AP Calculus AB Summer Assignment

There are 2 parts to the summer assignment:

1. Please purchase the Barron's Book, either with or without the CD. Read Chapter 2 do the following problems on Limits:

   From the 12th edition: P. 104-109 all

   If you do not have the 12th edition, do the Chapter 2 exercises. Most questions are the same...

   The answers are in the book, so check your work, and try to understand your mistakes. Do all work on loose-leaf. All work is due the first day of school. Hand in work, not just answers!

2. Complete the included assignment on Pre-Calc material, without the use of a calculator. Do all work on loose-leaf. All work is due the first day of school. Hand in work, not just answers! Chapter 1 in the Barrons book covers some of the same material. Use it as resource.

You will be tested on this material on the second "A" day of the year!!! No Calculator is allowed! If you do not understand something, ask about it during the first 2 days.

AP CALCULUS SUMMER ASSIGNMENT

Complete the following. <u>Show and attach all work (neatly) on a separate piece of paper.</u> Clearly indicate your final answer on the answer sheet provided. This must be handed in on the first day of school.

1. Are the following statements true? If not, explain clearly in words why not.

   a. $\dfrac{2k}{2x+h}=\dfrac{k}{x+h}$ b. $\dfrac{1}{p+q}=\dfrac{1}{p}+\dfrac{1}{q}$ c. $\dfrac{x+y}{2}=\dfrac{x}{2}+\dfrac{y}{2}$

   d. $3\dfrac{a}{b}=\dfrac{3a}{3b}$ e. $3\dfrac{a}{b}=\dfrac{3a}{b}$ f. $3\dfrac{a+b}{c}=\dfrac{3a+b}{c}$

2. Simplify: a. $\dfrac{x/2}{x/4}$ b. $h\div\dfrac{(x+h)}{h}$ c. $\dfrac{\sqrt{x-2}+\dfrac{5}{\sqrt{x-2}}}{x-2}$

3. Solve $xy'+y=1+y'$ for y'

4. Solve the equation: a) $4x^2-21x-18=0$ b) $2x^2-3x+3=0$ c) $x^4-9x^2+8=0$

5. Write as a single fraction with denominator in factored form: $\dfrac{7x^2+5x}{x^2+1}-\dfrac{5x}{x^2-6}$

6. (calc) Graph the equation $y=x^3-x$ and answer the following questions:
   a. Is the point (3,2) on the graph?
   b. Is the point (2,6) on the graph?
   c. Is the function even, odd, or neither? Why?
   d. What is the y-intercept?
   e. Find the x-intercept(s).

7. What are the coordinates of the point at which the line passing through the points (-1,3) and (-2,4) intersects the y-axis?

8. Given $f(x)=x^2-3x+4$, find $f(x+2)-f(2)$

9. Find the domain of each of the following functions (in interval notation):

   a) $h(x)=\dfrac{1}{4x^2-21x-18}$ b) $k(x)=\sqrt{x^2-5x-14}$ c) $p(x)=\dfrac{\sqrt[3]{x-6}}{\sqrt{x^2-x-30}}$ d) $y=\ln(2x-12)$

10. Given $f(x)=\dfrac{1}{x}$, find $\dfrac{f(x+h)-f(x)}{h}$

11. Sketch the graph of each function: a) $f(x)=\begin{cases}1, & x\le 0\\ -1, & x>0\end{cases}$ b) $f(x)=\begin{cases}2x, & (-\infty,-1)\\ 2x^2, & [-1,2)\\ -x+3, & (2,\infty)\end{cases}$

12. Given $f(x) = x - 3$ and $g(x) = \sqrt{x}$, complete the following:

a) f(g(x)) b) g(f(x)) c) f(f(x))

13. Let $f(x) = 2x - 2$, complete the following:

a. Sketch the graph of f(x)
b. Determine whether *f* has an inverse function
c. Sketch the graph of $f^{-1}(x)$
d. Give the equation for $f^{-1}(x)$

14. Simplify using only positive exponents. Do not rationalize the denominator.

a. $\dfrac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$ b. $\left(\dfrac{1}{x^{-2}} + \dfrac{2}{x^{-1}y^{-1}} + \dfrac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

15. If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of f(x).

a) f(x) – 4 b) f(x – 4) c) - f(x + 2)
d) 5f(x) + 3 e) f(2x) f) |f(x)|

16. Find the surface area of a box of height *h* whose base dimensions are *p* and *q*, and that satisfies the following condition:

a) The box is closed
b) The box has an open top
c) The box has an open top and a square base with side length *p*

17. A seven foot ladder, leaning against a wall, touches the wall *x* feet above the ground. Write an expression (in terms of *x*) for the distance from the foot of the ladder to the base of the wall.

