IB Review

Following the Data Packet

- The front matter
- Section by section formulas/problems
  - Linear kinematics
  - Dynamics (F = ma)
  - Circular Motion
  - Energy
  - Momentum
  - Waves
  - Thermal
  - Field Theory
  - Currents and Induction
- The rest of the formulas

Fundamental constants

Quantity	Symbol	Approximate value
Acceleration of free fall (Earth's surface)	g	$9.81\,\text{m}\,\text{s}^{-2}$
Gravitational constant	G	$6.67\times10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$
Avogadro's constant	N_A	$6.02\times10^{23}\,\text{mol}^{-1}$
Gas constant	R	$8.31\,\text{J}\,\text{K}^{-1}\,\text{mol}^{-1}$
Boltzmann's constant	k	$1.38\times10^{-23}\,\text{J}\,\text{K}^{-1}$
Stefan–Boltzmann constant	σ	$5.67\times10^{-8}\,\text{W}\,\text{m}^{-2}\,\text{K}^{-4}$
Coulomb constant	k	$8.99\times10^{9}\,\text{N}\,\text{m}^2\,\text{C}^{-2}$
Permittivity of free space	ε_0	$8.85\times10^{-12}\,\text{C}^2\,\text{N}^{-1}\,\text{m}^{-2}$
Permeability of free space	μ_0	$4\pi\times10^{-7}\,\text{T}\,\text{m}\,\text{A}^{-1}$
Speed of light in vacuum	c	$3.00\times10^{8}\,\text{m}\,\text{s}^{-1}$
Planck's constant	h	$6.63\times10^{-34}\,\text{J}\,\text{s}$
Elementary charge	e	$1.60\times10^{-19}\,\text{C}$
Electron rest mass	m_e	$9.110\times10^{-31}\,\text{kg} = 0.000549\,\text{u} = 0.511\,\text{MeV}\,\text{c}^{-2}$
Proton rest mass	m_p	$1.673\times10^{-27}\,\text{kg} = 1.007276\,\text{u} = 938\,\text{MeV}\,\text{c}^{-2}$
Neutron rest mass	m_n	$1.675\times10^{-27}\,\text{kg} = 1.008665\,\text{u} = 940\,\text{MeV}\,\text{c}^{-2}$
Unified atomic mass unit	u	$1.661\times10^{-27}\,\text{kg} = 931.5\,\text{MeV}\,\text{c}^{-2}$

Metric (SI) multipliers

Prefix	Abbreviation	Value
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deca	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

Unit conversions

$1\ \text{light year (ly)} = 9.46 \times 10^{15}\ \text{m}$

$1\ \text{parsec (pc)} = 3.26\ \text{ly}$

$1\ \text{astronomical unit (AU)} = 1.50 \times 10^{11}\ \text{m}$

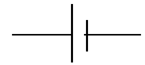
$1\ \text{radian (rad)} = \frac{180°}{\pi}$

$1\ \text{kilowatt-hour (kW h)} = 3.60 \times 10^{6}\ \text{J}$

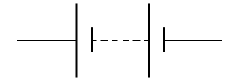
$1\ \text{atm} = 1.01 \times 10^{5}\ \text{N m}^{-2} = 101\ \text{kPa} = 760\ \text{mm Hg}$

Electrical circuit symbols

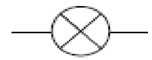
cell



battery



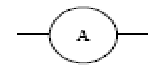
lamp



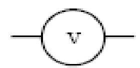
ac supply

switch

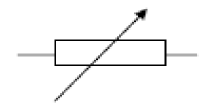
ammeter



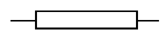
voltmeter



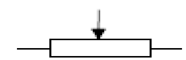
variable resistor



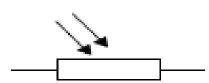
resistor



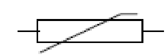
potentiometer



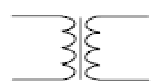
light-dependent resistor (LDR)



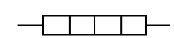
thermistor



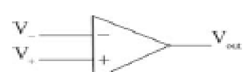
transformer



heating element



operational amplifier (op-amp)



Uncertainty and vector components

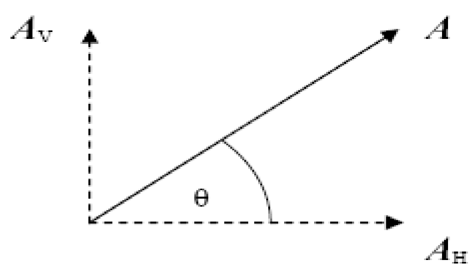
Core	AHL
Topic 1: Physics and physical measurement	

If $y = a \pm b$

then $\Delta y = \Delta a + \Delta b$

If $y = \frac{ab}{c}$

then $\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$



$A_H = A\cos\theta$

$A_V = A\sin\theta$

Linear Kinematics

Core	AHL
Topic 2: Mechanics	
$s = \frac{u+v}{2}t$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$	$v = u + at$?????
$F = ma$	
$p = mv$	
$F = \frac{\Delta p}{\Delta t}$	
$\text{Impulse} = F\Delta t = m\Delta v$	
$W = Fs\cos\theta$	
$E_K = \frac{1}{2}mv^2$	
$E_K = \frac{p^2}{2m}$	
$\Delta E_p = mg\Delta h$	
$\text{power} = Fv$	
$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$	

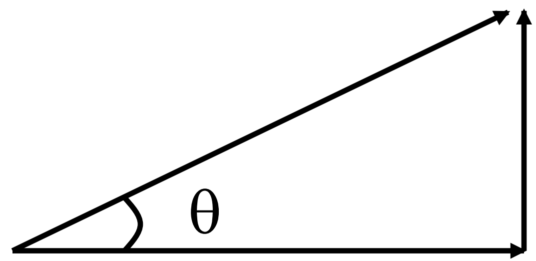
An air rocket leaves the ground straight up, and strikes the ground 4.80 seconds later.

1. What time does it take to get to the top?
2. How high does it go?
3. What was its initial velocity?
4. What is the velocity at elevation 21.0 m?

2.4 s, 28.2 m, 23.5 m/s, + or - 11.9 m/s

2-Dimensional Motion

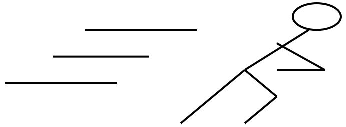
H	V
s	s
u	u
v	v
a	a
t	t



$A_V = A \sin\theta$

$A_H = A \cos\theta$

Pythagorean $x^2 + y^2 = hyp^2$



$V = 9.21$ m/s

$t = 2.17$ s

1. How far out does she land?
2. How high is the cliff?
3. What is the velocity of impact in VC Notation?
4. What is the velocity of impact? (in AM Notation)

20.0 m, 23.1 m,

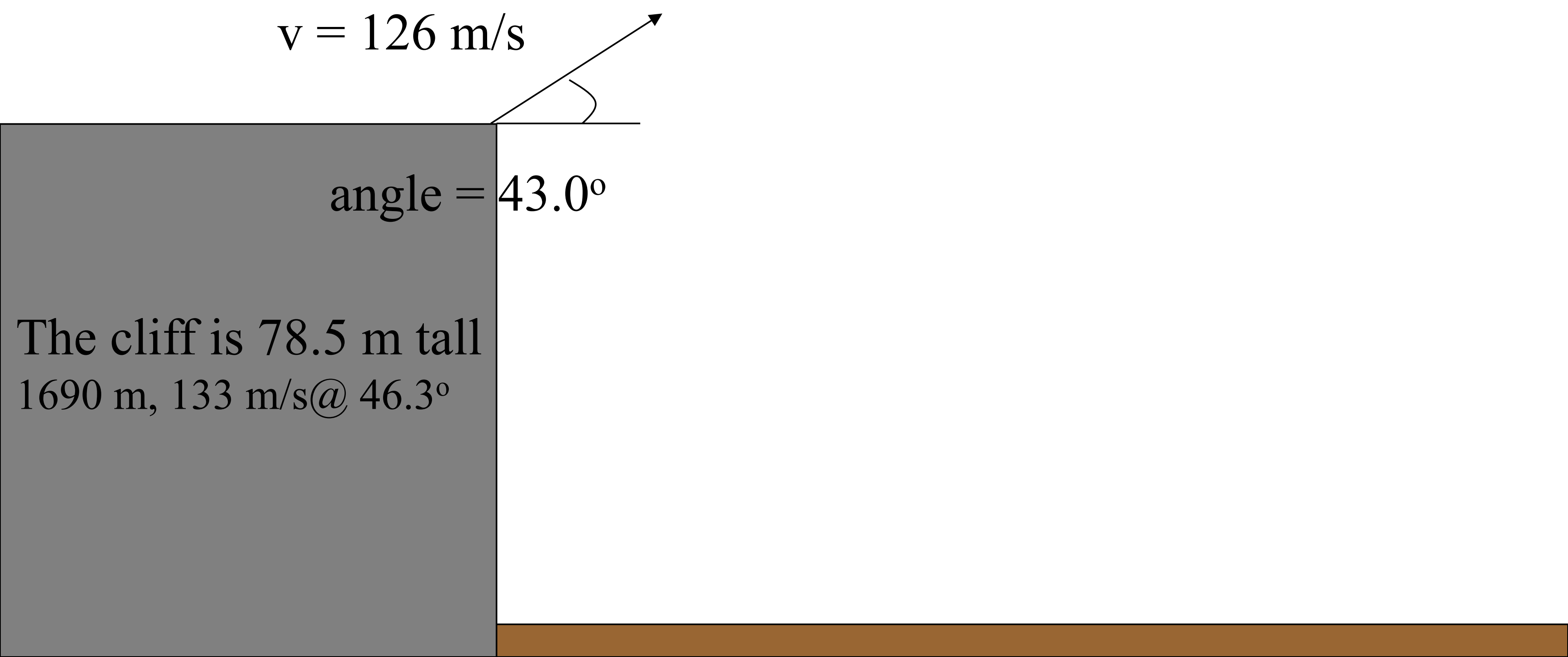
9.21 m/s x + -21.3 m/s y

23.2 m/s 66.6^{o} below horiz

Find vector components

Fill in your H/V table of suvat

1. Find the horizontal distance traveled
2. Find velocity of impact in angle magnitude



Dynamics

Core	AHL
Topic 2: Mechanics	
$s = \frac{u+v}{2}t$	
$s = ut + \frac{1}{2}at^2$	
$v^2 = u^2 + 2as$	
$F = ma$	
$p = mv$	
$F = \frac{\Delta p}{\Delta t}$	
$\text{Impulse} = F\Delta t = m\Delta v$	
$W = Fs\cos\theta$	
$E_K = \frac{1}{2}mv^2$	
$E_K = \frac{p^2}{2m}$	
$\Delta E_p = mg\Delta h$	
$\text{power} = Fv$	
$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$	

Find the force:

F = ma,

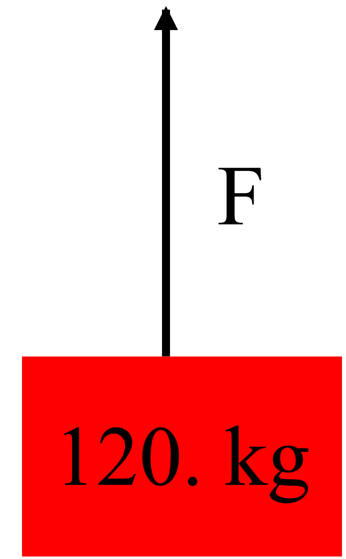
wt = 1176 N downward

<F – 1176 N> = (120. kg)(-4.50 m/s/s)

F – 1176 N = -540 N

F = <u>636 N</u>

…



a = -4.50 m/s/s

(DOWNWARD)

<u>636 N</u>

A 120 mW laser uses a wavelength of 656 nm.

What is the energy and momentum of a photon of light at this wavelength?

How many photons per second does it emit?

What force would it exert on an object that absorbs the photons?

How would that change if the photons were reflected?

3.030E-19 J, 1.010E-27 kg m/s, 3.960E17 photons/sec, 4.00E-10 N

Gravity and Circular Motion

Core	AHL

Topic 2: Mechanics

$s = \frac{u+v}{2}t$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$F = ma$

$p = mv$

$F = \frac{\Delta p}{\Delta t}$

$\text{Impulse} = F\Delta t = m\Delta v$

$W = Fs\cos\theta$

$E_K = \frac{1}{2}mv^2$

$E_K = \frac{p^2}{2m}$

$\Delta E_p = mg\Delta h$

$\text{power} = Fv$

$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$

Also on page 8:

Topic 6: Fields and forces

$F = G\frac{m_1 m_2}{r^2}$	$F = k\frac{q_1 q_2}{r^2}$
$g = \frac{F}{m}$	$E = \frac{F}{q}$

A Volkswagen can do .650 “g”s of lateral acceleration. What is the minimum radius turn at 27.0 m/s? (3)

$a = v^2/r$

1g= 9.81 m/s/s

a = (9.81 m/s/s)(.650) = 6.3765 m/s/s

6.3765 m/s/s = $(27.0 \text{ m/s})^2/r$

r = $(27.0 \text{ m/s})^2/(6.3765 \text{ m/s/s})$ = 114.326 m

r = 114m

114m

What should be the period of motion if you want 3.5 “g”s of centripetal acceleration 5.25 m from the center of rotation?

$a = 4\pi^2 r/T^2$
$a = (3.5)(9.8\ m/s/s) = 34.3\ m/s/s$
$34.3\ m/s/s = 4\pi^2(5.25\ m)/T^2$
$T = 2.5\ s$
…

2.5 s

Ice skates can give 420 N of turning force. What is r_{min} for a 50.kg skater @10.m/s?

$F=ma$, $a=v^2/r$

$F=mv^2/r$

$420\ N = (50\ kg)(10.m/s)^2/r$

$r = (50\ kg)(10.m/s)^2/(420\ N)$

$r = 11.9m$

11.9m

The moon has a mass of 7.36×10^{22} kg, and a radius of 1.74×10^{6} m. What does a 34.2 kg mass weight on the surface?

r = Center to center distance
m_1 = One of the masses
m_2 = The other mass
$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

$$F = \frac{Gm_1m_2}{r^2}$$

F = 55.5 N
…

55.5 N

At what distance from the moon's center is the orbital velocity 52.5 m/s?

$M_m = 7.36 \times 10^{22}$ kg

$$\frac{m_s v^2}{r} = \frac{G m_s m_c}{r^2}$$

$$r = \frac{G m_c}{v^2}$$

1781086621 m

1.78×10^9 m

Energy

Core	AHL

Topic 2: Mechanics

$$s = \frac{u+v}{2}t$$

$$s = ut + \tfrac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$F = ma$$

$$p = mv$$

$$F = \frac{\Delta p}{\Delta t}$$

$$\text{Impulse} = F\Delta t = m\Delta v$$

$$W = Fs\cos\theta$$

$$E_K = \tfrac{1}{2}mv^2$$

$$E_K = \frac{p^2}{2m}$$

$$\Delta E_p = mg\Delta h$$

$$\text{power} = Fv$$

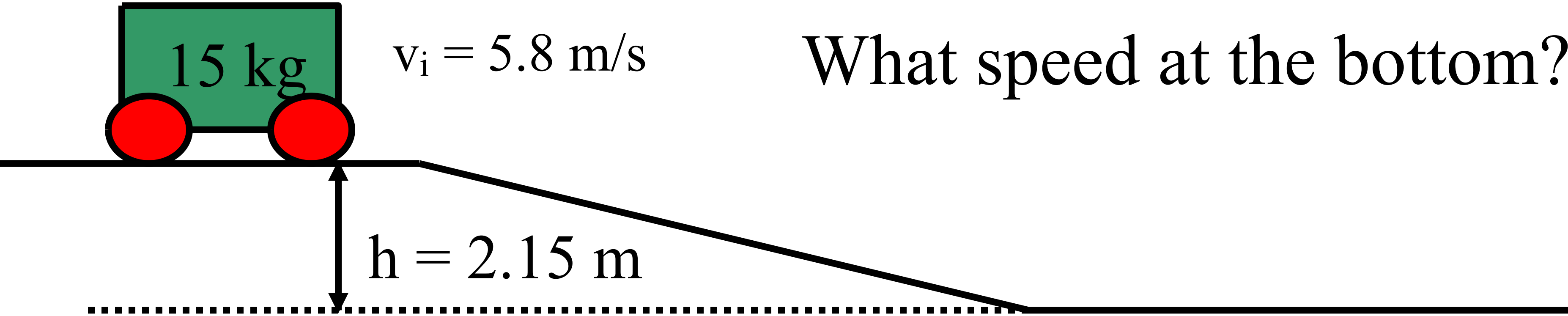
$$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

Also

Power = work/time

$E_{elas} = {}^1/_2 kx^2$

What speed at the bottom?



$$Fd + mgh + \tfrac{1}{2}mv^2 = Fd + mgh + \tfrac{1}{2}mv^2$$

$$0 + mgh + \tfrac{1}{2}mv^2 = 0 + 0 + \tfrac{1}{2}mv^2$$

$$(15\text{ kg})(9.8\text{ N/kg})(2.15\text{ m}) + \tfrac{1}{2}(15\text{ kg})(5.8\text{ m/s})^2 = \tfrac{1}{2}(15\text{ kg})v^2$$

$$v = 8.7\text{ m/s}$$

…

8.7 m/s

Ima Wonder can put out 127 W of power. What time will it take her to do 671 J of work?

$P = W/\Delta t$,
$\Delta t = W/P = (671\ J)/(127\ W) = 5.28\ s$

5.28 s

Frieda People can put out 430. W of power. With what speed can she push a car if it takes 152 N to make it move at a constant velocity?

$P = Fv$

$v = P/F = (430.\ W)/(152\ N) = 2.83\ m/s$

2.83 m/s

What must be the power rating of a motor if it is to lift a 560 kg elevator up 3.2 m in 1.5 seconds?

11700 W

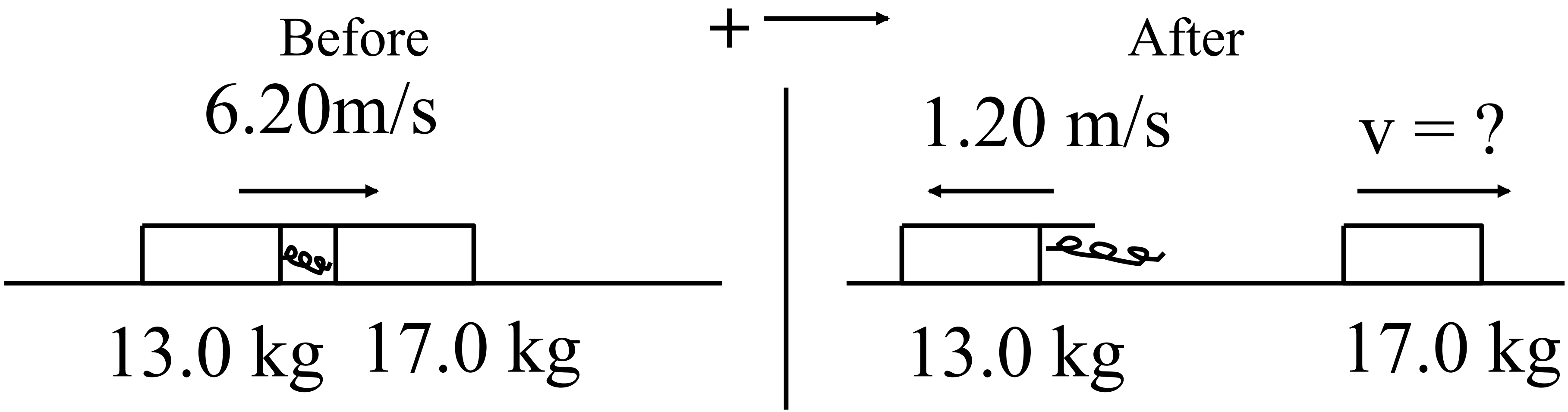
Momentum

Core	AHL
Topic 2: Mechanics $s = \frac{u+v}{2}t$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$ $F = ma$ $p = mv$ $F = \frac{\Delta p}{\Delta t}$ $\text{Impulse} = F\Delta t = m\Delta v$ $W = Fs\cos\theta$ $E_K = \frac{1}{2}mv^2$ $E_K = \frac{p^2}{2m}$ $\Delta E_p = mg\Delta h$ $\text{power} = Fv$ $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$	

Jolene exerts a 50. N force for 3.00 seconds on a stage set. It speeds up from rest to .25 m/s. What is the mass of the set?

