

Geometry

Unit 5: Triangles and Quadrilaterals

Time Frame: Approximately five weeks



Unit Description

This unit introduces the various postulates and theorems that outline the study of congruence and similarity. The focus is on congruence and similarity as a result of rigid motions and similarity transformations. It also includes the definitions of special segments in triangles, classic theorems that develop the total concept of a triangle, and relationships between triangles and quadrilaterals that underpin measurement relationships. The properties of the special quadrilaterals (parallelograms, trapezoids, and kites) are also developed and discussed.

Student Understandings

Students should know defining properties and basic relationships for all forms of triangles and quadrilaterals. They should also be able to discuss and apply the congruence postulates and theorems and compare and contrast them with their similarity counterparts. Students should be able to apply basic classical theorems, such as the Isosceles Triangle theorem, Triangle Inequality theorem, and others.

Guiding Questions

1. Can students illustrate the basic properties and relationships tied to congruence and similarity?
2. Can students develop and prove conjectures related to congruence and similarity?
3. Can students draw and use figures to justify arguments and conjectures about congruence and similarity?
4. Can students state and apply classic theorems about triangles, based on congruence and similarity patterns?
5. Can students construct the special segments of a triangle and apply their properties?
6. Can students determine the appropriate name of a quadrilateral given specific properties of the figure?
7. Can students apply properties of quadrilaterals to find missing angle and side measures?

Unit 5 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

Grade-Level Expectations	
GLE #	GLE Text and Benchmarks
Algebra	
6.	Write the equation of a line parallel or perpendicular to a given line through a specific point (A-3-H) (G-3-H)
Geometry	
9.	Construct 2- and 3-dimensional figures when given the name, description, or attributes, with and without technology (G-1-H)
16.	Represent and solve problems involving distance on a number line or in the plane (G-3-H)
19.	Develop formal and informal proofs (e.g., Pythagorean theorem, flow charts, paragraphs) (G-6-H)
CCSS for Mathematical Content	
CCSS#	CCSS Text
Congruence	
G.CO.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
G.CO.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
Similarity, Right Triangles, and Trigonometry	
G.SRT.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
Expressing Geometric Properties with Equations	
G.GPE.6	Find the point on a directed line segment between two given points that partitions the segment in the given ratio.
G.GPE.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
ELA CCSS	
CCSS #	CCSS Text
Reading Standards for Literacy in Science and Technical Subjects 6-12	
RST.9-10.3	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
Writing Standards for Literacy in History/Social Studies, Science and Technical Subjects 6-12	
WHST.9-10.1b	Write arguments focused on discipline-specific content. Develop claim(s) and counterclaims fairly, supplying data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form and in a manner that anticipates the audience's knowledge level and concerns.
WHST.9-10.10	Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.

Sample Activities

Teacher Note: Before beginning this unit, the teacher should make sure the students have a good understanding of the Angle Sum theorem and Exterior Angle theorem found in Unit 3 Activity 7. Students should also be able to classify triangles according to their side and angle measures. If necessary, the teacher may take a day or two to review these concepts; however, this should be kept to a minimum.

Activity 1: Corresponding Parts of Congruent Triangles (CPCTC) (CCSS: G.CO.7; CCSS: WHST.9-10.10)

Materials List: paper, pencil, graph paper, protractor, *learning logs*

Give students the coordinates of the vertices of a triangle (be sure to use a scalene triangle) and have them reflect the triangle over the x -axis. Students should remember that a reflection is an isometry and, therefore, the image should be congruent to the pre-image. Have students verify that the sides of the triangle are congruent by having them use the distance formula. Also, have them measure the angles using a protractor. Lead a brief discussion about corresponding parts. Have students identify the corresponding sides and the corresponding angles. Ask students, “How can we determine the corresponding sides and corresponding angles?” Students should understand that corresponding sides and corresponding angles should have the same measure if the triangles are congruent. Have students write a congruence statement with the triangles (i.e., $\triangle ABC \cong \triangle A'B'C'$). Discuss the importance of the order of letters naming the vertices. This portion of the activity can be repeated with other transformations and different triangles as many times as needed to be sure students can identify corresponding parts of triangles. Provide students with triangles that have already been transformed in some way and ask them to determine the corresponding sides and angles. Also, provide them with an incorrect congruence statement based on given diagrams and ask them if the given statement is correct for the diagram provided.

Provide students with congruence statements like $\triangle MET \cong \triangle PSA$ and ask them to name the corresponding sides and angles based on the statement alone. Also, have students create equivalent congruence statements (i.e., $\triangle TEM \cong \triangle ASP$ or $\triangle EMT \cong \triangle SPA$ are equivalent statements).

To end the activity, have students answer the following prompt in their *learning log* ([view literacy strategy descriptions](#)): “If you were given $\triangle SMA \cong \triangle THE$, what information must be true? If you are told $\overline{SK} \cong \overline{CA}$, $\overline{KY} \cong \overline{AT}$, $\overline{SY} \cong \overline{CT}$, $\angle S \cong \angle C$, $\angle K \cong \angle A$, and $\angle Y \cong \angle T$, what must be true? Explain your reasoning.” After giving students time to respond in their *learning logs*, ask a few students to share their responses to make sure there are no misconceptions or misunderstandings. Students should revise their responses as needed. Students should understand that if they are told two triangles are congruent, then the corresponding parts (sides and angles) are also congruent. Likewise, if they are given

statements that indicate the corresponding sides and angles are congruent, then the two triangles are congruent.

Activity 2: More about Congruent Triangles (CCSS: G.CO.7, G.CO.8)

Materials List: pencil, paper, Investigating Congruence BLM, ruler, protractor, patty paper, Sample Split-Page Notes BLM, Blank Split-Page Notes BLM, Triangle Split-Page Notes Model BLM

*Teacher Note: Safety compasses can be used in schools where sharp, pointed instruments are prohibited. Techniques for using patty paper to duplicate segments, construct perpendicular lines, angle bisectors, etc., can be found in *Patty Paper Geometry* by Michael Serra (Key Curriculum Press). Constructions may also be completed using dynamic drawing software, such as *The Geometer's Sketchpad*® or *GeoGebra*. *GeoGebra* can be found online at <http://www.geogebra.org> and provides a free web applet as well as a free download of dynamic geometry software. Directions for using the program can be found on the website as well.*

Begin with a *discussion* ([view literacy strategy descriptions](#)) on the question “Which criteria would need to be met to ensure that two triangles are congruent?” Use the Round Robin strategy to facilitate this *discussion*. Place students into groups of three to five. Pose the question to the class and have each student go around the circle, in a clockwise direction, quickly sharing ideas to answer the question. Students can be given one opportunity to “pass” on a response, but eventually every student must respond. Allow approximately one minute for students to share their ideas with their group. After the initial responses, have the students write down each of their responses, as a group. Then, have all groups report their thoughts to the class. It is okay if groups report the same ideas. Record all of the responses from each group, highlighting, in some way, those ideas which are repeated. Use the responses developed during the *discussion* to guide the remainder of the activity.

