

Grades Three through Five Number Talks

Based on *Number Talks* by Sherry Parrish, Math Solutions 2010

Number Talks is a ten-minute classroom routine included in this year's Scope and Sequence. Kindergarten through fifth grade teachers will facilitate Number Talks with all students three days a week.

Number Talks are designed to support proficiency with grade level fluency standards. The goal of Number Talks is for students to compute accurately, efficiently, and flexibly. This includes fluency with single-digit combinations in addition, subtraction, multiplication and division as well as procedural fluency with two or multi digit numbers.

In addition to developing efficient computation strategies, Number Talks encourages students to make sense of mathematics, be able to communicate mathematically, and reason and prove solutions.

The key components of successful Number Talks:

- ***A safe and accepting classroom environment and mathematical community***
- ***Classroom discussions (PROTOCOL)***
 1. Teacher provides the problem.
 2. Teacher provides students opportunity to solve problem mentally.
 3. Students show a visual cue when they are ready with a solution. Students signal if they have solved it in more than one way too. (Quiet form of acknowledgement allows time for students to think, while the process continues to challenge those that are already have an answer)
 4. Teacher calls for answers. S/he collects all answers- correct and incorrect- and records answers.
 5. Students share strategies and justifications with peers.
- ***The teacher's role as a "facilitator, questioner, listener, and learner"***
- ***Use of mental math to increase efficiency and knowledge of number relationships***
- ***Purposeful computation problems that support mathematical goals in number and operations***

Many of the number talks consist of three or more sequential problems. The sequence of problems within a given number talk allows students to apply strategies from previous problems to subsequent problems. You may:

- Select at random from each category; or
- navigate in a systematic order by selecting problems with smaller numbers from a specific category, then building to larger numbers.

You may also adjust the numbers according to your students' needs and responses.

Addition: Making Landmark or Friendly Numbers

When students understand that you can compensate in addition (remove a specific quantity from one addend and add that same quantity to another addend) without altering the sum, they can begin to construct powerful mental computation strategies from this concept. Telling them that this will always work is not sufficient; they need to have opportunities to test and prove this idea. Initially you may want to have students use manipulative to provide proof for their ideas. Numerical fluency (composing and decomposing numbers) is a key component of this strategy.

Category 1: Making Landmark or Friendly Numbers

The following number talks are carefully designed to use numbers that are one away from a landmark or friendly number.

$$9 + 8$$

$$19 + 5$$

$$9 + 26$$

$$16 + 19$$

$$25 + 25$$

$$25 + 26$$

$$24 + 26$$

$$26 + 49$$

$$37 + 69$$

$$79 + 26$$

$$89 + 28$$

$$99 + 19$$

$$7 + 19$$

$$16 + 29$$

$$19 + 18$$

$$29 + 33$$

$$49 + 8$$

$$49 + 23$$

$$49 + 37$$

$$49 + 51$$

$$99 + 5$$

$$99 + 15$$

$$88 + 26$$

$$99 + 51$$

Category 2: Making Landmark and Friendly Numbers

The following number talks consist of one addend that is two away from a multiple of ten or a landmark number

$$8 + 5$$

$$8 + 13$$

$$8 + 24$$

$$18 + 7$$

$$18 + 63$$

$$38 + 37$$

$$67 + 28$$

$$48 + 52$$

$$98 + 5$$

$$98 + 13$$

$$98 + 34$$

$$98 + 52$$

$$8 + 4$$

$$18 + 6$$

$$28 + 17$$

$$27 + 18$$

$$48 + 6$$

$$48 + 17$$

$$23 + 48$$

$$48 + 47$$

$$8 + 4 + 18$$

$$18 + 4 + 18$$

$$28 + 5 + 27$$

$$24 + 3 + 48$$

$$28 + 16$$

$$25 + 38$$

$$23 + 27$$

$$28 + 45$$

$$58 + 36$$

$$24 + 78$$

$$88 + 14$$

$$68 + 33$$

$$48 + 4 + 48$$

$$48 + 49 + 3$$

$$98 + 97 + 5$$

$$99 + 98 + 97 + 5$$

Category 3: Making Landmark or Friendly Numbers

The following number talks consist of computation problems with two- and three-digit addends. The addends are one or more away from a multiple of ten or landmark number. The further the addends are from the landmark numbers, the more challenging the strategy.

$$99 + 38$$

$$98 + 47$$

$$98 + 99$$

$$99 + 99 + 5$$

$$116 + 29$$

$$39 + 127$$

$$114 + 118$$

$$46 + 118$$

$$119 + 119$$

$$149 + 149$$

$$129 + 139$$

$$199 + 199$$

$$119 + 26$$

$$118 + 17$$

$$129 + 16$$

$$124 + 26$$

$$198 + 7$$

$$199 + 13$$

$$148 + 27$$

$$139 + 43$$

$$249 + 22$$

$$248 + 49$$

$$225 + 49$$

$$299 + 26$$

$$36 + 109$$

$$49 + 108$$

$$119 + 48$$

$$126 + 124$$

$$128 + 34$$

$$119 + 36$$

$$56 + 129$$

$$126 + 49$$

$$999 + 99$$

$$998 + 49$$

$$997 + 199$$

$$199 + 99 + 49$$

Addition: Doubles/ Near Doubles

Instructions

To foster the Doubles/Near-Doubles strategy, initially select number that are one away from doubles that students often use such as $5 + 5$, $25 + 25$, and $150 + 150$. If one addend is that targeted double and the other addend is just one away from that double, student's will begin to notice this relationship. For example, $25 + 26$ lends itself to students thinking about 26 as $25 + 1$, so they will more readily think about $25 + 25$.

Category 2: Doubles/Near-Doubles

The following number talks use doubles with two-digit numbers.

(Category 1 consists of doubles up to twenty)

$$20 + 20$$

$$19 + 19$$

$$19 + 18$$

$$19 + 17$$

$$30 + 30$$

$$29 + 29$$

$$29 + 28$$

$$28 + 27$$

$$45 + 45$$

$$46 + 45$$

$$46 + 46$$

$$45 + 47$$

$$25 + 25$$

$$24 + 25$$

$$25 + 26$$

$$26 + 27$$

$$35 + 35$$

$$35 + 36$$

$$34 + 35$$

$$36 + 37$$

$$50 + 50$$

$$49 + 49$$

$$48 + 49$$

$$49 + 52$$

$$25 + 25$$

$$25 + 28$$

$$24 + 27$$

$$24 + 28$$

$$40 + 40$$

$$39 + 39$$

$$39 + 38$$

$$38 + 37$$

$$100 + 100$$

$$99 + 99$$

$$99 + 98$$

$$99 + 97$$

Category 3: Doubles/Near-Doubles

The following number talks use doubles with two- and three-digit numbers.

$$100 + 100$$

$$99 + 99$$

$$98 + 99$$

$$97 + 99$$

$$200 + 200$$

$$199 + 199$$

$$198 + 199$$

$$198 + 198$$

$$400 + 400$$

$$399 + 399$$

$$398 + 399$$

$$398 + 398$$

$$125 + 125$$

$$124 + 126$$

$$126 + 127$$

$$124 + 128$$

$$250 + 250$$

$$249 + 249$$

$$249 + 248$$

$$248 + 248$$

$$500 + 500$$

$$499 + 499$$

$$498 + 499$$

$$498 + 497$$

$$150 + 150$$

$$149 + 149$$

$$148 + 149$$

$$148 + 148$$

$$300 + 300$$

$$299 + 299$$

$$298 + 299$$

$$298 + 297$$

$$1000 + 1000$$

$$999 + 999$$

$$998 + 999$$

$$998 + 998$$

Addition: Breaking Each Number into Its Place Value

Encourage students to break each number into its place value using numbers that do not have an obvious relationship to each other. By selecting numbers this characteristic, students are more likely to break numbers apart into their respective place values and work mentally from left to right. The categories in this strategy are based on the magnitude of the numbers.

The following number talks consist of three or more sequential problems. The sequence of problems within a given number talk allows students to apply strategies from previous problems to subsequent problems. These number talk problems may be used in two ways:

- Selected at random from each category; or
- navigated in a systematic order by selecting problems with smaller numbers from a specific category, then building to larger numbers.

