9-4 Compositions of Isometries

Objectives To find compositions of isometries, including glide reflections
To classify isometries

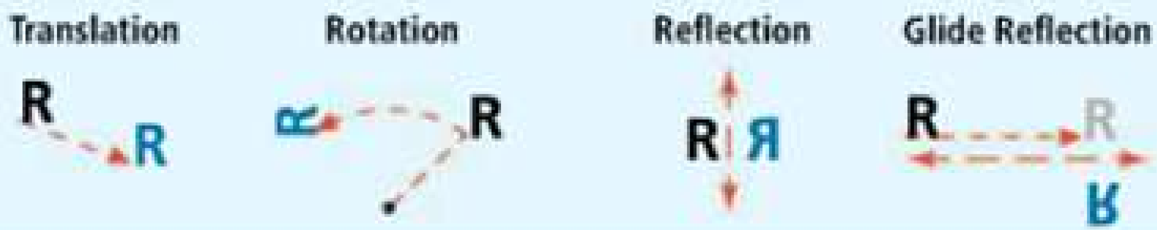
The term *isometry* means same distance. An **isometry** is a transformation that preserves distance or length. So, translations, reflections, and rotations are isometries.

SIZE

Theorem 9-1

The composition of two or more isometries is an isometry.

There are only four kinds of isometries.



take note

Theorem 9-2 Reflections Across Parallel Lines

A composition of reflections across two parallel lines is a translation.

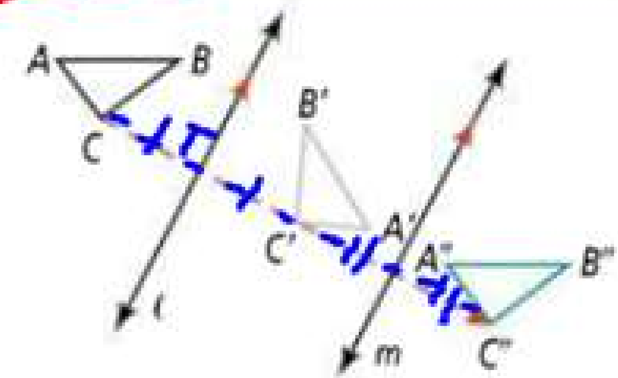
You can write this composition as

$(R_m \circ R_\ell)(\triangle ABC) = \triangle A''B''C''$

or $R_m(R_\ell(\triangle ABC)) = \triangle A''B''C''$.

Notation

$\overline{AA''}$, $\overline{BB''}$, and $\overline{CC''}$ are all perpendicular to lines ℓ and m.



2nd

1st

$(R_m \circ R_\ell)(\triangle ABC)$

$\triangle ABC \to \triangle A''B''C''$ is a translation.

Distance: twice the distance between ℓ and m

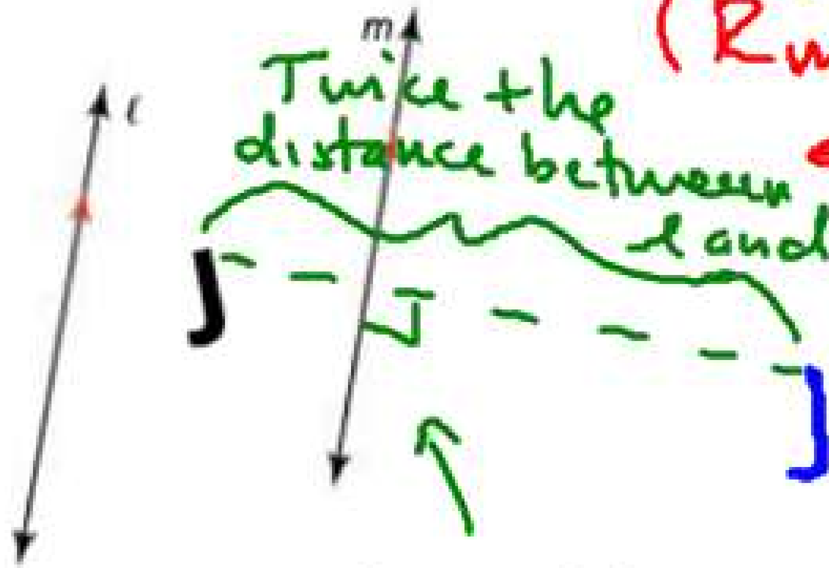
Direction: perpendicular to ℓ and m

Got It? **1. a.** Draw parallel lines ℓ and m as in Problem 1. Draw J between ℓ and m. What is the image of $(R_m \circ R_\ell)(J)$? What is the distance of the resulting translation? and direction

b. Reasoning Use the results of part (a) and Problem 1. Make a conjecture about the distance of any translation that is the result of a composition of reflections across two parallel lines.

2nd 1st

$(R_m \circ R_\ell)(J)$



Twice the distance between l and m

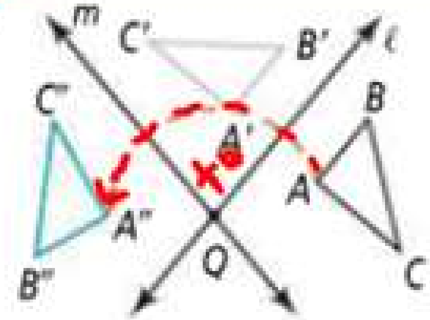
l

Direction is $\perp$ to l and m

take note

Theorem 9-3 Reflections Across Intersecting Lines

A composition of reflections across two intersecting lines is a rotation.