Give students a copy of the Investigating Congruence BLM and have students complete part one. Help students understand, through reviews of transformations and the definition of isometry, that when segment length is preserved, angle measure is also preserved. If students need help to understand this concept, have them copy the angles of the image using patty paper and lay the patty paper over the pre-image. Have students continue with parts two and three of the Investigating Congruence BLM stopping in between to discuss their conjectures, observations, and understandings. From the investigations and discussions, students should understand that they can be given information about the three sets of corresponding sides (SSS), two corresponding sides and the included corresponding angle (SAS), or two sets of corresponding angles and the corresponding included sides (ASA) to prove that two triangles are congruent.

To complete the activity, have students record the text of the postulates developed through these investigations using the *split-page notetaking* ([view literacy strategy descriptions](#)) format. *Split-page notetaking* is simply a different way for students to organize their notes

more effectively for study. Model the approach by placing the Sample Split-Page Notes BLM with other information about triangles on the board or overhead, or give each student a copy of the Sample Split-Page Notes BLM. Explain the value of taking notes in this format by showing how it helps them organize information and ideas from various sources. *Split-page notetaking* helps separate big ideas from supporting details, and it promotes active reading and listening. Show students how they can prompt recall by bending the sheet or using another sheet of paper so that information in the right or left column is covered. The uncovered information is then used as prompts for the information in the column that is covered. Next, ask students to use *split-page notetaking* to record the postulates developed through the investigations. Provide each student with a copy of the Blank Split-Page Notes BLM. Start the notetaking by revisiting the question posed at the beginning of the activity for the *discussion*. Pose the question again (Which criteria would need to be met to ensure that two triangles are congruent?), but this time, have students use the conjectures and understandings developed through the investigations to answer the question. Give students the text of each of the postulates developed in this activity (SSS, SAS, and ASA). The title of the postulate should be in the left column while the text of the postulate and example diagrams should be in the right column. See the Triangle Split-Page Notes Model BLM for clarification. As each postulate is discussed and recorded, refer to the ideas recorded at the beginning of the activity during the *discussion* identifying those which were correct. Discuss those ideas which were incorrect as well by helping students understand why those ideas cannot ensure the congruence of two triangles. Some of the incorrect ideas may be addressed in Activity 3.

As students progress through the unit, have them continue to use the *split-page notetaking* approach to record the information about triangles and quadrilaterals. When it is time for quizzes or tests, have students use their *split-page notes* to review the material.

Activity 3: Are They Congruent? (GLEs: 19)

Materials List: pencil, paper, ruler, protractor, compass, geometry drawing software (optional), computers (optional)

Give students a diagram of two congruent triangles but do not tell them they are congruent. Have two angles and the non-included side of the two triangles marked congruent. Ask students, “Are these two triangles congruent? Why or why not?” Guide students to develop and prove the Third Angle Theorem (If two angles in one triangle are congruent to two angles in another triangle, then the third angles in each triangle are also congruent). Once students have developed and proven the Third Angle Theorem, have them prove the two triangles congruent using ASA. Discuss with students whether being given two angles and a non-included side will always produce congruent triangles. Allow students to provide examples they believe will not work and work to show that if two angles and a non-included side of one triangle are congruent to two angles and the **corresponding** non-included side of a second triangle, then the two triangles are congruent (this is the text of the AAS theorem; once students have proven the AAS theorem, they can use it to prove triangles congruent). Be sure to show examples where the congruent sides are **not corresponding** and why those

triangles cannot be assumed to be congruent (this will have to be shown using the ASA theorem). Have students record these theorems in their notes with the other theorems from Activity 2.

Provide students with the measures of two sides and a non-included angle for a triangle, and the measures of three angles and no sides. Have students construct a triangle using the given measures (either with or without technology). Next, have students compare their constructions. Have students make a conjecture about the relationships of these constructed triangles. Repeat this activity with several sets of SSA or AAA measures. Students should make a conjecture about whether SSA and AAA can be used to justify two triangles being congruent. Students should understand that these criteria do not ensure congruent triangles and should record examples of why SSA and AAA do not work in their notes.

Activity 4: Proving Triangles Congruent (GLEs: 19; CCSS: WHST.9-10.1b)

Materials List: pencil, paper, Proving Triangles Congruent BLM

Have students work in groups. Provide each pair of students with one of the sheets in the Proving Triangles Congruent BLM that present diagrams involving congruent triangles. Each group should have different diagrams. Develop more diagrams with different given information to accommodate the class. The Proving Triangles Congruent BLM provides some samples of the types of diagrams teachers can use. Since students have not yet learned the properties of parallelograms, provide given information that allows students to prove two (or more) triangles congruent. Ask students to prove two of the triangles in the diagram are congruent or parts of congruent triangles are congruent. Allow students to use various methods of proof: two-column, flow, or paragraph.

Have groups share their proofs with the class. Using a modified *questioning the content (QtC)* ([view literacy strategy descriptions](#)) technique, have students ask questions about the proofs to clarify their own understanding. The goal of *QtC* is to help students construct meaning from the content from which they are expected to learn. Instead of students asking the questions during reading, in this activity, students will be asking questions after they have reviewed the proofs of other groups. Some possible questions might be:

- Does the flow of the proof make sense logically?
- What information in the diagram led you to that statement?
- Is the correct reason given for the statement presented?
- Are the statements and reasons necessary to complete the proof?
- Is there a step missing that would help the reasoning sound more logical?

Direct students to think about their classmates' presentations and develop questions that may highlight incorrect logic or missing information. Set rules that create an environment conducive to this process. Students should ask and answer most of the questions; however, where necessary ask questions and offer explanations to clarify any misconceptions and

incorrect answers. Reinforce to students that this questioning process should be used whenever they read others' proofs or write their own proofs.

To end the activity, have students work individually, employing techniques used in class to prove two triangles from a diagram are congruent. This individual work will show whether students have mastered the skill.

