Geometry
Unit 8: Circles and Spheres

Time Frame: Approximately six weeks

Unit Description

This unit focuses on justifications for circular measurement relationships in two and three dimensions, as well as the relationships dealing with measures of arcs, chords, secants, and tangents related to a circle. It also provides a review of formulas for determining the circumference and area of circles.

Student Understandings

Students can find the surface area and volume of spheres. Students can apply the relationship of the measures of minor and major arcs to the measures of central angles and inscribed angles, and to the circumference in various situations. They can also explain the relevance of tangents in real-life situations and the power of a point relationship for intersecting chords.

Guiding Questions

1. Can students provide an argument for the value of π and the way in which it can be approximated by polygons?
2. Can students provide convincing arguments for the surface area and volume formulas for spheres?
3. Can students apply the circumference, surface area, and volume formulas for circles, cylinders, cones, and spheres?
4. Can students apply geometric probability concepts using circular area models and using area of a sector?
5. Can students find the measures of inscribed and central angles in circles, as well as measures of sectors, chords, and tangents to a circle from an external point?
6. Can students use the power of a point theorem (intersecting chords and intersecting secants) to determine measures of intersecting chords in a circle?

Unit 8 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

Grade-Level Expectations	
GLE #	**GLE Text and Benchmarks**
Measurement	
7.	Find volume and surface area of pyramids, spheres, and cones (M-3-H) (M-4-H)
Geometry	
13.	Solve problems and determine measurements involving chords, radii, arcs, angles, secants, and tangents of a circle (G-2-H)
19.	Develop formal and informal proofs (e.g., Pythagorean theorem, flow charts, paragraphs) (G-6-H)
Data Analysis, Probability, and Discrete Math	
21.	Determine the probability of conditional and multiple events, including mutually and non-mutually exclusive events (D-4-H) (D-5-H)
22.	Interpret and summarize a set of experimental data presented in a table, bar graph, line graph, scatter plot, matrix, or circle graph (D-7-H)
CCSS for Mathematical Content	
CCSS #	**CCSS Text**
Congruence	
G.CO.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
Circles	
G.C.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
Expressing Geometric Properties with Equations	
G.GPE.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
G.GPE.2	Derive the equation of a parabola given the focus and directrix.
Conditional Probability and the Rules of Probability	
S.CP.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
ELA CCSS	
CCSS #	**CCSS Text**
Reading Standards for Literacy in Science and Technical Subjects 6-12	
RST.9-10.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

Sample Activities

Activity 1: Vocabulary Self-Awareness (GLE: <u>13</u>; CCSS: G.CO.1, RST.9-10.4)

Materials List: pencil, Vocabulary Self-Awareness BLM

Give each student a copy of the Vocabulary Self-Awareness BLM. *Vocabulary self-awareness* (view literacy strategy descriptions) is a literacy strategy which helps to assess students' knowledge of terms before reading text or other tasks. This awareness is helpful for students because it highlights what students already know, as well as what they still need to learn in order to fully understand the concept.

Throughout this unit on circles and spheres, have students maintain the *vocabulary self-awareness* chart on the Vocabulary Self-Awareness BLM. A target vocabulary has been provided in the BLM (additional words may be added if necessary). Have students complete self-assessments of their knowledge of the words using the chart provided on the BLM. Do not give students definitions or examples at this stage. Ask students to rate their understanding of each word with a "+" (understand well), a "✓" (limited understanding or unsure), or a "–" (don't know). Tell students to provide a definition and example for any term they feel they understand well (they would put a "+" for these terms). Over the course of the unit, students should be told to return often to the chart to add new information to it. Students will add precise definitions, examples, and other words, as well as revise definitions and examples from their initial self-assessment. The goal is to replace all the check marks and minus signs with a plus sign. Because students continually revisit their vocabulary charts to revise their entries, they have multiple opportunities to practice and extend their understanding of key terms related to circles and spheres.

Activity 2: Similar Circles (GLE: <u>19</u>)

Materials List: pencil, paper, overhead (optional), graph paper, safety compass, different sized circular objects, string or tailor's measuring tape, ruler

Begin this lesson by employing *student questions for purposeful learning (SQPL)* (view literacy strategy descriptions). To implement this strategy, develop a thought-provoking statement related to the topic about to be discussed. The statement does not have to be factually true, but it should generate some level of curiosity for the students. For this activity, pose the statement: "All circles are similar." This statement can be written on the board, projected on the overhead, or stated orally for the students to write in their notebooks. Allow the students to think about the statement for a moment and to develop some questions they might have that are related to the statement. After a minute or two, have students pair up and generate two or three questions they would like to have answered that relate to the statement. After another minute or so, have one member from

each pair share their questions with the class. As the questions are read aloud, write them on the board or overhead while students copy the questions in their notebooks. When questions are repeated or are very similar to others which have already been posed, those questions should be starred or highlighted in some way. Once all pairs' questions have been shared, look over the list to determine if other questions need to be added. The list should include the following questions:

- What is the definition of similar circles?
- Can circles be considered similar since they are not polygons?
- How would you prove that two circles are similar?
- What is the proof that all circles are similar?
- How can you create two similar circles?

At this point, be sure students have copied the questions in their notebooks. Continue with the lesson as follows. Be sure to tell the students to pay attention as the material is presented in order to find the answers to the questions posted on the board. Periodically, stop throughout the remainder of the lesson to allow pairs to discuss which questions have been answered. This may be followed by a whole class exchange of ideas to be sure everyone has the correct answers to each question.

Ask students to provide their definition of similar figures as they remember it from future study. This may require some probing questions and diagrams of similar figures to aid students in remembering what was discussed earlier in the year. Also, ask students to recall the transformations discussed in Unit 4—ask them to identify any transformation that was considered a similarity transformation. Students should remember that in order for two figures to be similar, 1) all corresponding angles in the figure should be congruent, and 2) the measures of the sides of the figure should be proportional. Also, students should identify dilation as a similarity transformation. Review with students what dilation is and why it produces a figure similar to the pre-image. Remind students that, in general, two figures are similar if there is a set of transformations that will move one figure exactly covering the other figure. The set of transformations could be any combinations of reflections, rotations, translations, and dilations.

