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12–1 Prisms

Objectives: Identify the parts of prisms. Find the lateral areas, total areas, and volumes of right prisms.

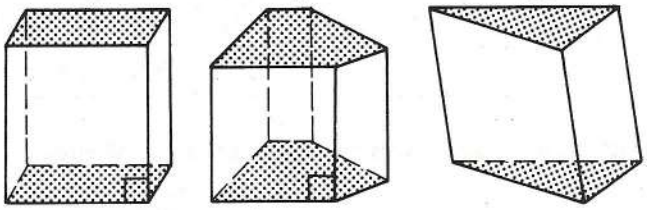
In this chapter you will be calculating surface areas and volumes of special solids. The first solid you will study is the **prism**.

base The shaded faces are called bases. Bases lie in parallel planes and are congruent polygons.

lateral faces The faces that are not bases are lateral faces. Lateral faces are parallelograms that intersect each other in parallel segments called

lateral edges lateral edges.

I II III



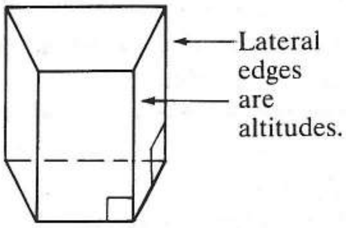
Right rectangular prism — Right pentagonal prism — Oblique triangular prism

Prisms I and II, in which all the lateral faces are rectangles, are called **right prisms**. Prism III is an **oblique prism**.

altitude A segment joining the two base planes of a prism and perpendicular to both is an altitude. In a right prism, each lateral edge is an altitude.

height The length of an altitude is the height, h, of a prism.

lateral area The lateral area (L.A.) of a prism is the sum of the areas of its lateral faces.

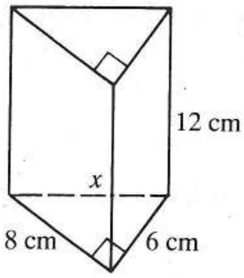


Right trapezoidal prism

> **The lateral area of a right prism equals the perimeter of a base times the height of the prism.** **L.A. = *ph***

Example 1

Find the lateral area of the right triangular prism.



Solution

First find the value of x:

$x^2 = 6^2 + 8^2 \qquad x = 10$

perimeter of base: $p = 6 + 8 + 10 = 24$

height of prism: $h = 12$

L.A. $= ph = 24 \cdot 12 = 288$

The lateral area is 288 cm^2.

total area The total area (T.A.) of a prism is the sum of the areas of all of its faces.

> **The total area of a right prism equals the lateral area plus the areas of both bases.** **T.A. = L.A. + 2*B***

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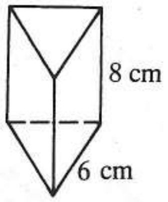
12–1 Prisms *(continued)*

Example 2

Find the total area of the right triangular prism with bases that are equilateral triangles.



Solution

T.A. = L.A. + $2B$

$p = 3 \cdot 6 = 18; h = 8$

L.A. = $ph = 18 \cdot 8 = 144$

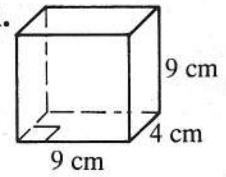
$B = \frac{1}{2}bh = \frac{1}{2} \cdot 6 \cdot 3\sqrt{3} = 9\sqrt{3}$

T.A. = L.A. + $2B = 144 + 18\sqrt{3}$

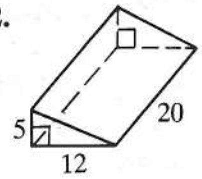
The total area is $(144 + 18\sqrt{3})$ cm^2.

Find (a) the lateral area and (b) the total area of each right prism.

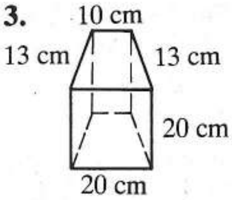
1.



2.



3.