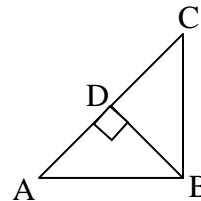
Activity 5: Proving Right Triangles Congruent (GLEs: 19; CCSS: WHST.9-10.1b)

Materials List: pencil, paper

Give students diagrams of pairs of right triangles. Each pair of right triangles should have different sides (either both legs or a leg and the hypotenuse) and corresponding acute angles marked congruent. Help them to determine which pairs of the triangles are congruent and to explain why they are congruent. Then have the students write congruence proofs for these triangles using methods already discussed, like SAS, AAS, and ASA. Connect these proofs and methods to the LL, HA, and LA theorems for right triangles. Also, introduce the HL Postulate. Provide opportunities for students to write proofs for different sets of right triangles formed by perpendicular bisectors and altitudes in triangles as well.

Example:

Given: \overline{BD} is perpendicular to \overline{AC}
 \overline{BD} bisects \overline{AC}



Prove: $\triangle ABD \cong \triangle CBD$

Solution: Since \overline{BD} is perpendicular to \overline{AC} , $\angle BDA$ and $\angle BDC$ are right angles because perpendicular lines form 4 right angles. That makes $\triangle ABD$ and $\triangle CBD$ right triangles. By the definition of segment bisector, D is the midpoint of \overline{AC} . By the definition of midpoint, $\overline{AD} \cong \overline{CD}$. Also, $\overline{BD} \cong \overline{BD}$ because congruence of segments is reflexive. Therefore, since two pairs of legs of $\triangle ABD$ and $\triangle CBD$ are congruent, $\triangle ABD \cong \triangle CBD$ by LL.

Encourage students to use the questions developed in Activity 4 to determine if the proofs they have created are logical and complete. Correct and incorrect proofs written by classmates should be shared with the class to allow students the opportunity to question their work and the work of their peers.

Activity 6: Analyzing Isosceles Triangles (GLEs: 9, 16, 19; CCSS: WHST.9-10.1b)

Materials List: pencil, paper, patty paper, compass, straightedge, ruler

Have students use patty paper or tracing paper to draw an acute, isosceles triangle. Have them start by drawing an acute angle and labeling it C . Then have them mark equal lengths on each side of the angle, label them A and B , and draw \overline{AB} . Have students use patty paper constructions, compass/straightedge constructions, or measurements with a ruler to mark equal lengths. Have students fold the triangle in half so that the equal sides lie on top of each other. Have students make observations about base angles A and B . Repeat the activity with obtuse and right isosceles triangles as well. Have a class discussion in which students form conjectures that lead to the Isosceles Triangle theorem (If two sides of a triangle are congruent, then the angles opposite those sides are congruent) and its converse.

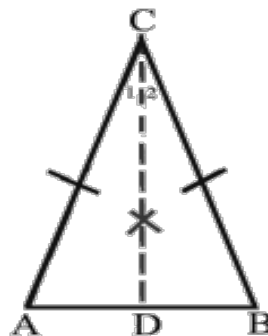
Next, have students prove the Isosceles Triangle Theorem using the methods learned in Activities 2 and 3. Begin by having them construct an isosceles triangle and then bisect the vertex angle. Discuss the definition of the term *angle bisector* and the importance that may play in the proof. Then, have them complete a proof of the theorem either in paragraph or two-column format.

Proof:

Given: $\overline{AC} \cong \overline{BC}$

\overline{CD} is an angle bisector

Prove: $\angle A \cong \angle B$



Given \overline{CD} is an angle bisector, $\angle ACD \cong \angle BCD$ by definition of angle bisector. $\overline{CD} \cong \overline{CD}$ by the reflexive property of congruence. So, $\overline{AC} \cong \overline{BC}$, $\angle ACD \cong \angle BCD$, and $\overline{CD} \cong \overline{CD}$ which provides two sides and the included angle of two triangles. Therefore, $\triangle ACD \cong \triangle BCD$ by SAS. Finally, $\angle A \cong \angle B$ because corresponding parts of congruent triangles are congruent (CPCTC).

Students should also be given the opportunity to prove the converse of the Isosceles Triangle Theorem (If two angles in a triangle are congruent, then the sides opposite those angles are also congruent). The proof would be very similar to the one above, so students should be given the opportunity to develop it individually. Have students record these theorems and definitions in their notes using a *split-page notetaking* ([view literacy strategy descriptions](#)) format as they did in Activity 2. The two-column proofs can also be recorded in *split-page notetaking* format with the given information, diagram, and statements in the left column and the reasons in the right column.

Have students practice their algebra skills to find the measures of sides and angles of isosceles triangles, when given information about the isosceles triangles.

For example:

- A. In isosceles triangle $\triangle ABC$ with base \overline{BC} , $m\angle ABC = 5x - 4^\circ$, and $m\angle ACB = 7x - 20^\circ$, find the measure of each angle.
Solution: $x = 8$, $m\angle ABC = 36^\circ$, $m\angle ACB = 36^\circ$, and $m\angle BAC = 108^\circ$.
- B. In isosceles triangle $\triangle DEF$, $\angle F$ is the vertex angle. If $DE = 5x$ inches, $EF = 4x - 3$ inches, and $DF = 2x + 7$ inches, find the length of the base.
Solution: $x = 5$, and $DE = 25$ inches.
- C. $\triangle ABC$ has vertices $A(2,5)$, $B(5,2)$, and $C(2,-1)$. Use the distance formula to show that $\triangle ABC$ is an isosceles triangle and name the pair of congruent angles.
Solution: $AB = 3\sqrt{2}$, $BC = 3\sqrt{2}$, $AC = 6$, $\overline{AB} \cong \overline{BC}$, and $\angle A \cong \angle C$. All distances are in linear units.

Activity 7: Angle Bisectors, Medians, Perpendicular Bisectors, and Altitudes of a Triangle (GLE: 9)

Materials List: pencil, paper, patty paper, ruler, protractor (optional), compass (optional), geometry drawing software, computers

Have students work in groups of three or four. Give each group 12 sheets of patty paper. Instruct students to draw the following types of triangles (one triangle on each sheet of patty paper): four acute scalene triangles, four right scalene triangles, and four obtuse scalene triangles. Have students label one sheet of patty paper from each group of triangles with one of the following: angle bisectors, medians, perpendicular bisectors, or altitudes.

Review the definition of *angle bisector* from Activity 5. Have students construct the angle bisectors for all the angles in each triangle labeled with angle bisector. This may be done via patty paper folding, measuring, compass and straightedge constructions, or with the use of a drawing program such as the *Geometer's Sketchpad*®. The purpose of this activity is to have students learn the definition by having to create angles of equal measure. Ask students to share their work with other class members. If done properly, the angle bisectors will intersect at one point. Have students discuss any differences they observed when constructing the angle bisectors on the different types of triangles (acute, right, and obtuse). For all three types, the angle bisectors should intersect inside the triangle. Tell them the name of the common intersection point for the three angle bisectors in a triangle is called the *incenter*. Have students record this definition in their notes. Have students measure the distance from the incenter to each side. Review with students the concept of distance between a point and a line. The distances from the incenter to each side of the triangle should be the same, indicating that the incenter is the center of a circle which can be inscribed in the triangle. Have students use a compass to draw the inscribed circle.

