Geometric optics

Light behaves as rays of light and also as electromagnetic waves. We are able to see because of light.

Luminous objects generate their own light and illuminated objects reflect light.

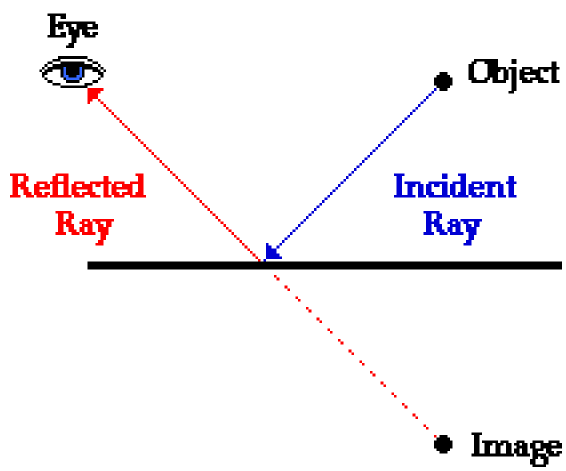
In order to see an object, it must be in our line of sight, so that light from the object comes to our eyes.

In geometric optics we will use the ray model.

Two main principals in this unit:

1) Light travels in straight lines
2) Light will bend according to Snell's law at the interface of 2 media.

Plane mirror:



For all plane mirrors, **Object distance = Image distance**

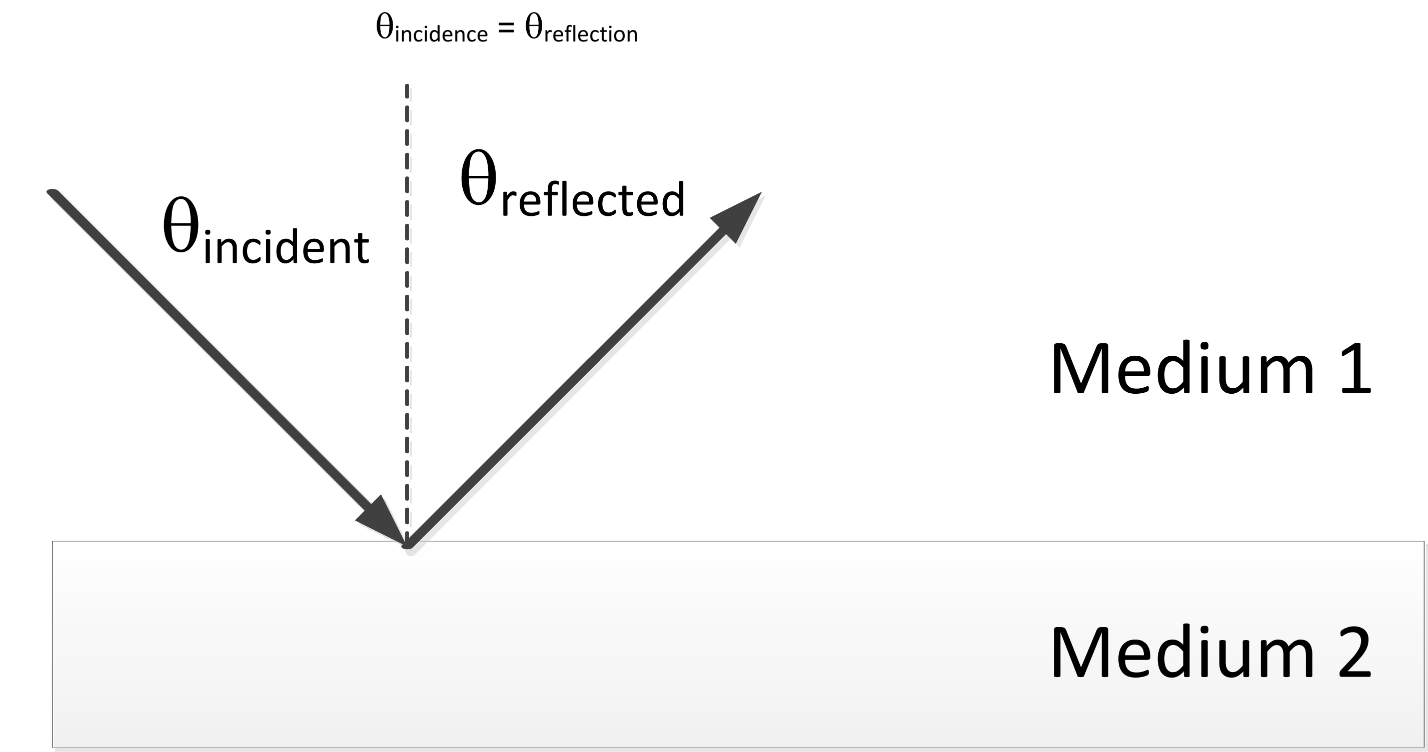
Define *The normal*:

When we discuss reflection and refraction of light, the normal is defined as the perpendicular line to the boundary of the interface between two media.

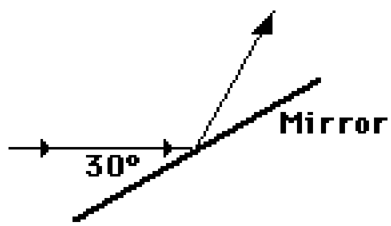
All angles are measured wrt the normal.

Below is a diagram showing light reflecting off a plane mirror, with the incident angle measured wrt the normal.

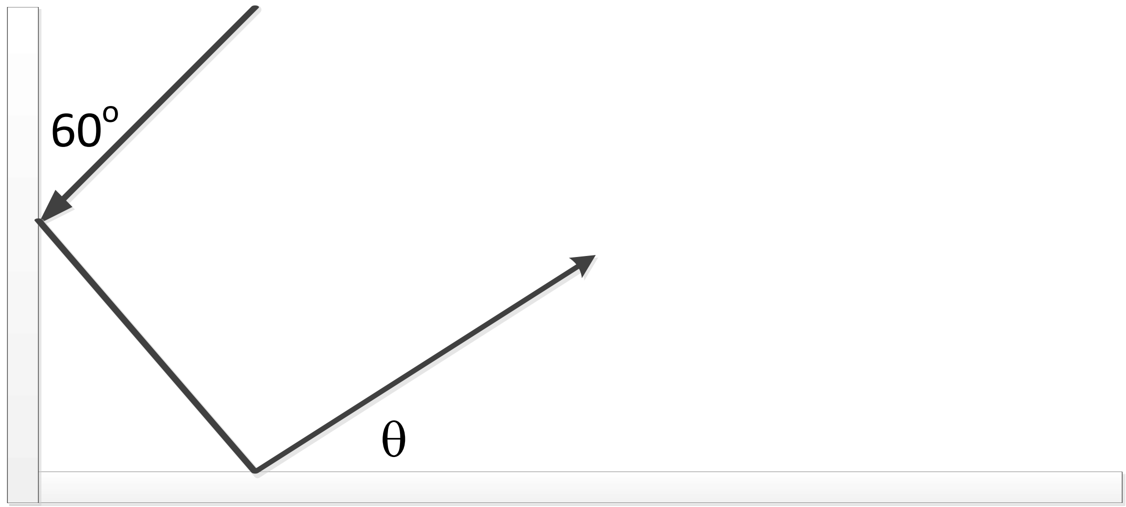
Law of reflection: The angle of incidence = angle of reflection



problem 1: A ray of light is incident towards a plane mirror at an angle of 30-degrees with the mirror surface. What will be the angle of reflection?

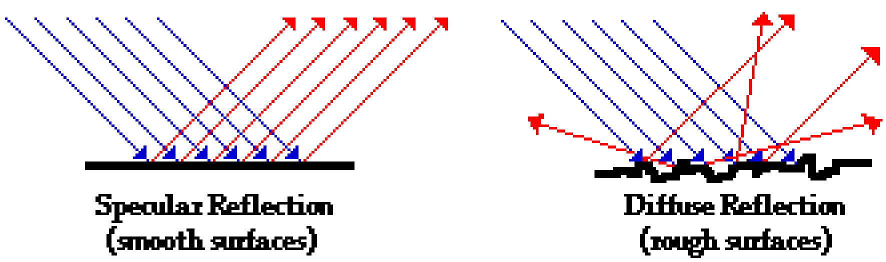


problem 2: A ray of light strikes a completely mirrored surface as shown in the diagram. [The mirrors meet at a right angle] Determine unknown angle θ.



problem 3: A ray of light is approaching a set of three mirrors as shown in the diagram. The light ray is approaching the first mirror at an angle of 45-degrees with the mirror surface. Trace the path of the light ray as it bounces off the mirror. Continue tracing the ray until it finally exits from the mirror system. How many times will the ray reflect before it finally exits?

light reflects off all surfaces:



question 4: Perhaps you have observed magazines which have glossy pages. The usual microscopically rough surface of paper has been filled in with a glossy substance to give the pages of the magazine a smooth surface. Do you suppose that it would be easier to read from rough pages or glossy pages? Explain your answer.

Snell's law and refraction

Refraction occurs when light traveling in one medium is incident on a second medium. The speed of light will change in the second medium. This causes the light to bend at the interface of the two media. The exception to the bending is when the angle of incidence is 0 degrees [wrt the normal]. Snell's law explains this behavior:

Snell's Law: $\mathbf{n_1 \sin\theta_{incidence} = n_2 \sin\theta_{refraction}}$

Also written $\mathbf{n_1 \sin\theta_1 = n_2 \sin\theta_2}$

or

$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1}$$

where $\mathbf{n_1}$ = index of refraction of medium 1

and $\mathbf{n_2}$ = index of refraction of medium 2

index of refraction can be thought of as *optical density*.

Speed of light $\mathbf{c = 3X10^8 m/sec}$ in a vacuum.

in a vacuum n= 1 also in air, n is ≈ 1

index of refraction for any medium is

$$\mathbf{n = c / v_{medium}}$$

where v_{medium} = velocity of light in that medium

problem 2: If the index of refraction for glass = 1.5, what is the speed of light in glass?

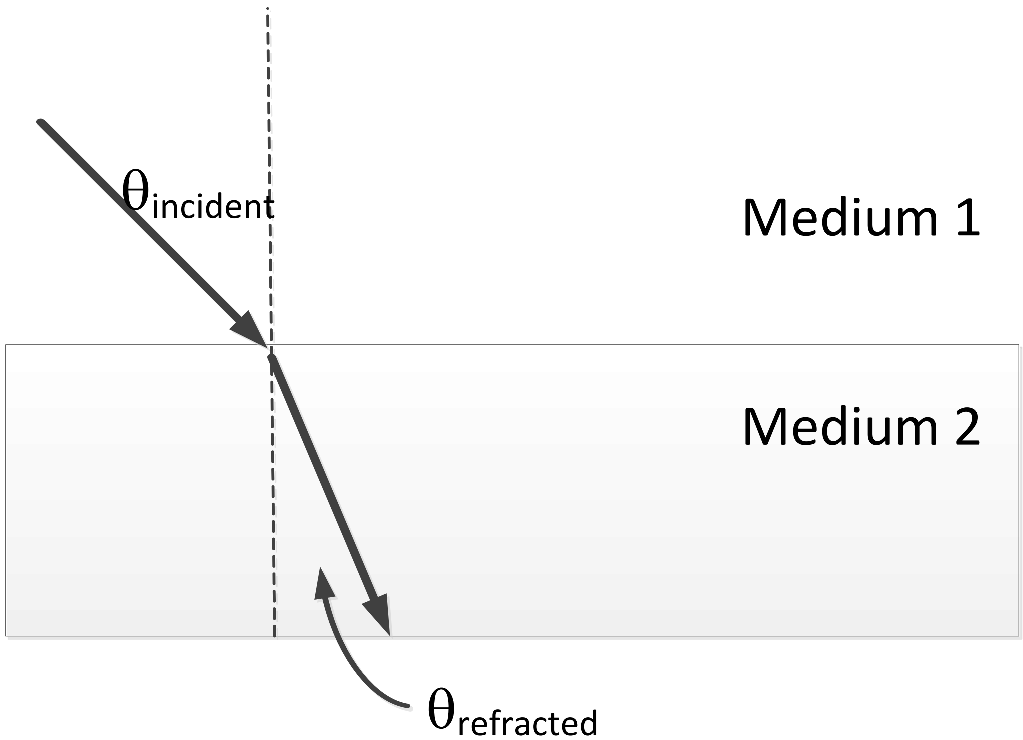
Note: the speed of light in any medium *can never* be greater than c.