The volume of a right prism equals the area of a base times the height of the prism. $V = Bh$

Example 3

Find the volume of the right triangular prism in Example 2.

Solution

$V = Bh$ and $B = 9\sqrt{3}$ (from Example 2), so

$V = 9\sqrt{3} \cdot 8 = 72\sqrt{3}$. The volume is $72\sqrt{3}$ cm^3.

4–6. Find the volume of each right prism in Exercises 1–3.

Example 4

Find the width of a rectangular solid with length 15 cm, height 8 cm, and lateral area 400 cm^2.

Solution

L.A. = ph and $p = 2(l + w)$, so

L.A. = $2(l + w) \cdot h$.

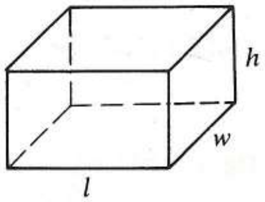
$400 = 2(15 + w) \cdot 8$

$25 = 15 + w$

$10 = w$ The width is 10 cm.

Exercises 7–10 refer to rectangular solids with dimensions *l*, *w*, and *h*. Complete the table.

	7.	8.	9.	10.
l	25	8	15	8
w	20	4	?	6
h	10	6	4	?
L.A.	?	?	216	?
T.A.	?	?	?	?
V	?	?	?	576



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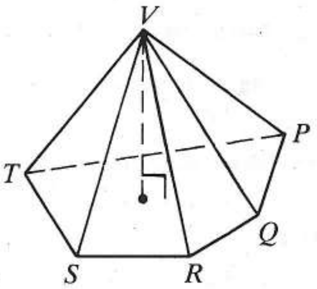
12–2 Pyramids

Objectives: Identify the parts of pyramids. Find the lateral areas, total areas, and volumes of regular pyramids.

A **pyramid** has only one base.



vertex Point V is the vertex of pyramid V-$PQRST$.

base Pentagon $PQRST$ is the base of the pyramid.

lateral faces The lateral faces of a pyramid are triangles. The segments in which the triangles intersect are the

lateral edges lateral edges.

altitude The segment from the vertex perpendicular to the plane of the base is the altitude of the pyramid.

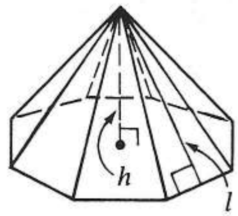
height The length of the altitude is the height, h, of the pyramid.

A **regular pyramid** has a regular polygon as its base. Regular pyramids have several important properties.

(1) All lateral faces are isosceles triangles. The **slant height**, l, of the pyramid is the length of an altitude of a triangle that is a lateral face.

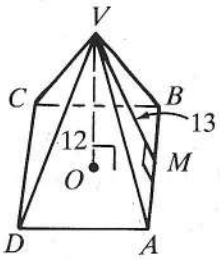
(2) All lateral edges are congruent.

(3) The altitude of the pyramid meets the base at the center of the base polygon.



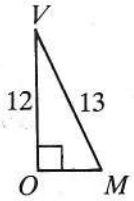
Example 1

The height of regular square pyramid V-$ABCD$ is 12 and its slant height is 13. Find the length of the base edges and the lateral edges.

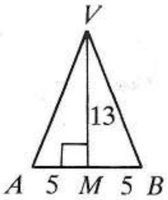


Solution

In right $\triangle VOM$,
$12^2 + (OM)^2 = 13^2$
A base edge $= 2 \cdot OM$
$= 2 \cdot 5 = 10$

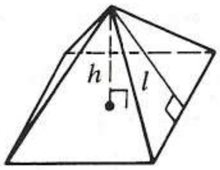


In right $\triangle VMA$,
$(VA)^2 = 13^2 + 5^2 = 194$
$VA = \sqrt{194}$
A lateral edge $= \sqrt{194}$



Complete the table for regular square pyramids.

