

End-of-year FRQ practice **DRAFT 2020/04/03 14h19 EDT**

USE AT YOUR OWN RISK. DO NOT ASSUME EVERY PROBLEM STATEMENT IS DEBUGGED.

2020/04/03 14h19 EDT: Approximate problem-type category codes (PLR, QQT, and LFRQ) added to problem titles.

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2 AP PHYSICS 1

2.1.1 QQT. Toy car.

(12 points, suggested time 25 minutes) (inspired by 2019 AP Physics 1 FRQ 2) A toy car is programmed to travel in the $+x$ direction in two phases of motion. In the first phase, the car travels with constant velocity v_1 for a time duration Δt_1 . In the second phase, the toy car continues to travel, but now with constant non-zero acceleration. At the end of the second phase of motion, which lasts for a time duration Δt_2 , the toy car reaches a final velocity v_2 that is greater than v_1 . In a set of experimental trials, time durations Δt_1 and Δt_2 are varied while total time $\Delta t_{\text{TOT}} = \Delta t_1 + \Delta t_2$ and velocities v_1 and v_2 are held constant.

(a)

- i. Suppose the duration Δt_1 is much greater than the duration Δt_2 . Estimate the x -displacement through which the car travels during the experiment. Express your answer in terms of v_1 , v_2 , Δt_{TOT} , and fundamental constants, as appropriate.

Briefly explain your reasoning without deriving or manipulating equations.

- ii. Now suppose the duration Δt_1 is much less than the duration Δt_2 . Estimate the x -displacement through which the car travels during the experiment. Express your answer in terms of v_1 , v_2 , Δt_{TOT} , and fundamental constants, as appropriate.

Briefly explain your reasoning without deriving or manipulating equations.

- (b) Now suppose neither time duration Δt_1 nor time duration Δt_2 is much greater than the other, but that they are not necessarily equal. On the axes below, draw a velocity vs. time graph for the car. Label axes and the values of times and velocities at the beginning and end of each phase of motion.



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- (c) Derive an equation for the x -displacement through which the car travels during the experiment in terms of v_1 , v_2 , Δt_1 , Δt_{TOT} , and fundamental constants, as appropriate. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

- (d) Consider the scenario from part (a)(ii), where the duration Δt_1 is much less than the duration Δt_2 . Does your equation for the x -displacement of the car from part (c) agree with your reasoning in part (a)(ii)?

☐ Yes ☐ No

Briefly explain your reasoning by addressing why, according to your equation, the x -displacement becomes (or approaches) a certain value when Δt_1 is much less than Δt_2 .

- (e) In one trial, the car travels through an overall displacement Δx , and the acceleration of the car during the second phase of motion is a_I . In a second trial, the car travels through the same overall displacement Δx through the same total time Δt_{TOT} and achieves the same final velocity v_2 , but starts out with a slightly greater initial velocity v_1 (still less than v_2). The acceleration of the car during the second phase of motion during the second experiment is a_{II} . How do the two accelerations a_I and a_{II} compare to each other?

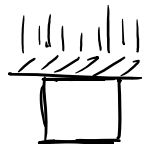
☐ $a_I > a_{II}$ ☐ $a_I = a_{II}$ ☐ $a_I < a_{II}$

Briefly explain your reasoning. You may use (a) diagram(s) to support your explanation.

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2.1.2 **PLR.** Not-proportional reasoning: Normal force on block

In the experiment below, a block near Earth's surface is pressed from above by a movable panel. The block is gaining speed while moving downward with a constant acceleration. The experiment is repeated, but now with the normal force provided by the panel only half as strong as before. Will the magnitude of the resulting acceleration of the block be greater than, less than, or equal to half the original magnitude of acceleration of the block? Explain.



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2.1.3 *ExptDes.* Equivalence principle

The equivalence principle states that for a given an object, the inertial mass and the gravitational mass have equal values. Students are asked to test the equivalence principle by working with resources listed in the table below.

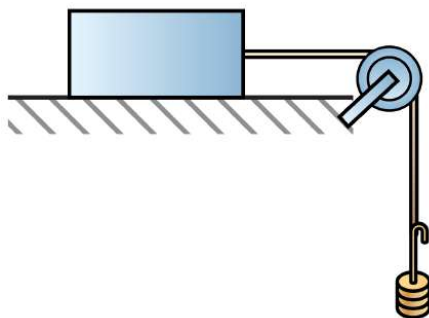
Resource	Description
Cart	Has massless wheels that spin without friction
Floor	Smooth, firm horizontal planar surface
Horizontally-mounted motion detector	Tracks horizontal position of object on floor, can be used to produce position-time, velocity-time, and acceleration-time plots on a computer.
Force probe	Displays the strength of the pull force applied by the force probe to whatever object is attached to the force probe's hook
Classmates	Other students can hold and read items for you

- (a) Provide an experimental procedure that could provide data that could be used to test whether the equivalence principle is satisfied by the cart. Give enough detail so that another student could carry out your procedure. Be sure to provide a way to reduce experimental uncertainty.
- (b) Describe an analytic procedure that could be applied to the data obtained using the method you described in part (a) to determine whether the equivalence principle is satisfied by the cart. Give enough detail so that another student could perform your analysis.
- (c) Using physics principles, explain why the analytic procedure you described in part (b) would work.

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2.1.4 QQT. Correct false reasoning: Block-string-pendant mass

(11 points, suggested time 22 min 55 sec) In the figure below, a block of mass M on a frictionless, horizontal tabletop is connected to a taut massless string that runs over a frictionless, massless pulley and connects to a slotted mass set of mass M that is initially located a height H above the floor. The block and slotted mass set are released from rest such that the string remains taut and all parts of the system consisting of the block, string, and slotted mass set have equal magnitude of acceleration a_1 until the slotted mass set hits the floor.



A second experiment is planned in which the mass of the slotted mass set will be doubled (now having mass $2M$). A student is asked to predict how the magnitude of system's acceleration will be affected.

Student: Doubling the mass of the slotted mass set doubles the component of the net force on the system along the direction of acceleration. The block is the portion of the system trying to resist being accelerated, so keeping the same block means keeping the inertial mass of the system the same. According to Newton's 2nd law, acceleration is proportional to net force and inversely proportional to inertial mass, so the magnitude of the acceleration will increase, specifically by doubling.

(a)

i. Which aspects of the student's reasoning, if any, are correct? Explain how you arrived at your answer.

ii. Which aspects of the student's reasoning, if any, are incorrect? Explain how you arrived at your answer.

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- (b) Use quantitative reasoning, including equations as needed, to develop an expression for the new magnitude of acceleration of the system. Express your answer in terms of a_1 .

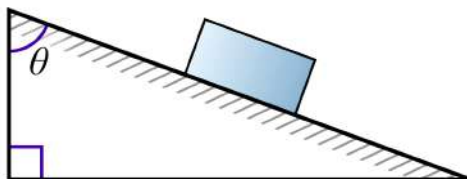
(c)

- i. Explain how any correct aspects of the student's reasoning identified in part (a) are expressed by your mathematical relationships in part (b). Your response should refer to the relationships you wrote in part (b), not only to the final answer you obtained by manipulating those relationships.
- ii. Explain how your relationships in part (b) correct any incorrect aspects of the student's reasoning identified in part (a). Your response should refer to the relationships you wrote in part (b), not only to the final answer you obtained by manipulating those relationships.

