

Geometry
Unit 4: Transformations

Time Frame: Approximately four weeks



Unit Description

This unit provides a deeper mathematical understanding and justification for transformations that students have seen in previous grades. The focus is providing justifications for the congruence and similarity relationships associated with translations, reflections, rotations, and dilations (centered at the origin).

Student Understandings

Students determine what transformations have been performed on a figure and can determine a composition of transformations that can be performed to mimic other transformations like rotations. They are also able to find new coordinates for transformations without actually performing the indicated transformation.

Guiding Questions

1. Can students find transformations and mappings that relate one congruent figure in the plane to another?
2. Can students provide an argument for the preservation of measures of figures under reflections, translations, and rotations?
3. Can students find the dilation (enlargement or reduction) centered at the origin of a specified figure in the plane and relate it to a similarity mapping?
4. Can students perform a composition of transformations and explain its relationship to single transformations or other compositions that produce the same image?

Unit 4 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

Grade-Level Expectations	
GLE #	GLE Text and Benchmarks
Geometry	
14.	Develop and apply coordinate rules for translations and reflections of geometric figures (G-3-H)
15.	Draw or use other methods, including technology, to illustrate dilations of geometric figures (G-3-H)
19.	Develop formal and informal proofs (e.g., Pythagorean theorem, flow charts, paragraphs) (G-6-H)
CCSS for Mathematical Content	
CCSS#	CCSS Text
Congruence	
G.CO.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
G.CO.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
ELA CCSS	
CCSS #	CCSS Text
Reading Standards for Literacy in Science and Technical Subjects 6-12	
RST.9-10.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
Writing Standards for Literacy in History/Social Studies, Science and Technical Subjects 6-12	
WHST.9-10.2d	Write informative/explanatory texts, including the narration of historical events, scientific procedures/experiments, or technical processes. Use precise language and domain-specific vocabulary to manage the complexity of the topic and convey a style appropriate to the discipline and context as well as to the expertise of likely readers.
WHST.9-10.10	Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.

Sample Activities

Activity 1: Vocabulary Self-Awareness (CCSS: G.CO.4; CCSS: RST.9-10.4)

Materials List: pencil, Vocabulary Self-Awareness BLM

Have students maintain a *vocabulary self-awareness chart* ([view literacy strategy descriptions](#)) for this unit. *Vocabulary self-awareness* is valuable because it highlights students' understandings of what they know, as well as what they still need to learn, in order to fully comprehend the concept. Students indicate their understanding of a term/concept, but then adjust or change the marking to reflect their change in understanding. The objective is to have all terms marked with a + at the end. A sample chart is included in the Blackline Masters. Be sure to allow students to revisit their *vocabulary self-awareness charts* often to monitor their developing knowledge about important concepts. Terms/concepts that should be included for this unit are given below:

- Transformation
- Pre-image
- Image
- Rigid transformation (rigid motion)
- Non-rigid transformation (non-rigid motion)
- Orientation
- Isometry
- Reflection
- Line of reflection
- Translation
- Rotation
- Center of rotation
- Degree of rotation
- Clockwise
- Counterclockwise
- Dilation
- Center of dilation
- Scale factor
- Similarity transformation
- Composite transformation
- Glide reflection

Have students complete self-assessments of their knowledge of the words using the Vocabulary Self-Awareness BLM prior to the start of the unit. Do not give students definitions or examples at this stage. Ask students to rate their understanding of each word with a "+" (student understands the term well and can provide an accurate example and definition related to the unit of study), a "✓" (student has a limited understanding, is unsure, or can only provide a definition or an example, not both), or a "-" (student does not know the term). Tell students to provide a definition and an example for any term they understand well (terms for which they put a plus sign). Over the course of the unit, students should be referred to the chart to add new information and adjust their understanding of each term. Students will add definitions, examples, and other words, as well as revise definitions and examples from their initial self-assessment. The goal is to replace all check marks and minus signs with plus signs. Since students will continually revisit their *vocabulary self-awareness charts*, they have multiple opportunities to practice and extend their understanding of key terms related to transformations.