Category 1: Breaking Each Number into Its Place Value

The following number talks consist of smaller two-digit numbers. The first column on the left consists of problems that do not require regrouping. The two columns on the right include problems that encourage students to combine the ten from the ones column with the tens from the tens column.

$28 + 11$

$14 + 35$

$22 + 15$

$18 + 31$

$15 + 27$

$23 + 18$

$17 + 25$

$16 + 27$

$17 + 33$

$24 + 38$

$16 + 38$

$37 + 18$

$36 + 22$

$12 + 37$

$13 + 14$

$24 + 32$

$26 + 28$

$23 + 27$

$27 + 25$

$28 + 24$

$27 + 15$

$35 + 26$

$17 + 33$

$25 + 38$

Category 2: Breaking Each Number into Its Place Value

The following number talks consist of two- and three- digit numbers, some of which require regrouping.

$$74 + 18$$

$$58 + 28$$

$$37 + 26$$

$$46 + 38$$

$$77 + 36$$

$$58 + 65$$

$$46 + 88$$

$$74 + 47$$

$$354 + 111$$

$$267 + 232$$

$$215 + 136$$

$$342 + 64$$

$$26 + 45$$

$$38 + 17$$

$$28 + 42$$

$$53 + 38$$

$$113 + 56$$

$$122 + 37$$

$$114 + 44$$

$$121 + 48$$

$$216 + 137$$

$$285 + 127$$

$$156 + 85$$

$$274 + 57$$

$$37 + 38$$

$$28 + 47$$

$$66 + 28$$

$$45 + 47$$

$$158 + 221$$

$$136 + 113$$

$$205 + 134$$

$$262 + 35$$

$$135 + 219$$

$$315 + 192$$

$$167 + 173$$

$$115 + 293$$

Category 3: Breaking Each Number into Its Place Value

The following number talks consist of computation problems with three-digit number that require regrouping.

$$365 + 247$$

$$138 + 292$$

$$168 + 254$$

$$292 + 139$$

$$238 + 184$$

$$361 + 292$$

$$515 + 127$$

$$209 + 136$$

$$444 + 177$$

$$333 + 277$$

$$276 + 258$$

$$518 + 265$$

$$275 + 147$$

$$386 + 137$$

$$246 + 356$$

$$377 + 340$$

$$146 + 277$$

$$216 + 188$$

$$255 + 267$$

$$185 + 146$$

$$386 + 147$$

$$216 + 388$$

$$424 + 193$$

$$370 + 267$$

$$240 + 392$$

$$150 + 186$$

$$230 + 284$$

$$310 + 192$$

$$240 + 195$$

$$360 + 275$$

$$109 + 256$$

$$218 + 293$$

$$111 + 999$$

$$222 + 888$$

$$333 + 777$$

$$444 + 777$$

Addition: Adding Up in Chunks

Instructions

Beginning midyear in second grade, we want students to be able to add ten and then multiples of ten to any number with ease. Adding up numbers in chunks builds upon adding multiples of ten by encouraging students to keep one number whole while adding “chunks” of the second addend.

The following number talks consist of three or more sequential problems. The sequence of problems within a given number talk allows students to apply strategies from previous problems to subsequent problems. These number talk problems may be used in two ways:

- Selected at random from each category; or
- navigated in a systematic order by selecting problems with smaller numbers from a specific category, then building to larger numbers.

Category 1: Adding Up in Chunks

The following number talks build gradually from adding multiples of ten to a number to adding in chunks.

$$16 + 10$$

$$16 + 20$$

$$16 + 40$$

$$16 + 42$$

$$35 + 10$$

$$35 + 20$$

$$35 + 40$$

$$35 + 42$$

$$46 + 20$$

$$46 + 30$$

$$46 + 50$$

$$46 + 53$$

$$26 + 10$$

$$26 + 30$$

$$26 + 50$$

$$26 + 53$$

$$32 + 10$$

$$32 + 30$$

$$32 + 50$$

$$32 + 55$$

$$57 + 10$$

$$57 + 20$$

$$57 + 30$$

$$57 + 33$$

$$24 + 10$$

$$24 + 30$$

$$24 + 50$$

$$24 + 55$$

$$44 + 10$$

$$44 + 20$$

$$44 + 30$$

$$44 + 35$$

$$53 + 20$$

$$53 + 25$$

$$53 + 40$$

$$53 + 42$$

Category 2: Adding Up in Chunks

The following number talks consist of adding multiples of ten while keeping one number whole and then breaking apart the ones into friendly combinations. For example, $28 + 24$ could be chunked as $28 + 20 = 48$; the $48 + 4$ could be added by breaking the 4 apart into $2 + 2$. The problem could then be solved as $(48 + 2) + 2 = 50 + 2 = 52$.

$18 + 10$

$18 + 13$

$18 + 20$

$18 + 23$

$29 + 10$

$29 + 15$

$29 + 20$

$29 + 24$

$57 + 10$

$57 + 14$

$57 + 30$

$57 + 36$

$16 + 20$

$16 + 25$

$16 + 30$

$16 + 36$

$38 + 20$

$38 + 26$

$38 + 30$

$38 + 33$

$65 + 30$

$65 + 36$

$65 + 50$

$65 + 57$

$17 + 10$

$17 + 14$

$17 + 30$

$17 + 35$

$45 + 30$

$45 + 38$

$45 + 40$

$45 + 46$

$73 + 30$

$73 + 38$

$73 + 50$

$73 + 58$

Category 3: Adding Up in Chunks

The following number talks consist of adding multiples of ten and one hundred while keeping one number whole.

$$56 + 40$$

$$56 + 50$$

$$156 + 40$$

$$156 + 43$$

$$37 + 40$$

$$37 + 46$$

$$237 + 40$$

$$237 + 48$$

$$345 + 200$$

$$345 + 400$$

$$345 + 450$$

$$345 + 457$$

$$256 + 100$$

$$256 + 300$$

$$256 + 340$$

$$256 + 342$$

$$25 + 60$$

$$25 + 66$$

$$125 + 60$$

$$125 + 68$$

$$134 + 100$$

$$134 + 300$$

$$134 + 380$$

$$134 + 387$$

$$117 + 200$$

$$117 + 400$$

$$117 + 420$$

$$117 + 426$$

$$47 + 80$$

$$47 + 84$$

$$247 + 70$$

$$247 + 74$$

$$218 + 200$$

$$218 + 400$$

$$218 + 450$$

$$218 + 456$$

Subtraction Number Talks

The following number talks are crafted to elicit specific subtraction strategies; students may also share other efficient methods to solve the problems. The overall purpose is to help students build a toolbox of efficient strategies based on numerical reasoning. The ultimate goal of number talks is for students to compute accurately, efficiently, and flexibly.

Subtraction: Removal or Counting Back

Instructions

Many students intuitively count back to solve subtraction problems. The key is to help them realize when this is and is not an efficient strategy. The closer the minuend and subtrahend are, the more likely students are to use Removal or Counting Back as a strategy. At times this can be an efficient strategy as evidenced by the following problem, $100 - 98$. For this problem, using the standard U.S. subtraction algorithm would be an inefficient strategy; counting back or up would be much more efficient. However, if the numbers were farther apart as in $100 - 81$, counting back by ones would become cumbersome. Counting back by chunks is more efficient as the numbers get farther apart. It is not necessary to develop a sequence of problems to foster the removal or counting back strategy. Instead, look for appropriate times to discuss when this strategy is and is not appropriate

Subtraction: Adding Up

Instructions

Two ideas to consider when crafting number talks to encourage the Adding Up strategy for subtraction are 1) keep the minuend and the subtrahend far apart, and 2) frame the problem in a context that implies distance. Specific examples of using a context for subtraction can be found in Chapter 5

The farther apart the subtrahend is from the minuend, the more likely it is that students will count or add up. The closer the two numbers are, the more the likelihood that students will count back. For example, if the problem is $50 - 47$, it would be more cumbersome and tedious to count back.

Creating a word problem that implies distance also gives students a mental image and action of counting up or moving forward from the smaller number to the larger number. The following story problem is an example of a context that implies distance for $50 - 17$

Martha's goal is to walk 50 laps on the school track. She has already walked 17 laps. How many more laps does Martha need to walk to reach her goal?

The scenario alone creates a mental picture and action of moving forward from 17 to 50 and lends itself to the student solving the problem in this manner.