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2.1.5 **PLR.** Labeling quantities in unusual places: Inclined ramp
(Short sample question):

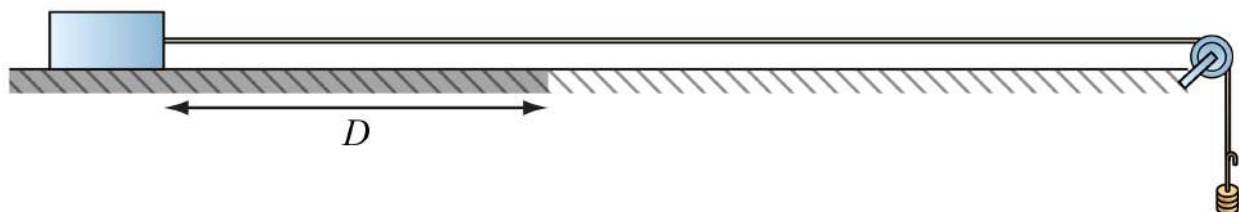
A block is sliding on a fixed frictionless inclined plane having an angle θ as illustrated in the figure below. The acceleration of the block has magnitude a . If the angle θ is decreased, will the magnitude of the block's acceleration increase, decrease, or stay the same? Explain your reasoning.



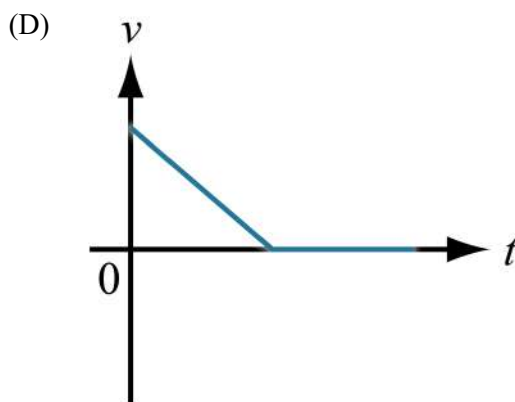
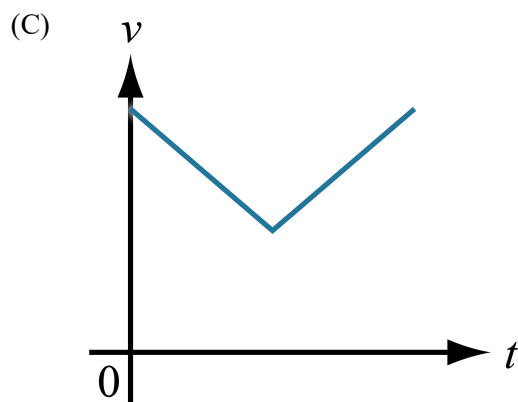
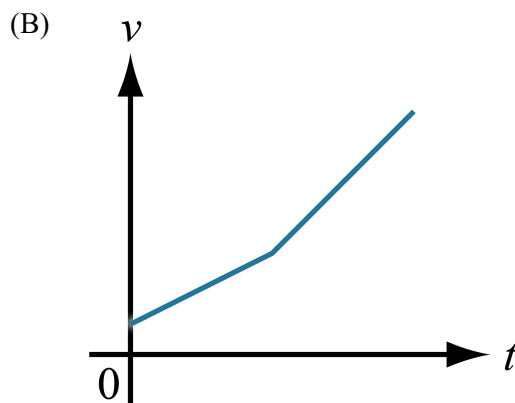
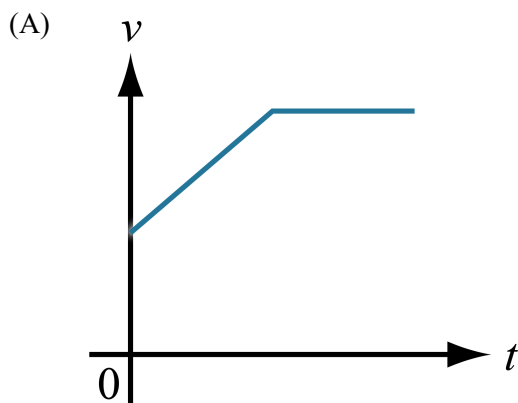
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2.1.6 Explaining a choice: Block, half-rough surface, pulley

In the figure below, a block of mass m is connected to a massless string draped over a massless, frictionless pulley. The other end of the string is connected to a slotted mass set, also of mass m . The block is on the rough portion (darker shading) of a horizontal tabletop, and the coefficients of kinetic and static friction between the block and the rough portion of the table are $\mu_K = \mu_S = 2.0$. The block is initially skidding to the right with speed $v = 10. \frac{\text{m}}{\text{s}}$ at a distance $D = 7.5 \text{ m}$ away from the location where the rough portion of the tabletop joins the frictionless portion of the tabletop (lighter shading).



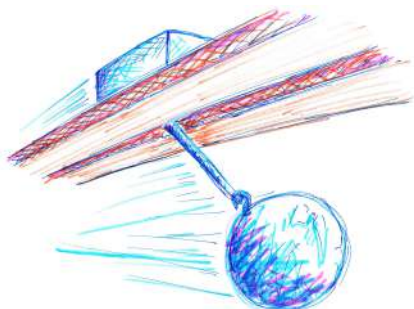
A video recording is analyzed to obtain a speed-time plot for the block. The tabletop is long, and the block does not crash into the pulley before the video recording is stopped. Which one of the following graphs represents the speed-time graph for the block? Explain how you arrived at your choice.



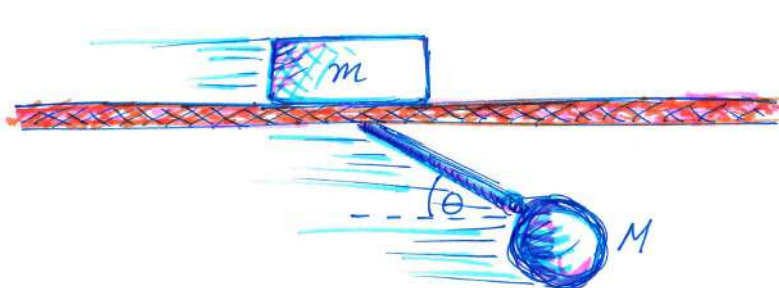
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2.1.7 QQT. Unfamiliar scenario: Block-String-Leading ball

As shown below, a block of mass m on two rough horizontal planks is attached to a massless string, the other end of which is connected to a sphere of mass M . The coefficient of kinetic friction between the block and planks is μ_K . It is possible (perhaps after some practice trials) to launch the block, string, and sphere so that the angle θ between the tight string and the horizontal has a constant value during the time interval when the block, string, and sphere are still moving together and have not yet come to rest. The acceleration of the block-string-sphere system before coming to rest has magnitude a .



Perspective view



Side view

(a) Suppose that the experiment is repeated, but now with mass M doubled.

- i. Does the magnitude a of the acceleration of the block-string-sphere system increase, decrease, or stay the same?