When light ray is refracted **from more dense medium to less dense** medium it ***bends away from the normal***

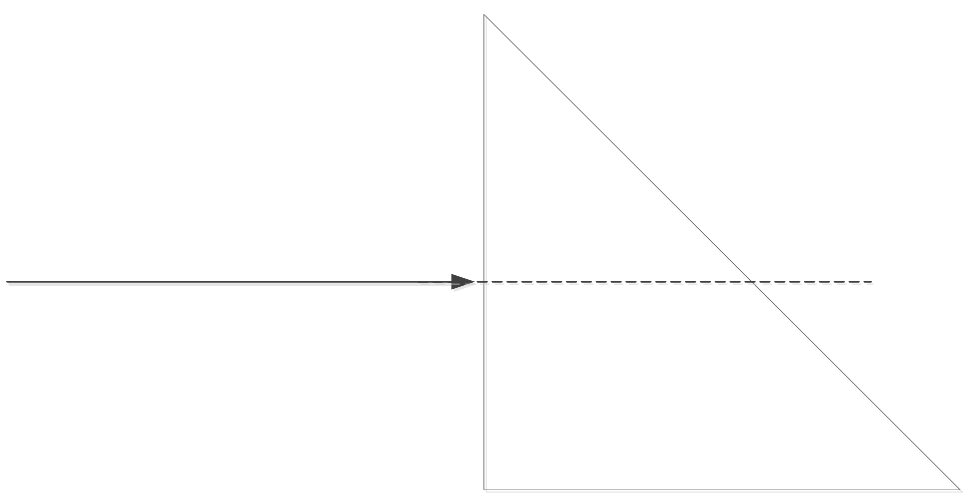
light ray is refracted when going from less dense medium to more dense medium,

it bends towards the normal

Problem 3: label the sketch indicating which medium is air and which medium is glass.



problem 4: Draw the light refracting through the following prism with is a 45-45-90 prism. **Calculate the angle** above or below the horizontal line that the beam emerges from the prism. Then index of refraction for the glass is 1.3.



Experimental facts about refraction observed in lab experiments:

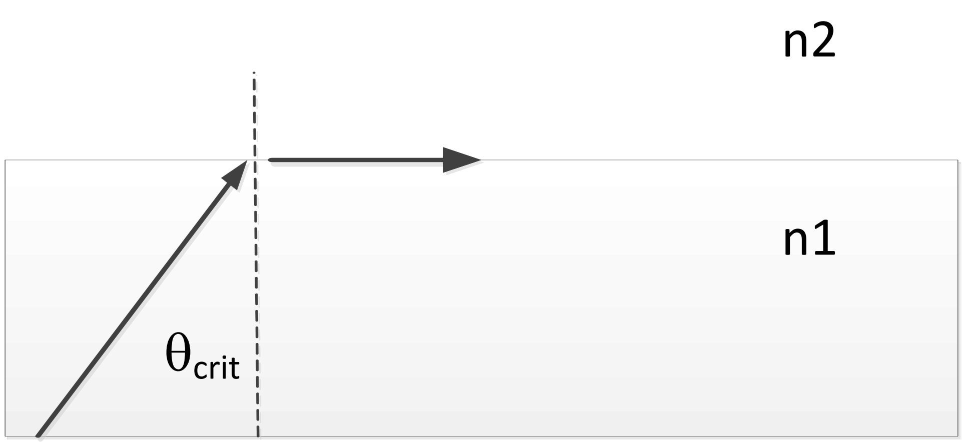
- When light strikes the boundary between two media, there will always be some reflected light and some refracted light. The amount of each depends on the media.

- The indices of refaction are wavelength dependent. Each color in the spectrum has a particular value for the index of refraction in a particular medium. The indices do not vary greatly , but at large angles of refraction , their wavelength dependence shows up in the phenomenon called dispersion. Dispersion occurs when refracted light is spread out into a spectrum.

Critical angle:

The critical angle is the angle at which all the incident light is reflected back by an interface boundary. This can only occur when going from a more dense to less dense medium. This critical reflection is called total internal reflection [TIR]. Here is a diagram of TIR:

Note $n_2 > n_1$



Determine $\boldsymbol{\theta_{crit}}$: $\mathbf{n_1 \sin\theta_1 = n_2 \sin\theta_2}$

becomes : $n_1 \sin\theta_{crit} = \qquad n_2 \sin 90$

$n_2 = n_1 \sin\theta_{crit}$

$n_2 / n_1 = \sin\theta_{crit}$

$\theta_{crit} = \sin^{-1} [n_2/n_1]$ **where** $n_2 < n_1$

if second medium is air, $n_2 = 1$, **then** $\theta_{crit} = \sin^{-1} [1/n_1]$

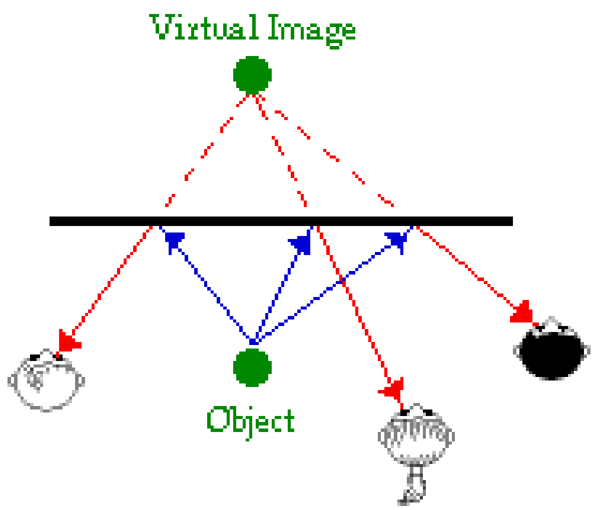
TIR is reason fiberoptic cables work. As long as cable is not bent into too small angle, light bounces off the boundary of the cable and there is no light refracted out.

Lenses and mirrors

Lenses and mirrors use the same mathematics and geometry

MIRRORS

In order to see the image of an object in a mirror, you must sight at the image; when you sight at the image, light will come to your eye along that line of sight.



All observers would perceive light to be diverging from the same point - the image point.

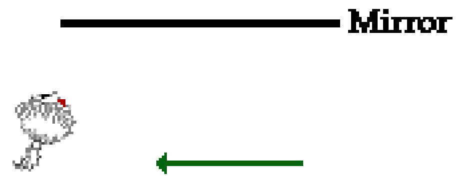
image in a plane mirror:

- is a ***virtual image***
- ***is upright [not inverted]***
- ***is left-right reversed***

- ***is same size as object [magnification = 0]***

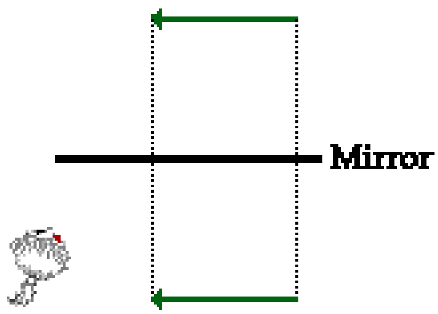
problem 5: If Suzie stands 3 feet in front of a plane mirror, how far from the person will her image be located?

RAY DIAGRAMS FOR PLANE MIRRORS



1) Draw the object as an arrow such as ↑ or ←

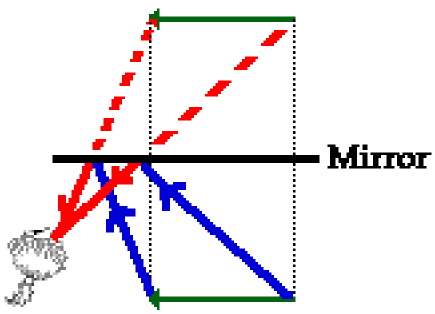
2) draw the image of the object. The object distance is the same as the image distance.



3. Pick one extreme on the image of the object and draw the reflected ray that will travel to the eye as it sights at this point.

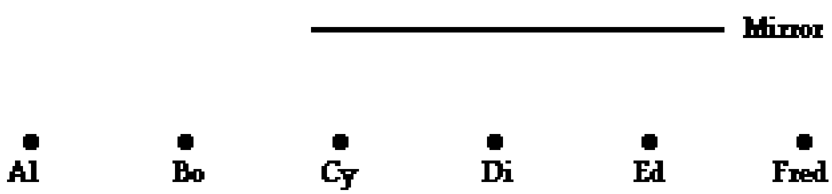
4. Draw the incident ray for light traveling from the corresponding extreme on the object to the mirror.

5. Repeat steps 3 and 4for all other extremities on the object.

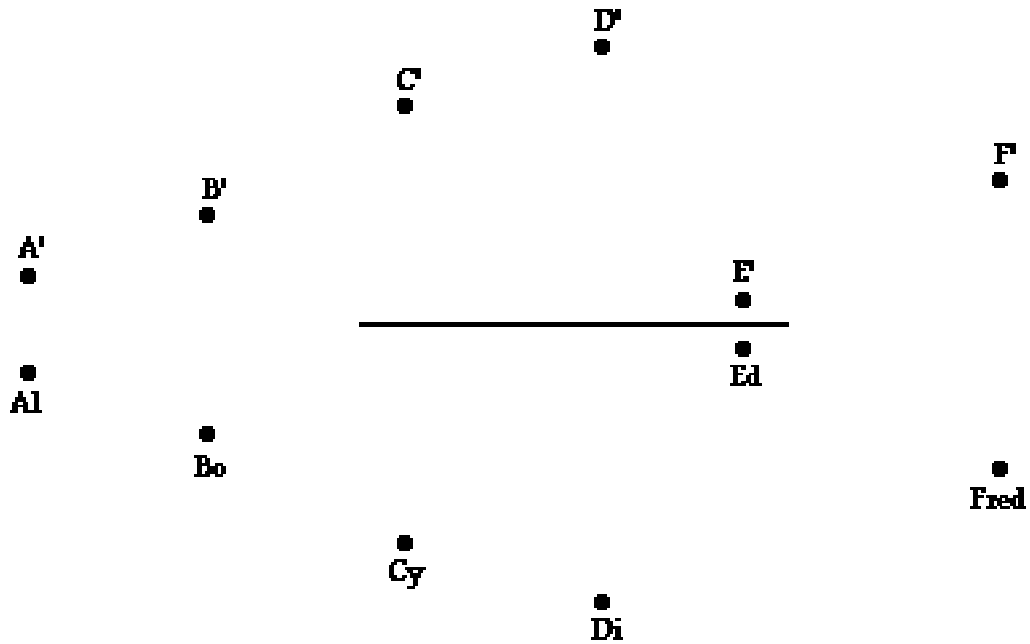


problem 6: draw the ray diagrams for theses plane mirrors:

problem 7: six students - Al, Bo, Cy, Di, Ed, and Fred sit *in front of* a plane mirror and attempt to see each other in the mirror. And suppose the exercise involves answering the following questions: Whom can Al see? Whom can Bo see? Whom can Cy see? Whom can Di see? Whom can Ed see? And whom can Fred see?



problem 8: Six students are arranged in front of a mirror. Their positions are shown below. The image of each student is also drawn on the diagram. Make the appropriate line of sight constructions to determine that students each individual student can see.

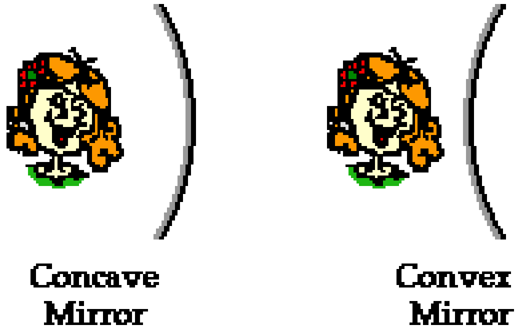


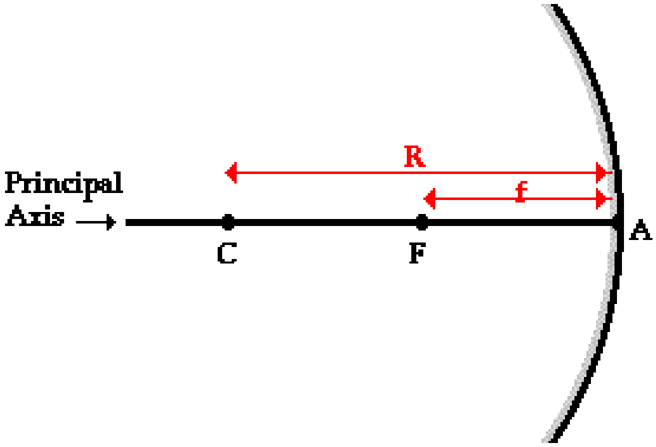
problem 9: A man is 1.8 meters tall. What is the smallest mirror he can use to see his entire reflection? How should the mirror be placed to do this?

Problem 10: does it matter how close to the mirror the man stands when he sees his entire reflection?

Problem 11: Ben Phooled is 6-feet tall. He is the tallest person in his family. It just so happens that Ben learned the important principle of the 2:1 relationship just prior to his family's decision to purchase a mirror that was to be used by the entire family. Enthused about the recent physics lesson, Ben decided to put it to *good* use. Ben convinced his parents that it would be a waste of money to buy a mirror longer than 3 feet. "After all," Ben argued, "I'm the tallest person in the family and only three feet of mirror would be required to view my image." Ben's parents conceded and they purchased a 3-foot tall mirror and mounted it on the bathroom wall.Comment on the wisdom behind the Phooled family decision.