	1.	2.	3.	4.
height	8	15	?	?
slant height, l	10	?	15	?
base edge	?	16	12	10
lateral edge	?	?	?	13



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12–2 Pyramids *(continued)*

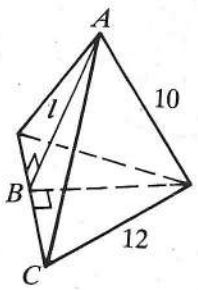
The lateral area of a regular pyramid with n lateral faces is n times the area of one lateral face. $\quad$ **L.A. = nF**

The lateral area of a regular pyramid equals half the perimeter of the base times the slant height. $\quad$ **L.A. = $\frac{1}{2}pl$**

The total area of a pyramid is its lateral area plus the area of its base.
T.A. = L.A. + B

Example 2

Find the lateral area and total area of the regular triangular pyramid.



Solution

To find the lateral area:

Method I

Each lateral face is an isosceles triangle with base 12 and legs 10. The height of each face is the slant height.

$6^2 + l^2 = 10^2$, so $l = 8$.

$F = \frac{1}{2}bl = \frac{1}{2} \cdot 12 \cdot 8 = 48$

L.A. $= nF = 3 \cdot 48 = 144$

Method 2

perimeter of the base = 36

In right $\triangle ABC$,

$6^2 + l^2 = 10^2$, so $l = 8$.

L.A. $= \frac{1}{2}pl$

$= \frac{1}{2} \cdot 36 \cdot 8 = 144$

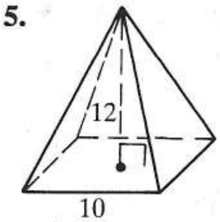
To find the total area:

T.A. = L.A. + B where $B = \frac{1}{2}bh = \frac{1}{2} \cdot 12 \cdot 6\sqrt{3} = 36\sqrt{3}$

$= 144 + 36\sqrt{3}$

Find the lateral area and the total area of each regular pyramid.

5.



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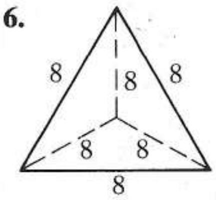

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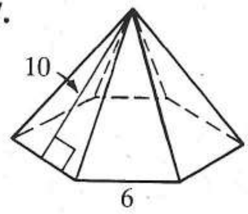

The volume of a pyramid equals one third the area of the base times the height of the pyramid. $\quad$ $V = \frac{1}{3}Bh$

Find the volume of each of the following.

8. The regular square pyramid in Exercise 5

9. A regular square pyramid with base edge 24 and lateral edge 24

10. A regular square pyramid with height 8 and slant height 17

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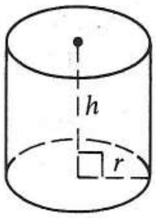
12–3 Cylinders and Cones

Objectives: Identify the parts of cylinders and cones. Find the lateral areas, total areas, and volumes of right cylinders and right cones.

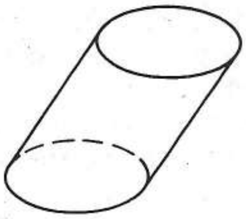
A **cylinder** is like a prism, but its bases are circles instead of polygons. We will only work with **right cylinders.**

The segment joining the centers of the bases is the altitude, with height h. The radius of a base is also the radius of the cylinder.

The formulas for the areas and volumes of cylinders are similar to the corresponding formulas for prisms. Remember that the circumference of a circle is $2\pi r$ and the area of a circle is πr^2.



Right cylinder


Oblique cylinder

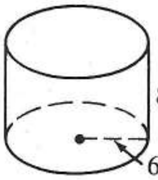
The lateral area of a cylinder equals the circumference of a base times the height of the cylinder. $\text{L.A.} = 2\pi rh$

The total area of a cylinder is the lateral area plus twice the area of a base. $\text{T.A.} = \text{L.A.} + 2B = 2\pi rh + 2\pi r^2$

The volume of a cylinder equals the area of a base times the height of the cylinder. $V = Bh = \pi r^2 h$

Example 1

Find the lateral area, total area, and volume of the cylinder.



