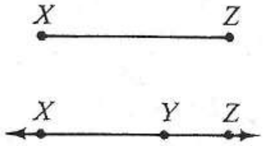
NAME ______________________________ DATE ______________

1–3 Segments, Rays, and Distance

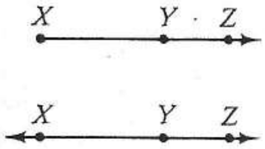
Objectives: Use symbols for lines, segments, rays, and distances; find distances. Use the Segment Addition Postulate.

In learning a new language, the first things you need to learn are vocabulary and rules of grammar. In geometry, you need vocabulary, symbols, and rules called *postulates.*

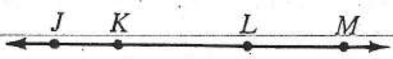
segment A segment is named by giving its endpoints. X and Z are the endpoints of $\overline{XZ}$. $\overline{XZ}$ and $\overline{ZX}$ are the same segment. Y is **between** X and Z. Y must be on $\overleftrightarrow{XZ}$.



ray A ray is named by giving its endpoint and another point on the ray. The endpoint of a ray is always named first.
$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are the same ray.
$\overrightarrow{XZ}$ and $\overrightarrow{ZX}$ are different rays.
$\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are **opposite rays.**

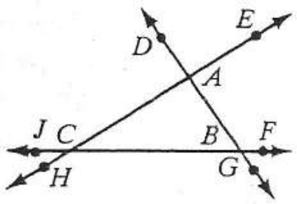


Refer to the diagram at the right.



1. Give several names for the line.
2. Name several segments in the figure.
3. Name several rays in the figure.
4. Name two pairs of opposite rays.

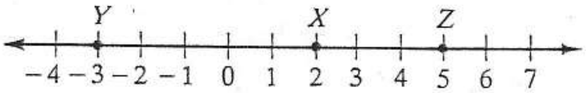
Classify each statement as true or false.



5. C is between A and B.
6. $\overrightarrow{AD}$ and $\overrightarrow{AG}$ are opposite rays.
7. $\overline{CB}$ is the same as $\overline{BC}$.
8. $\overrightarrow{CB}$ is the same as $\overrightarrow{BC}$.
9. $\overleftrightarrow{CB}$ is the same as $\overleftrightarrow{BC}$.
10. $\overline{CB}$ is the same as $\overleftrightarrow{BC}$.
11. $\overleftrightarrow{JF}$ is the same as $\overleftrightarrow{CB}$.
12. $\overrightarrow{BA}$ is the same as $\overrightarrow{BD}$.

length XZ is the length of $\overline{XZ}$ or the distance between point X and point Z. You can find the length of a segment on the number line by computing the absolute value of the difference of the coordinates of the endpoints. Length must be a positive number.

Example 1
Find XZ and YX.



Solution

$XZ = |2 - 5| = |-3| = 3$ $YX = |-3 - 2| = |-5| = 5$

or $XZ = |5 - 2| = |3| = 3$ *or* $YX = |2 - (-3)| = |5| = 5$

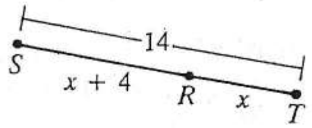
Segment Addition Postulate If B is between A and C, then $AB + BC = AC$.

NAME ______________________________ DATE ______________

1–3 Segments, Rays, and Distance *(continued)*

Example 2

R is between S and T, with $RT = x$,
$SR = x + 4$, and $ST = 14$.
Find the value of x. Then find RT and SR.



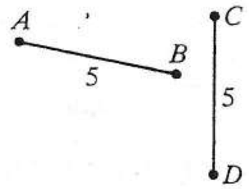
Solution

$SR + RT = ST$ (by the Segment Addition Postulate)

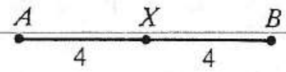
$$x + x + 4 = 14$$
$$2x + 4 = 14$$
$$2x = 10$$
$$x = 5 \qquad RT = x = 5$$

$$SR = x + 4 = 5 + 4 = 9$$

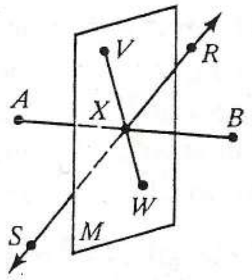
congruent Two objects that have the same size and shape are congruent ($\cong$). For example, **congruent segments** have the same length. If $AB = CD$, then $\overline{AB} \cong \overline{CD}$.



midpoint of a segment A midpoint divides a segment into two congruent segments. X is the midpoint of $\overline{AB}$, so $\overline{AX} \cong \overline{XB}$.

