

AP Calculus AB Summer Assignment

Complete the included assignment on Pre-Calc material, without the use of a calculator. It is recommended that you do not complete this assignment until a week or two before the start of school. The purpose is to refresh your memory before school starts in September, not see how much you know when school ends in June. Do all work on loose leaf. It is due the first day of school. There will be an assessment on this material during the first week of school. We will be reviewing it the first couple of days in class, so come in prepared with questions.

AP CALCULUS SUMMER ASSIGNMENT

Complete the following. **Show and attach all work (neatly) on a separate piece of paper.** Clearly indicate your final answer on the answer sheet provided. This must be handed in on the first day of school.

1. Are the following statements true? If not, explain clearly in words why not.

a. $\frac{2k}{2x+h} = \frac{k}{x+h}$ b. $\frac{1}{p+q} = \frac{1}{p} + \frac{1}{q}$ c. $\frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$

d. $3\frac{a}{b} = \frac{3a}{3b}$ e. $3\frac{a}{b} = \frac{3a}{b}$ f. $3\frac{a+b}{c} = \frac{3a+b}{c}$

2. Simplify: a. $\frac{\frac{x}{2}}{\frac{x}{4}}$ b. $h \div \frac{(x+h)}{h}$ c. $\frac{\sqrt{x-2} + \frac{5}{\sqrt{x-2}}}{x-2}$

3. Solve $xy' + y = 1 + y'$ for y'

4. Solve the equation: a) $4x^2 - 21x - 18 = 0$ b) $2x^2 - 3x + 3 = 0$ c) $x^4 - 9x^2 + 8 = 0$

5. Write as a single fraction with denominator in factored form: $\frac{7x^2 + 5x}{x^2 + 1} - \frac{5x}{x^2 - 6}$

6. (calc) Graph the equation $y = x^3 - x$ and answer the following questions:

- Is the point (3,2) on the graph?
- Is the point (2,6) on the graph?
- Is the function even, odd, or neither? Why?
- What is the y-intercept?
- Find the x-intercept(s).

7. What are the coordinates of the point at which the line passing through the points (-1,3) and (-2,4) intersects the y-axis?

8. Given $f(x) = x^2 - 3x + 4$, find $f(x+2) - f(2)$

9. Find the domain of each of the following functions (in interval notation):

a) $h(x) = \frac{1}{4x^2 - 21x - 18}$ b) $k(x) = \sqrt{x^2 - 5x - 14}$ c) $p(x) = \frac{\sqrt[3]{x-6}}{\sqrt{x^2 - x - 30}}$ d) $y = \ln(2x - 12)$

10. Given $f(x) = \frac{1}{x}$, find $\frac{f(x+h) - f(x)}{h}$

11. Sketch the graph of each function: a) $f(x) = \begin{cases} 1, & x \leq 0 \\ -1, & x > 0 \end{cases}$ b) $f(x) = \begin{cases} 2x, & (-\infty, -1) \\ 2x^2, & [-1, 2) \\ -x + 3, & (2, \infty) \end{cases}$

12. Given $f(x) = x - 3$ and $g(x) = \sqrt{x}$, complete the following:

- a) $f(g(x))$ b) $g(f(x))$ c) $f(f(x))$

13. Let $f(x) = 2x - 2$, complete the following:

- a. Sketch the graph of $f(x)$
b. Determine whether f has an inverse function
c. Sketch the graph of $f^{-1}(x)$
d. Give the equation for $f^{-1}(x)$

14. Simplify using only positive exponents. Do not rationalize the denominator.

a. $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$ b. $\left(\frac{1}{x^{-2}} + \frac{2}{x^{-1}y^{-1}} + \frac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

15. If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$.

- a) $f(x) - 4$ b) $f(x - 4)$ c) $-f(x + 2)$
d) $5f(x) + 3$ e) $f(2x)$ f) $|f(x)|$

16. Find the surface area of a box of height h whose base dimensions are p and q , and that satisfies the following condition:

- a) The box is closed
b) The box has an open top
c) The box has an open top and a square base with side length p

17. A seven foot ladder, leaning against a wall, touches the wall x feet above the ground. Write an expression (in terms of x) for the distance from the foot of the ladder to the base of the wall.

18. A piece of wire 5 inches long is to be cut into two pieces. One piece is x inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of x .