$(m)(\Delta v) = (F)(\Delta t)$
$(m)(.25\ m/s) = (50.\ N)(3.0\ s)$
$m = (50.\ N)(3.0\ s)/(.25\ m/s) =$
$600\ kg = 6.0 \times 10^2\ kg$
…

6.0×10^2 kg



$$(13kg+17kg)(6.2m/s) = (13kg)(-1.2m/s)+(17kg)v$$

$$186kgm/s = -15.6kgm/s+(17kg)v$$

$$201.6kgm/s = (17kg)v$$

$$(201.6kgm/s)/(17kg) = 11.9\ m/s = v$$

…

11.9 m/s

Topic 4: Oscillations and waves

Oscillations and waves

Core

Topic 4: Oscillations and waves

$$\omega = \frac{2\pi}{T}$$

$$x = x_0 \sin \omega t \,; \quad x = x_0 \cos \omega t$$

$$v = v_0 \cos \omega t \,; \quad v = -v_0 \sin \omega t$$

$$v = \pm \omega \sqrt{(x_0{}^2 - x^2)}$$

$$E_K = \tfrac{1}{2} m \omega^2 (x_0{}^2 - x^2)$$

$$E_{K(max)} = \tfrac{1}{2} m \omega^2 x_0{}^2$$

$$E_T = \tfrac{1}{2} m \omega^2 x_0{}^2$$

$$v = f\lambda$$

$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$$

$$\text{path difference} = n\lambda$$

$$\text{path difference} = \left(n + \tfrac{1}{2}\right)\lambda$$

AHL

Topic 11: Wave phenomena

$$f' = f\left(\frac{v}{v \pm u_s}\right) \quad \text{moving source}$$

$$f' = f\left(\frac{v \pm u_o}{v}\right) \quad \text{moving observer}$$

$$\Delta f = \frac{v}{c} f$$

$$\theta = \frac{\lambda}{b}$$

$$\theta = 1.22\frac{\lambda}{b}$$

$$I = I_0 \cos^2 \theta$$

$$n = \tan\phi$$

Simple Harmonic Motion - Kinematics

$\omega = \frac{2\pi}{T}$ $f = \frac{1}{T}$ $\omega = 2\pi f$

$x = x_o \sin(\omega t)$ or $x_o \cos(\omega t)$

$v = v_o \cos(\omega t)$ or $-v_o \sin(\omega t)$

ω – "Angular" velocity

T – Period of motion

x – Position (at some time)

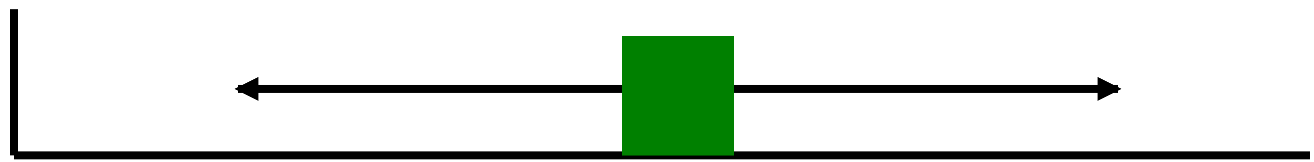
v – Velocity (at some time)

Draw on board:

x_o – Max Position (Amplitude)

v_o – Max Velocity

- x_o = Maximum displacement (AKA Amplitude)
- v_o = Maximum velocity
- a_o = Maximum acceleration



•x:	$-x_o$	0	$+x_o$
•v:	0	$^{+}/-v_o$	0
•a:	$+a_o$	0	$-a_o$

Simple Harmonic Motion - Energy

$$E_k = {}^1/_2 m\omega^2(x_o^2 - x^2)$$

$$E_{k\,(max)} = {}^1/_2 m\omega^2 x_o^2$$

$$E_T = {}^1/_2 m\omega^2 x_o^2$$

E_T	– Total Energy
$E_{k\,(max)}$	– Maximum Kinetic Energy
E_k	– Kinetic Energy
ω	– “Angular” velocity
T	– Period of motion
x	– Position (at some time)
v	– Velocity (at some time)
x_o	– Max Position (Amplitude)
v_o	– Max Velocity

Simple Harmonic Motion - Energy

$E_{k\,(max)} = {}^1/_2 m v_o^{\,2}$

$E_{p\,(max)} = {}^1/_2 k x_o^{\,2}$

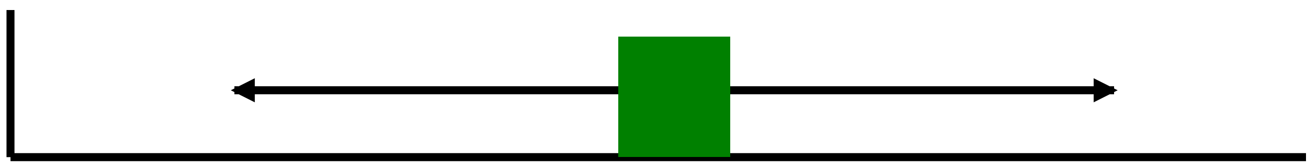
Where they happen

Derive the energy equations:

$E_k = {}^1/_2 m\omega^2(x_o^{\,2} - x^2)$

$E_{k\,(max)} = {}^1/_2 m\omega^2 x_o^{\,2}$

$E_T = {}^1/_2 m\omega^2 x_o^{\,2}$



•E_k:	0	max	0
•E_p:	max	0	max

What is the period of a guitar string that is vibrating 156 times a second? (156 Hz)

Use $f = 1/T$

0.00641 s

W

A mass on the end of a spring oscillates with a period of 2.52 seconds and an amplitude of 0.450 m. What is its maximum velocity? (save this value)

$v = \pm\omega \sqrt{(x_o^2 - x^2)}$, make $x = 0$, $\omega = 2\pi/2.52$, $|v| = 1.12199\ldots$ m/s

1.12 m/s

W

A SHO has an equation of motion of: (in m)

$x = 2.4\sin(6.1t)$

a) what is the amplitude and angular velocity of the oscillator?

b) what is its period?

c) what is its maximum velocity?

d) write an equation for its velocity.

$x_o = 2.4$ m, $\omega = 6.1$ rad/s

$T = 2\pi/6.1 = 1.03$ s

$v_o = (6.1 \text{ rad/s})(2.4 \text{ m}) = 14.64$

$v = 15\cos(6.1t)$

2.4 m – 6.1 rad/s
1.0 s
15 m/s
$v = 15\cos(6.1t)$

W

A loudspeaker makes a pure tone at 440.0 Hz. If it moves with an amplitude of 0.87 cm, what is its maximum velocity? (0.87 cm = .0087 m) (f = 1/T)

$v = \pm\omega \sqrt{(x_o^2 - x^2)}$, make $x = 0$, $\omega = 2\pi(440)$, $|v| = 24.052....$ m/s

24 m/s

W

A mass on the end of a spring oscillates with a period of 1.12 seconds and an amplitude of 0.15 m. Suppose it is moving upward and is at equilibrium at t = 0.

What is its **velocity** at t = 13.5 s?

use $v = v_o\cos(\omega t)$, $\omega = 2\pi/1.12$, $v_o = \omega\sqrt{(x_o^2)} = \omega x_o$, $v = +0.79427\ldots$ m/s

+0.79 m/s

W

An SHO has a mass of 0.259 kg, an amplitude of 0.128 m and an angular velocity of 14.7 rad/sec.
What is its total energy? (save this value in your calculator)

Use $E_T = {}^1/_2 m\omega^2 x_o^2$

0.458 J

W

An SHO has a mass of 0.259 kg, an amplitude of 0.128 m and an angular velocity of 14.7 rad/sec.
What is its kinetic energy when it is 0.096 m from equilibrium? What is its potential energy?

Use $E_k = {}^1\!/_2 m\omega^2(x_o^2 - x^2)$

0.20 J, 0.26 J

W

An SHO has a total energy of 2.18 J, a mass of 0.126 kg, and a period of 0.175 s.

a) What is its maximum velocity?

b) What is its amplitude of motion?

Use $E_k = {}^1/_2mv^2$

Then $\omega = 2\pi/T$

Use $E_{k\,(max)} = {}^1/_2m\omega^2x_o{}^2$

5.88 m/s J, 0.164 m

W

An SHO a maximum velocity of 3.47 m/s, and a mass of 0.395 kg, and an amplitude of 0.805 m. What is its potential energy when it is 0.215 m from equilibrium?

$\omega = 2\pi/T$

Use $E_k = {}^1/_2 mv^2$

Use $E_{k\ (max)} = {}^1/_2 m\omega^2 x_o^2$

Then Use $E_k = {}^1/_2 m\omega^2(x_o^2 - x^2)$

Subtract kinetic from max

0.170 J

W

A 1250 kg car moves with the following equation of motion: (in m)

$$x = 0.170\sin(4.42t)$$

a) what is its total energy?

b) what is its kinetic energy at t = 3.50 s?

Use $E_T = {}^1/_2 m\omega^2 x_o^2$

Then find x from the equation: (.04007…)

Then use Use $E_k = {}^1/_2 m\omega^2(x_o^2 - x^2)$

353 J, 333 J

W

Oscillations and waves

Core

Topic 4: Oscillations and waves

$$\omega = \frac{2\pi}{T}$$

$$x = x_0 \sin \omega t \,; \quad x = x_0 \cos \omega t$$

$$v = v_0 \cos \omega t \,; \quad v = -v_0 \sin \omega t$$

$$v = \pm\omega\sqrt{(x_0{}^2 - x^2)}$$

$$E_{\mathrm{K}} = \tfrac{1}{2} m\omega^2 (x_0{}^2 - x^2)$$

$$E_{\mathrm{K(max)}} = \tfrac{1}{2} m\omega^2 x_0{}^2$$

$$E_{\mathrm{T}} = \tfrac{1}{2} m\omega^2 x_0{}^2$$

$$v = f\lambda$$

$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$$

$$\text{path difference} = n\lambda$$

$$\text{path difference} = \left(n + \tfrac{1}{2}\right)\lambda$$

AHL

Topic 11: Wave phenomena

$$f' = f\left(\frac{v}{v \pm u_s}\right) \quad \text{moving source}$$

$$f' = f\left(\frac{v \pm u_o}{v}\right) \quad \text{moving observer}$$

$$\Delta f = \frac{v}{c} f$$

$$\theta = \frac{\lambda}{b}$$

$$\theta = 1.22\frac{\lambda}{b}$$

$$I = I_0 \cos^2\theta$$

$$n = \tan\phi$$

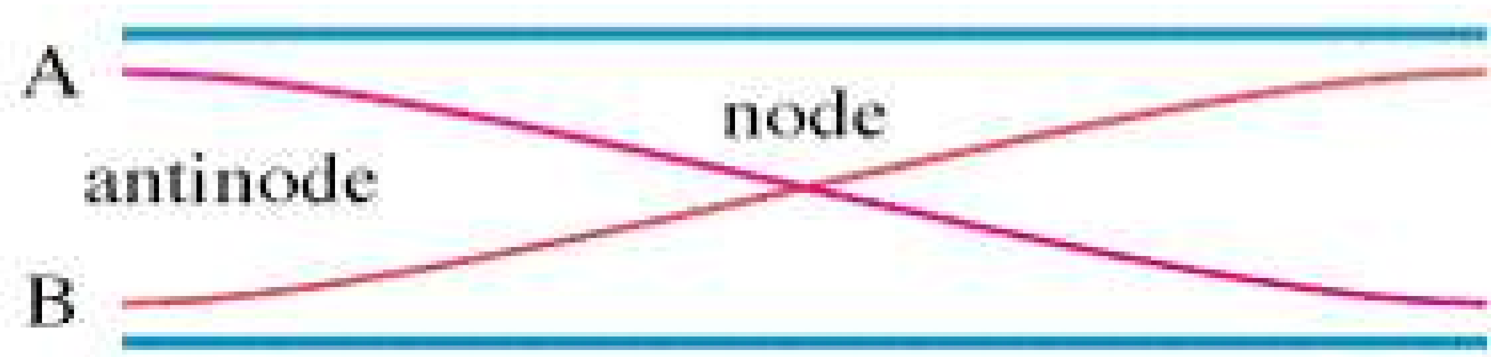
What is the frequency of a sound wave that has a wavelength of 45 cm, where the speed of sound is 335 m/s

$v = f\lambda$

$f = v/\lambda = (335\text{ m/s})/(.45\text{ m}) = 744.444 = 740\text{ Hz}$

…

740 Hz



The waveform is 62 cm long. What is the λ?

If it is a sound wave (v = 343 m/s), what is its frequency (v = fλ)

$L = {}^{2}/_{4}\ \lambda$

$\lambda = {}^{4}/_{2}(.62\text{ m}) = 1.24\text{ m}$

v = fλ, f = v/λ = (343 m/s)/(1.24 m) = 277 Hz

…

277 Hz

The waveform is 2.42 m long. What is the $\underline{\lambda}$?

If it is a sound wave (v = 343 m/s), what is its <u>frequency</u> ($v = f\lambda$)

$L = {}^{1}/_{4}\,\lambda$

$\lambda = {}^{4}/_{1}(2.42\text{ m}) = 9.68\text{ m}$

$v = f\lambda$, $f = v/\lambda = (343\text{ m/s})/(9.68\text{ m}) =$ 35.4 Hz

...

35.4 Hz

Oscillations and waves

Core	AHL
Topic 4: Oscillations and waves	**Topic 11: Wave phenomena**
$\omega = \frac{2\pi}{T}$	$f' = f\left(\frac{v}{v \pm u_s}\right)$ moving source
$x = x_0 \sin \omega t\,; \quad x = x_0 \cos \omega t$	$f' = f\left(\frac{v \pm u_o}{v}\right)$ moving observer
$v = v_0 \cos \omega t\,; \quad v = -v_0 \sin \omega t$	$\Delta f = \frac{v}{c} f$
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$v = f\lambda$	
$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$	
path difference $= n\lambda$	
path difference $= \left(n + \frac{1}{2}\right)\lambda$	

A person who is late for a concert runs at 18.0 m/s towards an A 440.0 Hz. What frequency do they hear? (use v sound = 343 m/s)

Moving observer
higher frequency
$f' = f\{1 \pm v_o/v\}$

f = 440.0 Hz, vo = 18.0 m/s, v = 343 m/s, and +
F = 463 Hz
…

463 Hz

A car with a 256 Hz horn is moving so that you hear 213 Hz. What is its velocity, and is it moving away from you or toward you?

(use v sound = 343 m/s)

Moving source

lower frequency

$$f' = f\left\{\frac{v}{\{v \pm u_s\}}\right\}$$

f' = 213 Hz, f = 256 Hz, v = 343 m/s, and +

69.2 m/s away from you

Oscillations and waves

Core	AHL
Topic 4: Oscillations and waves	**Topic 11: Wave phenomena**
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Two speakers 3.0 m apart are making sound with a wavelength of 48.0 cm.

If I am 2.12 m from one speaker, and 3.80 m from the other, is it loud, or quiet, and how many wavelengths difference in distance is there?

3.80 m - 2.12 m = 1.68 m

(1.68 m)/(.48 m) = 3.5 λ = destructive interference

…

3.5 λ = destructive interference

Oscillations and waves

Core

Topic 4: Oscillations and waves

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AHL

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$$\theta = \frac{\lambda}{b}$$

$$\theta = 1.22\frac{\lambda}{b}$$

$$I = I_0 \cos^2\theta$$

$$n = \tan\phi$$

What is the speed of light in diamond? n = 2.42

$n = {}^{c}/_{V}$

$n = 2.42$, $c = 3.00 \times 10^8$ m/s
$V = 1.24 \times 10^8$ m/s
…

1.24×10^8 m/s

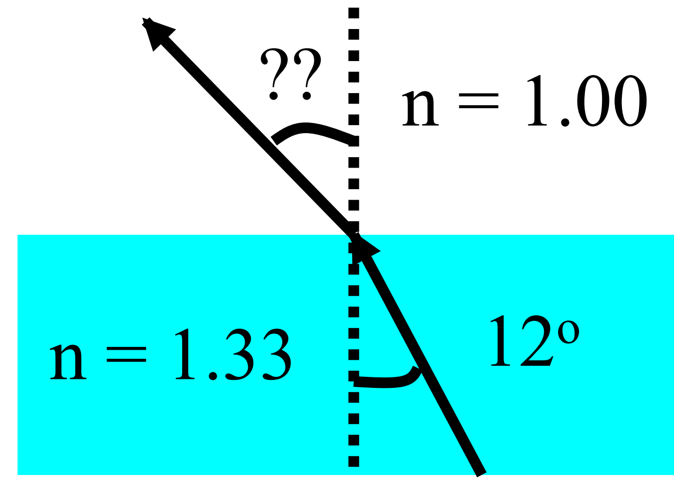
A ray of light has an incident angle of 12° with the underside of an air-water interface, what is the refracted angle in the air? (n = 1.33 for water, 1.00 for air)

$n_1 \sin \theta_1 = n_2 \sin \theta_2$

$n_1 = 1.33$, $\theta_1 = 12^o$, $n_2 = 1.00$

Angle = 16°

…



16°

Oscillations and waves

Core

Topic 4: Oscillations and waves

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Oscillations and waves

Core	AHL

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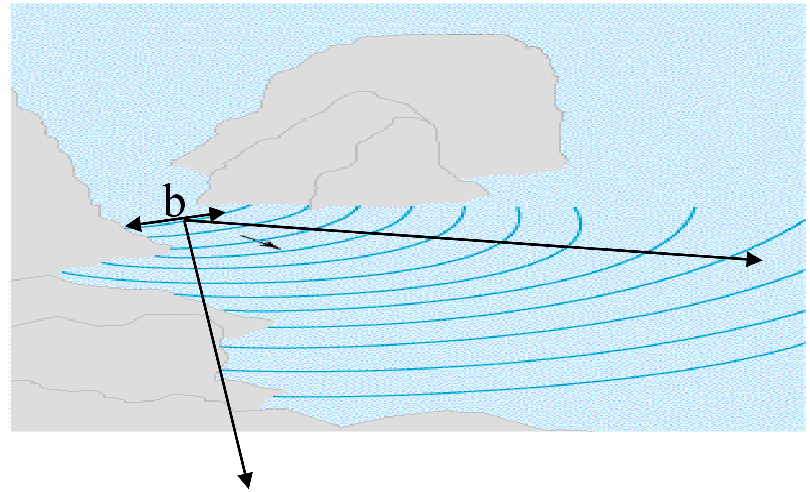
$$\theta = 1.22 \frac{\lambda}{b}$$

$$I = I_0 \cos^2 \theta$$

$$n = \tan \phi$$

$$\theta \approx \frac{\lambda}{b}$$

θ = Angular Spread
λ = Wavelength
b = Size of opening



Try this problem: Sound waves with a frequency of 256 Hz come through a doorway that is 0.92 m wide. What is the approximate angle of diffraction into the room? Use 343 m/s as the speed of sound.

Use $v = f\lambda$, so $\lambda = 1.340$ m

Then use

$$\theta \approx \frac{\lambda}{b}$$

$\theta \approx 1.5$ rad

What if the frequency were lower?

Sub Woofers

$\theta \approx 1.5$ rad

Oscillations and waves

Core

Topic 4: Oscillations and waves

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$$\theta = 1.22\frac{\lambda}{b}$$

$$I = I_0 \cos^2\theta$$

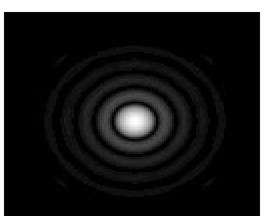
$$n = \tan\phi$$

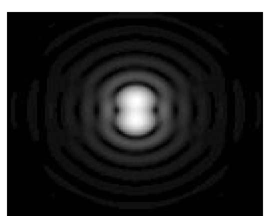
Rayleigh Criterion

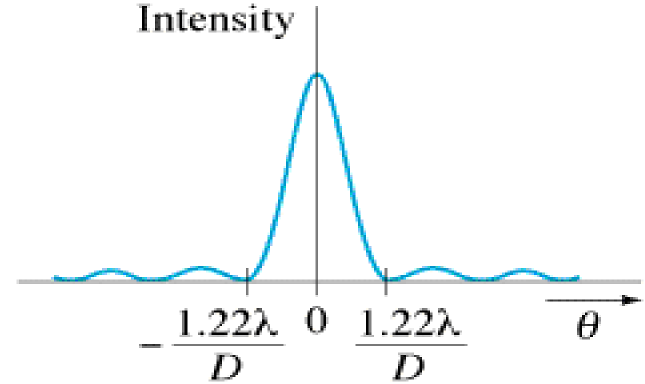
$$\theta = \frac{1.22\lambda}{b}$$

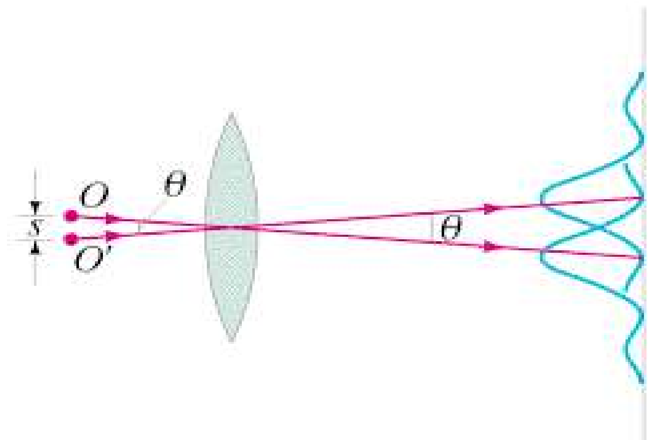
θ = Angle of resolution (Rad)
λ = Wavelength (m)
b = Diameter of circular opening (m)
(Telescope aperture)

the bigger the aperture, the smaller the angle you can resolve.