Provide students with the definition of *median*. Have students construct the three medians in each triangle labeled with median. This may be done via patty paper folding, measuring, compass and straightedge constructions, or with the use of a drawing program such as the *Geometer's Sketchpad*®. The purpose of this activity is to have students learn the definition by having to first find the midpoint of a side, and then draw a segment from the midpoint to the opposite vertex in the triangle. Ask students to share their work with other class members. If done properly, the medians will intersect in one point. Have students discuss any differences they observed when constructing the medians on the different types of triangles (acute, right, and obtuse). For all three types, the medians should intersect inside of the triangle. Tell students that the name of the common intersection point for the three medians is called the *centroid* and is the center of gravity for the triangle. Have them record this information in their notes. Have students transfer the location of the three vertices of the triangle and the centroid to a sheet of cardstock (an old manila file folder works well, too). This can be done by placing the sheet of patty paper on the card stock and making an indentation with a pencil or pen point through the patty paper onto the card stock. Have the students use a straight edge to draw the sides of their triangle on the cardstock and then cut it out with scissors. If done properly, the triangle should balance when the centroid is placed on the lead end of a sharpened pencil. Use the eraser end, if needed. Position the pencil at a location other than the centroid, and the triangle will tilt to one side and fall off. It will not stay balanced. An investigation and proof concerning the centroid will be presented in a later activity.

Provide students with the definition of *perpendicular bisector of a segment*. Have students construct the perpendicular bisectors for all sides in each triangle labeled with perpendicular bisector. This may be done via patty paper folding, measuring, compass and straightedge constructions, or with the use of a drawing program such as the *Geometer's Sketchpad*®. The purpose of this activity is to have students learn the definition by having to locate the midpoint of a side and then drawing a line through the midpoint so that the line is perpendicular to the side of the triangle. Ask students to share their work with other class members. If done properly, the perpendicular bisectors of the three sides of the triangle will intersect in one point. Have students discuss any differences they observed when constructing the perpendicular bisectors on the different types of triangles (acute, right, and obtuse). For the acute triangle, the perpendicular bisectors will intersect inside the triangle. For the right triangle, the perpendicular bisectors will intersect on the hypotenuse. For the obtuse triangle, the perpendicular bisectors will intersect outside the triangle. Tell them the name of the common intersection point for the three perpendicular bisectors in a triangle is called the *circumcenter* and have students record the definition in their notes. Have students measure the distance from the circumcenter to each vertex of the triangle. These distances should be the same indicating that the circumcenter is the center of a circle which passes through each vertex of the triangle. Have students use a compass to draw the circumscribed circle.

Provide students with the definition of *altitude* in a triangle. Have students construct the three altitudes in each triangle and label each line with the word altitude. This may be done via patty paper folding, measuring, compass and straightedge constructions, or with the use of a drawing program such as the *Geometer's Sketchpad*®. The purpose of this activity is to have

students learn the definition by creating a line that passes through a vertex of the triangle and is perpendicular to the opposite side. Ask students to share their work with other class members. If done properly, the altitudes will intersect in one point. Have students discuss any differences they observed when constructing the altitudes on the different types of triangles (acute, right, and obtuse). For the acute triangle, the altitudes will intersect inside the triangle. For the right triangle, the altitudes will intersect at the vertex of the right angle. For the obtuse triangle, the altitudes will intersect outside the triangle.

Teacher Note: Instructions for using patty paper to fold segments in this activity can be found in Patty Paper Geometry by Michael Serra (Key Curriculum Press). It is not recommended that these constructions be made with a compass/straightedge as students seldom remember the construction steps. If compass/straightedge constructions are used, time must be taken to explain and demonstrate how the constructions relate to congruent triangles (i.e., the construction of a perpendicular bisector of a segment is based on the creation of two triangles by SSS). Patty paper constructions or use of the Geometer's Sketchpad are much more intuitive for students, and their use does not present safety issues.

Have students practice constructing their own altitudes, perpendicular bisectors, medians, and angle bisectors to help internalize the definitions.

Activity 8: Altitudes, Medians, and Perpendicular Bisectors on the Coordinate Plane (GLEs: 6, 9)

Materials List: pencil, paper

Provide students with information that allows them to graph triangles on a coordinate plane. Have them draw the medians, perpendicular bisectors, and altitudes of those triangles. Ask students to write equations that represent those segments. Writing equations reinforces skills learned in Algebra.

Examples:

- A. $\square ABC$ has vertices $A(-3,10)$, $B(9,20)$, and $C(-2,21)$. Find the coordinates of P such that \overline{CP} is a median of $\square ABC$. Determine if \overline{CP} is an altitude of $\square ABC$.

Solutions: P is at $(3,15)$; \overline{CP} is an altitude of $\square ABC$.

- B. The following equations intersect to form a triangle. Identify the vertices of the triangle.

$$x - 2y = -6$$

$$3x + 2y = -2$$

$$9x - 2y = 26$$

Draw one of the perpendicular bisectors in the triangle and identify the slope and point used to draw it. Then write the equation for that perpendicular bisector.

Solutions: Vertices are $(-2,2)$, $(4,5)$, and $(2,-4)$. Students should have one of the following for the point, slope, and equation:

- A. $(1, 3.5), m = -2, y = -2x + 5.5$
- B. $(0, -1), m = \frac{2}{3}, y = \frac{2}{3}x - 1$
- C. $(3, \frac{1}{2}), m = -\frac{2}{9}, y = -\frac{2}{9}x + \frac{7}{6}$

Activity 9: More on Angle Bisectors, Medians, and Perpendicular Bisectors of a Triangle (GLEs: 9, 19; CCSS: WHST.9-10.1b)

Materials List: pencil, paper, geometry drawing software, computers

Have students complete this activity with a partner. They will need an automatic drawer.

1. Using an automatic drawer, such as that found in *Geometer's Sketchpad*®, draw scalene triangle ABC and measure the lengths of \overline{AB} and \overline{AC} .
2. Construct m , the angle bisector of $\angle BAC$.
3. Construct the midpoint D and the perpendicular bisector of \overline{BC} .
4. Draw the median from point A to \overline{BC} .
5. Move point A until the angle bisector, perpendicular bisector, and the median coincide. Record the lengths of \overline{AB} and \overline{AC} .
6. Drag point A to find two other positions for point A in which angle bisector, perpendicular bisector, and the median coincide. Again, record the lengths of \overline{AB} and \overline{AC} .

