

**Grade 2**

**Eureka  
Essentials**

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# Overview

## How to Read Eureka Essentials

### Module 2: Addition and Subtraction of Length Units

#### Summary:

- Measure (using different tools & units) and compare lengths (exactly & approximately/visually)
  - Key lesson: Lesson 6
- Apply understanding to solve word problems involving adding or subtracting lengths
  - Building on addition and subtraction from [Module 1](#)

#### Definitions:

- **To measure length** with (or using) a specified **length unit**: to count the number of length units that fit
  - (1) side by side (no overlap or extra space between) and
  - (2) from one endpoint to the other endpoint of the object
- **To estimate (approximate) length**: to find the number of units that is closest to the actual length of the object

Overview of module

Basic materials of module

(see Teacher Edition for

#### Materials:

- Centimeter cubes [Topic A]
- Rulers [Topics A-C]

### Module, Lesson,

#### M7 L3 Bar Graph

**Cut/consolidate:** Can consolidate with Lesson 4 (*Eureka Math's Notes on Pacing*)

#### Goals:

- **Build on Lesson 2** by representing categorical data as (vertical and horizontal) bar graphs
- Observe that the category count equals the length (or height) of the bar measured by the count axis (number line)

**Check:** Problem Set problem #2a

**Materials:** Lesson 3 Template 2 (p. 62 of Teacher Edition)

#### M4 L16 Word Problems (Continued)

#### Goals:

- **Echo** Lesson 5 with “place value” addition and subtraction strategies:
  - Solve one- and two-step word problems using tape diagrams and addition and subtraction strategies from [Lessons 7-8 and 12-13](#)

**Focus:** Concept Development problems #1, #2, #4

**Check:** Problem Set problem #3, #5

#### M4 L23 Subtracting Multiples of 10

#### Goals:

- **Build on Lesson 2** to subtract multiples of 10 from three-digit numbers

◦ Example:

$$127 - 70$$

$$= 100 + 27 - 70$$

$$= (100 - 70) + 27$$

$$= 30 + 27$$

$$= 57$$

**Key Ideas:** Subtract from any part of the minuend

#### M7 L5 Word Problems

#### Goals:

- **Review/reinforce** Lesson 3

**Check:** Problem Set problems #2a-b

Suggestions for omission or consolidation of lessons if pacing is a challenge

Additional materials to consider

Like repeating the identified lesson (Lesson 5) for the new concept (“place value” strategies)

Connections to other lessons in **purple** text



Review or reinforcement of identified lessons or

← Problem(s) suggested for checking understanding

← Problem(s) to focus on from the Concept Development

← Key idea(s) to highlight in lesson

## How to Approach Pacing

- Cover Major Cluster Standards more thoroughly and move through other Standards more quickly
  - **Major Clusters:** [Modules 1, 3-5](#)
    - *If student learning of Module 4 is solid, move through Module 5 more quickly because it applies the same ideas to larger numbers*
  - **Additional/Supporting Clusters:** [Modules 2, 6-8](#)
    - *If pacing is a challenge, save Module 6 to the end of the year because Grade 3 starts with multiplication*
- Use **lesson connections** (in **purple** text) to foresee when and how certain concepts will be revisited and further developed in later lessons
- Embed “reteaching” into the next lesson’s activities (Fluency Practice, Application Problems, Concept Development, etc.) rather than repeat a prior lesson when formative assessments indicate lack of student understanding
- Omit or differentiate lessons labelled  **Review/reinforce** based on students’ strengths and needs
- Follow suggestions for omission or consolidation of lessons labelled  **Cut/consolidate** based on students’ strengths and needs

## References

- Great Minds Eureka Math Teacher Edition version 3.0 (2015)
- [California Common Core State Standards](#) (2013)
- [Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve](#) (2016)

# Module 1: Sums and Differences to 100 ([Grade 1](#) Reviewed & Continued)

## Summary:

- Decomposition of two-digit numbers [Lessons 1-2]
- Addition of two-digit numbers [Lessons 3-5]
- Subtraction of two-digit numbers [Lessons 3, 6-8]

## Definitions:

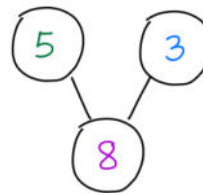
- **Addition** ([Grade 1](#) Module 1):

(# of things start with) + (# of things add on)

# of things all together

- Word sentences & equations
  - “8 = 5 + 3” means “8 is the same number as 5 and 3 all together”
- Number bonds & equations
  - Part + Part = Whole

$$5 + 3 = 8$$



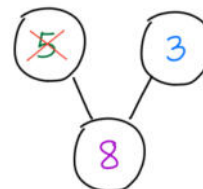
- **Subtraction** ([Grade 1](#) Module 1):

(# of things start with) - (# of things take away)

# of things leftover


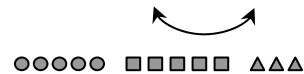

- Word sentences & equations
  - “8 - 5 = 3” means “the leftover of 8 take away 5 is the same number as 3”
- Number bonds & equations
  - Whole - Part = Part

$$8 - 5 = 3$$



## Key Ideas:

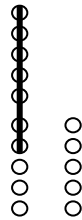
- **Addition** (Grade 1 Modules 2, 4, & 6): Because of the associative and commutative properties of addition, we can add numbers in *any order* and still keep the sum (total) the same
  - Example:

	$5 + 3 + 5$
	$= 5 + 5 + 3 \leftarrow$ <a href="#">commutative property of addition</a>
	$= (5 + 5) + 3 \leftarrow$ <a href="#">associative property of addition</a> $= 10 + 3$ $= 13$

- **Subtraction** (Grade 1 Module 2): We can subtract parts of the subtrahend in *any order* and from *any part* of the minuend (total) that is greater than or equal to the subtrahend part(s) and still keep the difference the same

- Example:

$$\begin{aligned}
 &15 - 7 \\
 &= 10 + 5 - 7 \\
 &= (10 - 7) + 5 \\
 &= 3 + 5 \\
 &= 8
 \end{aligned}$$



## Topic A: Foundations for Fluency with Sums and Differences Within 100



## Decomposition of 10-19

**Goals:**

- Represent decompositions of 10 with number bond and addition equation (e.g.,  $7 + 3 = 10$ ) or subtraction equation (e.g.,  $10 - 3 = 7$ )

✓ **Check:** Problem Set problem #1

- Decompose 11-19 into ten and ones (e.g.,  $17 = 10 + 7$ )



## Decomposition of Two-Digit Numbers

**Goals:**

- Decompose two-digit numbers into tens and ones (e.g.,  $57 = 50 + 7$ , 57 is 5 tens and 7 ones)

✓ **Check:** Problem Set problem #4

## Topic B: Initiating Fluency with Addition and Subtraction Within 100



**Suggestions:** Write continuous equations (see examples below) rather than separate equations to keep track of all parts of the expression



## Adding and Subtracting: Common Units (Place Value Strategy)

**Goals:**

- Use addition key idea to add common units (tens, ones)

○ Example:

$$51 + 20$$

$$= 50 + 1 + 20$$

$$= (50 + 20) + 1$$

$$= 70 + 1$$

$$= 71$$



**Key Ideas:** Add in any order



✓ **Check:** Problem Set problems #2a-b

- Use subtraction key idea to subtract common units (tens, ones)

- Example:

$$71 - 20$$

$$= 70 + 1 - 20$$

$$= (70 - 20) + 1$$

$$= 50 + 1$$

$$= 51$$



**Key Ideas:** Subtract from any part of the minuend

✓ **Check:** Problem Set problems #2c-d



## Adding Within 20: Making 10

### Goals:

- Decompose an addend to make 10 with the other addend (strategy to develop mental fluency for addition within 20)

- Example:

$$9 + 3$$

$$= (9 + 1) + 2$$

$$= 10 + 2$$

$$= 12$$



**Key Ideas:** Add in any order

✓ **Check:** Problem Set problems #2, #4



## Adding Within 100: Making Multiple of 10

### Goals:

- **Build on Lesson 4** to add within 100:
  - Decompose an addend to make multiple of 10 with the other addend

- Example:

$$39 + 4$$

$$= (39 + 1) + 3$$

$$= 40 + 3$$

$$= 43$$



**Key Ideas:** Add in any order

✓ **Check:** Problem Set problems #1d & f



## Subtracting from Multiple of 10: Taking from 10

### Goals:

- Decompose minuend (total) to take from 10

- Example:

$$40 - 9$$

$$= 30 + 10 - 9$$

$$= 30 + (10 - 9)$$

$$= 30 + 1$$

$$= 31$$



**Key Ideas:** Subtract from any part of the minuend



**Check:** Problem Set problems #1e-f



## Subtracting Within 20: Taking from 10

### Goals:

- Build on Lesson 6 to subtract within 20:

- Decompose minuend (total) into ten and ones to take from 10 (strategy to develop mental fluency for subtraction within 20)



- Example:

$$12 - 9$$

$$= 10 + 2 - 9$$

$$= (10 - 9) + 2$$

$$= 1 + 2$$

$$= 3$$



**Key Ideas:** Subtract from any part of the minuend



**Check:** Problem Set problems #1c-d



## Subtracting Within 100: Taking from 10

### Goals:


- Build on Lessons 6-7 to subtract within 100:

- Decompose minuend (total) to take from 10

- Example:


$$42 - 9$$

$$= 32 + 10 - 9$$

$$= 32 + (10 - 9)$$
  **Key Ideas:** Subtract from any part of the minuend

$$= 32 + 1$$

$$= 33$$

 **Check:** Problem Set problems #1c & f

**Notes to teacher:**

- [Grade 1](#) only covered subtraction within 40 (Module 4) and subtraction of tens from multiples of 10 (Module 6 Lesson 10)

## Module 2: Addition and Subtraction of Length Units

### Summary:

- Measure (using different tools & units) and compare lengths (exactly & approximately/visually)
  - Key lesson: Lesson 6
- Apply understanding to solve word problems involving adding or subtracting lengths
  - Building on addition and subtraction from [Module 1](#)

### Definitions:

- **To measure length** with (or using) a specified **length unit**: to count the number of length units that fit
  - (1) side by side (no overlap or extra space between) and
  - (2) from one endpoint to the other endpoint of the object
- **To estimate (approximate) length**: to find the number of units that is closest to the actual length of the object
- **Meter, centimeter**: 1 meter = 100 centimeter
  - There are a hundred (cent) centimeters in a meter.

### Key Ideas:

- **Relationship between comparing and measuring lengths:**

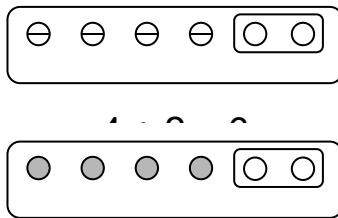
How much ...	means how many ...
<i>longer</i> is A than B	<i>more</i> centimeters (units) is the length of A than the length of B
<i>shorter</i> is A than B	<i>fewer</i> centimeters (units) is the length of A than the length of B

- **Relationship between measurement units and values:**

Measuring with a ...	yields a ...
<i>larger</i> unit	<i>smaller</i> value
<i>smaller</i> unit	<i>larger</i> value

**Reasoning:** one larger unit takes more space so fewer can fit in a given space; one smaller unit takes less space so it takes more to fit in a given space

- **Subtraction as addition with unknown addend:** “ $6 - 4 = \underline{\quad}$ ” also means “ $4 + \underline{\quad} = 6$ ”



Materials:

- Centimeter cubes [Topic A]
- Rulers [Topics A-C]
- Meter stick [Topic B, D]
- Meter tape [Topic B, D]

## Topic A: Understand Concepts About the Ruler



**Cut/consolidate:** Can consolidate Lesson 2 with Lesson 1 or 3 (*Eureka Math's Notes on Pacing*)



### Measuring with Multiple Centimeter Cubes

#### Goals:

- Use centimeter cubes to [measure](#) and [estimate length](#) ([Grade 1](#) Module 3) of linear objects (e.g., crayons) and their parts (e.g., pen cap)



**Check:** Problem Set problem #2

- For Problem Set problem #6b and Homework problem #7: Use the [relationship between comparing and measuring lengths](#) to find difference in lengths



**Suggestions:** Assign Problem Set problem #6b and Homework problem #7 as “challenge” problems; **Lesson 6** will revisit and focus on this concept



### Measuring with One Centimeter Cube

#### Goals:

- Use “mark and move forward” (iteration) technique to measure length with only one centimeter cube




**Check:** Problem Set problem #2



### Measuring with Ruler

#### Goals:

- Create and use centimeter ruler to measure length of objects
- Observe features and functions of ruler:
  - We label the “hash marks” (*not* the space between hash marks) with numbers to easily read length without counting spaces
  - The number under a hash mark tells the number of units between the leftmost hash mark or edge (0) and that hash mark