Central maximum of one is over minimum of the other

Rayleigh Criterion

$$\theta = \frac{1.22\lambda}{b}$$

θ = Angle of resolution (Rad)

λ = Wavelength (m)

b = Diameter of circular opening (m)

(b) A space shuttle orbits at a height of 300 km above the surface of the Earth. It carries two panels separated by a distance of 24 m. The panels reflect light of wavelength 500 nm towards an observer on the Earth's surface.

The observer views the panels with a telescope of aperture diameter 85 mm. The panels act as point sources of light for the observer.

(i) Describe what is meant by the Rayleigh criterion. *[2]*

. . . .

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(ii) Determine whether the images of the panels formed by the telescope will be resolved. *[3]*

. . . .

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Oscillations and waves

Core

Topic 4: Oscillations and waves

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AHL

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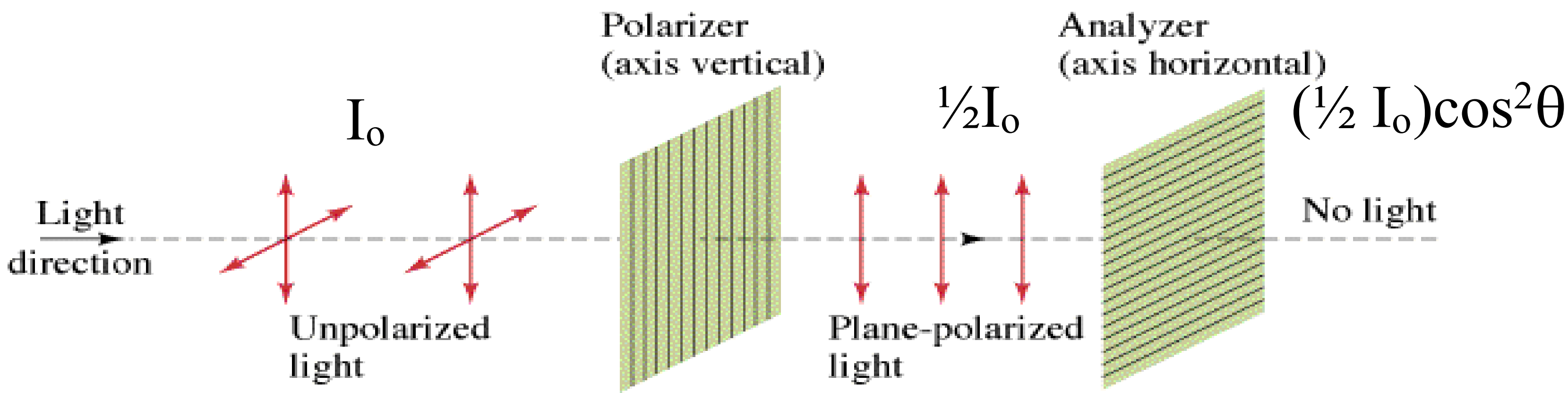
$$I = I_0 \cos^2\theta$$

$$n = \tan\phi$$

More than one polarizer:

$I = I_o \cos^2\theta$

- I_o – incident intensity of polarized light
- I – transmitted intensity (W/m^2)
- θ – angle twixt polarizer and incident angle of polarization



Two polarizers are at an angle of 37^o with each other. If there is a 235 W/m^2 beam of light incident on the first filter, what is the intensity between the filters, and after the second?

$I = I_o cos^2\theta$

After the first polarizer, we have half the intensity:

$I = 235/2 = 117.5\ W/m^2$

and then that polarized light hits the second filter at an angle of 37^o:

$I = (117.5\ W/m^2)\ cos^2(37^o) = 74.94 = 75\ W/m^2$

117.5 W/m^2 75 W/m^2

Oscillations and waves

Core	AHL

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$$E_\mathrm{T} = \tfrac{1}{2} m\omega^2 x_0{}^2$$

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$$\theta = 1.22\frac{\lambda}{b}$$

$$I = I_0 \cos^2\theta$$

$$n = \tan\phi$$

Brewster's angle:

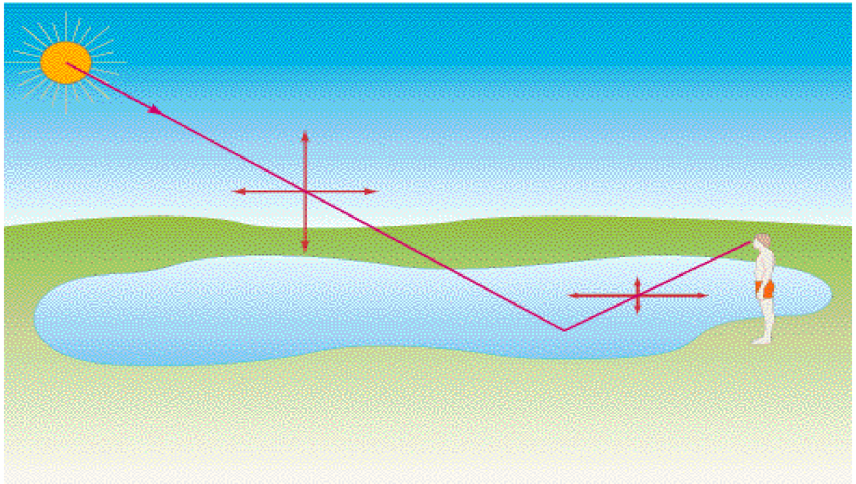
- non-metallic surface
- reflected light polarized parallel to surface.

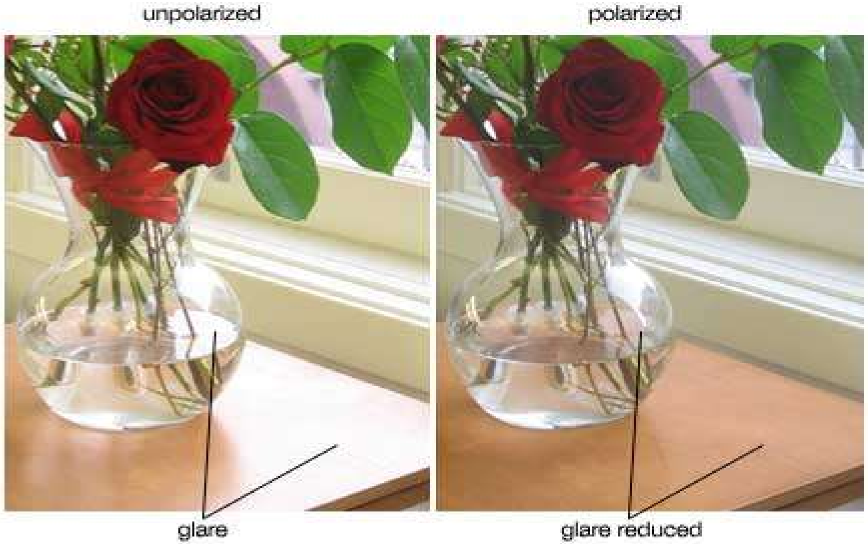
In general

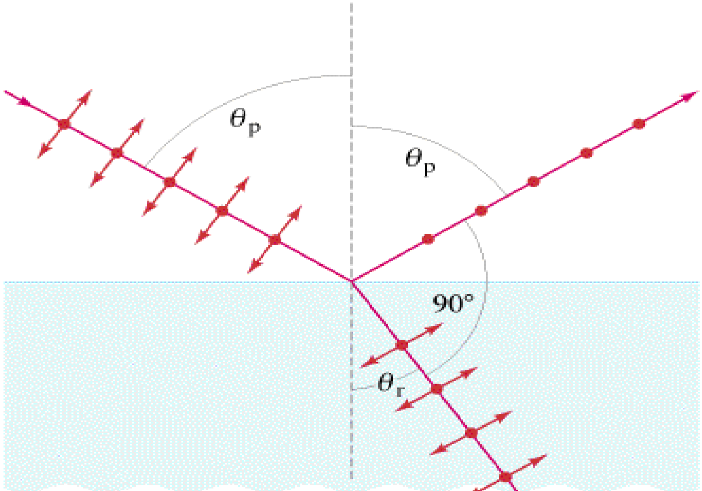
$$\frac{n_2}{n_1} = \tan\theta$$

For air ($n_1 = 1.00$) to something:

$n = \tan\theta$







What is Brewster's angle from air to water? (n = 1.33)

$n = \tan\theta$

$n = 1.33$, $\theta = ?$

$\theta = 53.06^{o}$

53.1^{o}

Topic 3 Thermal Physics

Thermal

Topic 3: Thermal physics	Topic 10: Thermal physics
$P = \frac{F}{A}$	$PV = nRT$
$Q = mc\Delta T$	$W = P\Delta V$
$Q = mL$	$Q = \Delta U + W$

What is the pressure of 42 N on a 20. cm x 32 cm plate?

$A = (.20\ m)(.32\ m) = .064\ m^2$
$P = F/A = (42\ N)/(.064\ m^2)$

660 Pa

Thermal

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$P = \frac{F}{A}$	$PV = nRT$
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$Q = mL$	$Q = \Delta U + W$

Example: A. Nicholas Cheep wants to calculate what heat is needed to raise 1.5 liters (1 liter = 1 kg) of water by 5.0 ^{o}C. Can you help him? ($c = 4186\ J\ ^{o}C^{-1}kg^{-1}$)

$Q = mc\Delta T$

$Q = ??$, $m = 1.5\ kg$, $c = 4186\ J\ ^{o}C^{-1}kg^{-1}$, $\Delta T = 5.0\ ^{o}C$

31,000 J

What is specific heat of the gaseous phase?

1480 J oC-1 kg-1

.112 kg of a mystery substance at 85.45 °C is dropped into .873 kg of water at 18.05 °C in an insulated Styrofoam container. The water and substance come to equilibrium at 23.12 °C. What is the c of the substance?

(c_{water} = 4186 $J^{o}C^{-1}kg^{-1}$)

2650 $J^{o}C^{-1}kg^{-1}$

Thermal

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$Q = mL$	$Q = \Delta U + W$

Aaron Alysis has a 1500. Watt heater. What time will it take him to melt 12.0 kg of ice, assuming all of the heat goes into the water at 0 ^oC

Some latent heats

(in J kg^{-1})	Fusion	Vaporisation
H_2O	3.33×10^5	22.6×10^5
Lead	0.25×10^5	8.7×10^5
NH_3	0.33×10^5	1.37×10^5

Q = mL, power = work/time (= heat/time)
Q = ??, m = 12.0 kg, L = 3.33×10^5 J^{o}kg^{-1}
3,996,000 J, power = heat/time
heat = 3,996,000 J, power = 1500. J/s

2660 seconds

What is the latent heat of fusion?

22,000 J kg-1

$\Delta Q = 10{,}000$, $m = .45$ kg, $L = ??$ $L_f = 22{,}000$ J kg^{-1}

Thermal

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$Q = mc\Delta T$	$W = P\Delta V$
$Q = mL$	$Q = \Delta U + W$

What is the volume of 1.3 mol of N_2 at 34 ^{o}C, and 1.0 atm? (1 atm = 1.013 x 10^5 Pa)

pV = nRT
p = 1.013 x 10^5 Pa, n = 1.3, T = 273 K + 34 K,
V = .033 m^3
…

.033 m^3

Thermal

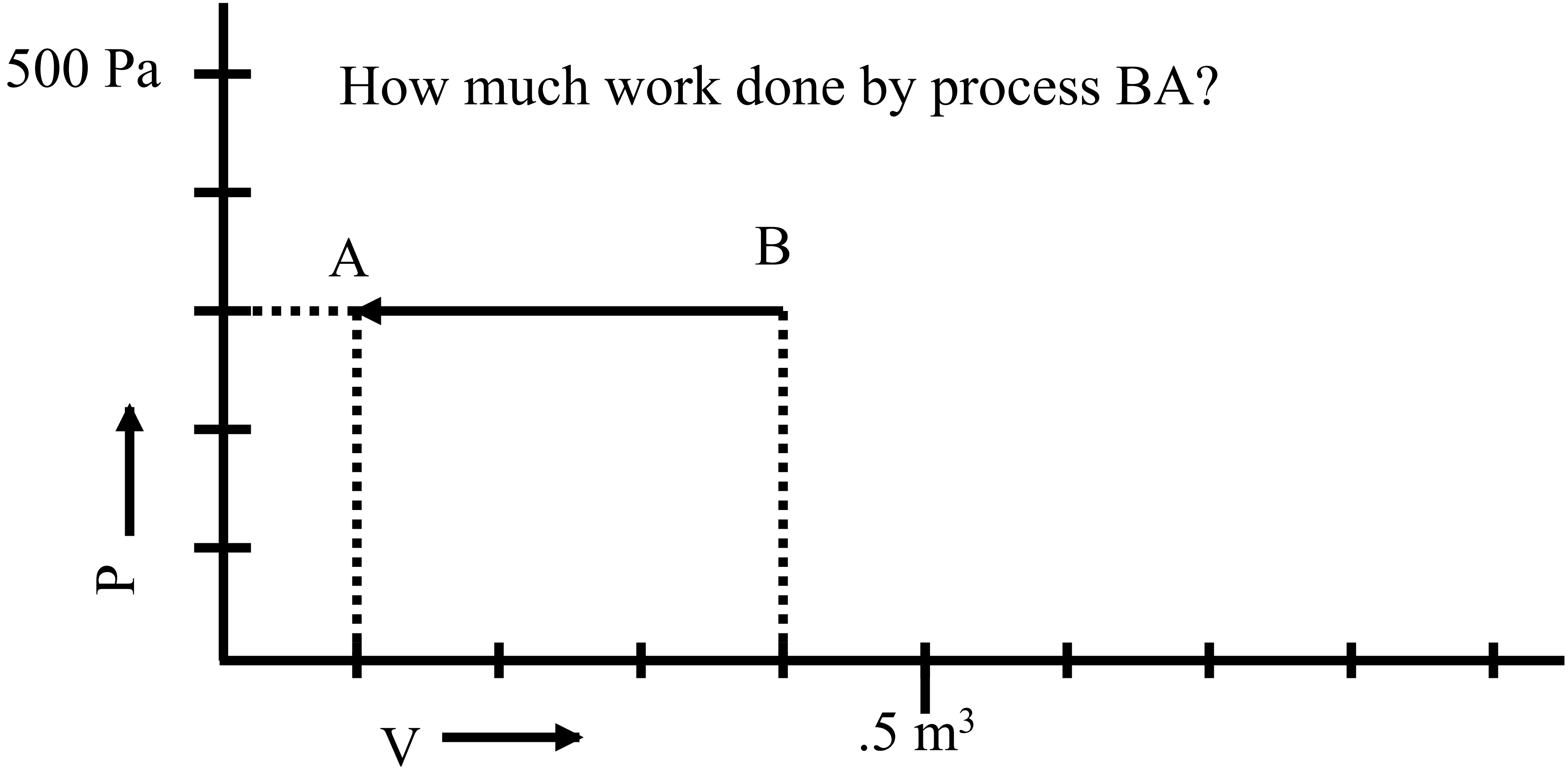
Topic 3: Thermal physics	Topic 10: Thermal physics
$P = \frac{F}{A}$	$PV = nRT$
$Q = mc\Delta T$	$W = P\Delta V$
$Q = mL$	$Q = \Delta U + W$

Mr. Fyde compresses a cylinder from .0350 m^3 to .0210 m^3, and does 875 J of work. What was the average pressure?

$W = P\Delta V$
$W = -875$, $\Delta V = .0350 - .0210 = -.0140\ m^3$
$P = 62500$ Pa

62.5 kPa

How much work done by process BA?

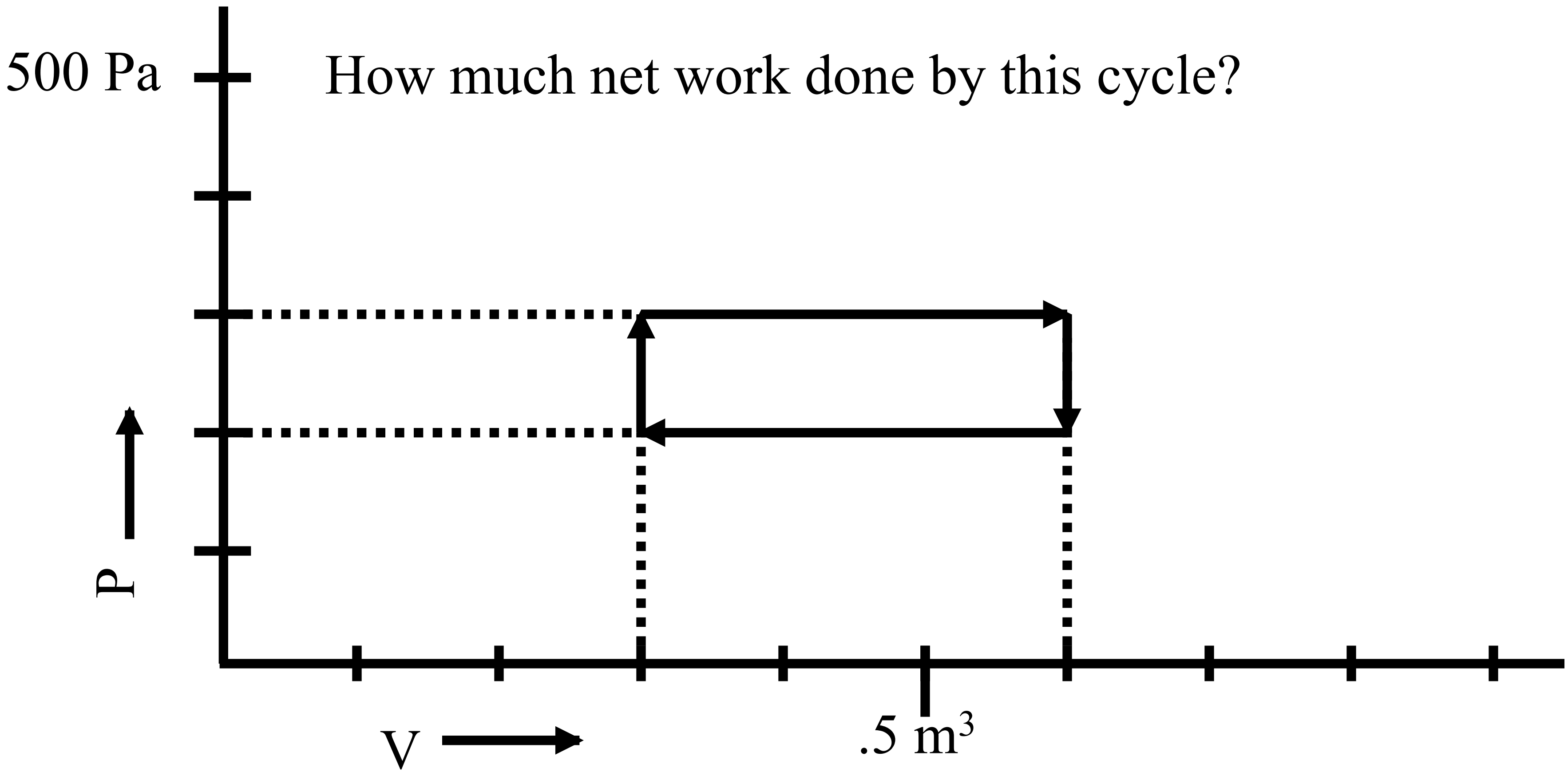


$W = P\Delta V$, $P = 300$ Pa, $\Delta V = .1 - .4 = -.3\ m^3$

-90. J

$W = -90$ J (work done <u>on</u> the gas)

How much net work done by this cycle?