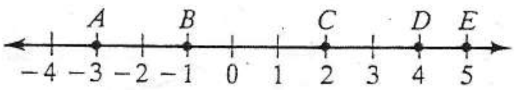


bisector of a segment A segment bisector is a line, segment, ray, or plane that intersects a segment at its midpoint.
$AX = XB$, so plane M, $\overleftrightarrow{RS}$, and $\overline{VW}$ are all bisectors of $\overline{AB}$.

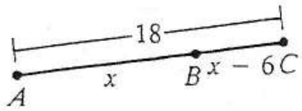


For Exercises 13–16, refer to the number line at the right.

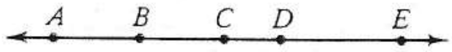
13. Find BD.
14. Find the length of $\overline{AC}$.
15. Find the distance between B and E.
16. Find the coordinate of the midpoint of $\overline{AE}$.



17. Find the value of x.



18. In the diagram, $\overline{AC} \cong \overline{CE}$ and B is the midpoint of $\overline{AC}$. $CD = 2$ and $AB = 3$. Find BC, AC, and DE.

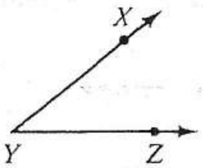


NAME ______________________________ DATE ______________

1–4 Angles

Objectives: Name angles and find their measures. Use the Angle Addition Postulate.

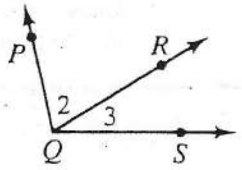
angle A figure formed by two rays with the same endpoint is an angle. In $\angle XYZ$, $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are the **sides** of the angle. Y is called the **vertex** of the angle. Another name for $\angle XYZ$ is $\angle Y$.



Example 1

a. Name three different angles in the diagram.

b. Give another name for $\angle PQR$ and for $\angle 3$.



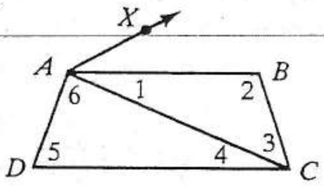
Solution

a. $\angle PQS$, $\angle PQR$, $\angle RQS$

b. $\angle PQR$ can be called $\angle 2$; $\angle 3$ can be called $\angle RQS$. In this figure, you could not call any of the angles $\angle Q$ because it would not be clear which angle with vertex Q you meant.

Refer to the diagram at the right.

1. Name the vertex and sides of $\angle XAC$.
2. How many angles have A as the vertex? List them.
3. Give another name for $\angle 6$, $\angle ABC$, $\angle ADC$, and $\angle 4$.



measure of an angle You can use a protractor to find a number associated with each side of an angle. To find the measure in degrees of an angle ($m\angle XYZ$), compute the absolute value of the difference of these numbers.

Example 2

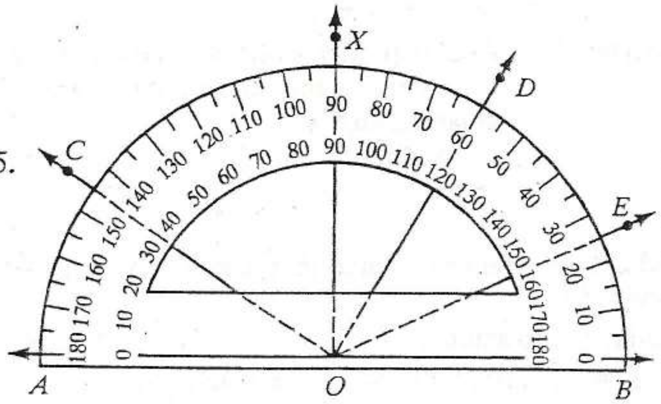
Find $m\angle COD$ and $m\angle BOE$.

Solution

As you can see, $\overrightarrow{OD}$ is on 60 and $\overrightarrow{OC}$ is on 145.