 **Check:** Problem Set problem #2



**Materials:** See p. 34 of Teacher Edition for instructions to make ruler

## Topic B: Measure and Estimate Length Using Different Measurement Tools



**Cut/consolidate:** Can consolidate Lessons 4 and 5 (*Eureka Math's Notes on Pacing*)



### Measuring with Appropriate Tool

**Goals:**

- Define [meter](#)
- Measure large or long object (e.g., white board) with larger unit like meter stick (more efficient than with centimeter ruler)
- Measure round object (e.g., globe) with meter tape (soft tool)

 **Check:** Problem Set problems #2, #3



### Estimating with Mental Benchmarks

**Goals:**

- Use “mental benchmarks” by “picturing them in your head to estimate the length of an object”:
  - pinky: about 1 cm
  - 3-ring binder (long side): about 30 cm
  - floor to doorknob: about 1 m

 **Check:** Problem Set problems #2, #6a

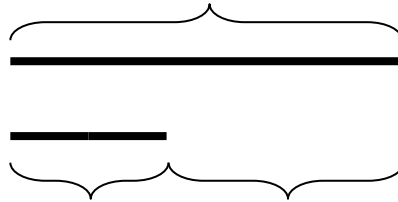
## Topic C: Measure and Compare Lengths Using Different Length Units




## Comparing Lengths

### Goals:

- Use the [relationship between comparing and measuring lengths](#) and definition of [subtraction](#) (or [addition with unknown addend](#)) to find difference in lengths
  - Example:



 **Check:** Problem Set problems #1, #2




**Suggestions:** If objects can be physically moved and aligned, students may just directly measure the length of the difference, so include an example that requires measuring each object (e.g., sides of desktop) and *calculating* the difference



## Measuring with Different Units

### Goals:

- Measure objects with units of different size (e.g., small and large paper clips) and observe the [relationship between measurement units and values](#)
- Recognize that:
  - Measurement needs to be done with a unit of one size (e.g., cannot use both large and small paper clips together to measure length by paper clips)
  - Measurement values can only be compared for the same unit (e.g., 7 small paper clips > 5 small paper clips but 7 small paper clips  $\nrightarrow$  5 large paper clips)
-  **Review/reinforce** Lesson 6: Find difference in lengths



**Check:** Problem Set problem #1

- For Problem Set problem #3 and Homework problem #3: Draw a line segment of given length





## Topic D: Relate Addition and Subtraction to Length



## Adding and Subtracting Lengths



**Goals:**

- Find total length of a “bending” path (e.g., the length of ribbon to frame a door, the length of Christmas lights to string along steps) by adding the lengths of the parts  
 **Check:** Problem Set problem #4
- For Problem Set and Homework: Find final length or location on ruler by adding and subtracting lengths  
 **Check:** Problem Set problem #2
- For Concept Development and Exit Ticket: Find length when the left endpoint of the object is above zero by subtracting the length between zero and the left endpoint



## Measuring Curves

**Goals:**

- **Build on Lesson 4** by measuring curve with string (e.g., lay string on top of zig-zag tape and then mark end of curve on string and measure length of (straightened) string with ruler)  
 **Check:** Problem Set problem #2
- **Build on Lessons 6 and 8** by using tape diagram (or number line) to represent and compare lengths  
 **Check:** Problem Set problem #2c



**Materials:** string (about 50 cm) per student



## Two-Step Word Problems

### Goals:

- Build on Lesson 9 to use tape diagram (or number line) to find an unknown length and then the sum of two or more lengths

✓ Check: Problem Set problem #2

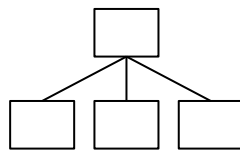
## Module 3: Place Value, Counting, and Comparison of Numbers to 1,000 ([Grade 1](#) Module 6 Continued)

### Summary:

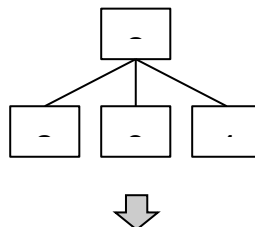
- Counting [Lessons 1-15]
  - Place value
  - How to count to a number
  - How to count from one number to another number
  - Counting ten or more of a unit
  - Different forms of a number (standard form or base-ten numeral, word form or number name, expanded form, unit form)
- Comparing [Lessons 16-18]
  - Greater than, less than, equal
- Increasing/decreasing by 1, 10, and 100 [Lessons 19-21]
  - “1, 10, 100 more” and “less”

### Definitions:

- **Unit form of a number:** “unit form of 234” is “2 hundreds 3 tens 4 ones”



- Note to teacher: Reading/writing numbers in “unit form” is not a state standard but is central to Eureka Math’s theme of units and serves as a bridge between the [standard form \(base-ten numeral\)](#) and [expanded form](#)
- **Word form of a number (number name\*):** “word form (or number name) of 234” is “two hundred thirty-four”
- **Number form of a number (base-ten numeral\*):** “standard form (or base-ten numeral) of 234” is “234”
- **Expanded form of a number:** “expanded form of 243” is “200 + 40 + 3”



\* State standard’s terminology

- **Greater than, less than, equals:**

Word form	Symbolic form	Meaning (Definition)
3 is <u>greater</u> (bigger, more) than 2	$3 > 2$	3 comes <u>after</u> 2 when counting
2 is <u>less</u> (smaller, fewer) than 3	$2 < 3$	2 comes <u>before</u> 3 when counting
$3 + 0$ <u>equals</u> $2 + 1$	$3 + 0 = 2 + 1$	$3 + 0$ counts to the <u>same</u> number as $2 + 1$

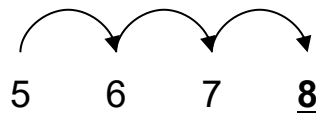
- **“1 (or 10 or 100) more than”:** “1 (or 10 or 100) more than 429” means “ $429 + 1$  (or 10 or 100)”
- **“1 (or 10 or 100) less than”:** “1 (or 10 or 100) less than 429” means “ $429 - 1$  (or 10 or 100)”

### Key Ideas:

- **Place value:**

In our number system (base ten), we only have **ten** symbols (the digits 0 to 9) to represent **zero to nine** ones, tens, hundreds, etc. (a unit) so to represent **ten** ones, tens, hundreds, etc. we use the next “place” to the left and make it 1 of that larger unit (e.g., ten 1s = 10, ten 10s = 100, ten 100s = 1000)

- **Counting to a number:** begin with largest base-ten unit (left-most digit)
  - Example: 476
    1. Count from 0 to 400 by hundreds
    2. Count from 400 to 470 by tens
    3. Count from 470 to 476 by ones
- **Counting from one number to another number:** begin with smallest base-ten unit (right-most nonzero digit) of starting number
  - Example: 476 to 600
    1. Count from 476 to 480 by ones
    2. Count from 480 to 500 by tens
    3. Count from 500 to 600 by hundred(s)
- **“Count on” method:** “ $5 + 3$ ” is the same number as (equals) “starting at 5 and counting on 3 (6, 7, 8)”



- “1 more” or “less” changes the ones digit
- “10 more” or “less” changes the tens digit
- “100 more” or “less” changes the hundreds digit

### Materials:

- Base ten units (e.g., Dienes base ten blocks, bundled straws/sticks) [Topics A-C]
- Dollar bills: \$1, \$10, \$100 [Topic D]
- Place value disks [Topics E-F]



**Suggestions:** Incorporate place value chart in Lessons 1-3 (then Lesson 4 can be omitted or used as review)



**Resources:** [Alternative approach](#) to Lessons 1-4

## Topic A: Forming Base Ten Units of Ten, a Hundred, and a Thousand



### Counting to 1,000

**Goals:**

- Use [place value](#) to form base ten units (ten, hundred, thousand) and [count to a number](#) up to 1,000

## Topic B: Understanding Place Value Units of One, Ten, and a Hundred



### Counting between Numbers with Ones and Tens

**Goals:**

- Use [place value](#) to [count to a number](#) and [count from one number to another](#) with ones and tens

✓ **Check:** Problem Set problem #2



### Counting between Numbers with Ones, Tens, and Hundreds

**Goals:**

- Use [place value](#) to [count to a number](#) and [count from one number to another](#) with ones, tens, and hundreds

✓ **Check:** Problem Set problems #1, #3

## Topic C: Three-Digit Numbers in Unit, Standard, Expanded, and Word Forms



## Counting between Numbers with Ones, Tens, and Hundreds (Continued)

**Goals:**

- Build on **Lessons 2-3** with the place value chart

✓ **Check:** Problem Set problem #3

**Notes to teacher:**

- Will revisit/reinforce counting between numbers in **Lessons 9 and 12** with dollar bills, empty number line, and place value disks



## Unit Form, Word Form, Standard Form

**Goals:**

- Say, write, and/or match numbers in [unit form](#), [word form](#), and [number form](#)



## Expanded Form

**Goals:**

- Use “[count on](#)” method of addition to write a number in [expanded form](#) and vice versa

✓ **Check:** Problem Set problems #4, #5, #9



## Different Forms of a Number

**Goals:**

- ↻ **Review/reinforce** Lessons 5-6

✓ **Check:** Problem Set “Match Part 1” problems C & D

- Use place value and unit form of a number to count ten or more of a unit
  - Example: 120  
If counting by tens and not bundling 10 tens into 100, then 120 is 12 tens.

✓ **Check:** Problem Set “Match Part 2” problem F

**Notes to teacher:**

- Will revisit/reinforce counting ten or more of a unit in **Lesson 14**

## Topic D: Modeling Base Ten Numbers Within 1,000 with Money



### Representing Numbers with Dollar Bills

**Goals:**

- Represent numbers to 1,000 using \$1, \$10, and \$100 bills
- ↻ **Review/reinforce** Lessons 5-7 by representing value of money in different number forms (number, expanded, unit)

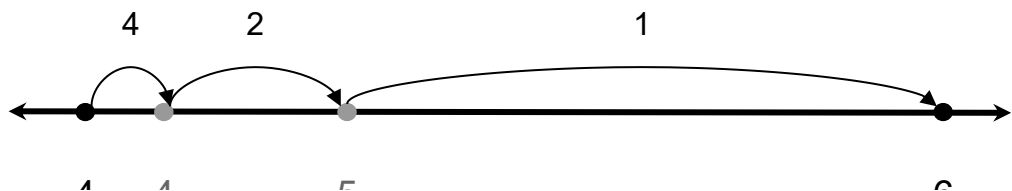
✓ **Check:** Problem Set problem #2



### Counting between Numbers with Dollar Bills and Empty Number Line

**Goals:**

- Build on **Lessons 2-4** by counting between numbers with dollar bills and an empty number line
  - Example: 476 to 600



✓ **Check:** Problem Set problems #3, #5

**Notes to teacher:**

- Counting between numbers with an empty number line lays groundwork for addition/subtraction strategies (“the arrow way”, counting on) in **Modules 4-5**



## Decomposition of \$1,000 into \$10 bills



**Cut/consolidate:** Can omit lesson or use as “challenge” task

### Goals:

- **Build on Lesson 7** to find how many \$10 bills make \$1,000 (answer: 100)

## Topic E: Modeling Numbers Within 1,000 with Place Value Disks



**Suggestions:** Incorporate drawing place value disks in Lessons 11-12 (then Lesson 13 can be omitted or used as review)



## Representing Numbers with Place Value Disks



**Cut/consolidate:** Can omit use of Dienes base ten blocks

### Goals:

- Represent numbers to 1,000 using place value disks



**Check:** Problem Set problems #2a-b



## Counting between Numbers with Place Value Disks

### Goals:

- **Build on Lessons 2-4 and 9** by counting between numbers with place value disks
  - Example:
    - Count from 186 to 300 by counting
      - 4 ones disks from 186 to 190
      - 1 ten disk from 190 to 200
      - 1 hundred disk from 200 to 300





**Suggestions:** Omit the Problem Set and Homework if the language is confusing





## Representing Numbers with Place Value Disks (Continued)


### Goals:

-  **Review/reinforce** Lessons 5-7 and 11 by drawing place value disks
-  **Check:** Problem Set problems #1, #2



## Counting Ten or More of a Unit

### Goals:

- **Build on Lesson 7** by counting ten or more ones or tens with place value disks
  - Example: 140  
If counting by tens disks and not exchanging 10 tens disks for a hundred disk, then 140 is 14 tens disks
-  **Check:** Problem Set problems #1a & c, #2a & c



## Word Problems with Counting Tens

### Goals:

- **Build on Lessons 2-4, 7, 9, 12 and 14** to solve word problems that involve counting tens and counting between numbers