W = Area = LxW = (.3 m^3)(100 Pa) = +30 J (CW)

+30 J

Thermal

Topic 3: Thermal physics	Topic 10: Thermal physics
$P = \dfrac{F}{A}$	$PV = nRT$
$Q = mc\Delta T$	$W = P\Delta V$
$Q = mL$	$Q = \Delta U + W$

The "system"

Gas/cylinder/piston/working gas

ΔU - Increase in internal energy

($U \alpha T$)

Q - Heat added to system

Heat flow in (+) / heat flow out (-)

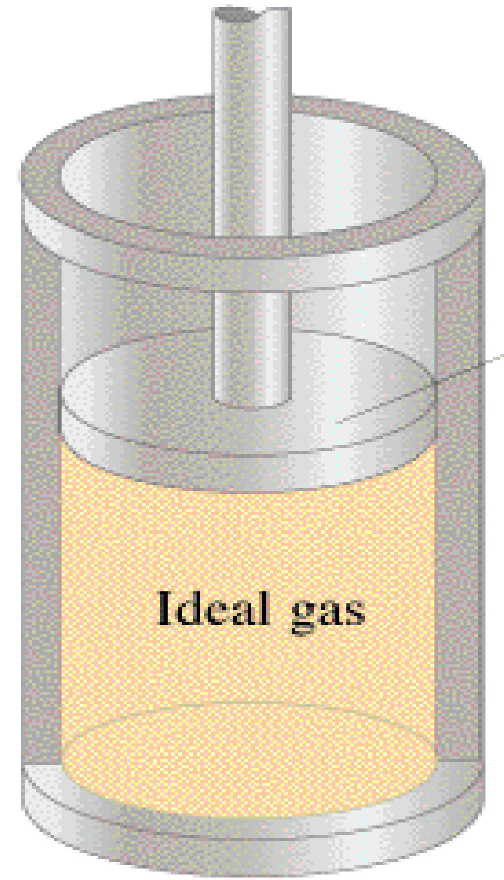
W - Work done by the system

piston moves out = work by system (+)

piston moves in = work on system (-)

$$Q = \Delta U + W$$

(conservation of energy)



Ben Derdundat lets a gas expand, doing 67 J of work, while at the same time the internal energy of the gas goes down by 34 J. What heat is transferred to the gas, and does the temperature of the gas increase, or decrease?

$Q = \Delta U + W$

$Q = -34\ J + 67\ J$

$Q = 33\ J$

Temperature decreases as it is intrinsically linked to internal energy. (the system does more work than the thermal energy supplied to it)

+33 J, decreases

End of first year stuff

Field Theory

Topic 6: Fields and forces

$F = G\dfrac{m_1 m_2}{r^2}$	$F = k\dfrac{q_1 q_2}{r^2}$
$g = \dfrac{F}{m}$	$E = \dfrac{F}{q}$

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

$$F = qvB\sin\theta$$

$$F = BIL\sin\theta$$

Topic 9: Motion in fields

$\Delta V = \dfrac{\Delta E_p}{m}$	$\Delta V = \dfrac{\Delta E_p}{q}$
$V = -\dfrac{Gm}{r}$	$V = \dfrac{kq}{r} = \dfrac{q}{4\pi\varepsilon_0 r}$
$g = -\dfrac{\Delta V}{\Delta r}$	$E = -\dfrac{\Delta V}{\Delta x}$

All of these equations are well explained on the Wiki:
http://tuhsphysics.ttsd.k12.or.us/wiki/index.php/Field_Theory_Worksheet

Ido Wanamaker places an electron 1.32×10^{-10} m from a proton. What is the force of attraction?

$$F = \frac{kq_1q_2}{r^2}$$

$k = 8.99 \times 10^{9}\ Nm^2C^{-2}$, $q_1 = -1.602 \times 10^{-19}$ C,
$q_2 = +1.602 \times 10^{-19}$ C, $r = 1.32 \times 10^{-10}$ m
$F = -1.32 \times 10^{-8}$ N

-1.32×10^{-8} N

W

Ishunta Dunnit notices that a charge of -125 μC experiences a force of .15 N to the right. What is the electric field and its direction?

$E = F/q = (.15\ N)/(-125 \times 10^{-6}\ N) = -1200\ N/C$ right or 1200 N/C left

1200 N/C left

W

Amelia Rate measures a gravitational field of 3.4 N/kg. What distance is she from the center of the earth? (Me = 5.98 x 10^{24} kg.)

g for a point mass:

$$g = \frac{Gm}{r^2}$$

G = 6.67x10^{-11} Nm^2kg^{-2}, g = 3.4 N/kg, m = 5.98x10^{24} kg

r = 10831137.03 m = 10.8 x 10^6 m (r_e = 6.38 x 10^6 m)

1.1 x 10^7 m

W

Lila Karug moves a 120. μC charge through a voltage of 5000. V. How much work does she do?

$\Delta V = \Delta E_p/q$, $q = 120\text{x}10^{-6}$ C, $\Delta V = 5000.$ V

$\Delta E_p = 0.600$ J

.600 J

W

Art Zenkraftz measures a 125 V/m electric field between some || plates separated by 3.1 mm. What must be the voltage across them?

$E = -\Delta V/\Delta x$, $\Delta x = 3.1\text{x}10^{-3}$ m, $E = 125$ V/m

$\Delta V = 0.3875\text{ V} = 0.39\text{ V}$

.39 V

W

Brennan Dondahaus accelerates an electron ($m = 9.11 \times 10^{-31}$ kg) through a voltage of 1.50 V. What is its final speed assuming it started from rest?

$\Delta V = \Delta E_p/q$, $\Delta E_p = \Delta V q = {}^1/_2 mv^2$

$\Delta V = 1.50$ V, $m = 9.11 \times 10^{-31}$ kg, $q = 1.602 \times 10^{-19}$ C

$v = 726327.8464 = 726{,}000$ m/s

726,000 m/s

W

Ashley Knott reads a voltage of 10,000. volts at what distance from a 1.00 μC charge?

$V = kq/r$, $V = 10{,}000$ V, $q = 1.00 \times 10^{-6}$ C

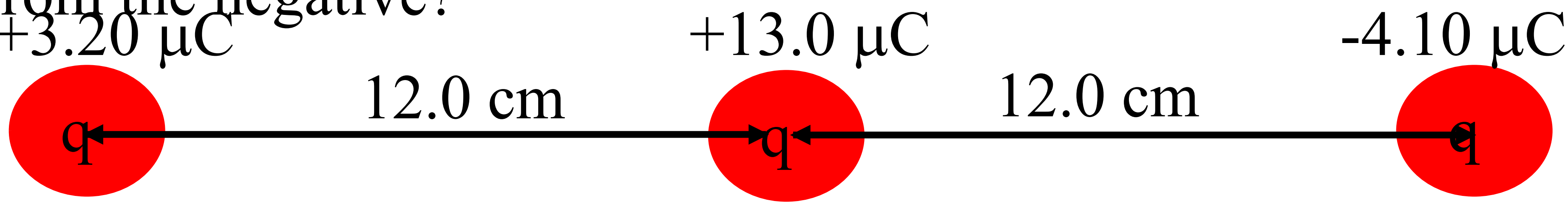
$r = .899$ m

.899 m

W

Try this one

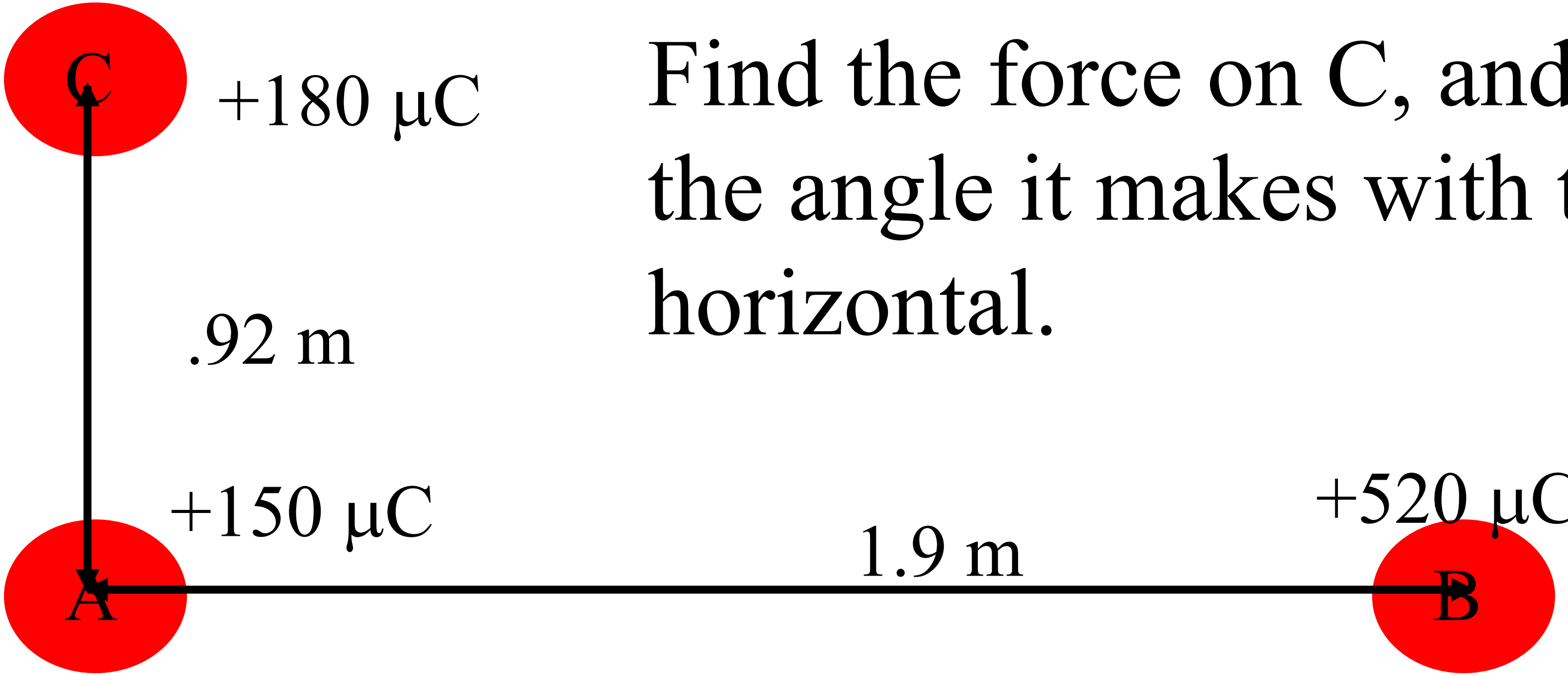
What work to bring a 13.0 μC charge from halfway between the other two charges to 6.0 cm from the positive and 18 cm from the negative?



Initial V	-67425 V
Final V	274700. V
Change in V	342100. V
Work	4.448 V

+4.4 J

Find the force on C, and the angle it makes with the horizontal.



$F_{AC} = 286.8$ N, $F_{BC} = 188.8$ N

$\theta_{ABC} = \text{Tan}^{-1}(.92/1.9) = 25.84^o$

$F_{AC} = 0 \text{ N } x \quad + 286.8 \text{ N } y$

$F_{BC} = -188.8\cos(25.84^o)\, x \quad + 188.8\sin(25.84^o) y$

$F_{total} = -170.\, x \quad + 369\, y$

410 N, 65° above x axis (to the left of y)

Current and Induction

Core	AHL
Topic 5: Electric currents	**Topic 12: Electromagnetic induction**
$Ve = \frac{1}{2}mv^2$	$\Phi = BA\cos\theta$
$I = \frac{\Delta q}{\Delta t}$	$\mathcal{E} = Bvl$
$R = \frac{V}{I}$	$\mathcal{E} = -N\frac{\Delta \Phi}{\Delta t}$
$R = \frac{\rho L}{A}$	$\frac{I_s}{I_p} = \frac{V_p}{V_s} = \frac{N_p}{N_s}$
$P = VI = I^2R = \frac{V^2}{R}$	$I_{rms} = \frac{I_0}{\sqrt{2}}$
$\mathcal{E} = I(R + r)$	$V_{rms} = \frac{V_0}{\sqrt{2}}$
$R = R_1 + R_2 + \cdots$	$R = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}}$
$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$	$P_{max} = I_0V_0$
	$P_{av} = \frac{1}{2}I_0V_0$

What current flows through a 15 ohm light bulb attached to a 120 V source of current? What charge passes through in a minute? What is the power of the light bulb?

$I = 120/15 = 8.0$ Amps

$q = It = (8\ C/s)(60\ s) = 480$ Coulombs

$P = V^2/R = 1800\ W$

8.0 A, 480 C, 960 W

W

A copper wire is 1610 m long (1 mile) and has a cross sectional area of 4.5×10^{-6} m^2. What is its resistance? (This wire is about 2.4 mm in dia)

$$R = \frac{\rho L}{A}$$

and

$A = \pi r^2$

$R = ??$

$\rho = 1.68E\text{-}8\ \Omega m$

$L = 1610$ m

$A = 4.5E\text{-}6$ m^2

$R = 6.010666667 = 6.0\ \Omega$

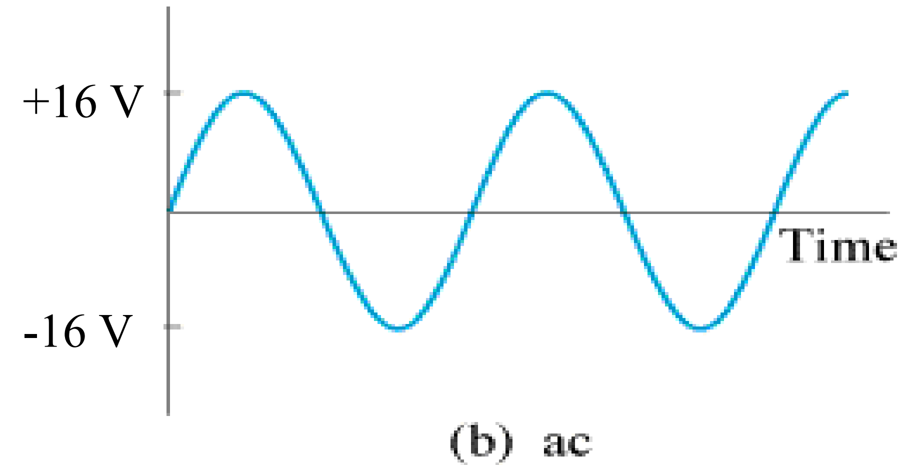
Silver	1.59E-8
Copper	1.68E-8
Gold	2.44E-8
Aluminium	2.65E-8
Tungsten	5.6 E-8
Iron	9.71E-8
Platinum	10.6E-8
Nichrome	100E-8

W

6.0 Ω

What's the rms voltage here?

$$I_{rms} = \frac{I_o}{\sqrt{2}} \qquad V_{rms} = \frac{V_o}{\sqrt{2}}$$



Given:

$$V_{rms} = \frac{V_o}{\sqrt{2}}$$

$V_o = 16$ V

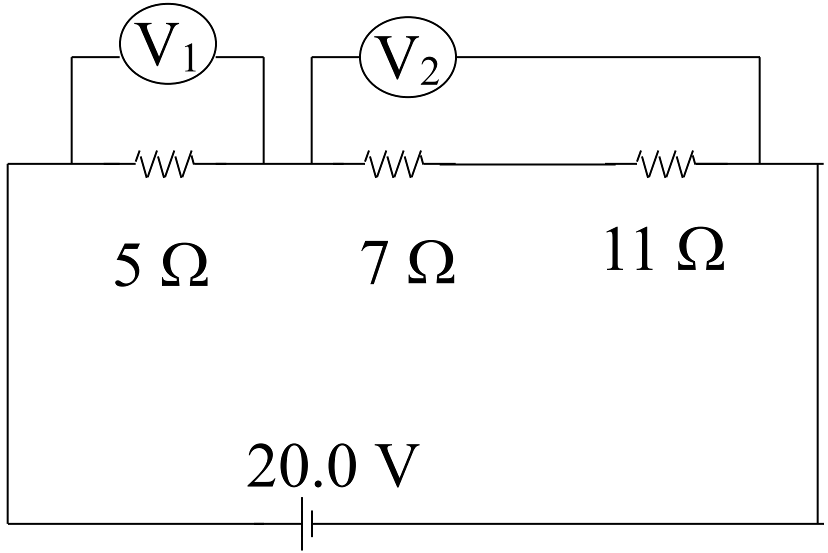
$V_{rms} = ??$

$V_{rms} = 11.3 = 11$ V

11 V

W

What do the voltmeters read? (3 SF)



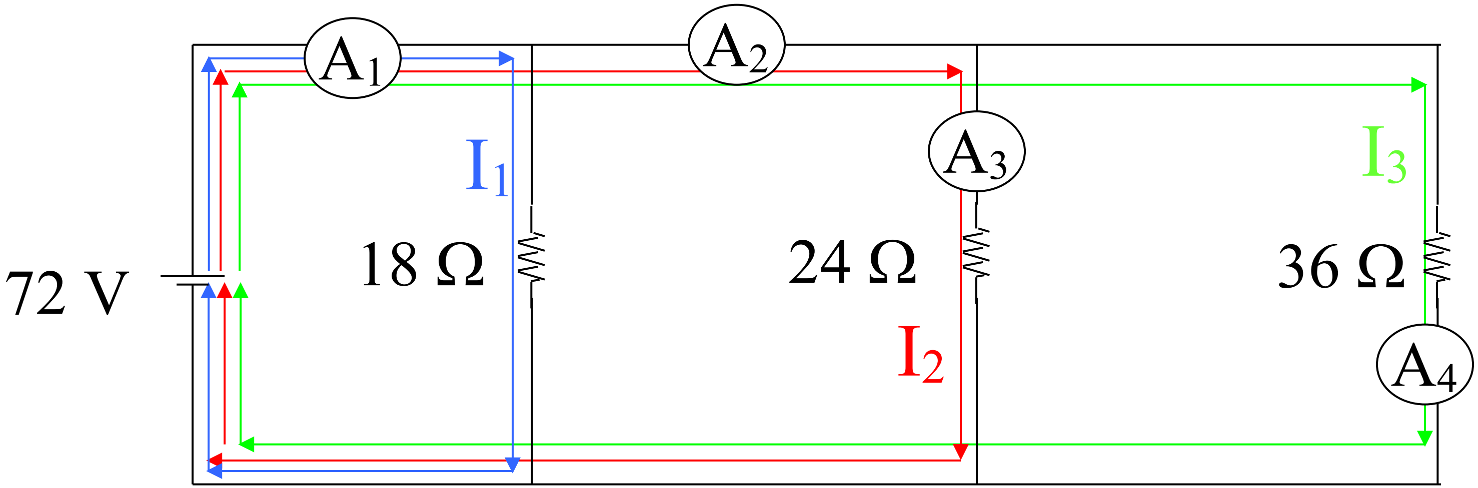
$$V = IR$$

$$V_1 = (5\ \Omega)(.8696\ \text{A}) = 4.35\ \text{V}$$

$$V_2 = (18\ \Omega)(.8696\ \text{A}) = 15.7\ \text{V}$$

4.35 V, 15.7 V

What are the readings on the meters? (2 SF)