You can write this composition as $(R_m \circ R_\ell)(\triangle ABC) = \triangle A''B''C''$ or $R_m(R_\ell(\triangle ABC)) = \triangle A''B''C''$.

The figure is rotated about the point where the two lines intersect. In this case, point Q.

$\triangle ABC \rightarrow \triangle A''B''C''$ is a rotation.

Center of Rotation : Q (Pt. where lines intersect)

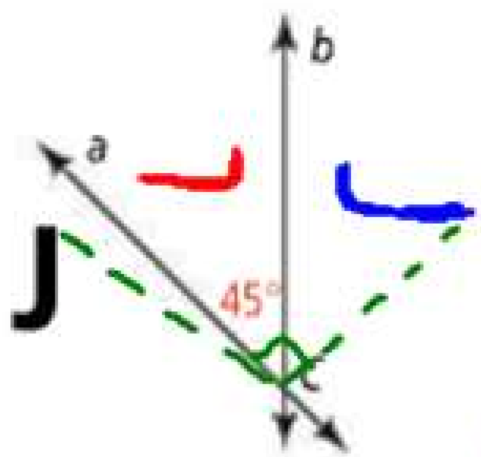
Angle of Rotation : $2x°$ (Twice the Acute Angle)

Got It? 2. a. Use the diagram at the right. What is $(R_b \circ R_a)(J)$? What are the center and the angle of rotation for the resulting rotation?

b. **Reasoning** Use the results of part (a) and Problem 2. Make a conjecture about the center of rotation and the angle of rotation for any rotation that is the result of any composition of reflections across two intersecting lines.

2nd 1st

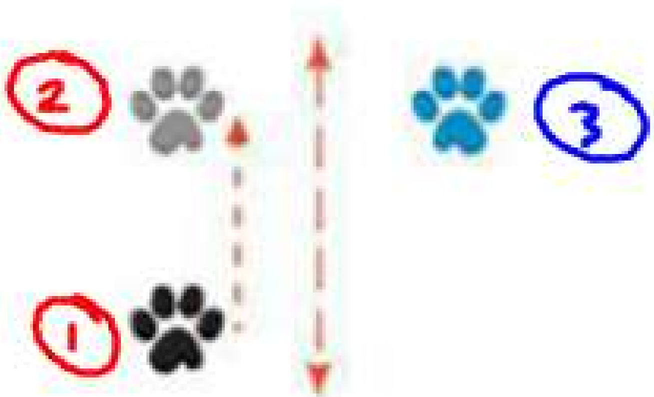
$(R_b \circ R_a)(J)$



Center of Rotation: C

∠ of Rotation: $2 \cdot 45° = 90°$

Any composition of isometries can be represented by either a reflection, translation, rotation, or glide reflection. A **glide reflection** is the composition of a translation (a glide) and a reflection across a line parallel to the direction of translation. You can map a left paw print onto a right paw print with a glide reflection.



Translation: 1 → 2

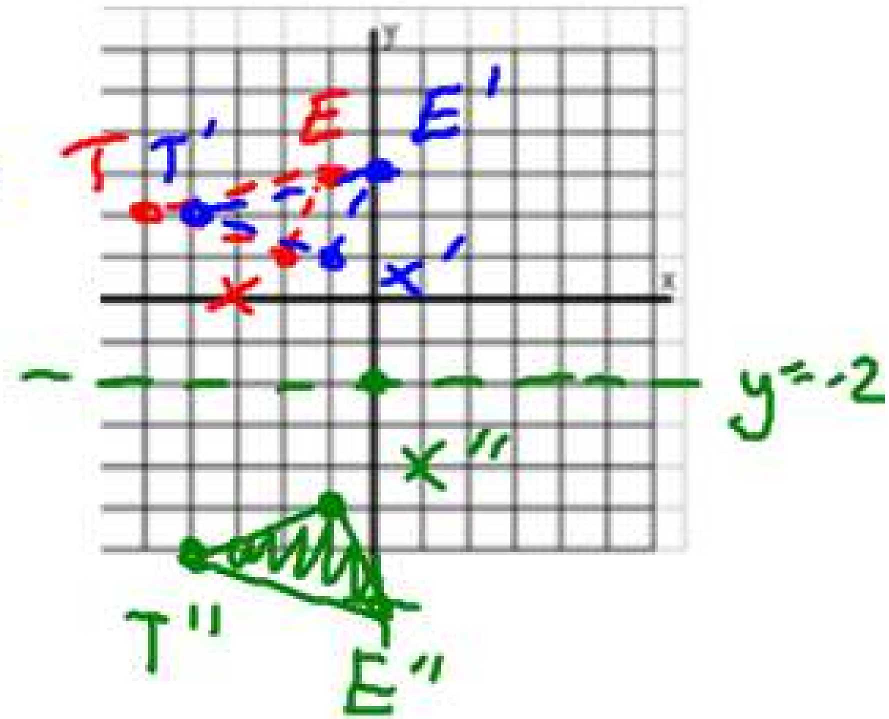
Reflection: 2 → 3

Got It? **3.** Graph $\triangle TEX$ from Problem 3. What is the image of $\triangle TEX$ for the glide reflection $(R_{y=-2} \circ T_{<1,0>})(\triangle TEX)$?

T(-5, 2), E(-1, 3), X(-2, 1)

2nd Horiz. 1st R+1

$$\left(R_{y=-2} \circ T_{<1,0>} \right)$$



Inclass: p. 574 #12, 18
Homework: p. 574-575 #7-25(odd)
Interactmath: #7, 9, 13, 17, 19, 25