## Topic F: Comparing Two Three-Digit Numbers



**Suggestions:** Use a 0-999 chart or 0-99, 100-199, ..., 900-999 charts to help compare numbers (determine which number comes before or after when counting)




## Comparing Numbers

### Goals:

- Review definitions of [greater than, less than, equals](#) ([Grade 1](#) Modules 4 & 6)
- Use above definitions to compare and order numbers
  - Example:
    - Compare 105 and 135
      - To count to 105, we count from 0 to 100 by hundred, then count by ones to 105
      - To count to 135, we count from 0 to 100 by hundred, then count by tens (past 105) to 130, then count by ones to 135
      - 105 comes before 135 when counting so  $105 < 135$

 **Check:** Problem Set problem #2g, #3d

 **Suggestions:** Represent numbers with Dienes base ten blocks or bundled straws/sticks if students need to “see” how a number is greater or less



## Comparing Numbers (Continued)

 **Cut/consolidate:** Can consolidate with Lesson 18 (*Eureka Math's Notes on Pacing*)

### Goals:

- **Build on Lesson 16** to compare numbers expressed as ten or more of a unit
- ✓ **Check:** Problem Set problems #2a & e



## Ordering Numbers in Different Forms

### Goals:

-  **Review/reinforce** Lessons 5-7, 16-17
- ✓ **Check:** Problem Set problems #2b-d

## Topic G: Finding 1, 10, and 100 More or Less Than a Number



## Patterns with 1, 10, and 100 More or Less

**Goals:**

- Review definitions of [“1 \(or 10 or 100\) more than”](#), [“1 \(or 10 or 100\) less than”](#) ([Grade 1 Modules 4 & 6](#))
- Use above definitions to observe:

**Key Ideas:**

- “1 more” or “less” changes the ones digit
- “10 more” or “less” changes the tens digit
- “100 more” or “less” changes the hundreds digit

**Check:** Problem Set problems #2b-d, g-h

## Patterns with 1, 10, and 100 More or Less (Continued)

**Goals:**

- **Build on Lesson 19** to observe:
  - “1 more” than a number with 9 in the ones place changes the ones digit to 0 and makes the tens digit increase by 1 (e.g., 1 more than 59 is 60)
  - “10 more” than a number with 9 in the tens place changes the tens digit to 0 and makes the hundreds digit increase by 1 (e.g., 10 more than 90 is 100)
  - “1 less” than a number with 0 in the ones place makes the tens digit decrease by 1 and changes the ones digit to 9 (e.g., 1 less than 230 is 229)
  - “10 less” than a number with 0 in the tens place makes the hundreds digit decrease by 1 and changes the tens digit to 9 (e.g., 10 less than 503 is 493)

**Check:** Problem Set problems #1a-b, #2a & d**Notes to teacher:**

- Will revisit/reinforce 1 and 10 more or less in **Module 4 Lesson 1**



## Skip-Counting Patterns

**Cut/consolidate:** Can omit lesson or use as “challenge” task**Goals:**

- Find skip-counting pattern to fill in missing numbers

**Check:** Problem Set problems #2a-c



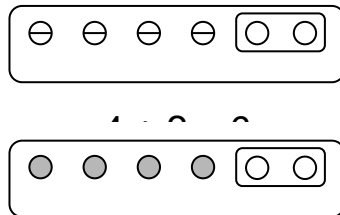
# Module 4: Addition and Subtraction Within 200 with Word Problems to 100

Summary:

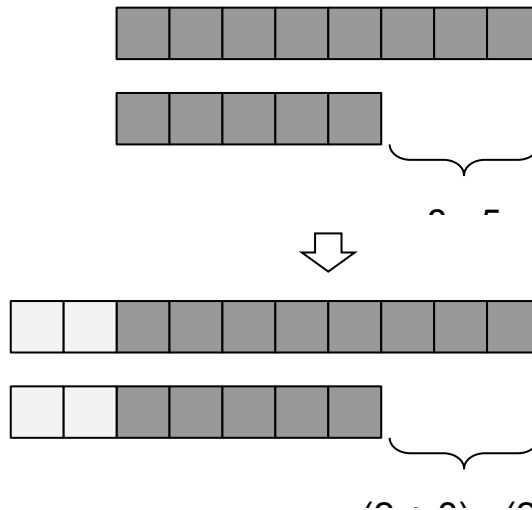
Strategy [Lesson(s)]	Addition	Subtraction	Key Ideas/Notes
Place value (Common units) [Lessons 2, 6 and on]	$\begin{aligned} &26 + 30 \\ &= \underline{20} + 6 + 30 \\ &= \underline{20} + 30 + 6 \\ &= 50 + 6 \\ &= 56 \end{aligned}$ $\begin{aligned} &35 + 26 \\ &= (30 + 5) + (20 + 6) \\ &= 30 + 20 + (5 + 6) \\ &= 30 + 20 + 11 \\ &= \underline{30} + 20 + \underline{10} + 1 \\ &= 60 + 1 \\ &= 61 \end{aligned}$	$\begin{aligned} &56 - 30 \\ &= \underline{50} + 6 - 30 \\ &= \underline{50} - 30 + 6 \\ &= 20 + 6 \\ &= 26 \end{aligned}$ $\begin{aligned} &46 - 18 \\ &= 40 + 6 - 10 - 8 \\ &= \underline{40} - 10 + 6 - 8 \\ &= \underline{30} + \underline{10} - 10 + 6 - 8 \\ &= 30 - 10 + 10 + 6 - 8 \\ &= \underline{30} - \underline{10} + (16 - 8) \\ &= 20 + 8 \\ &= 28 \end{aligned}$	<ul style="list-style-type: none"> <li>• <a href="#">Addition</a> and <a href="#">subtraction</a> key ideas</li> <li>• Basis for vertical form (standard algorithms)</li> </ul>
Make multiple of 10 [Lesson 4]	$\begin{aligned} &28 + 36 \\ &= \underline{28} + \underline{34} + 2 \\ &= \underline{28} + 2 + 34 \\ &= 30 + 34 \\ &= 64 \end{aligned}$	$\begin{aligned} &78 - 39 \\ &= \underline{78} - \underline{30} - 8 - 1 \\ &= \underline{78} - 8 - 30 - 1 \\ &= 70 - 30 - 1 \\ &= 39 \end{aligned}$	<ul style="list-style-type: none"> <li>• <a href="#">Addition</a> (and <a href="#">subtraction</a>) key ideas</li> </ul>
In parts, method 1 ("The arrow way") [Lessons 2-3]	$\begin{aligned} &19 + 32 \\ &= \underline{19} + \underline{30} + 2 \\ &= 49 + 2 \\ &= 51 \end{aligned}$	$\begin{aligned} &90 - 51 \\ &= \underline{90} - \underline{50} - 1 \\ &= 40 - 1 \\ &= 39 \end{aligned}$	<ul style="list-style-type: none"> <li>• Use arrows or number line</li> </ul>
In parts, method 2 [Lesson 3]	$\begin{aligned} &19 + 29 \\ &= \underline{19} + \underline{30} - 1 \\ &= 49 - 1 \\ &= 48 \end{aligned}$	$\begin{aligned} &90 - 39 \\ &= \underline{90} - \underline{40} + 1 \\ &= 50 + 1 \\ &= 51 \end{aligned}$	<ul style="list-style-type: none"> <li>• Use number line</li> </ul>
Count on [Lesson 2]		$\begin{aligned} &56 - 30 = \underline{\quad} \\ &\rightarrow 30 + \underline{\quad} = 56 \\ &\rightarrow 30 + \underline{20} + \underline{6} = 56 \\ &\rightarrow 56 - 30 = 26 \end{aligned}$	<ul style="list-style-type: none"> <li>• Subtraction as addition with unknown addend</li> <li>• Use (empty) number line (<a href="#">Module 3</a> Lesson 9)</li> </ul>
Compensation [Lesson 4]		$\begin{aligned} &34 - 28 \\ &= (34 + 2) - (28 + 2) \\ &= 36 - 30 \end{aligned}$	<ul style="list-style-type: none"> <li>• Show simple case with linking cubes</li> <li>• Use tape diagram</li> </ul>

## Key Ideas:

- **Addition:** Because of the associative and commutative properties of addition, we can add numbers in *any order* and still keep the sum (total) the same
- **Subtraction:** We can subtract parts of the subtrahend in *any order* and from *any part* of the minuend (total) that is greater than or equal to the subtrahend part(s) and still keep the difference the same
- **Subtraction as addition with unknown addend:** “ $6 - 4 = \underline{\quad}$ ” also means “ $4 + \underline{\quad} = 6$ ”



- **Compensation:** Addition (or subtraction) of the same number to the minuend and subtrahend keeps the difference the same
  - Example:  
 $8 - 5 = (2 + 8) - (2 + 5)$



## Materials:

- Linking/snap cubes [Topic A]
- Place value disks [Topics B-E]

## Topic A: Sums and Differences Within 100



## Sequences of 1 or 10 More or Less

**Goals:**

- Build on [Module 3 Lessons 19-20](#) to:
  - Identify rule in sequence as +1, -1, +10, or -10
  - Determine next number in sequence based on given rule

✓ **Check:** Problem Set problems #2a, #3b



## Adding and Subtracting Multiples of 10

**Goals:**

- Build on [Module 1](#) to develop strategies for adding multiples of 10
  - “Place value” (common units) strategy
 
$$\begin{aligned} &26 + 30 \\ &= 20 + 6 + 30 \\ &= (20 + 30) + 6 \\ &= 50 + 6 \\ &= 56 \end{aligned}$$

🔑 **Key Ideas:** Add in any order
  - “In parts” method 1 (“the arrow way”) strategy
 
$$\begin{aligned} &26 + 30 \\ &= (26 + 10) + 10 + 10 \\ &= (36 + 10) + 10 \\ &= 46 + 10 \\ &= 56 \end{aligned}$$

🔑 **Key Ideas:** Add in any order

✓ **Check:** Problem Set problem #1c

- Build on [Module 1](#) to develop strategies for subtracting multiples of 10
  - “Place value” (common units) strategy
 
$$\begin{aligned} &56 - 30 \\ &= 50 + 6 - 30 \\ &= (50 - 30) + 6 \\ &= 20 + 6 \\ &= 26 \end{aligned}$$

🔑 **Key Ideas:** Subtract from any part of the minuend
  - “In parts” method 1 (“the arrow way”) strategy

$$56 - 30$$

$$= (56 - 10) - 10 - 10$$

$$= (46 - 10) - 10$$

$$= 36 - 10$$

$$= 26$$

- “Count on” strategy

$$56 - 30 = \underline{\quad}$$

- $30 + \underline{\quad} = 56$

- $30 + \underline{20} + \underline{6} = 56$

- so  $56 - 30 = 26$



**Key Ideas:** Subtract subtrahend parts in any order



**Key Ideas:** Subtraction as addition of unknown addend



**Check:** Problem Set problem #2b



## Adding and Subtracting Two-Digit Numbers

### Goals:

- **Build on Lesson 2** to add two-digit numbers

- “In parts” method 1 (“the arrow way”) strategy

$$48 + 21$$

$$= 48 + (20 + 1)$$

$$= (48 + 20) + 1$$

$$= 68 + 1$$

$$= 69$$



**Key Ideas:** Add in any order

**Lesson 2:**  $48 + 20 = 40 + 20 + 8$

- “In parts” method 2 strategy

$$48 + 19$$

$$= 48 + (20 - 1)$$

$$= (48 + 20) - 1$$

$$= 68 - 1$$

$$= 67$$



**Key Ideas:** Add (subtract from minuend) in any order

**Lesson 2:**  $48 + 20 = 40 + 20 + 8$



**Check:** Problem Set problem #1b

- **Build on Lesson 2** to subtract two-digit numbers

- “In parts” method 1 (“the arrow way”) strategy

$$68 - 21$$

$$= 68 - 20 - 1$$

$$= (68 - 20) - 1$$

$$= 48 - 1$$

$$= 47$$



**Key Ideas:** Subtract subtrahend parts in any order

**Lesson 2:**  $68 - 20 = 60 - 20 + 8$

- “In parts” method 2 strategy



$$68 - 19$$

$$= 68 - 20 + 1$$

$$= (68 - 20) + 1$$

$$= 48 + 1$$

$$= 49$$

← subtracting 20 is too much by 1 so we add back 1



**Key Ideas:** Subtract from minuend (add) in any order

**Lesson 2:**  $68 - 20 = 60 - 20 + 8$

✓ **Check:** Problem Set problem #1c



## Adding and Subtracting Two-Digit Numbers (Continued)

### Goals:

- Observe [compensation](#) (with linking/snap cubes) and use compensation to subtract two-digit numbers