$I_1 = 4.0\text{A}$

$I_2 = 3.0\text{ A}$

$I_3 = 2.0\text{ A}$

$A_1 = 4 + 3 + 2 = 9\text{ A}$

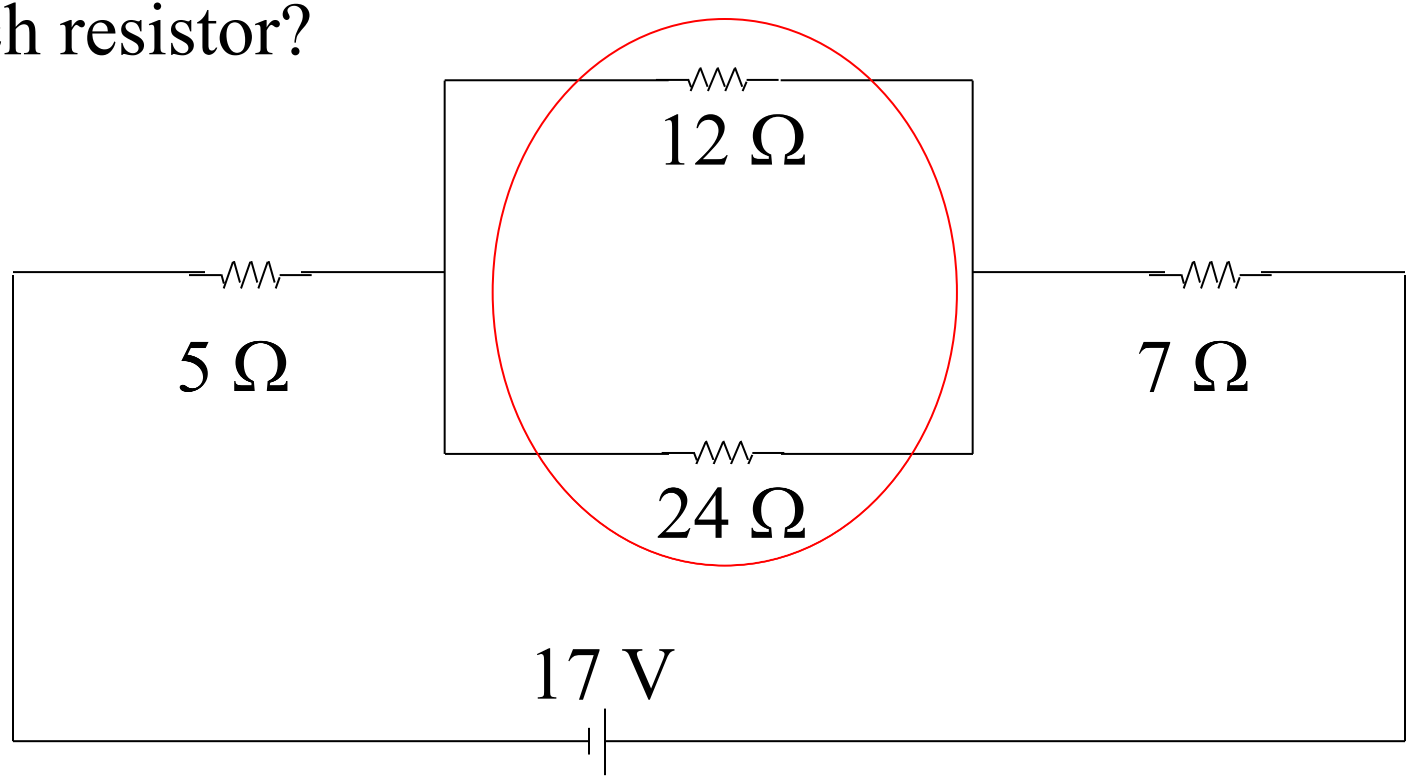
$A_2 = 3 + 2 = 5\text{ A}$

$A_3 = 3\text{ A}$

$A_4 = 2\text{ A}$

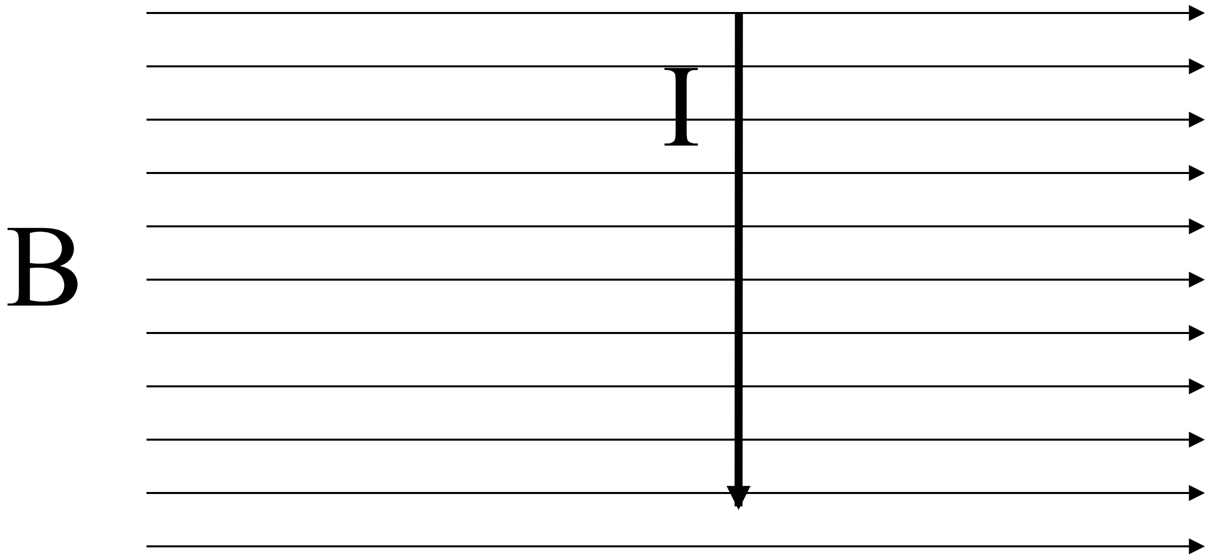
9.0, 5.0, 3.0, 2.0 A

What is the current through and the power dissipated by each resistor?



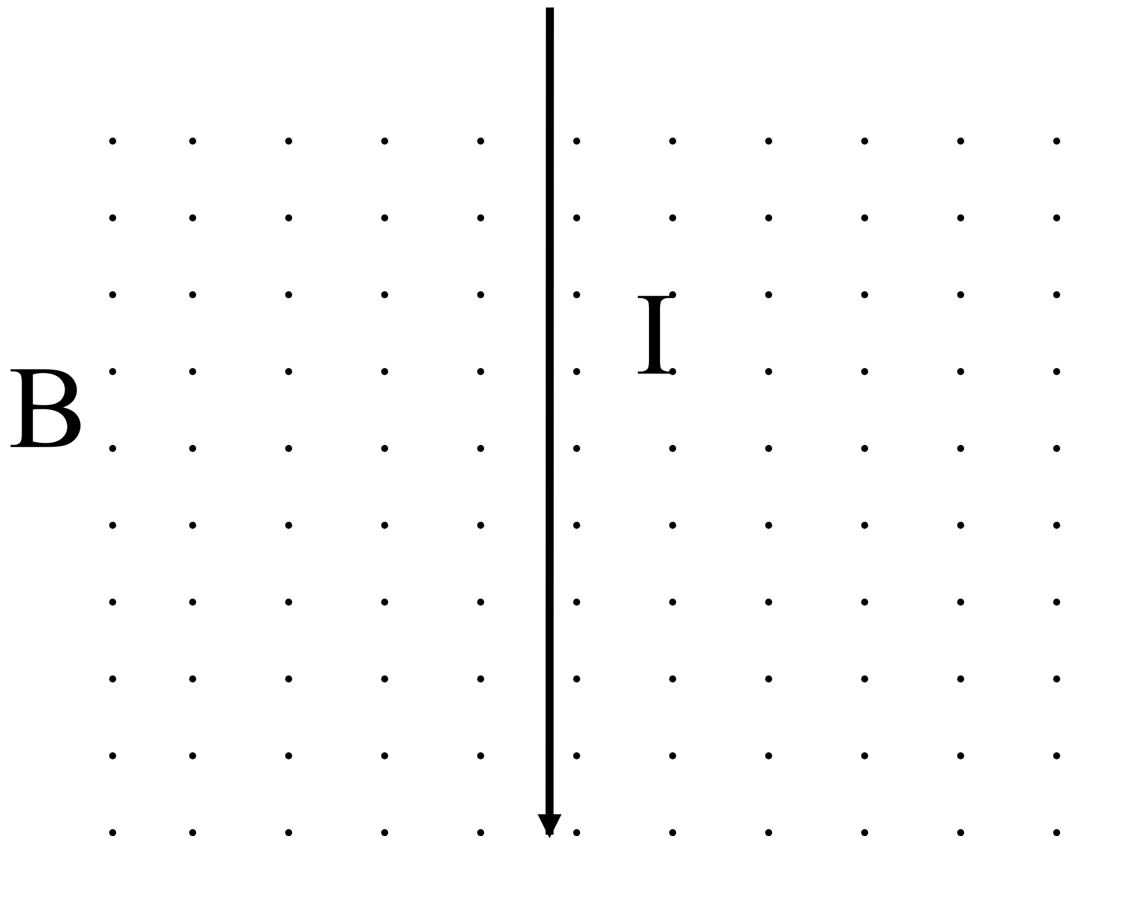
Step 1 - reduce until solvable

Which way is the force?



outa the page

Which way is the force?



left

A 0.15 T magnetic field is 17° east of North What's the force on a 3.2 m long wire if the current is 5.0 A to the West?

$\theta = 90^\circ + 17^\circ = 117^\circ$

$F = IlB\sin\theta$

$F = (5.0\ A)(3.2\ m)(0.15\ T)\sin(117^\circ) = 2.1\ N$

W x NE = Down (Into this page)

N

W E

S

2.1 N vertically downward

What is the force acting on a proton moving at 2.5×10^8 m/s perpendicular to a .35 T magnetic field?

$q = 1.602 \times 10^{-19}$ C

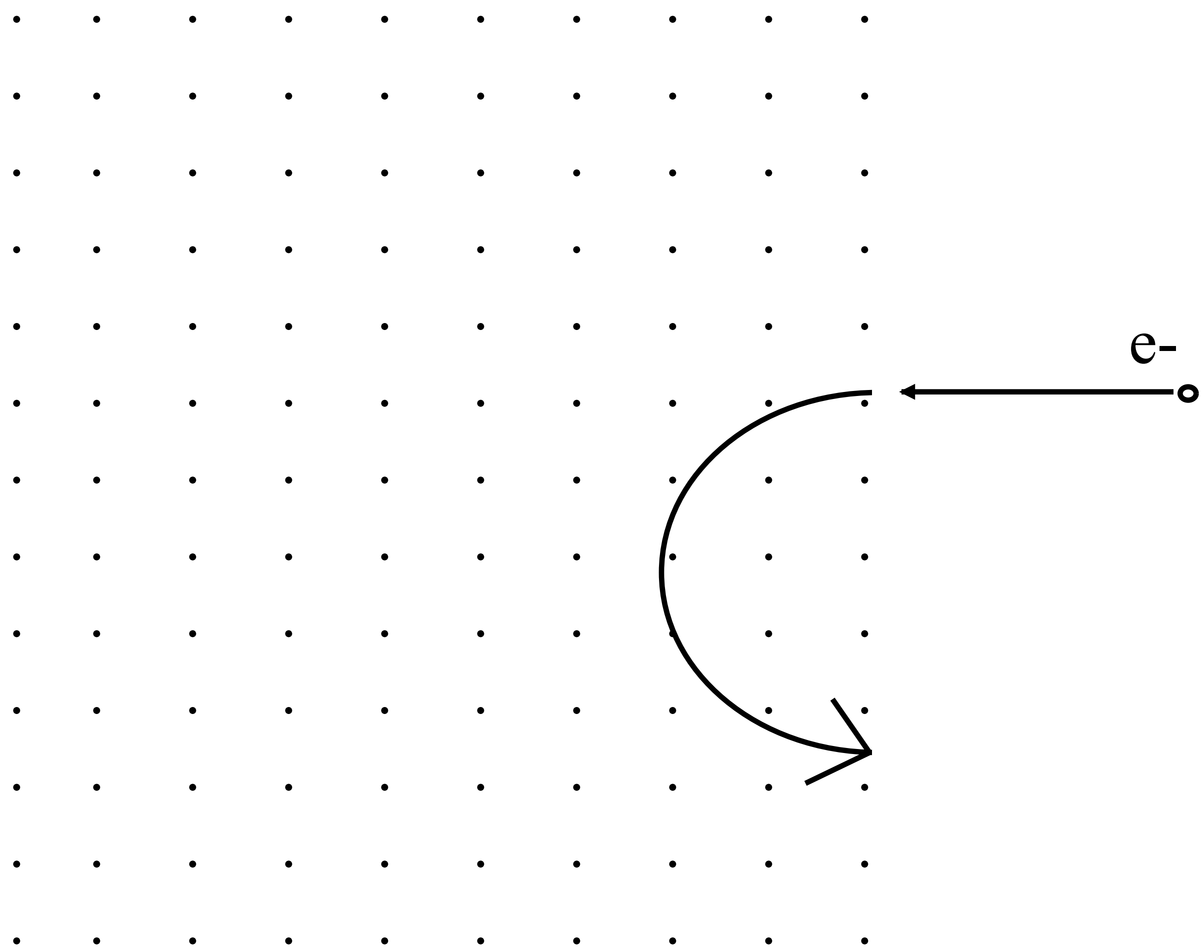
$F = qvB\sin\theta$

$F = qvB\sin\theta$

$F = (1.602 \times 10^{-19}\ C)(2.5 \times 10^{8}\ m/s)(.35\ T)\sin(90^{o}) = 1.4 \times 10^{-11}\ N$

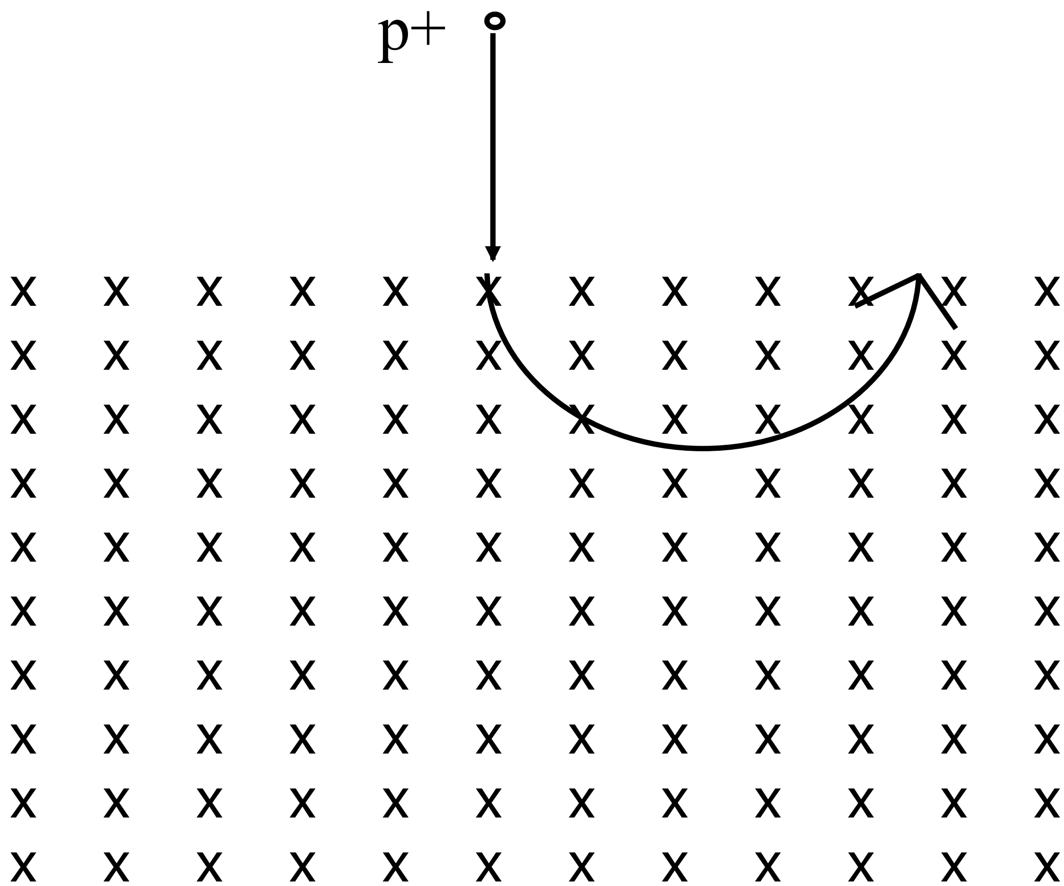
1.4×10^{-11} N

What is the path of the electron in the B field?



ACW

What is the path of the proton in the B field?



ACW

If the electron is going 1.75×10^6 m/s, and the magnetic field is .00013 T, what is the radius of the path of the electron?