Solution

$$\begin{aligned}\text{L.A.} &= 2\pi rh \\ &= 2\pi \cdot 6 \cdot 8 \\ &= 96\pi \text{ (cm}^2\text{)}\end{aligned}$$

$$\begin{aligned}\text{T.A.} &= \text{L.A.} + 2B \\ &= 96\pi + 2(\pi \cdot 6^2) \\ &= 96\pi + 72\pi \\ &= 168\pi \text{ (cm}^2\text{)}\end{aligned}$$

$$\begin{aligned}V &= \pi r^2 h \\ &= \pi \cdot 6^2 \cdot 8 \\ &= 288\pi \text{ (cm}^3\text{)}\end{aligned}$$

Find the (a) lateral area, (b) total area, and (c) volume of each cylinder.

1. $r = 5; h = 8$ **2.** $r = 6; h = 9$ **3.** $r = 5; h = 4$ **4.** $r = 3; h = 7$

5. The lateral area of a cylinder is 96π. If $h = 12$, find r.

6. The volume of a cylinder is 375π. If $h = 15$, find the lateral area.

7. The lateral area of a cylinder is 96π. If $r = 8$, find the volume.

8. The total area of a cylinder is 256π cm^2. If $r = h$, find r.

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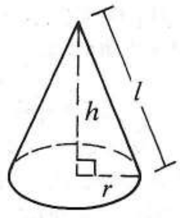
12–3 Cylinders and Cones *(continued)*

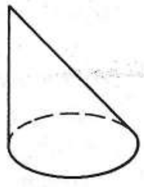
A **cone** is like a pyramid, but its base is a circle. We will only work with **right cones.**

The segment joining the vertex of a cone to the center of its base is the altitude, which has height h. The slant height l is the hypotenuse of a right triangle formed by the altitude and a radius.

The formulas for the areas and volumes of cones are similar to the corresponding formulas for pyramids.



Right cone



Oblique cone

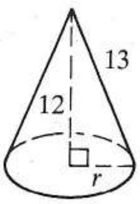
The lateral area of a cone equals half the circumference of the base times the slant height. $\text{L.A.} = \frac{1}{2} \cdot 2\pi r \cdot l = \pi r l$

The total area of a cone equals the lateral area plus the area of the base. $\text{T.A.} = \text{L.A.} + B = \pi r l + \pi r^2$

The volume of a cone equals one third the area of the base times the height of the cone. $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$

Example 2

Find the lateral area, total area, and volume of the cone.



Solution

Use the Pythagorean Theorem to find r.

$h^2 + r^2 = l^2$

$12^2 + r^2 = 13^2$

$r = 5$

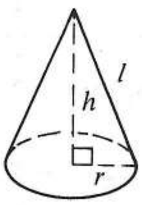
$\text{L.A.} = \pi r l = \pi \cdot 5 \cdot 13 = 65\pi$

$\text{T.A.} = \text{L.A.} + B = 65\pi + \pi(5^2) = 90\pi$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(5^2)(12) = 100\pi$

Complete the table for the cone shown.

	r	h	l	L.A.	T.A.	V
9.	3	?	5	?	?	?
10.	10	24	?	?	?	?
11.	8	?	?	136π	?	?
12.	7	?	?	?	?	392π



13. A cone has volume 432π cm^3 and height 9 cm. Find its slant height.

14. A cone has lateral area 100π and total area 136π. Find its radius.

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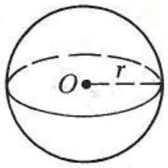
12–4 Spheres

Objective: Find the area and volume of a sphere.

The area of a sphere equals 4π times the square of the radius. $A = 4\pi r^2$

The volume of a sphere equals $\frac{4}{3}\pi$ times the cube of the radius. $V = \frac{4}{3}\pi r^3$



Notice that the formula for volume includes cubes of numbers. Some common cubes are listed. These will be useful when solving some exercises, such as Example 1(b) below.