Have students work in pairs. Have students graph the following circles: Circle C with center (-1, 2) and radius 3 and Circle D with center (3,4) and radius 5. The easiest way to have students graph a circle on a coordinate plane is to have them plot the center, then count the units for the radius length to the left and right, and up and down from the center to give four points to connect onto the graph. Using the safety compass set to the measure of the radius, place the tip of the compass on the center and then draw the circle by connecting all four points. It might be helpful for students to graph the circles in different colors. Once each pair has the circles graphed, ask students what transformations seem to have been performed on the circles. Students should see that the circle was translated and dilated. Then, ask students to identify the translation for the center and the dilation factor for the radius. It may be helpful to students to identify circle C as the pre-image and circle D as the image so they can identify the correct dilation factor. In this case, the center of dilation is NOT the origin, so students should use the linear measures of the radii to determine the scale factor. *Solution: The translation is 4 units to the right and 2*

units up; the dilation scale factor is $\frac{5}{3}$ *.If students are having difficulty seeing the scale factor, have then translate both circles so that the center of each circle is the origin. At that point, students should be able to see that if they multiply the coordinates of Circle C by* $\frac{5}{3}$ *they will get the coordinates of Circle D.* Ask students which other measures they might expect to have the same ratio if these two circles are similar. They should identify other linear measures such as the diameter and circumference. Have them verify that the diameters and circumferences of the two circles have the same ratio as the radii. *Solution: Circle C diameter is 6 units; Circle D diameter is 10 units; scale factor is* $\frac{5}{3}$ *. Circle C circumference is 6π units; Circle D circumference is 10π units; scale factor is* $\frac{5}{3}$ *.* Then have each pair graph a different circle of their choosing. Have pairs form groups of four and determine if there is a set of transformations (a translation and a dilation) that will show that the two circles are similar. Students should be able to determine the dilation factors for each and explain to the class how they know what the dilation factors are and why their circles are similar. At this point, have students write their understanding of the definition of similar circles in their notebooks. Have students share their definitions and explain their reasoning. Students should realize they can show two circles are similar if they can identify a dilation factor, and possibly a translation, that carries the pre-image onto the image.

Next, have students remain in groups of four and give each group a different sized circular object and either string or a tailor's measuring tape. Have students find the circumference and diameter of the circle. Then have them write the ratios of circumference to the diameter. Have each group record their data in a table on the board for the whole class to see. There should be a column for the circumference, the diameter, and the ratio (it is best to write the ratio in decimal form). Ask students to identify any patterns they see developing. Depending on the amount of human error in the measurements the students have taken, the ratios should be approximately 3.14. Ask students to identify the formula for circumference. If they were to rewrite the formula as $\frac{C}{d} = \pi$, they should see that the ratio of the circumference of a circle to its diameter is π for all circles. Since the ratio of circumference to diameter is the same for all circles, all circles are similar. Identify the value of π as the constant of proportionality which dictates that the circumference of a circle is proportional to the diameter of a circle by the factor of π. This means that as the diameter increases, the circumference will also increase by a factor of π.

Be sure to revisit the questions from *SQPL* at the end of the lesson to make sure students have all of the correct answers.

Activity 3: Derivation of the Area of a Circle Formula (GLE: <u>13</u>)

Materials List: pencil, paper, paper circles, scissors, automatic drawing program

Lead students in an exercise to show how the formula for the area of a circle can be developed. Have students cut a circle into 8 or 16 sectors, rearrange the sectors to form a parallelogram, and then use algebra to generate the area formula from the formula for the area of the parallelogram. Using this process allows students to review the circumference formula and area formula of a parallelogram.

Another way to show the derivation is to increase the number of sides of a regular polygon inscribed in a circle. Using an automatic drawing program such as *The Geometer's Sketchpad*®, have students draw a circle and inscribe an equilateral triangle. Have them find the area of the triangle and compare it to the area of the circle. Then have the students inscribe a square, regular pentagon, regular hexagon, regular octagon, regular decagon, and a regular dodecagon. Students should notice that the length of the apothem of the regular polygon approaches the length of the radius of a circle as the number of sides in the polygon increases. Have students derive the area of a circle formula by looking at the process for calculating the area of a regular polygon (i.e., area of one triangle times number of triangles or by using the formula discussed in Unit 6 Activity 12), and generalize this formula as the length of the polygon's sides gets smaller and smaller (as the number of sides increases). Students should see that as the number of sides of the regular polygon increases, the perimeter of the polygon approaches the circumference of the circle circumscribed around the polygon. Using this technique, allows students to review the process and/or formula for finding the area of a regular polygon.

Finally, have students develop an informal argument for the area of a circle using either of the methods above. In their argument, students should use precise terminology and be careful to indicate the correct units as necessary. Students should be able to justify any claims they make using the information they discovered during this activity.

Activity 4: Central Angles and Arcs (GLE: <u>13</u>; CCSS: RST.9-10.4)

Materials List: Sample Split-Page Notes BLM, Split-Page Notes Model BLM, paper, pencil, circle diagrams, string or tailor's tape,

In this activity, students will use a *split-page notetaking* ([view literacy strategy descriptions](#)) format to take notes on central angles and arcs. *Split-page notetaking* is simply a different way for students to organize their notes to help them use their notes more effectively for study. Model the approach by placing on the board or overhead sample split-page notes from the topic of circles. Explain the value of taking notes in this format by saying it organizes information and ideas from various sources. It helps separate big ideas from supporting details, and it promotes active reading and listening.

Next, ask students to use *split-page notetaking* while listening to a brief presentation on central angles and arcs. This presentation should define the terms central angles, arcs, major arcs, minor arcs, and semicircles. Tell students to draw a line from the top to bottom of their pages about one-third from the left side of the page (or provide them with a copy of the Blank Split-Page Notes BLM from Unit 5). On the left side, they will write the terms or concepts and on the right side, they will write the definition or explanation of the term. They can also draw examples on the right side of the page. After the presentation, have students compare notes with a partner, then answer questions and provide clarification using the Split-Page Notes Model BLM as a guide. Show students how they can prompt recall by bending the sheet of notes so that the information in the right or left column is covered. Then proceed with the remainder of the activity reminding students to add information to the concepts they have recorded already (like formulas and other examples).

Provide pairs of students with a diagram containing a circle with a given radius length and with central angles labeled 1, 2, and 3. The measures of the central angles should be in the ratio of 2:3:4. Have students determine the measures of the central angles and measures of the respective arcs. Students should realize the measure of the arc is equal to the measure of the central angle. Repeat this activity for other circles and other ratios. In addition, have students identify major and minor arcs given a central angle. Students should realize that the measure of the major arc is equal to 360 minus the measure of the minor arc. To help students visualize major and minor arcs, ask them to determine the type of arc associated with various times of day displayed on an analog clock.

Following this activity, have students revisit both their *split-page notes* and their *vocabulary self-awareness* charts from Activity 1 to update and revise their entries.

2013-2014
Activity 5: Measures Involving Arcs (CCSS: G.C.5, RST.9-10.4)

Materials List: Circular Flower Bed BLM, Arc Length and Sector Area Part I BLM, Arc Length and Sector Area Part II BLM, cotton string, protractors, rulers, safety compasses, calculators, measuring tapes, cans of various sizes (soup, vegetables, sodas, etc), document camera or overhead projector, transparencies (optional), markers (optional)

Have students work in cooperative groups of three to four students per group. Give each student a copy of the Circular Flower Bed BLM. Give students time to read the questions and discuss as a group possible plans to answer the questions. Tell students they are not attempting to find a solution at this point—they should only try to develop a plan for their solution. Then have groups report to the whole class parts of their plans. Record these ideas and discuss what information students still need to know in order to carry out their plans for solving the problems. Throughout the discussion, identify those pieces of the circles that would be identified as arcs and sectors. Also, introduce the terms arc length

and area of a sector. Do not provide formulas or methods for calculating these measures yet as students will derive these on their own.