___ Increase

___ Decrease

___ Remain the same

Qualitatively explain your reasoning without algebraic manipulations (you can write, cite, and describe features of algebraic equations, but do not merely write, “I solved the system of 3 equations over there, and the final expression shows that ...”).

- ii. Does the angle θ increase, decrease, or stay the same?

___ Increase

___ Decrease

___ Remain the same

Qualitatively explain your reasoning without algebraic manipulations (you can write, cite, and describe features of algebraic equations, but do not merely write, “I solved the system of 3 equations over there, and the final expression shows that ...”).

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- (b)
- Mathematically derive an expression for the magnitude a of the acceleration in terms of m , M , μ_K , and fundamental constants, as needed.
 - Mathematically derive an expression for the angle θ in terms of m , M , μ_K , and fundamental constants, as needed.

A class of students attempted to perform the experiment described at the beginning of this problem. There was only one set of equipment (block, string, sphere, planks, and video recording equipment), so students had to share (only one group was able to perform the experiment at a time; groups of students had to wait their turn to use the equipment). The table below shows the angle θ measured by each group using video analysis.

Group	Time group finished experiment	Angle θ ($\pm 2^\circ$)
1	10:02 am	22
2	10:07 am	21
3	10:12 am	24
4	10:18 am	25
5	10:22 am	28
6	10:30 am	33
7	10:37 am	31

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(c)

- i. The students notice that the angle θ is mostly increasing from group to group. One student accuses the class of sabotaging the equipment. The student proposes that each group shaved some mass off of the block, causing the block to be pressed against the planks less and less tightly as the class period continued, which in turn reduced the strength of the kinetic friction force applied by the planks on the block. The student argues that weaker friction on the block resulted in less acceleration of the block-string-sphere system, and, thus, a more vertically oriented string. Is the student's accusation consistent with the mathematical work in part (b)? Explain.
- ii. Provide an alternative reason for the trend of increasing angle θ . Use your mathematical result in part (b) to support your alternative reason.

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2.1.8 *ExptDes.* Test false claim: Static friction

A student claims that the maximum static friction force f_S^{MAX} that can be provided before two surfaces slip increases in proportion to the normal force between the surfaces until reaching a maximum strength and that the f_S^{MAX} then has this same maximum strength for all greater magnitudes of normal force.

- (a) Describe an experimental setup that could be used to test the student's claim for a wooden block and a wooden board. You may use any equipment ordinarily available in a typical physics classroom laboratory. Include a diagram of your setup.
- (b) How could you use the experimental setup you described in (a) to obtain data to test the student's claim? Be sure to indicate which equipment is used to measure each relevant quantity. Provide enough detail so that another student could carry out your experimental procedure. Provide a method to reduce uncertainty.

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(c) How could you analyze the data you would obtain according to the procedure you described in (b) to test the student's claim? Include a way to take measurement uncertainty into account.

(d) Using physical principles and reasoning, explain how you know that your analysis method could be used to test the student's claim.

The student performs an experiment using a wood block and wooden board and obtains the data in the table below.

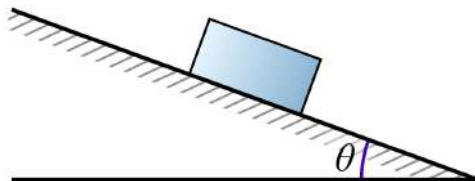
Normal force (± 0.02 N)	Maximum static friction force (± 0.1 N)
2.00	1.6
4.00	3.0
8.00	5.9
12.00	8.9
16.00	12.0
20.00	15.1

(e) Are the data in the table consistent with the student's claim? Justify your answer.

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2.1.9 Warm-up/short PLRs. Kinetic friction, inclined ramp

In the figure below, a block is sliding on an inclined plane having angle of inclination θ . Because “whooshies” are not drawn, it might be the case that the block is sliding up the plane. It also might be the case that the block is sliding down the plane.

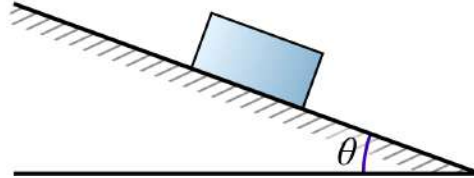


- (a) If the plane is frictionless, is the magnitude of the acceleration of the block greater while sliding up the plane, greater while sliding down the plane, or equal for both cases? Explain your reasoning.
- (b) Assuming now that there is friction between the plane and the box, of equal strength while the box slides up the plane and while the box slides down the plane, is the magnitude of the acceleration of the block greater while sliding up the plane, greater while sliding down the plane, or equal for both cases? Explain your reasoning.

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2.1.10 **PLR.** Non-proportional reasoning with trigonometry: Block on accelerating wedge

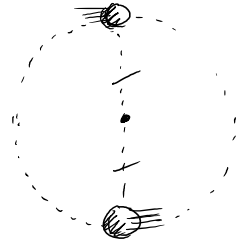
In the figure below, the frictionless inclined plane has angle of inclination θ and is accelerating, purely horizontally to the right, with an acceleration of magnitude a . As a result, the box remains in contact with the plane at a constant height (rather than sliding up or down along the plane). In a second experiment, the magnitude of acceleration a is doubled. In order for the box to remain in contact with the plane at a constant height (again, rather than sliding up or down along the plane), will the angle of inclination need to more than double, less than double, or exactly double? Explain your reasoning.



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2.1.11 QQT. Explaining scaling: Orbits

A pair of stars, each having mass m , orbit a point midway between them. The distance between the stars is ℓ .



- (a) If the mass of each star is increased by the same factor, how will the orbital speed of each star be affected? Indicate increase, decrease, or remain the same.

___ Increase ___ Decrease ___ Remain the same

Justify your answer.

- (b) Derive an expression for the orbital speed v of each star in terms of m , ℓ , and fundamental constants, as needed.

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2.1.12 *Data Analysis*. Deviation from Kepler's 3rd law

An astronomer discovers a new planet that is circularly orbited by four moons. The astronomer obtains the orbital radii and periods for the four moons in the table below.

Moon	Orbital radius $R \text{ (m)} \pm 3 \times 10^7$	Orbital period $T \text{ (s)} \pm 2 \times 10^3$		
A	6.2×10^8	2.66×10^5		
B	9.2×10^8	4.80×10^5		
C	1.07×10^9	4.26×10^5		
D	1.20×10^9	5.06×10^5		

One of the astronomer's colleagues claims that practically all the mass in the vicinity of the newly discovered planet and its four moons is concentrated in a point-like fashion in the center of the planet itself.

- (a) Provide a method for analyzing the data to test the colleague's claim. Be sure to provide enough detail so that another physics student could carry out your analysis procedure. Provide a method to take experimental uncertainty into account.
- (b) Use physics principles to explain why the analysis method you described in part (a) can be used to test the colleague's claim.

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- (c) Carry out the method you described in part (a) and report whether the colleague's claim is correct. You may use the gridded space below and/or empty space in the data table in the problem statement if needed for your analysis method.

A 10x10 grid of 100 small squares, each containing a different geometric shape or symbol. The symbols are arranged in a complex, non-repeating pattern, representing a complex pattern.

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2.1.13 **PLR.** False-reasoning task: Momentum.