Ask students to make a conjecture about $\triangle ABC$ when the angle bisector of $\angle BAC$, the median from A to \overline{BC} , and the perpendicular bisector of \overline{BC} coincide. Have students write a proof to show that if the median, angle bisector, and perpendicular bisector of the same triangle coincide, then the triangle is isosceles.

Activity 10: The Medians and Centroid of a Triangle (CCSS: G.GPE.6, G.GPE.7)

Materials List: pencil, paper, scissors, graph paper, ruler

Have students draw a scalene triangle on a piece of paper and cut it out. Students should then find the midpoint of each side by folding each side, vertex to vertex, and pinching the paper in the middle. Have students draw the medians from the vertices to the midpoints they found after folding the sides. Students should label the vertices of the triangle A , B , and C . The midpoint of \overline{AB} should be labeled E ; the midpoint of \overline{BC} should be labeled D ; and the midpoint of \overline{AC} should be labeled F . The centroid of the triangle (the point where the three medians meet) should be labeled as P . Have students measure the lengths of the segments in the table below and record the measurements.

Length of median	$AD=?$	$BF=?$	$CE=?$
Length of segment from P to vertex	$AP=?$	$BP=?$	$CP=?$
Median length multiplied by $\frac{2}{3}$.	$\frac{2}{3}AD = ?$	$\frac{2}{3}BF = ?$	$\frac{2}{3}CE = ?$

In the table above, the answers will vary from student to student depending on the triangle they drew. However, in each column the second and third rows should be approximately equal.

Next, have students answer the question, “Does there seem to be a relationship between the distance from P to a vertex and the distance from that vertex to the midpoint of the opposite side? If so, state what that relationship seems to be.” *Solution: Yes; the distance from P to a vertex is equal to two-thirds the distance from that vertex to the midpoint of the opposite side.*

Have students compare their answers with one another to determine if the size or shape of the triangle makes a difference in the results. They should find that this should work for any triangle.

Next, have students write the ratios $\frac{AP}{PD}$, $\frac{BP}{PF}$, and $\frac{CP}{PE}$. Students should see that the ratios of the two smaller parts of the median created by the centroid are equal to 2:1. Discuss with students how this ratio relates to the centroid’s being located $\frac{2}{3}$ of the distance from the vertex to the midpoint of the opposite side. *If the centroid is $\frac{2}{3}$ of the distance from the vertex to the midpoint of the opposite side, then the median has been divided into three equal parts, and the centroid is located at a point which is two of the three parts away from the vertex.* Review with students the work completed in Unit 3 with finding the location of a point that divides a segment into a given ratio.

Then, have students plot the points $H(-9,3)$, $A(-3,7)$, and $T(3,5)$ on graph paper and draw $\square HAT$. Students should find the midpoints of each side of the triangle and label the midpoints as follows: midpoint of \overline{HA} is G ; midpoint of \overline{AT} is O ; and midpoint of \overline{HT} is D . Instruct the students **not** to draw the medians at this time. *Solutions: The midpoints are $D(-3,4)$, $O(0,6)$, and $G(-6,5)$.*

For the next step, have one-third of the class find the point that divides \overline{HO} into a ratio of 2:1; another third should find the point that divides \overline{AD} into a ratio of 2:1; and the remaining third should find the point that divides \overline{TG} into a ratio of 2:1 (this can also be accomplished by having students in groups of three and having each student in a group find the point on one of the segments). When students have completed the work, have them compare answers. If all work were done correctly, all students should have calculated the same answer. Have

them plot the point and label it C . *Solution:* The location of the centroid is $C(-3,5)$. Ask students what this point represents and have them draw the medians. Have them verify by observation that the three medians pass through C . Then, have students verify that the centroid is located two-thirds of the distance from the vertex to the midpoint of the opposite side for each median using the distance formula.

Give students more problems where they are asked to find the location of the centroid but do not require them to produce the graph. Students should be given enough practice to become proficient in finding the location of the centroid without looking at the graph of the triangle.

Next, give students the coordinates $T(-4,3)$, $R(-2,9)$, and $I(4,3)$ and have them plot the points on the coordinate system. Then, have students find the midpoint of \overline{TI} , plot the midpoint, and label it M . Review with students what the altitude of the triangle is and have them find the altitude of the triangle from R . Students should draw the segment for the altitude and label the point where the altitude intersects \overline{TI} as A . Review the formula for area of a triangle and have students find the area of the two smaller triangles ($\triangle TRM$ and $\triangle MRI$). Students should find that the areas of the two smaller triangles are the same. *Solution:* $M(0,3)$, $A(-2,3)$, $RA = 6$, area of $\triangle TRM = 12 \text{ units}^2$ and area of $\triangle MRI = 12 \text{ units}^2$. Using the information from this investigation, have students generalize what they know about the altitudes and medians of a triangle and provide a convincing argument that the median of a triangle creates two triangles with the same area.

Activity 11: Inequalities for Sides and Angles in a Triangle (GLEs: 9, 16, 19; CCSS: WHST.9-10.1b, WHST.9-10.10)

Materials List: pencil, paper, Angle and Side Relationships BLM, patty paper, ruler, protractor, Angle and Side Relationships Proof BLM, *learning logs*

Have students work in groups of three. Give each student a copy of the Angle and Side Relationships BLM. In each group, have each student draw one of the following types of triangles on a sheet of patty paper: acute, obtuse, and right (they should all be scalene—if they are isosceles, the activity will not work as well). Each triangle in the group should be labeled differently. For instance, the acute triangle can be labeled $\triangle ABC$, the obtuse triangle can be labeled $\triangle DEF$, and the right triangle can be labeled $\triangle GHI$. Make sure each triangle has the group members' names on it so it can be returned after the activity. Ask groups to exchange triangles so that they do not measure their own triangles. Using the procedure listed below, have the students in each group measure each side and each angle, then record the measurements on the Angle and Side Relationships BLM. Have students record which group the triangles came from, and the name of the triangle they measured (i.e., $\triangle ABC$, $\triangle DEF$, etc.).

- Each student should measure ONE of the angles in a triangle, record the measurement in the correct place on the Angle and Side Relationships BLM, then pass the triangle to the next group member. This step should be repeated with the remaining two

angles in the triangles. (If students are in groups of three, then each student will have measured three angles.)