Other examples of contextual problems to promote Adding Up for subtraction are shown in the examples that follow. Notice how each problem uses number that are relatively far apart, and the context implies a distance to be bridged by starting with a part and working towards the whole.

Contextual Problems to Promote the Adding Up Strategy

Paul plans to read 90 pages each day. So far, he has read only 16 pages. How many more pages does Paul need to read to reach his goal?

Rebekah wants to buy an MP3 player that costs \$182. She has saved \$53 so far. How much more money does Rebekah need before she can buy the MP3 player?

If Green Acres School raises \$5,000 for new books for the library, the students will receive a pizza party. So far the students have raised \$1,238. How much more money do they need to reach their goal?

Category 1: Adding Up

The following number talks include computation problems that foster the Adding Up strategy by incorporating two ideas: 1) the whole is a multiple of ten or one hundred, and 2) the subtrahend is close to a multiple of ten or a landmark number.

$20 - 15$

$20 - 14$

$20 - 9$

$20 - 8$

$50 - 44$

$50 - 39$

$50 - 29$

$50 - 24$

$80 - 69$

$80 - 59$

$80 - 49$

$80 - 39$

$30 - 24$

$30 - 19$

$30 - 15$

$30 - 12$

$60 - 54$

$60 - 49$

$60 - 39$

$60 - 29$

$90 - 79$

$90 - 74$

$90 - 49$

$90 - 44$

$40 - 34$

$40 - 29$

$40 - 24$

$40 - 19$

$70 - 59$

$70 - 49$

$70 - 39$

$70 - 34$

$100 - 89$

$100 - 74$

$100 - 49$

$100 - 44$

Category 2: Adding Up

The following number talks include computation problems where the whole is a multiple of ten or one hundred, and the subtrahend is close to a multiple of ten or a landmark number.

$$100 - 89$$

$$100 - 69$$

$$100 - 49$$

$$100 - 37$$

$$250 - 224$$

$$250 - 219$$

$$200 - 199$$

$$200 - 149$$

$$500 - 449$$

$$500 - 419$$

$$500 - 299$$

$$500 - 249$$

$$150 - 124$$

$$150 - 99$$

$$150 - 74$$

$$150 - 49$$

$$300 - 269$$

$$300 - 249$$

$$300 - 99$$

$$300 - 149$$

$$750 - 709$$

$$750 - 599$$

$$750 - 449$$

$$750 - 324$$

$$200 - 174$$

$$200 - 149$$

$$200 - 124$$

$$200 - 99$$

$$400 - 349$$

$$400 - 299$$

$$400 - 274$$

$$400 - 199$$

$$1000 - 899$$

$$1000 - 749$$

$$1000 - 624$$

$$1000 - 499$$

Category 3: Adding Up

The following number talks consist of computation problems where the whole is no longer an exact multiple of ten or one hundred, and the subtrahend is a farther distance from the whole.

$$50 - 29$$

$$55 - 29$$

$$55 - 48$$

$$55 - 37$$

$$100 - 75$$

$$120 - 75$$

$$125 - 75$$

$$125 - 83$$

$$300 - 174$$

$$315 - 174$$

$$335 - 219$$

$$335 - 287$$

$$80 - 59$$

$$84 - 59$$

$$81 - 48$$

$$81 - 36$$

$$100 - 80$$

$$140 - 80$$

$$146 - 80$$

$$146 - 89$$

$$400 - 329$$

$$420 - 329$$

$$423 - 318$$

$$444 - 298$$

$$70 - 49$$

$$73 - 49$$

$$76 - 67$$

$$76 - 39$$

$$200 - 149$$

$$250 - 149$$

$$223 - 186$$

$$245 - 198$$

$$500 - 249$$

$$525 - 249$$

$$1000 - 499$$

$$1000 - 671$$

Subtraction: Removal

Instruction

A primary consideration in helping students think about the Removal strategy is to create a context that implies taking or removing an amount out of the whole. By structuring the following story problem for $50 - 17$, we can create a removal action

Bethany has 50 marbles. She decides to give her friends Marco 17 of her marbles. How many marbles will Bethany have left?

Notice how the following story problems also lend themselves to a Removal strategy by starting with the whole and taking out a part.

Contextual Problems to Promote a Removal Strategy

Richard has 90 paperback books. He plans to donate 18 of them to his neighborhood library. How many books will Richard have left?

Mia saved \$182. She bought a video game for \$53. How much money does Mia have left?

The students of Green Acres School raised \$5,000 for new books for the school library. So far the librarian has purchased \$1,238 in new books. How much money does the school have left to purchase books?

The following number talks consist of three or more sequential problems. The sequence of problems within a given number talk allows students to apply strategies from previous problems to subsequent problems. These number talk problems may be used in two ways:

- Selected at random from each category; or
- navigated in a systematic order by selecting problems with smaller numbers from a specific category, then building to larger numbers.

To help your students be successful with this strategy, you may wish to introduce each of the problems embedded in a context similar to the situations discussed previously. It is also important to encourage students to keep the minuend intact and remove the subtrahend in parts; otherwise, it is easy for them to lose sight of the whole and the part. The following number talk sequences help promote this idea

Removal

The following number talks include computation problems with two-digit numbers that require regrouping or decomposing.

$23 - 10$

$23 - 14$

$23 - 18$

$23 - 15$

$42 - 30$

$42 - 33$

$43 - 10$

$43 - 14$

$72 - 50$

$72 - 54$

$72 - 30$

$72 - 36$

$33 - 10$

$33 - 14$

$33 - 20$

$33 - 25$

$51 - 30$

$51 - 35$

$55 - 20$

$55 - 26$

$81 - 50$

$81 - 55$

$83 - 70$

$83 - 74$

$45 - 20$

$45 - 26$

$45 - 10$

$45 - 16$

$64 - 40$

$64 - 45$

$64 - 20$

$64 - 28$

$91 - 60$

$91 - 63$

$94 - 50$

$94 - 56$

Category 3: Removal

The following number talks utilize two- and three-digit numbers; some require decomposing.

$100 - 50$

$100 - 52$

$100 - 60$

$100 - 64$

$150 - 20$

$150 - 28$

$155 - 20$

$155 - 28$

$150 - 15$

$150 - 100$

$150 - 115$

$153 - 115$

$100 - 80$

$100 - 86$

$100 - 30$

$100 - 37$

$200 - 100$

$200 - 150$

$200 - 153$

$210 - 153$

$345 - 200$

$345 - 220$

$345 - 222$

$345 - 234$

$100 - 50$

$120 - 50$

$126 - 50$

$126 - 55$

$200 - 60$

$270 - 60$

$270 - 65$

$276 - 65$

$543 - 20$

$543 - 100$

$543 - 120$

$543 - 240$

Subtraction: Place Value and Negative Numbers

Instructions

Place Value and Negative Numbers is optional! Use this only if your students are ready for these ideas.

Many of us were told that in subtraction, "You can't take a bigger number from a smaller number, so you must go next door and borrow from your neighbor." Mathematically, this is an incorrect statement. You can subtract a larger number from a smaller number; you will be left with a negative amount. This idea, accompanied by an understanding of place value, is the core of the Place Value and Negative Number strategy.

Category 1: Place Value and Negative Numbers

The following number talks consist of computation problems that begin the discussion of what happens when you remove a larger number from a smaller quantity. The number line will be an important tool to use when reasoning with this strategy.

$$5 - 5$$

$$5 - 6$$

$$5 - 7$$

$$5 - 8$$

$$4 - 4$$

$$4 - 5$$

$$4 - 6$$

$$4 - 7$$

$$20 - 10$$

$$2 - 4$$

$$22 - 14$$

$$0 - 1$$

$$0 - 2$$

$$0 - 3$$

$$0 - 5$$

$$20 - 10$$

$$0 - 4$$

$$20 - 14$$

$$20 - 10$$

$$3 - 5$$

$$23 - 15$$

$$6 - 6$$

$$6 - 7$$

$$6 - 8$$

$$6 - 9$$

$$20 - 10$$

$$1 - 3$$

$$21 - 13$$

$$20 - 10$$

$$5 - 6$$

$$25 - 16$$

Category 2: Place Value and Negative Numbers

The following number talks consist of two-digit computation problems to continue the work with this strategy. A deliberate sequence is used to support thinking for the initial problem in each section. The last problem in each section allows students to test their thinking with a similar problem.

$$20 - 10$$

$$4 - 6$$

$$24 - 16$$

$$23 - 15$$

$$40 - 20$$

$$8 - 9$$

$$48 - 29$$

$$44 - 26$$

$$70 - 40$$

$$2 - 5$$

$$72 - 45$$

$$77 - 28$$

$$30 - 10$$

$$2 - 5$$

$$32 - 15$$

$$35 - 17$$

$$50 - 10$$

$$3 - 7$$

$$53 - 17$$

$$54 - 26$$

$$80 - 30$$

$$1 - 8$$

$$81 - 38$$

$$83 - 44$$

$$30 - 10$$

$$6 - 9$$

$$36 - 19$$

$$33 - 16$$

$$60 - 30$$

$$6 - 8$$

$$66 - 38$$

$$64 - 29$$

$$90 - 50$$

$$6 - 7$$

$$96 - 57$$

$$93 - 68$$

Category 3: Place Value and Negative Numbers

This is optional! You should decide if this is appropriate for your students.