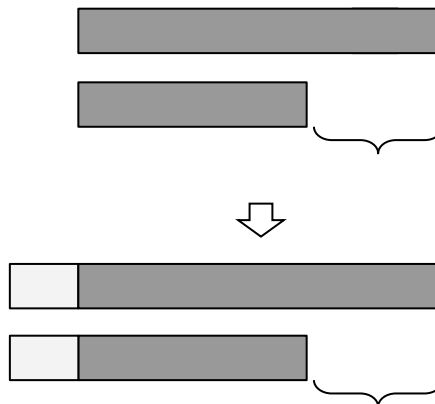
- Compensation strategy

$$34 - 28$$

$$= (2 + 34) - (2 + 28)$$

$$= 36 - 30$$

**Lesson 2:**  $36 - 30 = 30 - 30 + 6$



✓ **Check:** Problem Set problem #1b

- Build on [Module 1 Lessons 4-5](#) (“make 10” strategy) to add two-digit numbers

- “Make multiple of 10” strategy

$$28 + 36$$

$$= 28 + (34 + 2)$$

$$= (28 + 2) + 34$$

$$= 30 + 34$$

$$= 64$$



**Key Ideas:** Add in any order

**Lesson 2:**  $30 + 34 = 30 + 30 + 4$

✓ **Check:** Problem Set problem #2b



## Word Problems

### Goals:

- Solve one- and two-step word problems using tape diagrams and addition and subtraction strategies from **Lessons 2-4**



**Focus:** Concept Development problems #1, #4



**Check:** Problem Set problems #3, #4

### Notes to teacher:

- Will revisit/reinforce one- and two-step word problems in **Lesson 16** with “place value” strategies

## Topic B: Strategies for Composing a Ten



**Suggestions:** Incorporate drawing place value disks in Lessons 6-7 (then Lesson 8 can be omitted or used as review)



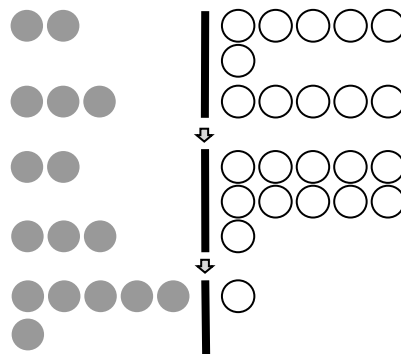
## Adding Two-Digit Numbers: Disks

### Goals:

- Build on Lesson 2** (and Grade 1 Modules 4 & 6) by using place value disks to add two two-digit numbers involving composing a ten

○ Example:

$$\begin{aligned}
 &26 + 35 \\
 &= (20 + 6) + (30 + 5) \\
 &= 20 + 30 + (6 + 5) \\
 &= 20 + 30 + 11 \\
 &= \underline{20 + 30 + 10} + 1 \\
 &= \underline{60} + 1 \\
 &= 61
 \end{aligned}$$



**Check:** Problem Set problem #2c

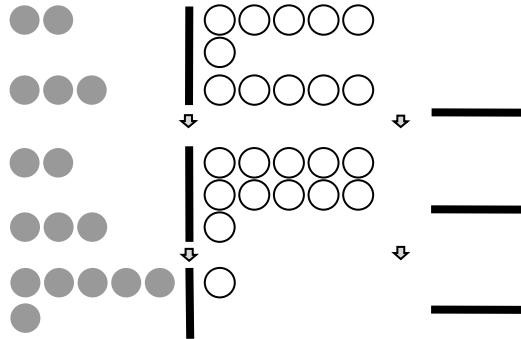


## Adding Two-Digit Numbers: Disks and Written Vertical Form

### Goals:

- **Build on Lesson 6** by using place value disks and written vertical form to add two two-digit numbers involving composing a ten

○ Example:



✓ **Check:** Problem Set problem #1c



## Adding Two-Digit Numbers: Drawings and Written Vertical Form

### Goals:

- **Review/reinforce** Lesson 7 by drawing place value disks

✓ **Check:** Problem Set problem #1a



## Adding Two-Digit Number to Three-Digit Number

### Goals:

- **Build on Lessons 7-8** by using place value disk drawings and written vertical form to add a two-digit number to a three-digit number involving composing a ten

 **Focus:** Concept Development problem #2

✓ **Check:** Problem Set problem #1c



## Adding Two-Digit Number to Three-Digit Number (Continued)

### Goals:

- **Review/reinforce** Lesson 9
- **Check:** Problem Set problem #1d

## Topic C: Strategies for Decomposing a Ten



### Suggestions:

- Incorporate drawing place value disks in Lessons 11-12 (then Lesson 13 can be omitted or used as review)
- Use / for exchange (decomposition) and -- for subtraction in place value drawings



## Subtracting Two-Digit Numbers: Disks

### Goals:

- **Echo** Lesson 6 for subtraction:
  - **Build on Lesson 2** (and Grade 1 Modules 4 & 6) by using place value disks to subtract two-digit numbers involving decomposing a ten

#### Example:

$$46 - 18$$

$$= 40 + 6 - 10 - 8$$

$$= \underline{40} - 10 + 6 - 8 \leftarrow \text{not enough to subtract}$$

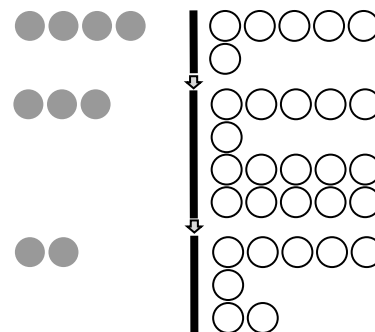
$$= \underline{30} + \underline{10} - 10 + 6 - 8$$



$$= 30 - 10 + 10 + 6 - 8$$

$$= 30 - 10 + (16 - 8)$$

$$= 20 + 8$$

$$= 28$$




-  **Focus:** Concept Development problem #1 or #2
-  **Check:** Problem Set problem #2c-d

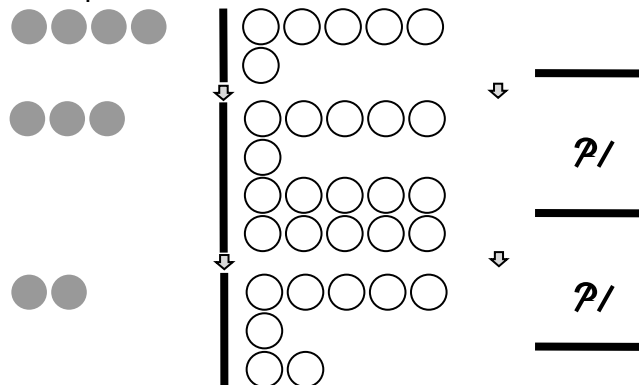




## Subtracting Two-Digit Numbers: Disks and Written Vertical Form

### Goals:

-  **Echo** Lesson 7 for subtraction:
  - **Build on Lesson 11** by using place value disks and written vertical form to subtract two-digit numbers involving decomposing a ten

#### ■ Example:







-  **Focus:** Concept Development problem #2
-  **Check:** Problem Set problem #1d-e



## Subtracting Two-Digit Numbers: Drawings and Written Vertical Form

### Goals:



-  **Echo** Lesson 8 for subtraction:
  -  **Review/reinforce** Lesson 12 by drawing place value disks

-  **Focus:** Concept Development problem #1
-  **Check:** Problem Set problem #1c



## Subtracting Two-Digit Number from Three-Digit Number



### Goals:

-  **Echo** Lesson 9 for subtraction:
  - **Build on Lessons 12-13** by using place value disk drawings and written vertical form to subtract a two-digit number from a three-digit number involving decomposing a ten
-  **Check:** Problem Set problem #1c



## Subtracting Two-Digit Number from Three-Digit Number (Continued)




### Goals:

-  **Review/reinforce** Lesson 14
-  **Check:** Problem Set problem #1c
- For Concept Development: Use [subtraction as addition with unknown addend](#) to “check” answer (e.g., to check  $172 - 48 = 124$ , see if  $48 + 124 = 172$ )



## Word Problems (Continued)

### Goals:

-  **Echo** Lesson 5 with “place value” addition and subtraction strategies:
  - Solve one- and two-step word problems using tape diagrams and addition and subtraction strategies from **Lessons 7-8 and 12-13**
-  **Focus:** Concept Development problems #1, #2, #4
-  **Check:** Problem Set problem #3, #5

### Notes to teacher:

- Will revisit/reinforce two-step word problems in **Lesson 31**

## Topic D: Strategies for Composing Tens and Hundreds



### Suggestions:

- Embed Lesson 29’s subtotals (“Totals Below”) into Lessons 19-21 as an intermediate step to transitioning to the vertical form (then Lessons 29-30 can be omitted)

- Incorporate drawing place value disks in Lessons 18-19 (then Lessons 20-21 can be omitted or used as review)



## Composing Ten or Hundred when Adding Ones or Tens

### Goals:

- Use [place value](#) and addition within 20 ([Module 1](#) Lesson 4) to compose ten when adding ones and compose hundred when adding tens

✓ **Check:** Problem Set problems #1d, #2a



## Adding Two Numbers: Disks

### Goals:

- **Echo** Lesson 6 for two compositions:
  - Use place value disks to add two two-digit numbers involving composing a ten and a hundred
    - Example:
 
$$\begin{aligned}
 &49 + 73 \\
 &= (40 + 9) + (70 + 3) \\
 &= 40 + 70 + (9 + 3) \\
 &= 40 + 70 + 12 \\
 &= \underline{40 + 70 + 10} + 2 && \leftarrow \text{compose 10 ones into a unit of ten} \\
 &= \underline{12 \text{ tens}} + 2 && \leftarrow 4 \text{ tens} + 7 \text{ tens} + 1 \text{ ten} = 12 \text{ tens} \\
 &= 100 + 20 + 2 && \leftarrow \text{compose 10 tens into a unit of hundred} \\
 &= 122
 \end{aligned}$$



**Focus:** Concept Development problem #2 or #3

✓ **Check:** Problem Set problem #1c



## Adding Two Numbers: Disks and Written Vertical Form

### Goals:

- **Echo** Lesson 7 for two compositions:

- **Build on Lesson 18** by using place value disks and written vertical form to add two two-digit numbers involving composing a ten and a hundred

tens	ones
..	...
....	.....

$$\begin{array}{r} 23 \rightarrow 20 + 3 \\ + 48 \rightarrow 40 + 8 \\ \hline 60 + 11 \end{array}$$

$$\begin{array}{r} 23 \\ + 48 \\ \hline 11 \\ 60 \\ \hline 71 \end{array}$$

✓ **Check:** Problem Set problem #1d



## Adding Two Numbers: Drawings and Written Vertical Form

### Goals:

- **Echo** Lesson 8 for two compositions:
  - **Review/reinforce** Lesson 19 by drawing place value disks
  - **Focus:** Concept Development problem #1
- ✓ **Check:** Problem Set problem #1c



## Adding Two Numbers: Drawings and Written Vertical Form (Continued)

### Goals:

- **Review/reinforce** Lesson 20
- ✓ **Check:** Problem Set problem #1d



## Adding Up to Four Numbers

### Goals:

- Use Module 1 [addition](#) key idea to make multiples of 10 when adding numbers

$$31 + 23 + 19 + 47 = 120$$

$$50 + 60 + 10 = 120$$



$$31 + 23 + 19 + 47 =$$

$$31 + 19 + 23 + 47 = 12 \text{ tens} = 120$$

**Focus:** Concept Development problems #2, #5

**Check:** Problem Set problem #1c

## Topic E: Strategies for Decomposing Tens and Hundreds



### Suggestions:

- Incorporate drawing place value disks in Lessons 24-25 (then Lesson 26 can be omitted or used as review)
- Use / for exchange (decomposition) and -- for subtraction in place value drawings



## Subtracting Multiples of 10

### Goals:

- **Build on Lesson 2** to subtract multiples of 10 from three-digit numbers

○ Example:

$$127 - 70$$

$$= 100 + 27 - 70$$

$$= (100 - 70) + 27$$

$$= 30 + 27$$

$$= 57$$



**Key Ideas:** Subtract from any part of the minuend






**Focus:** Concept Development problem #2

**Check:** Problem Set problem #1c



## Subtracting from a Three-Digit Number: Disks




**Goals:**

-  **Echo** Lesson 11 for two decompositions:
  - Use place value disks to subtract a two-digit number from a three-digit number involving decomposing a ten and a hundred
    -  **Focus:** Concept Development problems #1, #3
    -  **Check:** Problem Set problems #2a-b, e