On a horizontal, frictionless track, cart A is launched at and collided with identical cart B. The two carts immediately stick upon contact. Charlie tried to use conservation of momentum to determine the final speed of the stuck-together carts, but Charlie misremembered the formula for the x -momentum as $p_x = \frac{1}{2}mv_x^2$. Indicate whether the speed of the stuck-together carts Charlie calculated was greater than, less than, or equal to the speed of the stuck-together carts Charlie should have calculated.

____ Greater than ____ Less than ____ Equal

Explain your reasoning.

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2.1.14 *ExptDes*. Testing a false claim: Momentum

(25 min) Students are provided with two carts that they can collide and a track on which the carts move with practically no friction. Students are allowed to use equipment ordinarily found in a high school physics laboratory except for instruments for measuring forces. Alice claims that there is a special magnitude of momentum P_C so that as long as the total pre-collision momentum of the system's magnitude is less than P_C , the total post-collision momentum will equal the total pre-collision momentum but whenever, instead, the total pre-collision momentum of the system is greater than or equal to P_C the total post-collision momentum will have the same direction as the total pre-collision momentum but have magnitude equal to P_C .

- (a) Describe an experimental procedure that can be used to test Alice's claim. You can include a labeled diagram of your setup. Indicate which equipment is used to make each measurement. Give enough detail so that another student could carry out your procedure. Specify how you will minimize and estimate experimental uncertainty.
- (b) Describe a method that can be used to analyze the data you would obtain in part (a) above in order to test Alice's claim. Provide enough detail so that another student could carry out your analysis. Take into account experimental uncertainty.

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(c) Explain how you know that your analytic method can be used to test Alice's claim.

(d) In a later class, students collided two carts of mass $m_1 = 0.30$ kg and $m_2 = 0.53$ kg on a horizontal, frictionless track and obtained the following (x -)velocity data. The velocities of cart k before and after the collision are $v_{k,i}$ and $v_{k,f}$, respectively.

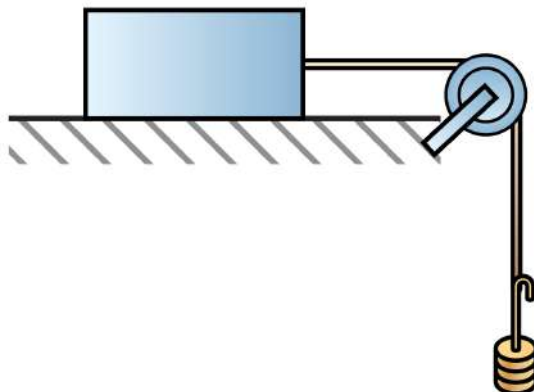
Trial	Pre-collision		Post-collision			
	$v_{1,i} \left(\frac{\text{m}}{\text{s}}\right)$	$v_{2,i} \left(\frac{\text{m}}{\text{s}}\right)$	$v_{1,f} \left(\frac{\text{m}}{\text{s}}\right)$	$v_{2,f} \left(\frac{\text{m}}{\text{s}}\right)$		
1	0.21	0.00	0.10	0.07		
4	1.32	-0.43	-0.25	0.47		
3	0.84	-0.12	-0.32	0.54		
2	0.39	0.20	-0.30	0.60		

Taking into account that all velocities were measured with an uncertainty of about $\pm 0.02 \frac{\text{m}}{\text{s}}$, are the data consistent with Alice's claim? Justify your answer. You may use the blank columns in the table above, if needed for your analysis.

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2.1.15 **PLR.** Correcting false reasoning: Impulse-momentum

A group of students is shown a block, which is connected to a taut massless string that is draped around a massless and frictionless pulley and which is connected to a slotted mass set, as shown in the figure below. The block is on a horizontal, frictionless tabletop.



Initially, an instructor holds the block in place. The students are asked to predict how the horizontal momentum of the system consisting of the block, string, and slotted mass set will (or won't) change after the instructor releases the block. A group of students complains that there is an apparent contradiction:

Without the gravitational force that the Earth exerts on the slotted mass set, the system would remain at rest, so the gravitational force that the Earth exerts on the slotted mass set is the cause of the motion of the system. The gravitational force that the Earth exerts on the slotted mass set is directed purely in the vertically downward direction, so the horizontal component of the impulse on the system is zero. Thus, the total horizontal momentum of the system should stay constant. However, the block moves faster and faster toward the right (while the slotted mass set moves faster and faster toward the bottom of the page) after the block is released, so the total horizontal momentum of the system actually increases in magnitude.

In a clear, coherent paragraph-length response that refers to the horizontal component of the impulse on the system, explain why the total horizontal momentum of the system changes after the block is released even though the gravitational force that the Earth exerts on the slotted mass set is purely vertical.

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2.1.16 *ExptDes.* Design so simple, easy to forget to mention principle: KE of cart

The following data were obtained for a cart sliding across a horizontal table. The table coincides with the xy plane.

Mass of cart (kg)	x -component of velocity (m/s)	y -component of velocity (m/s)
0.33 kg	0.30 m/s	0.40 m/s

- (a) Describe an analytic procedure for using these data to obtain the kinetic energy of the cart. Give enough detail so that another student who was absent from school on the day kinetic energy was learned can carry out your procedure.
- (b) Explain how you know that the method you described in part (a) will give the kinetic energy of the cart.

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2.1.17 **PLR.** Simple explanation: Roller coaster

A cart of mass m moves along a frictionless roller coaster track. At one position, the height of the cart above the ground is H , and at another position, the height of the cart is $h < H$. Indicate how the speeds v_h and v_H of the cart, at those respective positions, compare.

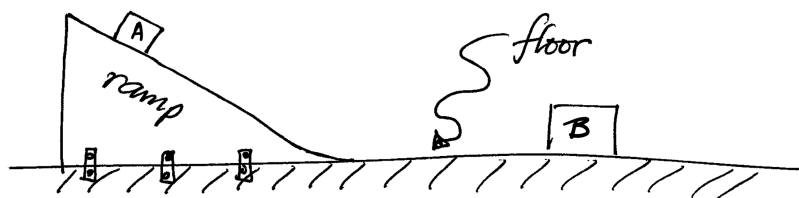
___ $v_h > v_H$ ___ $v_h < v_H$ ___ $v_h = v_H$

In a clear, coherent paragraph-length response, explain your choice.

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2.1.18 **PLR.** Tempt student to misapply conservation principle: Block, incline, block

Block A of mass m_A is released from a height H on a frictionless inclined ramp that is attached to the horizontal, frictionless floor, upon which a block B of mass m_B initially rests.



- (a) At the moment that the block A is released, how do the gravitational potential energy $U_{G,A\&E}$ of the block A-Earth system and the gravitational potential energy $U_{G,B\&E}$ of the block B-Earth system compare?

_____ $U_{G,A\&E} > U_{G,B\&E}$

_____ $U_{G,A\&E} < U_{G,B\&E}$

_____ $U_{G,A\&E} = U_{G,B\&E}$

Briefly explain your reasoning.

The foot of the ramp is smoothly curved (so that block A smoothly transfers onto, rather than slams into, the floor). After leaving the ramp, block A collides with and sticks to block B. The resulting speed of the stuck-together blocks $v_{A\&B}$ is measured. The experiment is repeated, but now with block A having a greater mass m_A .