19. Evaluate each of the following. Answer must be in radians where appropriate.

- a) $\cos 0$ b) $\sin 0$ c) $\tan \frac{\pi}{2}$ d) $\cos \frac{\pi}{4}$ e) $\sin \frac{\pi}{2}$ f) $\sin \pi$
g) $\arccos \frac{\sqrt{3}}{2}$ h) $\tan^{-1} 1$

20 – 22 Find the solution(s) of the equations for $0 \leq x \leq 2\pi$

20. $2 \sin^2 x = 1 - \sin x$ 21. $2 \tan x - \sec^2 x = 0$ 22. $\sin 2x + \sin x = 0$

23. Which of the following expressions are identical?

- a) $\cos^2 x$ b) $(\cos x)^2$ c) $\cos x^2$

24. Which of the following expressions are identical?

a) $(\sin x)^{-1}$

b) $\arcsin x$

c) $\sin x^{-1}$

d) $\frac{1}{\sin x}$

25. Prove each of the following trig identities:

a) $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$

b) $\cot \theta = \frac{\sin 2\theta}{2 \sin^2 \theta}$

c) $\frac{1 + \cos \theta + \cos 2\theta}{\sin \theta + \sin 2\theta} = \cot \theta$

d) $\frac{2 \sin \beta}{\sin 2\beta \cos \beta} = \sec^2 \beta$

e) $\sec x - \sin x \tan x = \cos x$

f) $\frac{\cos(\alpha - \beta)}{\sin \alpha \cos \beta} = \cot \alpha + \tan \beta$

g) $\csc^4 x - 1 = \frac{\cos^2 x (1 + \sin^2 x)}{\sin^4 x}$

h) $(\sin \theta + \cos \theta)^2 = 1 + \sin 2\theta$

i) $\frac{\sin 2x}{\cos 2x} = \frac{2 \tan x}{1 - \tan^2 x}$

26. Solve for x.

a. $\ln e^3 = x$

b) $\ln e^x = 4$

c) $\ln x + \ln x = 0$

d) $e^{\ln 5} = x$

e. $\ln 1 - \ln e = x$

f) $\ln 6 + \ln x - \ln 2 = 3$

g) $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$

HP Calc. Summer Assignment

$$a) \frac{2k}{2x+h} = \frac{k}{x+h}$$

False. They would be = if $\frac{2k}{2x+2h}$.

$$b) \frac{1}{p+q} = \frac{1}{p} + \frac{1}{q}$$

$$\text{False } \frac{1}{p+q} = \frac{q+p}{qp}$$

$$c) \frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$$

True

$$d) 3 \frac{a}{b} = \frac{3a}{3b}$$

$$\text{False } 3 \frac{a}{b} = \frac{3(a)}{1(b)} = \frac{3a}{1b}$$

$$e) 3 \frac{a}{b} = \frac{3a}{b}$$

True

$$f) 3 \frac{a+b}{c} = \frac{3a+b}{c}$$

$$\text{False } 3 \frac{a+b}{c} = \frac{3(a+b)}{1(c)} = \frac{3a+3b}{c}$$

$$2a) \frac{\frac{x}{2} \cdot \frac{4}{1}}{\frac{x}{4} \cdot \frac{4}{1}} \Rightarrow \frac{2x}{x} = \boxed{2}$$

$$b) h \div \frac{(x+h)}{h}$$

$$h \cdot \frac{h}{x+h} = \boxed{\frac{h^2}{x+h}}$$

$$c) \frac{\sqrt{x-2} \cdot \frac{(\sqrt{x-2})^5}{1(\sqrt{x-2})}}{x-2} \Rightarrow$$

$$\frac{x-2+5}{\sqrt{x-2}} \Rightarrow \frac{x+3}{\sqrt{x-2}}$$

$$\Rightarrow \frac{x+3}{\sqrt{x-2}} \cdot \frac{1}{x-2} = \boxed{\frac{x+3}{\sqrt{(x-2)^3}}}$$

$$3) \begin{matrix} xy' + y = 1 + y' \\ -y' \quad -y \quad -y' \quad -y \end{matrix}$$