## Subtracting from a Three-Digit Number: Disks and Written Vertical Form





**Goals:**

-  **Echo** Lesson 12 for two decompositions:
  - **Build on Lesson 24** by using place value disks and written vertical form to subtract from a three-digit number involving decomposing a ten and a hundred
    -  **Focus:** Concept Development problem #1 or #2
    -  **Check:** Problem Set problems #1b & g



## Subtracting from a Three-Digit Number: Drawings and Written Vertical Form


**Goals:**

-  **Echo** Lesson 13 for two decompositions:
  -  **Review/reinforce** Lesson 25 by drawing place value disks
    -  **Focus:** Concept Development problem #1 or #2
    -  **Check:** Problem Set problem #1a-b



## Subtracting from a Three-Digit Number with Zero(s)

**Goals:**




- **Build on Lessons 23-26** to subtract from three-digit numbers with zero in the tens (and ones place)
  -  **Focus:** Concept Development problems #3, #4

 **Check:** Problem Set problems #2a & d



## Subtracting from a Three-Digit Number with Zero(s) (Continued)

### Goals:

-  **Review/reinforce** Lesson 27
-  **Focus:** Concept Development problem #1 or #2
-  **Check:** Problem Set problems #1b-c

## Topic F: Student Explanations of Written Methods




**Cut/consolidate:** Can omit Lessons 29-30 (see Suggestion under [Topic D](#)) or consolidate Lessons 29-31 (see *Eureka Math's Notes on Pacing*)



## “Totals Below” Written Method

### Goals:

- Record addition process with the “totals below” written method
-  **Check:** Problem Set problem #2b



## Adding Two Numbers (Continued)




### Goals:

-  **Review/reinforce** Lessons 17-21, 29
-  **Check:** Problem Set problems #3a-b



## Word Problems (Continued)

**Goals:**

-  **Review/reinforce** Lesson 16: Solve two-step word problems using tape diagrams and addition and subtraction strategies from **Lessons 7-8 and 12-13**
-  **Focus:** Concept Development problem #3
-  **Check:** Problem Set problem #3

# Module 5: Addition and Subtraction Within 1,000

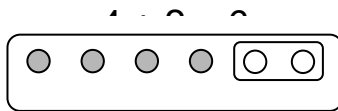
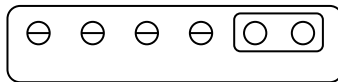
## with Word Problems to 100

Summary:

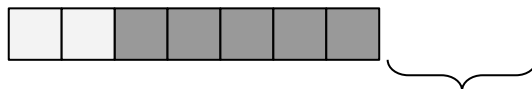
Strategy [Lesson(s)]	Addition	Subtraction	Key Ideas/Notes
Place value (Common units) [Lessons 2, 8 and on]	$450 + 300$ $= 400 + 50 + 300$ $= 400 + 300 + 50$ $= 700 + 50$ $= 750$	$582 - 300$ $= 500 + 82 - 300$ $= 500 - 300 + 82$ $= 200 + 82$ $= 282$	<ul style="list-style-type: none"> <li>• <a href="#">Addition</a> and <a href="#">subtraction</a> key ideas</li> <li>• Basis for vertical form (standard algorithms)</li> </ul>
Make multiple of 100 [Lessons 3-5]	$280 + 230$ $= 280 + 200 + 20 + 10$ $= 280 + 20 + 200 + 10$ $= 300 + 200 + 10$ $= 510$	$780 - 390$ $= 780 - 300 - 80 - 10$ $= (780 - 80) - 300 - 10$ $= 700 - 300 - 10$ $= 390$	<ul style="list-style-type: none"> <li>• <a href="#">Addition</a> and <a href="#">subtraction</a> key ideas</li> </ul>
In parts, method 1 ("The arrow way") [Lessons 2-4]	$450 + 300$ $= 450 + 100 + 100 + 100$ $450 \xrightarrow{+100} 550 \xrightarrow{+100} 650 \xrightarrow{+100} 750$ $= 750$  $280 + 230$ $= 280 + 200 + 30$ $280 \xrightarrow{+200} 480 \xrightarrow{+30} 510$ $= 510$	$582 - 300$ $= 582 - 100 - 100 - 100$ $582 \xrightarrow{-100} 482 \xrightarrow{-100} 382 \xrightarrow{-100} 282$ $= 282$  $780 - 390$ $= 780 - 300 - 50 - 30 - 10$ $780 \xrightarrow{-300} 480 \xrightarrow{-50} 430 \xrightarrow{-30} 400 \xrightarrow{-10} 390$ $= 390$	<ul style="list-style-type: none"> <li>• Use arrows or number line</li> </ul>
In parts, method 2	$120 + 190$ $= 120 + 200 - 10$ $= 320 - 10$ $= 310$	$780 - 390$ $= 780 - 400 + 10$ $= 380 + 10$ $= 390$	<ul style="list-style-type: none"> <li>• Use number line</li> </ul>
Count on [Lessons 2 & 18]		$582 - 300 = \underline{\quad}$ $\rightarrow 300 + \underline{\quad} = 582$ $\rightarrow 300 + 200 + 82 = 582$ $\rightarrow 582 - 300 = \underline{282}$	<ul style="list-style-type: none"> <li>• Subtraction as addition with unknown addend</li> <li>• Use (empty) number line (<a href="#">Module 3</a> Lesson 9)</li> </ul>
Compensation [Lessons 6 & 18]		$780 - 390$ $= (780 + 10) - (390 + 10)$ $= 790 - 400$ $= 390$	<ul style="list-style-type: none"> <li>• Show simple case with linking cubes (<a href="#">Module 4</a> Lesson 4)</li> <li>• Use tape diagram</li> </ul>

## Key Ideas:

- **Addition:** Because of the associative and commutative properties of addition, we can add numbers in *any order* and still keep the sum (total) the same
- **Subtraction:** We can subtract parts of the subtrahend in *any order* and from *any part* of the minuend (total) that is greater than or equal to the subtrahend part(s) and still keep the difference the same
- **Subtraction as addition with unknown addend:** “ $6 - 4 = \underline{\quad}$ ” also means “ $4 + \underline{\quad} = 6$ ”



- **Compensation:** Addition (or subtraction) of the same number to the minuend and subtrahend keeps the difference the same
  - Example:  
 $8 - 5 = (2 + 8) - (2 + 5)$



## Materials:

- Place value disks [Topics A-C]



**Suggestions:** If student learning of Module 4 is solid, move through Module 5 more quickly because it applies the same ideas to larger numbers

## Topic A: Strategies for Adding and Subtracting Within 1,000



### Sequences of 10 or 100 More or Less

#### Goals:

- **Echo** [Module 4](#) Lesson 1 for tens and hundreds:
  - **Build on** [Module 3 Lessons 19-20](#) to:
    - Identify rule in sequence as +10, -10, +100, or -100
    - Determine next number in sequence based on given rule



**Check:** Problem Set problems #2b-c



**Suggestions:** Omit (or assign as “challenge problems”) Problem Set problem #4, Exit Ticket, and Homework problem #4; these types of problems will be covered in later lessons of the module



### Adding and Subtracting Multiples of 100

#### Goals:

- **Echo** [Module 4](#) Lesson 2 for multiples of 100:
  - **Build on** [Module 1](#) to develop strategies for adding multiples of 100

- “Place value” (common units) strategy

$$450 + 300$$

$$= 400 + 50 + 300$$

$$= (400 + 300) + 50$$

$$= 700 + 50$$

$$= 750$$



**Key Ideas:** Add in any order

- “In parts” method 1 (“the arrow way”) strategy

$$450 + 300$$

$$= (450 + 100) + 100 + 100$$

$$= (550 + 100) + 100$$



**Key Ideas:** Add in any order

$$= 650 + 100$$

$$= 750$$

$$450 \xrightarrow{+100} 550 \xrightarrow{+100} 650 \xrightarrow{+100} 750$$

✓ **Check:** Problem Set problem #1c

- **Build on** [Module 1](#) to develop strategies for subtracting multiples of 100

- “Place value” (common units) strategy

$$582 - 300$$

$$= 500 + 82 - 300$$

$$= (500 - 300) + 82$$

$$= 200 + 82$$

$$= 282$$



**Key Ideas:** Subtract from any part of minuend

- “In parts” method 1 (“the arrow way”) strategy

$$582 - 300$$

$$= (582 - 100) - 100 - 100$$

$$= (482 - 100) - 100$$

$$= 382 - 100$$

$$= 282$$



**Key Ideas:** Subtract parts in any order

$$582 \xrightarrow{-100} 482 \xrightarrow{-100} 382 \xrightarrow{-100} 282$$

- “Count on” strategy

$$582 - 300 = \underline{\quad}$$

- $300 + \underline{\quad} = 582$



**Key Ideas:** Subtraction as addition of unknown addend

- $300 + \underline{200} + \underline{82} = 582$

- so  $582 - 300 = 282$

✓ **Check:** Problem Set problem #2c



## Adding Three-Digit Numbers

### Goals:

- **Echo** [Module 4](#) Lessons 3-4 for three-digit numbers:

- **Build on** [Lesson 2](#) to add three-digit numbers

- “In parts” method 1 (“the arrow way”) strategy

$$280 + 230$$

$$= 280 + 200 + 30$$

$$= (280 + 200) + 30$$

$$= 480 + 30$$

$$= 510$$



**Key Ideas:** Add in any order

$$280 \xrightarrow{+200} 480 \xrightarrow{+30} 510$$

- **Build on** [Module 4 Lesson 4](#) (“make 10” strategy) to add three-digit numbers



- “Make multiple of 100” strategy

$$280 + 230$$

$$= \mathbf{280} + 200 + \mathbf{20} + 10$$

$$= (\mathbf{280} + \mathbf{20}) + 200 + 10$$

$$= 300 + 200 + 10$$

$$= 510$$



**Key Ideas:** Add in any order



**Focus:** Concept Development problems #3, #5



**Check:** Problem Set problems #2b & d



## Subtracting Three-Digit Numbers

### Goals:

- **Echo** [Module 4](#) Lessons 3-4 for three-digit numbers:
  - **Build on Lesson 2** to subtract three-digit numbers

- “In parts” method 1 (“the arrow way”) strategy

$$780 - 390$$

$$= 780 - 300 - 90$$

$$= (780 - 300) - 90$$

$$= 480 - 50 - 40$$

$$= 390$$



**Key Ideas:** Subtract parts in any order

$$780 \xrightarrow{-300} 480 \xrightarrow{-50} 430 \xrightarrow{-30} 400 \xrightarrow{-10} 390$$

- **Build on Lesson 3** to subtract three-digit numbers

- “Make multiple of 100” strategy

$$780 - 390$$

$$= \mathbf{780} - 300 - \mathbf{80} - 10$$

$$= (\mathbf{780} - \mathbf{80}) - 300 - 10$$

$$= 700 - 300 - 10$$

$$= 390$$



**Key Ideas:** Subtract parts in any order



**Focus:** Concept Development problem #3




**Check:** Problem Set problems #2a & c




## Adding Three-Digit Numbers (Continued)

**Goals:**

-  **Review/reinforce** Lesson 3: “Make multiple of 100” strategy


- Example:

$$\begin{aligned}
 &199 + 25 \\
 &= 199 + (1 + 24) \\
 &= (199 + 1) + 24 \\
 &= 200 + 24 \\
 &= 224
 \end{aligned}$$

 **Key Ideas:** Add in any order

 **Focus:** Concept Development Parts 2-3


 **Check:** Problem Set problems #2c & f

 **Suggestions:** Write continuous equations (see examples above) rather than separate equations to keep track of all parts of the expression




## Subtracting Three-Digit Numbers (Continued)

**Goals:**

-  **Echo** [Module 4](#) Lesson 4 for three-digit numbers:

- Use [compensation](#) to subtract three-digit numbers
    - Compensation strategy
 
$$\begin{aligned}
 &514 - 290 \\
 &= (514 + 10) - (290 + 10) \\
 &= 524 - 300 \\
 &= 224
 \end{aligned}$$

 **Focus:** Concept Development problems #2, #3

 **Check:** Problem Set problems #1b & d, #2c



## Adding and Subtracting Three-Digit Numbers (Continued)

**Goals:**

-  **Review/reinforce** Lessons 2-6
-  **Check:** Problem Set problem #1

## Topic B: Strategies for Composing Tens and Hundreds Within 1,000



**Cut/consolidate:** Can omit Lessons 8-9 if students are ready to use place value disk drawings without first using the disks/chart