18. A piece of wire 5 inches long is to be cut into two pieces. One piece is *x* inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of *x*.

19. Evaluate each of the following. Answer must be in radians where appropriate.

a) $\cos 0$ b) $\sin 0$ c) $\tan\dfrac{\pi}{2}$ d) $\cos\dfrac{\pi}{4}$ e) $\sin\dfrac{\pi}{2}$ f) $\sin \pi$

g) $\arccos\dfrac{\sqrt{3}}{2}$ h) $\tan^{-1} 1$

20 – 22 Find the solution(s) of the equations for $0 \le x \le 2\pi$

20. $2\sin^2 x = 1 - \sin x$ 21. $2\tan x - \sec^2 x = 0$ 22. $\sin 2x + \sin x = 0$

23. Which of the following expressions are identical?

a) $\cos^2 x$ b) $(\cos x)^2$ c) $\cos x^2$

24. Which of the following expressions are identical?

a) $(\sin x)^{-1}$ b) arcsin x c) $\sin x^{-1}$ d) $\dfrac{1}{\sin x}$

25. Prove each of the following trig identities:

a) $\dfrac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$

b) $\cot\theta = \dfrac{\sin 2\theta}{2\sin^2\theta}$

c) $\dfrac{1+\cos\theta+\cos 2\theta}{\sin\theta+\sin 2\theta} = \cot\theta$

d) $\dfrac{2\sin\beta}{\sin 2\beta\cos\beta} = \sec^2\beta$

e) $\sec x - \sin x\tan x = \cos x$

f) $\dfrac{\cos(\alpha-\beta)}{\sin\alpha\cos\beta} = \cot\alpha + \tan\beta$

g) $\csc^4 x - 1 = \dfrac{\cos^2 x(1+\sin^2 x)}{\sin^4 x}$

h) $(\sin\theta+\cos\theta)^2 = 1+\sin 2\theta$

i) $\dfrac{\sin 2x}{\cos 2x} = \dfrac{2\tan x}{1-\tan^2 x}$

26. Solve for x.

a. $\ln e^3 = x$ b) $\ln e^x = 4$ c) $\ln x + \ln x = 0$ d) $e^{\ln 5} = x$

e. $\ln 1 - \ln e = x$ f) $\ln 6 + \ln x - \ln 2 = 3$ g) $\ln(x+5) = \ln(x-1) - \ln(x+1)$

AP Calc. Summer Assignment

a) $\frac{2k}{2x+h} = \frac{k}{x+h}$

False. They would be = if $\frac{2k}{2x+2h}$.

b) $\frac{1}{p+q} = \frac{1}{p} + \frac{1}{q}$

False. $\frac{1}{p} + \frac{1}{q} = \frac{q+p}{qp}$

c) $\frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$

True

d) $3\frac{a}{b} = \frac{3a}{3b}$

False. $3\frac{a}{b} = \frac{3}{1}\left(\frac{a}{b}\right) = \frac{3a}{1b}$

e) $3\frac{a}{b} = \frac{3a}{b}$

True

f) $3\frac{a+b}{c} = \frac{3a+b}{c}$

False. $3\frac{a+b}{c} = \frac{3}{1}\left(\frac{a+b}{c}\right) = \frac{3a+3b}{c}$

2a) $\dfrac{\frac{x}{2}\cdot\frac{4}{1}}{\frac{x}{4}\cdot\frac{4}{1}} \Rightarrow \frac{2x}{x} \Rightarrow \boxed{2}$

b) $h \div \frac{(x+h)}{h}$

$h \cdot \frac{h}{x+h} \Rightarrow \boxed{\frac{h^2}{x+h}}$

c) $\dfrac{\sqrt{x-2}\,\frac{(\sqrt{x-2})}{1(\sqrt{x-2})} + \frac{5}{\sqrt{x-2}}}{x-2} \Rightarrow$

$\dfrac{\frac{x-2+5}{\sqrt{x-2}}}{x-2} \Rightarrow \dfrac{\frac{x+3}{\sqrt{x-2}}}{x-2}$