The following number talks include three-digit computation problems to continue the work with this strategy. A deliberate sequence is used to support thinking for the initial problem in each section. The last problem in each section allows students to test their thinking with a similar problem.

$$0 - 6$$

$$100 - 40$$

$$100 - 46$$

$$100 - 19$$

$$300 - 100$$

$$50 - 80$$

$$7 - 8$$

$$357 - 188$$

$$321 - 233$$

$$60 - 300$$

$$50 - 60$$

$$0 - 5$$

$$650 - 365$$

$$612 - 248$$

$$100 - 0$$

$$20 - 80$$

$$5 - 7$$

$$125 - 87$$

$$114 - 75$$

$$400 - 200$$

$$40 - 50$$

$$4 - 6$$

$$444 - 256$$

$$413 - 135$$

$$800 - 600$$

$$50 - 90$$

$$3 - 4$$

$$853 - 694$$

$$826 - 437$$

$$200 - 100$$

$$10 - 20$$

$$5 - 6$$

$$215 - 126$$

$$223 - 134$$

$$500 - 300$$

$$0 - 50$$

$$0 - 8$$

$$500 - 358$$

$$500 - 263$$

$$1000 - 700$$

$$0 - 10$$

$$0 - 5$$

$$1000 - 715$$

$$1000 - 674$$

Subtraction: Adjusting One Number to Create an Easier Problem

Instructions

In students do not have a strong understanding of the part-whole relationship in subtraction; they will be limited in the strategies they can use. When either the minuend or subtrahend is adjusted to make a friendlier number, the strategy will warrant that the remainder or answer also be adjusted. For the problem $50 - 24$, some students changed the problem to $49 - 24$. Since the child changed the whole by removing 1, she has to add back one to the answer of 25 to get 26 (adjust the minuend): $(50 - 1) - 24 = 26 = 25 + 1 = (49 - 24)$.

For the same problem, other students might change the 24 to 25 to think about doubles or money. They have removed one too many and will need to add back one to the answer (adjust the subtrahend): $50 - (24 + 1) = 26 = (50 - 24) = 25 + 1$. These are the types of discussions that will need to occur when using this strategy.

Category 1: Adjusting One Number to Create an Easier Problem

The following number talks consist of smaller quantities- even basic facts- to help students consider what happens when numbers are adjusted in a subtraction problem. The following problems focus on adjusting the whole or the minuend

$$9 - 4$$

$$10 - 4$$

$$19 - 14$$

$$20 - 14$$

$$31 - 15$$

$$30 - 15$$

$$29 - 15$$

$$32 - 15$$

$$30 - 19$$

$$29 - 19$$

$$40 - 19$$

$$39 - 19$$

$$15 - 5$$

$$14 - 5$$

$$10 - 5$$

$$11 - 5$$

$$51 - 25$$

$$50 - 25$$

$$49 - 25$$

$$52 - 25$$

$$37 - 18$$

$$38 - 18$$

$$44 - 25$$

$$45 - 25$$

$$20 - 15$$

$$21 - 15$$

$$19 - 15$$

$$22 - 15$$

$$50 - 28$$

$$49 - 28$$

$$60 - 28$$

$$59 - 28$$

$$99 - 73$$

$$100 - 73$$

$$100 - 64$$

$$100 - 82$$

Category 2: Adjusting One Number to Create an Easier Problem

The following number talks include problems that focus on adjusting the subtrahend- the part being removed- to create an easier problem.

$$20 + 10$$

$$20 - 9$$

$$20 - 11$$

$$21 - 9$$

$$30 - 15$$

$$30 - 16$$

$$30 - 14$$

$$30 - 19$$

$$70 - 30$$

$$70 - 31$$

$$70 - 29$$

$$70 - 49$$

$$25 - 15$$

$$25 - 16$$

$$25 - 18$$

$$25 - 19$$

$$50 - 25$$

$$50 - 24$$

$$50 - 26$$

$$50 - 19$$

$$80 - 40$$

$$80 - 39$$

$$80 - 41$$

$$80 - 49$$

$$40 - 20$$

$$40 - 19$$

$$40 - 21$$

$$40 - 18$$

$$60 - 30$$

$$60 - 29$$

$$60 - 31$$

$$60 - 39$$

$$100 - 50$$

$$100 - 51$$

$$100 - 49$$

$$100 - 52$$

Category 3: Adjusting One Number to Create an Easier Problem

The following number talks require students to make decisions about which number might be adjusted to create an easier problem.

$$49 - 28$$

$$50 - 30$$

$$50 - 28$$

$$53 - 28$$

$$52 - 40$$

$$49 - 39$$

$$52 - 39$$

$$51 - 37$$

$$149 - 118$$

$$151 - 120$$

$$151 - 118$$

$$155 - 128$$

$$39 - 19$$

$$38 - 20$$

$$38 - 19$$

$$35 - 18$$

$$75 - 40$$

$$79 - 39$$

$$75 - 39$$

$$77 - 39$$

$$172 - 60$$

$$169 - 59$$

$$172 - 59$$

$$179 - 88$$

$$59 - 47$$

$$60 - 50$$

$$60 - 47$$

$$62 - 45$$

$$99 - 69$$

$$100 - 70$$

$$100 - 69$$

$$101 - 68$$

$$199 - 98$$

$$200 - 100$$

$$200 - 98$$

$$203 - 99$$

Subtraction: Keeping a Constant Difference

Instructions

With the Constant Difference strategy, both the minuend and the subtrahend are adjusted by the same amounts. This strategy can be efficient and beneficial because it allows the students to adjust the numbers to make a friendlier easier problem.

An example of the effectiveness of this strategy can be seen with the problem $51 - 26$. If both numbers are adjusted by subtracting 1, the problem is $50 - 25$, a common money problem. The answer of 25 is the same for either problem, because the numbers have both shifted the same amount.

Number talks in Categories 1 and 2 provide a progression of problems designed to help the students notice the relationships between the problems. For example, in Category 1, the problem $14 - 10$ is listed to help students notice $13 - 9$ could be solved by adjusting both numbers up by 1. Keep all problems in each sequence posted for the duration of the number talk. This will give your students the maximum opportunity to notice the relationships between the problems. These number talk problems may be used in two ways:

- Selected at random from each category; or
- navigated in a systematic order by selecting problems with smaller numbers from a specific category, then building to larger numbers.

Category 1: Keeping a Constant Difference

The following number talks consist of computation problems that use numbers up to one hundred and are focused on adjusting both numbers by adding or subtracting one or two.

$14 - 10$

$13 - 9$

$14 - 7$

$15 - 6$

$42 - 20$

$39 - 17$

$41 - 19$

$51 - 19$

$61 - 29$

$62 - 30$

$59 - 27$

$49 - 17$

$20 - 15$

$19 - 14$

$21 - 16$

$41 - 16$

$50 - 25$

$49 - 24$

$51 - 26$

$71 - 36$

$90 - 45$

$89 - 44$

$91 - 46$

$98 - 52$

$30 - 15$

$29 - 14$

$31 - 16$

$51 - 16$

$35 - 20$

$30 - 15$

$34 - 19$

$44 - 29$

$100 - 51$

$99 - 50$

$100 - 36$

$100 - 48$

Category 2: Keeping a Constant Difference

The following number talks include computation problems with numbers above one hundred.