Distribute kits of materials to each group. Each kit should include cotton string, a protractor, a ruler, a safety compass, a calculator, a measuring tape, and a can. Each group should get a different sized can. Also, give each student a copy of the Arc Length and Sector Area Part I BLM. Give an extra copy to each group for them to record the group's answers to share at the end of this portion of the activity. If you do not have access to a document camera, give each group transparencies and overhead markers so they can plan for their presentation to the class.

Review the instructions for the activity and answer any initial questions students may have. However, do not provide any guidance about how to solve the problems. Allow students to use their prior knowledge and attempt to solve the problems on their own. This is an exploration so students should devise their own method for determining the answers to the questions on the BLM. The kit provided to each group will allow students the opportunity to choose the tools they wish to use to help them answer the questions. Students will be required to explain their answers/reasoning to the class when they have completed the activity. This explanation will include which tools they chose to help them accomplish the task as well as why they chose those tools. As students are completing the exploration, circulate through the room listening to student discussions and answering questions or asking probing questions as needed. Some questions might be:
- How do you use the length of the diameter or radius to find the circumference of the circle?
- What information other than the diameter or radius is used to compute the circumference of the circle?
- How are you sure you have divided the circle into four equal parts?
- How is arc measure related to arc length?
- Is the answer what you expected? Why?
- How will you explain this to the class?

After about 20 minutes, begin group presentations. Students should explain the answers to the questions on the Arc Length and Sector Area Part I BLM either using the document camera to show their work or the overhead and transparencies. It may be necessary to set the order of the presentations allowing groups that may have struggled to go last so they might identify mistakes and make corrections before presenting, thereby allowing them to be more confident in their presentations. Otherwise allow groups to volunteer or choose groups at random for the presentations. Students should be encouraged to ask each group questions about their work for clarification following the group's presentation. Some questions that may need to be answered by groups at the present may be:
- How did your group arrive at the answer to #_____?
- Have you found an easier way to find the answer to # _____?
- What information helped you find the answer to #_____?

This is time for the students to explain their reasoning and critique each other's work (in a controlled, positive environment). Let the students explain and summarize their work and only step in to provide an explanation in extreme circumstances. As the groups

present, record the answers each group provides on a chart for the class to see. The chart may look something like the following:

Can	Circumference	Central Angle	Ratio #6	Arc Length	Ratio #8	Area	Sector Area	Ratio #11

The data can be recorded by one of the students in each group as they are presenting. This collection of data will allow students to see the patterns that emerge for all of the different cans used in the exploration. Students should copy the data in their notes to help them complete the next part of the activity. Discussions may need to occur to explain why the ratios in questions 8 and 11 are slightly different for each group—this will be due to human error in measuring; however, all ratios should be close to 0.25 when written in decimal form.

After group presentations are complete, have students complete the Arc Length and Sector Area Part II BLM individually. If necessary, this can be assigned as homework. Once students have completed this BLM, allow students time to share their formulas with group members and then discuss the results with the entire class. During the discussions about the formulas, be sure students are explaining their reasoning. After all ideas about the formulas have been expressed, have students come to a consensus to determine the formula. If necessary, give students the formula found in the answer key for the BLM and ask them to identify differences between the formula(s) they developed and the "actual" formula. The completed BLMs can also be collected as a formative assessment to gauge student understanding at this point.

Teacher Note*: The formulas given on the answer key are written intuitively and have not been simplified. Depending on the level of the students in the classroom, it may be beneficial to discuss what these formulas may look like once they have been simplified.*

Finally, have students revisit the Circular Flower Bed BLM which began the activity. Working in groups, students should work on solving the problems presented. The answers provided on the answer key for the second and third questions are sample answers as there are some assumptions and approximations that will need to be made. This will take time for students to complete, and their work with sector area and arc length may be applied here. The goal of this assignment is to have students persevere in solving a problem with no one correct answer. After giving students sufficient time to work on these problems, have groups present their answers to the class and defend their work as necessary. Encourage students to attempt to use sector area and arc length in their calculations.

Activity 6: Concentric Circles (GLE: <u>13</u>)

Materials List: Concentric Circles BLM, protractor, ruler, string or a tailor's tape measure, *learning log*

Provide students with copies of the Concentric Circles BLM. Working in groups of 3-4, have students find the measure of one of the central angles using a protractor by assigning each group a different central angle. Have students state the arc measure for the four minor arcs created by the assigned central angle. Make certain that students understand that the measure of an arc is the same as the measure of its central angle (definitions provided in Activity 3). Next, have students measure the radii of the concentric circles and calculate the circumference of the four circles. Discuss with students how they might find the length of the intercepted arc. Students should understand that arc length is a portion of the circumference. Remind students that the sum of the central angles of a circle is 360°. Give students the formula to calculate arc length:

$$\text{Arc Length} = \frac{N}{360}(2\pi r)$$

, where N is the measure of the central angle which creates the arc.

Teacher Note*: This formula is written intuitively and has not been simplified. Depending on the level of the students in the classroom, it may be beneficial to discuss what this formula may look like once it has been simplified.*

 Have students record the ratio of the length of the arc to the circumference and the ratio of the measure of the central angle to the total sum of the central angles. Students should see that the ratios are equal (or very close to equal, using decimal approximations) based on possible human measurement error). If needed, have students use string or a tailor's tape measure to verify their calculations for circumference and arc length to assist in internalizing the formula. Students should record data in a table similar to the one below.

Measure of Assigned Central Angle: $\angle BAI = 50°$ Ratio of Central Angle to 360°: $\frac{5}{36} \approx 0.139$

Minor Arc	Radii	Circumference	Arc Length	Ratio Arc Length to Circumference
$\overset{\frown}{BI} = 50°$	$AI = 1.5$ in	$C = 3\pi \approx 9.42$ in	≈ 1.31 in	≈ 0.139
$\overset{\frown}{CH} = 50°$	$AH = 2$ in	$C = 4\pi \approx 12.57$ in	≈ 1.75 in	≈ 0.139
$\overset{\frown}{DG} = 50°$	$AG = 2.5$ in	$C = 5\pi \approx 15.71$ in	≈ 2.18 in	≈ 0.139
$\overset{\frown}{EF} = 50°$	$AF = 3$ in	$C = 6\pi \approx 18.85$ in	≈ 2.62 in	≈ 0.139

Have students discuss in groups their observations about the arcs they have measured. Then have groups share their measures with the class and record class data for the various arcs. Have students make observations about the lengths of the arcs for the same circle

and for different circles. During this discussion, students should begin to see that arcs which have the same degree measure do not have the same arc length if they are parts of circles whose radii are different lengths. Ask students if the arc length is proportional to any other measure of the circle. After some discussion and probing, students should see that arc length is proportional to the radius of the circle by a factor of $2\pi\left(\dfrac{N}{360}\right)$, meaning that if the measure of the central angle is known, then that becomes the constant of proportionality for all arcs associated with that central angle and as the radius increases (or decreases) so will the arc length.