$m = 9.11 \times 10^{-31}$ kg

$q = 1.602 \times 10^{-19}$ C

$F = qvB\sin\theta$

$F = ma$, $a = v^2/r$

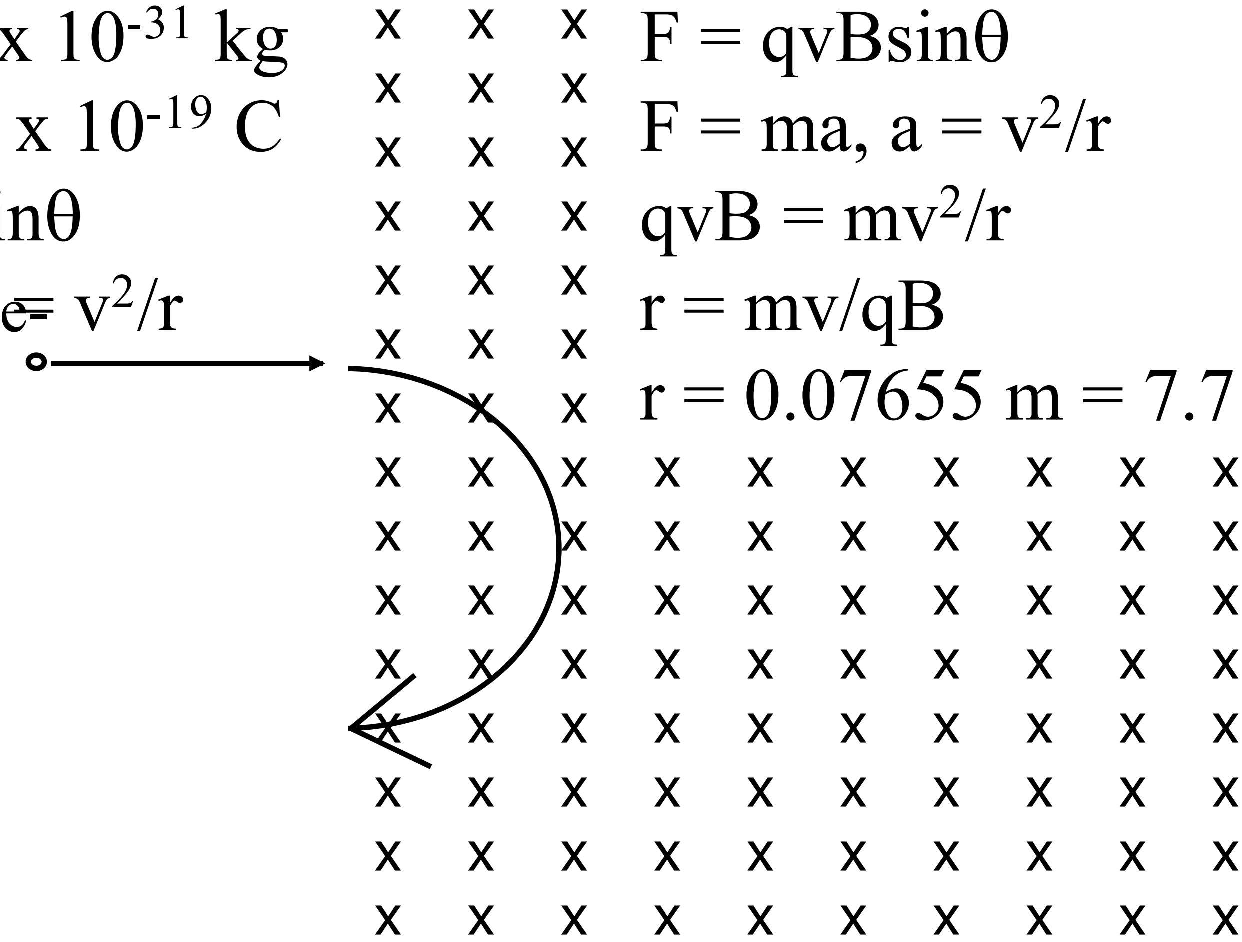
$F = qvB\sin\theta$

$F = ma$, $a = v^2/r$

$qvB = mv^2/r$

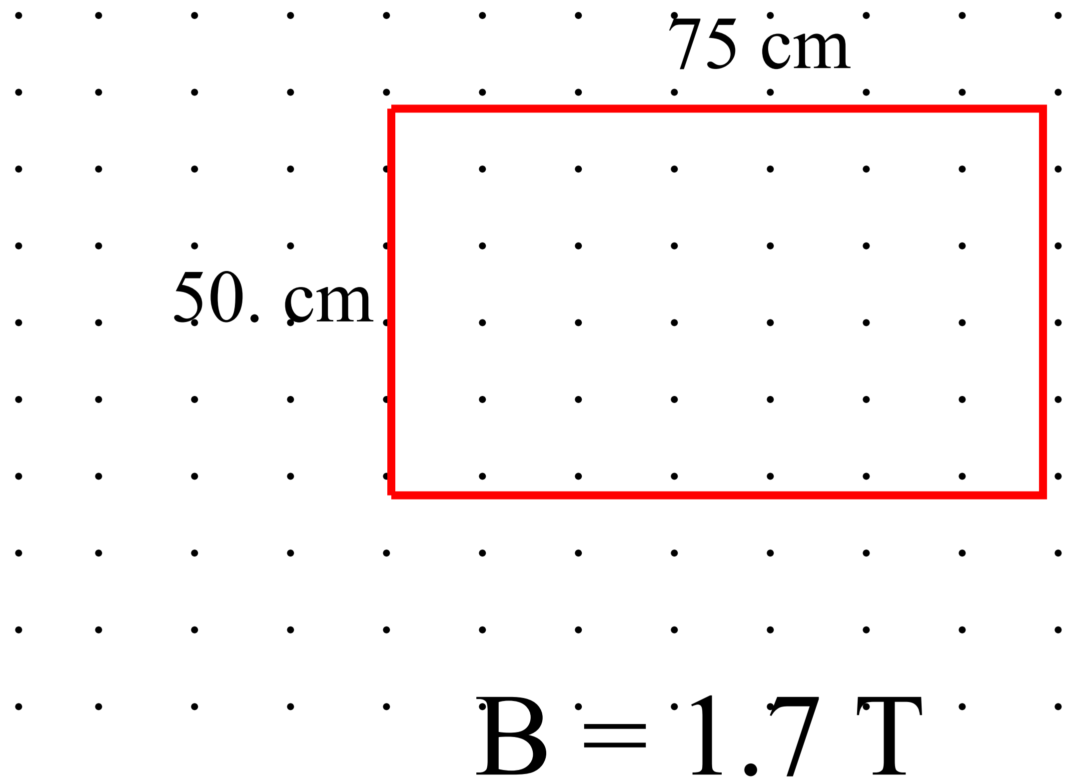
$r = mv/qB$

$r = 0.07655$ m $= 7.7$ cm



7.7 cm

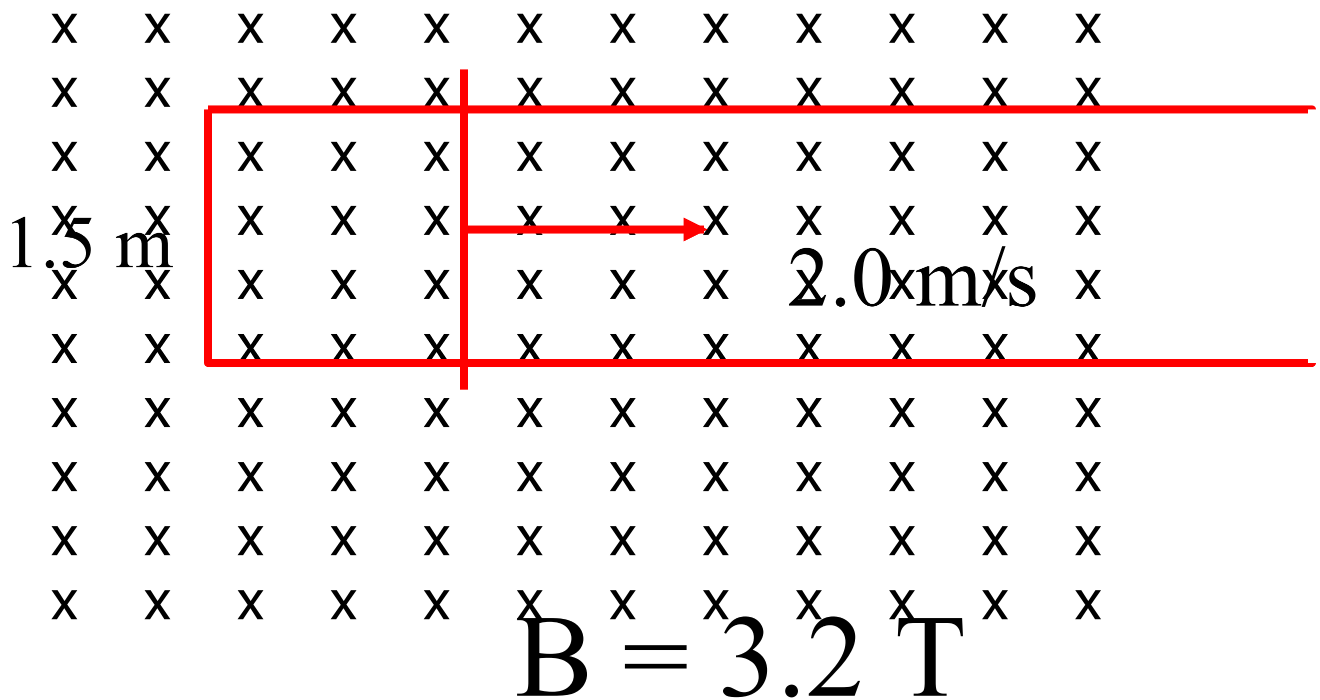
The loop is removed in .012 s. What is the EMF generated? Which way does the current flow? (N = 1)



53 V, acw

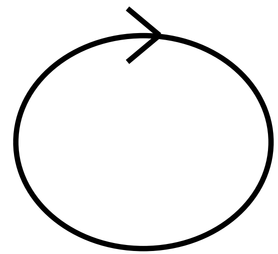
Three ways for direction, resist change, magnet, qvb

The bar moves to the right at 2.0 m/s, and the loop is 1.5 m wide. What EMF is generated, and which direction is the current?



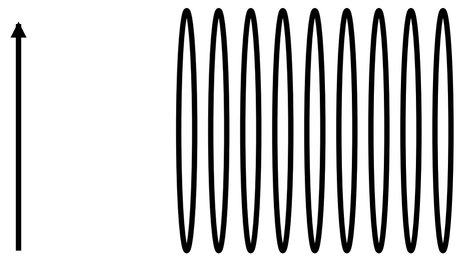
9.6 V, acw

Where da North Pole?



into da page

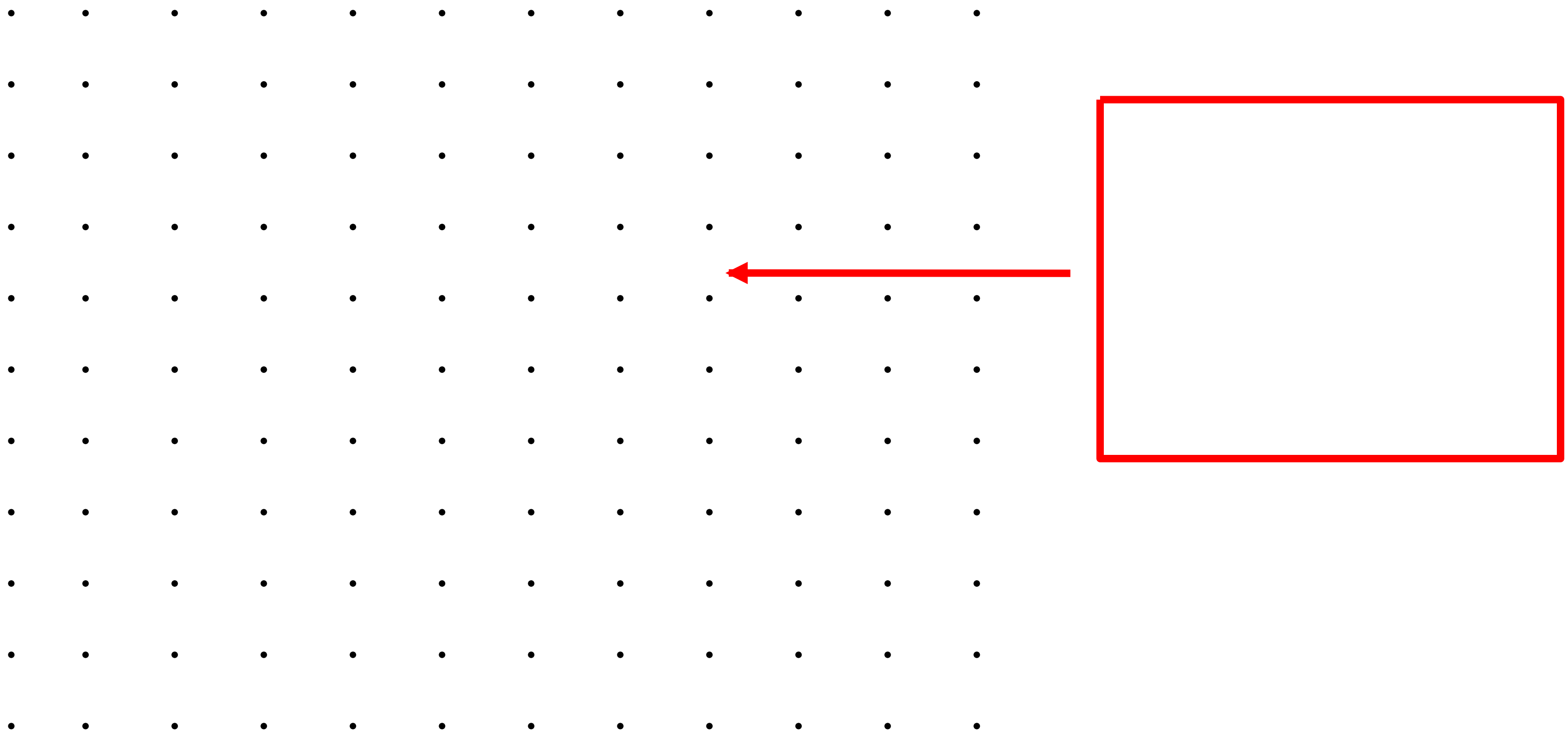
Where da North Pole?



Current goes up in the front of the coil

left side

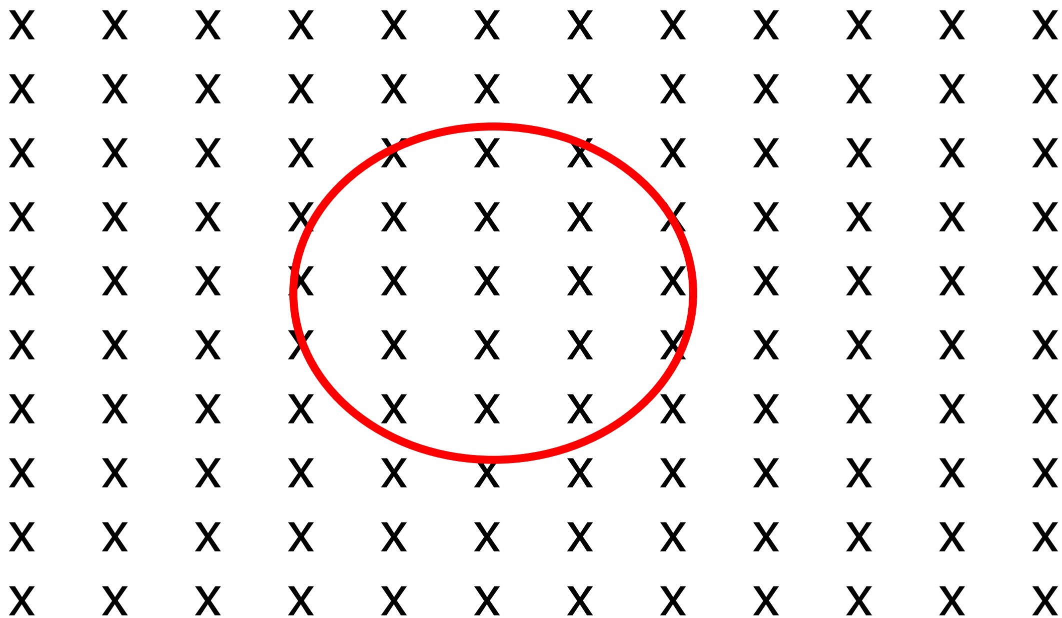
Which way is the current? (When does it stop flowing?)



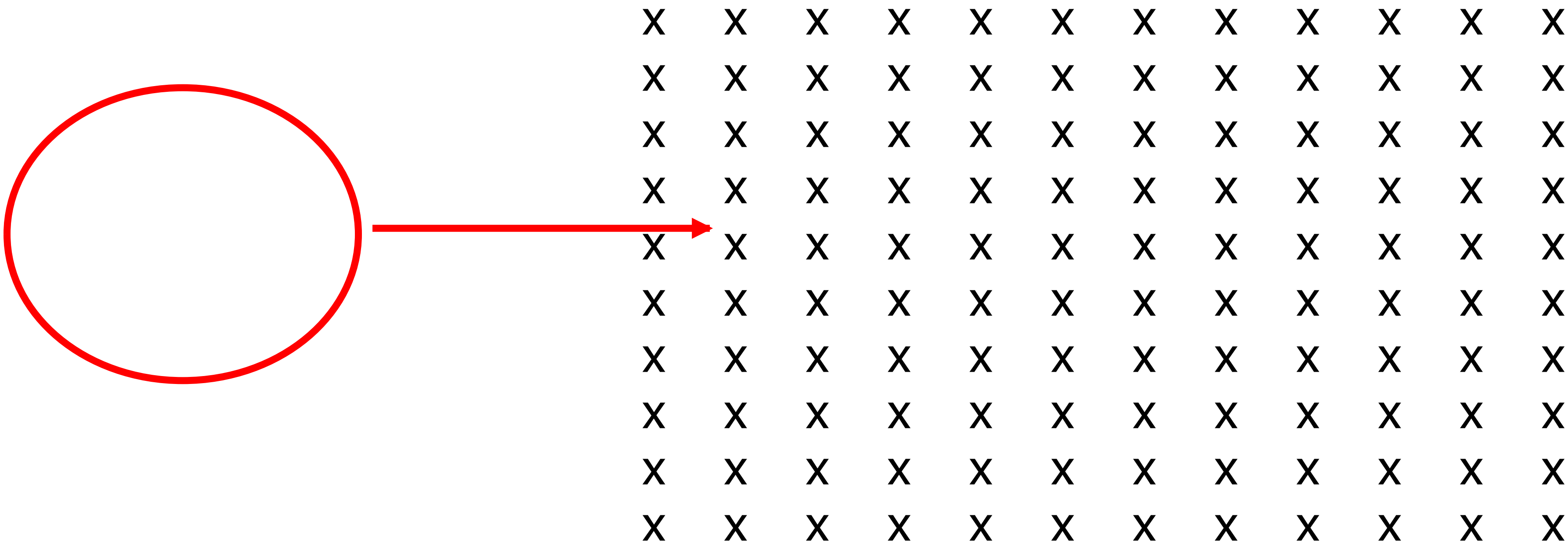
CW

Which way is the current?

B increases into page

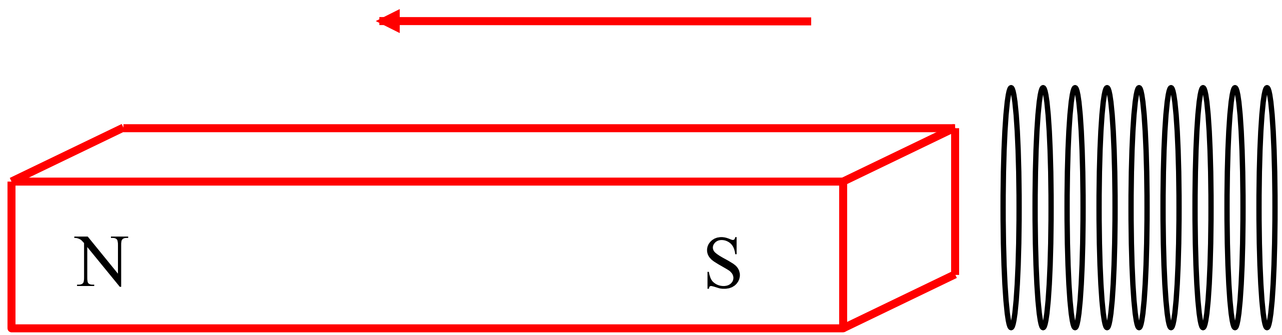


Which way is the current?



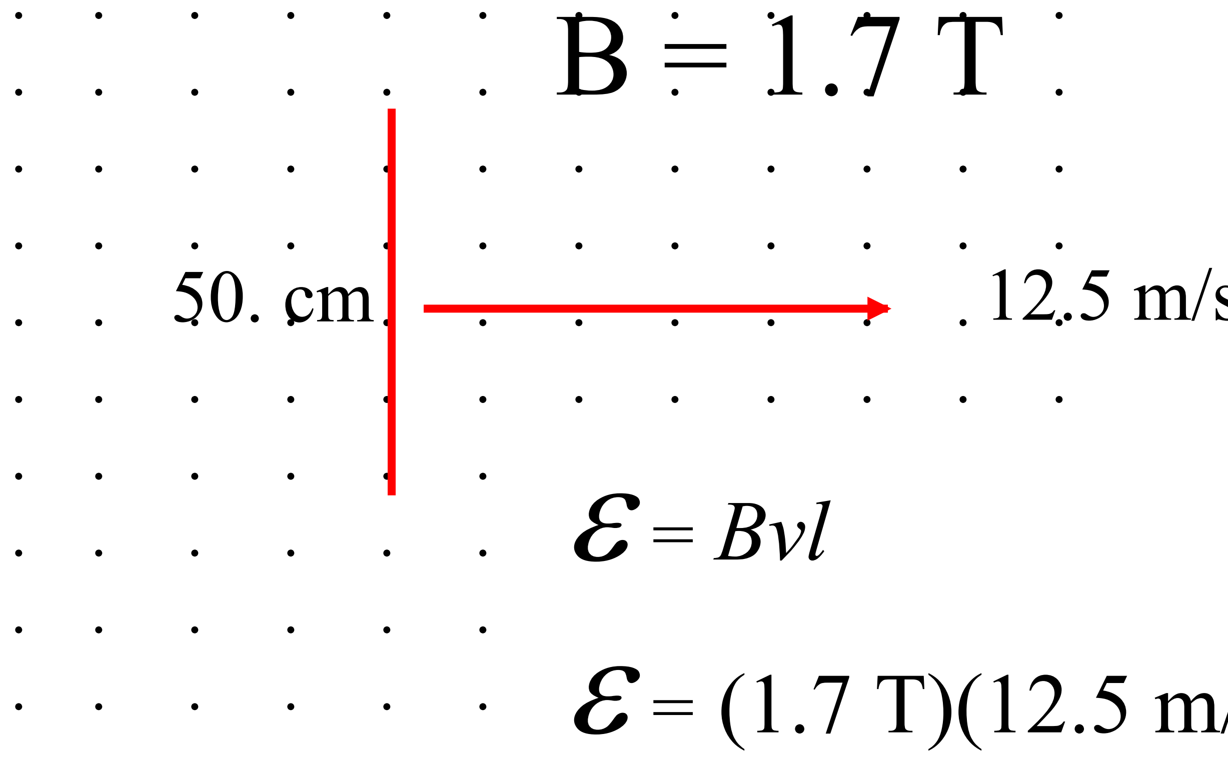
ACW

Which way is the current on the front of the coil? (up or down)



Up the front

The wire moves to the right at 12.5 m/s. What is the EMF generated? Which end of the wire is the + end?



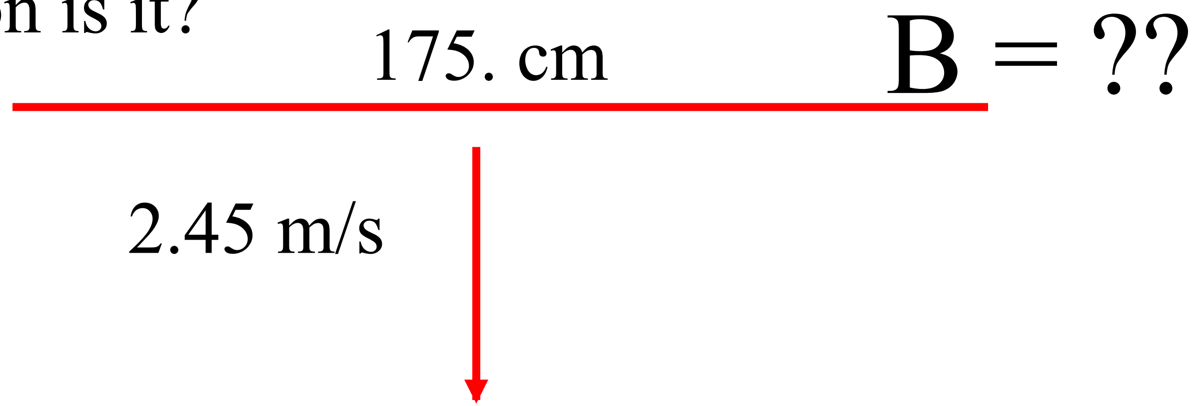
$$\mathcal{E} = Bvl$$

$$\mathcal{E} = (1.7\ \text{T})(12.5\ \text{m/s})(.50\ \text{m})$$

$$= 10.625\ \text{V} = 11\ \text{V}$$

11 V , Bottom

The wire has a potential of .215 V, and the right end is positive. What is the magnetic field, and which direction is it?



$$\varepsilon = Bvl$$

.215 V = B(2.45 m/s)(1.75 m)

B = 0.050145773 = .0501 T

.0501 T ,into page

A transformer has 120 primary windings, and 2400 secondary windings. If there is an AC voltage of 90. V , and a current of 125 mA in the primary, what is the voltage across and current through the secondary?