$101 - 50$

$99 - 48$

$100 - 49$

$109 - 51$

$150 - 125$

$149 - 124$

$151 - 126$

$171 - 136$

$342 - 120$

$339 - 117$

$341 - 119$

$351 - 119$

$139 - 60$

$138 - 59$

$114 - 90$

$112 - 88$

$153 - 100$

$151 - 98$

$173 - 160$

$171 - 158$

$199 - 90$

$200 - 91$

$299 - 150$

$300 - 151$

$135 - 120$

$130 - 115$

$134 - 119$

$164 - 119$

$261 - 129$

$262 - 130$

$259 - 127$

$249 - 117$

$498 - 310$

$500 - 312$

$499 - 366$

$500 - 367$

Category 3: Keeping a Constant Difference

The following number talks consist of computation problems that do not build one upon the others. Instead, each problem offers opportunities for students to choose the best method for keeping a constant difference. Many of the problems can be adjusted up or down to create easier problems.

$$32 - 19$$

$$48 - 29$$

$$35 - 18$$

$$41 - 13$$

$$111 - 56$$

$$134 - 68$$

$$127 - 88$$

$$122 - 77$$

$$234 - 119$$

$$271 - 158$$

$$251 - 158$$

$$209 - 151$$

$$35 - 17$$

$$53 - 29$$

$$62 - 37$$

$$44 - 26$$

$$133 - 95$$

$$114 - 89$$

$$123 - 105$$

$$100 - 34$$

$$391 - 146$$

$$359 - 127$$

$$251 - 116$$

$$315 - 106$$

$$86 - 47$$

$$90 - 36$$

$$78 - 59$$

$$52 - 35$$

$$236 - 119$$

$$200 - 137$$

$$287 - 118$$

$$151 - 98$$

$$300 - 214$$

$$500 - 289$$

$$700 - 477$$

$$1000 - 674$$

Multiplication Number Talks

The following number talks are crafted to elicit specific multiplication strategies; however, you may find that students also share other efficient methods. Keep in mind that the overall purpose is to help students build a toolbox of efficient strategies based on numerical reasoning. The ultimate goal of number talks is for students to compute accurately, efficiently, and flexibly.

Multiplication: Repeated Addition or Skip Counting

Repeated Addition is an entry-level strategy for multiplication and will naturally occur when students are first presented with multiplication problems. Since we want to encourage students to move towards multiplicative thinking and away from an additive approach to multiplication, we do not present specific number talks to foster this strategy. If students share this method as their strategy, honor their thinking; however, always make connections to multiplication. Possible ways to make this explicit are shared using the problem 4×9 .

If students share their strategy as $9 + 9 + 9 + 9 = 18 + 18 = 36$

Scaffold to multiplication with $(2 \times 9) + (2 \times 9) = 18 + 18 = 36$

If students share their thinking strategy as $(9 + 1) + (9 + 1) + (9 + 1) + (9 + 1) = 40$

$10 + 10 + 10 + 10 = 40$, $40 - 1 - 1 - 1 - 1 = 40 - 4 = 36$

Scaffold to multiplication with $(4 \times 10) - 4 = 36$

If students share their strategy as $4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 = 36$

Scaffold to multiplication by looking at clusters of multiplication problems embedded in the problem, such as $(5 \times 4) + (2 \times 4) + (2 \times 4) = 20 + 8 + 8 = 36$

Repeated Addition also affords an excellent vehicle for discussing efficiency: Is it more efficient to add four 9's or nine 4's? Is there a ways we can build on something we know, such as 5×4 , to make the problem more efficient? Which is more efficient, to add four 9's or four 10's? Each situation offers opportunities to help students think flexibly, fluently, and efficiently.

Multiplication: Making Landmark or Friendly Numbers

Instructions

A common error students make when changing one of the factors to a landmark number is to forget to adjust the number of groups. The problem 9×25 can help us consider the common errors children make when making this adjustment. If 9 had been changed to 10, then the product of 250 would need to be adjusted not just by 1 but by one groups of 25.

This common error arises when children are applying what works with addition to multiplication. They do not consider that they have changed the problem by adding on one of 25 instead of 1.

$$(25 \times (9 + 1)) = (25 \times 10) = 250$$
$$250 - 1 = 249$$

When students understand that one group of 25 has been added, they will adjust the answer accordingly.

$$(25 \times (9 + 1)) = (25 \times 10) = 250$$
$$250 - 25 = 225$$

The following number talks consist of three or more sequential problems. The sequence of problems within a given number talk allows students to apply strategies from previous problems to subsequent problems. These number talk problems may be used in two ways:

- Selected at random from each category; or
- navigated in a systematic order by selecting problems with smaller numbers from a specific category, then building to larger numbers.

Category 1: Making Landmark or Friendly Numbers

The following number talks consist of 1 x 2-digit problems and have a connection to U.S. coin values. The problems in each section are purposefully ordered to help students build their knowledge from one problem to the next. This allows them to use the relationships from the initial problem in the final problem in the sequence. For example, 6×25 could be solved by using $(2 \times 25) + (4 \times 25)$, or by using 4×25 twice and then removing 2×25 from that product

2×25

4×25

6×25

7×5

7×10

7×9

2×25

4×20

2×50

4×50

2×50

4×50

8×50

5×5

5×10

5×20

5×19

5×5

5×10

5×30

5×29

3×5

3×10

3×9

2×5

2×10

2×20

2×19

4×5

4×10

4×50

4×49

Category 2: Making Landmark or Friendly Numbers

The following number talks are intentionally ordered to help students use relationships from the sequence to solve the final 1 x 3-digit problem.

$$4 \times 25$$

$$4 \times 200$$

$$4 \times 250$$

$$4 \times 249$$

$$3 \times 10$$

$$3 \times 50$$

$$3 \times 100$$

$$3 \times 149$$

$$3 \times 100$$

$$3 \times 200$$

$$3 \times 199$$

$$6 \times 20$$

$$6 \times 100$$

$$6 \times 120$$

$$6 \times 119$$

$$6 \times 50$$

$$6 \times 300$$

$$6 \times 349$$

$$5 \times 100$$

$$5 \times 300$$

$$5 \times 60$$

$$5 \times 539$$

$$3 \times 50$$

$$3 \times 100$$

$$3 \times 149$$

$$4 \times 60$$

$$5 \times 300$$

$$4 \times 359$$

$$8 \times 50$$

$$8 \times 100$$

$$8 \times 200$$

$$8 \times 199$$

Category 3: Making Landmark or Friendly Numbers

The following number talks consist of computation problems that are ordered to help students use relationships from the sequence to solve the final 2 x 2-digit problems.

$$6 \times 20$$

$$30 \times 20$$

$$36 \times 20$$

$$36 \times 19$$

$$6 \times 40$$

$$10 \times 40$$

$$16 \times 40$$

$$16 \times 39$$

$$2 \times 150$$

$$10 \times 150$$

$$12 \times 150$$

$$12 \times 149$$

$$3 \times 50$$

$$50 \times 50$$

$$53 \times 50$$

$$53 \times 48$$

$$2 \times 25$$

$$4 \times 25$$

$$8 \times 25$$

$$10 \times 25$$

$$16 \times 25$$

$$5 \times 200$$

$$20 \times 200$$

$$25 \times 200$$

$$25 \times 199$$

$$10 \times 10$$

$$10 \times 30$$

$$2 \times 30$$

$$12 \times 29$$

$$5 \times 10$$

$$5 \times 50$$

$$10 \times 50$$

$$15 \times 50$$

$$15 \times 49$$

$$6 \times 600$$

$$10 \times 600$$

$$16 \times 600$$

$$16 \times 599$$

Multiplication: Partial Products

Instructions

The partial Products strategy can be used with any multiplication problem. This strategy is based on breaking one or both factors into addends through using expanded notation and the distributive property. While both factors can be represented with expanded notation, keeping one number whole is often more efficient. Several ways to solve the problem 8×25 using Partial Products follow:

Breaking 8 into Addends

$$(4 + 4) \times 25 = (4 \times 25) + (4 \times 25)$$

$$(2 + 2 + 4) \times 25 = (2 \times 25) + (2 \times 25) + (4 \times 25)$$

Breaking 25 into Addends

$$8 \times (20 + 5) = (8 \times 20) + (8 \times 5)$$

$$8 \times (10 + 10 + 5) = (8 \times 10) + (8 \times 10) + (8 \times 5)$$

Breaking Both Factors into Addends

$$(4 + 4) \times (20 + 5) = (4 \times 20) + (4 \times 5) + (4 \times 20) + (4 \times 5)$$

The following number talks consist of three or more sequential problems. The sequence of problems within a given number talk allows students to apply strategies from previous problems to subsequent problems. These number talk problems may be used in two ways:

- Selected at random from each category; or
- navigated in a systematic order by selecting problems with smaller numbers from a specific category, then building to larger numbers.

