Answers

20. Let f be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the x-coordinates of the points of inflection of the graph of f?
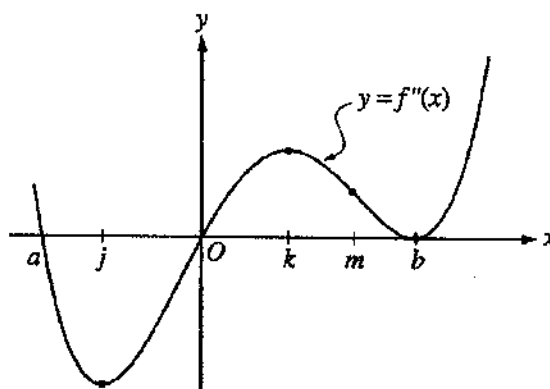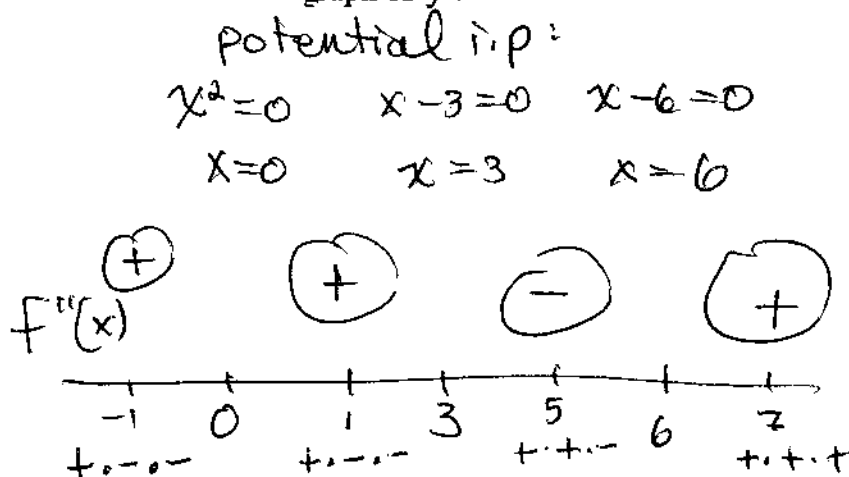
(A) 0 only

(B) 3 only

(C) 0 and 6 only

(D) 3 and 6 only

(E) 0, 3, and 6

Potential i.p:

$x^2 = 0 \qquad x - 3 = 0 \qquad x - 6 = 0$

$x = 0 \qquad x = 3 \qquad x = 6$

$f''(x)$

$\boxed{+}$ $\qquad$ $\boxed{+}$ $\qquad$ $\boxed{-}$ $\qquad$ $\boxed{+}$

$-1 \qquad 0 \qquad 1 \qquad 3 \qquad 5 \qquad 6 \qquad 7$

$+ \cdot - \cdot - \qquad + \cdot - \cdot - \qquad + \cdot + \cdot - \qquad + \cdot + \cdot +$

21. The second derivative of the function f is given by $f''(x) = x(x - a)(x - b)^2$. The graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?
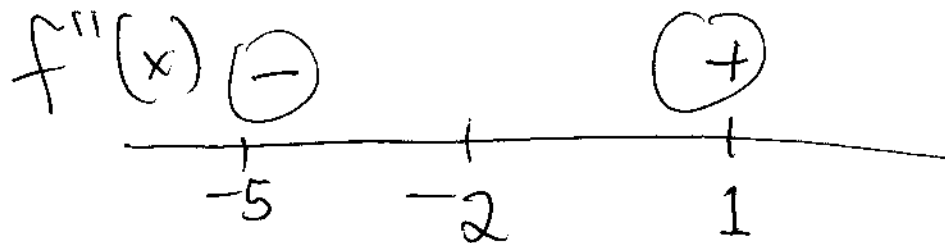
(A) 0 and a only $\qquad$ (B) 0 and m only $\qquad$ (C) b and j only $\qquad$ (D) 0, a, and b $\qquad$ (E) b, j, and k

$$f''(x) \quad \ominus \qquad \qquad \oplus$$

$$\xrightarrow{\qquad | \qquad | \qquad\qquad | \qquad}$$
$$\qquad -5 \qquad -2 \qquad\qquad 1$$

17. Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

(A) $x < -2$ (B) $x > -2$ (C) $x < -1$ (D) $x > -1$ (E) $x < 0$

$$f'(x) = 2e^x + 2xe^x$$

$$f''(x) = 2e^x + 2e^x + 2xe^x$$

$$0 = 4e^x + 2xe^x$$

$$2e^x(2+x) = 0$$

$$2e^x = 0 \qquad x = -2$$

never   potential

i.p.

Challenge Problems:

87. The function f has first derivative given by $f'(x) = \dfrac{\sqrt{x}}{1 + x + x^3}$. What is the x-coordinate of the inflection point of the graph of f?

(A) 1.008   (B) 0.473   (C) 0   (D) -0.278 (E) The graph of f has no inflection point.

$$f''(x) = \frac{\left(\frac{1}{2}x^{-\frac{1}{2}}\right)\left(1+x+x^3\right) - x^{1/2}\left(1+3x^2\right)}{\left(1+x+x^3\right)^2}$$

80. The derivative of the function f is given by $f'(x) = x^2\cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$?

(A) One   (B) Two   (C) Three   (D) Four   (E) Five