$1^3 = 1$ $2^3 = 8$ $3^3 = 27$ $4^3 = 64$ $5^3 = 125$ $6^3 = 216$ $7^3 = 343$

$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$ $\left(\frac{1}{4}\right)^3 = \frac{1}{64}$ $8^3 = 512$ $9^3 = 729$ $10^3 = 1000$

Example 1

a. Find the area and volume of a sphere with radius 5.

b. The volume of a sphere is 972π. Find its area.

Solution

a. $A = 4\pi r^2$

$= 4\pi \cdot 5^2 = 4\pi \cdot 25 = 100\pi$

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \cdot 5^3 = \frac{4}{3}\pi \cdot 125 = \frac{500}{3}\pi$

b. $V = \frac{4}{3}\pi r^3$ $972\pi = \frac{4}{3}\pi r^3$

$\left(\frac{3}{4}\right)(972) = r^3$

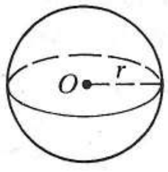
$729 = r^3$

$9 = r$

$A = 4\pi r^2 = 4\pi \cdot 9^2 = 324\pi$

Complete the table for spheres.

	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
radius	3	6	2	$\frac{1}{3}$	$\sqrt{3}$	?	?
area	?	?	?	?	?	576π	?
volume	?	?	?	?	?	?	$\frac{1372}{3}\pi$



8. The area of a sphere is $\frac{\pi}{4}$. Find its diameter.

9. The area of a sphere is 9π. Find its volume.

10. Refer to Exercises 2 and 3 above. If the radius of a sphere is multiplied by 3, then the area is multiplied by __?__ and the volume is multiplied by __?__.

11. Refer to Exercises 1 and 2 above. A hemisphere ("half" a sphere) has radius 6. A sphere has radius 3. Compare their volumes.

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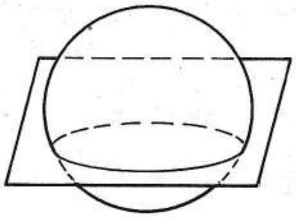
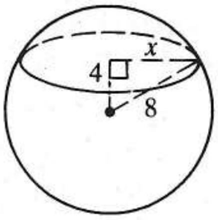
12–4 Spheres *(continued)*

When a sphere and a plane intersect, their intersection is either a tangent point or a circle called the **circle of intersection.**



Example 2

A plane passes 4 cm from the center of a sphere with radius 8 cm. Find the area of the circle of intersection.



Solution

Let x = the radius of the circle.

$4^2 + x^2 = 8^2$

$x^2 = 64 - 16 = 48$

$A = \pi x^2 = 48\pi \quad (\text{cm}^2)$

A plane passes h cm from the center of a sphere with radius r cm. Find the area of the circle of intersection.

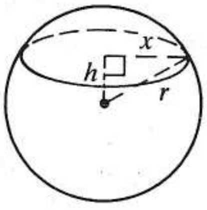
12. $r = 5;\ h = 3$

13. $r = 13;\ h = 12$

14. $r = 8;\ h = 2$

15. $r = 10;\ h = 6$

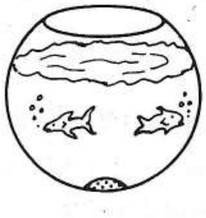
16. $r = 12;\ h = 10$



17. When a plane passes 5 cm from the center of a sphere, the radius of the circle of intersection is 12 cm. Find the volume of the sphere.

18. A scoop of ice cream with radius 4 cm is placed on an ice-cream cone with radius 3 cm and height 15 cm. Is the cone big enough to hold the ice cream if it melts?

19. A spherical fishbowl has diameter 24 cm. To fill the fishbowl three-fourths full, about how many liters of water will you need? Give your answer to the nearest 0.1 L. Use $\pi \approx 3.14$. ($1000 \text{ cm}^3 = 1 \text{ L}$)



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12–5 Areas and Volumes of Similar Solids

Objective: Apply the properties of similar solids.