As an entry in the students' math *learning logs* (**view literacy strategy descriptions**), have students explain how arc measure and arc length differ, and under what conditions two arcs can have the same measure, but different lengths. Ask for students to volunteer their explanations and lead a class discussion on the topic.

2013-2014
Activity 7: Radian Measure (CCSS: G.C.5, RST.9-10.4)

Materials List: pencil, paper, string, circular objects from Activity 6, scissors, rulers, calculators, tables with Concentric Circles measurements from Activity 6

This activity should immediately follow Activity 6 when implemented (Activities 6 and 7 should be combined).

Have students add the word *radian* to their *vocabulary awareness charts* (view literacy strategy descriptions) they started in Activity 1. Have students rate their level of knowledge of this new term with a plus, check, or minus. Most students will probably not have an understanding of the word. Then define the term *radian* as "the measure of the angle made at the center of a circle by an arc whose length is equal to the radius of the circle." Ask students to think about the definition for a moment, then give their interpretations of what the definition means using the understanding of central angles and arc length from previous activities. Allow students to state observations as well as ask questions based on the definition they were given. Students may ask how degrees compare to radians, whether the size of a radian changes, whether radians can be measured with a tool like a protractor, and when radians are used for calculations. Make a list of the questions and observations for the class to see and continue with the remainder of the lesson. Keep the questions and observations visible during the lesson so students can refer to them as they find answers to their questions.

Using the circular objects from Activity 6, have students, working in groups, cut a length of string equal to the measure of the circumference of their object. Ask students to imagine taking a length of string equal to the measure of the radius and wrapping it around the circumference of the object. Ask them to estimate how many times the radius would fit around the circle they have been given. Once groups have made their predictions, instruct them to cut the string that represents the circumference into pieces

that are the same measure as the radius of their circle. Groups should cut 6 radii from the total circumference with a small piece left over. Have students measure the "left over" piece and divide its measure by the measure of the radius. If the measurements were close to accurate, the decimal value of the "left-over" piece should be approximately 0.28. Record the results from each group on the board for students to make some observations. Ask students, "What do the pieces which you cut from the circumference represent?" Students should understand that the six large pieces represent an arc length which is equal to the length of the radius and that each of those pieces represents 1 radian. The remaining small piece is approximately 0.28 radians (28 hundredths of the length of the radius). Have students generalize that there are approximately 6.28 radians, or exactly 2π radians, in the circumference of a circle, regardless of the length of the radius or the measure of the circumference. In order to help students understand that radians are a unit of measure for central angles, have students determine how many degrees are in 1 radian. First, ask them to make a prediction about whether 1 radian is larger or smaller than 1 degree and provide a justification for their reasoning. Then have students recall the sum of the measures of all central angles of a circle in degrees (360°). Since radians are units of angle measure, the total radian measure 2π, is equivalent to the total of the central angles, 360°. Therefore, $2\pi = 360°$. In order to find the value of 1 radian, divide both sides by 2π. The answer is $1 \text{ radian} \approx 57.296°$. Explain to students that when an angle measure is stated, if the measure is in degrees, the degree symbol must be used. When an angle measure is given with no degree symbol (or no notation at all), it is assumed to be in radians. Sometimes the abbreviation "rad" may be used to indicate a radian measure.

The exact degree measure of a radian is $1 \text{ radian} = \dfrac{180}{\pi}$. This equivalence can be used to convert radian measures to degree measures while $1° = \dfrac{\pi}{180}$ can be used to convert degree measures to radian measures.

Next, using the tables the students created for Activity 6: Concentric Circles, have students determine the ratio of the arc length to the radius. For example, using the table provided below (copied from Activity 6), the ratios for arc length to radius are all approximately 0.873. Have students use the exact arc length rather than the approximation found in Activity 6.

Measure of Assigned Central Angle: $\angle BAI = 50°$

Ratio of Central Angle to 360°: $\dfrac{5}{36} \approx 0.139$

Minor Arc	Radii	Circumference	Arc Length	Arc Length to Circumference	Arc Length to Radius
$\overline{BI} = 50°$	$AI = 1.5$ in	$C = 3\pi \approx 9.42$ in	$\dfrac{5\pi}{12}$	≈ 0.139	$\dfrac{5\pi}{18} \approx 0.873$
$\overline{CH} = 50°$	$AH = 2$ in	$C = 4\pi \approx 12.57$ in	$\dfrac{5\pi}{9}$	≈ 0.139	$\dfrac{5\pi}{18} \approx 0.873$
$\overline{DG} = 50°$	$AG = 2.5$ in	$C = 5\pi \approx 15.71$ in	$\dfrac{25\pi}{36}$	≈ 0.139	$\dfrac{5\pi}{18} \approx 0.873$

$\widehat{EF} = 50°$	$AF = 3$ in	$C = 6\pi \approx 18.85$ in	$\dfrac{5\pi}{6}$	≈ 0.139	$\dfrac{5\pi}{18} \approx 0.873$

Ask students to then interpret what the ratio means. Give students a moment to think about the relationship and then ask them to share their ideas with the class. Students should realize that if they multiply the ratio by the measure of the radius, they will obtain the measure of the arc length. Also, some students may realize that if they divide the arc length by the ratio, they will get the length of the radius. Therefore, the students should have the following three relationships:

$$\text{Ratio} = \frac{\text{Arc Length}}{\text{Radius}} \qquad \text{Arc Length} = \text{Ratio} * \text{Radius} \qquad \text{Radius} = \frac{\text{Arc Length}}{\text{Ratio}}$$

Refer students to the circumference formula, $C = 2\pi r$. The circumference formula is written in the same format as the second relationship above. Have students identify the pieces of the formula in terms of the stated relationship. Discuss with students that the circumference is the arc length around the whole circle, 2π is the ratio, and r is the radius. Earlier in the lesson, 2π was identified as the radian measure for the whole circle. Ask students, "If 2π is the ratio in the equation and it is the radian measure for the circle, what can we generalize about the ratio of the arc length to the radius for any central angle?" Students should realize that the ratio is the radian measure of the central angle. Therefore, the formula for arc length can be written as $s = \theta r$ where s is the arc length, θ is the measure of the central angle in radians, and r is the radius. Review with students that the arc length is proportional to the radius. In this formula, the constant of proportionality is the radian measure. To further assist students in seeing the connection between radians and degrees, have them equate the two formulas for arc length discussed in this unit and simplify the equation. *Solution:* $\dfrac{N}{360} 2\pi r = \theta r$. *Since the radius would be equal in these formulas, the equation can become* $\dfrac{N}{360} 2\pi = \theta$ *and simplified, that equivalence becomes* $\dfrac{N\pi}{180} = \theta$. The resulting equivalence can then be used to convert radians to degrees and vice versa.