Concave and convex mirrors





C is center of curvature

F is focal pont

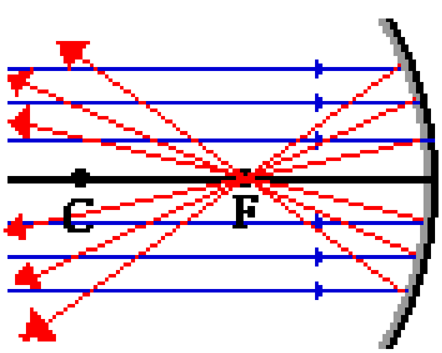
f is focal length

R is radius of curvature

A is the vertex where the principal axis meets the mirror

The focal length = ½ the radius of curvature $f = R/2$

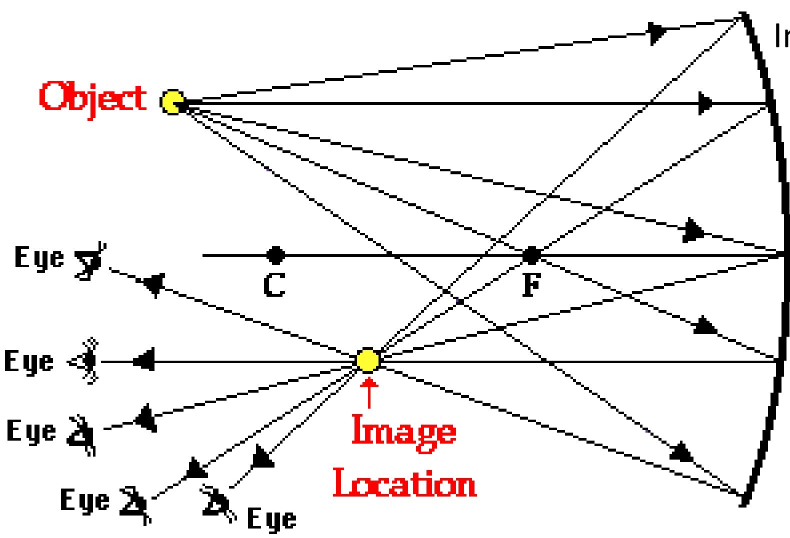
The focal point is the point in space at which light incident towards the mirror and traveling parallel to the principal axis will meet after reflection.



problem 12: The surface of a concave mirror is *pointed* towards the sun. Light from the sun hits the mirror and converges to a point. How far is this *converging point* from the mirror's surface if the radius of curvature (R) of the mirror is 150 cm?

problem 13: It's the early stages of a concave mirror lab. Your teacher hands your lab group a concave mirror and asks you to find the focal point. What procedure would you use to do this?

Light always follows the law of reflection. For a concave mirror, the normal at the point of incidence on the mirror surface is a line that extends through the center of curvature. Once the normal is drawn the angle of incidence can be measured and the reflected ray can be drawn with the same angle.



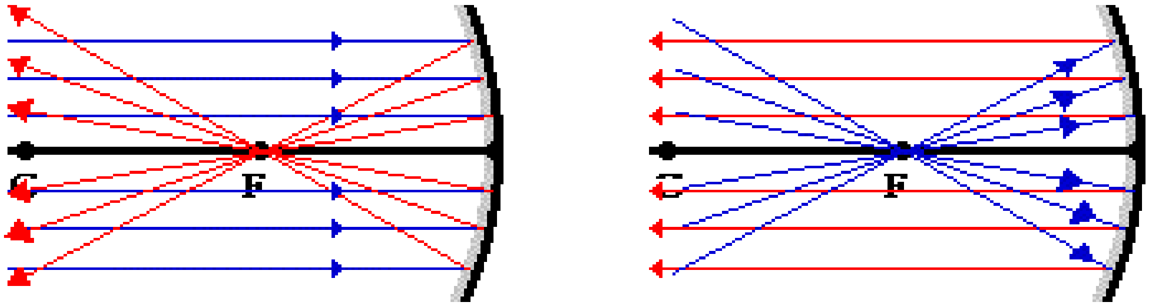
Images of concave mirrors are:

- real or virtual
-
- inverted
-

rules of reflection for concave mirrors:

- Any incident ray traveling parallel to the principal axis on the way to the mirror will pass through the focal point upon reflection.

• Any incident ray passing through the focal point on the way to the mirror will travel parallel to the principal axis upon reflection.

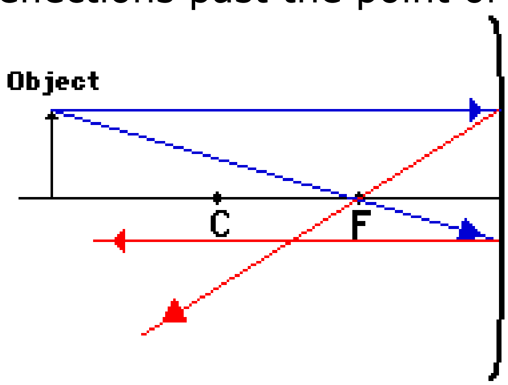


RAY DIAGRAMS FOR CONCAVE MIRRORS

The goal of a ray diagram is to determine the ***location, size, orientation, and type of image*** that is formed by the concave mirror.

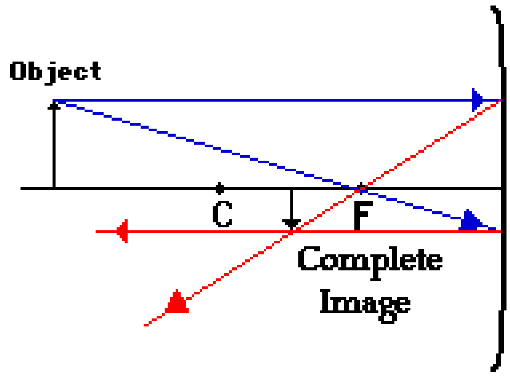
1) Pick a point on the top of the object and draw two incident rays traveling towards the mirror: one through the focal point and one parallel to principal axis.

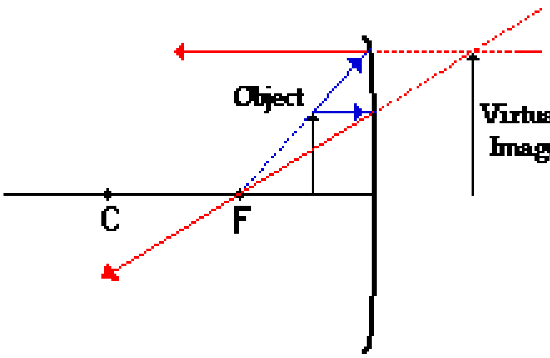
2. reflect these rays back using the 2 rules of reflection. Extend the reflections past the point of intersection.



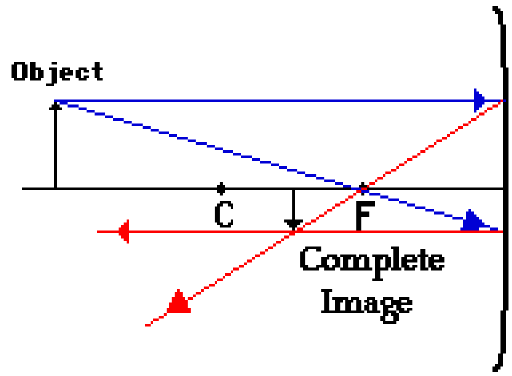
3. the top of the image is the point of intersection.

4. repeat steps 1 and 2 for the bottom of the object.





Ray Diagram for Object Located in Front of F



object between F and mirror:

IMAGE is upright, **virtual,** magnified

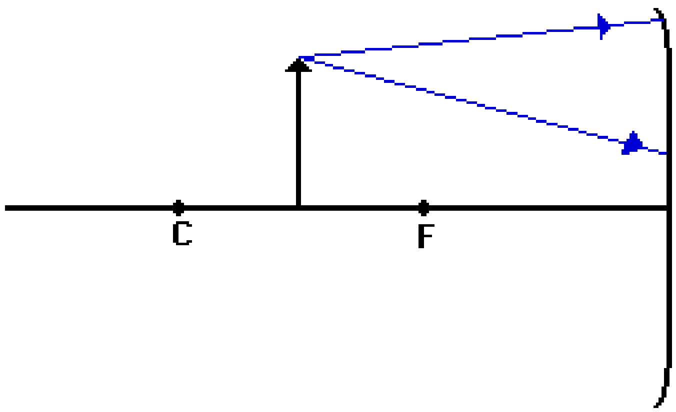
object beyond C: image is between C and F,

inverted, **real,** smaller

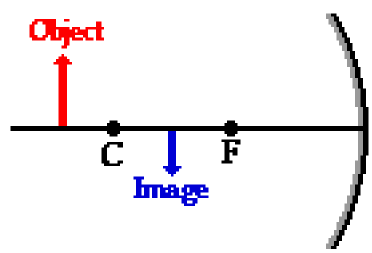
object at C, image is same size, at C; **real,** inverted,

object between C and F. image is: inverted, located beyond C, **real,** inverted, magnified

problem 13: The diagram below shows two light rays emanating from the top of the object and incident towards the mirror. Describe how the reflected rays for these light rays can be drawn without actually using a protractor and the law of reflection.

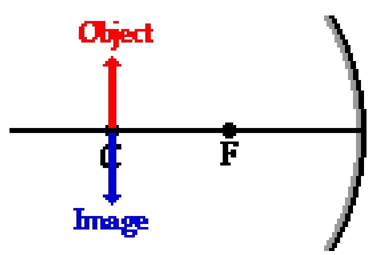


images using concave mirrors:



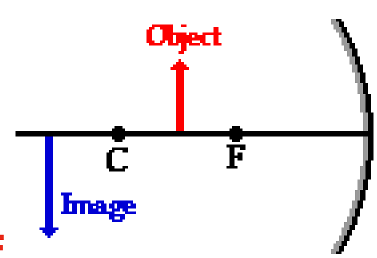
Case 1: The object is located *beyond* C

Inverted, reduced, located between C and F, REAL



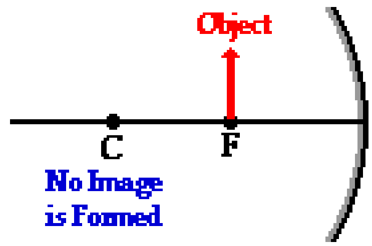
Case 2: The object is located at C

Image is inverted, same size, located at C, real



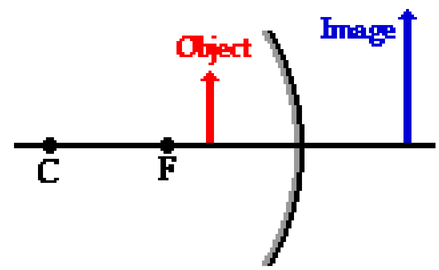
Case 3: The object is located between C and F

Image is inverted, magnified, beyond C, real

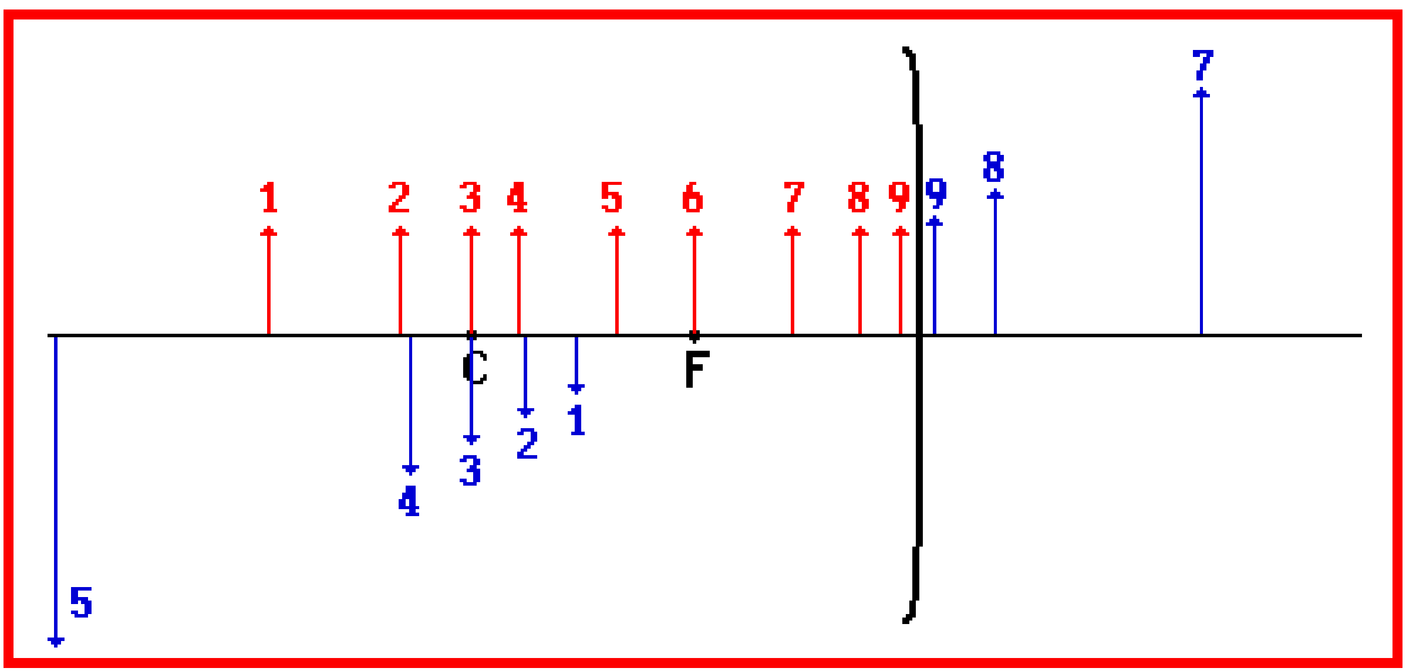


Case 4: The object is located at F

No image forms



case 5: The object is located *in front of* F
image is virtual, magnified, upright



problem 14 : Identify the means by which you can use a concave and/or a plane mirror to form a real image.