$m\angle COD = |60 - 145|$
$= |-85|$
$= 85$

$m\angle BOE = |25 - 0|$
$= 25$



Referring to the protractor and angles in Example 2 above:

$\angle XOC$ is **acute** since $0 < m\angle XOC < 90$.

$\angle AOX$ is a **right** angle since $m\angle AOX = 90$.

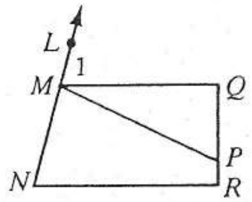
$\angle AOE$ is **obtuse** since $90 < m\angle AOE < 180$.

$\angle AOB$ is a **straight** angle since $m\angle AOB = 180$.

NAME ______________________________ DATE ______________

1–4 Angles *(continued)*

State whether each angle appears to be acute, right, obtuse, or straight. Then estimate its measure.

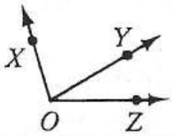


4. $\angle Q$ **5.** $\angle 1$

6. $\angle RPM$ **7.** $\angle LMN$

8. $\angle PMQ$ **9.** $\angle PML$

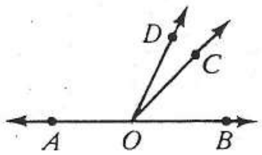
Angle Addition Postulate **If a point Y lies in the interior of $\angle XOZ$, then $m\angle XOY + m\angle YOZ = m\angle XOZ$.**



Example 3

Given: $m\angle AOD = 4y - 8; m\angle DOC = y - 11;$
$m\angle COB = y + 13$

Find the measure of $\angle AOD$.



Solution

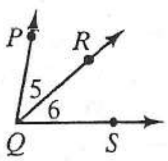
$m\angle AOD + m\angle DOC + m\angle COB = 180$ (by the Angle Addition Postulate)

$$4y - 8 + y - 11 + y + 13 = 180$$
$$6y - 6 = 180$$
$$6y = 186$$
$$y = 31$$

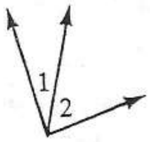
$m\angle AOD = 4y - 8 = 4(31) - 8 = 116$

congruent angles Two angles with equal measures are congruent. If $m\angle 5 = m\angle 6$, then $\angle 5 \cong \angle 6$.

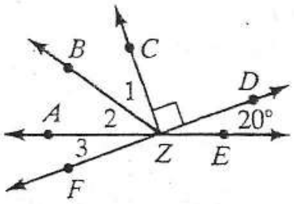
bisector of an angle The ray that divides an angle into two congruent angles is the angle bisector. If $\overrightarrow{QR}$ bisects $\angle PQS$, then $\angle 5 \cong \angle 6$, or $m\angle 5 = m\angle 6$.



adjacent angles Coplanar angles with a common vertex and a common side but no common interior points are adjacent angles. $\angle 1$ is adjacent to $\angle 2$.



Complete.



10. $\angle 2$ and $\angle 3$ are adjacent. Name their common vertex and common side.

11. $\angle 1$ is adjacent to acute $\angle$ _?_.

12. $m\angle DZE$ = _?_, $m\angle CZD$ = _?_, $m\angle AZC$ = _?_

13. If $\overrightarrow{ZB}$ bisects $\angle AZC$, then $m\angle$ _?_ $= m\angle$ _?_ $=$ _?_.

14. $m\angle 1 + m\angle 2 + m\angle 3$ = _?_

15. If $m\angle 3 = 20$, $m\angle 2 = 3x - 5$, and $m\angle 1 = 2x + 10$, find the value of x.

NAME ______________________________ DATE ______________

1–5 Postulates and Theorems Relating Points, Lines, and Planes

Objective: Use postulates and theorems relating points, lines, and planes.

postulate	a basic assumption accepted without proof
theorem	a statement that can be proved using postulates, definitions, and previously proved theorems
exists	there is at least one
unique	there is no more than one
one and only one	exactly one
determine	to define or specify

Example

Restate Postulate 6 using the word *determine.*
Postulate 6: Through any two points there is exactly one line.

Solution

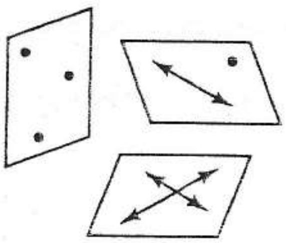
Two points determine a line.