Activity 8: Probability and Circle Graphs (GLEs: 13; CCSS: S.CP.4, RST.9-10.4)

Materials List: Anticipation Guide BLM, Relative Frequency and Probability BLM, results from group surveys, protractors, calculators, pencil, paper, color pencils or colors, teacher-created data sets and matching circle graphs, the Internet access, magazines, newspapers (The last three items are optional and for teacher use)

Start by giving students a copy of the Anticipation Guide BLM and have students read the statements and respond by circling either "Agree" or "Disagree" to indicate how they

feel about the statement. This BLM uses an *anticipation guide* (<ins>view literacy strategy descriptions</ins>) to activate students' prior knowledge of the two-way tables and conditional probabilities. It will also help set the purpose for learning the material to be presented, which makes *anticipation guides* especially helpful to struggling and reluctant learners. Tell students to respond to the statements individually under the column with the heading "Before Learning" and be prepared to explain their reasoning (suggest to students that they write thoughts about their reasoning in the space at the bottom of the page or on the back of the page). At this point, students should not be concerned about whether their answers are correct. After students have had the opportunity to respond individually, have students get into pairs to discuss their responses. Emphasize that there is no "correct" answer at this stage and that students should discuss their choices freely. Then have students share their responses with the whole class. After the discussion, tell students they are about to learn information about the topics included on the *anticipation guide*. Tell them to pay particular attention to the content that is related to the statements on the *anticipation guide*. Proceed with the lesson below.

Give students a copy of the Relative Frequency and Probability BLM. This BLM will introduce the concepts of two-way frequency tables, relative frequencies, and conditional probabilities. Have students read and work through the information in the Relative Frequency and Probability BLM stopping periodically to have students consider the statements from the *anticipation guide* and have students reconsider their "Before Learning" responses. Students should revise, if necessary, their original responses to reflect their new learning. Also, as students work to answer the questions on the Relative Frequency and Probability BLM, have students compare their answers with a partner. Discuss correct answers to the questions on the Relative Frequency and Probability BLM to avoid misconceptions and misunderstandings as they progress through the material. After the material has been presented, engage students in a discussion around the statements in the *anticipation guide*. This will provide an opportunity to clarify any lingering misconceptions about issues, information, and concepts.

Have students work in groups to conduct surveys about favorite TV shows, foods, colors, etc. The surveys should be completed outside of class. They can be completed before the beginning of the lesson, so the data is ready when the students complete the initial introduction to two-way frequency tables. Students should ask the questions to a variety of students from different grades, not just the people in their class. Each group can ask two different questions: "What is your favorite TV show? What is your favorite food? What is your favorite color?" Each group should compile its results and create a two-way frequency data table to organize their data. The two-way table can compare answers of the two questions (compare responses of favorite TV Show and Favorite Food) or it can compare the age ranges of the survey participants to the answers to the questions. If class size may be an issue, compile the class data to create a larger data set. Using the data students collect and the two-way tables they create, have students create probability problems about their data sets. Then have groups exchange their problem and data sets and have the groups find the requested probabilities.

Next, have groups find the relative frequencies for the data they collected with their surveys. Provide the students with the definition of the term *sector*. Discuss with students how a circle graph is created and why the percentage, or relative frequency they have found, is important when creating a circle graph. Students should understand that in order to create accurate circle graphs, the measure of the central angle should be proportional to the percentage for the category in the data set. Also, circle graphs can only be created based on one set of categorical data, so the circle graphs will have to represent either the marginal frequencies in the rows or the marginal frequencies in the columns of the two-way tables, not the joint frequencies in the body of the table. Have groups of students create circle graphs to represent their data. When creating their graphs, allow students to use colors or color pencils to shade the sectors of their graphs. Make sure that students use protractors to calculate the correct angle measures based on their data. Once groups create their circle graphs, have them share the graphs with the whole class.

Prior to the start of this activity, search the Internet or magazines and newspapers for data which can be presented as a circle graph. Another option is to survey the classes on various topics like their favorite colors, movies, foods, hobbies, etc., months of the students' birthdays, ages of the students (if there is a variety among all of the classes). This survey can be accomplished as an interest survey.

Using the data collected either from the Internet or the survey, provide the students with the data in table form (one-way or two-way tables) and matching circle graphs that are already drawn. Make sure some of the circle graphs are constructed incorrectly. Have students discuss the data that is presented to them in the graph and compare to the data provided in the table. Using their knowledge of central angles, instruct students to determine if the circle graph has been constructed correctly. If any graph is incorrectly constructed, indicate that students should develop a new graph based on the given data. Have students determine the kind of arc associated with each category of data (e.g., major, minor, semicircle). Instruct students to determine what each of the sectors represents. For instance, if one of the circle diagrams is a survey about favorite television programs and 24% of the 250 people surveyed like *The Frugal Millionaire*, have students determine the number of people who like that show.

Activity 9: Geometric Probability (CCSS: S.CP.4)

Materials List: paper, pencil, teacher-created "dart" boards, calculators, *learning logs*, transparencies (optional), images of circles divided into various sized pieces, spinners, calculators, Conditional Geometric Probability BLM

Provide students with several "dart" boards made of circles. For example, use circles on squares with the circles cut up into quarter pieces and placed in the corners of a square, or use several concentric circles. Have students shade in some of the circular regions, determine the areas of these regions, and then figure the probability of a dart which is thrown randomly at a dartboard's landing in a shaded region (assume each dart hits the

board). Have students create dartboards that possess specific probabilities for a randomly thrown dart's landing in a shaded region.

As an entry in the students' math *learning logs* (view literacy strategy descriptions), have the students explain the process they should employ to find the probability of a dart's hitting a certain region of the board. The prompt could be:

> Create a dart board that includes at least one circle and one other polygon. Shade at least one region of the dart board. Calculate the probability of a dart's hitting the shaded region(s). Explain the process you used to find the probability for your dart board.

After students have written their entries, allow students to draw a diagram of the boards they described in their *learning log* entry on transparencies or some other device and share them with the class. Have the whole class determine if the calculated probability matches what the student said it should. This can also be done as a group activity.

Have students practice finding the area of a sector using the formula $A = \dfrac{N}{360}\pi r^2$, where N is the measure, in degrees of a central angle. Give students circle spinners divided into unequal sectors. Have students find the probability of spinning and landing on a certain sector. Allow students to play simulated games to see if there is a way always to win the game if there are points allotted to certain sectors. Give each student a copy of the Conditional Geometric Probability BLM and have them attempt to answer the question. Then pose more problems involving conditional probabilities. Discuss in each case whether the probability being requested is independent or dependent.

As an entry in the students' math *learning logs* (view literacy strategy descriptions), have the students answer the following prompt: "Is the game Jim and Susan are playing fair? Why? How would you change the game? Explain your reasoning for the changes" After students have answered the prompt, have students share their responses with the whole class and discuss the reasoning used.

Activity 10: Arcs and Chords (GLE: <u>13</u>)

Materials List: geometry software program (or compass and straightedge), pencil, paper, centimeter ruler, protractor, Diameters and Chords BLM, scissors, patty paper

Have students use a geometry software program (or compass and straightedge) to inscribe a variety of polygons in circles. Next, have students determine the measure and length of each arc of the circle subtended by a chord. For example, inscribe a stop sign in a circle and then determine the two measures (arc measure and arc length) of each of the 8 arcs.