$\Rightarrow \frac{x+3}{\sqrt{x-2}} \cdot \frac{1}{x-2} = \boxed{\frac{x+3}{\sqrt{(x-2)^3}}}$

3) $xy' + y = 1 + y'$

$-y' \quad -y \quad -y' - y$

$xy' - y' = 1 - y$

$y'(x-1) = 1 - y$

$\boxed{y' = \frac{1-y}{x-1}}$

b) $2x^2 - 3x + 3 = 0$

$a = 2 \quad b = -3 \quad c = 3$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(3)}}{2(2)}$

$x = \frac{3 \pm \sqrt{-15}}{4}$

$\boxed{x = \frac{3 \pm i\sqrt{15}}{4}}$

4a) $4x^2 - 21x - 18 = 0$ $\quad -72$

$4x^2 - 24x + 3x - 18 = 0$

$4x(x-6) + 3(x-6) = 0$

$(x-6)(4x+3) = 0$

$\boxed{x = 6}$ $\quad \boxed{x = -3/4}$

4c) $x^4 - 9x^2 + 8 = 0$

$(x^2 - 8)(x^2 - 1) = 0$

$x = \pm\sqrt{8}$ $\quad \boxed{x = \pm 1}$

$\boxed{x = \pm 2\sqrt{2}}$

5) $\frac{7x^2 + 5x}{x^2 + 1} - \frac{5x}{x^2 - 6}$

$\frac{(7x^2 + 5x)(x^2 - 6) - 5x(x^2 + 1)}{(x^2 + 1)(x^2 - 6)}$

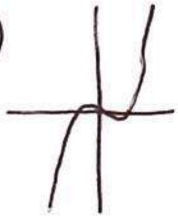
$\frac{7x^4 - 42x^2 + 5x^3 - 30x - 5x^3 - 5x}{x^4 - 6x^2 - x^2 - 6}$

$\frac{7x^4 \cancel{-42x^2} - \cancel{5x^3} - 42x^2 - 35x}{x^4 - 7x^2 - 6}$

$\frac{7x^4 - \cancel{5x^3} - 42x^2 - 35x}{(x^2 + 1)(x^2 - 6)}$

$\boxed{\frac{7x^4 - 42x^2 - 35x}{(x^2 + 1)(x^2 - 6)}}$

a)



a) (3,2) No

b) (2,6) Yes

c) Odd: Symmetry wrt origin

d) 0

e) $x^3 - x = 0$ when $y = -1, 0, 1$

7) (-1,3) (-2,4)