This one steps up

V = 90*(2400/120) = 1800 V

Current gets less: Power in = power out

IV = IV

(0.125 A)(90. V) = (I)(1800) = .00625 A = 6.25 mA

1800 V, 6.25 mA

Atomic and Nuclear

Core	AHL
Topic 7: Atomic and nuclear physics	**Topic 13: Quantum physics and nuclear physics**
$E = mc^2$	$E = hf$
	$hf = \phi + E_{max}$
	$hf = hf_0 + eV$
	$p = \frac{h}{\lambda}$
	$E_K = \frac{n^2 h^2}{8 m_e L^2}$
	$\Delta x \Delta p \geq \frac{h}{4\pi}$
	$\Delta E \Delta t \geq \frac{h}{4\pi}$
	$N = N_0 e^{-\lambda t}$
	$A = -\frac{\Delta N}{\Delta t}$
	$A = \lambda N = \lambda N_0 e^{-\lambda t}$
	$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Energy, Power and Climate

Core	AHL
Topic 8: Energy, power and climate change $\text{power} = \frac{1}{2} A \rho v^3$ $\text{power per unit length} = \frac{1}{2} A^2 \rho g v$ $I = \frac{\text{power}}{A}$ $\text{albedo} = \frac{\text{total scattered power}}{\text{total incident power}}$ $C_s = \frac{Q}{A \Delta T}$ $\text{power} = \sigma A T^4$ $\text{power} = e \sigma A T^4$ $\Delta T = \frac{(I_{in} - I_{out}) \Delta t}{C_s}$	

Sankey diagrams

Figure 805 Sankey diagram for a torch

(torch is British for flashlight)

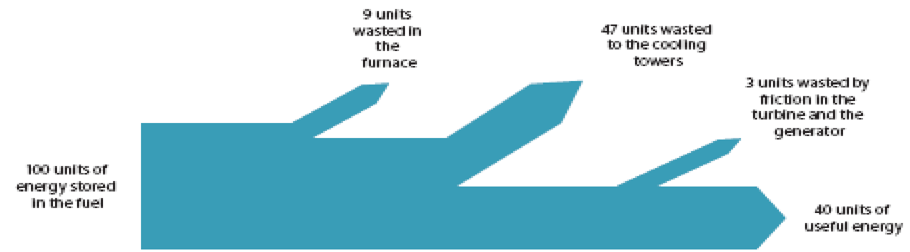


Figure 806 Sankey diagram for a coal-fired power station

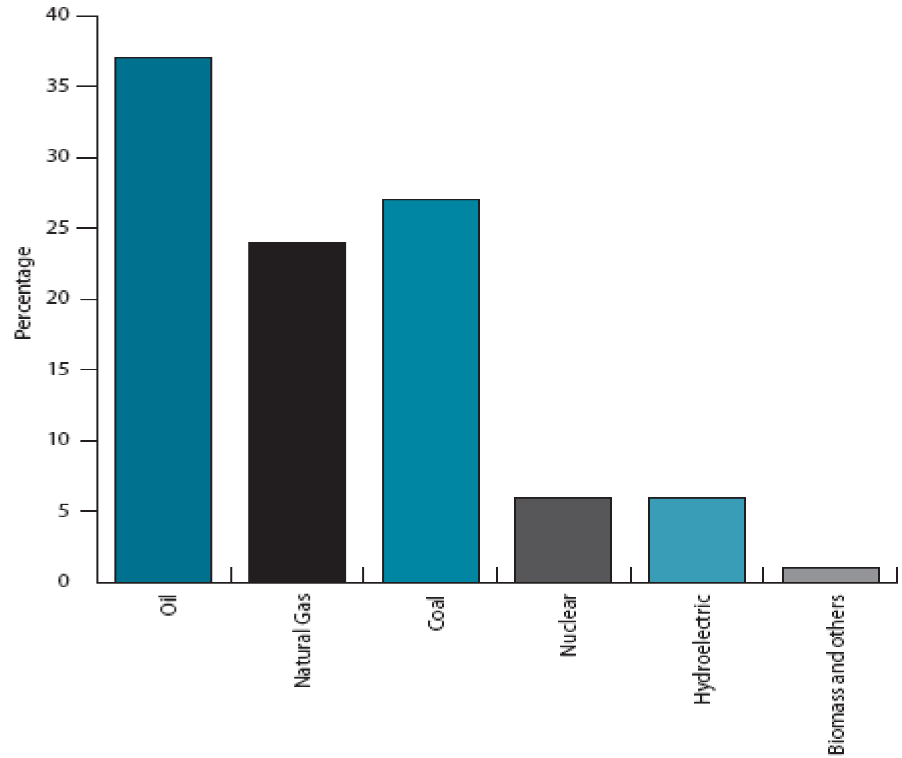


Figure 817 World fuel consumption

$$= \frac{1}{2} m v^2 = \frac{1}{2} (\rho v A) v^2 = \frac{1}{2} \rho A v^3$$

Power available $= \frac{1}{2} \rho A v^3$

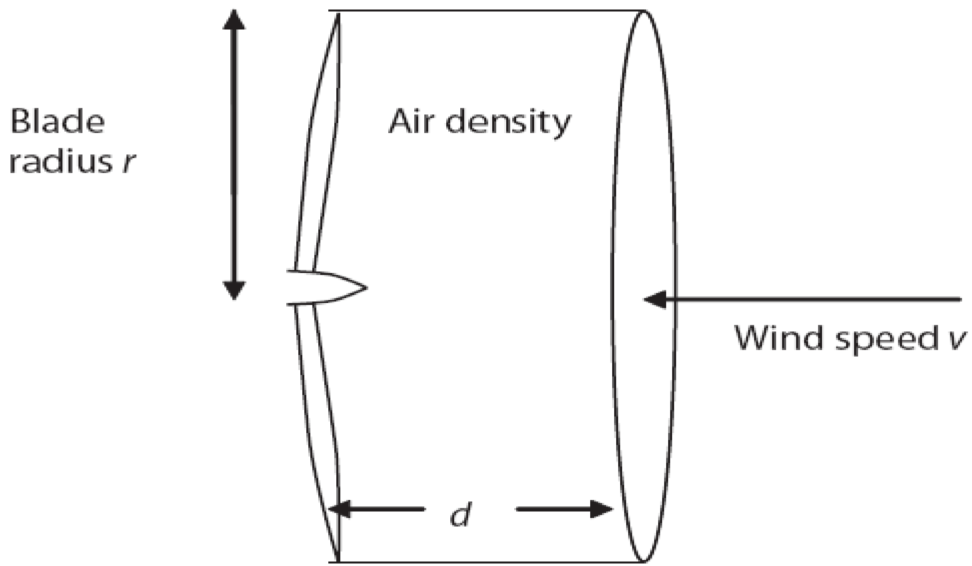


Figure 847 Power output of a wind generator

Air with a density of 1.3 kg m^{-3} is moving at 13.5 m/s across a wind turbine with a radius of 32.1 m. What is the theoretical wind power available to this turbine? If the generator actually generates 2.8 MW, what is the efficiency?

power = $^{1}/_{2}A\rho v^3$ = $^{1}/_{2}\pi(32.1)^2(1.3\text{kg m}^{-3})(13.5\text{ m/s})^3$ = 5,176,957.499 W = 5.2 MW
efficiency = 2.8E6W/ 5,176,957.499 W = 0.540858216 = 0.54 or 54%

5.2 MW, 0.54

You have a wind turbine that is 49% efficient at a wind speed of 8.5 m/s. How long do the blades need to be so that you can generate 1.8 MW of electricity. Use the density of air to be 1.3 kg m^{-3}.

$0.49 = 1.8E6W/P_{theoretical}$, $P_{theoretical} = 3.6735E+06$ W
power = $^1/_2 A\rho v^3 = {}^1/_2\pi(32.1)^2(1.3\text{kg m}^{-3})(13.5 \text{ m/s})^3$ = 5,176,957.499 W = 5.2 MW
3.6735E+06 W = $^1/_2\pi r^2(1.3\text{kg m}^{-3})(8.5 \text{ m/s})^3$
r = 54.12 = 54 m.

54 m

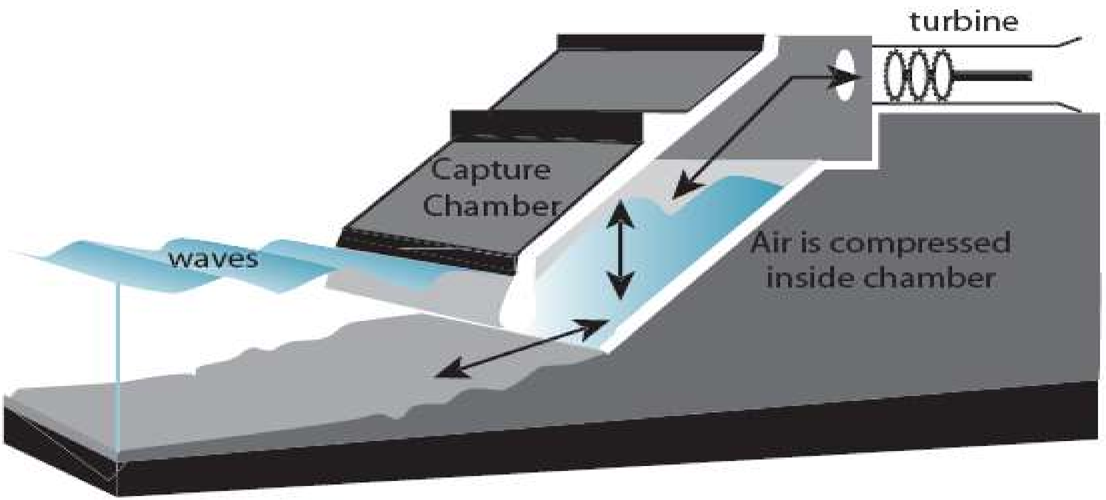


Figure 850 Onshore oscillating water column

Power per metre = ½ $\rho g A^2 v$

ρ = water density kg/m^3

g = 9.81 N/kg

A = wave amplitude in m

v = wave speed

$v = f\lambda$, $f = 1/T$

Wave energy .

Wave energy solution

yah

8.5.2 Albedo

The term **albedo** (α) (Latin for white) at a **surface is the** ratio between the incoming radiation and **the amount** reflected expressed as a coefficient or as a percentage.

Surfaces	Albedo %
Oceans	10
Dark soils	10
Pine forests	15
Urban areas	15
Light coloured deserts	40
Deciduous forests	25
Fresh snow	85
Ice	90
Whole planet	31

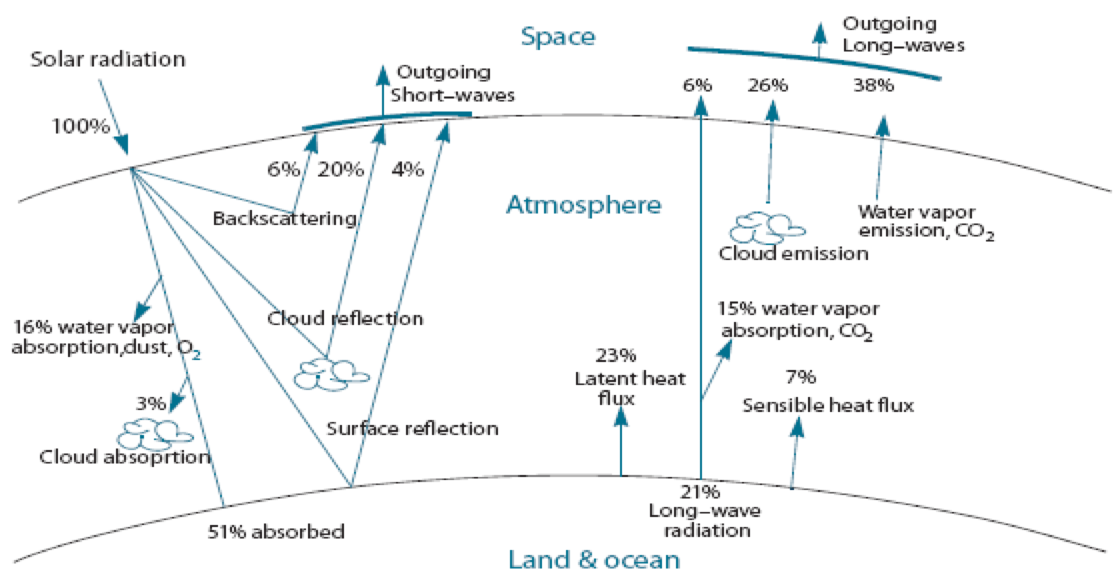


Figure 853 Solar radiation energy input and output

8.5.12 Surface heat capacity

Surface heat capacity C_s is the energy required to raise the temperature of a unit area of a planet's surface by one degree Kelvin and is measured in $Jm^{-2}K^{-1}$.

$$C_s = Q / A\,\Delta T$$

$$C_S = f\,\rho\,c\,h$$

where f = 0.7 (fraction of Earth covered by water),

ρ = the density of sea water 1023 kgm^{-3},

c = the specific heat capacity of water 4186 $Jkg^{-1}K^{-1}$

h = the depth of seawater that stores thermal energy.

So $C_S = 0.7 \times 1023\ kgm^{-3} \times 4180\ Jkg^{-1}K^{-1} \times 70\ m = 2.1 \times 10^8\ Jm^{-2}K^{-1}$

Astophysics

Core (SL and HL)	Extension (HL only)
Option E: Astrophysics	
$L = \sigma A T^4$	$L \propto m^n$ where $3 < n < 4$
$\lambda_{\max}\,(\text{metres}) = \dfrac{2.90\times10^{-3}}{T\,(\text{kelvin})}$	$\dfrac{\Delta\lambda}{\lambda} \cong \dfrac{v}{c}$
$d\,(\text{parsec}) = \dfrac{1}{p\,(\text{arc-second})}$	$v = H_0 d$
$b = \dfrac{L}{4\pi d^2}$	
$m - M = 5\lg\left(\dfrac{d}{10}\right)$	

Concept 0 – Total power output

Luminosity $L = \sigma A T^4$

Luminosity L = The star's power output in Watts

σ = Stefan Boltzmann constant = $5.67 \times 10^{-8} W/m^2K^4$

A = The star's surface area = $4\pi r^2$

T = The star's surface temperature in Kelvins

A star has a radius of 5×10^8 m, and Luminosity of 4.2×10^{26} Watts, What is its surface temperature?

Luminosity $L = \sigma A T^4$,

$T = (L/(\sigma 4\pi(5x10^8)^2))^{.25} = 6968\ K = 7.0x10^3\ K$

$7.0x10^3$ K

Astophysics

Core (SL and HL)	Extension (HL only)
Option E: Astrophysics	
$L = \sigma A T^4$	$L \propto m^n \quad \text{where} \quad 3 < n < 4$
$\lambda_{\max}\,(\text{metres}) = \dfrac{2.90 \times 10^{-3}}{T\,(\text{kelvin})}$	$\dfrac{\Delta\lambda}{\lambda} \cong \dfrac{v}{c}$
$d\,(\text{parsec}) = \dfrac{1}{p\,(\text{arc-second})}$	$v = H_0 d$
$b = \dfrac{L}{4\pi d^2}$	
$m - M = 5\lg\left(\dfrac{d}{10}\right)$	

Concept -1 – Temperature

$$\lambda_{max}\ (\text{metres}) = \frac{2.90 \times 10^{-3}\ \text{m k}}{T\ (\text{Kelvin})}$$

λ_{max} = Peak black body wavelength

T = The star's surface temperature in Kelvins

A star has a λ_{max} of 940 nm, what is its surface temperature?

$\lambda_{max} = (2.90 \times 10^{-3}\ \text{m K})/T$,

$T = (2.90 \times 10^{-3}\ \text{m K})/\ \lambda_{max}$

$= (2.90 \times 10^{-3}\ \text{m K})/\ (940\text{E-}9) = 3100\ \text{K}$

3100 K

Astophysics

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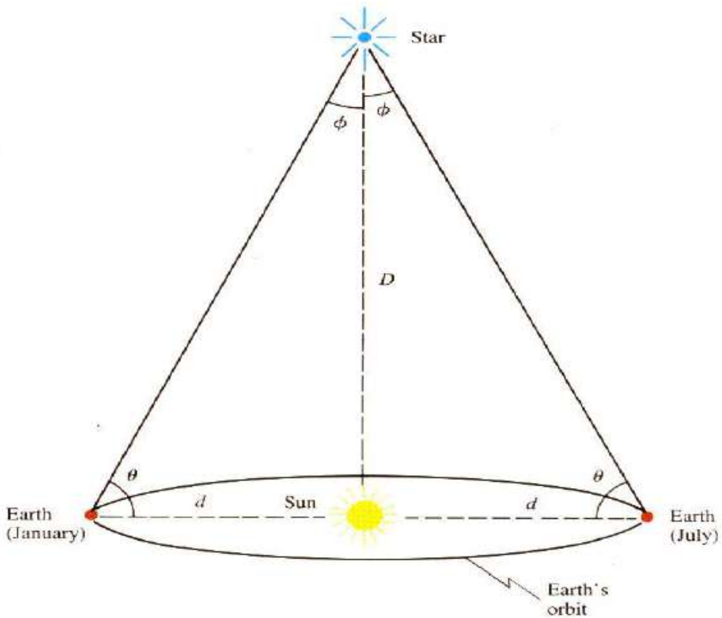
$L = \sigma A T^4$	$L \propto m^n$ where $3 < n < 4$
$\lambda_{max}\ (\text{metres}) = \dfrac{2.90 \times 10^{-3}}{T\ (\text{kelvin})}$	$\dfrac{\Delta\lambda}{\lambda} \cong \dfrac{v}{c}$
$d\,(\text{parsec}) = \dfrac{1}{p\,(\text{arc-second})}$	$v = H_0 d$
$b = \dfrac{L}{4\pi d^2}$	
$m - M = 5\lg\left(\dfrac{d}{10}\right)$	

Parsecs - Parallax Seconds

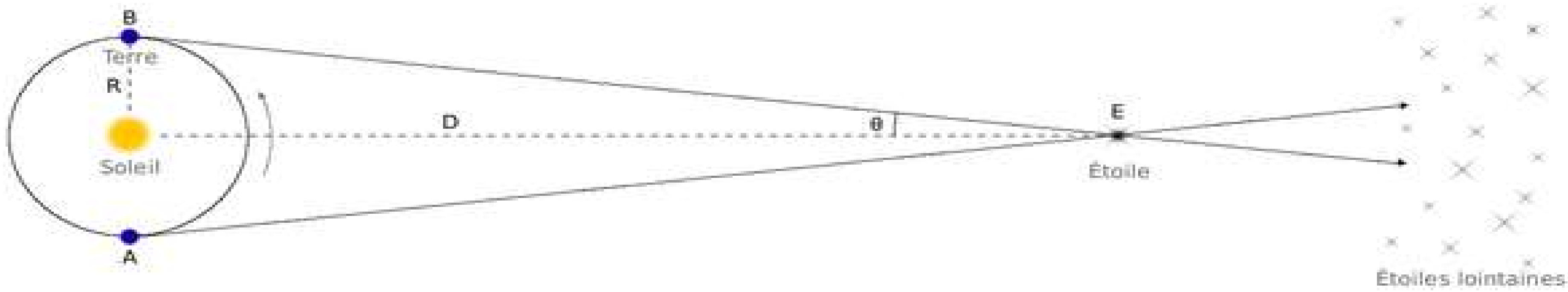


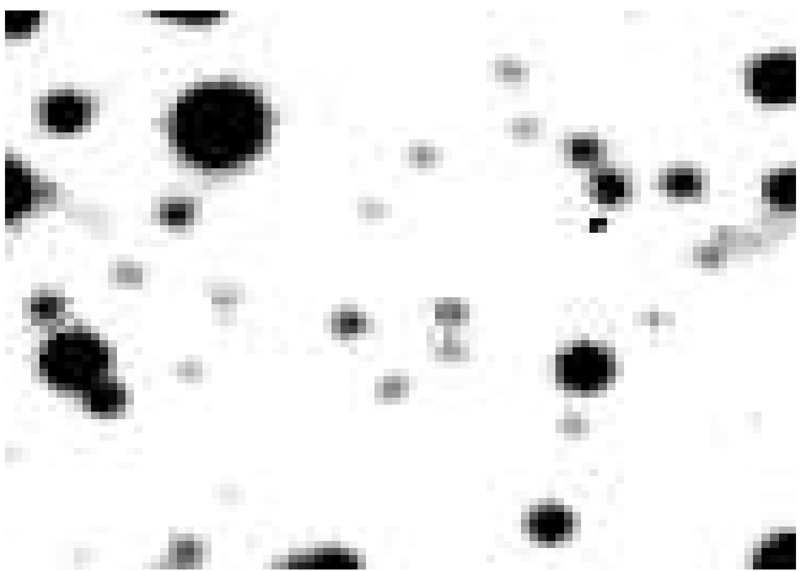
$$d\ (\text{parsec}) = \frac{1}{p\ (\text{arc-second})}$$

p = parallax angle in seconds

(1 second = 1/3600 of a degree)

From Douglas Giancoli's Physics


B
Terre
R
Soleil
A
D
θ
E
Étoile
Étoiles lointaines



If a star has a parallax of .12”, what is its distance in parsecs?