- (b) Indicate whether the kinetic energy K_2 of the stuck-together blocks in the new experiment is greater than, less than, or equal to the kinetic energy K_1 of the stuck-together blocks in the original experiment.

_____ $K_2 > K_1$

_____ $K_2 < K_1$

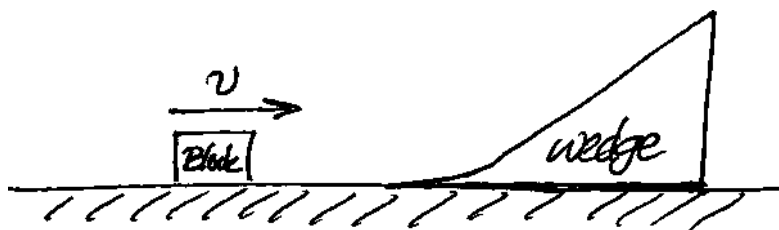
_____ $K_2 = K_1$

Explain your reasoning.

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2.1.19 PLR. Block at unanchored wedge

As shown in the figure below, a block of mass m slides across a horizontal frictionless surface with speed v toward a wedge of mass $M > m$ that is initially at rest on the horizontal surface.



The inclined top surface of the wedge is frictionless. The block slides up the wedge and eventually reaches a maximum height H . The experiment is repeated, but now with the mass of the block equal to M and the mass of the wedge equal to m . Will the new maximum height of the block on the wedge be greater than, less than, or equal to H from the first experiment?

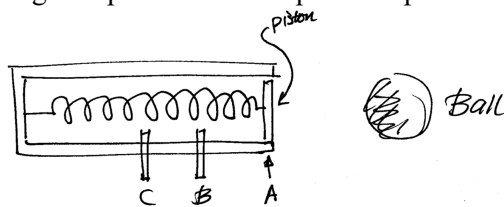
☐ Greater than ☐ Less than ☐ Equal

Explain your reasoning.

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2.1.20 **PLR.** Tempt student to misassume system: Piston, spring, ball launcher

A horizontally-oriented launcher (below) consists of a massless spring and an attached thin, massless piston housed in a capped frictionless tube. When the piston is at position A, the spring is relaxed. The spring can be compressed by using thin pins to hold the piston at positions B and C.



- (a) In one experiment, a ball is shoved into the launcher, and a pin is used to keep the spring compressed with the ball at rest, touching the piston, with the piston staying at position B. In a second experiment, the same procedure is used, except with the ball resting with the piston at position C, not B. Indicate how the mechanical energy $E_{\text{MECH},B}$ of the ball at the end of the first experiment compares with the mechanical energy $E_{\text{MECH},C}$ of the ball at the end of the second experiment.

☐ $E_{\text{MECH},C} > E_{\text{MECH},B}$ ☐ $E_{\text{MECH},C} < E_{\text{MECH},B}$ ☐ $E_{\text{MECH},C} = E_{\text{MECH},B}$

Briefly explain your reasoning.

The launcher is now used to launch the ball. First, the ball is horizontally launched from rest with the piston released from position B. The ball exits the launcher with speed $v_{\text{B,HOR}}$ as the piston passes through position A. In a second trial, the ball is still launched from rest with the piston released from position B, but now with the launcher oriented vertically (to launch the ball upward). The ball again exits the launcher, now with speed $v_{\text{B,VER}}$ as the piston passes through position A.

- (b) Indicate how the muzzle speeds $v_{\text{B,HOR}}$ and $v_{\text{B,VER}}$ compare.

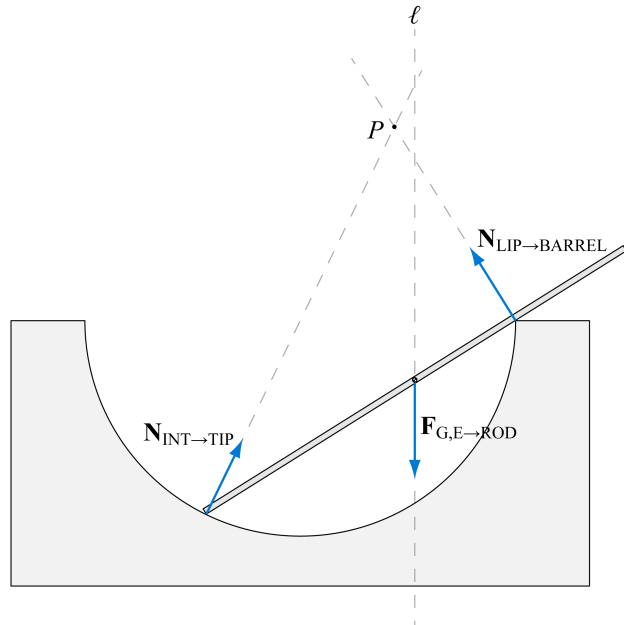
☐ $v_{\text{B,HOR}} > v_{\text{B,VER}}$ ☐ $v_{\text{B,HOR}} < v_{\text{B,VER}}$ ☐ $v_{\text{B,HOR}} = v_{\text{B,VER}}$

Explain your reasoning.

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2.1.21 PLR. Freedom to choose unusual axis of rotation: Adaptation of classic rod-in-bowl

The figure below shows a cutaway view of a frictionless hemispherical bowl and a rod. The tips of the rod



and the lip of the bowl are rounded and *slightly* elastic. As a result, the normal force $\vec{N}_{\text{INT} \rightarrow \text{TIP}}$ that the interior of the bowl exerts on a tip of the rod is perpendicular to the interior of the bowl, and the normal force $\vec{N}_{\text{LIP} \rightarrow \text{BARREL}}$ that the lip of the bowl exerts on the barrel of the rod is perpendicular to the rod. Lines drawn through these two normal force vectors intersect at point P . Vertical line ℓ is drawn through the gravitational force vector $\vec{F}_{\text{G,E} \rightarrow \text{ROD}}$ acting from the center-of-mass of the rod. The forces are not drawn to scale, and the figure might not show the rod at static equilibrium.

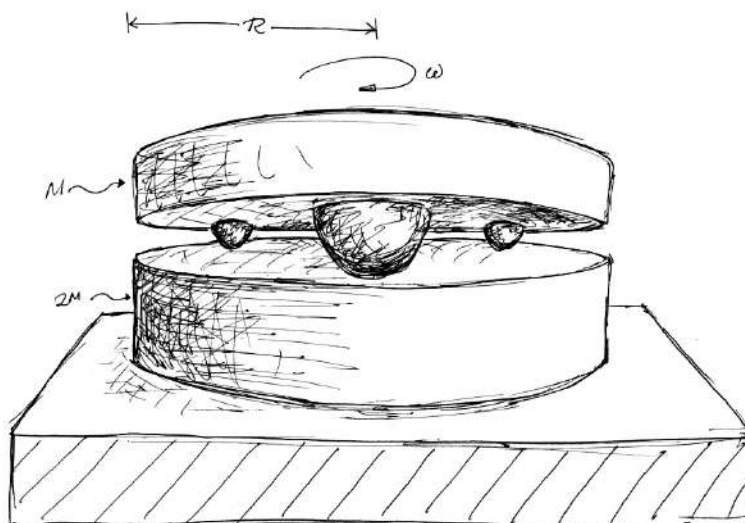
When the rod is at static equilibrium, will point P lie to the left of line ℓ (as shown), to the right of line ℓ , or on line ℓ ? Indicate your choice below.