Use a *process guide* (view literacy strategy descriptions) to help students examine the relationship between a chord and its arc when a diameter is perpendicular to the chord,

and the relationship between two chords that are equidistant from the center. *Process guides* are used to guide students in processing new information and concepts. They are used to scaffold students' comprehension and are designed to stimulate students' thinking during and after reading. *Process guides* also help students focus on important information and ideas. In this activity, students will be given *process guides* that will lead them through the steps to discover the relationships created by chords and diameters of a circle.

Provide each student with a copy of the Diameters and Chords BLM. Have students work as pairs to complete the investigation. Have the pairs share their findings with the rest of the class. Students should be told that they are required to support their statements, conjectures, and answers, with evidence from their investigations with the *process guide*. Lead a summary discussion of the conjectures formed through the investigation. Also encourage students to add these conjectures to their split-page notes to help them organize their new information.

At this point, students should be given the opportunity to apply their understanding of the conjectures by having the students find measures of segments in circles. These problems should include algebra skills as well.

To complete the activity, pose the problem of finding the center of a circular picnic table in order to cut a hole for an umbrella. Challenge students to use their knowledge of chords, lines perpendicular to a chord at its midpoint, and the intersection of these lines to find the center of the circle.

Activity 11: Inscribed angles (GLEs: 13, 19)

Materials List: paper, pencil, ruler, protractor

Have each student draw a circle and then inscribe a regular hexagon in the circle. Students should label the hexagon *ABCDEF* and the center of the circle *P*. Draw radii $\overline{PA}$, $\overline{PF}$, and $\overline{PB}$. Label the following angles: $\angle FPA$ as $\angle 1$, $\angle APB$ as $\angle 2$, $\angle FAP$ as $\angle 3$, and $\angle BAP$ as $\angle 4$. Have students measure each numbered angle with a protractor, then find $m\overset{\frown}{FA}$ and $m\overset{\frown}{AB}$. *Solution: The measure of each numbered angle is 60 degrees. The arc measure for the arcs listed are each 60 degrees as well.* Next, instruct students to find $m\angle FAB$ and $m\overset{\frown}{BDF}$ and make a conjecture about the relationship between $m\angle FAB$ and $m\overset{\frown}{BDF}$. *Solution:* $m\angle FAB = 120$ and $m\overset{\frown}{BDF} = 240$. *The measure of major arc BDF is found by subtracting the measure of minor arc FB from 360. A conjecture that students may produce is that the measure of the major arc subtended by the inscribed angle is half of the measure of the inscribed angle.* Have students prove this conjecture and consider all three cases: the center of the circle lies on the side of the angle, the center of the circle is in the interior of the angle, and the center of the circle is in the exterior of the angle. Ask students to investigate the

measure of an angle inscribed in a semicircle, and the measures of the angles in a quadrilateral inscribed in a circle.
The theorems students should develop in this activity are as follows:
- If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc (or the measure of the intercepted arc is twice the measure of the inscribed angle).
- If an inscribed angle intercepts a semicircle, the angle is a right angle.
- If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

Activity 12: Tangents and Secants (GLE: <u>13</u>)

Materials List: Tangents and Secants BLM, compass, pencil, paper, straightedge, information on satellites

Provide students with copies of the Tangents and Secants BLM. Use the BLM to discuss the three theorems listed below:
1. If a tangent and a secant intersect at a point on a circle, then the measure of each angle formed is half of the measure of its intercepted arc.
2. If two secants intersect in the interior of a circle, then the measure of each angle is half the sum of the measures of the arcs intercepted by the angle and its vertical angle.
3. If a tangent and a secant, two tangents, or two secants intersect in the exterior of a circle, then the measure of the angle formed is half the difference of the measures of the intercepted arcs.

Throughout the discussion, have students write down the relationships formed by each pair of intersecting lines.

The first theorem is based solely on the inscribed angles discussed in Activity 11. To help illustrate the second theorem, have students use a compass to draw a circle of any radius and choose any point on the interior of the circle (not the center) and label it *V*. Then, using a straightedge, have students draw two secants that intersect at *V*. Label one secant as $\overset{\frown}{WY}$ and the other secant as $\overset{\frown}{XZ}$. Label $\angle ZVY$ as $\angle 1$. Next have them draw $\overline{XY}$ and label $\angle VYX$ as $\angle 2$ and $\angle VXY$ as $\angle 3$. Students should see that $\angle 2$ and $\angle 3$ are inscribed angles so $m\angle 2 = \frac{1}{2} m\overset{\frown}{WX}$ and $m\angle 3 = \frac{1}{2} m\overset{\frown}{ZY}$. Lead the students in a discussion about why $m\angle 1 = m\angle 2 + m\angle 3$ and help them write an equation based on that understanding. Students should be able to understand the second theorem with this illustration.

Provide students with information about geostationary satellites and their orbits. Geostationary satellites move in a circular orbit about 26,000 miles above the Earth's center. For a satellite whose orbit is directly over the equator, have students determine the measure of the arc along the equator that is "visible" to the satellite when given the

measure of the exterior angle formed by the two tangent lines drawn from the satellite to the earth. For a satellite that is 26,000 miles from Earth's center, the angle formed by two tangent lines measures approximately $17.7°$. Ask students to apply the third theorem to find the measure of the required arc. Next, provide students with an arc measure along Earth's equator that is less than $\frac{1}{2}$ of Earth's circumference (so that they have two secant lines intersecting at the satellite's location). Have students determine the measure of an angle of view of the satellite.
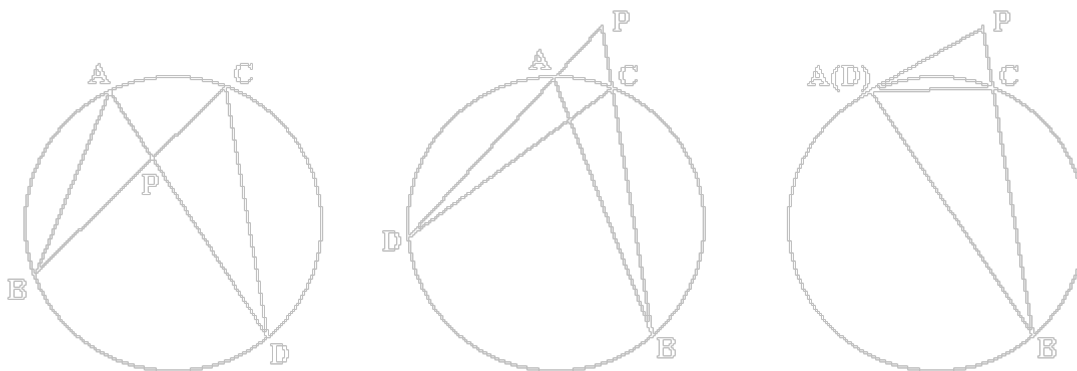
Activity 13: Intersecting Chords and Secants (GLE: <u>13</u>)

Materials List: drawing program, pencil, paper

Have students use a drawing program such as *The Geometer's Sketchpad*® to construct two intersecting chords, two intersecting secants, or a tangent and secant for a circle. These three cases are the basis of the Power of a Point theorem. When point P lies inside the circle, the theorem is called the Intersecting Chords theorem; when point P lies outside the circle, the theorem is called the Intersecting Secants theorem.