Parsecs = 1/arcseconds = 1/.12 = 8.3 pc

8.3 pc

Astophysics

Core (SL and HL)	Extension (HL only)
Option E: Astrophysics	
$L = \sigma A T^4$	$L \propto m^n \quad \text{where} \quad 3 < n < 4$
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$d\,(\text{parsec}) = \dfrac{1}{p\,(\text{arc-second})}$	$v = H_0 d$
$b = \dfrac{L}{4\pi d^2}$	
$m - M = 5\lg\left(\dfrac{d}{10}\right)$	

Concept 1 – Apparent Brightness

Apparent Brightness $b = \frac{L}{4\pi d^2}$

b = The apparent brightness in W/m^2
L = The star's Luminosity (in Watts)
d = The distance to the star

L is spread out over a sphere..

Another star has a luminosity of 3.2 x 10^{26} Watts. We measure an apparent brightness of 1.4 x 10^{-9} W/m^2. How far are we from it?

$b = L/4\pi d^2$, $d = (L/4\pi b)^{.5} = 1.3\text{x}10^{17}$ m

1.3x10^{17} m

Astophysics

Core (SL and HL)	Extension (HL only)
Option E: Astrophysics	
$L = \sigma A T^4$	$L \propto m^n$ where $3 < n < 4$
$\lambda_{\max}\,(\text{metres}) = \dfrac{2.90 \times 10^{-3}}{T\,(\text{kelvin})}$	$\dfrac{\Delta\lambda}{\lambda} \cong \dfrac{v}{c}$
$d\,(\text{parsec}) = \dfrac{1}{p\,(\text{arc-second})}$	$v = H_0 d$
$b = \dfrac{L}{4\pi d^2}$	
$m - M = 5\lg\left(\dfrac{d}{10}\right)$	

Absolute Magnitude: $m - M = 5 \log_{10}(d/10)$

M = The Absolute Magnitude

d = The distance to the star in parsecs

m = The star's Apparent Magnitude

Example: 100 pc from an m = 6 star, M = ?

(10x closer = 100x the light = -5 for m)

$M = 6 - 5 \log10(100/10) = 1$

You are 320 pc from a star with an absolute magnitude of 6.3. What is its apparent magnitude?

$M = m - 5\ \log_{10}(d/10)$,
$m = M + 5\ \log_{10}(d/10) = 6.3 + 5$
$\log_{10}(320/10) = 14$

Astophysics

Core (SL and HL)	Extension (HL only)
Option E: Astrophysics $L = \sigma A T^4$ $\lambda_{max}\,(\text{metres}) = \dfrac{2.90\times10^{-3}}{T\,(\text{kelvin})}$ $d\,(\text{parsec}) = \dfrac{1}{p\,(\text{arc-second})}$ $b = \dfrac{L}{4\pi d^2}$ $m - M = 5\lg\left(\dfrac{d}{10}\right)$	$L \propto m^n$ where $3 < n < 4$ $\dfrac{\Delta\lambda}{\lambda} \cong \dfrac{v}{c}$ $v = H_0 d$

Redshift:

If v << c:

$$\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$$

$\Delta\lambda$ - Change in wavelength

λ - original wavelength

v - recession velocity

c - speed of light

What is the recession rate of a galaxy whose 656 nm line comes in at 691 nm?

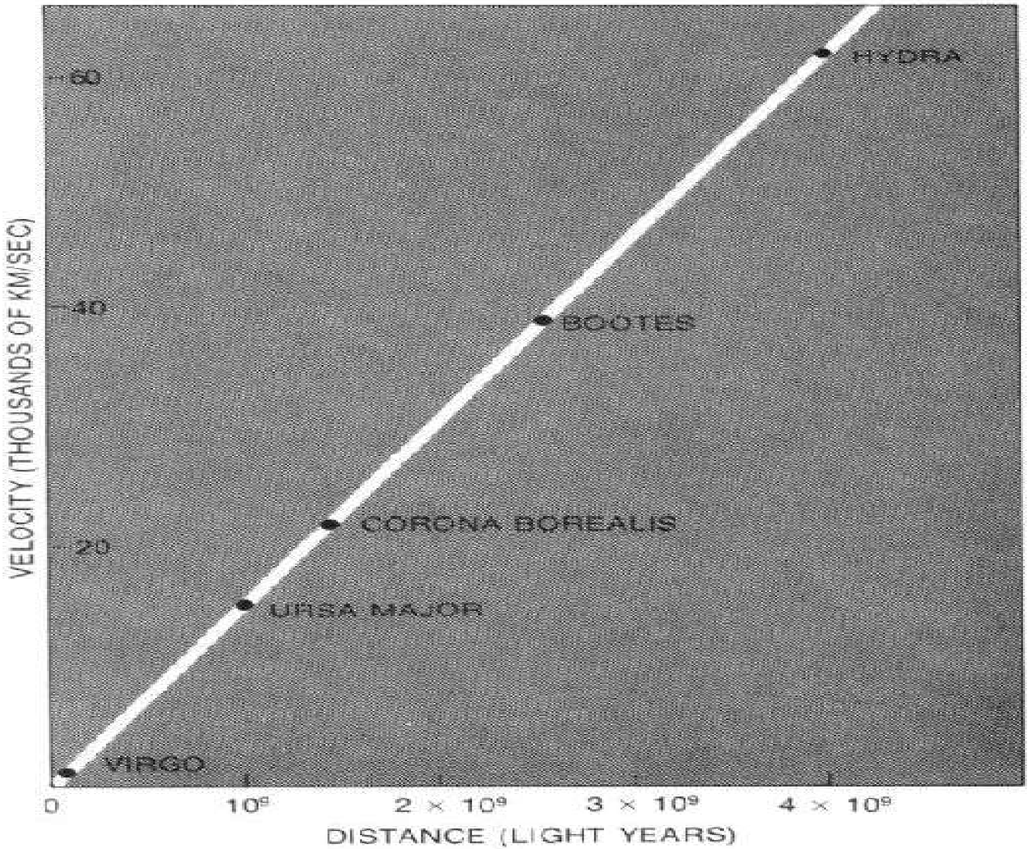
(691-656)/656*3E5 = 16,000 km/s

16,000 km/s

Astophysics

Core (SL and HL)	Extension (HL only)
Option E: Astrophysics	
$L = \sigma A T^4$	$L \propto m^n \quad \text{where} \quad 3 < n < 4$
$\lambda_{\max}\,(\text{metres}) = \dfrac{2.90 \times 10^{-3}}{T\,(\text{kelvin})}$	$\dfrac{\Delta\lambda}{\lambda} \cong \dfrac{v}{c}$
$d\,(\text{parsec}) = \dfrac{1}{p\,(\text{arc-second})}$	$v = H_0 d$
$b = \dfrac{L}{4\pi d^2}$	
$m - M = 5\lg\left(\dfrac{d}{10}\right)$	

Hubble's Law:



$v = Hd$

- v = recession velocity in km/s
- d = distance in Mpc
- H = 71 $^{km/s}/_{Mpc}$ (± 2.5)

Mpc = Mega parsecs

The greater the distance, the greater the recession velocity.

What is the recession rate of a galaxy that is 26 Mpc away?
(Use $H = 71 \ {}^{km/s}/_{Mpc}$)

$26 \text{ Mpc} * (71 \ {}^{km/s}/_{Mpc}) = 1846 \text{ km/s}$

1800 km/s

Relativity

Option H: Relativity

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Delta t = \gamma \Delta t_0$$

$$L = \frac{L_0}{\gamma}$$

also mass dilates

$$u_x' = \frac{u_x - v}{1-\frac{u_x v}{c^2}}$$

$$E_0 = m_0 c^2$$

$$E = \gamma m_0 c^2$$

$$p = \gamma m_0 u$$

$$E_K = (\gamma - 1) m_0 c^2$$

$$E^2 = p^2 c^2 + m_0^2 c^4$$

$$\frac{\Delta f}{f} = \frac{g \Delta h}{c^2}$$

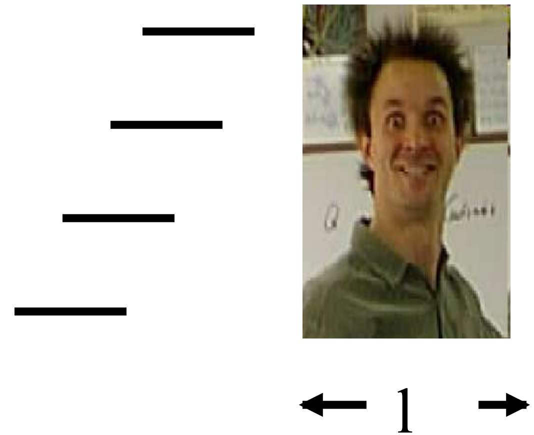
$$R_s = \frac{2GM}{c^2}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{R_s}{r}}}$$

Lorentz factor:

You can safely ignore relativistic effects to about .2 c

Length Contraction



Moving objects shrink in the direction of motion

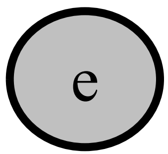
This reconciles the frames of reference

TOC

Mass Dilation

Electron at rest

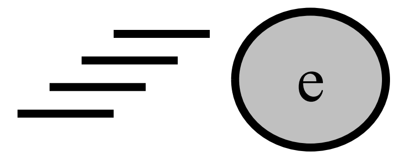
$m_o = 0.511$ MeV



Electron with 1.0 MeV Ke:

m = 1.00 MeV + 0.511 MeV

=1.511 MeV



Moving objects gain mass

(The gained mass is energy mass as in $E = mc^2$)

TOC

Moe and Joe have clocks that tick every 10.00 seconds. Joe is flying by at .85 c. What time does Moe see Joe's clock take to tick? (trick with c)

19 s

W

An electron has a rest mass of 0.511 MeV, and a moving mass of 1.511 MeV. What is its speed ?

answer

0.941 c

W

What speed does a 45 foot long bus need to go to fit exactly into a tunnel that is 40. feet long?

answer

0.46 c

W

Relativity

Option H: Relativity

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t = \gamma \Delta t_0$$

$$L = \frac{L_0}{\gamma}$$

$$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$$E_0 = m_0 c^2$$

$$E = \gamma m_0 c^2$$

$$p = \gamma m_0 u$$

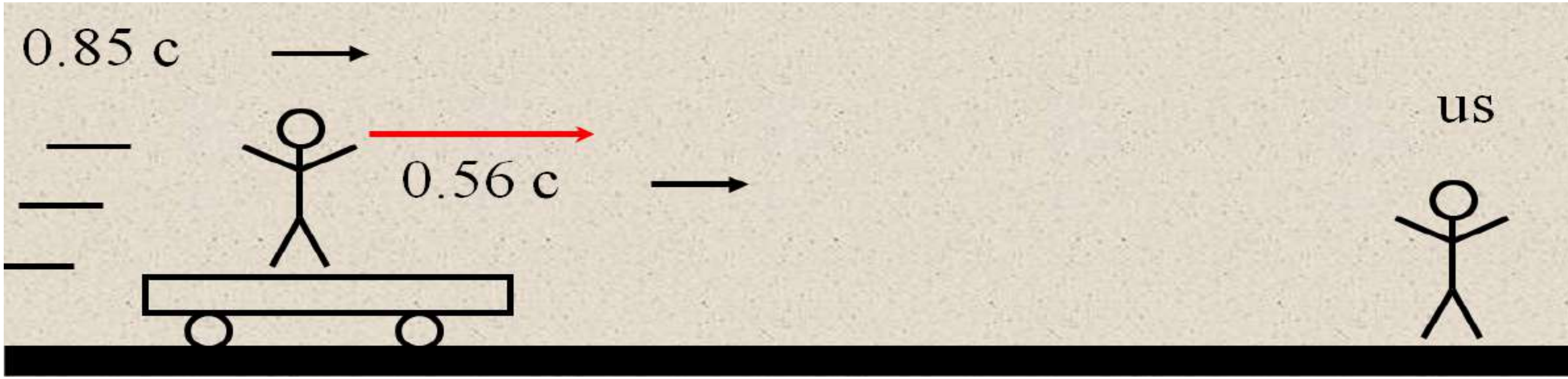
$$E_K = (\gamma - 1) m_0 c^2$$

$$E^2 = p^2 c^2 + m_0^2 c^4$$

$$\frac{\Delta f}{f} = \frac{g \Delta h}{c^2}$$

$$R_s = \frac{2GM}{c^2}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{R_s}{r}}}$$



Example – Tom is on a flatbed car going 0.85 c to the east. He throws a javelin at 0.56 c forward (relative to him, in the direction he is going) How fast is the javelin going with respect to us? (why Galilean doesn't work, lay out what is what)

in general – when you want to subtract velocities, use the left, add, right

Use the addition formula

This is about 0.96 c

Rob the hamster rides to the right on a cart going 0.36 c. He throws a baseball at 0.68 c relative to him in the direction he is going. How fast is the baseball going in the earth frame?

Use addition:

$u_x = (0.36 + 0.68\ c)/(1+(0.36\ c)(0.68\ c)/c^2) = 0.8355\ c$

0.84 c

W

Rob rides to the right on a cart going 0.36 c. He throws a baseball at 0.68 c relative to him opposite the direction he is going. How fast is the baseball going in the earth frame?

Use subtraction:

$u_x = (0.36 - 0.68\ c)/(1-(0.36\ c)(0.68\ c)/c^2) = -0.4237\ c$

-0.42 c

W

Relativity

Option H: Relativity

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Delta t = \gamma \Delta t_0$$

$$L = \frac{L_0}{\gamma}$$

$$u_x{}' = \frac{u_x - v}{1-\frac{u_x v}{c^2}}$$

$$E_0 = m_0 c^2$$

$$E = \gamma m_0 c^2$$

$$p = \gamma m_0 u$$

$$E_{\mathrm{K}} = (\gamma - 1) m_0 c^2$$

$$E^2 = p^2 c^2 + m_0^2 c^4$$

$$\frac{\Delta f}{f} = \frac{g \Delta h}{c^2}$$

$$R_s = \frac{2GM}{c^2}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{R_s}{r}}}$$

Kinetic Energy

Mass increase is energy

Example: What is the kinetic energy of a 10.0 kg object going .60 c?

Example: What is the kinetic energy of a 10.0 kg object going .60 c?

Dilated mass is $10.0/\sqrt{(1-.6^2)} = 12.5$ kg
So its mass has increased by 2.5 kg, this mass is energy.
2.5 kg represents $(2.5 \text{ kg})(3.00\text{E}8 \text{ m/s})^2 = 2.25\text{E}17$ J

Kinetic Energy

Example – A 0.144 kg baseball has 2.0×10^{15} J of kinetic energy. What is its mass, what is its velocity?

TOC

Example – A 0.144 kg baseball has $2.0\text{x}10^{15}$ J of kinetic energy. What is its mass, what is its velocity?

Well – the increase of mass is $(2.0\text{E}15\ \text{J})/(3\text{E}8)^2 = .022222$ kg so the new mass is 0.16622 kg
and
$v = c\ \sqrt{(1\text{-small}^2/\text{big}^2)} = c\ \sqrt{(1-0.144^2/0.16622^2)} \approx .50c$

Kinetic Energy

Example – An electron (rest mass 0.511 MeV) is accelerated through 0.155 MV, What is its velocity?

Example – An electron (rest mass 0.511 MeV) is accelerated through 0.155 MeV, What is its velocity?

Well – the new mass is 0.511 + 0.155 = 0.666 MeV

$v = c\sqrt{(1-\text{small}^2/\text{big}^2)} = c\sqrt{(1-0.511^2/0.666^2)} = .64c$

Relativity

Option H: Relativity

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Delta t = \gamma \Delta t_0$$

$$L = \frac{L_0}{\gamma}$$

$$u_x' = \frac{u_x - v}{1-\frac{u_x v}{c^2}}$$

$$E_0 = m_0 c^2$$

$$E = \gamma m_0 c^2$$

$$p = \gamma m_0 u$$

$$E_K = (\gamma - 1) m_0 c^2$$

$$E^2 = p^2 c^2 + m_0^2 c^4$$

$$\frac{\Delta f}{f} = \frac{g \Delta h}{c^2}$$

$$R_s = \frac{2GM}{c^2}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{R_s}{r}}}$$

Clocks and gravitation:

Approximate formula for small changes of height:

$$\frac{\Delta f}{f} = \frac{g\Delta h}{c^2}$$

Δf - change in frequency

f - original frequency

g - gravitational field strength

Δh - change in height

Two trombonists, one at the top of a 215 m tall tower, and one at the bottom play what they think is the same note. The one at the bottom plays a 256.0 Hz frequency, and hears a beat frequency of 5.2 Hz. What is the gravitational field strength?? For us to hear the note in tune, should the top player slide out, or in? (Are they sharp or flat)

$\Delta f/f = g\Delta h/c^2$, $g = \Delta f c^2/f\Delta h$

8.5×10^{12} m/s/s

8.5×10^{12} m/s/s, out, sharp

Relativity

Option H: Relativity

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Delta t = \gamma \Delta t_0$$

$$L = \frac{L_0}{\gamma}$$

$$u_x' = \frac{u_x - v}{1-\frac{u_x v}{c^2}}$$

$$E_0 = m_0 c^2$$

$$E = \gamma m_0 c^2$$

$$p = \gamma m_0 u$$

$$E_K = (\gamma - 1) m_0 c^2$$

$$E^2 = p^2 c^2 + m_0^2 c^4$$

$$\frac{\Delta f}{f} = \frac{g \Delta h}{c^2}$$

$$R_s = \frac{2GM}{c^2}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{R_s}{r}}}$$

Black Holes:

Gravitational Potential per unit mass:

$V = \frac{-GM}{r}$ so PE = Vm

At escape velocity, kinetic = potential

$\frac{1}{2}mv^2 = \frac{GMm}{r}$ substituting c for v:

$r = \frac{2GM}{c^2}$ where r is the Schwarzschild radius

What is the mass of a black hole the size of the earth?

$r = 6.38 \times 10^6$ m

$M = rc^2/(2G) =$

6.38E6*3E82/(2*6.67E-11) = 4.3E33 kg

4.3E33 kg

Relativity

Option H: Relativity

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Delta t = \gamma \Delta t_0$$

$$L = \frac{L_0}{\gamma}$$

$$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$$E_0 = m_0 c^2$$

$$E = \gamma m_0 c^2$$

$$p = \gamma m_0 u$$

$$E_K = (\gamma - 1) m_0 c^2$$

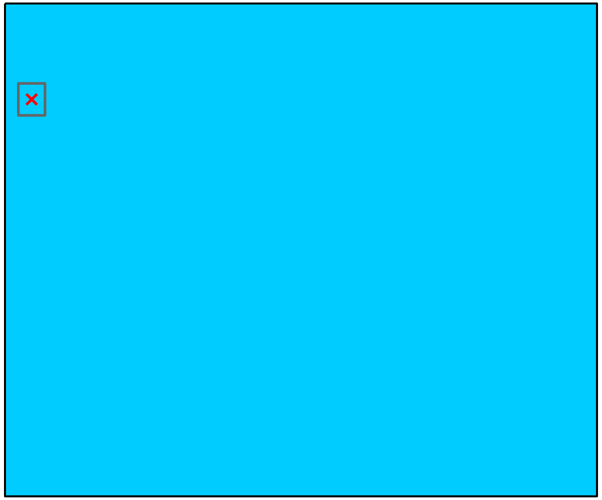
$$E^2 = p^2 c^2 + m_0^2 c^4$$

$$\frac{\Delta f}{f} = \frac{g \Delta h}{c^2}$$

$$R_s = \frac{2GM}{c^2}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{R_s}{r}}}$$

Gravitational Time Dilation



Δt - Dilated time interval

Δt_o - Original time interval

R_s - Schwarzschild radius

r - Distance that the clock is from the black hole

A graduate student is in orbit 32.5 km from the center of a black hole. If they have a beacon that flashes every 5.00 seconds, and we (from very far away) see it flashing every 17.2 seconds, what is the Schwarzschild radius of the black hole?

$17.2 = 5.00/\sqrt{(1 - R_s/32.5)}$

$R_s = 32.5(1-(5.00\text{ s})^2/(17.2\text{ s})^2) = 29.8\text{ km}$

29.8 km