## Adding Three-Digit Numbers: Disks and Written Vertical Form

**Goals:**

- **Echo** [Module 4](#) Lessons 6-7 for three-digit numbers:
  - **Build on Lesson 2** by using place value disks and written vertical form to add two three-digit numbers involving composing a hundred
    - Example:
 
$$\begin{aligned} &211 + 95 \\ &= (200 + 10 + 1) + (90 + 5) \\ &= 200 + (10 + 90) + (1 + 5) \\ &= 200 + 100 + 6 \\ &= 300 + 6 \\ &= 306 \end{aligned}$$



**Focus:** Concept Development problem #2



**Check:** Problem Set problem #1g



## Adding Three-Digit Numbers: Disks and Written Vertical Form (Continued)





**Goals:**

- **Echo** [Module 4](#) Lessons 18-19 for three-digit numbers:
  - **Build on Lesson 8** by using place value disks and written vertical form to add two three-digit numbers involving composing a hundred and composing a ten
    - **Focus:** Concept Development problem #1 or #2
    - **Check:** Problem Set problem #1c



## Adding Three-Digit Numbers: Drawings and Written Vertical Form





**Goals:**

-  **Echo** [Module 4](#) Lesson 8 for three-digit numbers:
  -  **Review/reinforce** Lesson 8 (one composition) by drawing place value disks
  -  **Focus:** Concept Development problem #2
  -  **Check:** Problem Set problem #1c



## Adding Three-Digit Numbers: Drawings and Written Vertical Form (Continued)




**Goals:**

-  **Echo** [Module 4](#) Lesson 20 for three-digit numbers:
  -  **Review/reinforce** Lesson 9 (two compositions) by drawing place value disks
  -  **Focus:** Concept Development problem #1
  -  **Check:** Problem Set problem #1c



## Adding Three-Digit Numbers (Continued)

**Goals:**

-  **Review/reinforce** Lessons 2-11
-  **Focus:** Concept Development problem #2 or #3
-  **Check:** Problem Set problem #2

## Topic C: Strategies for Decomposing Tens and Hundreds Within 1,000



## Subtracting Three-Digit Numbers: Disks and Written Vertical Form



**Cut/consolidate:** Can omit if students are ready to use place value disk drawings without first using the disks/chart

**Goals:**

- **Echo** [Module 4](#) Lessons 11-12 for three-digit numbers:
  - **Build on Lesson 2** by using place value disks and written vertical form to subtract three-digit numbers involving decomposing a ten or hundred

■ **Example:**

$$\begin{aligned} 254 - 135 \\ = 200 + 50 + 4 - 100 - 30 - 5 \end{aligned}$$



$$= 200 - 100 + \underline{50} - 30 + 4 - 5 \leftarrow \text{not enough to subtract}$$

$$\begin{aligned} &= 200 - 100 + \underline{40} + \underline{10} - 30 + 4 - 5 \\ &= 200 - 100 + 40 - 30 + \underline{14} - 5 \end{aligned}$$



$$\begin{aligned} &= 100 + 10 + 9 \\ &= 119 \end{aligned}$$



**Focus:** Concept Development problems #1, #2



**Check:** Problem Set problem #2c-d



## Subtracting Three-Digit Numbers: Drawings and Written Vertical Form (Continued)

**Goals:**

- **Echo** [Module 4](#) Lesson 13 for three-digit numbers:
  - **Review/reinforce** Lesson 13 (one decomposition) by drawing place value disks
  - Verify answer by checking equivalent addition equation



**Focus:** Concept Development problem #1






**Check:** Problem Set problem #1b



## Subtracting Three-Digit Numbers: Drawings and Written Vertical Form (Continued)




### Goals:

-  **Echo** [Module 4](#) Lessons 24-26 for three-digit numbers:
  - **Build on Lesson 14** by using drawings and written vertical form to subtract three-digit numbers involving decomposing a ten and decomposing a hundred
  -  **Focus:** Concept Development problem #3
  -  **Check:** Problem Set problem #1d



## Subtracting from a Three-Digit Number with Zero(s)





### Goals:

-  **Echo** [Module 4](#) Lesson 27 for three-digit numbers:
  - **Build on Lessons 13-15** to subtract from three-digit numbers with zero in the tens (and ones place)
  -  **Focus:** Concept Development problems #1, #2
  -  **Check:** Problem Set problems #1a-b & e



## Subtracting from a Three-Digit Number with Zero(s) (Continued)

### Goals:


-  **Echo** [Module 4](#) Lesson 28 for three-digit numbers:
  -  **Review/reinforce** Lesson 16
  -  **Focus:** Concept Development problem #2, #3
  -  **Check:** Problem Set problems #1c & e




## Subtracting from a Three-Digit Number with Zero(s) (Continued)

**Goals:**

- **Build on Lessons 2 and 6** by using [compensation](#) and “place value” strategies to subtract from multiple of 100
  - Example:
 
$$300 - 159$$


$$= (300 - 1) - (159 - 1)$$
 **Key Ideas:** Compensation
 
$$= 299 - 158$$

$$= 200 + 90 + 9 - 100 - 50 - 8$$

$$= 200 - 100 + 90 - 50 + 9 - 8$$
 **Key Ideas:** Subtract from any part of minuend
 
$$= 100 + 40 + 1$$


$$= 141$$

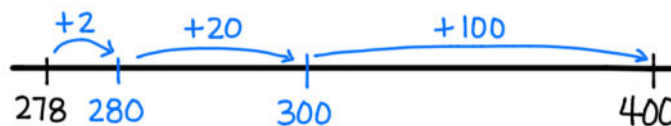
 **Focus:** Concept Development problem #1

 **Check:** Problem Set problem #5a

- **Build on Lesson 2** (“count on” strategy) to subtract from multiple of 100
  - Example:
 
$$400 - 278 = \underline{\quad}$$


$$\blacksquare 278 + \underline{\quad} = 400$$

 **Key Ideas:** Subtraction as addition of unknown addend



- $278 + \underline{2} + \underline{20} + \underline{100} = 400$
- so  $400 - 278 = \underline{122}$

 **Focus:** Concept Development problem #2

 **Check:** Problem Set problem #1b

## Topic D: Student Explanations for Choice of Solution Methods



## Adding and Subtracting Three-Digit Numbers

**Goals:**

- ↺ **Review/reinforce** Lessons 2-18



**Adding and Subtracting Three-Digit Numbers (Continued)**

**Goals:**

- ↺ **Review/reinforce** Lessons 2-18



# Module 6: Foundations of Multiplication and Division

## Summary:

- Addition of equal groups (sum of equal addends):  $n + n + \dots + n$
- Progression of learning:
  - objects in equal groups  $\rightarrow$  objects in array  $\rightarrow$  squares in rectangle
  - groups  $\leftrightarrow$  rows or columns
- To prep students for Grade 3:
  - Multiplication will be *shorthand* for addition of equal groups
    - Example:  $7 \times 2 = 2 + 2 + 2 + 2 + 2 + 2 + 2$  (total of 7 groups of 2)
  - Area of rectangle: counting total number of unit squares that fit inside rectangle

## Definitions:

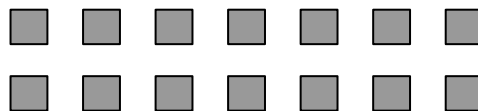
- **Even number:** a number equal to  $n + n$  for some whole number  $n$  (“twice a whole number,” “doubles”).
  - Example:  $14 = 7 + 7$
- **Odd number:** a number that is not even

## Key Ideas:

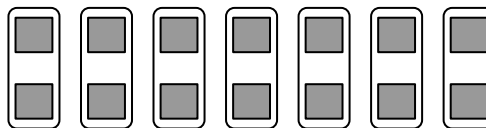
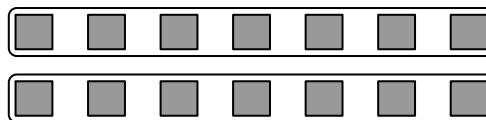
- **Interpretations of an even number:**
  - An even number is equal to  $2 + 2 + \dots + 2$

### Reasoning:

14



$= 7 + 7$   
(so by  
definition of even  
number 14 is



- An even number is in the sequence of skip-counting by twos: 2, 4 ( $= 2 + 2$ ), 6 ( $= 4 + 2 = 2 + 2 + 2$ ), etc.
- An even number is the total when all objects are paired up with none left unpaired.
  - An even number has last digit 0, 2, 4, 6, or 8.

- **Interpretations of an odd number:**

- An odd number is the total when all objects are paired up with one left unpaired.  
Example:  $15 = 7 \text{ pairs} + 1 \text{ unpaired}$
- An odd number is equal to *even number* + 1 (or *even number* – 1).  
Example:  $15 = 14 + 1$ ,  $15 = 16 - 1$

Materials:

- Counters (up to 30 per student/pair) [Topics A-B, D]
- Color Tiles (up to 25 per student/pair) [Topics B-D]
- Ruler [Topic C]

## Topic A: Formation of Equal Groups



## Equal Groups: Objects



**Cut/consolidate:** Can consolidate with Lesson 2 (*Eureka Math's Notes on Pacing*)

**Goals:**

- Arrange a set of objects into equal groups



**Check:** Problem Set problems #2, #4



## Equal Groups: Drawings and Addition Equation

**Goals:**

- Write the addition equation that corresponds to the drawing and statement “\_\_ groups of \_\_ equals \_\_”
  - Example:



“3 groups of 5 equals 15”

$$\underline{\quad} + \underline{\quad} + \underline{\quad} \rightarrow \underline{5} + \underline{5} + \underline{5} \rightarrow \underline{5} + \underline{5} + \underline{5} = 15$$



**Check:** Problem Set problem #1



## Equal Groups: Drawings and Addition Equation (Continued)

**Goals:**

- **Review/reinforce** Lesson 2
- **Check:** Problem Set problem #1a
- Add pairs of addends (“doubles”) as a strategy to find sum more efficiently



## Equal Groups: Tape Diagrams

### Goals:

- **Build on Lesson 2** by representing equal groups with tape diagram (or number line)

✓ **Check:** Problem Set problems #1a & c

## Topic B: Arrays and Equal Groups



**Suggestions:** Incorporate Lesson 8's Concept Development into Lesson 5 or 6 and Problem Set into Lesson 7 (then Lesson 8 can be omitted or used as review)



## Composing Arrays: Objects

### Goals:

- Organize a set of equal groups into an array of rows and columns
  - "Group" is now "row" or "column"



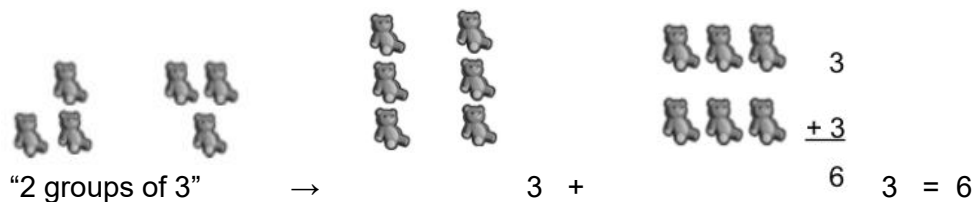
**Focus:** Concept Development problems #1, #2



✓ **Check:** Problem Set problem #2



**Suggestions:** Include the addition equation that corresponds with each array



## Decomposing Arrays: Objects

### Goals:

- Decompose array into rows or columns and write the corresponding addition equation

✓ **Check:** Problem Set problems #1c-d



### Arrays: Drawings

#### Goals:


- Make array drawings and use lines between columns or rows to distinguish the equal groups by columns or rows

✓ **Check:** Problem Set problem #2



### Arrays: Square Tiles

#### Goals:

-  **Review/reinforce** Lessons 5-7 with square tiles

✓ **Check:** Problem Set problem #3



### Word Problems

#### Goals:

- Solve word problems involving addition of equal groups
  - Draw groups, array, or tape diagram
  - Write corresponding addition equation

 **Focus:** Concept Development problems #2, #3

✓ **Check:** Problem Set problems #1, #5

## Topic C: Rectangular Arrays as a Foundation for Multiplication and Division



### Composing Rectangles: Square Tiles



**Cut/consolidate:** Can omit or consolidate with Lesson 11

**Goals:**

- Compose rectangle or square from array of square tiles
- Write addition equation to find total number of square tiles in rectangle
- Note there can be no overlap or gap

✓ **Check:** Problem Set problems #2, #3a



## Composing Rectangles: Square Tiles (Continued)

**Goals:**

- Compose rectangles with different numbers of rows and columns for a given total number of square tiles

✓ **Check:** Problem Set problem #2



## Drawing Rectangles

**Goals:**

- Use one square tile (and ruler) to draw rectangle composed of square tiles

✓ **Check:** Problem Set problem #3



**Suggestions:** For Problem Set problems #1 and #2, allow students to use one square tile to be consistent with Concept Development and Homework