Activity 2: Understanding Congruence, Similarity, and Symmetry Using Transformations and Interactive Figures: Visualizing Transformations (Using Technology) (GLE: 14)

Materials List: pencil, paper, the Internet access

Begin by using *student questions for purposeful learning (SQPL)* ([view literacy strategy descriptions](#)). *SQPL* is a strategy designed to help students develop the ability to learn with a purpose. *SQPL* promotes purposeful learning by prompting students to ask and answer their own questions about content. To implement this strategy, look over all of the material to be covered in the lesson, then develop a thought-provoking statement related to the topic about to be discussed. The statement does not have to be factually true, but it should generate some level of curiosity for the students. For this activity, pose the statement, “All transformations are rigid transformations.” This statement can be written on the board, projected on the overhead, or stated orally for the students to write in their notebooks. Allow the students to ponder the statement for a moment and ask them to think of some questions they might have related to the statement. After a minute or two, have students pair up and generate two or three questions they would like to have answered that relate to the statement. When all of the pairs have developed their questions, have one member from each pair share their questions with the class. As the questions are read aloud, write them on the board or overhead. Students should also copy these in their notebooks. When questions are repeated or are very similar to others which have already been posed, those questions should be starred or highlighted in some way. Once all of the students’ questions have been shared, look over the list and determine if additional questions should be added. The list should include the following questions:

- What is a transformation?
- What types of transformations are there?
- What does the phrase “rigid transformations” mean?
- Are there non-rigid transformations?
- What are non-rigid transformations called?

At this point, be sure the students have copied all of the questions in their notebooks and continue with the lesson as follows. Tell students to pay attention as the material is presented to find the answers to the questions posted on the board, focusing on those questions which have been starred or highlighted. Students should refer to these questions throughout the entire unit as all questions will not be completely answered until the end of the unit.

The website, <http://www.nctm.org/standards/content.aspx?id=26885>, allows students to visualize transformations and compositions of transformations, while working interactively with various geometric figures. Students explore the effects of applying reflections, translations, and rotations to any one of three shapes. There are four sections students can explore.

- 6.4.1 Visualizing Transformations – Students can investigate what happens in a reflection, rotation, and translation as the figure is moved around the plane.

Students can also change the orientation of the figure and change its shape and make observations about what happens to the image.

- 6.4.2 Identifying Unknown Transformations – In this applet, students are challenged to put what they learned in the first section to determine which transformations have been applied to the pre-image to obtain the image.
- 6.4.3 – Composing Reflections – This applet allows students to investigate what happens when a figure is reflected successively over two different lines. This investigation is important to help students understand two important theorems to be discussed later in the unit (Parallel Lines Theorem and Intersecting Lines Theorem).
- 6.4.4 – Composing Transformations – Students are asked to develop equivalent transformations in two different ways. Students can investigate here what happens when figures are reflected over more than two lines, what happens when a reflection is followed by a rotation or vice versa, and what happens when a reflection is followed by a translation or vice versa.

Periodically stop throughout the lesson to allow the student pairs to discuss which questions have been answered from the list created at the beginning of the lesson. Ask questions, such as “How does the original shape compare to the shape after the transformation?” and “What is the effect of the transformation on the side lengths and angle measures of the original shape?” Students should begin to realize that reflections, translations, and rotations do not change the size or shape of the figures they use, which means they are congruence transformations, or isometries. This concept will be reinforced in the following activities. This exploration may be followed with a whole class discussion, so all students are sure to have the correct answers to each question. Be sure to add information to the *vocabulary self-awareness chart* started in Activity 1 that may have been discussed in this activity.

Activity 3: A Basic Look at Transformations (GLE: 14; CCSS: G.CO.3)

Materials List: A Basic Look at Transformations BLM, patty paper or tracing paper, pencil, paper, ruler, protractor

Begin by providing students with a copy of A Basic Look at Transformations BLM and 5 pieces of patty paper or tracing paper. If patty paper or tracing paper is inaccessible, make two copies per student on two different colors of paper. Have students use a straightedge to trace the polygons on the patty paper or tracing paper or cut out the shapes from one of the colored copies. Then have students work in pairs to perform reflections and rotations using their patty paper as the image. Students should place the patty paper over the corresponding shape and turn the patty paper and flip the patty paper. The goal is to have students identify any reflections and/or rotations that would carry the polygons onto themselves (i.e., the patty paper shape is an exact copy of the shape on the BLM). Students should record the transformations that work describing the lines of reflection and angles of rotation as best they can. After students have had time to investigate, lead a whole class discussion where students talk about which transformations work. Have