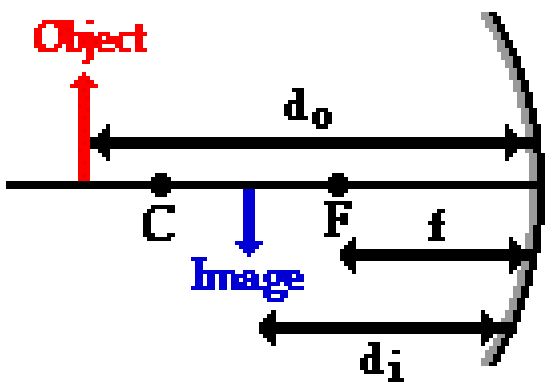
Problem 15: Identify the means by which you can use a concave and/or a plane mirror to form a virtual image

Problem 16: Identify the means by which you can use a concave and/or a plane mirror to produce an upright image.

Problem 17: 6. Are all real images larger than the object?

Problem 18: A magician conducts a classic magic trick utilizing a concave mirror with a focal length of 1.6 m. He is able to use the mirror in such a manner as to produce an image of a light bulb at the same location and of the same size as the actual light bulb itself. Use complete sentences to explain how he is able to accomplish this magic trick. Be specific about the light bulb location.

The mirror equation: $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$ the magnification equation: $M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$



The +/- Sign Conventions

The sign conventions for the given quantities in the mirror equation and magnification equations are as follows:

- f is + if the mirror is a concave mirror
- f is - if the mirror is a convex mirror
- d_i is + if the image is a real image and located on the object's side of the

mirror.

- d_i is - if the image is a virtual image and located behind the mirror.
- h_i is + if the image is an upright image (and therefore, also virtual)
- h_i is - if the image an inverted image (and therefore, also real)

Problem 19: A 4.00-cm tall light bulb is placed a distance of 45.7 cm from a concave mirror having a focal length of 15.2 cm. Determine the image distance and the image size.

A **negative sign** for height indicates image is INVERTED

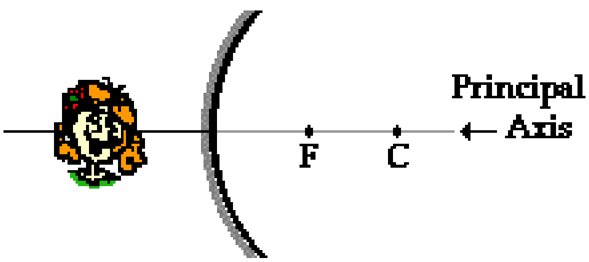
Problem 20: A 4.0-cm tall light bulb is placed a distance of 8.3 cm from a concave mirror having a focal length of 15.2 cm. (NOTE: this is the same object and the same mirror, only this time the object is placed closer to the mirror.) Determine the image distance and the image size.

Problem 21: Determine the image distance and image height for a 5.00-cm tall object placed 45.0 cm from a concave mirror having a focal length of 15.0 cm.

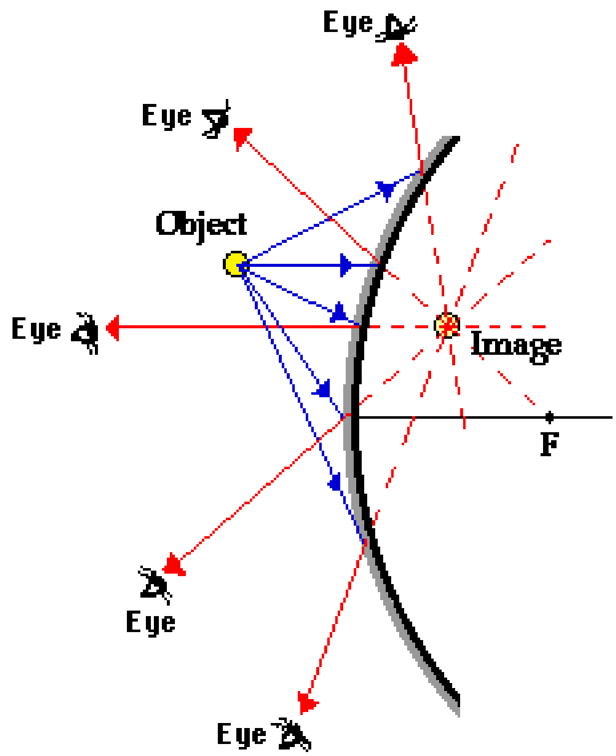
Problem 22: A magnified, inverted image is located a distance of 32.0 cm from a concave mirror with a focal length of 12.0 cm. Determine the object distance and tell whether the image is real or virtual.

Problem 23: . An inverted image is magnified by 2 when the object is placed 22 cm in front of a concave mirror. Determine the image distance and the focal length of the mirror.

Convex lenses: produce virtual images.



A Convex Mirror with its Negative Focal Length

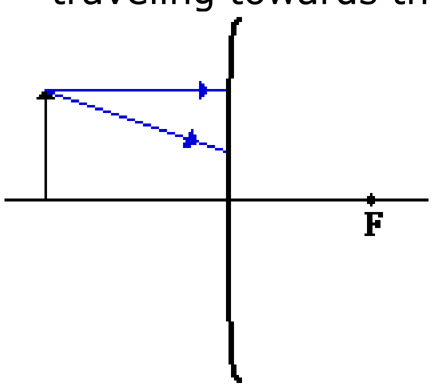


rules of reflection for convex mirrors:

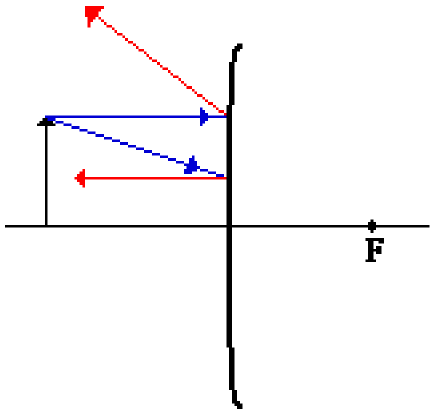
- Any incident ray traveling parallel to the principal axis on the way to a convex mirror will reflect in such a manner that its extension will pass through the focal point.
- Any incident ray traveling towards a convex mirror such that its extension passes through the focal point will reflect and travel parallel to the principal axis.

RAY DIAGRAMS FOR CONVEX MIRRORS

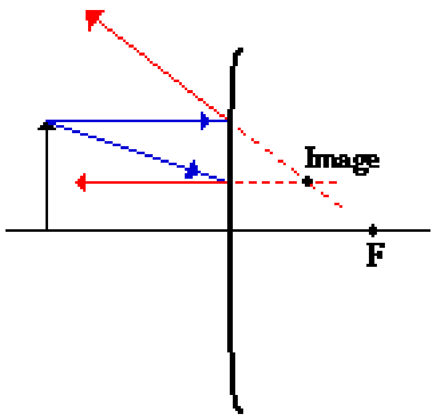
1. Pick a point on the top of the object and draw two incident rays traveling towards the mirror.



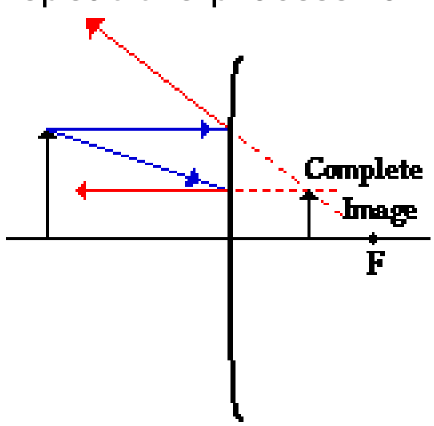
2. Once these incident rays strike the mirror, reflect them according to the two rules of reflection for convex mirrors.



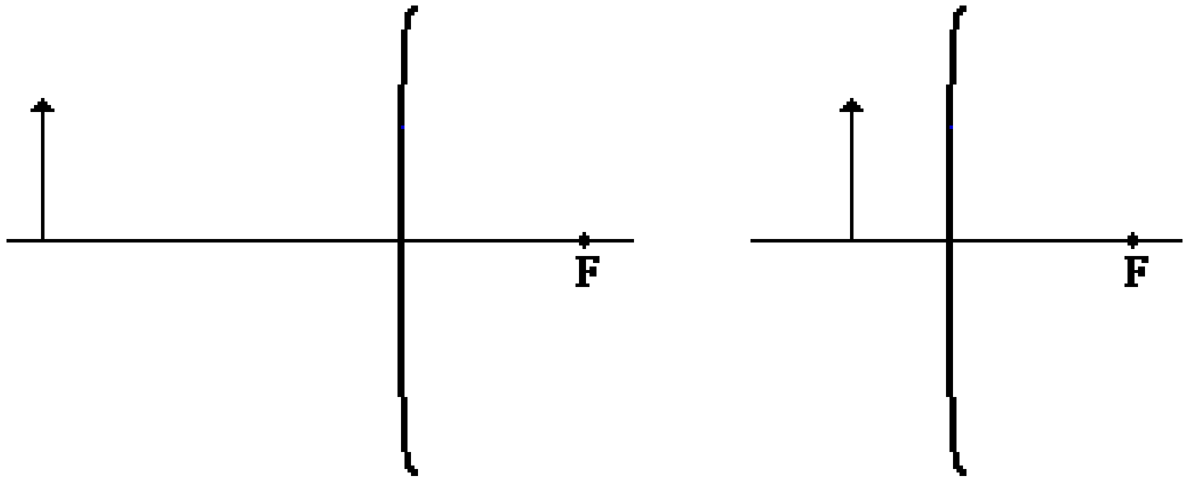
2. Locate and mark the image of the top of the object.



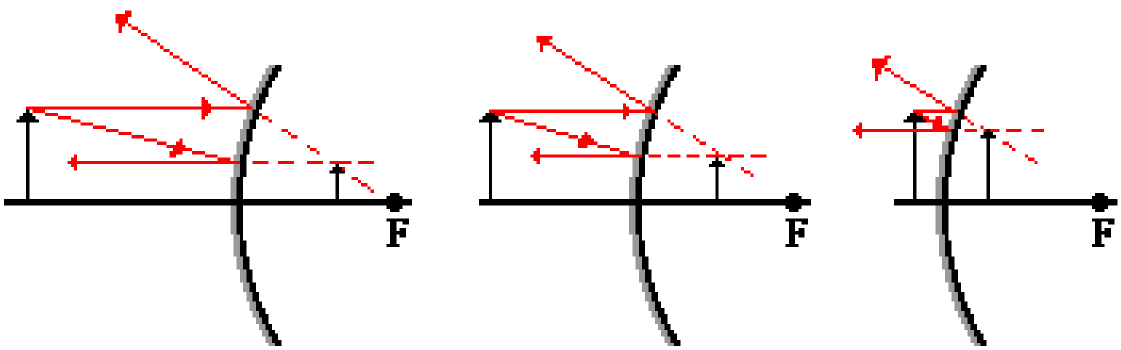
3. Repeat the process for the bottom of the object.