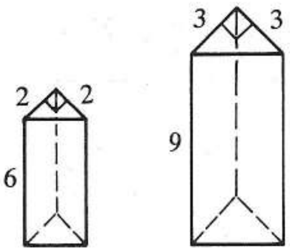
Similar polygons are polygons that have the same shape but not necessarily the same size. Similar solids are solids that have the same shape but not necessarily the same size. Just as all circles are similar, all spheres are similar.

Two solids are similar if and only if their bases are similar and their corresponding lengths are proportional.

Example 1 Are the two solids similar?

a. Two cylinders have radii 4 cm and 3 cm. The heights are 12 cm and 8 cm, respectively.

b.



Solution

a. The bases are similar because all circles are similar. The corresponding lengths are not in proportion since $\frac{4}{3} \neq \frac{12}{8}$. The cylinders are not similar.

b. The bases are similar triangles because all 45°-45°-90° triangles are similar. The corresponding lengths are in proportion because $\frac{2}{3} = \frac{6}{9}$.
The right triangular prisms are similar.

Are the given solids similar?

1. Two regular square pyramids have heights 10 and 12. The bases are squares with sides 4 and 4.8, respectively.
2. One rectangular solid has length 7, width 5, and height 3. Another rectangular solid has length 14, width 10, and height 9.
3. Two right triangular prisms have heights 4 and 6. Their bases are triangles with sides of 3, 4, 5, and 6, 8, 10, respectively.

If the scale factor of two similar solids is $a:b$, then
(1) the ratio of corresponding perimeters is $a:b$.
(2) the ratio of the base areas, of the lateral areas, and of the total areas is $a^2:b^2$.
(3) the ratio of the volumes is $a^3:b^3$.

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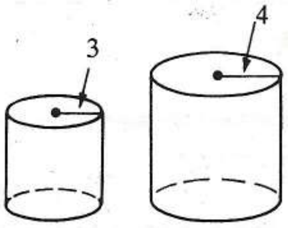
12–5 Areas and Volumes of Similar Solids *(continued)*

Example 2

a. The two cylinders are similar. Find the scale factor, the ratio of the heights, the ratio of the total areas, and the ratio of the volumes.



b. The ratio of the volumes of two similar pyramids is 27:8. Find the scale factor, the ratio of the base perimeters, and the ratio of the lateral areas.

Solution

a. Their scale factor is $\frac{3}{4}$.
The ratio of their heights is the same as their scale factor, $\frac{3}{4}$.
The ratio of their total areas is $\frac{3^2}{4^2} = \frac{9}{16}$.
The ratio of their volumes is $\frac{3^3}{4^3} = \frac{27}{64}$

b. $\frac{a^3}{b^3} = \frac{27}{8} = \frac{3^3}{2^3}$, so the scale factor is $\frac{3}{2}$.
The ratio of the base perimeters equals the scale factor, $\frac{3}{2}$.
The ratio of the lateral areas is $\frac{3^2}{2^2} = \frac{9}{4}$.

Complete the table, which refers to two similar prisms.

	4.	**5.**	**6.**	**7.**	**8.**
scale factor	2:5	?	?	?	?
ratio of base perimeters	?	?	?	?	?
ratio of heights	?	1:3	?	?	?
ratio of lateral areas	?	?	4:49	?	?
ratio of total areas	?	?	?	?	?
ratio of volumes	?	?	?	125:216	27:1000

9. Two similar cones have volumes 27π and 64π. Find the ratio of:
a. the radii **b.** the slant heights **c.** the lateral areas

10. Two spheres have radii 5 cm and 7 cm. Find the ratio of:
a. the areas **b.** the volumes

11. Two spheres have diameters 12 and 28. Find the ratio of:
a. the areas **b.** the volumes

12. Two foam plastic balls have scale factor 2:3.
a. If the smaller ball has radius 6 cm, what is the radius of the larger ball?
b. If the area of the larger ball is 36π cm^2, what is the area of the smaller ball?
c. If the larger ball weighs 12 g, about how much does the smaller ball weigh? (*Hint:* Weight is related to volume.)