$$xy' - y' = 1 - y$$

$$y'(x-1) = 1-y$$

$$\boxed{y' = \frac{1-y}{x-1}}$$

$$4a) 4x^2 - 21x - 18 = 0$$

$$4x^2 - 24x + 3x - 18 = 0$$

$$4x(x-6) + 3(x-6) = 0$$

$$(x-6)(4x+3) = 0$$

$$\boxed{x=6} \quad \boxed{x=-3/4}$$

$$4c) x^4 - 9x^2 + 8 = 0$$

$$(x^2-8)(x^2-1) = 0$$

$$x = \pm\sqrt{8} \quad \boxed{x = \pm 1}$$

$$\boxed{x = \pm 2\sqrt{2}}$$

$$b) 2x^2 - 3x + 3 = 0$$

$$a=2 \quad b=-3 \quad c=3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{-15}}{4}$$

$$\boxed{x = \frac{3 \pm i\sqrt{15}}{4}}$$

$$5) \frac{7x^2+5x}{x^2+1} - \frac{5x}{x^2-6}$$

$$\frac{(7x^2+5x)(x^2-6) - 5x(x^2+1)}{(x^2+1)(x^2-6)}$$

$$\frac{7x^4 - 42x^2 + 5x^3 - 30x - 5x^3 - 5x}{x^4 - 6x^2 - x^2 - 6}$$

$$\frac{7x^4 - 42x^2 - 35x}{x^4 - 7x^2 - 6}$$

$$\frac{7x^4 - 42x^2 - 35x}{(x^2+1)(x^2-6)}$$

$$\boxed{\frac{7x^4 - 42x^2 - 35x}{(x^2+1)(x^2-6)}}$$

a) (3,2) No
 b) (2,6) Yes
 c) Odd: Symmetry wrt origin
 d) 0
 e) $x^3 - x = 0$ when $y = -1, 0, 1$

7) $(-1, 3) (-2, 4)$
 $m = \frac{4-3}{-2-1} = \frac{1}{-1} = -1$

$y - 3 = -1(x + 1)$ or $y - 3 = x - 1$
 $y - 3 = -1(0 + 1)$ $y = x + 2$
 $y - 3 = -1$
 $y = 2$ $(0, 2)$

$f(x) = x^2 - 3x + 4$
 3) $f(x+2) - f(2)$
 $f(x+2) = (x+2)^2 - 3(x+2) + 4$
 $= x^2 + 4x + 4 - 3x - 6 + 4$
 $= x^2 + x + 2$
 $f(2) = (2)^2 - 3(2) + 4$
 $= 4 - 6 + 4 = 2$
 $f(x+2) - f(2) =$
 $x^2 + x + 2 - 2 = \boxed{x^2 + x}$

9) a) $h(x) = \frac{1}{4x^2 - 21x + 18}$
 $4x^2 - 21x + 18 \neq 0$
 $4x^2 - 24x + 3x - 18 \neq 0$
 $4x(x-6) + 3(x-6) \neq 0$
 $(x-6)(4x+3) \neq 0$
 $x \neq 6 \quad x \neq -3/4$

$\boxed{(-\infty, -3/4) \cup (-3/4, 6) \cup (6, \infty)}$

b) $k(x) = \sqrt{x^2 - 5x - 14}$
 $x^2 - 5x - 14 \geq 0$

$(x-7)(x+2) \geq 0$
 $x = 7 \quad x = -2$



$\boxed{(-\infty, -2] \cup [7, \infty)}$

4) $\sqrt[3]{x-6}$
 $x = \sqrt{x^2 - x - 30}$
 $x^2 - x - 30 > 0$
 $(x-6)(x+5) > 0$
 $x = 6 \quad x = -5$

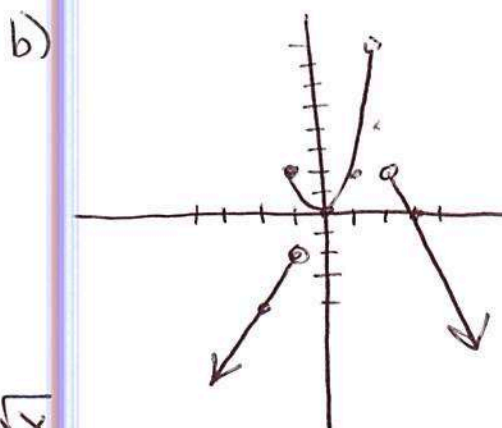
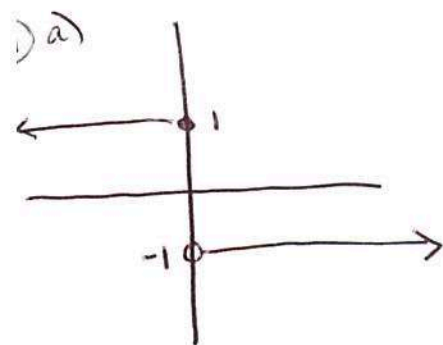
$\boxed{(-\infty, -5) \cup (6, \infty)}$