PROBLEM 24: draw ray diagrams for these mirrors

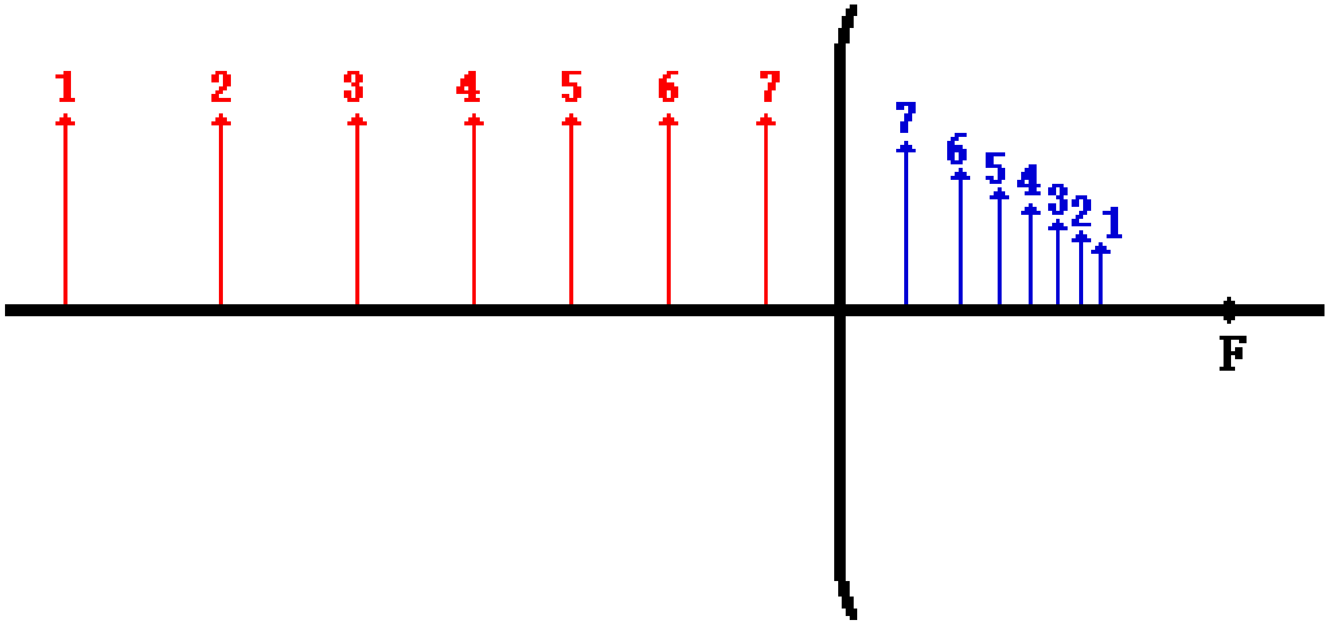


Images of convex mirrors:

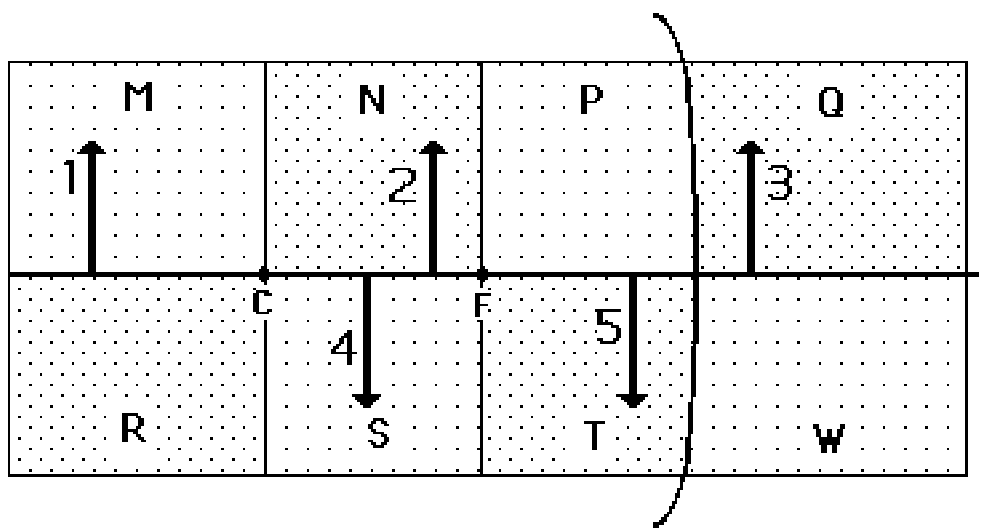


the image in a convex mirror is   UNLIKE concave mirrors, CONVEX IMAGES are always virtual:

- located behind the convex mirror
- a virtual image
- an upright image
- reduced in size (i.e., smaller than the object)



problem 25: The diagram below shows a spherical surface that is silvered on both sides. Thus, the surface serves as double-sided mirror, with one of the sides being the concave and one being the convex side. The principal axis, focal point, and center of curvature are shown. The region on both sides of the mirror is divided into eight sections (labeled M, N, P, Q, R, S, T, and W). Five objects (labeled 1, 2, 3, 4, and 5) are shown at various locations about the double-sided mirror. Use the diagram to answer the questions #1-6.



26 The image of object 1 would be located in section _______.

M N P Q R S T W

27. the image of object 2 would be located in section _______.

M N P Q R S T W

28. The image of object 3 would be located in section _______.

M N P Q R S T W

29. The image of object 4 would be located in section _______.

M N P Q R S T W

30. he image of object 5 would be located in section _______.

M N P Q R S T W

31. The double-sided mirror would cause virtual image to be formed of objects _________.

a. 1, 2, and 4 b. 1, 2, and 3 c. 3 and 5 d. 4 and 5 e. 3 only

32. How can a plane mirror, concave mirror, and/or convex mirror be used to produce an image that has the same size as the object?

33. How can a plane mirror, concave mirror, and/or convex mirror be used to produce an upright image?

34. How can a plane mirror, concave mirror, and/or convex mirror be used to produce a real image?

10. The image of an object is found to be upright and reduced in size. What type of mirror is used to produce such an image?

The mirror equation for Convex mirrors: $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$ magnification: $M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$

11. A 4.0-cm tall light bulb is placed a distance of 35.5 cm from a convex mirror having a focal length of -12.2 cm. Determine the image distance and the image size.

A convex mirror has a focal length of -10.8 cm. An object is placed 32.7 cm from the mirror's surface. Determine the image distance.

2. Determine the focal length of a convex mirror that produces an image that is 16.0 cm behind the mirror when the object is 28.5 cm from the mirror.

3. A 2.80-cm diameter coin is placed a distance of 25.0 cm from a convex mirror that has a focal length of -12.0 cm. Determine the image distance and the diameter of the image.

4. A focal point is located 20.0 cm from a convex mirror. An object is placed 12 cm from the mirror. Determine the image distance.

LENSES

The lens enables images of objects to be formed in two ways: real and virtual.

Real images: can be projected on a screen

Virtual images: can be seen only by our eyes, and cannot be projected on a screen

Ray diagrams:

Draw an object, a lens and the principal axis. The principal axis is perpendicular to the lens, through the center of the lens.

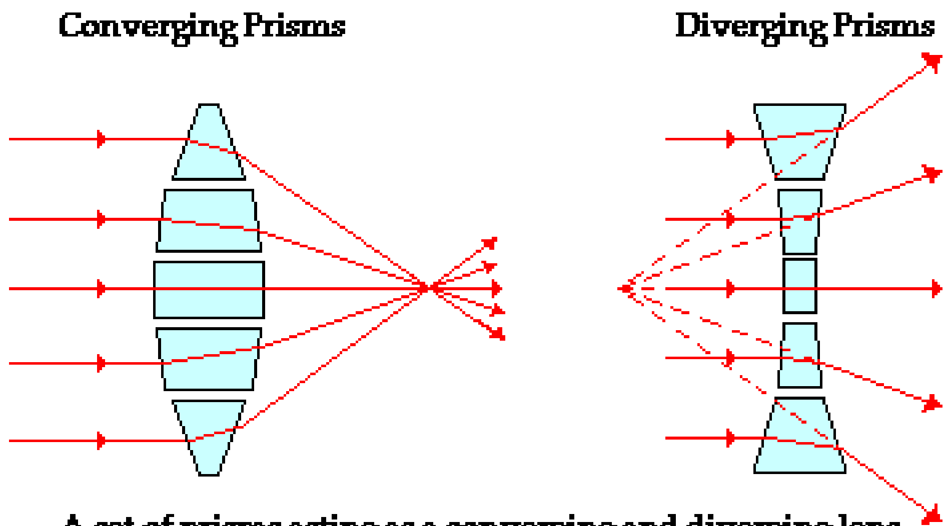
Draw three **principal rays** through a lens to show **location d_i,** of image and **size h_i** of image.

1) First ray goes through the center of the lens without bending.
2) The second ray goes through the focal point on the *incident side* of the lens and is refracted parallel to the principal axis on the real side of the lens.
3) The third ray is parallel to the principal axis on the incident side of the lens and refracts through the focal point on the real side of the lens.

if a piece of glass or other transparent material is the appropriate shape, it is possible that parallel incident rays would either converge to a point or appear to be diverging from a point.

A piece of glass that has such a shape is referred to as a lens.

Therefore, a lens is a piece of glass that will bend parallel light rays through an exact point called the **focal point.**



A set of prisms acting as a converging and diverging lens.

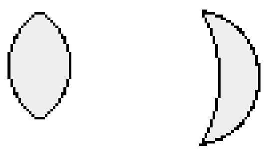
A **lens** is merely a carefully ground or molded piece of transparent material that refracts light rays in such as way as to form an image. Lenses can be thought of as a series of tiny refracting prisms, each of which refracts light to produce their own image. When these prisms act together, they produce a bright image focused at a point.

There are a variety of types of lenses. Lenses differ from one another in terms of their shape and the materials from which they are made. Our focus will be upon lenses that are symmetrical across their horizontal axis - known as the **principal axis**. In this unit, we will categorize lenses as converging lenses and diverging lenses.

A **converging lens** is a lens that converges rays of light that are traveling parallel to its principal axis. Converging lenses can be identified by their shape; they are relatively thick across their middle and thin at their upper and lower edges.

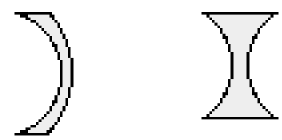
A **diverging lens** is a lens that diverges rays of light that are traveling parallel to its principal axis. Diverging lenses can also be identified by their shape; they are relatively thin across their middle and thick at their upper and lower edges.

Converging Lenses



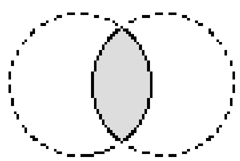
thicker across the middle
thinner at its edges
serves to converge light

Diverging Lenses

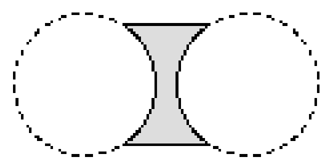


thinner across the middle
thicker at its edges
serves to diverge light

we will only discuss symmetrical double convex and double concave lenses

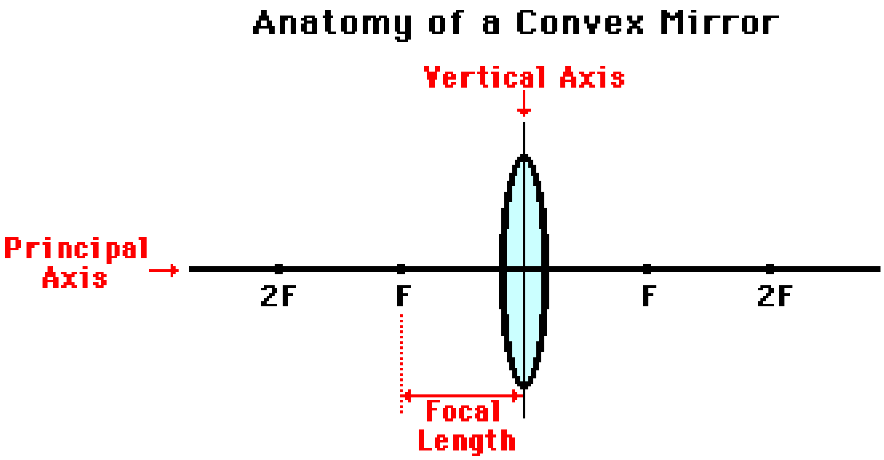


A double convex lens



A double concave lens

DOUBLE CONVEX LENSES



Think of a symmetrical lens as a slice of a sphere

The line passing through the center of the sphere and attaching to the mirror in the exact center of the lens. This imaginary line is known as the

principal axis line perpendicular to the lens, passing through center of lens.

vertical axis that bisects the symmetrical lens into halves.

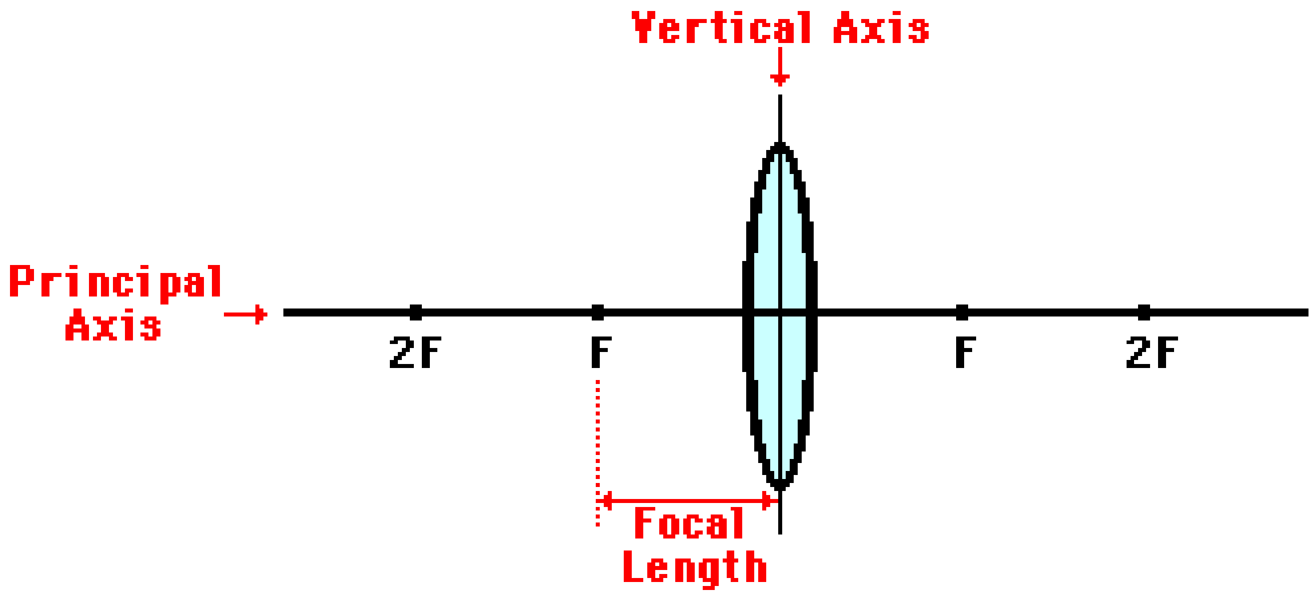
light rays incident towards either face of the lens and traveling parallel to the principal axis will either converge or diverge.