students demonstrate their suggestions on the board/overhead for the other students in class. Ask students if they know they have found all possible transformation and how they were able to make that determination. Be sure to record the information on the board/overhead for all students to record. *Solution: The rectangle has two lines of reflection: a vertical line through the center of the rectangle and a horizontal line also through the center of the rectangle. The rectangle can be rotated 180 degrees. The parallelogram has no lines of reflection but can be rotated 180 degrees. The trapezoid has one line of reflection—vertical through the center of the trapezoid. The trapezoid has no rotations that will carry it onto itself. The hexagon has six lines of reflection: 3 that include the diagonals of the hexagon, and the other three intersect opposite sides of the hexagon at their midpoints. The hexagon can be rotated through 60 degrees, 120 degrees, 180 degrees, 240 degrees and 300 degrees (the central angles of the hexagon measure 60 degrees; rotating around the center of the hexagon will produce rotations through these angles). The equilateral triangle has three lines of reflection which pass through each of the vertices and the midpoint of the side opposite the vertex. There are two rotations which will carry the triangle onto itself: 120 degrees and 240 degrees. **Note: for all figures, a 360 degree rotation will carry the figure onto itself. For this reason, it was omitted from the answer as effectively a rotation of 360 degrees is the same as no transformation. This should be discussed with the class.*

Activity 4: Defining Transformations (CCSS: G.CO.4, RST.9-10.4)

Materials List: pencil, paper, ruler, protractor, What Are Transformations? BLM

This activity will use the *questioning the content (QtC)* ([view literacy strategy descriptions](#)) technique to help students extend their prior knowledge of transformations from earlier grades. The goal of QtC is to teach student to use a questioning process to construct meaning of content and to think at higher levels about the content from which they are expected to learn. For this activity, provide students with a copy of the What Are Transformations? BLM. Students will read the material provided in this BLM and develop questions about the content. Give the students the following types of questions as examples of the types they should ask about the content with which they have been presented.

- What is the content about?
- It says _____, but what does it mean?
- Why was this word used?
- How does that connect with what I already know?
- How does that connect with what was said earlier?
- Does that make sense?
- Is this explained clearly? Why or why not?
- What do I need to figure out or find out?
- Did the content tell me that?
- Did the source provide the answer for that?

Students may develop such questions as:

- What new content will “What Are Transformations?” contain?
- It says that a rotation can be clockwise or counterclockwise, but what does that mean?
- Why is the word “prime” used when listing the vertices of a transformed figure?
- How does the definition of reflection given here connect with what I already know about reflections?
- Does the definition of rotation make sense?
- Does the content tell me how to determine the angle of rotation for a given diagram?

As students read through the material, have them record their questions about the content. If they can locate the answer to the content, they should also record the answer. Once students have had time to read through the entire BLM, and develop and answer their questions, lead a whole class discussion about the content. Have students share the questions they developed throughout their reading and share their answers. Take this opportunity to address any misconceptions or misunderstandings and clarify the knowledge students have obtained from the text they have read.

After discussing the text information and questions raised by students, model performing transformations on various figures without the use of the coordinate system to help students understand the definitions provided in the text they read. Allow students time to practice these transformations on their own as well. After practicing the transformations, have students revisit the *vocabulary self-awareness chart* they began in Activity 1 to update their definitions, examples, and their understandings of the terms discussed.

Activity 5: Reflections on the Coordinate Plane (GLE: 14)

Materials List: pictures of reflections, pencil, paper, straightedge, protractor

Give students pictures of various reflections using the x -axis, y -axis, the origin, and the line $y = x$. Have students develop a chart that names the type of reflection, the change of the original to the image, a statement about how to find the coordinates of the image, and a numerical example in the coordinate plane. For example, a chart might look like the following:

Reflection	Over x -axis	Over y -axis	Around Origin	Over $y = x$
Original to image	$(a, b) \rightarrow (a, -b)$	$(a, b) \rightarrow (-a, b)$	$(a, b) \rightarrow (-a, -b)$	$(a, b) \rightarrow (b, a)$
Statement	Multiply y -coordinate by -1			
Example				

Provide students with multiple examples for various different figures with different properties to complete this chart. Next, provide students with coordinates of a figure and

have them determine the coordinates of the image points for a given reflection without graphing any of the points. Allow students to use the chart to determine what the coordinates of the image points should be. After they have identified the new coordinates for each reflection, have them graph their original images and the reflected images to check their work. Have students confirm that a reflection represents a congruence transformation. Students may use the distance formula to confirm congruent sides and a protractor to confirm congruent angles. Reinforce the definition of *isometry* and *rigid transformation* and have students update their *vocabulary self-awareness charts* as needed. Be sure to have students refer to the questions posed in Activity 2 to be sure all answers are complete.

Activity 6: Rotations on the Coordinate Plane (GLE: 14)

Materials List: image on a coordinate plane, pencil, paper, straightedge, protractor

Teacher Note: While GLE 14 does not refer to rotations, rotations are addressed in the Common Core State Standards and shall remain in the Geometry curriculum so as to avoid any gaps in knowledge for subsequent courses.