___ Pt. P is to the left of line ℓ ___ Pt. P is to the right of line ℓ ___ Pt. P is on line ℓ

In a clear, coherent paragraph-length response that incorporates physics principles, explain your choice.

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2.1.22 *Calculation.* Using scaling/graphical reasoning to avoid “I substituted”: Vinyl record
A student is listening to a song on a record (a record is a historical technology in which sound waves were mechanically represented using indentations on a vinyl disk) that is on a turntable spinning with angular velocity ω . The student finishes listening to the song and turns the record player off at time $t = 0$. The turntable slows down with constant angular acceleration until it is no longer spinning. Let time $t_{\frac{1}{2}}$ denote the time at which the angular velocity has decreased to $\frac{1}{2}$ its original value, and let t_{STOP} denote the time at which the turntable comes to rest. What is the ratio of the angular displacement of the turntable from time $t_{\frac{1}{2}}$ to time t_{STOP} to the angular displacement of the turntable from time $t = 0$ to time $t_{\frac{1}{2}}$?

2.1.23 PLR. Adaptation of linear problem in Liao 2018 TPT article: Stacked disks

The apparatus in the figure above consists of a horizontal surface, a uniform disk of mass $2M$ in contact with the surface, and a second uniform disk of mass M that makes contact with the first disk via three identical rubber “feet” equally spaced around the disks’ outer rims. Both disks have a radius of R . The rotational inertia of a uniform disk having mass M and radius R is $\frac{1}{2}MR^2$. There is no friction between the horizontal surface and the disk of mass $2M$, but there is friction between the disk of mass $2M$ and the rubber feet of the disk of mass M . Initially, the disk of mass M is spinning with angular speed ω while the underlying disk is stationary. The rubber feet leave skid mark arcs on the lower disk. The distances between the rubber feet are sufficiently great so that the skid marks do not overlap. After scraping concludes, the disks are collected, and the lengths of the skid marks are measured. In a second experiment, the masses M and $2M$ are both increased by one same factor. In a clear, coherent, paragraph-length response that incorporates relevant principles of physics, explain whether the skid mark lengths in the second experiment will be longer than, shorter than, or as long as the skid marks in the first experiment.

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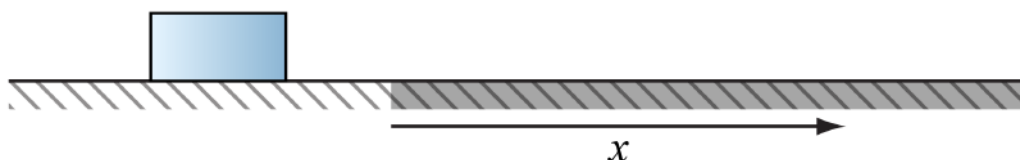
2.1.24 *ExptDes*. Placeholder: Correct false experimental procedure

[Alice claims that by ... one can test whether Identify an aspect of Alice's procedure that is incorrect. Why is the aspect you identified incorrect? How could you change the aspect of Alice's procedure you identified so that Alice's procedure can be used to test whether ...?]

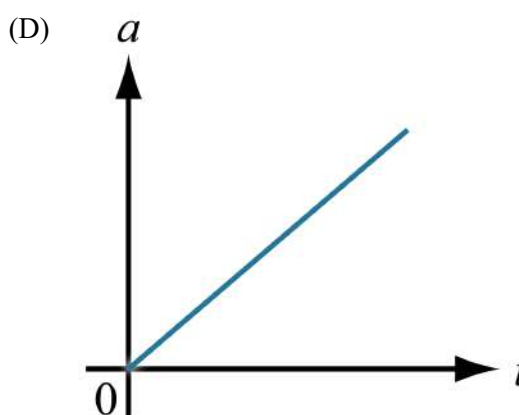
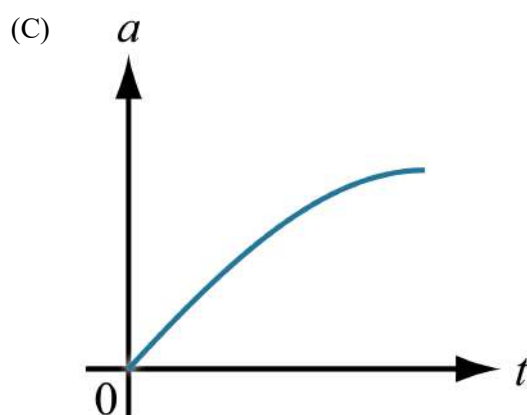
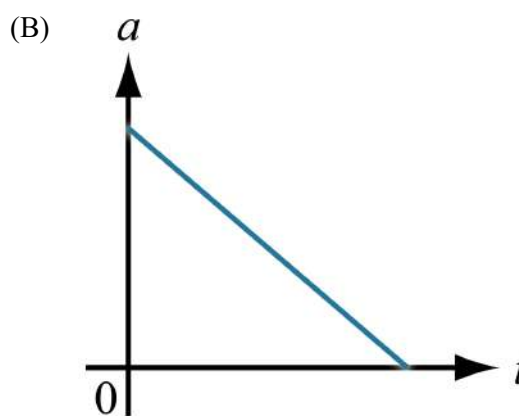
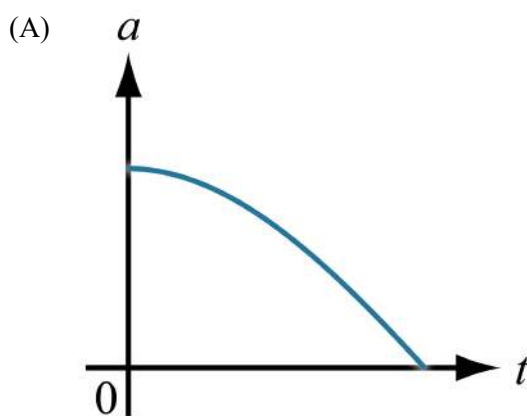
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2.1.25 PLR. Hidden SHM

In the figure below, a block is sliding to the right across a frictionless portion of the floor (less shading) and then encounters a second portion of the floor (darker shading). In the darker region, the coefficient of kinetic friction μ_K between the block and the floor increases linearly with distance from the location on the floor where the lighter and darker regions meet according to $\mu_K = bx$, where b is a positive constant with appropriate units. Even though the block is drawn large, consider the block very small (like a point particle), so that at almost any given moment, the block is either entirely on the frictionless surface or entirely on the dark-shaded surface.

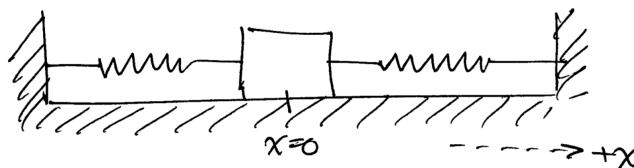


Which one of the following graphs represents the magnitude of the block's acceleration as a function of time, beginning at the instant the block first encounters the darker portion of the floor? Explain how you arrived at your choice.