Category 1: Partial Products

The following number talks are ordered by section to help students use relationships from the sequence to solve the final 1-digit by 1-digit and 1-digit by 2-digit problems.

2×7

4×7

4×8

3×8

8×7

2×15

3×15

6×5

6×10

6×15

2×16

8×5

8×10

8×6

8×16

3×8

2×6

6×8

3×26

6×26

9×26

2×36

4×10

4×6

4×36

2×7

4×7

3×7

7×7

2×45

4×45

2×40

2×5

8×45

8×5

8×2

8×50

8×56

Category 2: Partial Products

The following number talks are ordered so that students can use the relationships from the sequence to solve these 3-digit problems.

$$2 \times 125$$

$$4 \times 25$$

$$6 \times 100$$

$$6 \times 20$$

$$6 \times 125$$

$$2 \times 124$$

$$6 \times 100$$

$$6 \times 20$$

$$6 \times 4$$

$$6 \times 124$$

$$2 \times 45$$

$$5 \times 100$$

$$5 \times 40$$

$$5 \times 5$$

$$5 \times 245$$

$$2 \times 100$$

$$2 \times 15$$

$$4 \times 5$$

$$4 \times 10$$

$$4 \times 115$$

$$2 \times 150$$

$$5 \times 100$$

$$5 \times 10$$

$$5 \times 50$$

$$5 \times 150$$

$$2 \times 500$$

$$4 \times 500$$

$$4 \times 30$$

$$4 \times 2$$

$$4 \times 532$$

$$8 \times 100$$

$$8 \times 10$$

$$8 \times 2$$

$$4 \times 100$$

$$4 \times 12$$

$$8 \times 112$$

$$4 \times 250$$

$$4 \times 6$$

$$8 \times 200$$

$$8 \times 50$$

$$8 \times 6$$

$$8 \times 256$$

$$2 \times 325$$

$$2 \times 300$$

$$6 \times 300$$

$$4 \times 25$$

$$6 \times 25$$

$$6 \times 325$$

Category 3: Partial Products

The following number talks consist of multiplication problems designed to help students use the relationships from the sequence to solve the final 2-digit by 2-digit problems.

3×15

10×15

13×10

13×5

13×15

2×16

10×16

10×14

2×14

14×16

5×30

10×30

3×15

10×15

15×33

15×10

15×1

10×11

5×11

15×11

25×10

25×4

14×10

14×5

25×14

35×10

35×2

35×20

35×24

4×22

6×11

3×22

6×22

10×22

16×22

4×25

5×25

10×25

20×25

25×25

10×36

50×36

2×36

10×54

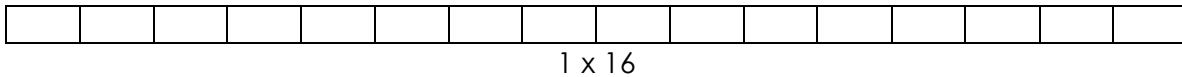
2×54

54×36

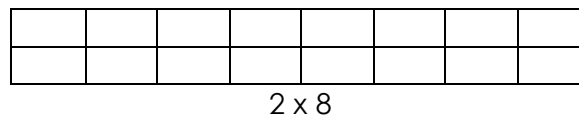
Multiplication: Doubling and Halving

Instructions

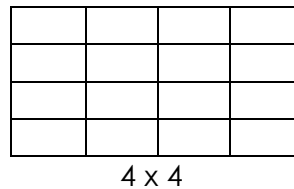
Halving and doubling is an excellent strategy to restructure a problem with multiple digits and make it easier to solve. Helping students notice the relationship between the two factors and the dimensions of the accompanying array is important to understanding this strategy. An equally important idea in this strategy is that the factors can adjust while the area of the array stays the same. Take, for instance, the problem 1×16 . It can be represented with the following arrays.



But if we cut the length of the array in half and attach the second half below the first half, we have created a different array with the same area.



We can repeat this process and half the length and double the width and still keep the same area.



As students begin their initial investigations of this strategy, choose smaller numbers that have a number of factors and have children build the arrays, list the accompanying multiplication sentences for each, and look for patterns that occur- just like we did for 16. As students become more familiar with this strategy and when and how it works, they will look for opportunities to apply it. The following number talks consist of three or more sequential problems. The sequence of problems within a given number talk allows students to apply strategies from previous problems to subsequent problems. These number talk problems may be used in two ways:

- Selected at random from each category; or
- navigated in a systematic order by selecting problems with smaller numbers from a specific category, then building to larger numbers.

Category 1: Doubling and Halving

The following number talks investigate doubling and halving with basic facts.

1×16

2×8

4×4

8×2

16×1

1×24

2×24

4×6

8×3

1×12

2×6

4×3

1×36

2×18

4×9

1×48

2×24

4×12

8×6

16×3

1×32

2×16

4×8

8×4

16×2

32×1

1×20

2×10

4×5

1×40

2×20

4×10

8×5

1×56

2×28

4×14

8×7

Category 2: Doubling and Halving

The following number talks investigate doubling and halving with 1-digit by 3-digit numbers.

$$8 \times 16$$

$$4 \times 32$$

$$2 \times 64$$

$$8 \times 125$$

$$4 \times 250$$

$$2 \times 500$$

$$125 \times 8$$

$$250 \times 4$$

$$500 \times 2$$

$$8 \times 32$$

$$4 \times 64$$

$$2 \times 128$$

$$84 \times 5$$

$$42 \times 10$$

$$21 \times 20$$

$$345 \times 8$$

$$690 \times 4$$

$$1380 \times 2$$

$$36 \times 5$$

$$18 \times 10$$

$$9 \times 20$$

$$35 \times 8$$

$$70 \times 4$$

$$140 \times 2$$

$$8 \times 29$$

$$4 \times 58$$

$$2 \times 116$$

Category 3: Doubling and Halving

The following number talks investigate doubling and halving with 2-digit by 2-digit numbers.

3×60

6×30

12×15

104×3

52×6

26×12

112×2

56×4

28×8

14×16

9×56

18×28

36×14

4×120

8×60

16×30

32×15

360×3

180×6

90×12

45×24

2×280

4×140

8×70

16×35

100×4

50×8

25×16

2×1440

4×720

8×360

16×180

32×90

64×45

Category 4: Doubling and Halving

The following number talks are included for classes who may wish to investigate what happens with you third and triple or quarter and quadruple numbers. The problems also provide an opportunity to investigate whether halving and doubling will work with odd numbers. While these strategies are not common for children, they afford an opportunity to investigate why this principle works.

$$9 \times 12$$

$$3 \times 36$$

$$1 \times 108$$

$$15 \times 16$$

$$60 \times 4$$

$$240 \times 1$$

$$18 \times 12$$

$$9 \times 24$$

$$4.5 \times 36$$

$$2.25 \times 72$$

$$27 \times 15$$

$$9 \times 45$$

$$3 \times 135$$

$$1 \times 405$$

$$25 \times 48$$

$$100 \times 12$$

$$400 \times 3$$

$$1200 \times 1$$

$$6 \times 12$$

$$3 \times 24$$

$$1.5 \times 48$$

$$36 \times 18$$

$$72 \times 9$$

$$216 \times 3$$

$$648 \times 1$$

$$64 \times 35$$

$$16 \times 140$$

$$4 \times 560$$

$$1 \times 2240$$

$$6 \times 24$$

$$3 \times 48$$

$$1.5 \times 96$$

Multiplication: Breaking Factors into Smaller Factors

Instructions

Because of the focus on place value, students have many experiences breaking numbers into expanded form that lead to the Partial Products strategy. However, they tend to have limited experiences breaking factors into smaller factors and applying the associative property. Providing number talk opportunities for students to grapple with equivalence with problems such as

$$8 \times 25 = 2 \times 4 \times 25 \text{ or}$$

$$8 \times 25 = 8 \times 5 \times 5 \text{ or}$$

$$8 \times 25 = 2 \times 4 \times 5 \times 5$$

These problems are critical to building understanding of the associative property and its real-life applications with multiplication

The following number talks consist of three or more sequential problems. The sequence of problems within a given number talk allows students to apply strategies from previous problems to subsequent problems. These number talk problems may be used in two ways:

- Selected at random from each category; or
- navigated in a systematic order by selecting problems with smaller numbers from a specific category, then building to larger numbers.