- After the third angle has been measured and recorded in all three triangles, each student in the group should write down all of the measurements from the group. Then each student should check to be sure the angle measures are sensible (i.e., the sum of the measures of the angles is 180, and angles which are acute/obtuse by sight have acute/obtuse measures). Once the measures have been deemed acceptable, the students should pass the triangles again.
- Each student should measure ONE of the sides of the triangle, record the measurement in the correct place on the Angle and Side Relationships BLM, then pass the triangle to the next group member. This step should be repeated with the remaining two sides of the triangles.
- After the third side has been measured and recorded in all three triangles, each student in the group should write down all of the measurements from the group. Then, each student should take one triangle and check to be sure the side measures are sensible (check units, be sure the stated measure “looks” close).
- Have the students list the angles in order from largest to smallest to help identify the relationships between the angles and sides.
- Have students work together with their group to determine what conclusions they can draw from the activity they completed and answer the three questions at the end of the Angle and Side Relationships BLM.

Have students share their findings with the whole class. Students should hear that all groups have the same observation: The longest side is opposite the largest angle and the shortest side is opposite the smallest angle, and vice versa. If there are any groups that found other observations, discuss their validity and have the class help those groups realize their mistake(s), if necessary. Ask students to produce, if they can, a triangle where the observations they made are not true. Have students experiment with constructing triangles and changing the lengths of the sides or the measures of the angles and watching the effect on the side/angle opposite the part they are changing.

Next, provide students with a copy of the Angle and Side Relationships Proof BLM which illustrates a proof of the theorem “If one side of a triangle is longer than a second side, then the angle opposite the first side is greater than the angle opposite the second side.” Using the *questioning the content (QtC)* ([view literacy strategy descriptions](#)) technique, have students read the proof individually and create questions about the text they have read. The following are examples of some of the questions students may develop:

- What is the theorem being proven?
- Why is $m\angle BPC > m\angle A$ true?
- What is an exterior angle?
- Why is the exterior angle important?
- What is the transitive property of inequalities?
- What is the importance of the last statement in the proof?

After students have read the proof and developed their questions, have them share their questions with a partner and discuss possible answers to the questions. Have pairs of students

report to the class and discuss the answers to the questions students have developed. Students should record answers to their questions to help them understand the proof better. Ask students how this proof relates to their findings from the activity at the beginning of the lesson (does the proof support their conjectures?). Have students answer a *SPAWN writing* ([view literacy strategy descriptions](#)) prompt from the *What if?* category. Give the students the prompt, “How does the proof change if the theorem changes to ‘If one side of a triangle is shorter than a second side, then the angle opposite the first side is smaller than the angle opposite the second side?’” Students should realize that the change in the words of the theorem does not change the proof because in order to prove the shorter side is opposite the smaller angle, it must be proven that the longer side is opposite the larger angle. Students may write this as an entry in their *learning logs* ([view literacy strategy descriptions](#)) or they may turn it in as a separate assignment. Students may also discuss their responses to the prompt (after developing them individually) with a partner or by sharing them with the class.

Next, have students solve problems that require an understanding of this concept. Provide students with diagrams showing triangles and their angle measures. Ask students to list the sides in order from longest to shortest or shortest to longest. Provide other diagrams showing triangles and the lengths of the sides. Have students list the angles in order from least to greatest or vice versa. Incorporate a review of algebra skills and coordinate geometry as indicated in the examples below.

Examples:

- A. Find the value of x and list the length of the sides of $\triangle ABC$ in order from shortest to longest if $m\angle A = 5x - 5^\circ$, $m\angle B = 4x^\circ$, and $m\angle C = 17x + 3^\circ$.

Solution: $x = 7$, $AC < BC < AB$

- B. $\triangle DEF$ has vertices $D(-2,1)$, $E(5,3)$, and $F(4,-3)$. List the angles in order from greatest to least.

Solution: $\angle F$, $\angle E$, $\angle D$

End this activity with another *SPAWN writing* prompt from the *Problem Solving* category. Give the students the prompt “What relationships between sides and angles occur if the triangle is isosceles? Do these relationships still fit with the observations you found completing this activity? Explain.” Since students have already studied isosceles triangles in Activity 6, they should realize that if the measures of the base angles are greater than the vertex angle, then the legs of the isosceles triangle are longer than the base and vice versa. This does fit with the conjectures they will make through this discovery activity. They should also understand that when listing the sides of an isosceles triangle in order by its lengths, two of the sides will need to be designated as equal, (e.g., $(AC = BC) > AC$). Students may write this as an entry in their math *learning logs* or they may turn it in as a separate assignment.

Activity 12: The Triangle Inequality (GLE: 9, 19; CCSS: WHST.9-10.1b)

Materials List: long, thin pasta (such as spaghetti or linguine); rulers; pencil; The Triangle Inequality Activity Sheet (from website)

In this activity, students will informally prove the Triangle Inequality by forming a conjecture based on measurements they create and then explain their observations. This activity will use the activity sheet found on NCTM's Illuminations site. The page is titled *Illuminations: Inequalities in Triangles*. The website is <http://illuminations.nctm.org/lessondetail.aspx?ID=L681> and discusses both the Triangle Inequality and Inequalities for Sides and Angles in a Triangle. For this activity, use the information provided for the Triangle Inequality (first portion of the website) only.

Students will use the pasta to create triangles and non-triangles and record the measurements on the chart provided. Then students will compare the sums of the small and medium sides to the measure of the large sides for each triangle and each non-triangle and describe their observations. They will form and test a conjecture and answer questions which will get them to think critically about their conjecture. The last question asks them to formally state the Triangle Inequality with side lengths a , b , and c .

The website provides information for teachers including the Learning Objectives, Materials, Instructional Plan, Questions for Students, Assessment Options, Extensions, Teacher Reflection and NCTM Standards and Expectations. The Instructional Plan provides information about where students may encounter problems, expectations for what students should discover when answering the questions on the activity sheet, and suggested examples for helping students understand the concepts.

The last part of the website can be used to reinforce the information presented in Activity 11.