Students should first prove that there are two similar triangles created if additional segments are added. ςAPB and ςCPD, in each case in the diagrams provided below, are the similar triangles. Once the similarity of the triangles is established, then students should be able to write the proportion $\dfrac{AP}{CP} = \dfrac{BP}{DP}$, which is equivalent to the statement of the theorem: AP·DP = BP·CP. *Note that in the case of the tangent, points A and D coincide and are the same point.*


Have students work various problems involving intersecting chords and secants and using the theorems discussed in the activity to find missing segment measures.

Activity 14: Surface Area of a Sphere (GLE: <u>7</u>)

Materials List: Surface Area of a Sphere BLM, small spheres, wrapping paper, pencil, paper, scissors, tape

Here is a concrete way to show students why the surface area of a sphere formula is $4\pi r^2$.

Students should already understand that the surface area of an object can be represented by how much wrapping paper it would take to cover it. Ask them to picture a sphere (a balloon or ball) and a piece of paper that is cut as wide as its diameter and as long as its circumference.

Paper is same height as ball's diameter
and same length as circumference.
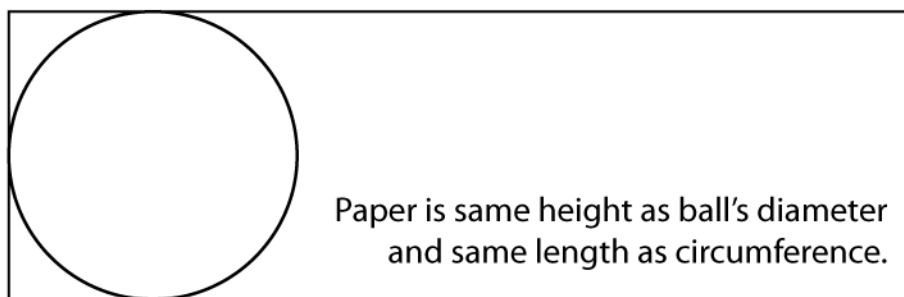
If the ball were wrapped with the paper, the paper would cover the entire sphere except for all the overlaps (which would fit into the gaps if they were cut out). If possible, provide each student with a small sphere such as a tennis ball, softball, or golf ball. Have each student cut a rectangle from wrapping paper that will match the specifications listed for his/her ball and then test the concept.

Lead students through the algebraic development of the formula for the surface area of a sphere using this model as a starting point. The formula for the surface area of the paper is

> Length $^{\times}$ width which can be written as circumference $^{\times}$ diameter

This is easily understood by looking at the picture above. Now, substitute formulas we know:

$$C = 2\pi \times r \text{ and } d = 2r$$

$$C \times d$$
$$2\pi r \times 2r$$
$$4\pi r^2 \quad \text{is equal to the surface area}$$

Provide practical applications problems which require the use of the surface area of a sphere for students to work.

Activity 15: Surface Area and Volume of Spheres (GLEs: 7, 10)

Materials List: the Internet access for research, paper, pencil, scissors, tape, centimeter or inch cubes, several types of balls

Spherical balls are used in many sports (e.g., golf, soccer, baseball, basketball). Have students research the various dimensions for selected balls either by searching the Internet or by checking the sizes of various balls at the store. Students will then create a circle on paper that represents a great circle associated with the sphere (ball). Using the

circle pattern, have students cut the circle into fractional sectors, each of which represents $\frac{1}{8}$ of the circle. Next, have students "cover" a quarter of the sphere (ball) with these sectors. Students will then make a conjecture about the surface area formula for a sphere. To conceptualize the volume formula, have students use centimeter or inch cubes and create a large cube that approximates the size of the ball.

Following the development of the formulas, provide students with several types of balls (e.g., baseball, golf ball, basketball, soccer ball, tennis ball, etc.). Working in groups, students will determine the surface areas and volumes for each type of ball by first making appropriate measurements and then using those measurements in the correct formula.

2013-2014
Activity 16: Circles on the Coordinate Plane (CCSS: G.GPE.1)

Materials List: paper, pencil, graph paper, automatic drawing program

Begin the lesson by having students create a *lesson impression* (view literacy strategy descriptions) for this activity. *Lesson impressions* capitalize on students' curiosity and create situational interest in the content to be covered. By asking student to form a written impression of the topic to be discussed, they become eager to determine how closely their impression text matches the actual content. Provide students with the following list of key terms/phrases: circle, radius, center, Pythagorean theorem, equation, standard form, completing the square, and general form. Ask student to use the list of terms/phrases to write a short descriptive passage to guess what will be covered in the lesson. After giving students a few minutes to create their *lesson impression* texts, have a few students share their descriptions with the class. Then continue with the remainder of this activity, directing students to compare their texts with the actual information presented throughout the lesson. One way students could keep track of the similarities and differences between their *lesson impression* texts and the actual content is to create a Venn diagram with one circle's representing the student's ideas, one circle's representing the actual content, and the overlapping area's representing the common ideas.

Have students graph Circle C on graph paper with the center located at $C(3,4)$ and having a radius of 3 units. Ask students to identify as many points on the circle as they can. Students will only be able to accurately identify four points: $(6,4)$, $(3,7)$, $(0,4)$, and $(3,1)$. Some students will think that the points $(5,6)$ or $(1,6)$ are also on the circle. Ask them how they could verify whether a point was on the circle. Students should understand that if a point lies on the circle, the distance between the point and the center should be equal to the measure of the radius. Review with students how they can use either the Pythagorean theorem or the distance formula to find the distance between two points. To use the Pythagorean theorem, students would need to imagine, or draw, a right triangle radius of the circle as the hypotenuse. To find the length of each legs, students would subtract the *x*-values to find the length of the horizontal leg and subtract the *y*-

values to find the length of the vertical leg. These values would represent the values of *a* and *b* in the Pythagorean theorem. Have students determine whether $(5,6)$ or $(1,6)$ are truly on the circle. *Solution: Both segments have a measure of $2\sqrt{2}$ units or approximately 2.83 units. Since the measure found is not equal to the measure of the radius (3 units), these points cannot be on the circle.* If students identify any other points they believe are on the circle, have them complete the same process to verify whether the point is on the circle. Then repeat this process with a circle with the center located at $(2,-7)$ and a radius of 5 units. This time, students should be able to identify a total of 12 points on the circle and verify the measure of the radius to support their findings. *Solution: The 12 points are* $(7,-7),(6,-4),(5,-3),(2,-2),(-1,-3)$, $(-2,-4),(-3,-7),(-2,10),(-1,-11),(2,-12),(5,-11),(6,-10)$. Then ask students, "What would help to describe the set of all points that lie on a circle?" Lead students through a discussion to develop the idea of writing an equation for the circle if the center and radius of a circle are known. Students my need to be reminded of the purpose of a linear equation and that it represents the set of all ordered pairs that lie on the graph of the line for that equation. Guide students through the process of writing the equation of the circles graphed above using the Pythagorean theorem. First, discuss with students the fact that any arbitrary point on the circle can be represented by the ordered pair (x,y). Then have them determine the lengths of the legs of a right triangle formed by using the center $(3,4)$ and the arbitrary point (x,y) as the endpoints of the hypotenuse. Then using the Pythagorean theorem, have students write the equation of the circle. *Solutions: For the first circle with center (3,4), the equation is* $(x-3)^2+(y-4)^2=9$. *For the second circle with center (2,-7), the equation is* $(x-2)^2+(y+7)^2=25$. Have students find the equations of the circles described below.