Give students a pre-image on the coordinate plane with the vertices labeled. Have students make a chart like the one below and rotate the pre-image 90° , 180° , 270° , and 360° using the origin as the center of rotation. They should record the new coordinates each time and then graph the new images.

Original	90°	180°	270°	360°
(a,b)				
(c,d)				
(e,f)				
(g,h)				
(x,y)				

After drawing the new figures, have students analyze the coordinates to determine a pattern of changes in coordinates. Provide opportunities to practice performing rotations both with and without this chart on the coordinate plane. Have students confirm that rotations represent a congruence transformation. Students may use the distance formula to confirm congruent sides and a protractor to confirm congruent angles. Reinforce the definition of *isometry* and *rigid transformation* and have students update their *vocabulary self-awareness charts* as needed. Be sure to have students refer to the questions posed in Activity 2 to be sure all answers are complete.

Activity 7: Slide It! (GLE: 14; CCSS: WHST.9-10.2d, WHST.9-10.10)

Materials List: polygons on a coordinate plane, pencil, paper, straightedge, protractor, learning log

Provide students with diagrams showing various polygons on a coordinate plane. Give specific instructions about the directions in which to move the figure (3 units up and 4 units right, 2 units down and 4 units left, etc). Have students record the original and new coordinates of the vertices and analyze the results. Lead a discussion to summarize how the x -coordinate is affected when a point is translated left/right, and how it is affected if the point is moved up/down. Have similar discussion about the change in the y -coordinate when the point is moved left/right and up/down. Assuming that a represents a horizontal translation and b represents a vertical translation, the effect of the translation is point $(x, y) \rightarrow (x+a, y+b)$.

Provide the opportunity for students to draw a translation that moves in the same direction as a given vector. Since students have not yet studied vectors, the vector should be already drawn on the coordinate plane so that students can identify the number of units to move. The focus of the lesson is not on understanding what a vector is, but instead on using the visual of the vector (a ray) to determine the direction and distance a polygon will move on the coordinate system. Again, they should analyze their ordered pairs and determine if the pattern is still true.

Have students confirm that a translation represents a congruence transformation. Students may use the distance formula to confirm congruent sides and a protractor to confirm congruent angles. Reinforce the definition of *isometry* and *rigid transformation* and have students update their *vocabulary self-awareness charts* as needed. Have students refer to the questions posed in Activity 2 to be sure all answers are complete.

To end this activity, have students create an entry in their math *learning logs* ([view literacy strategy descriptions](#)). Present the following prompt:

Explain how you could determine the new coordinates of the vertices of a polygon if it were reflected, rotated, or translated without graphing the transformations on the coordinate plane. Be sure to explain your reasoning for each transformation to be performed.

The students' responses should reflect an ability to apply the coordinate rules developed in Activities 5, 6, and 7.

A *learning log* is typically a notebook students keep in order to record their ideas, questions, and new understandings. Students should keep this as a separate notebook or as a separate section in their binders. Students should use their math *learning log* other times in class, in addition to those listed throughout the curriculum to provide opportunities to assess understanding.

Activity 8: Magnify It! (GLE: 15)

Materials List: figure graphed in a coordinate plane, pencil, paper, drawing program

Have students work in groups of two to develop a specific dilation of a figure that has been graphed in the plane. All dilations in this activity assume the center of dilation to be (0,0). First, have them create a dilation that is 1.5 times the size of the original figure. Next, instruct students to create a dilation that is .75 times the size of the original figure. Be sure to instruct students to specify the coordinates of the dilated figures. If available, use *The Geometer's Sketchpad*® or other drawing program to perform dilations. Have students make statements about the similarity of the original figure and its dilation and determine the attributes of the original figure that remain unchanged after the dilation is performed. Students should recognize that dilation is not a congruence transformation; however, the figures are similar. Have students refer to the questions from Activity 2 to be sure all answers are complete. Also, have students update the *vocabulary self-awareness chart* from Activity 1 to reflect this new understanding.

Activity 9: Composite Transformations (CCSS: G.CO.3)

Materials List: graph paper, pencil, straightedge, protractor (optional)

Provide students with a sheet of graph paper with the four quadrants marked. Have students construct a parallelogram with vertices located at (3,10), (6,8), (9,14), and (12,12). Next, ask students to predict what they think will happen when they reflect the pre-image over the x-axis, then rotate the image 90° counterclockwise with the origin as the center of rotation. After listening to students' responses, have students perform those transformations and discuss the results. Ask them if the resulting image is the same as any single transformation discussed throughout the unit. Students should respond that it is the same as a reflection across the line $y = x$. Provide students with more polygons and a variety of series of transformations, ask for predictions and then discuss the results. Any polygon can be used, but this activity should be repeated with rectangles, parallelograms, trapezoids, any other regular polygons. Include at least one example of a glide reflection which is a reflection followed by a translation or vice versa. Discuss the term *glide reflection* with the class and have them add the information to their *vocabulary self-awareness charts* as needed.