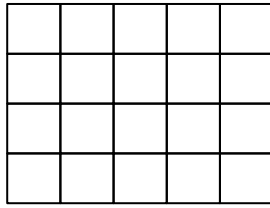
## Decomposing Rectangles: Square Tiles



**Cut/consolidate:** Can omit because concept is a Grade 3 Standard (2.MD.7)

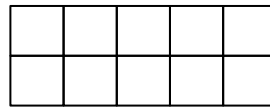
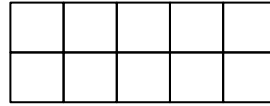
**Goals:**

- Decompose rectangle into smaller rectangles and express total number of square tiles as sum of two repeated addition sums



$$5 + 5 + 5 + 5$$

=



$$(5 + 5) + (5 + 5)$$

✓ **Check:** Problem Set problem #2

- Find the total number of square tiles in a rectangle when one row or column is removed



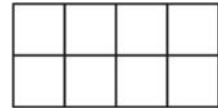
## Decomposing and Composing Rectangles: Paper Squares

### Goals:

- Decompose rectangle into rows and columns, and write corresponding addition equations for total number of square tiles in rectangle
  - Example:

- 2 rows of 4 squares  $\rightarrow 4 + 4 = 8$

- 4 columns of 2 squares  $\rightarrow 2 + 2 + 2 + 2 = 8$



### Notes to teacher:

- Flexibility in switching between rows and columns is necessary for reasoning and understanding the relationship between the two main interpretations of even numbers (**Lessons 17-18**)



**Materials:** Lesson 14 Template (p. 198 of Teacher Edition), scissors



## Drawing Rectangles (Continued)

### Goals:

- Draw rectangle with given dimensions (“\_\_ rows of \_\_” or “\_\_ columns of \_\_”) and write corresponding addition equation for total number of square tiles in rectangle



## Tessellation Designs



**Cut/consolidate:** Can omit or use as enrichment because not directly relevant to Grade 2 Standards

### Goals:

- Use colored squares (and triangles) to create a rectangular design (“core unit”) and repeat “core unit” to create pattern (“tessellation”)





**Materials:** Lesson 16 Template (p. 221 of Teacher Edition), colored pencils/crayons, scissors

## Topic D: The Meaning of Even and Odd Numbers



### Even Numbers


#### Goals:

- Define [even number](#)  
 **Check:** Problem Set problem #1c
- Observe and give reasoning for why an even number is also  $2 + 2 + \dots + 2$  and skip-counting by twos ([interpretations of an even number](#) #1 and #1a)  
 **Check:** Problem Set problem #2b



### Even Numbers (Continued)

#### Goals:

- **Build on Lesson 17** to observe and give reasoning for why an even number is the total when all objects are paired up with none left unpaired ([interpretations of an even number](#) #1b)
- Use definition and interpretations of an even number to determine if a number is even or not  
 **Check:** Problem Set problems #1, #6





## Even and Odd Numbers

### Goals:

- Define [odd number](#)
- ✓ **Check:** Problem Set problem #3b
- Observe and give reasoning for additional [interpretations of an odd number](#)
- Use definitions and interpretations to determine if a number is even or odd
  - Note: If you want to show a number is odd by using the definition (“not even”) or interpretation of an odd number #2 (“even number + 1” or “even number - 1”), then you **also** need to show WHY the number is not even or WHY an addend or the minuend (total) is even
    - Example: 19 is odd because  $19 = 18 + 1 = \text{even number} + 1$  **and** 18 is even because  $18 = 9 + 9$  (or  $18 = 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$ ).

✓ **Check:** Problem Set problems #5a-b



## Sums of Even or Odd Numbers

### Goals:

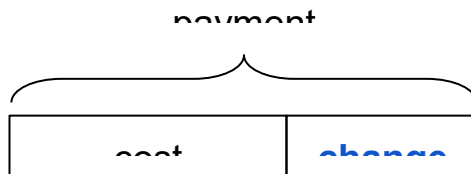
- Use definitions and interpretations of even, odd numbers and reasoning to conclude the following:
  - even + even = even
  - even + odd = odd, odd + even = odd
  - odd + odd = even

✓ **Check:** Problem Set problems #2b-d

# Module 7: Problem Solving with Length, Money, and Data

Definitions:

- To “make change”:
  - $\text{cost} + \text{change} = \text{payment}$
  - $\text{change} = \text{payment} - \text{cost}$



- To **measure length** with (or using) a specified **length unit**: to count the number of length units that fit
  - (1) side by side (no overlap or extra space between) and
  - (2) from one endpoint to the other endpoint of the object
- To **estimate (approximate) length**: to find the number of units that is closest to the actual length of the object
- **Inch, foot, yard**: 12 inches = 1 foot, 3 feet = 1 yard
- “More than”, “less than” on the number line:

Word form	Meaning (Definition)
20 yards more than 35 yards	$35 + 20$
20 yards less than 35 yards	$35 - 20$

## Key Ideas:

- **Relationship between measurement units and values:**

Measuring with a ...	yields a ...
<i>larger</i> unit	<i>smaller</i> value
<i>smaller</i> unit	<i>larger</i> value

**Reasoning:** one larger unit takes more space so fewer can fit in a given space; one smaller unit takes less space so it takes more to fit in a given space

## Materials:

- Money [Topic B]
- Centimeter cubes [Topic C]
- Color tiles [Topic C]
- Ruler [Topics D & F]
- Tape measure [Topic D]

## Topic A: Problem Solving with Categorical Data



## Categories and Table with Counts



**Cut/consolidate:** Can consolidate with Lesson 2 (*Eureka Math's Notes on Pacing*)

**Goals:**

- Sort animals into categories based on characteristics such as classification (bird, mammal, reptile, fish), habitat (arctic, woodland, ocean)
- Record category counts into table
- Solve word problems involving addition, subtraction, or comparison of category counts (see questions on p. 17 of Teacher Edition)



**Check:** Problem Set problems #1, #3a-b



**Materials:** See p. 15 of Teacher Edition; will continue to use in Lesson 2



## Picture Graph

**Goals:**

- **Build on Lesson 1** by representing categorical data as (vertical and horizontal) “picture graphs”



**Check:** Problem Set problems #1a-c



**Materials:** Lesson 2 Templates 1-2 (pp. 46-47 of Teacher Edition)



## Bar Graph



**Cut/consolidate:** Can consolidate with Lesson 4 (*Eureka Math's Notes on Pacing*)

**Goals:**

- **Build on Lesson 2** by representing categorical data as (vertical and horizontal) bar graphs

- Observe that the category count equals the length (or height) of the bar measured by the count axis (number line)

✓ **Check:** Problem Set problem #2a



**Materials:** Lesson 3 Template 2 (p. 62 of Teacher Edition)



### Bar Graph (Continued)

**Goals:**

- ↺ **Review/reinforce** Lesson 3

✓ **Check:** Problem Set problems #1a-b



### Word Problems

**Goals:**

- ↺ **Review/reinforce** Lesson 3

✓ **Check:** Problem Set problems #2a-b

## Topic B: Problem Solving with Coins and Bills



### Coins

**Goals:**

- Use coin values and addition strategies to find total value of a set of coins

✓ **Check:** Problem Set problems #4, #12



### Word Problems with Coins

**Goals:**

- **Build on Lesson 6** to solve word problems involving addition (and/or subtraction) of values of coins

✓ **Check:** Problem Set problems #2, #4



## Word Problems with Dollar Bills

**Goals:**

- Solve word problems involving addition (and/or subtraction) of values of dollar bills

✓ **Check:** Problem Set problems #2, #3



## Different Coin Combinations

**Goals:**

- Compose different coin combinations with the same total value



**Focus:** Concept Development Part 1



**Check:** Problem Set problem #2

- Solve word problems involving composition of different coin combinations with the same total value



**Focus:** Concept Development Part 2 Problem #1



**Check:** Problem Set problem #6



## Fewest Number of Coins

**Goals:**

- Find the coin combination that makes the given total value with the fewest number of coins and observe:



**Key Ideas:**

- Concept Development Part 2: Combination with fewest coins is reached when there is no group of coins that can be exchanged for fewer coins
- Concept Development Part 3: Strategy to find combination with fewest coins - start with highest coin value possible and then work down

- Example:  $27¢ = 25¢ + 1¢ + 1¢$

✓ **Check:** Problem Set problem #3



## Making \$1 or Making Change from \$1



**Cut/consolidate:** Can omit or consolidate with Lesson 12 (*Eureka Math's Notes on Pacing*)

### Goals:

- Define [to “make change”](#)
- Use strategies for subtraction or addition with unknown addend to “make change” from \$1 (payment = 100¢)



**Focus:** Concept Development Part 1

✓ **Check:** Problem Set problem #2b-c



## Making \$1 or Making Change from \$1 Word Problems

### Goals:

- **Build on Lesson 11** to solve word problems involving making \$1 or making change from \$1



**Focus:** Concept Development Parts 1 & 3

✓ **Check:** Problem Set problem #2



## Word Problems with Coins or Dollar Bills

### Goals:

- Solve two-step word problems involving addition and/or subtraction of coin or dollar bill values

✓ **Check:** Problem Set problems #1, #2

## Topic C: Creating an Inch Ruler




**Cut/consolidate:** Can consolidate Lessons 14 and 15 (*Eureka Math's Notes on Pacing*)



## Measuring Length with One Square-Inch Tile

**Goals:**


-  **Echo** [Module 2](#) Lesson 2 (iterated length units) for inches:
  - Use the “mark and move forward” technique with one square-inch tile to [measure](#) and [estimate](#) lengths of linear objects

✓ **Check:** Problem Set problem #1



## Measuring with Ruler

**Goals:**

-  **Echo** [Module 2](#) Lesson 3 (create and measure with ruler) for inches:
  - Define [inch, foot](#)
  - Create and use inch ruler to measure lengths of objects
  - Compare and add or subtract lengths

✓ **Check:** Problem Set problems #6, #7b



## Topic D: Measuring and Estimating Length Using Customary and Metric Units



## Measuring with Customary Units

**Goals:**

- Define [yard](#)
- Use inch ruler and yardstick (or tape measure) to measure lengths of objects
- Determine the appropriate unit (inch, foot, yard) for measuring an object




**Focus:** Concept Development Centers 3-4 (Recording Sheet)



## Estimating with Mental Benchmarks

**Goals:**

-  **Echo** [Module 2](#) Lesson 5 (mental benchmarks) for inches/feet/yards:
  - Use “mental benchmarks” by “picturing them in your head to estimate the length of an object”
    - Sample mental benchmarks:
      - width of quarter: about 1 inch
      - length of paper: about 1 foot
      - width of door: about 1 yard




**Check:** Problem Set problems d-e



## Measuring with Different Units

**Goals:**

-  **Echo** [Module 2](#) Lesson 7 (comparison of different unit measurements) for centimeters and inches:
  - Measure objects with units of different size (e.g., centimeters and inches) and observe the [relationship between measurement units and values](#)




**Check:** Problem Set problems #4, #5



## Comparing Lengths

## Goals:

-  **Echo** [Module 2](#) Lesson 6 for inches/feet/yards:
  - Use the [relationship between comparing and measuring lengths](#) and definition of [subtraction](#) (or [addition with unknown addend](#)) to find difference in lengths

✓ **Check:** Problem Set problem #2


## Topic E: Problem Solving with Customary and Metric Units



## Word Problems

## Goals:

- **Build on Lesson 19** to solve one-step word problems involving subtraction (or addition with unknown addend) of lengths

 **Focus:** Concept Development Part 1 (Problem Set problem #1)

✓ **Check:** Problem Set problem #2

- Solve two-step word problems involving addition (and subtraction) of lengths

 **Focus:** Concept Development Part 2 (Problem Set problem #3)

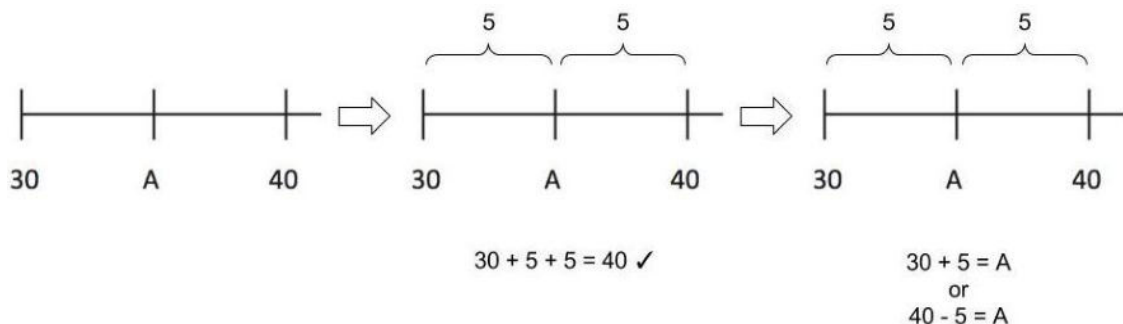
✓ **Check:** Problem Set problem #4



## Finding Unlabeled Point on Number Line

## Goals:

- Use definitions of addition or subtraction to determine if number line has been marked by “units” of 5 or 10 and then find value of unlabeled point