d) $y = \ln(2x-12)$
 $2x-12 > 0$
 $x > 6$
 $\boxed{(6, \infty)}$

10) $f(x) = \frac{1}{x}$
 $f(x+h) = \frac{1}{x+h}$
 $\frac{f(x+h) - f(x)}{h}$

$= \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{x - (x+h)}{x(x+h)}}{h} = \frac{-h}{h x(x+h)} = \frac{-1}{x(x+h)}$

or $\frac{x - (x+h)}{h x(x+h)} \Rightarrow \frac{x-x-h}{h(x^2+xh)} = \frac{-h}{h(x^2+xh)} \Rightarrow \frac{-1}{x^2+xh}$



x	2x
-2	-4
-1	-2

 open

x	2x^2
-1	2
0	0
1	2
2	8

 closed

x	-x+3
2	1
3	0

 open

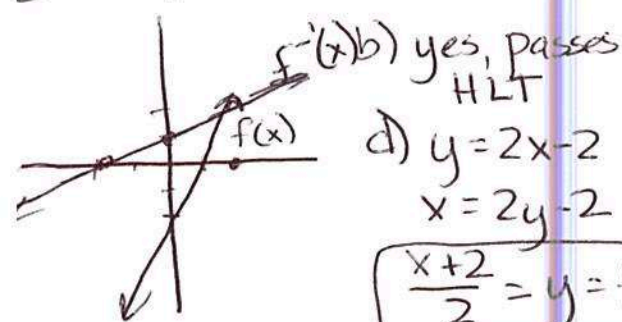
12) $f(x) = x - 3 \quad g(x) = \sqrt{x}$

a) $f(g(x)) = \boxed{\sqrt{x} - 3}$

b) $g(f(x)) = \boxed{\sqrt{x-3}}$

c) $f(f(x)) = x - 3 - 3 = \boxed{x - 6}$

13) $f(x) = 2x - 2$



d) $y = 2x - 2$
 $x = 2y - 2$

$\boxed{\frac{x+2}{2} = y = f^{-1}(x)}$

15a) shift down 4

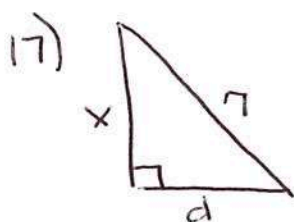
b) shift right 4

c) ref in y-axis, shift left 2

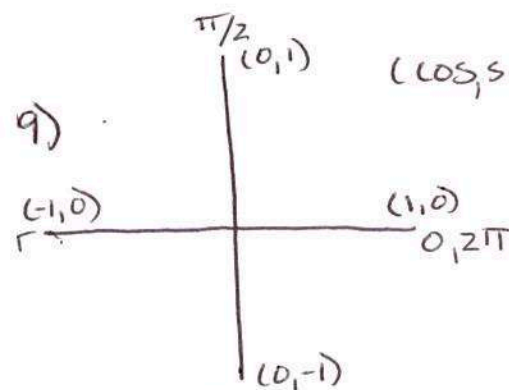
d) dilation of 5, shift up 3

e) dilation in x of 2 (double slope)

f) all negs become pos.