Activity 13: Similar or Not? (GLEs: 9, 19; CCSS: G.SRT.3; G.GPE.7)

Materials List: pencil, paper, graph paper, patty paper (optional)

Begin by using *student questions for purposeful learning (SQPL)* ([view literacy strategy descriptions](#)). To implement this strategy, develop a thought-provoking statement related to the topic about to be discussed. The statement does not have to be factually true, but it should generate some level of curiosity for the students. For this activity, pose the statement, "All triangles are similar." This statement can be written on the board, projected on the overhead, or stated orally for the students to write in their notebooks. Allow the students to ponder the statement for a moment and ask them to think of some questions they might have related to the statement. After a minute or two, have students pair up and generate two or three questions they would like to have answered that relate to the statement. When all of the pairs have developed their questions, have one member from each pair share their questions with the class. As the questions are read aloud, write them on the board or overhead. Students should also copy them in their notebooks. When questions are repeated or are very similar to others which have already been posed, those questions should be starred or highlighted in

some way. Once all of the students' questions have been shared, look over the list and determine if other questions need to be added. The list should include the following questions:

- What is the definition of similar triangles?
- Can isosceles triangles be similar to scalene or equilateral triangles?
- Can acute triangles be similar to obtuse or right triangles?
- Are congruent triangles similar?
- What must be true about two triangles in order for them to be similar?
- How can you prove two triangles are similar?
- What is the proof that all triangles are similar?

At this point, see that the students have copied all of the questions in their notebooks and continue with the lesson as follows. Tell the students to pay attention as the material is presented to find the answers to the questions posted on the board. Focus on those questions which have been starred or highlighted. Periodically, stop throughout the lesson to allow the student pairs to discuss which questions have been answered from the list. This may be followed with a whole class discussion so all students are sure to have the correct answers to each question.

Have students review dilations from Unit 4. Give students the coordinates of the vertices of a triangle and have them perform multiple dilations, including both enlargements and reductions on the same pre-image. Have students recall the properties of a dilation (the degree measures are preserved, but the side measures have changed in proportion to the dilation or scale factor). Ask students to look at all of the images and determine what each of them have in common. Students should see that in every triangle they have drawn, even though the segments have different measures, the corresponding angles are all congruent. If students need help visualizing this, have them trace the angles of the pre-image on patty paper and then lay those angles over the corresponding angles of the transformed images. Define similar triangles as two triangles whose corresponding angles are congruent. Then ask students, "Do you have to know that all three pairs of corresponding angles are congruent to know that two triangles are similar?" After a few moments, have students begin to give their responses and discuss their ideas. Students should recall the Third Angle Theorem (Activity 3) and realize that knowing two pairs of corresponding angles are congruent implies the third pair of corresponding angles is congruent. Then give students the text of the AA Similarity Theorem (If two angles of one triangle are congruent to two angles of a second triangle, the triangles are similar).

Next, have students find the perimeters of each triangle they have drawn on graph paper using the distance formula. Then, have students set up the ratios of the sides of the pre-image to the corresponding sides of the image, as well as the ratio of the perimeter of the pre-image to the perimeter of the image. Have students compare the ratios of the corresponding sides and the perimeters. They should notice for each pair of triangles, the ratios for all sets of corresponding sides and the perimeters are equal. Expand the definition of similar triangles to:

Similar triangles—two triangles are said to be similar if and only if all pairs of corresponding angles are congruent and if all pairs of corresponding sides are proportional.

Then, pose the question, “Are congruent triangles considered similar?” Give students an opportunity to think about their response for a few moments. Have them share their ideas with a partner. After about 2 minutes, have the students report to the whole class. Students should realize that congruent triangles are considered similar triangles because all pairs of corresponding angles are congruent and the ratio of corresponding sides is 1:1. Students should also understand that while congruence implies similarity, similarity does not imply congruence (this should be obvious after the investigation completed earlier). Using their understanding of congruence, lead them to investigate SSS Similarity and SAS Similarity for triangles, providing the text for these theorems as needed.

Once students have developed the definition of similar triangles, provide students with diagrams that have pairs of similar triangles. Have students prove that these triangles are similar using the SSS similarity, SAS similarity, and AA similarity. Some pairs of triangles should also be congruent.

To end the lesson, review the questions posed by the students at the beginning of the activity based on the *SQPL* prompt and be sure all students have the answers to the questions for further study.

Activity 14: Conjectures about Quadrilaterals (GLE: 9, 19; CCSS: RST.9-10.3, WHST.9-10.1b)

Materials List: geometry drawing software, computers (depending on type of automatic drawer), pencil, paper, Quadrilateral Process Guide BLM, Quadrilateral Family BLM

Have students work in groups of two (preferred), three or four using an geometry drawing software, such as that found in *The Geometer's Sketchpad*® software. The purpose of this activity is to allow students to investigate the properties of special convex quadrilaterals.

Use a *process guide* ([view literacy strategy descriptions](#)) to help students develop conjectures about quadrilaterals. *Process guides* are used to guide students in processing new information and concepts. They are used to scaffold students' comprehension and are designed to stimulate students' thinking during and after reading. *Process guides* also help students focus on important information and ideas. In this activity, students will be given a *process guide* that will lead them through the steps to discover the relationships inherent in all quadrilaterals. Create a *process guide* by reviewing the information to be studied and by deciding how much help students will need to construct and to use meaning. Copy the Quadrilateral Process Guide BLM for each student. Make one copy for each convex quadrilateral that will be studied. The BLM provided is generic as the process will be the same, but the answers will be different for the various quadrilaterals. An example of the process to be used is provided below using a kite as the convex quadrilateral.

- Provide students with an electronic file in which a kite has been drawn. Have students measure the four angles and the four sides, then record the measures.
- Instruct students to resize the quadrilateral by dragging the vertices of the kite. Measure the angles and sides of the resized kite and record the information.
- Have students resize and make measurements until they can form conjectures about the measures of the angles and the lengths of the sides in any kite. For example, a kite has two pairs of congruent and adjacent sides. A kite has one pair of congruent angles which are formed by a pair of non-congruent sides.
- Instruct students to construct the diagonals of the kite and then answer questions relative to the behavior of the diagonals. Are diagonals perpendicular? Do the diagonals bisect the angles of the quadrilateral? Do the diagonals bisect each other?

Repeat the process using trapezoids, isosceles trapezoids, parallelograms, squares, rectangles and rhombi. Have the pairs/groups share their findings with the rest of the class. Students should be told that they are required to be able to support their statements, conjectures, and answers with evidence from their investigation with the *process guide*. Have students compare the properties of the various convex quadrilaterals used in the investigation.