F = focal point of the converging lens = location where light rays converge

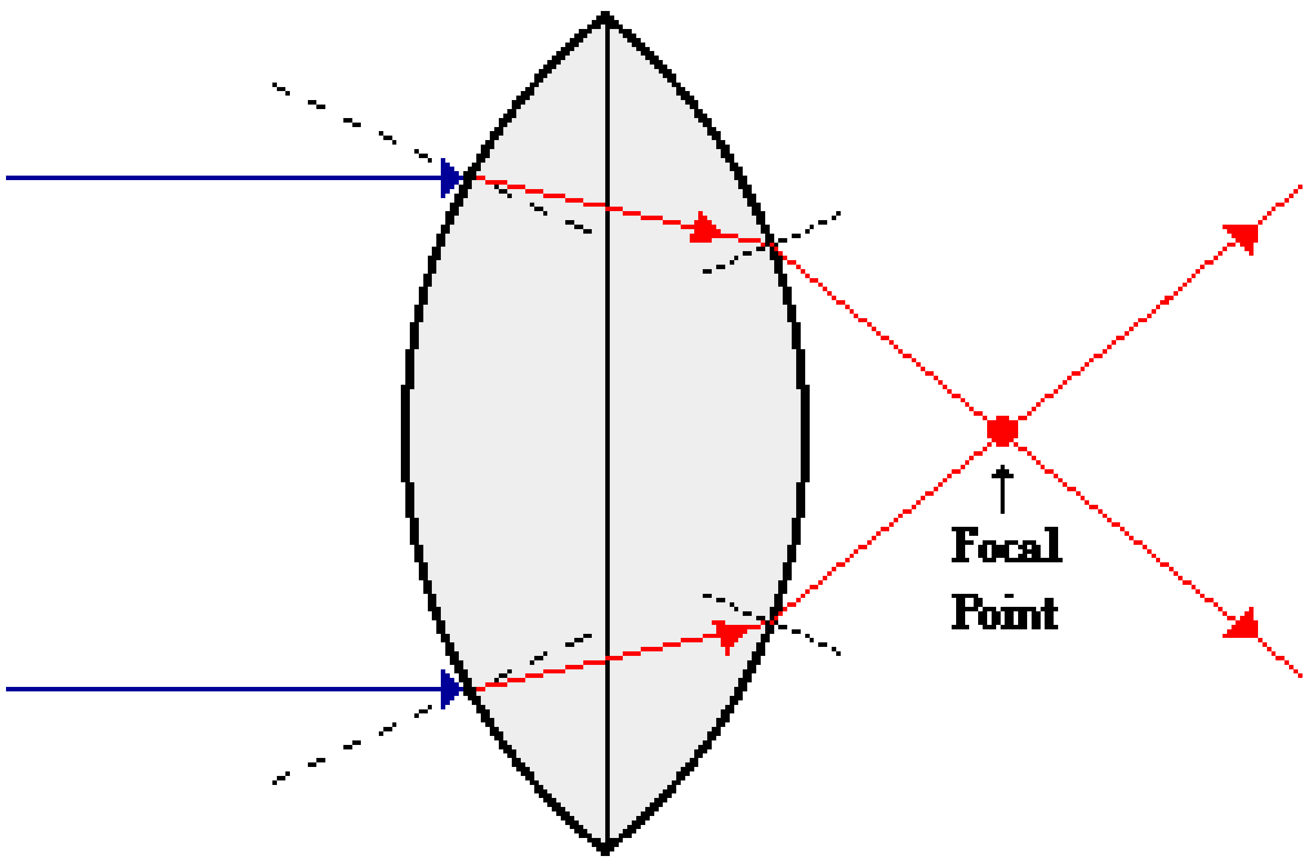
F = focal point of the diverging lens = location where diverging rays can be traced backwards until they intersect at a point.

2F point. This is the point on the principal axis that is twice as far from the vertical axis as the focal point is [in a mirror , this is the radius of curvature].

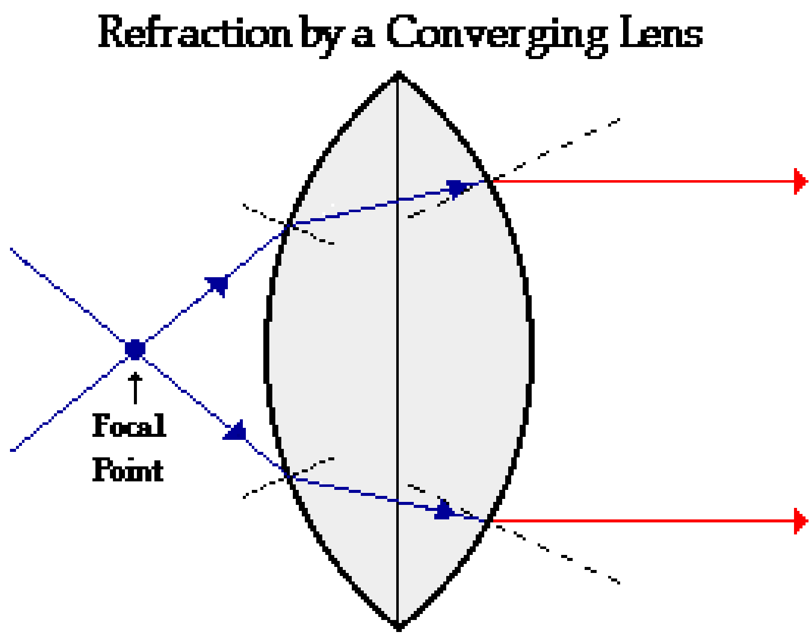
Anatomy of a Convex Mirror



Refraction by a Converging Lens



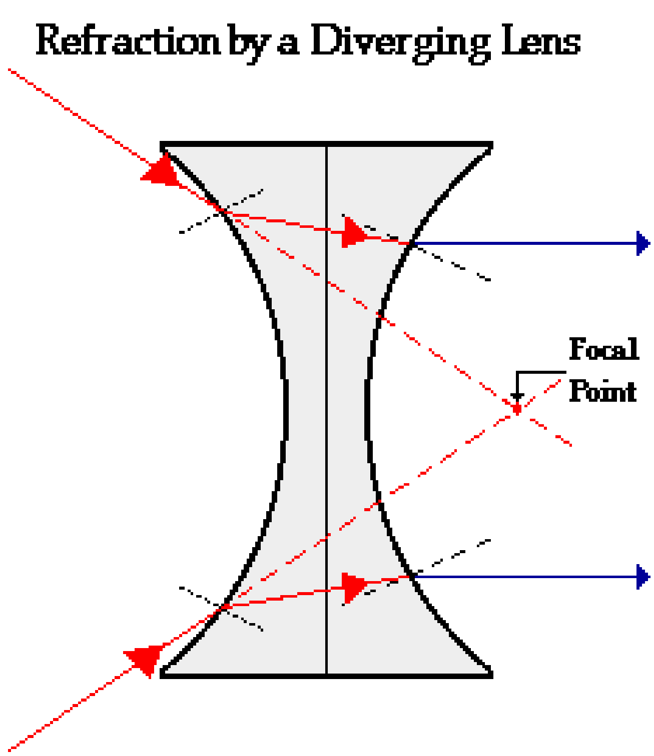
Incident rays which travel parallel to the principal axis will refract through the lens and converge to a point.



Incident rays which travel through the focal point will refract through the lens and travel parallel to the principal axis.

We are assuming lenses are "thin lenses"= thickness of lens is negligible compared do the focal length.

DOUBLE CONCAVE LENSES



Incident rays traveling towards the focal point will refract and travel parallel to the principal axis.

Refraction Rules for a Converging Lens

- Any incident ray traveling **parallel to the principal axis** of a converging lens will refract through the lens and travel through the *focal point on the opposite side of the lens.*
- Any incident ray traveling through the **focal point** on the way to the lens will refract through the lens and travel *parallel to the principal axis.*
- An incident ray that passes through the **center of the lens** will continue **in the same direction** that it had when it entered the lens.

Refraction Rules for a Diverging Lens

- Any incident ray traveling **parallel to the principal axis** of a diverging lens will refract through the lens and travel *in line with* the focal point (i.e., in a direction such that its *extension will pass through the focal point*).

- Any incident ray traveling towards the **focal point on the way to the lens** will refract through the lens and travel *parallel to the principal axis.*
- An incident ray that passes through the **center of the lens** will **continue in the same direction** it had when it entered the lens.

Converging Lens Image Formation

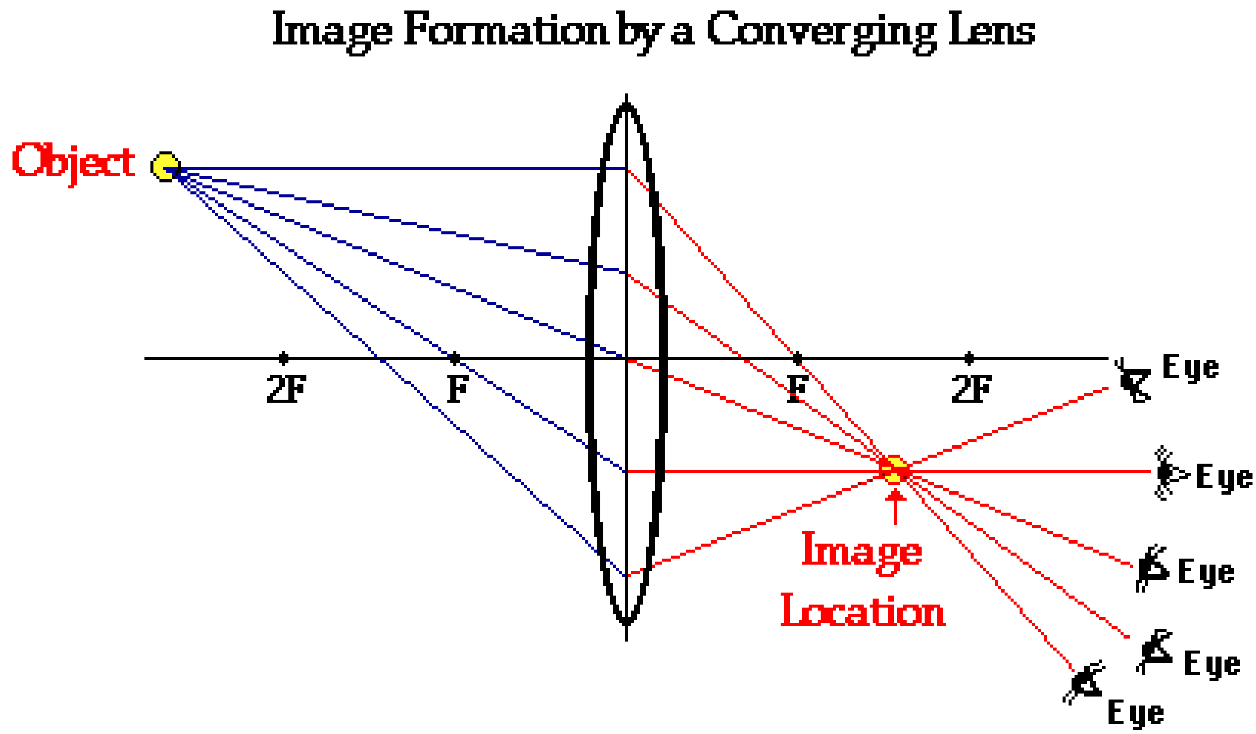
Converging[convex] lenses can produce **both** real and virtual images