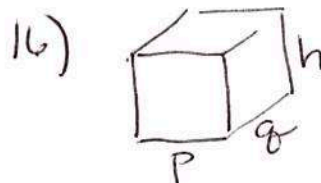
$x^2 + d^2 = 7^2$
 $d^2 = 49 - x^2$
 $d = \sqrt{49 - x^2}$



	$\pi/6$ 30°	$\pi/4$ 45°	$\pi/3$ 60°
sin	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$
cos	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2
tan	$\sqrt{3}/3$	1	$\sqrt{3}$

14a) $\frac{(4(x-4))^{1/2}}{(x-4)^{3/4}}$
 $4^{1/2} = 2$
 $\frac{2(x-4)^{1/2}}{(x-4)^{3/4}}$
 $\boxed{\frac{2}{(x-4)^{1/4}}}$

b) $(x^2 + 2xy + y^2)^{-1/2}$
 $(x^2 + 2xy + y^2)^{-1/2}$
 $\frac{1}{\sqrt{x^2 + 2xy + y^2}}$
 $\frac{1}{\sqrt{(x+y)^2}}$
 $\boxed{\frac{1}{x+y}}$



a) $2pq + 2ph + 2qh$

b) $pq + 2ph + 2qh$

c) $(q=p) p^2 + 2ph + 2ph$
 $= p^2 + 4ph$

18) $\frac{x}{4} + \frac{5-x}{4}$
 square: $A = \left(\frac{x}{4}\right)^2$
 circle: $C = 5 - x = 2\pi r$

$r = \frac{5-x}{2\pi}$

$A = \pi \left(\frac{5-x}{2\pi}\right)^2$
 $\boxed{A = \frac{(5-x)^2}{4\pi}}$

Total = $\left(\frac{x}{4}\right)^2 + \frac{(5-x)^2}{4\pi}$

a) 1

b) 0

c) undef

d) $\sqrt{3}/2$

e) 1

f) 0

g) $\pi/6$

h) $\pi/4$

20) $2\sin^2 x = 1 - \sin x$

$2\sin^2 x + \sin x - 1 = 0$

$2\sin^2 x + 2\sin x - \sin x - 1 = 0$

$2\sin x(\sin x + 1) - 1(\sin x + 1) = 0$

$(\sin x + 1)(2\sin x - 1) = 0$

$\sin x = -1 \quad \sin x = 1/2 \leftarrow \text{Q1, II}$

$\boxed{x = 3\pi/2 \quad x = \pi/6, 5\pi/6}$

$$\begin{aligned}
 21) \quad & 2\tan x - \sec^2 x = 0 \quad \tan^2 x + 1 = \sec^2 x \\
 & 2\tan x - (\tan^2 x + 1) = 0 \\
 & -\tan^2 x - 1 + 2\tan x = 0 \\
 & \quad \quad \quad -1 \\
 & \tan^2 x - 2\tan x + 1 = 0 \\
 & (\tan x - 1)(\tan x - 1) = 0 \\
 & \tan x = 1 \leftarrow \text{QI, III} \\
 & x = \pi/4, 5\pi/4
 \end{aligned}$$

$$\begin{aligned}
 23) \quad & a) \cos^2 x = (\cos x)(\cos x) \\
 & b) (\cos x)^2 = (\cos x)(\cos x) \\
 & c) \cancel{\cos} x^2 = \cos(x \cdot x)
 \end{aligned}$$

$$25a) \frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$$

$$\frac{2\sin\theta\cos\theta}{1 + 2\cos^2\theta - 1} = \tan\theta$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\tan\theta = \tan\theta \checkmark$$

$$c) \frac{1 + \cos\theta + \cos 2\theta}{\sin\theta + \sin 2\theta} = \cot\theta$$

$$\frac{1 + \cos\theta + 2\cos^2\theta - 1}{\sin\theta + 2\sin\theta\cos\theta} = \cot\theta$$

$$\frac{\cos\theta(1 + 2\cos\theta)}{\sin\theta(1 + 2\cos\theta)} = \cot\theta$$

$$\cot\theta = \cot\theta \checkmark$$

$$\begin{aligned}
 22) \quad & \sin 2x + \sin x = 0 \\
 & 2\sin x \cos x + \sin x = 0 \\
 & \sin x (2\cos x + 1) = 0 \\
 & \sin x = 0 \quad 2\cos x + 1 = 0 \\
 & \quad \quad \quad \cos x = -1/2 \\
 & x = 0, \pi \quad x = \frac{2\pi}{3}, \frac{4\pi}{3} \quad \uparrow \text{Q II, III}
 \end{aligned}$$

$$\begin{aligned}
 24) \quad & a) (\sin x)^{-1} = \frac{1}{\sin x} \\
 & b) \arcsin x = \sin^{-1} x \\
 & c) \sin x^{-1} = \sin \frac{1}{x} \\
 & d) \frac{1}{\sin x}
 \end{aligned}$$