Category 1: Breaking Factors into Smaller Factors

The following number talks consist of problems that focus on breaking basic facts into smaller factors.

$$2 \times 3 \times 4$$

$$4 \times 3 \times 2$$

$$6 \times 4$$

$$2 \times 3 \times 8$$

$$4 \times 2 \times 6$$

$$6 \times 8$$

$$4 \times 3 \times 4$$

$$2 \times 2 \times 12$$

$$8 \times 3 \times 2$$

$$2 \times 2 \times 3 \times 4$$

$$12 \times 4$$

$$3 \times 2 \times 6$$

$$3 \times 3 \times 3 \times 2$$

$$2 \times 2 \times 9$$

$$6 \times 6$$

$$2 \times 3 \times 2 \times 3$$

$$3 \times 3 \times 4$$

$$9 \times 2 \times 2$$

$$9 \times 4$$

$$5 \times 2 \times 6$$

$$5 \times 4 \times 3$$

$$2 \times 2 \times 3 \times 5$$

$$5 \times 12$$

$$2 \times 3 \times 3 \times 3$$

$$3 \times 6 \times 3$$

$$9 \times 3 \times 2$$

$$6 \times 9$$

$$5 \times 2 \times 4$$

$$4 \times 5 \times 2$$

$$2 \times 2 \times 5 \times 2$$

$$8 \times 5$$

$$4 \times 4 \times 2$$

$$2 \times 8 \times 2$$

$$2 \times 2 \times 2 \times 2 \times 2$$

$$4 \times 8$$

Category 2: Breaking Factors into Smaller Factors

The following number talks use the associative property to solve 1-digit by 2-digit multiplication problems.

$$3 \times 5 \times 4$$

$$2 \times 2 \times 15$$

$$15 \times 4$$

$$5 \times 5 \times 8$$

$$2 \times 4 \times 25$$

$$2 \times 25 \times 4$$

$$25 \times 8$$

$$2 \times 4 \times 35$$

$$8 \times 5 \times 7$$

$$8 \times 35$$

$$2 \times 10 \times 5$$

$$4 \times 5 \times 5$$

$$20 \times 5$$

$$8 \times 9 \times 3$$

$$3 \times 24 \times 3$$

$$9 \times 12 \times 2$$

$$24 \times 9$$

$$8 \times 10 \times 5$$

$$50 \times 2 \times 4$$

$$25 \times 4 \times 4$$

$$50 \times 8$$

$$4 \times 2 \times 16$$

$$4 \times 4 \times 8$$

$$2 \times 4 \times 8 \times 2$$

$$8 \times 8 \times 2$$

$$16 \times 8$$

$$32 \times 4 \times 2$$

$$16 \times 4 \times 4$$

$$32 \times 8$$

$$9 \times 10 \times 3$$

$$5 \times 6 \times 3 \times 3$$

$$15 \times 2 \times 9$$

$$30 \times 9$$

Category 3: Breaking Factors into Smaller Factors

*The following number talks use the associative property to solve 2 * 2-digit multiplication problems.*

$$3 \times 4 \times 25$$

$$5 \times 12 \times 5$$

$$5 \times 2 \times 25$$

$$12 \times 25$$

$$2 \times 45 \times 8$$

$$5 \times 16 \times 9$$

$$4 \times 5 \times 4 \times 9$$

$$16 \times 45$$

$$2 \times 15 \times 4 \times 3 \times 3$$

$$8 \times 5 \times 9 \times 3 \times 9$$

$$24 \times 5 \times 9$$

$$72 \times 15$$

$$2 \times 15 \times 6$$

$$5 \times 12 \times 3$$

$$4 \times 5 \times 3 \times 3$$

$$12 \times 15$$

$$6 \times 6 \times 3 \times 5$$

$$6 \times 5 \times 3 \times 6$$

$$4 \times 15 \times 9$$

$$36 \times 15$$

$$6 \times 5 \times 7 \times 3$$

$$9 \times 5 \times 2 \times 7$$

$$7 \times 5 \times 2 \times 9$$

$$18 \times 35$$

$$4 \times 4 \times 25$$

$$8 \times 2 \times 25$$

$$16 \times 5 \times 5$$

$$16 \times 25$$

$$2 \times 12 \times 3 \times 5$$

$$4 \times 15 \times 6$$

$$24 \times 5 \times 3$$

$$24 \times 15$$

$$2 \times 35 \times 6$$

$$12 \times 5 \times 7$$

$$3 \times 4 \times 5 \times 7$$

$$12 \times 35$$

Division Number Talks

The following number talks are crafted to elicit specific division strategies; however, you may find that students also share other efficient methods. Keep in mind that the overall purpose is to help students build a toolbox of efficient strategies based on numerical reasoning. The ultimate goal of number talks is for students to compute accurately, efficiently, and flexibly.

Division: Repeated Subtraction or Sharing/Dealing Out

Instructions

Repeated Subtraction is an entry-level strategy for division and will naturally occur when students are presented with initial division problems. Since we want to encourage students to move toward multiplicative thinking and away from a removal approach to division, specific number talks are not presented to foster this strategy. If students share this method as their strategy, honor their thinking; however, always make connections to multiplication. Possible ways to make this explicit are suggested in the example that follows using the problem $12 \div 2$.

If students share their strategy as $12 - 2, -2, -2, -2, -2, -2$.

Scaffold to multiplication with $3 \times 2 = 6, 3 \times 2 = 6$

So... $6 \times 2 = 12$

So... $12 \div 2 = 6$ or $\frac{12}{2} = 6$

Repeated Subtraction also affords an excellent vehicle for discussing efficiency: Is it more efficient to subtract 2's or to multiply groups of 2? Is there a way we can build on something we know, such as 3×2 , to make this problem more efficient? Each situation offers opportunities to help students think flexibly, fluently, and efficiently.

Division: Partial Quotients

Instructions

The Partial Quotients strategy maintains the integrity of place value and allows the students to approach the problem by building on multiplication problems with friendly multipliers such as 2, 5, 10, powers of 10, and so on. This strategy allows the student to navigate through the problem by building on what they know, understand, and can implement with ease.

As we look at the problem $550 \div 15$ we can see how students could approach this problem using the Partial Quotients strategy. While we might say that the third example is more efficient than the other two, it is important to note that regardless of how the student scaffolds her thinking, access and opportunities are there to build on individual understanding. Whether the student multiplies 2×15 over and over or uses higher multiples of ten efficiently, she can reach a correct solution. For this reason, Partial Quotient strategy will work with any division problem.

When comparing this strategy to the standard U.S. Long Division Algorithm, note that the place value of the numbers remains intact. When using this algorithm, students are taught to test if the divisor will divide into the first digit of the dividend. For the problem $550 \div 15$, students would be asked to consider whether 15 could “go into” 5, and they would be prompted to respond that it could not. When the 500 is treated as five 1’s, teachers are in essence asked students to ignore place value. This is mathematically incorrect information since groups of 15 can be found in 500.

The following number talks consist of three or more sequential problems. The sequence of problems within a given number talk allows students to apply strategies from previous problems to subsequent problems. These number talk problems may be used in two ways:

- Selected at random from each category; or
- navigated in a systematic order by selecting problems with smaller numbers from a specific category, then building to larger numbers.