After students have presented their conjectures, have groups of students work on proving the conjectures using facts about parallel lines, congruent triangles, vertical angles, etc. One way to complete these proofs is to give one conjecture to each group of students and have them develop a proof and present it to the class. Have the groups use a *text chain* ([view literacy strategy descriptions](#)) to complete the proofs. A *text chain* process involves a small group of students writing a short composition, in this case a proof, using the information and concepts being learned. In this activity, the *text chain* will include the logical statements and reasons of the proof, either in two-column or paragraph format. By writing out new understandings in a collaborative context, students provide themselves and the teacher a reflection of their developing knowledge. Give each group of students a conjecture (theorem) with a diagram, given information, and the information they need to prove (more advanced students can be given the theorem only). Then, explain that each person in the group will take turns adding a statement and reason to the proof until they have reached the end. Once the group believes the proof is complete, every member should check the proof and ask questions similar to those in Activity 4 (see below) to determine if the proof is complete and logical. After each group has completed their proof, have the groups present their proof to the rest of the class. Each group should be prepared for questions from other groups. Some questions groups may ask are the following:

- Does the flow of the proof make sense logically?
- What information in the diagram (or theorem) led you to that statement?
- Is the correct reason given for the statement presented?
- Are the statements and reasons given necessary to complete the proof?
- Is there a step missing that would help the reasoning sound more logical?

These questions were developed in Activity 4 as part of the *questioning the content* ([view literacy strategy descriptions](#)) strategy and should be used any time students develop a proof.

To end the activity, lead a summary discussion of the conjectures and help students to organize the results by using classifications of quadrilaterals (e.g., any quadrilateral that is a parallelogram has congruent opposite angles and supplementary consecutive angles). Using the *graphic organizer* ([view literacy strategy descriptions](#)) provided in the Quadrilateral Family BLM, fill in the names of the quadrilaterals so that each of the following is used exactly once:

- | | |
|-----------------------|---------------|
| • PARALLELOGRAM | KITE |
| • SQUARE | QUADRILATERAL |
| • TRAPEZOID | RECTANGLE |
| • ISOSCELES TRAPEZOID | RHOMBUS |

Explanation: Following the arrows: The properties of each figure are also properties of the figure that follows it. Reversing the arrows: Every figure is also the one that precedes it.

Have students complete the *graphic organizer* given in the Quadrilateral Family BLMs, and then lead a class discussion to summarize how different quadrilaterals are related to one another. Students should be able to identify a square as being a rectangle, rhombus, parallelogram, and quadrilateral and be able to justify their reasoning.

Activity 15: Quadrilaterals on the Coordinate Plane (GLEs: 6, 9, 16)

Materials List: pencil, paper, graph paper

Present students with sets of ordered pairs which form various quadrilaterals on the coordinate plane. For each of these sets, have students determine which quadrilateral is presented by using the distance formula, midpoint formula, and slope formula to determine which properties, if any, apply to the given quadrilateral. Students should be given time to explore this on their own, at first, to see how they would begin to determine whether the given quadrilateral is a parallelogram or not. After some time, lead a discussion about which properties might be most important to show first, in order to determine if the quadrilateral is a square. This would mean it is also a rectangle, rhombus, and parallelogram. Also, discuss which formulas would be used to show congruent segments, perpendicular segments, and that segments are bisected.

Once students have been able to determine the type of quadrilateral, have students write the equations of the lines that will produce the quadrilateral. Depending on the type of quadrilateral, some of the lines will be parallel and perpendicular, while others may not be related at all. Be sure that students write the equations of the lines that form the diagonals, especially on rectangles, rhombi, and squares.

Sample Assessments

General Assessments

- The student will complete *learning log* entries for this unit. Grade the *learning log*. Topics could include:
 - Explain the statement, “A square is a rectangle, but a rectangle is not a square.”
 - In an isosceles triangle, is a perpendicular bisector drawn from any vertex always the same segment as altitude and median? Explain your reasoning.
 - Suppose you have three different positive numbers arranged in order from greatest to least. Which sum is it most crucial to test to see if the numbers could be the lengths of the sides of a triangle? Explain your answer and use examples if necessary.
- The teacher will provide the student with a net which gives some of the angle measures, and a net in which other angles are labeled with variables representing the measurements of the angles. The student will find all the missing angle measures using the angle relationships learned in the unit, and will defend his/her answers by identifying the property or properties used to determine each missing value. The nets should have special quadrilaterals and other polygons embedded within the diagram so that the properties learned must be used to find some of the angle measures.
- The student will write proofs of congruent or similar triangles and properties of parallelograms using information provided by the teacher. Evaluate proofs for accuracy (use of correct postulates and theorems) and completeness (not missing any steps in the reasoning process), allowing the student to use any method of proof desired.

Activity-Specific Assessments

- Activity 4: The student will complete a product assessment in which he/she designs a 5 by 5 inch tile using various types of triangles. The triangle will be correctly marked to show an understanding of methods used to determine triangle congruency. See Activity 4 Specific Assessment BLM and the Activity 4 Specific Assessment Rubric BLM.
- Activity 5: Critical Thinking Writing:

The Hypotenuse-Leg Theorem is one example of when a Side-Side-Angle will work to show two triangles are congruent. Give a proof of why the HL Theorem works.

Rubric:

1 pts.	-answers in paragraph form in complete sentences with proper grammar and punctuation
1 pts.	-correct use of mathematical language
1 pts.	-correct use of mathematical symbols
5 pts.	-correct proof including all logical steps and correct reasons

- Activity 7: Provide the student with different triangles and have him/her draw the angle bisectors, perpendicular bisectors, medians, and altitudes for the triangles. Provide one isosceles, one obtuse, one right, and one scalene triangle and have the student draw different special segments on each. The student will draw the altitude on the right and obtuse triangles, all three special segments on the isosceles triangle (one segment should satisfy this), and the angle bisectors on any type triangle. The student will also explain the processes used to make the drawings.
- Activity 8: Given the coordinates $P(5,6)$, $N(11,2)$, and $M(-1,-2)$ find the following.
 - Find the coordinates of Q , the midpoint of \overline{MN} .
 - Find the length of the median \overline{PQ} .
 - Find the coordinates of T , the centroid.
 - Find the coordinates of R , the midpoint of \overline{MP} . Show that the quotient $\frac{NT}{NR}$ is $\frac{2}{3}$.

Solutions:

- $Q(5, 0)$
- $PQ = 6$
- $T(5,2)$
- $R(2,2); NT=6; NR=9; \frac{NT}{NR} = \frac{6}{9} = \frac{2}{3}$

- Activity 14: The student will complete a Venn diagram to demonstrate understanding of the properties of the *parallelograms* discussed in class. See Activity 14 Specific Assessment BLM. This example is only a guide and may be expanded to include trapezoids, isosceles trapezoids, and kites by drawing a larger rectangle around the Venn Diagram shown, thereby eliminating the requirement that properties numbers be shown. Isosceles trapezoids have congruent diagonals and are not parallelograms, so they would have to be drawn within a quadrilateral set and outside the parallelogram set. As a result, the congruent diagonal characteristic would need to be repeated.