$$b) \cot\theta = \frac{\sin 2\theta}{2\sin^2\theta}$$

$$\cot\theta = \frac{2\sin\theta\cos\theta}{2\sin^2\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\cot\theta = \cot\theta \checkmark$$

$$d) \frac{2\sin\beta}{\sin 2\beta \cos\beta} = \sec^2\beta$$

$$\frac{2\sin\beta \cancel{1}}{2\sin\beta \cos\beta \cos\beta} = \sec^2\beta$$

$$\frac{1}{\cos^2\beta} = \sec^2\beta$$

$$\sec^2\beta = \sec^2\beta \checkmark$$

$$e) \sec x - \sin x \tan x = \cos x$$

$$\frac{1}{\cos x} - \frac{\sin x \sin x}{\cos x} = \cos x$$

$$\frac{1 - \sin^2 x}{\cos x} = \cos x$$

$$\frac{\cos^2 x}{\cos x} = \cos x$$

$$\cos x = \cos x \checkmark$$

$$g) \csc^4 x - 1 = \frac{\cos^2 x (1 + \sin^2 x)}{\sin^4 x}$$

$$\csc^4 x - 1 = \frac{\cos^2 x (\cos^2 x)}{\sin^4 x}$$

$$\csc^4 x - 1 = \frac{\cos^4 x}{\sin^4 x}$$

$$\csc^4 x - 1 = \cot^4 x$$

$$\csc^4 x - 1 = \csc^4 x - 1 \checkmark$$

$$26) \cancel{\ln e}^3 = x$$

$$\boxed{3 = x}$$

$$b) \cancel{\ln e}^x = 4$$

$$\boxed{x = 4}$$

$$c) \ln x + \ln x = 0$$

$$2 \ln x = 0$$

$$\ln x^2 = 0$$

$$x^2 = e^0$$

$$x^2 = 1$$

$$\boxed{x = 1}$$

$$d) \cancel{e}^{4.5} = x$$

$$\boxed{5 = x}$$

$$e) \ln 1 - \ln e = x$$

$$0 - 1 = x$$

$$\boxed{-1 = x}$$

$$f) \ln 6 + \ln x + \ln 2 = 3$$

$$\ln \frac{6x}{2} = 3$$

$$\ln 3x = 3$$

$$3x = e^3$$

$$\boxed{x = \frac{e^3}{3}}$$

$$f) \frac{\cos(\alpha - \beta)}{\sin \alpha \cos \beta} = \cot \alpha + \tan \beta$$

$$\frac{\cos \alpha \cos \beta + \sin \alpha \sin \beta}{\sin \alpha \cos \beta} = \cot \alpha + \tan \beta$$

$$\frac{\cos \alpha \cos \beta}{\sin \alpha \cos \beta} + \frac{\sin \alpha \sin \beta}{\sin \alpha \cos \beta} = \cot \alpha + \tan \beta$$

$$\cot \alpha + \tan \beta = \cot \alpha + \tan \beta \checkmark$$

$$h) (\sin \theta + \cos \theta)^2 = 1 + \sin 2\theta$$

$$\sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta = 1 + \sin 2\theta$$

$$1 + 2 \sin \theta \cos \theta = 1 + \sin 2\theta$$

$$1 + \sin 2\theta = 1 + \sin 2\theta \checkmark$$

$$i) \frac{\sin 2x}{\cos 2x} = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\frac{\sin 2x}{\cos 2x} = \frac{2 \sin x (\cos^3 x)}{\cos x \left(\frac{\cos^2 x}{1} - \frac{\sin^2 x}{\cos^2 x} \left(\frac{\cos^2 x}{1} \right) \right)}$$

$$\frac{\sin 2x}{\cos 2x} = \frac{2 \sin x \cos x}{\cos^2 x - \sin^2 x}$$

$$\frac{\sin 2x}{\cos 2x} = \frac{\sin 2x}{\cos 2x} \checkmark$$

$$g) \ln(x+5) = \ln(x-1) - \ln(x+1)$$

$$\ln(x+5) = \ln \frac{(x-1)}{(x+1)}$$

$$x^2 + 6x + 5 = x - 1$$

$$x^2 + 5x + 6 = 0$$

$$(x+3)(x+2) = 0 \quad \cancel{x+3} \quad \cancel{x+2}$$

No solution