$m = \frac{4-3}{-2--1} = \frac{1}{-1} = -1$

$y - 3 = -1(x+1)$ or $y - 3 = x - 1$

$y - 3 = -1(0+1)$ $\quad y = x + 2$

$y - 3 = -1$

$y = 2$ $\boxed{(0,2)}$

$f(x) = x^2 - 3x + 4$

8) $f(x+2) - f(2)$

$f(x+2) = (x+2)^2 - 3(x+2) + 4$

$= x^2 + 4x + 4 - 3x - 6 + 4$

$= x^2 + x + 2$

$f(2) = (2)^2 - 3(2) + 4$

$= 4 - 6 + 4 = 2$

$f(x+2) - f(2) =$

$x^2 + x + 2 - 2 = \boxed{x^2 + x}$

9 a) $h(x) = \frac{1}{4x^2 - 21x - 18}$

$4x^2 - 21x - 18 \neq 0$

$4x^2 - 24x + 3x - 18 \neq 0$

$4x(x-6) + 3(x-6) \neq 0$

$(x-6)(4x+3) \neq 0$

$x \neq 6 \quad x \neq -3/4$

$\boxed{(-\infty, -3/4) \cup (-3/4, 6) \cup (6, \infty)}$

b) $k(x) = \sqrt{x^2 - 5x - 14}$

$x^2 - 5x - 14 \geq 0$

$(x-7)(x+2) \geq 0$

$x = 7 \quad x = -2$

-2 7

$\boxed{(-\infty, -2] \cup [7, \infty)}$

c) $f(x) = \frac{\sqrt[3]{x-6}}{\sqrt{x^2 - x - 30}}$

$x^2 - x - 30 > 0$

$(x-6)(x+5) > 0$

$x = 6 \quad x = -5$

-5 6

$\boxed{(-\infty, -5) \cup (6, \infty)}$

d) $y = \ln(2x - 12)$

$2x - 12 > 0$

$x > 6$

$\boxed{(6, \infty)}$

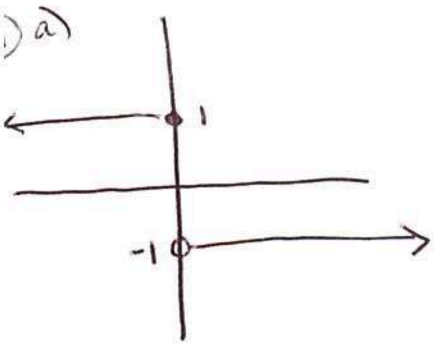
10 $f(x) = \frac{1}{x}$

$f(x+h) = \frac{1}{x+h}$

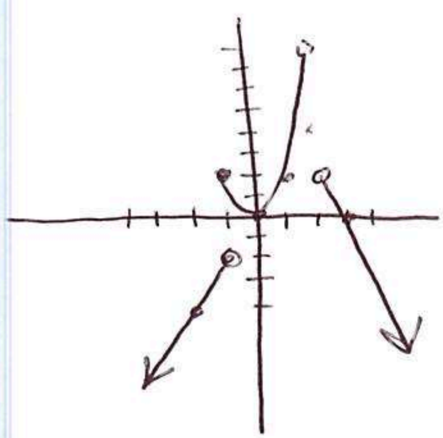
$\frac{f(x+h) - f(x)}{h} = \boxed{\frac{\frac{1}{x+h} - \frac{1}{x}}{h}}$

or $\frac{x - (x+h)}{h(x)(x+h)} \Rightarrow \frac{x - x - h}{h(x^2 + xh)} = \Rightarrow \frac{-h}{h(x^2+xh)} \rightarrow \boxed{\frac{-1}{x^2 + xh}}$

11) a)



b)



x	2x
-2	-4
-1	-2 open

x	$2x^2$
-1	2 closed
0	0
1	2
2	8 open

x	-x + 3
2	1 open
3	0

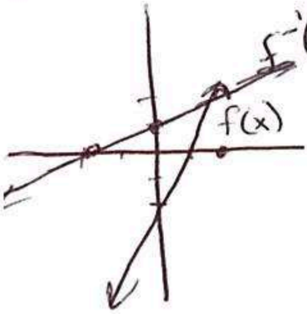
12) $f(x) = x - 3 \quad g(x) = \sqrt{x}$

a) $f(g(x)) = \boxed{\sqrt{x} - 3}$

b) $g(f(x)) = \boxed{\sqrt{x-3}}$

c) $f(f(x)) = x - 3 - 3 = \boxed{x - 6}$

13) $f(x)=2x-2$



b) yes, passes HLT

d) $y=2x-2$

$x=2y-2$

$\boxed{\frac{x+2}{2}=y=f^{-1}(x)}$

14a) $\dfrac{(4(x-4))^{1/2}}{(x-4)^{3/4}}$ $\quad 4^{1/2}=2$

$\dfrac{2(x-4)^{1/2}}{(x-4)^{3/4}}$

$\boxed{\dfrac{2}{(x-4)^{1/4}}}$

b) $\left(x^{-2}+\frac{2}{x^{-1}y^{-1}}+\frac{1}{y^{-2}}\right)$

$(x^2+2xy+y^2)^{-1/2}$

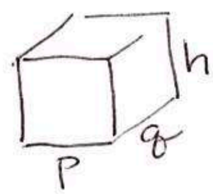
$\dfrac{1}{\sqrt{x^2+2xy+y^2}}$

$\dfrac{1}{\sqrt{(x+y)^2}}$

$\boxed{\dfrac{1}{x+y}}$

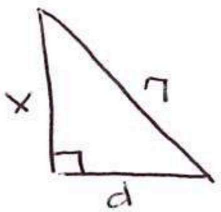
15a) shift down 4
b) shift right 4
c) ref in y-axis, shift left 2
d) dilation of 5, shift up 3
e) dilation in y in x of 2 (double slope)
f) all negs become pos.