Next, have students create their own polygon and instruct them to develop a series of translations, reflections, and/or rotations that combine to produce the original shape in its original location (avoiding, if possible, a rotation of 360 degrees). Have students record the series of transformations used. Discuss with students the meaning of composite transformations and have them revisit their *vocabulary self-awareness charts* from Activity 1 to update their definitions as necessary. To conclude the activity, have students pair up and work together to create a series of transformations to produce a new figure. Students may use any transformations, including dilations. Once they have

developed their series, have groups swap instructions. Each group should be provided with a list of ordered pairs to create the polygon and the series of transformations to be performed with the polygon. Groups should not give the ordered pairs of the final image. Once groups have received their instructions, they should graph the polygons and perform the indicated transformations. Then, they should check with their partner groups to see if they created the correct image.

Activity 10: Make a Conjecture and Prove It! (GLEs: 14, 15, 19; CCSS: G.CO.4)

Materials List: drawing program, pencil, paper, diagrams of transformed images

Using a geometry software package such as *The Geometer's Sketchpad*®, have students create several translations, reflections, rotations, or dilations and combinations of these, and then examine the properties of the transformed figures compared to original figures. Using these inspections, have students make conjectures about the effects of these transformations including conjectures concerning congruence and similarity. Instruct students to prove their conjectures.

For example, “Are any of the combinations of transformations the same as a single type of transformation? Are transformations commutative, that is, can the order of two transformations be changed and get the same result?” Allow students to use inductive and deductive reasoning while comparing conjectures and accompanying proofs. To enhance student understanding, have students begin by reflecting their figures over two parallel lines and compare this with a translation. Next, have students reflect their figures over a pair of intersecting lines (e.g., the x- and y-axes) and compare this with a rotation. Have students perform this activity through several iterations. Each iteration should focus on a specific set of combined transformations. In addition, provide students with a pre-image and an image in the coordinate plane with vertices labeled and require students to determine the transformation or set of transformations that produced the image.

To end the activity, provide students with the text for the Parallel Lines Theorem and the Intersecting Lines Theorem.

Parallel Lines Theorem – A) A composition of reflections across two parallel lines is equivalent to a translation (or across any even number of parallel lines).
B) A composition of reflections across three parallel lines (or across any odd number of parallel lines) is equivalent to a single reflection.

Intersecting Lines Theorem – A composition of reflections across two intersecting lines is equivalent to a rotation about the point of intersection through twice the acute or right angle between the intersecting lines.

Complete the activity by having students revisit their *vocabulary self-awareness chart* from Activity 1 to be sure students have the correct definitions and correct examples so they can use the chart to study for assessments.

Sample Assessments

General Assessments

- The teacher will provide the student with a polygon in the coordinate plane and instruct the student to perform various transformations on it.
- The student will investigate the transformations that are used in a board game, such as “checkers” or “chess.”
- The student will create a portfolio containing samples of work from the activities. Portfolio entries will include copies of the transformations performed in class with explanations about the procedures used to complete the transformation.
- The student will write entries in his/her *learning log* that are graded. Topics might include:
 - Which transformations produce congruent figures? Why are these figures congruent to their originals?
 - How did your knowledge of rotations, reflections, translations, and dilations change throughout this unit? In your response, discuss your initial thoughts when you completed the *vocabulary self-awareness chart* at the beginning of the unit and how you have changed that information.
- The student will create a “Transformation Album.” He/she will create a figure and perform at least one of each of the different transformations on the figure in the coordinate plane. Assess the work based on the accuracy of the transformations.

Activity-Specific Assessments

- Activity 5: Ask the student to demonstrate his/her ability to do the following:
 1. Given $A(-2,3)$, $B(-5,7)$, and $C(-1, 10)$, graph triangle ABC .
 2. Reflect triangle ABC over the x -axis. Label the image as $A'B'C'$.
 3. Find the area of triangle $A'B'C'$.
 4. Explain how the area of triangle $A'B'C'$ compares to the area of triangle ABC .
- Activity 6: Provide the student with the coordinates of an image that has already been rotated 90° , 180° , or 270° . The student will find the coordinates of the pre-image using either the coordinate plane or the chart developed in this activity.
- Activity 9: The student will design a tessellation using a composition of transformations. If materials are available, the student will transfer the design to a cloth square in order to make a class quilt. This can be done using special crayons which can be found in some school and/or art supply stores.