Relationships between points

Two points must be collinear. (Postulate 6)
Three points may be collinear or noncollinear.
Three points must be coplanar. (Postulate 7)
Three noncollinear points determine a plane. (Postulate 7)
Four points may be coplanar or noncoplanar.
Four noncoplanar points determine space. (Postulate 5)
Space contains at least four noncoplanar points.

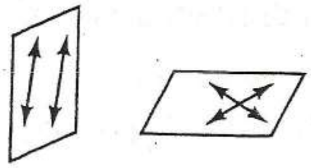
Three ways to determine a plane

Three noncollinear points determine a plane. (Postulate 7)
A line and a point not on the line determine a plane. (Theorem 1–2)
Two intersecting lines determine a plane. (Theorem 1–3)



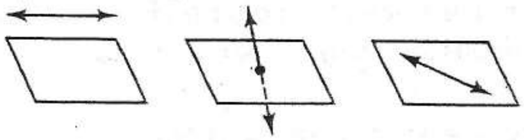
Relationships between two lines in the same plane

Either two lines are parallel, or they intersect in exactly one point.



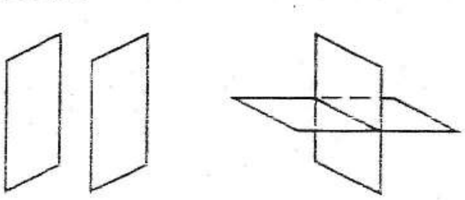
Relationships between a line and a plane

Either a line and a plane are parallel, or they intersect in exactly one point, or the plane contains the line.



Relationships between two planes

Either two planes are parallel, or they intersect in a line.



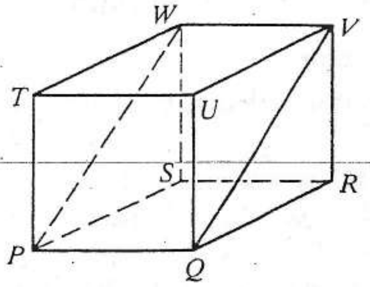
NAME ______________________________ DATE ______________

1–5 Postulates and Theorems Relating Points, Lines, and Planes *(continued)*

Draw a figure to represent each of the following:

1. Three coplanar lines that do not intersect
2. Three coplanar lines that intersect in exactly one point
3. A line that intersects each of two coplanar non-intersecting lines
4. Three planes that do not intersect
5. Three planes that intersect in one line
6. A plane that intersects two non-intersecting planes
7. Three planes that intersect in exactly one point

In Exercises 8–15 you will have to visualize certain lines and planes not shown in the diagram. When you name a plane, name it by using four points, no three of which are collinear. Often, more than one answer is possible.



8. Name a plane that contains $\overleftrightarrow{PR}$.
9. Name a plane that contains $\overleftrightarrow{PR}$ but is not shown in the diagram.
10. Name a plane that contains $\overleftrightarrow{PQ}$ and $\overleftrightarrow{WV}$.
11. Name a plane that contains $\overleftrightarrow{TW}$ and $\overleftrightarrow{RQ}$.
12. Name the intersection of plane *TWRQ* and plane *PSWT*.
13. Name five lines in the diagram that don't intersect plane *UVRQ*.
14. Name one line that is not shown in the diagram that does not intersect plane *UVRQ*.
15. Name three planes that don't intersect $\overleftrightarrow{SR}$ and don't contain $\overleftrightarrow{SR}$.

Classify each statement as true or false.

16. The intersection of a line and a plane may be the line itself.
17. Three noncollinear points determine exactly one line.
18. Two points can determine two lines.
19. Two lines can intersect in exactly one point.
20. Postulates are statements to be proved.
21. Two points determine a plane.
22. A line and a point not on it determine one plane.
23. A plane contains at least three noncollinear points.
24. Line *l* always has at least two points on it.
25. Theorems are statements to be proved.
26. Any three points are always coplanar.
27. It is possible that points *P* and *Q* are in plane *M* but $\overleftrightarrow{PQ}$ is not.
28. Two intersecting lines determine a plane.
29. Two planes can intersect in two lines.
30. If points *A*, *B*, *C*, and *D* are noncoplanar, then no one plane contains all four of them.
31. Two planes can intersect in exactly one point.
32. Three planes can intersect in exactly one point.
33. A line and a plane can intersect in exactly one point.