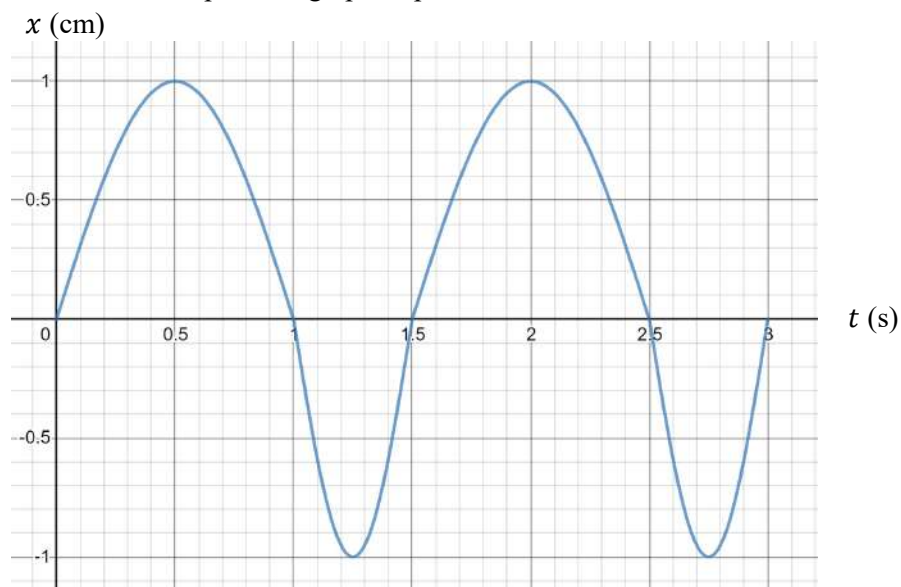
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2.1.26 QQT. Superficially similar to class question: Spring-mass-spring



In the figure above, a block of mass m is on a frictionless, horizontal floor. The block is in contact with a relaxed massless spring on the left, of spring constant k_L , and in contact with a relaxed massless spring on the right, of spring constant k_R . The other ends of the springs are fixed to immovable walls. The horizontal lengths of the springs can change, but the springs do not bend (the springs remain horizontally oriented). The block is not attached to either spring.

- (a) Suppose k_R is much, much greater than k_L . The block is set into oscillatory motion along the x -axis. Derive an algebraic expression for the period of oscillation of the block. Explain how you obtained your result.
- (b) Now suppose that k_R is not necessarily much, much greater than k_L , but that k_R and k_L are still not equal. A student sketches a possible graph of position x vs. time t for the block, shown below.



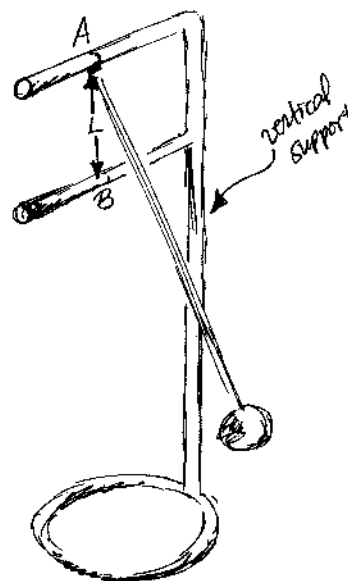
- i. Identify two correct geometric features of the student's sketch and explain, using physics principles, why those features are correct.
- ii. Identify two incorrect geometric features of the student's sketch and explain, using physics principles, why those features are incorrect.

2.1.27 **PLR.** Unfamiliar scenario: Nasty pendulum thing.

In the apparatus shown to the right, two thin horizontal rods are fixed to an immovable vertical support. One end of a massless string is attached to the upper rod at point A, and the other end of the string is attached to a massive, small “bob” (sphere). The vertical distance between the horizontal rods is L . The figure shows the bob immediately after being released from rest. As the bob swings, the string will come into contact with the lower rod at position B. After passing its lowest position, the bob will then ascend and momentarily come to rest. Assume that the bob is always released from a height lower than the height of position B and that the angle between any slanted segment of the string and the vertical is always “small.” The duration of time that elapses between the moment the bob is released from rest and the moment that the bob soonest then again comes to rest is Δt . If L increases, how does Δt change?

☐ Increase ☐ Decrease ☐ Stay the same

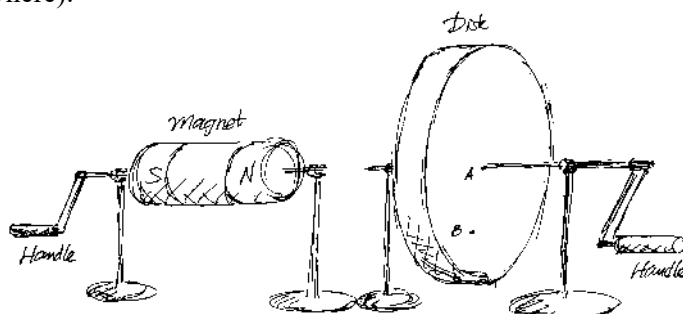
Explain your reasoning.



3 AP PHYSICS 2

3.1.1 **PLR.** Basically Faraday disk.

(Adapted from physical apparatus my PhD advisor had me play with after seeing scenario described online—I don't know where).



The apparatus above consists of a bar magnet and a copper disk, mounted coaxially. The magnet and, separately, the disk, can be spun using the attached handles. The supporting structures, including the rods threading the magnet and disk, are made of an electrically insulating material. Two dots are inked onto the copper disk, one at position A (near center of disk) and another at position B (farther out from center of disk). For each of the scenarios in the table below, each object is either stationary (not spinning) or in the midst of spinning with constant angular velocity. For each scenario, indicate whether an emf is being sustained between inked positions A and B on the copper disk and explain your reasoning.

Magnet	Disk	Magnitude of emf between inked dots A and B	Explanation of your reasoning (presumably a textbox would be provided) (there is a clever way to write a single paragraph that justifies all four choices)
Still	Still	<input type="checkbox"/> Zero <input type="checkbox"/> Nonzero	
Still	Spinning	<input type="checkbox"/> Zero <input type="checkbox"/> Non zero	
Spinning	Still	<input type="checkbox"/> Zero <input type="checkbox"/> Nonzero	
Spinning	Spinning	<input type="checkbox"/> Zero <input type="checkbox"/> Non zero	

4 AP PHYSICS C MECHANICS

4.1.1 **LFRQ** (FBD, PLR). No need to antidifferentiate: v_y - t graph for object falling with drag.

At initial time $t = 0$, an object of mass m is released from rest. Let the $+y$ direction point vertically downward, with the origin of the y -axis at the location from which the object is released. A drag force from the surrounding fluid has a magnitude $F_{\text{DRAG}}(v)$ that is an increasing function of the speed v with which the object moves relative to the background fluid. The exact form of $F_{\text{DRAG}}(v)$ is not known; in particular, it cannot be assumed that the function $F_{\text{DRAG}}(v)$ is proportional to a power of v .

(a) On the dot below, draw a free-body diagram for the object while the object is falling.



