Indefinite Integration and Motion- Practice for Quiz Name: Answers
AP Calculus

Determine the anti-derivative of each function below.

1) $f'(x) = \frac{1}{x^4} = x^{-4}$

f(x) = $-\frac{1}{3}x^{-3} + C$

or $\frac{-1}{3x^3} + C$

2) $f'(x) = \frac{5}{\sqrt{x}} = 5x^{-1/2}$

$f(x) = 2 \cdot 5x^{1/2} + C$

$f(x) = 10x^{1/2} + C$

3) $f'(x) = \frac{x+1}{\sqrt[3]{x}} = \frac{x}{x^{1/3}} + \frac{1}{x^{1/3}}$

$f'(x) = x^{2/3} + x^{-1/3}$

$f(x) = \frac{3}{5}x^{\frac{5}{3}} + \frac{3}{2}x^{\frac{2}{3}} + C$

4) $f'(x) = (2-x)^2 = 4 - 4x + x^2$

$f(x) = 4x - 2x^2 + \frac{1}{3}x^3 + C$

or

$f(x) = \frac{1}{3}x^3 - 2x^2 + 4x + C$

Evalute each indefinite integral.

5) $\int \frac{x^6 - 2x^4 + 1}{x^2} dx = \int (x^4 - 2x^2 + x^{-2}) dx$

$= \frac{1}{5}x^5 - \frac{2}{3}x^3 - x^{-1} + C$

or $\frac{1}{5}x^5 - \frac{2}{3}x^3 - \frac{1}{x} + C$

6) $\int (\cos(x) + \sin(x)) dx =$

$= \sin(x) - \cos(x) + C$

7) $\int \sec\theta(\sec\theta + \tan\theta) d\theta = \int \sec^2\theta + \sec\theta \cdot \tan\theta$

$= \tan\theta + \sec\theta + C$

8) $\int \frac{dx}{x} = \int \frac{1}{x} dx$

$= \ln|x| + C$

9) $\int (3a^2) dx$

$= 3a^2x + C$

10) $\int (\frac{a}{b}x^2 - \sqrt[e]{x}) dx = \int (\frac{a}{b}x^2 - x^{\frac{1}{e}}) dx$

$= \frac{1}{3} \cdot \frac{a}{b} x^3 - (\frac{e}{e+1}) x^{\frac{e}{e} + \frac{1}{e}} + C$

$= \frac{a}{3b}x^3 - \frac{e}{e+1}x^{\frac{e+1}{e}} + C$

11) Given a particle is traveling in a straight line and its position from the origin at any time in the interval $[3,7]$ is defined by the function $p(t) = 3\ln(t^4) - 5\cos^2(t)$ answer the following…

a. Which expression would determine the particles average velocity in the time interval given?

a. $p(7) - p(3)$ b. $v(7) - v(3)$ c. $\frac{v(7)-v(3)}{4}$ d. $\frac{p(7)-p(3)}{4}$

average velocity is $\frac{\Delta \text{position}}{\Delta \text{time}}$ or slope of secant line to $p(t)$

b. What is the velocity function for this particle?

$$v(t) = 3\frac{1}{t^4}\cdot 4t^3 - 10\cos(t)(-\sin t) = \frac{12}{t} + 10\cos(t)\sin(t)$$

c. What is the acceleration function for this particle?

$$a(t) = -12t^{-2} + -10\sin(t)\sin(t) + 10\cos(t)\cdot\cos(t)$$

d. Is the particle moving to the right, the left, or neither at t = 4? Justify

$$v(4) = \frac{12}{4} + 10\cos(4)\sin(4) = 7.947$$

Velocity is positive so particle moves right at $t=4$.

e. Is the particle speeding up or slowing down at t = 4? Justify

$$a(4) = -12(4)^{-2} - 10\sin(4)\sin(4) + 10\cos(4)\cos(4) = -1.424$$

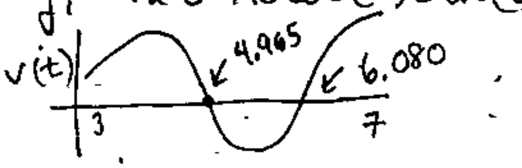
Since velocity is positive + acceleration is negative, the particle is slowing down at $t=4$.

f. Determine the total distance traveled by the particle in the time interval given.

$v(t) = 0$

Use calculator to find zeros

$y_1 = 12t^{-1} + 10\cos(t)\sin(t)$



$$\text{distance} = |p(4.965) - p(3)| + |p(6.080) - p(4.965)| + |p(7) - p(6.080)|$$

$$= |18.9166 - 8.2829| + |16.8636 - 18.9166| + |20.509 - 16.8636|$$

$$= \boxed{16.332 \text{ units}}$$

12) A mass is oscillating at the end of a spring. Let $s(t)$ be the displacement of the mass from the equilibrium position at time t. Assuming that the mass is located at the origin at t = 0 and has velocity $v(t) = \sin(\frac{\pi t}{2})$ meters/second, find $s(t)$.

$(0,0)$

$$s(t) = -\cos\left(\frac{\pi}{2}t\right)\cdot\frac{2}{\pi} + C = -\frac{2}{\pi}\cos\left(\frac{\pi}{2}t\right) + C$$

$$0 = -\frac{2}{\pi}\cos(0) + C$$

$$\frac{2}{\pi} = C$$

$$\boxed{S(t) = -\frac{2}{\pi}\cos\left(\frac{\pi}{2}t\right) + \frac{2}{\pi}}$$