16)



a) $2pq+2ph+2qh$
b) $pq+2ph+2qh$
c) $(q=p)\ p^2+2ph+2ph$
$=p^2+4ph$

17)



$x^2+d^2=7^2$

$d^2=49-x^2$

$d=\sqrt{49-x^2}$

18) $\underline{\ x\ }\ \underline{\ 5-x\ }$

square $\boxed{A=\left(\frac{x}{4}\right)^2}$

circle: $C=5-x=2\pi r$

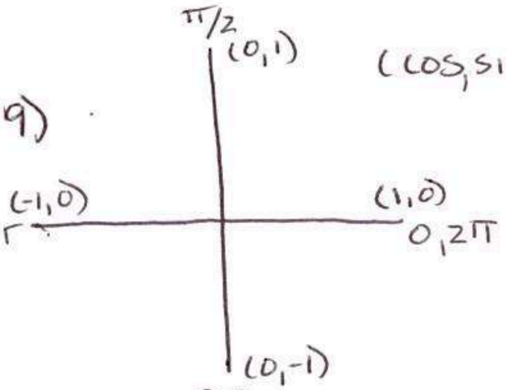
$r=\frac{5-x}{2\pi}$

$A=\pi\left(\frac{5-x}{2\pi}\right)^2$

$\boxed{A=\frac{(5-x)^2}{4\pi}}$

Total $=\left(\frac{x}{4}\right)^2+\frac{(5-x)^2}{4\pi}$

9)



(cos, sin) $\tan=\frac{\sin}{\cos}$

a) 1
b) 0
c) undef
d) $\sqrt{2}/2$
e) 1
f) 0
g) $\pi/6$
h) $\pi/4$

	$\pi/6$ 30°	$\pi/4$ 45°	$\pi/3$ 60°
sin	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
cos	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$
tan	$\sqrt{3}/3$	1	$\sqrt{3}$

20) $2\sin^2x=1-\sin x$

$2\sin^2x+\sin x-1=0$

$2\sin^2x+2\sin x-\sin x-1=0$

$2\sin x(\sin x+1)-1(\sin x+1)=0$

$(\sin x+1)(2\sin x-1)=0$

$\sin x=-1 \quad \sin x=1/2 \leftarrow$ QI, II

$\boxed{x=3\pi/2 \quad x=\pi/6,\ 5\pi/6}$

21) $2\tan x - \sec^2 x = 0$ $\quad \tan^2 x + 1 = \sec^2 x$

$2\tan x - (\tan^2 x + 1) = 0$

$$\frac{-\tan^2 x - 1 + 2\tan x = 0}{-1}$$

$\tan^2 x - 2\tan x + 1 = 0$

$(\tan x - 1)(\tan x - 1) = 0$

$\tan x = 1 \leftarrow QI, III$

$x = \pi/4,\ 5\pi/4$

22) $\sin 2x + \sin x = 0$

$2\sin x \cos x + \sin x = 0$

$\sin x (2\cos x + 1) = 0$

$\sin x = 0 \quad 2\cos x + 1 = 0$

$\cos x = -1/2$

$\boxed{x = 0, \pi \quad x = \frac{2\pi}{3}, \frac{4\pi}{3}} \leftarrow Q\ II, III$

23) $\boxed{\begin{array}{l} a)\ \cos^2 x = (\cos x)(\cos x) \\ b)\ (\cos x)^2 = (\cos x)(\cos x) \end{array}}$

c) $\cos x^2 = \cos(x \cdot x)$

24) a) $(\sin x)^{-1} = \frac{1}{\sin x}$

b) $\arcsin x = \sin^{-1} x$

c) $\sin x^{-1} = \sin \frac{1}{x}$

d) $\frac{1}{\sin x}$

25a) $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan\theta$

$\frac{\cancel{2}\sin\theta \cancel{\cos\theta}}{\cancel{1} + \cancel{2}\cos^{\cancel{2}}\theta - \cancel{1}} = \tan\theta$

$\frac{\sin\theta}{\cos\theta} = \tan\theta$

$\tan\theta = \tan\theta$ ✓

b) $\cot\theta = \frac{\sin 2\theta}{2\sin^2\theta}$

$\cot\theta = \frac{\cancel{2\sin\theta}\cos\theta}{\cancel{2}\sin^{\cancel{2}}\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\cot\theta = \cot\theta$ ✓

c) $\frac{1 + \cos\theta + \cos 2\theta}{\sin\theta + \sin 2\theta} = \cot\theta$

