AP Calculus AB Summer Assignment

For each of the following questions show all work in the space provided next to the question. You must show your work. There will be a test on the material on Friday September 8, 2017. You will be able to ask questions during class September 5-7, 2017

Multiple Choice Questions

No calculator is allowed for these questions.

1. The zeros of the polynomial function $f(x) = x^4 - 3x^3$ are

   (A) 0 and 3
   (B) 0 only
   (C) 3 only
   (D) 3 and 4

2. Arctan $\sqrt{3}$ is equal to

   (A) 1
   (B) $\frac{\pi}{4}$
   (C) $\frac{\pi}{6}$
   (D) $\frac{\pi}{3}$

3. Find the number of solutions of the equation $\cos^2 x - 1 = 0$ for values of x in the interval $[0, 2\pi]$.

   (A) 0
   (B) 1
   (C) 2
   (D) 3

4. Solve for x: $e^{2x} = 9$.

   (A) ln 9
   (B) ln 4.5
   (C) ln 3
   (D) ± 4.5

5. Find the range of the piecewise function defined by $f(x) = \begin{cases} (x-1)^2, x < 1 \\ 2x - 3, \ x > 1 \end{cases}$.

   (A) {all real numbers}
   (B) $\{y > -1\}$
   (C) $\{y \geq -1\}$
   (D) $\{y \neq 1\}$

6. Find the equation of the horizontal asymptote of $y = \frac{5x}{x-1}$.

(A) $y = 0$
(B) $x = 1$
(C) $x = 5$
(D) $y = 5$

7. Find the equation of the vertical asymptote of $y = \frac{5x}{x-1}$.

(A) $y = 1$
(B) $y = 0$
(C) $x = 1$
(D) $x = 5$

8. Given $f(x) = 2x - 3$, find $f(x + h)$.

(A) $2x + 2h - 3$
(B) $2x + h - 3$
(C) $x + h$
(D) $x + h - 3$

9. If $f(x) = (2x - 1)(x^2 + 1)(x - 5)^2$, then $f(x)$ has how many real roots?

(A) 0
(B) 1
(C) 2
(D) 3

10. Solve for x: $\log_9 x^2 = 9$.

(A) 1
(B) 3^3
(C) 3^9
(D) $\pm 3^9$

11. $2 \ln e^{5x} =$

(A) $10x$
(B) $5x^2$
(C) $25x^2$
(D) e^{10x}

12. The values of x that are solutions to the equation $\cos^2 x = \sin 2x$ in the interval $[0, \pi]$ are:

(A) $\arctan \frac{1}{2}$ only.

(B) $\arctan \frac{1}{2}$ and π.

(C) $\arctan \frac{1}{2}$ and 0.

(D) $\arctan \frac{1}{2}$ and $\frac{\pi}{2}$.

13. The graph of $f(x) = \frac{x^2 - 1}{x - 1}$ has

(A) a hole at $x = 1$.
(B) a hole at $x = -1$.
(C) a vertical asymptote at $x = 1$.
(D) a vertical asymptote at $x = -1$.

14. If $\ln x^2 = 6$, then $x =$

(A) $\pm e^6$

(B) $9^{\sqrt{6}}$

(C) $e^{\sqrt{6}}$

(D) $\pm e^3$

Free Response Questions

A graphing calculator is required for some questions.

1. Find the domain and range and sketch the graph of $y = e^{\ln x}$.

2. The rational function $y = \frac{ax}{bx + c}$ has a vertical asymptote at $x = 2$ and a horizontal asymptote at $y = 3$.

   (a) Find a and c in terms of b, and express y in simplest form.

   (b) Graph the function, showing the vertical and horizontal asymptotes.

3. Solve the trigonometric equation $4 \sin^2 x - \cos x = 1$ for values of x in the interval $(0, \pi)$.

Multiple Choice Questions

A graphing calculator is required for some questions.

1. If $f(x) = x^2 - x + 1$, then $f(x+1)=$

   (A) $x^2 - x + 2$

   (B) $x^2 + x + 1$

   (C) $x^2 + x + 3$

   (D) 1

2. Find the domain of $f(x) = \sqrt{x^3 - x^2}$.

   (A) $\{x \geq 1\}$

   (B) $\{x \geq 1, x = 0\}$

   (C) $\{x \leq 1\}$

   (D) $\{|x| \geq 1\}$

3. Which of the following is an even function with a domain of all real numbers?

   (A) $\ln x^2$

   (B) $e^{x^2} - x$

   (C) $e^{x^2} - x^2$

   (D) $e^{x^3} + 1$

4. Find the domain of $f(x) = \ln(\tan x)$ on the interval $[-\pi, \pi]$.

   (A) all x in $(-\pi, \pi)$

   (B) all x in $(0, \pi)$

   (C) all x in $\left(0, \frac{\pi}{2}\right)$

   (D) all x in $\left(-\pi, -\frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$

5. $f(x) = \frac{(x-1)^2}{x^2 - 1}$ has

   (A) holes at $x = -1$ and $x = 1$.