(b) Write, but do not solve, a differential equation that could be numerically solved, were $F_{\text{DRAG}}(v)$ known, to obtain the y -velocity of the object as a function of time. Explain how you used physical principles to arrive at your differential equation.

(c) Without directly integrating the differential equation you wrote down in part (b), explain why the graph of v_y vs. t must be both increasing and concave down for all $t > 0$.

5 AP PHYSICS C ELECTRICITY & MAGNETISM

5.1.1 **LFRQ** (PLR). No need to do definite integral: Potential difference outside conducting sphere
 A conducting sphere of radius a contains positive charge Q . The electric potential difference $V(a) - V(c)$ between a point outside the sphere at radius $c > a$ and a point on the surface of the sphere is V_0 . Suppose a thin spherical sheet of insulating material is constructed at radius $b = \frac{a+c}{2}$ so that the electric field strength at each position between the sphere and the sheet remains unchanged, but so that the electric field strength at each location outside the sheet is now doubled. Will the new potential difference $(V(a) - V(c))_f$ in this final situation be greater than, less than, or equal to $\frac{3}{2}V_0$?

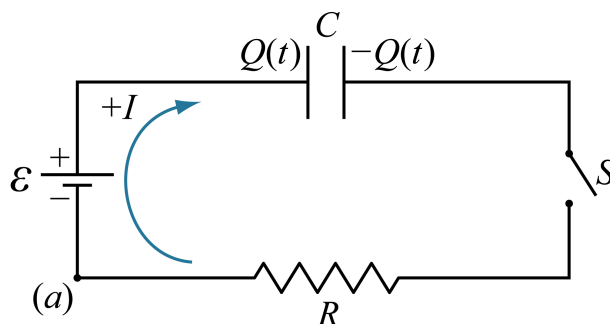
___ $(V(a) - V(c))_f > \frac{3}{2}V_0$ ___ $(V(a) - V(c))_f < \frac{3}{2}V_0$ ___ $(V(a) - V(c))_f = \frac{3}{2}V_0$

Explain your reasoning.

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5.1.2 **LFRQ** (PLR). No need to antidifferentiate: RC circuit curve sketching

The circuit below consists of a battery of emf \mathcal{E} connected to a capacitor of capacitance C connected to a switch S connected to a resistor of resistance R , which is connected to the battery. The switch is closed at time $t = 0$, at which point the charge on the capacitor is $Q = 0$.

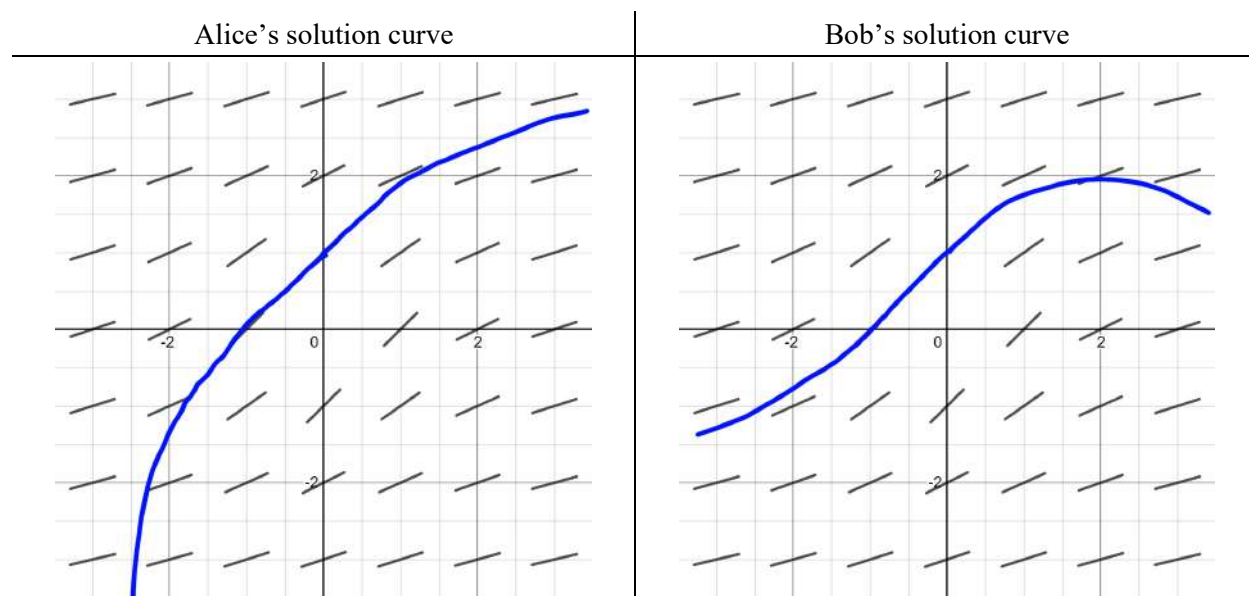


- (a) Write, but do not solve, a differential equation that could be solved to find the capacitor charge $Q(t)$ as a function of time. Explain how you used physical principles to arrive at your differential equation.
- (b) Without referring to the explicit formula for $Q(t)$ that would be obtained by integrating the differentiation equation you wrote in part (a) above, explain why the graph of Q vs. t must be concave down. Merely integrating the differential equation you wrote in part (a) and narrating features of the resultingly derived formula $Q(t)$ will not receive much credit.

6 AP CALCULUS AB

6.1.1 Correcting false reasoning: Solution curve through slope field

Alice and Bob drew the following particular solutions through $(0,1)$ on the provided slope field.



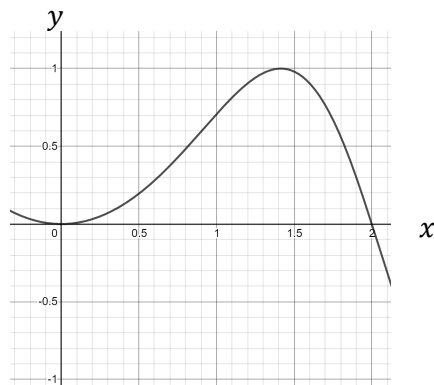
(a) Indicate one feature of each student's solution that is correct and explain why it is correct.

(b) Indicate one feature of each student's solution that is incorrect and explain why it is incorrect.

7 AP CALCULUS BC

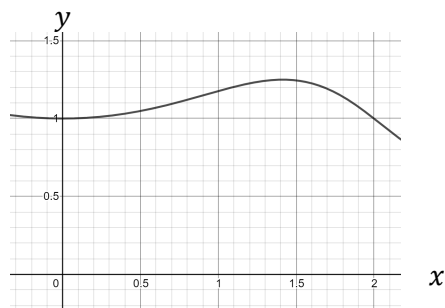
7.1.1 Connecting representations: Graph of f' and Taylor polynomial

Alice shows Bob the graph of f' below and tells Bob the value of $f(1)$. Bob uses the information Alice provides to write down an approximate 2nd-order Taylor polynomial for f centered at $x = 1$.



Alleged graph of f'

Alice realizes she gave Bob the wrong graph and gives Bob the corrected graph of f' below.



Corrected graph of f'

When Bob reconstructs the 2nd-order Taylor polynomial for f centered at $x = 1$, which terms in the Taylor polynomial, if any, change, and why?