapter. We examine the most recent developments in physics of fundamental particles) and in the cosmological history of the universe. The content-based learning objectives of this chapter are listed below.

**Students should be able to:**

1. Describe the contemporary model of fundamental interactions.
2. Discuss what families of particles are and why are they important.
3. Explain how beta decay of a neutron was instrumental for finding a neutrino.
4. Explain why we can't detect individual quarks.
5. Explain how and why we know about the existence of dark matter.

The chapter is broken into three parts:

- I. *Particle physics*
- II. *Fundamental interactions and the standard model*
- III. *Cosmology*

The ALG activities for this material are mostly qualitative and focus on explanations not the construction of knowledge and in general we do not treat the material in this chapter quantitatively. It is under development at the forefront of present physics research. We suggest that you assign students to read specific sections of the chapter and then have whole class discussions with them using the ALG Activities, Review Questions and EOC questions and problems.

## I. Particle physics

The focus in Section 30.1 is on the early proposal of the existence of anti-particles, on the discovery of these particles, and on the interesting applications of anti-particles in our lives. On the basis of his theory of relativistic quantum mechanics, Paul Dirac first predicted the existence of virtual electrons in negative energy states, which then became positive holes (positive electrons) when in excited positive states. Carl Anderson detected positive anti-electrons called positrons in 1931 (see Figure 30.1). Positrons could be produced by the radioactive decay of medium light nuclei, such as  $^{11}\text{C}$ ,  $^{13}\text{N}$ ,  $^{15}\text{O}$ ,  $^{22}\text{Na}$ ,  $^{40}\text{K}$ ; by pair production due to a high-energy gamma ray; and by the transformation of a proton into a neutron in a nucleus. The annihilation of positrons by pair annihilation is analyzed in Conceptual Exercise 30.1 (see Figure 30.2). Positron emission tomography (PET) is analyzed at the end of Section 30.1 and in Figure 30.3. Other anti-particles are introduced at the end of the section. Students can work on the ALG activities in this section, EOC Questions 5 and 6, and Problems 3–9.

## II. Fundamental interactions and the standard model

After a brief introduction to the four fundamental interactions, students focus on the electromagnetic interaction. We compare the action-at-a-distance model to the field interaction, which is mediated by the exchange of a virtual photon. Because the interaction can be over a long distance and involve a relatively long time interval  $\Delta t$  to occur, according to the uncertainty principle the energy of interaction  $\Delta E \geq \hbar/\Delta t$  is small. We then introduce the other interactions with their particle mediators. The size in terms of the energy of these other mediators depends on the distance and time interval for the interaction. See the summary in Table 30.1. We summarize the units used in particle physics in Quantitative Exercise 30.2.

The classification of elementary particles in terms of leptons, such as electrons (Table 30.2), and hadrons, such as protons and neutrons, is the subject of Section 30.3. The hadrons (Table 30.3) are further broken into those with two quarks (mesons such as pions) and those with three quarks (protons, neutrons, and so on). We describe experimental support for the idea of hadrons being comprised of quarks (i.e., the experiments in which electrons are scattered from the hadrons) similar to the alpha particle scattering from protons and neutrons in gold nuclei. We follow with the review of the development and summary of the standard model near the end of Section 30.3. For Sections 30.2 and 30.3 students can work on the ALG activities in the same sections and EOC Questions 1–4, 8–12 and Problems 9–14 and 15–27.

### III. Cosmology

A 13.7-billion-year history of the universe since the Big Bang is provided in Section 30.4. Students should know that their bodies are composite objects constructed from particles that have been around for 13.7 billion years; the heavy elements such as carbon, iron, potassium, and so forth were produced during supernova events and made their way to Earth over that time interval.

The recent unsolved mysteries involving dark matter and dark energy are the subjects of Section 30.5. Students can easily understand the need for dark matter by using circular motion dynamics to analyze the speed of our own Earth about the galactic center. This can be extended to the motion of galaxies in the Coma Cluster. The source of the missing mass that is needed to cause the higher speed circulation is an unsolved problem. An even greater problem is the source of dark energy that is causing the universe to expand at an accelerating rate. Together the dark matter and dark energy must amount to about 94 percent of the mass energy in the universe. There are many problems in physics left to understand. The pursuit of this knowledge is the subject of the brief concluding Section 30.6 of the textbook. Problems that students can work to help appreciate the need for dark matter and dark energy are the ALG activities in this section and EOC Problems 28–32 and 33–38.

The Reading Passage is dedicated to the famous solar neutrino problem.