   (B) vertical asymptotes at $x = 1$ and $x = -1$.

   (C) a horizontal asymptote at $y = -1$.

   (D) a hole at $x = 1$ and a vertical asymptote at $x = -1$.

6. $f(x)$ is an odd function and the graph of f contains the point (6, 5). Which of the following points is also on the graph of f?

(A) $(-6,5)$
(B) $(6, -5)$
(C) $(-6, -5)$
(D) $(-5, -6)$

7. If $f(x) = \sqrt{x-2}$, then $\dfrac{f(x+h)-f(x)}{h} =$

(A) $\dfrac{\sqrt{x-2}+\sqrt{h-2}}{h}$

(B) $\dfrac{\sqrt{xh-2}+\sqrt{x-2}}{h}$

(C) $\dfrac{\sqrt{x-2+h}-\sqrt{x-2}}{h}$

(D) $\dfrac{\sqrt{x+h}-\sqrt{2}}{h}$

8. If $f(x) = \dfrac{1}{x+2}$, then $\dfrac{f(x+h)-f(x)}{h} =$

(A) $\dfrac{h+4}{h(x+2)(x+h+2)}$

(B) $\dfrac{-1}{(x+2)(x+h+2)}$

(C) $\dfrac{-1}{(x+h)(x+h-2)}$

(D) $\dfrac{1}{h(x+2)(x+h+2)}$

9. Which of the following functions are odd?

I. $y = \ln(x^3)$
II. $y = |x^3|$
III. $y = e^{x^3}$

(A) None
(B) II only
(C) I and II
(D) II and III

10. Which of the following functions are even?

I. $y = \ln |x|$
II. $y = |\ln x|$
III. $y = \left|\frac{1}{x}\right|$

(A) None
(B) II only
(C) I and II
(D) II and III

Free-Response Questions

A graphing calculator is required for some questions.

1. Find all the zeros (real and complex) of $f(x) = x^3 + 2x - 3$.

2. Sketch $f(x) = \frac{x - 1}{x^2 - 3x + 2}$, and state the vertical asymptote(s), horizontal asymptote(s), and holes, if any.

3. Find the zeros and describe the end behavior of $f(x) = 2x(x - 1)(x + 1)$. Is $f(x)$ odd, even, or neither? Explain.

4. For each of the following functions, graph $f(x)$, $|f(x)|$, and $f(|x|)$. Using these graphs, write a statement about the relationship between the graphs of $f(x)$, $|f(x)|$, and $f(|x|)$.

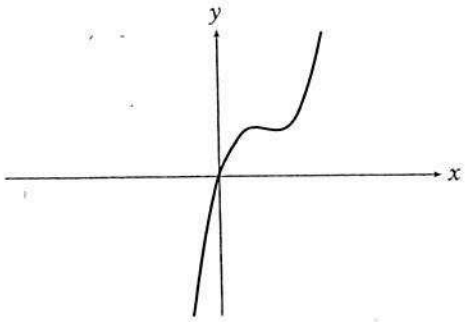
(a) $f(x) = \cos x$
(b) $f(x) = \sin x$
(c) $f(x) = x^2 - 2x$

Multiple-Choice Questions

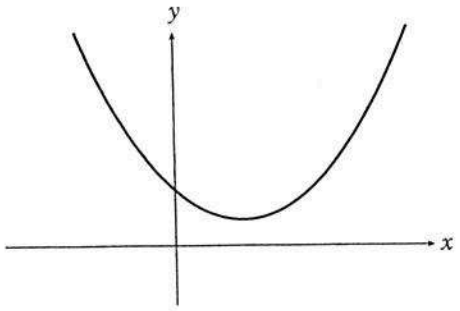
No calculator is allowed for these questions.