**Focus:** Concept Development problem #1



**Check:** Problem Set problem #2

- Use definitions of addition or subtraction to find values of unlabeled points on the number line marked by “units” of 5 or 10



**Focus:** Concept Development problems #2, #3



**Check:** Problem Set problems #4, #6



## Finding Distance on Number Line

### Goals:

- **Build on Lesson 21** to find [“more than”, “less than” on the number line](#)



**Focus:** Concept Development problems #1, #2



**Check:** Problem Set problems #1a & c, #2a & c

- Use definition of addition or subtraction to find distance (or length) between two points on number line (or to find values of points given distance in between)



**Focus:** Concept Development problem #3



**Check:** Problem Set problem #4

## Topic F: Displaying Measurement Data



**Cut/consolidate:** Can omit Lesson 26 or consolidate with Lesson 25 (*Eureka Math's Notes on Pacing*)



**Suggestions:** Use grid paper to display line plots in Lessons 24-26



## Table with Counts

**Goals:**

- Record data (measurements) as tally marks in a table and solve comparison problems using information in the table

✓ **Check:** Problem Set problems #1, #2




## Line Plot

**Goals:**

- Display measurement data as line plot

 **Focus:** Concept Development Part 1

✓ **Check:** Problem Set problem #1

-  **Review/reinforce** Lesson 23 and Lesson 24 Part 1

 **Focus:** Concept Development Parts 2-3

✓ **Check:** Problem Set problem #2

**Notes to teacher:**

- For line plots:
  - Label horizontal axis (scale) with unit of measurement
  - Use double hash mark to skip numbers



## Line Plot (Continued)

**Goals:**

- Build on Lesson 24** by displaying measurement data as line plot, solving problems involving comparison and totals, and drawing conclusions or making inferences based on the data





 **Focus:** Concept Development Part 1 or 2

✓ **Check:** Problem Set problem #2



## Line Plot (Continued)

**Goals:**

-  **Review/reinforce** Lesson 25
  -  **Focus:** Concept Development Part 1
  -  **Check:** Problem Set problem #2
- Display measurement data as line plot with vertical scale
  -  **Focus:** Concept Development Part 2

# Module 8: Time, Shapes, and Fractions as Equal Parts of Shapes

## Summary:

- Geometry: attributes, classification, construction/drawing, and compositions of shapes [Lessons 1-6]
- Fractions: “equal” partitions of shapes [Lessons 7-12]
- Telling time: half, quarter-partitions of circle [Lessons 13-16]

## Definitions:

- **Angle**: figure formed where two sides of a shape meet
  - **Right angle** (“square corner”): “L-shaped” corner; angle formed by sides of index card
- **Parallel lines**: lines that never cross or touch, even if they kept going in both directions
- **Polygon**: closed shape (no gap or overlap) with straight sides
- **Triangle**: polygon with three sides (and three angles)
- **Quadrilateral**: polygon with four sides (and four angles)
  - **Trapezoid**: quadrilateral with *at least one* pair of parallel sides
  - **Parallelogram**: quadrilateral with *two* pairs of parallel sides
  - **Rectangle**: quadrilateral with four right angles (special parallelogram)
  - **Rhombus**: quadrilateral with four equal sides (special parallelogram)
  - **Square**: quadrilateral with four equal sides and four right angles (rhombus and rectangle)
- **Pentagon**: polygon with five sides (and five angles)
- **Hexagon**: polygon with six sides (and six angles)
- **Cube**: solid shape with equal edges (and square faces)
- **Half/Third/Fourth**:
  - To “cut a shape into halves/thirds/fourths” means “to cut a shape into two/three/four parts with equal space (‘equal parts’)”
  - “1 half/third/fourth of a shape” means “1 part when a shape is cut into halves/thirds/fourths”
- **To tell time to the half or quarter hour**

Time	Analog Clock	Digital Clock (hour:minutes)
<b>Three-fifteen</b> <i>Quarter past 3 o'clock</i>	hour (short) hand pointed <b>between 3 and 4, closer to 3</b> minute (long) hand pointed <i>at 3</i>	<b>3:15</b>
<b>Three-thirty</b> <i>Half past 3 o'clock</i>	hour (short) hand pointed <b>halfway between 3 and 4</b> minute (long) hand pointed <i>at 6</i>	<b>3:30</b>

<b>Three-forty-five</b> <i>Quarter to 4</i> o'clock	hour (short) hand pointed <b>between 3 and 4, closer to 4</b> minute (long) hand pointed <i>at 9</i>	<b>3:45</b>
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- **a.m., p.m.**
  - “a.m.” for times starting at 12:00 midnight to right before 12:00 noon
  - “p.m.” for times starting at 12:00 noon to right before 12:00 midnight

#### Materials:

- Attribute blocks [Topics A-B]
- Geoboards and rubber bands [Topics A, C]
- Rulers [Topic A]
- Index cards [Topic A]
- Tangrams [Topic A]
- Pattern blocks [Topic B]
- Clocks [Topic D]

## Topic A: Attributes of Geometric Shapes



### Shapes

#### Goals:

- Define [angle](#)
- Construct closed shapes with 3-6 sides or angles
- Identify number of sides and angles in a shape

✓ **Check:** Problem Set problems #1d-f



### Polygons

#### Goals:

- Construct polygons with specified attributes
- Name two-dimensional shapes based on attributes (see definitions above)
  - Review from Grades K-1: [triangle](#), [hexagon](#)
  - New in Grade 2: [polygon](#), [quadrilateral](#), [pentagon](#)

✓ **Check:** Problem Set problems #1c-f



**Materials:** See p. 25 of Teacher Edition for materials and instruction note



### Polygons (Continued)

#### Goals:

- Draw polygons with specified attributes
  - 🔍 **Focus:** Concept Development Part 1
  - ✓ **Check:** Problem Set problems #1b-c
- Sort polygons or draw specified polygons
  - 🔍 **Focus:** Concept Development Part 2
  - ✓ **Check:** Problem Set problems #2c-d





## Right Angles, Parallel Lines, and Quadrilaterals

### Goals:

- Draw and identify [right angles](#) (“square corners”) with index card, [parallel lines](#) with ruler



**Focus:** Concept Development Part 1



**Check:** Problem Set problems #1, #3a-d

- Draw [quadrilaterals](#) with specified attributes using ruler and index card
  - New in Grade 2: [parallelogram](#) (other quadrilaterals were introduced in Grades K-1 but were not drawn using ruler and index card in those grades)



**Focus:** Concept Development Parts 2-4



**Check:** Problem Set problems #4, #5



## Cubes

### Goals:

- Construct and draw [cubes](#)



**Check:** Problem Set problem #6

### Notes to teacher:

- Problem Set problem #1 and Homework problem #1 assume “when looking *straight* at the face of a cube”



**Materials:** See p. 70 of Teacher Edition for materials and instruction note

## Topic B: Composite Shapes and Fraction Concepts



## Composite Shapes

### Goals:

-  **Review/reinforce** Lessons 2-4: Name and describe attributes of polygons

- Cut tangram pieces (each student needs a set for the Problem Set, Homework, and Lesson 7 Concept Development Part 1)



**Focus:** Concept Development Part 1



**Check:** Problem Set problem #1

- Create composite shapes with tangrams



**Focus:** Concept Development Part 2



**Check:** Problem Set problem #2



## Composite Shapes with Same (Congruent) Shapes

### Goals:

- Create composite shapes made with the same (congruent) shapes



**Check:** Problem Set problems #1a-e

- Use definition of [half, third, fourth](#) to identify if a shape has been cut into halves, thirds, or fourths



**Check:** Problem Set problem #2



## Half, Third, Fourth of a Shape

### Goals:

- Use definition of [half, third, fourth](#) to:
  - Identify the shape that would cover one half, third, or fourth of the given shape
  - Identify the fraction corresponding to the shaded part(s) of the shape



**Materials:** Students need pattern blocks to complete Homework problems #1-4, #6

## Topic C: Halves, Thirds, and Fourths of Circles and Rectangles



**Suggestions:**

- To check “equal parts” (parts that take up the same amount of space (area)), trace a part onto wax/patty paper and then see if it can fit exactly on top of the other part(s) after sliding/rotating/flipping
- Use finger to trace or outline “parts” and “whole” as count or identify parts, whole
- Consider the materials for each lesson



## Halves



**Cut/consolidate:** Can consolidate with Lesson 10

### Goals:

- Partition circle and rectangle into halves, and shade in one half of the shape
- **Review/reinforce** Lesson 7: Identify if a shape has been cut into halves



**Check:** Problem Set problems #2c-f, #3



## Halves, Thirds, and Fourths

### Goals:

- **Echo** Lesson 9 to include thirds and fourths:
  - Partition circle and rectangle into halves, thirds, or fourths, and shade in one half, third, or fourth of the shape
  - **Review/reinforce** Lesson 7: Identify if a shape has been cut into halves, thirds, or fourths



**Check:** Problem Set problems #1, #4a-e



## One (“Whole”)

### Goals:

- Identify the number of halves, thirds, or fourths to complete a rectangle or circle, which represents the number 1 (“whole”)

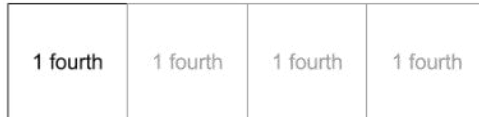


**Focus:** Concept Development Parts 1-2



**Check:** Problem Set problems #1a-c, #2c-d

- Use the identified half, third, or fourth to draw 1 (“whole”)
  - Note: more than one configuration is possible (preview of **Lesson 12 Part 3**)



**Focus:** Concept Development Part 3



**Check:** Problem Set problem #3b



## Equal Parts with Different Shapes

### Goals:

- Observe and confirm the equality of halves, thirds, or fourths with different shapes



**Focus:** Concept Development Parts 1-2



**Check:** Problem Set problem #1

## Topic D: Application of Fractions to Tell Time



### Telling Time to the Half or Quarter Hour

#### Goals:

- **Build on** [Grade 1 Module 5 Lessons 10-12](#) to [tell time to the half or quarter hour](#)



**Check:** Problem Set problem #2

- Reason that quarter (fourth) of an hour is 15 minutes by
  - observing that the minute hand has traveled a quarter (fourth) of the clock's face when it starts at 12 and ends at 3, and
  - counting the minute marks from 12 to 3 by ones or groups of five minutes



**Suggestions:** Skip Concept Development Part 2 by borrowing clocks from other grades/classrooms



## Telling Time to Nearest Five Minutes

### Goals:

- Tell time to the nearest five minutes

✓ **Check:** Problem Set problem #3



## a.m. and p.m.

### Goals:

- Use [a.m.](#), [p.m.](#) to distinguish first twelve hours from second twelve hours in day

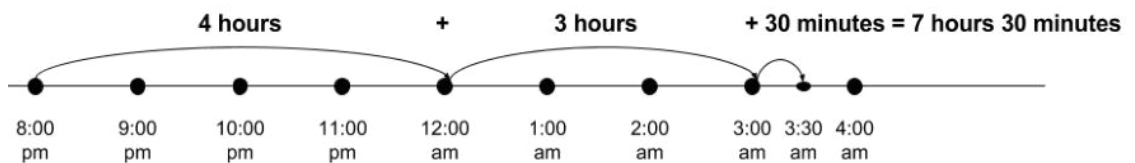
✓ **Check:** Problem Set problems #1a-b, #3



## Time Elapsed

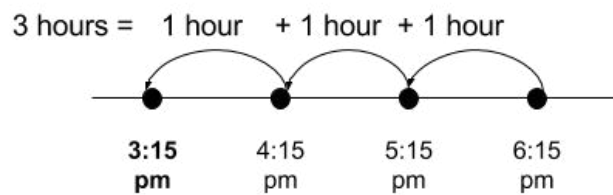
### Goals:

- Find time elapsed between hours and/or half hours and solve word problems that involve finding elapsed time



✓ **Check:** Problem Set problems #1c-f, #2a

- For Problem Set problems #2b-c and Homework problems #2b-c: Solve word problems that involve finding start or end time given time elapsed and end or start time, respectively



**Suggestions:** Use the number line to solve elapsed time problems as an additional or alternative tool to the physical clock (see examples above)