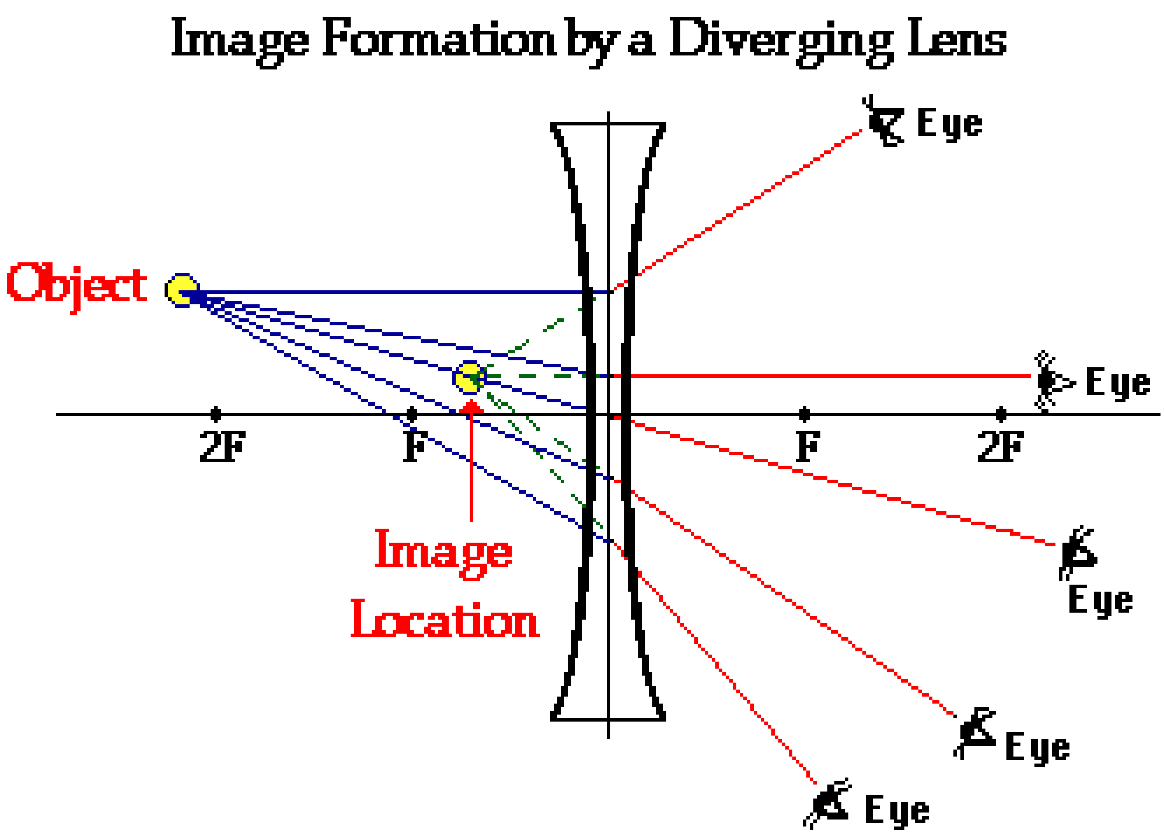
Diverging[concave] lenses can only produce **virtual** images.

The process by which images are formed for lenses is the same with curved mirrors.

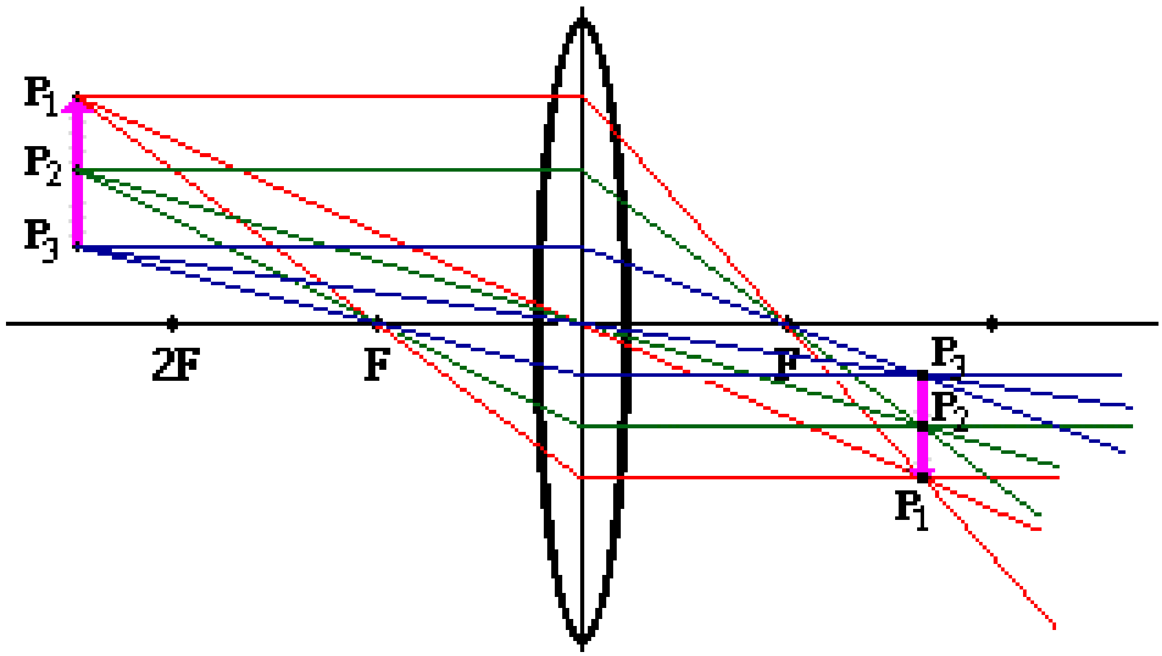
Images are formed where any observer views the object through a lens

Each observer must sight in the direction of this point in order to view the image of the object images of "point objects"





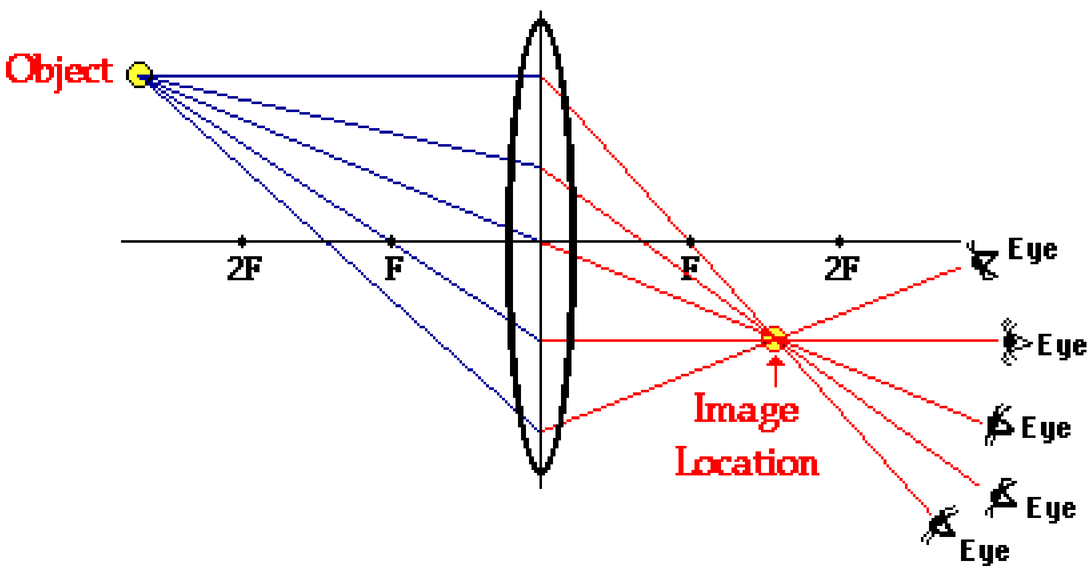
images of non-points.



All the rays of light emanating from each individual point on the object will refract and intersect at a single point in space. An image is created – the image is merely a replica or reproduction of the object.

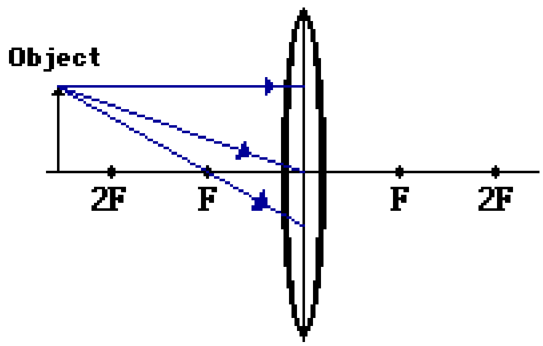
RAY DIAGRAMS

Image Formation by a Converging Lens



Drawing Ray Diagrams *using 3 rays.*

Case 1: object located *beyond* the 2F point of a double convex lens.

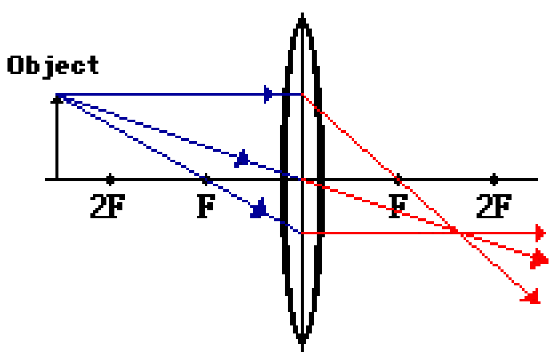


1. Pick a point on the top of the object and draw three incident rays traveling towards the lens.

2. Using a straight edge, accurately draw one ray so that it passes exactly through the focal point on the way to the lens.

3.Draw the second ray such that it travels exactly parallel to the principal axis.

4. Draw the third incident ray such that it travels directly to the exact center of the lens. Place arrowheads upon the rays to indicate their direction of travel.



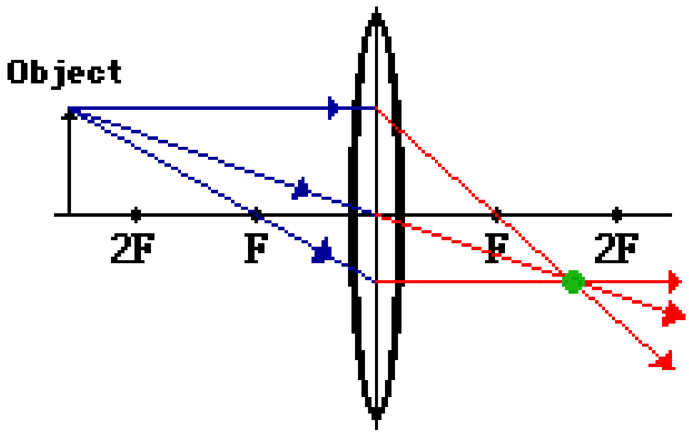
5. Once these incident rays strike the lens, refract them according to the three *rules* of refraction for converging lenses:

The ray that passes through the focal point on the way to the lens will refract and travel parallel to the principal axis.

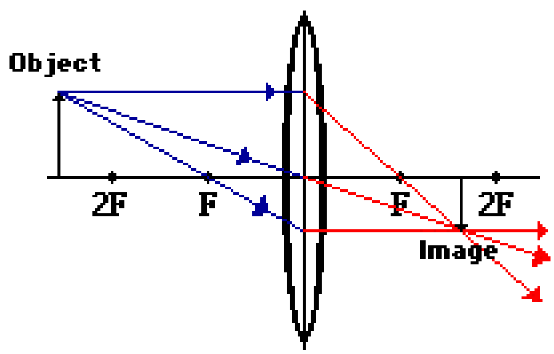
The ray that traveled parallel to the principal axis on the way to the lens will refract and travel through the focal point.

The ray that traveled to the exact center of the lens will continue in the same direction.

Place arrowheads upon the rays to indicate their direction of travel. Extend the rays past their point of intersection.



6. mark the Top of the image: where the 3 rays intersect.

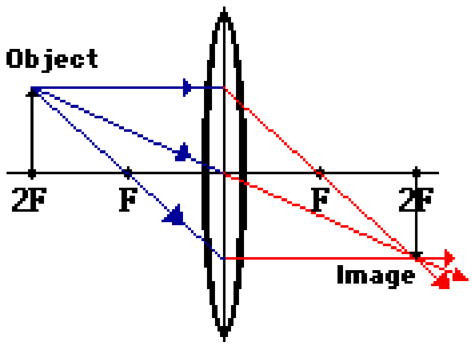


7. the bottom of object is on the principal axis, so fill in the object

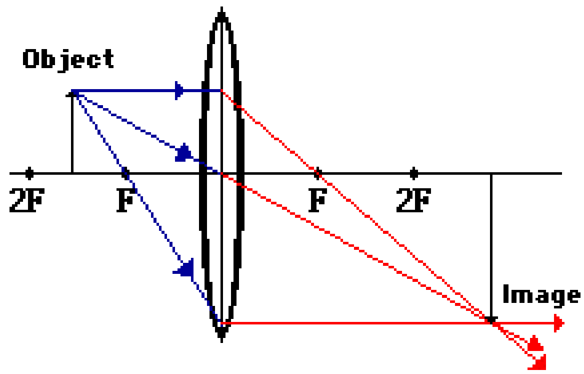
Fortunately, a shortcut exists. If the object is a vertical line, then the image is also a vertical line. For our purposes, we will only deal with the simpler situations in which the object is a vertical line that has its bottom located upon the principal axis. For such simplified situations, the image is a vertical line with the lower extremity located upon the principal axis.