1. Which of the following graphs show(s) a function that has an inverse?

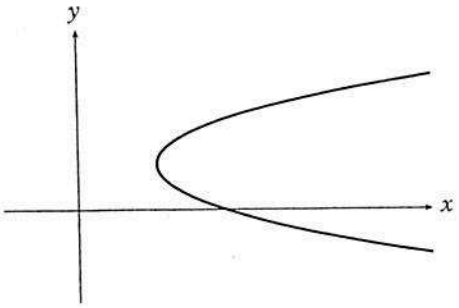
(I)



(II)



(III)



(A) None
(B) I only
(C) II only
(D) I and II

2. Find the inverse of the equation $y = 2x^3 + 1$.

(A) $y^{-1} = \frac{2}{x^3} + 1$

(B) $y^{-1} = -2x^3 + 1$

(C) $y^{-1} = \sqrt[3]{\frac{x-1}{2}}$

(D) $y^{-1} = \frac{\sqrt[3]{x-1}}{2}$

3. The graphs of a function and its inverse are reflections of each other across

   (A) the x-axis.
   (B) the y-axis.
   (C) the origin.
   (D) $y = x$.

4. The composition of a function f and its inverse is equal to

   (A) -1
   (B) 0
   (C) 1
   (D) x

Free-Response Questions

A graphing calculator is required for some questions.

1. (a) Sketch the graph of $y = -e^{-x}$. State its domain and range.

   (b) Solve algebraically for the inverse of $y = -e^{-x}$.

2. (a) Find the domain and range of the function $y = \sqrt{x - 2 + 1}$, and sketch the graph.

   (b) Find the domain and range of the of the inverse of y, and solve algebraically for the equation of the inverse.

No calculator is allowed for these questions.

1. The following functions have been shifted as described. Circle the equation that matches each description, then sketch its graph.

   (a) $y = \ln x$ shifted right 2 units.

   $y = \ln(x + 2)$ $\quad$ $y = \ln(x - 2)$

   $y = \ln x + 2$ $\quad$ $y = \ln x - 2$

   (b) $y = 2^x$ shifted down 1 unit.

   $y = 2^x - 1$ $\quad$ $y = 2^{x-1}$

   $y = 2^{x+1}$ $\quad$ $y = 2^x + 1$

   (c) $y = |x|$ shifted left 3 units.

   $y = |x + 3|$ $\quad$ $y = |x - 3|$

   $y = |x| + 3$ $\quad$ $y = |x| - 3$

   (d) $y = x^2$ shifted up 2 units and right 4 units.

   $y = (x - 2)^2 - 4$ $\quad$ $y = (x - 2)^2 + 4$

   $y = (x^2 + 4) + 2$ $\quad$ $y = (x - 4)^2 + 2$

   (e) $y = \sin x$ reflected across the x-axis.

   $y = \sin(-x)$ $\quad$ $y = \sin(x - 1)$

   $y = -\sin x$ $\quad$ $y = -\sin(-x)$

2. Write the domain for each of the following functions. Then sketch the graph.

   (a) $y = \ln x^2$

   (b) $y = |x + 2|$

   (c) $y = -\ln(x - 1)$

Multiple Choice Questions

A graphing calculator is required for some of these questions.

1. The graph of $y = x^2$ first reflected across the x-axis and then shifted down one unit is

   (A) $y = 1 - x^2$

   (B) $y = 1 + x^2$

   (C) $y = 1 - x^{-2}$

   (D) $y = -1 - x^2$

2. The graph of $y = x^2$ first shifted down one unit and then reflected in the x-axis is

   (A) $y = x^2 - 1$.

   (B) $y = 1 - x^2$.

   (C) $y = 1 + x^2$.

   (D) $y = 1 - x^{-2}$.

11.

3. The inverse of the function $y = x^2$ with domain $= \{x \leq 0\}$ has equation.

(A) $y = x^2$.
(B) $y = \sqrt{x}$.
(C) $y = \pm\sqrt{x}$.
(D) $y = -\sqrt{x}$.

Free-Response Questions

A graphing calculator is required for some questions.

1. Sketch the graph of $y = \frac{1}{x}$, and then use it to sketch the graphs of the following functions without a calculator. Check your results by graphing each equation in the calculator.

(a) $y = \frac{1}{x - 1}$
(b) $y = \frac{1}{x} - 1$
(c) $y = \frac{1}{x + 2} + 2$
(d) the inverse of $y = \frac{1}{x}$

2. Describe the translations and/or reflections that transform $y = x^{2/3}$ into the following:

(a) $y = x^{2/3} + 2$
(b) $y = -x^{2/3} - 3$
(c) $y = (-x)^{2/3}$
(d) $y = (x - 1)^{2/3} + 1$