Circles: *Solutions*

Center: $(-4,1)$; Radius: 2 units $(x+4)^2+(y-1)^2=4$

Center: $(5,0)$; Radius: 10 units $(x-5)^2+y^2=100$

Center: $(-2,-4)$; Radius: 1 unit $(x+2)^2+(y+4)^2=1$

Next, ask students to use the same process to find the standard form of the equation of any circle using (h,k) as the center and *r* as the radius. The standard form of the equation is $(x-h)^2+(y-k)^2=r^2$. Have students make connections between the standard form and the equations they wrote earlier in the activity given specific information about the center and radius. Be sure to discuss how students can use the standard form to determine the center and radius of a circle given the standard form of the

equation. Provide students with multiple examples of the equation of a circle in standard form and have them practice finding the center and radius.

Next, define the general form of the equation of a circle as $Ax^2 + By^2 + Cx + Dy + E = 0$ where $A = B$. Then, provide students with the equation $x^2 + y^2 - 6x - 8y + 16 = 0$. To find the center and radius, students will need to employ the process of completing the square. Have students follow the steps below to complete the square for the general form of the equation of a circle:

    (1) Rearrange the terms to group variables and move all constants to the right side.
    (2) If the coefficient on the squared terms is not 1, factor out the coefficient and divide both sides by the coefficient.
    (3) Take ½ the coefficient of linear terms (x and y terms) and square the result to find the constants for each group, adding the same quantities to the both sides of the equation.
    (4) Write the perfect square trinomials as binomials squared and simplify the right side.
    (5) Identify the center and radius of the circle.

Solution:

1) *rearrange terms to group variables and move constants to the right side* $\left(x^2 - 6x\right) + \left(y^2 - 8y\right) = -16$

2) *Take ½ of 6 and 8, square them, and add to both sides.* $\left(x^2 - 6x + 9\right) + \left(y^2 - 8y + 16\right) = -16 + 9 + 16$

3) *Write the perfect square trinomials as binomials squared.* $(x - 3)^2 + (y - 4)^2 = 9$

4) *identify the center and radius*   *center: (3,4); radius: 3*

This process will take a lot of development for students, so provide many opportunities for practice with completing the square and identifying the center and radius.

2013-2014
Activity 17: Equations of Parabolas (CCSS: G.GPE.2, RST.9-10.4)

Materials List: pencil, paper, the Internet, graphs of various parabolas,

Have students visit the following website to read the information provided about the terms parabola, focus, and directrix:
http://www.mathwarehouse.com/quadratic/parabola/focus-and-directrix-of-parabola.php.
Students should add these terms to their *vocabulary self-awarness* (view literacy strategy descriptions) charts they began in Activity 1.

Explain to students that the parabola has one focus point and the graph of the parabola wraps around the focus. The equation of a parabola can be created using a combination of distances from the focus and directrix to the graph of the parabola. Continue to explain

that parabolas are used in many situations in the real-world. One application is lighting. For instance, an automobile headlight is constructed of a reflective surface which has a parabolic shape. The actual light bulb is placed at the focus of the parabola. The light beams are then reflected in beams that are "focused" directly forward from the car.

Have students work in pairs and provide each pair with a graph of the equation $y = \frac{1}{4}x^2$, but do not give students the equation. On the graph, have the focus and the directrix identified. The focus of this parabola is (0,1) and the directrix is $y = -1$. Remind students that every point on the parabola, by definition, must be the same distance from the directrix and the focus of the parabola. Have students choose any point on the parabola and label it (x,y). Then have students draw a segment to connect the focus to the point they have chosen. Next, have students draw a perpendicular line from (x,y) to the directrix and label the point $(x, -1)$. Discuss with students why this is the ordered pair for the point where the perpendicular line intersects the directrix. Then, ask students how they would find the distance from that point (x,y) to the focus (0,1) and the directrix $(x, -1)$. Students should realize they can use the distance formula to find these distances. Since the definition of a parabola says these distances are equal, $\sqrt{(x-0)^2+(y-1)^2} = \sqrt{(x-x)^2+(y+1)^2}$. Have students expand this equation and isolate y to write the equation of the parabola.

$$\sqrt{(x-0)^2+(y-1)^2} = \sqrt{(x-x)^2+(y+1)^2}$$
$$x^2 + y^2 - 2y + 1 = y^2 + 2y + 1$$
$$x^2 = 4y$$
$$\frac{1}{4}x^2 = y$$

Solution:

Have students test some points in the equation they just wrote and verify they are on the parabola by plotting them on the graph they were given. Complete this activity by providing students many more opportunities to practice writing the equation of a parabola when given the focus and directrix. Each equation that will be written using this method will result in the standard form of the equation of a parabola, $y = ax^2 + by + c$.

Sample Assessments

General Assessments

- The student will complete entries in his/her *learning log* for this unit. Topics could include:

- o Explain why the formula for the surface area of a sphere is $4\pi r^2$ based on the activity performed in class.
  - o Explain the differences between the secant of a circle and the tangent of a circle.
  - o ςABC is inscribed in a circle so that $\overline{BC}$ is the diameter. What type of triangle is ςABC? Explain your reasoning.
- The student will find pictures in magazines or newspapers of diagrams that show tangents, secants, and chords. The student will explain what the picture is and why it represents the term he/she is defining. These pictures could be included in a portfolio.
- The student will construct various circles with different areas. The student will construct specified inscribed angles, arcs of given measures, secants, and tangents.

Activity-Specific Assessments

- Activity 8: The student will find examples of circle graphs in magazines or newspapers. He/she will write a paragraph that describes the information presented in the graph and find the measures of the central angles based on the information provided in the graph. He/she will determine if the graph has been constructed correctly.

- Activity 9: The student will create a dartboard game using area properties of circles or other figures. The student will determine the probabilities involved with the game and then play the games to determine the experimental probability.

- Activity 15: The student will:
  1. Find the volume of a tennis ball can. The student will either take measurements from an actual can or the information will be provided in a diagram.
  2. Assuming that the tennis balls are tightly packed, find the total volume of the three tennis balls.
  3. Determine the percentage of the volume of the can taken up by the tennis balls.
  4. Determine the volume of sand needed if sand is poured to fill the remaining air space in the can.
  5. Calculate the percentage of the can's volume taken up by the sand if one of the tennis balls is removed and sand is poured in to replace it.