$\frac{\cancel{1} + \cos\theta + 2\cos^2\theta - \cancel{1}}{\sin\theta + 2\sin\theta\cos\theta} = \cot\theta$

$\frac{\cos\theta(\cancel{1 + 2\cos\theta})}{\sin\theta(\cancel{1 + 2\cos\theta})} = \cot\theta$

$\cot\theta = \cot\theta$ ✓

d) $\frac{2\sin\beta}{\sin 2\beta \cos\beta} = \sec^2\beta$

$\frac{\cancel{2\sin\beta}\ 1}{\cancel{2\sin\beta}\cos\beta\cos\beta} = \sec^2\beta$

$\frac{1}{\cos^2\beta} = \sec^2\beta$

$\sec^2\beta = \sec^2\beta$ ✓

e) $\sec x - \sin x \tan x = \cos x$

$$\frac{1}{\cos x} - \frac{\sin x \sin x}{\cos x} = \cos x$$

$$\frac{1-\sin^2 x}{\cos x} = \cos x$$

$$\frac{\cos^{\cancel{2}} x}{\cancel{\cos x}} = \cos x$$

$$\cos x = \cos x \checkmark$$

f) $\dfrac{\cos(\alpha-\beta)}{\sin\alpha\cos\beta} = \cot\alpha + \tan\beta$

$$\frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\sin\alpha\cos\beta} = \cot\alpha + \tan\beta$$

$$\frac{\cos\alpha\cancel{\cos\beta}}{\sin\alpha\cancel{\cos\beta}} + \frac{\cancel{\sin\alpha}\sin\beta}{\cancel{\sin\alpha}\cos\beta} = \cot\alpha + \tan\beta$$

$$\cot\alpha + \tan\beta = \cot\alpha + \tan\beta \checkmark$$

g) $\csc^4 x - 1 = \dfrac{\cos^2 x(1+\sin^2 x)}{\sin^4 x}$

$$\csc^4 x - 1 = \frac{\cos^2 x(\cos^2 x)}{\sin^4 x}$$

$$\csc^4 x - 1 = \frac{\cos^4 x}{\sin^4 x}$$

$$\csc^4 x - 1 = \cot^4 x$$

$$\csc^4 x - 1 = \csc^4 x - 1 \checkmark$$

h) $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$

$$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = 1 + \sin 2\theta$$

$$1 + 2\sin\theta\cos\theta = 1 + \sin 2\theta$$

$$1 + \sin 2\theta = 1 + \sin 2\theta \checkmark$$

i) $\dfrac{\sin 2x}{\cos 2x} = \dfrac{2\tan x}{1-\tan^2 x}$

$$\frac{\sin 2x}{\cos 2x} = \frac{\dfrac{2\sin x}{\cancel{\cos x}}\left(\dfrac{\cos^2 x}{1}\right)}{\left(\dfrac{\cos^2 x}{1}\right)1 - \dfrac{\sin^2 x}{\cos^2 x}\left(\dfrac{\cos^2 x}{1}\right)}$$

$$\frac{\sin 2x}{\cos 2x} = \frac{2\sin x\cos x}{\cos^2 x - \sin^2 x}$$

$$\frac{\sin 2x}{\cos 2x} = \frac{\sin 2x}{\cos 2x} \checkmark$$

26) $\cancel{\ln e}^3 = x$

$\boxed{3 = x}$

b) $\cancel{\ln e}^x = 4$

$\boxed{x = 4}$

c) $\ln x + \ln x = 0$

$2\ln x = 0$

$\ln x^2 = 0$

$x^2 = e^0$

$x^2 = 1$

$\boxed{x = 1}$

d) $\cancel{e}^{\ln 5} = x$

$\boxed{5 = x}$

e) $\ln 1 - \ln e = x$

$0 - 1 = x$

$\boxed{-1 = x}$

f) $\ln 6 + \ln x - \ln 2 = 3$

$\ln \frac{6x}{2} = 3$

$\ln 3x = 3$

$3x = e^3$

$\boxed{x = e^3/3}$

g) $\ln(x+5) = \ln(x-1) - \ln(x+1)$

$$\frac{\cancel{\ln}(x+5)}{1} = \cancel{\ln}\frac{(x-1)}{(x+1)}$$

$x^2 + 6x + 5 = x - 1$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$ ~~x=-3~~ ~~x=-2~~

No solution