The ray diagram above illustrates that when the object is located at a position *beyond* the 2F point, the image will be located at a position between the 2F point and the focal point on the opposite side of the lens. Furthermore, the image will be inverted, reduced in size (smaller than the object), and real. This is the type of information that we wish to obtain from a ray diagram. These characteristics of the image will be discussed in more detail in the next section of Lesson 5.

Once the method of drawing ray diagrams is practiced a couple of times, it becomes as natural as breathing. Each diagram yields specific information about the image. The two diagrams below show how to determine image location, size, orientation and type for situations in which the object is located at the 2F point and when the object is located between the 2F point and the focal point.



Ray Diagram for Object Located at 2F

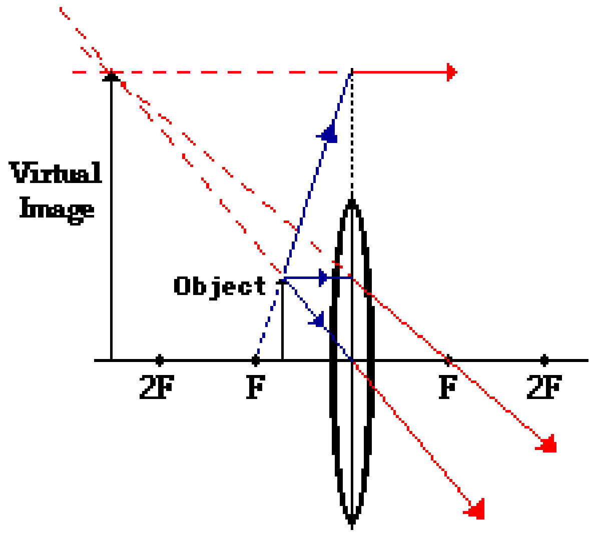


Ray Diagram for Object Located Between F and 2F

It should be noted that the process of constructing a ray diagram is the same regardless of where the object is located. While the result of the ray diagram (image location, size, orientation, and type) is different, the same three rays are always drawn. The three rules of refraction are applied in order to determine the location where all refracted rays appear to diverge from (which for real images, is also the location where the refracted rays intersect).

For object between 2f and f: real image

For object inside focal point : virtual image



Ray Diagram for Object Located in Front of F

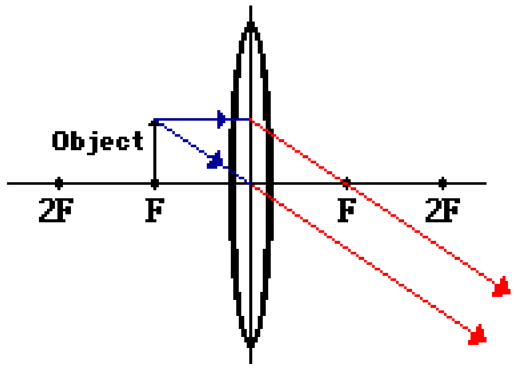
the light rays diverge after refracting through the lens.

When refracted rays diverge, a virtual image is formed.

The image location can be found by tracing all light rays backwards until they intersect.

For every observer, the refracted rays would seem to be diverging from this point; thus, the point of intersection of the extended refracted rays is the image point. Since light does not actually pass through this point, the image is referred to as a virtual image

For object located at f:

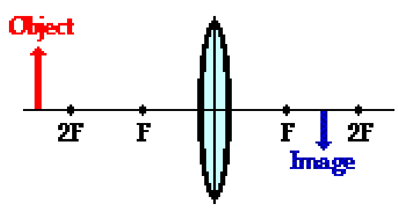


Ray Diagram for Object Located at F
(an image is not formed)

no image forms, lines never converge

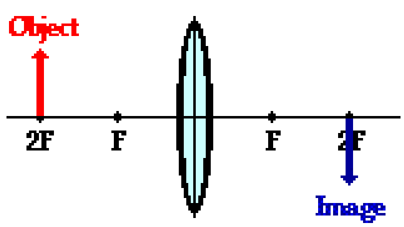
summary:

object beyond 2F

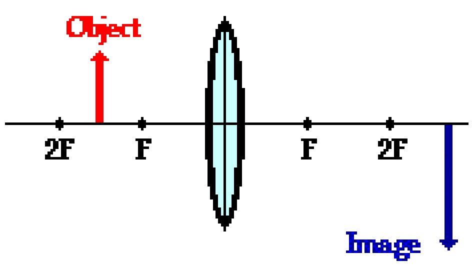


real, inverted, M<1

object at 2F

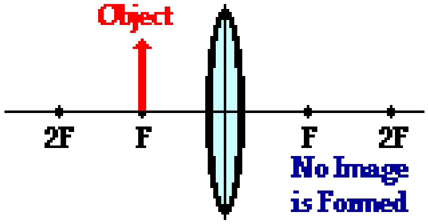


object between 2F and F   **real, inverted, M=1**

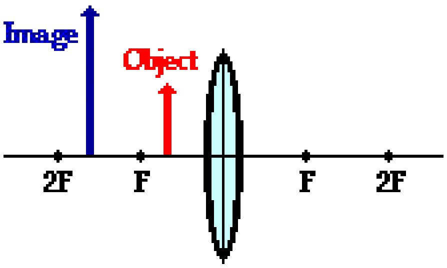


real, inverted, M>1

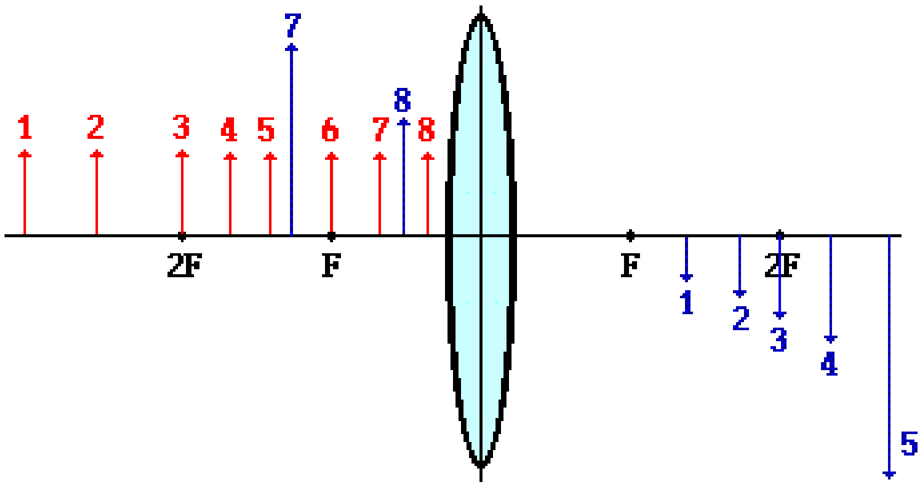
object at F



object between F and lens



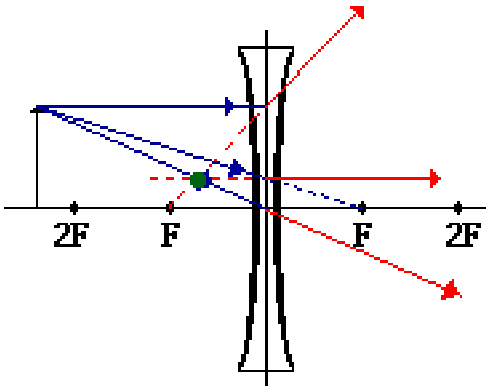
virtual, upright M>1



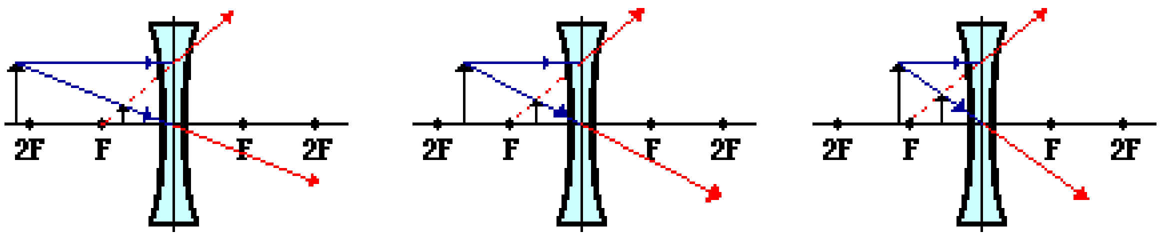
Each of the numbered objects (except #6) has an image with the corresponding number; its relative location, size, and orientation are shown.

problem: A converging lens is sometimes used as a magnifying glass. Explain how this works; specifically, identify the general region where the object must be placed in order to produce the magnified effect.

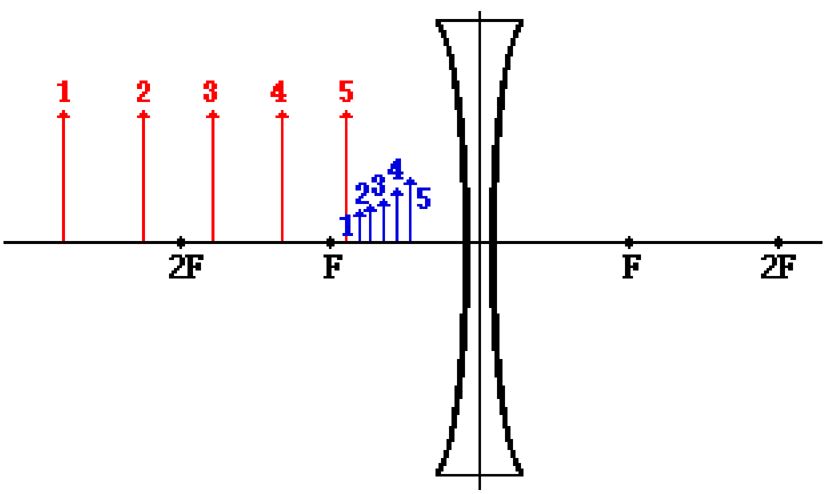
Diverging lenses:
the three refracted rays intersect at the [virtual] image



Draw 3 rays from the top of object and 3 from the bottom if necessary:
Object beyond 2F object between 2F and F



all images are virtual, upright, M<1



problem: The image of an object is found to be upright and reduced in size. What type of mirror and/or lens is used to produce such an image?

The thin lens equation: $1/F = 1/d_i + 1/d_o$

magnification: $M = h_i/h_o = -d_i/d_o$

Sign Conventions

The sign conventions for the given quantities in the lens equation and magnification equations are as follows:

- f is + if the lens is a double convex lens (converging lens)
- f is - if the lens is a double concave lens (diverging lens)
- d_i is + if the image is a real image and located on the opposite side of the lens.
- d_i is - if the image is a virtual image and located on the object's side of the lens.
- h_i is + if the image is an upright image (and therefore, also virtual)

h_i is - if the image an inverted image (and therefore, also real)

SUMMARY:

- hi = REAL image

-di = **virtual** image [on same side as object]

-F = diverging lens [**concave**]

ALL IMAGES OF DIVERGING LENS ARE VIRTUAL

IMAGES OF CONVERGING CAN BE VIRTUAL OR REAL

Problem 1 A 4.00-cm tall light bulb is placed a distance of 35.5 cm from a diverging lens having a focal length of -12.2 cm. Determine the image distance and the image size.

Problem 2: A 4.00-cm tall light bulb is placed a distance of 8.30 cm from a double convex lens having a focal length of 15.2 cm. (NOTE: this is the same object and the same lens, only this time the object is placed closer to the lens.) Determine the image distance and the image size.

PROBLEM 3: A 4.00-cm tall light bulb is placed a distance of 45.7 cm from a double convex lens having a focal length of 15.2 cm. Determine the image distance and the image size.

PROBLEM 4: Determine the image distance and image height for a 5-cm tall object placed 45.0 cm from a double convex lens having a focal length of 15.0 cm.

PROBLEM 5: Determine the image distance and image height for a 5-cm tall object placed 30.0 cm from a double convex lens having a focal length of 15.0 cm.

PROBLEM 6: Determine the image distance and image height for a 5-cm tall object placed 20.0 cm from a double convex lens having a focal length of 15.0 cm.

PROBLEM 7 Determine the image distance and image height for a 5-cm tall object placed 10.0 cm from a double convex lens having a focal length of 15.0 cm.

PROBLEM 8: A magnified, inverted image is located a distance of 32.0 cm from a double convex lens with a focal length of 12.0 cm. Determine the object distance and tell whether the image is real or virtual.

PROBLEM 9: An inverted image is magnified by 2 when the object is placed 22 cm in front of a double convex lens. Determine the image distance and the focal length of the lens.

PROBLEM 10 A double concave lens has a focal length of -10.8 cm. An object is placed 32.7 cm from the lens's surface. Determine the image distance.

PROBLEM 11 Determine the focal length of a double concave lens that produces an image that is 16.0 cm behind the lens when the object is 28.5 cm from the lens.

PROBLEM 12 A 2.8-cm diameter coin is placed a distance of 25.0 cm from a double concave lens that has a focal length of -12.0 cm. Determine the image distance and the diameter of the image.

PROBLEM 13 The focal point is located 20.0 cm from a double concave lens. An object is placed 12 cm from the lens. Determine the image distance.