Category 1: Partial Quotients

The following number talks consist of computation problems that help students to build on multiples of ten find easy multiples of the divisor within the dividend. The following problems focus on double-digit numbers with a single digit divisor.

$$40 \div 4$$

$$16 \div 4$$

$$56 \div 4$$

$$40 \div 4$$

$$24 \div 4$$

$$67 \div 4$$

$$30 \div 3$$

$$18 \div 3$$

$$48 \div 3$$

$$30 \div 3$$

$$24 \div 3$$

$$54 \div 3$$

$$40 \div 4$$

$$80 \div 4$$

$$4 \div 4$$

$$88 \div 4$$

$$30 \div 3$$

$$90 \div 3$$

$$92 \div 3$$

$$50 \div 5$$

$$30 \div 5$$

$$80 \div 5$$

$$40 \div 4$$

$$80 \div 4$$

$$16 \div 4$$

$$96 \div 4$$

$$5 \div 5$$

$$10 \div 5$$

$$25 \div 5$$

$$50 \div 5$$

$$77 \div 5$$

Category 2: Partial Quotients

The following number talks include problems that encourage students to build on multiples of ten and one hundred and find easy multiples of the divisor within the dividend. The problems that follow focus on three-digit numbers with a single-digit divisor.

$$300 \div 3$$

$$120 \div 3$$

$$420 \div 3$$

$$400 \div 4$$

$$80 \div 4$$

$$16 \div 4$$

$$496 \div 4$$

$$120 \div 6$$

$$18 \div 6$$

$$60 \div 6$$

$$300 \div 6$$

$$180 \div 6$$

$$500 \div 6$$

$$30 \div 6$$

$$18 \div 6$$

$$300 \div 6$$

$$347 \div 6$$

$$100 \div 4$$

$$40 \div 4$$

$$24 \div 4$$

$$4 \div 4$$

$$124 \div 4$$

$$100 \div 5$$

$$200 \div 5$$

$$30 \div 5$$

$$5 \div 5$$

$$235 \div 5$$

$$100 \div 4$$

$$200 \div 4$$

$$40 \div 4$$

$$16 \div 4$$

$$256 \div 4$$

$$160 \div 8$$

$$16 \div 8$$

$$400 \div 8$$

$$80 \div 8$$

$$496 \div 8$$

$$900 \div 3$$

$$300 \div 3$$

$$240 \div 3$$

$$12 \div 3$$

$$852 \div 3$$

Category 3: Partial Quotients

The following number talks include computation problems that help students build on multiples of ten and one hundred to find easy multiples of the divisor within the dividend. These problems focus on three-digit numbers with a two-digit divisor.

$$100 \div 25$$

$$250 \div 25$$

$$500 \div 25$$

$$120 \div 12$$

$$240 \div 12$$

$$368 \div 12$$

$$70 \div 35$$

$$105 \div 35$$

$$350 \div 35$$

$$525 \div 35$$

$$150 \div 15$$

$$300 \div 15$$

$$600 \div 15$$

$$130 \div 13$$

$$26 \div 13$$

$$52 \div 13$$

$$195 \div 13$$

$$100 \div 25$$

$$200 \div 25$$

$$500 \div 25$$

$$75 \div 25$$

$$675 \div 25$$

$$120 \div 12$$

$$240 \div 12$$

$$360 \div 12$$

$$36 \div 12$$

$$396 \div 12$$

$$100 \div 20$$

$$200 \div 20$$

$$400 \div 20$$

$$500 \div 20$$

$$30 \div 15$$

$$90 \div 15$$

$$300 \div 15$$

$$150 \div 15$$

$$540 \div 15$$

Division: Multiplying Up

Instructions

Following the same principle as Adding Up to Subtraction, Multiplying Up is an accessible division strategy that capitalizes on the relationship between multiplication and division. Similar to Partial Quotients, this strategy provides an opportunity for students to gradually build on multiplication problems they know until they reach the dividend. A subtle distinction between the two strategies can be seen with problem $550 \div 15$.

Notice how the student is building up to the dividend through multiplication in each example. While the third example is much more efficient, the other examples allow students to build on their understanding of the relationship between multiplication and division and scaffold this understanding to reach an accurate solution.

The following number talks consist of three or more sequential problems. The sequence of problems within a given number talk allows students to apply strategies from previous problems to subsequent problems. These number talk problems may be used in two ways:

- Selected at random from each category; or
- navigated in a systematic order by selecting problems with smaller numbers from a specific category, then building to larger numbers.

Category 1: Multiplying Up

The following number talks consist of computation problems that build on using multiples of ten with two-digit numbers with single digit divisors.

$$4 \times 10$$

$$4 \times 5$$

$$4 \times 4$$

$$56 \div 4$$

$$5 \times 5$$

$$5 \times 10$$

$$5 \times 2$$

$$79 \div 5$$

$$3 \times 10$$

$$3 \times 20$$

$$3 \times 3$$

$$3 \times 2$$

$$68 \div 3$$

$$3 \times 10$$

$$3 \times 20$$

$$3 \times 30$$

$$3 \times 1$$

$$96 \div 3$$

$$2 \times 10$$

$$2 \times 5$$

$$2 \times 2$$

$$38 \div 2$$

$$5 \times 5$$

$$5 \times 10$$

$$5 \times 2$$

$$85 \div 5$$

$$4 \times 10$$

$$4 \times 5$$

$$4 \times 2$$

$$48 \div 4$$

$$4 \times 10$$

$$4 \times 5$$

$$4 \times 8$$

$$4 \times 4$$

$$72 \div 4$$

$$6 \times 10$$

$$6 \times 5$$

$$6 \times 6$$

$$6 \times 2$$

$$99 \div 6$$

Category 2: Multiplying Up

The following number talks include three-digit numbers with single-digit divisors that encourage student's build on multiples of ten and one hundred.

$$3 \times 100$$

$$3 \times 50$$

$$3 \times 1$$

$$453 \div 3$$

$$4 \times 25$$

$$4 \times 100$$

$$4 \times 20$$

$$999 \div 4$$

$$4 \times 25$$

$$4 \times 50$$

$$4 \times 100$$

$$500 \div 4$$

$$4 \times 25$$

$$4 \times 50$$

$$4 \times 3$$

$$215 \div 4$$

$$3 \times 100$$

$$3 \times 20$$

$$3 \times 30$$

$$960 \div 3$$

$$8 \times 100$$

$$8 \times 50$$

$$8 \times 10$$

$$792 \div 8$$

$$6 \times 100$$

$$6 \times 50$$

$$6 \times 60$$

$$6 \times 5$$

$$536 \div 6$$

$$4 \times 25$$

$$4 \times 100$$

$$4 \times 10$$

$$4 \times 20$$

$$484 \div 4$$

$$7 \times 100$$

$$7 \times 10$$

$$7 \times 5$$

$$7 \times 2$$

$$836 \div 7$$

Category 3: Multiplying Up

The following number talks consist of three-digit numbers with two-digit divisors that build on using multiples of ten and one hundred.

$$50 \times 2$$

$$50 \times 5$$

$$50 \times 10$$

$$900 \div 50$$

$$35 \times 2$$

$$35 \times 10$$

$$35 \times 20$$

$$755 \div 35$$

$$25 \times 10$$

$$25 \times 4$$

$$25 \times 2$$

$$840 \div 25$$

$$15 \times 10$$

$$15 \times 20$$

$$5 \times 40$$

$$15 \times 2$$

$$658 \div 15$$

$$24 \times 5$$

$$24 \times 10$$

$$24 \times 20$$

$$756 \div 24$$

$$10 \times 15$$

$$20 \times 15$$

$$2 \times 15$$

$$1 \times 15$$

$$498 \div 15$$

$$10 \times 17$$

$$20 \times 17$$

$$40 \times 17$$

$$2 \times 17$$

$$699 \div 17$$

$$5 \times 21$$

$$10 \times 21$$

$$2 \times 21$$

$$321 \div 21$$

$$10 \times 27$$

$$20 \times 27$$

$$30 \times 27$$

$$825 \div 17$$

Division: Proportional Reasoning

The following number talks consist of division problems that can be solved using proportional reasoning.

$$100 \div 4$$

$$200 \div 8$$

$$400 \div 16$$

$$720 \div 36$$

$$360 \div 18$$

$$60 \div 3$$

$$800 \div 40$$

$$80 \div 4$$

$$40 \div 2$$

$$100 \div 4$$

$$200 \div 8$$

$$400 \div 16$$

$$250 \div 2$$

$$500 \div 4$$

$$1000 \div 8$$

$$384 \div 16$$

$$96 \div 4$$

$$48 \div 2$$

$$172 \div 3$$

$$144 \div 6$$

$$288 \div 12$$

$$46 \div 2$$

$$92 \div 4$$

$$184 \div 8$$

$$308 \div 7$$

$$308 \div 14$$

$$308 \div 28$$